Pathways to Astronomy, 2025 Release
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Instructor's Manual

Solutions to End of the Unit Quantitative Problems
Part I: The Cosmic Landscape

Unit 1

10. If you use a volleyball as a model of Earth, how big would 1 kilometer be on it? Assume a volleyball has a circumference of 68 cm.

10. (0.0017 cm) Use the value of 40,000 km for the circumference of the Earth given in the chapter, and create a scale factor for the model:

Circumference (Volleyball) / *Circumference* (Earth) = 68 cm / 40,000 km = 0.0017 cm/km

This means 1 km on Earth would be represented by just 0.0017 cm on the volleyball, or just 0.017 mm. This is about the diameter of extremely fine hair.

11. What would be the circumference and diameter (circumference = π × diameter) of a ball that would represent the Moon if Earth were a volleyball? What kind of ball or object matches this size? (Note: For this and the following questions you will probably need to look up the sizes and distances of astronomical objects in the set of tables in the Appendix.)

11. There are several ways to solve this problem. One easy way is to compare the Earth's radius or diameter to the Moon's radius or diameter. The chapter notes the Moon's diameter is about ¼ the Earth's. Since $C = \pi \times D$, then the circumference is also ¼ the Earth's, so in the model where Earth is a volleyball, the Moon must have a circumference 0.25 × 68 cm = 17 cm. The Moon's diameter is then C/π = 17 cm / π = 5.4 cm. This is smaller than a tennis ball.

A slightly more accurate calculation using the values of the Earth and Moon's size from the Appendix (the Moon's radius is 1738 km; the Earth is 6378) gives the Moon's radius is 0.273 times the Earth's; therefore the diameter and circumference are also 0.273 times the Earth's; this gives a circumference for the model of the Moon of 18.6 cm and a diameter of 5.9 cm.
Either way, the model of the Moon would be a little smaller than a tennis ball or about the size of a small orange.

12. If Earth were a volleyball, what would be the diameter of the Sun? What object matches this size?

12. (About 24 m, the size of a house). The volleyball is about 22 cm across (use *Diameter* = *Circumference*/π = 68 cm /π = 21.6 cm). The diameter of the Sun is about 109 times that of Earth (using the ratio of its radius to the radius of Earth $R_{\mathrm{Sun}}/R_{\mathrm{Earth}}$ = 696,000 km/6378 km = 109) so the model of the Sun would be 22 cm × 109 = 2398 cm = 23.98 m, or about 24 meters. This would be roughly the size of a house. (Also okay to use $R_{\mathrm{Sun}}/R_{\mathrm{Earth}} \sim 100$ as given in chapter, for 2200 cm).

13. How many astronomical units away is the Moon from Earth?

13. (0.0026 AU) An AU is approximately 1.5×10^8 km, while the Moon is d = 384,400 km away from Earth (Appendix Value). Performing a standard unit conversion gives a distance of

$$d = (384{,}400\ \text{~~km~~}) \times (1\ \text{AU}/1.5 \times 10^8\ \text{~~km~~}) = 0.0026\ \text{AU}.$$

This is about 1/400th of an AU, or put another way, the Moon is about 400 times closer to Earth than the Sun.

14. During the 1960s and 1970s, the Apollo spacecraft took humans to the Moon in three days. Traveling to Mars requires a trip of about 2 astronomical units in total. How long would this trip take, traveling at the same speed as to the Moon?

14. (6.4 years). The Moon is 384,400 km away from Earth (Appendix value). A three-day (72-hour) flight gives an average speed of 384,400 km/3 day ~ 128,000 km/day = 1.28×10^5 km/day. A trip to Mars on a trajectory 2 AU long would cover a distance $2 \times (1.5 \times 10^8\ \text{km}) = 3.0 \times 10^8$ km. At the speed of an *Apollo* spacecraft, a trip to Mars would therefore take:

$$\textit{time} = \textit{distance/speed} = 3.0 \times 10^8\ \text{km} / (1.28 \times 10^5\ \text{km/day}) = 2.34 \times 10^3\ \text{days}$$

This is about 6.4 years to reach Mars. (Note: if using the rounded value of 400,000 km distance to the Moon in the chapter, the speed is about 130,000 km/day, which yields about 6 yr.)

15. Using the same assumptions as in the previous question, how long would it take to travel to Pluto, about 40 astronomical units away?

15. (128 years). Based on the speed calculated in the last problem, a 2 AU flight took 6.4 years. Pluto is 20 times farther away, so the flight would take 20 times longer: 20×6.4 yr = 128 yr.

Unit 2

10. If the Milky Way were the size of a nickel (about 2 centimeters in diameter): (a) How big would the Local Group be? (b) How big would the Local Supercluster be? (c) How big would the visible universe be? (The data in Table 2.1 may help you here.)

10. (a) 60 cm; (b) 20 m; (c) 5.5 km.
Create a model scale relating the size of a nickel to the size of the Milky Way:

$$\textit{Diameter}(\text{nickel}) / \textit{Diameter}\ (\text{Milky Way}) = 2\ \text{cm} / 100{,}000\ \text{ly} = 2 \times 10^{-5}\ \text{cm/ly}.$$

Using the approximate radius sizes in table 2.1, and multiplying by 2 to get diameters:

a) The Local Group is about 3 million ly in diameter, which in the model is:

3×10^6 ly $\times 2 \times 10^{-5}$ cm/ly = 60 cm. 60 cm is about the size of a beach ball.

b) The Local Supercluster is 50 million ly × 2 = 100 million ly in diameter, scaled to the model this is 100×10^6 ly $\times 2 \times 10^{-5}$ cm/ly = 2000 cm = 20 m. 20 m is about the size of a house.

c) The visible universe would be about 13.7 billion ly × 2 = 27.4 billion ly in diameter, which in the model is: 27.4×10^9 ly × 2×10^{-5} cm/ly = 5.5×10^5 cm = 5500 m = 5.5 km
5.5 km is about the size of a town; vertically, it is more than 18,000 feet, or the height of many mountains in the Andes or Himalayas.

11. If we detected radio signals (which travel at the speed of light) of intelligent origin from the Andromeda Galaxy (M31) and immediately responded with a radio message of our own, how long would we have to wait for a reply?

11. (5 million years) Radio waves, like all forms of electromagnetic radiation, travel at the speed of light. We'd have to wait for the message to get to Andromeda, and for the response to make it back to us—twice the distance (2.5 million light years), divided by the speed of light. Because light travels 1 light year in a year, so:

time = *distance / speed* = (2 × 2.5 million ly) / (1 ly/yr) = 5 million years.

There is an introduction to solving distance-velocity-time problems at the beginning of the appendix.

12. How long does it take light to travel from the Sun to Earth?

12. (About 8.3 minutes). An AU is approximately 1.5×10^8 km, and the speed of light is about 3.0×10^5 km/sec. Therefore, the time it takes light to travel from the Sun to Earth is

time = *distance / speed* = 1.5×10^8 km / 3.0×10^5 km/sec = 500 sec,

which is about 8.3 minutes.

13. If the Milky Way is moving away from the Virgo Cluster at 1000 kilometers per second, how long does it take for the distance between them to increase by 1 light-year?

13. (About 300 years). This is another distance-velocity-time problem:

distance = 1 ly = 9.46×10^{15} m = 9.46×10^{12} km; *Velocity* = 1000 km/sec.

Therefore, we find the time for the distance to increase by 1 light-year is:

time = *distance/Velocity* = 9.46×10^{12} km / (1000 km/sec) = 9.46×10^9 sec.

Converting this to years:

9.46×10^9 sec × 1hr/(3600 sec) × 1 day / (24 hr) × 1 yr/(365.25 day) = 300 yr.

14. The Milky Way is moving toward the larger galaxy M31 at about 100 kilometers per second. M31 is about 2.5 million light-years away. How long will it take before the Milky Way collides with M31 if it continues at this speed?

14. (7.5×10^9 yr). This problem is like the last one, but with d = 2.5×10^6 ly and V = 100 km/sec:

$\underline{t}$ = (2.5×10^6 ly × 9.46 × 10^{12} km/ly) / (100 km/sec) = 2.37 × 10^{17} sec.

Converting this to years gives:

1.89 × 10^{17} sec × 1hr/(3600 sec) × 1 day / (24 hr) × 1 yr/(365.25 day) = 7.5 × 10^9 yr.

So the Milky Way and Andromeda will "collide" in 7.5 billion years they continued constantly at that speed. But since the force of gravity increases in strength significantly as distances between objects become smaller, the Milky Way will accelerate towards Andromeda faster and faster, and collide in less than 5 billion years.

15. Some distant galaxies seen by the Hubble Space Telescope are 12 billion light-years away. Using the scale in Figure 2.4 (where the Milky Way is about the size of a large city), how far would that be in the model

15. (0.08 AU). At a scale of Figure 2.4, 1 light year = 1 meter, so 12 billion light years is equivalent to 12 billion meters = 12 × 10^9 m = 12 × 10^6 km = 12 million km. So, on the scale where the Milky Way is about the size of a large metropolitan area, the most distant galaxies we can see are about 30 times the Moon's distance [12 × 10^6 km/384,000 km = 31], or about a tenth of an AU distant:

12 × 10^9 m/(1.5 × 10^{11} m/AU) = 0.08 AU.

Unit 3

9. The radius of the Sun is 7 × 10^5 kilometers, and that of Earth is about 6.4 ×
10^3 kilometers. Show that the Sun's radius is approximately 100 times Earth's radius.

9. The ratio of the Sun's radius to the Earth's radius is 7 10^5 km / (6.4 10^3 km) = 7/6.4 10^{5-3} ≈ 1 10^2 or about 100 times larger.

10. Hypergiant stars can have radii as large as 1500 times that of the Sun. Estimate the radius of a hypergiant star in kilometers. How many AU is this equivalent to?

10. (1.05 × 10^9 km = 7.0 AU) From Table 3.3 a solar radius is 6.97 × 10^8 m, or measured in kilometers 6.97 × 10^5 km. The radius of a hypergiant star would be about:

1500 × 6.97 × 10^5 km = 1.05 × 10^9 km.

An astronomical unit is 1.50 × 10^8 km making the radius of the hypergiant star:

(1.05 × 10^9 km) × (1 AU/1.50 × 10^8 km) = 6.97 AU ≈ 7.0 AU,

which is larger than Jupiter's orbit!

11. Using scientific notation, show that it takes sunlight about 8 ½ minutes to reach Earth from the Sun.

11. The light travel time to Earth from the Sun can be calculated from the distance-speed-time formula $t = d/V$, where $V = c = 3.0 \times 10^5$ km sec^{-1}, and $d = 1$ AU $= 1.496 \times 10^8$ km:

$$t = 1.496 \times 10^8 \text{ km}/ (3.0 \times 10^5 \text{ km sec}^{-1}) = (1.5/3.0) \times 10^{8-5} \text{ sec}$$
$$= 0.50 \times 10^3 \text{ sec} = 5.0 \times 10^2 \text{ sec}$$

Next we convert this to minutes, also using scientific notation:

$$t = 5.0 \times 10^1 \text{ sec} \times 1 \text{ min} / (6 \times 10^1 \text{ sec}) = 50/6 \text{ min} = 8.3 \text{ min (about 8 1/2 min).}$$

12. How many 1-kiloton bombs would need to be exploded to produce 3.86×10^{26} joules, the amount of energy emitted by the Sun in 1 second?

12. (9.23×10^{13} 1-kiloton bombs per second).

The Sun's luminosity is 3.86×10^{26} W $= 3.86 \times 10^{26}$ J/sec. Divide the by the energy output of a 9.23 kiloton bomb (from Table 3.3) to get the equivalent number of these explosions each second:

$$(3.86 \times 10^{26} \text{ J/sec}) / (4.18 \times 10^{12} \text{ J/kt}) = 9.23 \times 10^{13} \text{ kt per second.}$$

13. If I want to work out the distance to the Andromeda Galaxy, do I need to be concerned about whether the distance is from the center of the Milky Way or from the Sun's location? Why or why not?

13. The Sun is 25,000 light years ($2.5 \times 10^4 = 0.025 \times 10^6 = 0.025$ million light years) from the center of the Milky Way galaxy. The Andromeda galaxy is 2.5 million light years away–or about 100 times farther. In other words, the distance to the center of the Milky Way is only 1% of the distance to the Andromeda galaxy.

Even if the Andromeda galaxy was located in the exact opposite direction from the center of the Milky Way (and it isn't), its distance would be 2.5×10^6 ly - 0.025×10^6 ly $= 2.475 \times 10^6$ ly and rounded up to just two digits of precision, this is still $\approx 2.5 \times 10^6$ ly. So measuring from the center of the Milky Way or from our location would not affect the distance estimate to within the two-digit precision of the distance of the Andromeda galaxy.

14. Suppose two galaxies move away from each other at a speed of 6000 km/sec and are
300 million (3×10^8) light-years apart. If their speed has remained constant, how long has
it taken them to move that far apart? Express your answer in years.

14. (1.50×10^{10} years). The time for the galaxies to move this distance is given by the distance-speed-time equation:

$$t = d/V = (3 \times 10^8 \text{ ly})/(6000 \text{ km/sec}) = (3 \times 10^8 \text{ ly} \times 9.46 \times 10^{12} \text{ km/ly})/(6000 \text{ km/sec})$$
$$= 4.73 \times 10^{17} \text{ sec} = 4.73 \times 10^{17} \text{ sec} \times 1\text{y}/(3.16 \times 10^7 \text{ sec}) = 1.50 \times 10^{10} \text{ years.}$$

This time of 15 billion years is longer than the present age of our universe, but it assumes the galaxies have moved at a constant speed.

15. The equation $E = m \times c^2$ tells us how much energy, E, is stored in a mass of m. Use this equation to calculate the energy stored in a 3.10-gram penny.

15. (2.79×10^{14} J). To get the units to work our properly in the MKS system, the 3.10 gram mass of the penny must be expressed in kilograms:

$$E = m\,c^2 = (3.10 \times 10^{-3}\text{ kg}) \times (3.00 \times 10^8\text{ m/sec})^2 = 2.79 \times 10^{14}\text{ kg m}^2/\text{sec}^2 = 2.79 \times 10^{14}\text{ J}$$

Note: this means the conversion of a penny to pure energy is equivalent to a 66.7 kiloton bomb.

16. Using scientific notation, evaluate $(1.4 \times 10^9)^3 / (9.3 \times 10^8)^2$.

16. Using scientific notation:

$$(1.4 \times 10^9)^3 / (9.3 \times 10^8)^2 = (1.4^3 \times 10^{9 \times 3}) / (9.3^2 \times 10^{8 \times 2})$$
$$= 2.74 \times 10^{27} / 86.5 \times 10^{16} = 2.74/86.5 \times 10^{27-16} = 0.0317 \times 10^{11} = 3.2 \times 10^9$$

Note: as you do the calculation, keep at least one extra digit in all of the intermediate values, and only round off at the end. Because the original numbers (1.4×10^9 and 9.3×10^8) are shown with two digits of precision, we round the final answer up and record it also with just two digits.

17. Napoleon Bonaparte is often said to have been short, but contemporaries described him as being of average or slightly above-average height for his time. This mistake was apparently made because the height reported in Paris feet was misinterpreted. Bonaparte was reported to be about 5 feet 2 inches tall at his autopsy. If these were in Paris units, 6.63% longer than English inches and feet, how tall was he actually in (a) English units, and (b) metric units.

17. (a) 5 ft 6 in; (b) 1.68 m) Denote measurements made in Paris with (P). So Bonaparte's height of 5 ft 2 in (P) would be $5 \times 12 + 2 = 62$ in (P). Since the Parisian units are 6.63% larger, 1 Parisian inch = 1.0663 English inches. To convert this:

$$62\text{ in(P)} \times 1.0663\text{ in} / 1\text{ in(P)} = 66\text{ in} = (5 \times 12 + 6)\text{ in}$$

Thus in the English system, Bonaparte was 5 ft 6 in tall.

(b) To convert to the Metric system from the English measurement of 66 in, use the fact that 1 in = 2.54 cm and calculate:

$$66\text{ in} \times (2.54\text{ cm} / 1\text{ in}) = 168\text{ cm} = 1.68\text{ m}.$$

Unit 4

11. If a proton were the size of a large apple (about 10 cm in diameter), how large would a hydrogen atom be?

11. A hydrogen atom is 10^{-10} m in diameter while a proton is about 10^{-15} m, so the atom is about 10^5 times larger than the size of the proton. If the proton were blown up in size of a large apple (10 cm), then the size of the atom would become $(10 \times 10^{-2}$ m$) \times 10^5 = 10^4$ m = 10 km, about the size of a city.

12. Imagine that Earth and the Moon were both positively charged by an amount proportional to their masses, so that their electrical repulsion canceled out their gravitational attraction. Using the relative strengths of gravity and the electromagnetic force, how much mass in Earth would have to be positively charged?

12. (6×10^{-12} kg). The force of gravity is 10^{-36} times weaker than the force of electromagnetism (see Table 4.1 in chapter 4). So only 10^{-36} as much matter is needed for the electromagnetic force as for the gravitational force, for them to be equal. The mass of the Earth is 5.974×10^{24} kg, so only ~ 6×10^{24} kg $\times 10^{-36} = 6 \times 10^{-12}$ kg of mass needs to be charged! That is just 6 *billionth* of a gram! And only about 1/80th of this value would be needed for the Moon.

Note that this is about the mass of a single cell in your body, so if all the electrons could somehow disappear from a single cell on Earth, and similarly from an even smaller single cell on the Moon, the electrical repulsion from those two cells would rival the whole Earth's attractive gravitational pull.

13. Explain at a fundamental level what is happening when you take clothes out of a dryer and find them clinging to each other. What do you think is happening when you hear a crackling sound as you pull a sock and shirt apart?

13. While the atoms in your clothes typically are electrically neutral, meaning that they have an equal number of positive and negative charges, friction between tumbling clothes can allow certain materials to capture electrons from other materials. When this happens, one material becomes more negative and the other more positive. This is what we mean when we say something has a static charge. Since opposite charges attract, a sock that has gained a negative charge in this process is attracted to and clings to a sweater that has developed a positive charge. The crackling sound is that of charges jumping from one fabric to the other in tiny sparks as you pull them apart (you may be able to see these in the dark).

Under humid conditions, surfaces become covered with water molecules that are conductive and allow electrical charges to flow away relatively easily, so large charges rarely build up. That's why the static charge builds up on clothes in the dryer and not in the wash. Dryer sheets often also contain chemicals that are conductive and coat the clothes, thereby reducing the ability of tumbled clothes to build up a large electric charge.

14. It is estimated that a neutrino has to pass within a distance of about 10^{-24} meters of a quark for a high likelihood of interaction. If an atom has a radius of 10^{-10} meters and
12 nuclear particles in its nucleus, what is the probability that a neutrino

passing through the atom will undergo an interaction? (Hint: Think of the atom as a target and the area around each quark as a bull's-eye.)

14. The probability that a neutrino passing through an atom will interact is about one out of 10^{27}! Explanation: Assume that the atom is a sphere and the 12 nuclear particles are each spheres with circular cross-sections like circular targets. Using the given radii, calculate the cross-sectional areas for the atom as a whole (A_1) and for the nuclear particles (A_2). Each area = $\pi \times r^2$, so the ratio of areas is:

$$12 \times A_2 / A_1 = 12 \times \pi \times (10^{-24}\text{m})^2 / (\pi \times (10^{-10}\text{m})^2) = 12 \times (10^{-24})^2 / (10^{-10})^2$$

Note that the pi's (and meters) cancel out, so the fraction of the area that the neutrino might hit is $= 12 \times 10^{-48} / 10^{-20} = 1.2 \times 10^{-27}$.

Thus, a neutrino might be expected to pass through something in the order of 10^{27} atoms before there is a strong likelihood of it interacting! This means that a neutrino has a high probability of passing through a solid layer of material that is even a light year (about 10^{16} meters) thick, since that's "only" 10^{26} atoms thick.

Unit 5

10. What are your latitude and longitude? What are the latitude and longitude toward your nadir at the point on Earth exactly opposite you? What geographic features are located there?

10. (Adjust answer to match your location). Method: If your latitude and longitude are 40 N and 100 W (at the southern edge of the state of Nebraska, in the United States), then the points on the opposite side of the Earth would have to be somewhere on 40 S (reflected through the center of the Earth). The longitude is slightly trickier, because it is 180 away on the other side of the Prime Meridian, so it is (180−100) E = 80 E. This location (40S,80E) is in the middle of the Indian Ocean. [Look at a globe or a 360 protractor to understand why it's (180−value) in the opposite hemisphere.]

11. If your observatory is located at a latitude of 45°N and a longitude of 90°W, what range of declinations are visible from your observatory? If you wanted to see the most stars, at what latitude should you build your observatory?

11. (From–45° up to +90° declinations). At a latitude of 45° N, stars at declination +45° pass through your zenith, so the horizon blocks off stars more than 90° north or south of that. You can therefore observe declinations from–45° and up to +90°. To see the most stars you would want to build your observatory on the equator, and then you would be able to observe stars across all the declinations.

12. The constellations Ursa Major (the Great Bear), in the Northern Hemisphere, and Crux (the Southern Cross), in the Southern Hemisphere, can be used to locate the north and south celestial poles, respectively. Using the star charts at the end of the book, can you show how this is done?

12. In the Northern hemisphere, the end of the “bowl” of the Big Dipper “points” at the north celestial pole and the north star, Polaris (itself at the handle end of Ursa Minor or the Little Dipper). Continuing the line made by Merak and Dubhe (going “up” out of the bowl) leads the eye to Polaris about 30° away.

In the Southern hemisphere, the two bright stars on the longer arm of the “Southern Cross” (the constellation Crux) also point toward the south pole about 30° away, however there are no naked-eye stars close to the pole, so finding the position of the celestial pole is trickier. The distance from Acrux (the brightest star in Crux) to the pole is about four times the distance between Gamma Crucis and Acrux, or about 2/3rds the distance from Acrux to Beta Hydri, the first relatively bright star encountered along the line on the other side of the pole. The south celestial pole might also be pictured as the missing point needed to make a trapezoid out of the three brightest stars in the constellation Octans.

13. Buenos Aires and Sydney are both at latitude 34°S, but Buenos Aires is at longitude 58°W and Sydney at 151°E. Using the star chart at the back of the book, find a bright star that passes through the zenith of Buenos Aires and Sydney. How many hours after passing through the zenith in Buenos Aires will it pass through the zenith in Sydney?

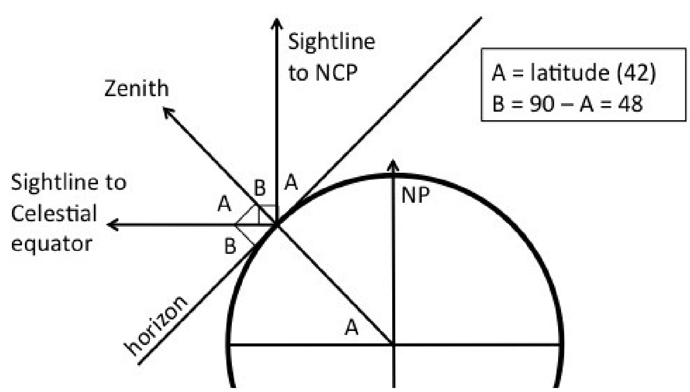
13. At 34°S, a star with a declination of–34° will pass through the zenith. Some examples you can find in the charts in the back of the book are □ Scorpii (Wei), □ Sagittarii (Kaus Australis), or □ Columbae (Phaet). As the Earth turns, the star will pass over Buenos Aires at 58° W, the rest of South America, the Pacific Ocean, the 180° line of longitude separating the western and eastern hemispheres, and then on to Sydney, Australia at 151° E. The separation between these cities and longitude 180° is:

Buenos Aires west to 180° longitude line = 180°□58° = 122°
180° longitude line west to Sydney = 180°□151° = 29°

The total change in longitude is therefore 122° + 29° = 151°. Since the Earth turns 15° per hour, it will take 151°/(15°/hr) or just a bit over 10 hours between the star being overhead in Buenos Aires and being overhead in Sydney.

14. How far is the celestial equator from your zenith if your latitude is 31°N? 15°S?

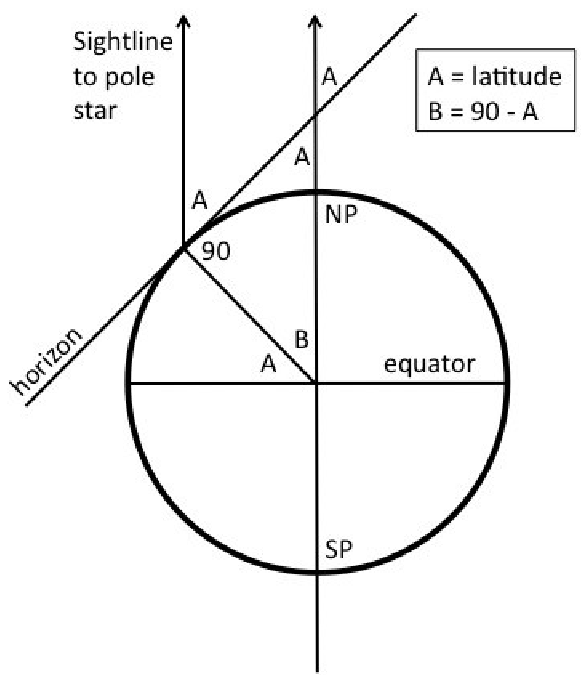
14. (31□ south of your zenith; 15□ north of your zenith). At 31□ N, a star at declination +31□ will pass through your zenith, so the celestial equator is 31□ south of your zenith. (This is illustrated



as angle A in the figure at right for latitude 42° N.) Likewise, at 15° S, the celestial equator is 15° north of your zenith.

15. Using a compass and protractor, draw a diagram of Earth. Mark the poles, the equator, and your own latitude. With a ruler, find the line that is "tangent" to your location—that is, the line should touch Earth's surface at your location while matching the surface at this point. This tangent line represents your horizon. Draw a line extending from pole to pole to indicate Earth's axis, and extend both this line and your horizon line until they cross. Show that your latitude is the same as the angle of the celestial pole with respect to your horizon.

15. The drawing should look like this: (Northern hemisphere location; reverse NP and SP for Southern hemisphere.).



Note that this geometric demonstration makes use of several principles: a line (in this case the horizon) crossing parallel lines makes the same angle A to each, the sum of the angles in a triangle is 180°, and the tangent line to a point on a circle is at a 90° angle to the radius line to that point.

Some students are concerned that the different sightlines to the pole star are not exactly parallel for a star that is at a finite distance. While true for any physical star, the difference is extremely small, because stars are so far away. For example, even for the star Polaris, which is about 50 times larger than the Sun, but more than 400 light years away, the difference in sightlines from being parallel is thousands of times smaller than the seemingly pointlike appearance of Polaris in the night sky.

16. If you are observing the sky from a latitude of 39°N, what range of declinations do circumpolar stars fall within?

16. (All stars north of declination +51°). From a latitude of 39° N, the north celestial pole is 39° above the northern horizon. Therefore, stars within 39° of the pole (which is at declination +90°) never set. This makes all stars north of declination (+90°–39°) = +51° circumpolar.

Unit 6

12. Suppose that instead of Earth's axis being tilted at 23.5°, it was not tilted at all. Where would the tropics and arctic regions be? How would seasons be different?

12. If the Earth's axis had no tilt instead of 23.5°, the tropic and arctic regions would shrink to their smallest possible size. In the tropics the Sun passes directly overhead at least once during the year. With 0° tilt, the Sun would always be directly over just the equator. In arctic regions the Sun remains below the horizon for more than 24 hours during some parts of the year, but with 0° tilt, observers at the north and south pole would find that the Sun always remained right on the horizon. Without a tilt there would not be the seasonal changes we experience as the angle of sunlight to the ground changes.

13. Suppose Earth's axis were tilted by 50° instead of 23.5°. Where would the tropics and arctic regions be? How would seasons be different?

13. If the Earth's axis had a 50° tilt, geographically the tropics would extend to 50° N and S latitudes. The arctic regions would reach all the way to 40° N and S latitudes, giving a 10° overlap between the two. If you lived in the overlap region, there would be times of the year when the Sun would pass straight overhead as in the tropics, but half a year later, there would be days when the Sun never rose. This would lead to much greater variations in seasonal temperatures than we experience on Earth.

14. Suppose Earth's axis were tilted by 90° instead of 23.5°. Where would the tropics and arctic regions be? How would the seasons be different?

14. If the Earth's axis was tilted by 90°, the Sun would appear to lie directly over the North Pole once a year, then move down toward the equator and eventually lie directly over the South Pole 6 months later before moving back to the North Pole over the course of a year. In this situation (which is similar to what the planet Uranus experiences), there would be no tropics or arctic regions, or you might say they were both everywhere. The polar regions would sometimes be warmer than the equator, and the seasons would be extreme, varying between "North Pole hot and in constant sunlight, equator cool with the Sun on the northern horizon all day long, South Pole dark for months and extremely cold," "equator warm with 12 hour days and nights, and both poles cool" and "South Pole hot and in constant sunlight, equator cool, North Pole cold."

15. Describe the daily motion of the Sun that you would see on the solstices and the equinoxes if you were observing from the Arctic Circle, at a latitude of 66.5°N.

15. On the Arctic Circle (latitude 66.5° N), it remains true on the equinoxes that night and day would both be about 12 hours long, and the Sun rises due east and sets due west. The Sun would not get very high in the sky, reaching a maximum altitude of 23.5° above the southern horizon at noon. On the December solstice, the Sun would barely skim the southern horizon around noon, with an hour or two of twilight before and after. It would be fully dark from about mid-afternoon to mid-morning the following day. On the June Solstice, the Sun would be up 24 hours, reaching a height of 47 degrees in the South at noon, and just flirting with the northern horizon at midnight.

16. If you wished to observe a star with a right ascension of 12 h, what would be the best time of year to observe it? What would be the best time to observe a star with a right ascension of 6 h? (You may find Unit 5.5 helpful in answering this problem.)

16. (For a star with R.A. = 12 h, around March 20; for a star with R.A. = 6 h, around Dec. 21) For the best opportunity to observe a star with a right ascension (R.A.) of 12 h, you would wish the Sun to be as far away from it as possible, at the opposite side of the sky, so that when the star is above the horizon and visible, the Sun would be below. This would best occur with the Sun at R.A. = 0 h (see also Unit 5.5). This would occur near March 20 (although much of the year would also be acceptable, as well). For the star at R.A. = 6 h, it's best if the Sun was again 12 hours away, at R.A. = 18 h. This happens ¾ of the year after March 21–around the winter solstice (Dec. 21).

Unit 7

10. Philadelphia is 5° east of Pittsburgh. In the early 1800s, both cities synchronized their clocks with apparent noon—that is, all the local clocks read noon when the Sun passed through the meridian. By how many minutes did clocks in the two cities differ from one other?

10. To find how the 5° longitude difference affects time measurement, we need to know how much time it takes the Earth to turn each degree. Since the Earth takes 24 hr to rotate 360°, the time per degree is: 24 hr / 360° = 1 hr / 15° = 60 min / 15° = 4 min / 1°.

Therefore, back in the days when clocks were set using a sundial, Philadelphia's clocks were 5° × 4 min/1° = 20 minutes ahead the clocks in Pittsburgh.

11. If Russia had all clocks set to Moscow time (time zone +3 in Figure 7.4) instead of using many time zones, what time would the Sun rise at the easternmost tip of Siberia on the equinoxes? Examining Figure 7.4, what places in the world have the Sun crossing the meridian earliest according to local time? What places have it latest?

11. (Around 8 pm the previous evening). Siberia is in the +12 h time zone; Moscow is in the +3 h. They appear to be 9 h apart, but are actually on opposite ends of their time zones—based on longitude, Siberia is nearly at what would be +13 h (and in fact Moscow is very close to the start of +3 h, although it is not obvious on the map). If the Sun rose at 6 am local time in Siberia, the Moscow time would be 10 h earlier, 8 pm the previous evening.

For the second part of the question, local noon moves from east to west across the map (or right to left; it's noon in England before it's noon in America). Time zones are accurate to local solar time in the middle of the time zone—the middle of "0" runs through Greenwich, England (except for political zig-zags). So if one is at the eastern (or right side) edge of a time zone, the Sun crosses the meridian (local solar noon) at 11:30 am, half an hour before the clock reads noon. In the middle, it crosses at the same time, and at the western edge (left on map) it crosses the meridian (local solar noon) when the clock reads 12:30 pm. Looking carefully at the map, in some places the time zones have been moved from being centered on the lines of longitude, and stretch east and west over other time zones. All of Greenland is-3 h, but the easternmost edge is nearly in the middle of the-1 h time zone based strictly on longitude. That means that the Sun will cross the meridian there 1 hour after it crosses the meridian in Greenwich, but the clock will be set to 3 hours earlier than Greenwich—it will be "noon" at 10:00 am! This is one of the places where the Sun crosses the meridian at the earliest local (clock) time.
11. (cont.) In China, there is one of the latest crossings. All of China, for example, is in the +8 h time zone, but the westernmost edge of China is actually in line with the middle of the +5 h time zone based on longitude alone. Therefore, in Western China, the clock will read 3 h later than local solar time (noon); it will read 3 pm when the Sun crosses the meridian!

12. One city is at a longitude of 21°E. A second city is at 104°W. How many hours apart will a star cross the meridian in the first city and then in the second? If the time zone of each city is based on the closest longitude that is a multiple of 15°, and the star passes overhead in the first city exactly at midnight, at what time (by the clock) will it pass over the second city?

12. For cities at 21° E (Warsaw in Poland would be an example) and 104° W (near Denver, Colorado would be close), the longitude difference = 21° + 104° = 125°. A star that passes overhead in the eastern city first, would move westward as the Earth turns towards the East, and next goes overhead at 0° longitude, and then continue on to the western city. The amount of time between meridian crossings = 125° × 24 hours/360° = 8.267 hours = 8 hours, 16 minutes.

The eastern city is +21°/(15°/hr) = +1.4 hr ahead of the prime meridian, so it would be located in +1 hr time zone. The second city is located-104°/(15°/h) = −6.9 hr behind the prime meridian, so it is in the–7 hr time zone. The clock near Denver at 104°W is set 8 h earlier than the clock in Poland at the 21°E, so when it is midnight at 21°E (and the star is overhead there), it will be take 8 more hours to reach midnight at 104°□□□□□ut the star will not yet be exactly overhead yet, because only 8 hr have passed, and the star won't quite be overhead for another 16 minutes (part 1). The star will be overhead at 104°W 16 minutes past midnight, at 12:16 am. When that happens, it would already be 8:16 am in Poland; the Sun would be up and the day started!

13. Suppose Earth's spin slowed down until there were just 180 days in a year. Compare the length of a sidereal and a solar day in this new situation.

(Do not redefine units of time—just express them in terms of our current hours, minutes, and seconds.)

13. (Sidereal day = 48.4 hr; solar day = 48.7 hr). If Earth's spin slowed down, a day would be longer, but our planet would still orbit the Sun in the same amount of time. At present there are 8766 hr in a year (365.25 day/year × 24 hr/day). This same total number of hours divided into 180 solar days is 8766 hr / 180 day = 48.7 hr/day, or 48 hours 42 minutes. So that is the length of a "solar day" if our planet's spin was slower.
13. (cont.) During each of these new solar days, the Earth would still move 2° in its orbit, so that it completed a full revolution of 360° around the Sun in 180 "new" days. This means that the Earth must be rotating an extra 2° compared to the stars during each solar day. Therefore the Earth takes 48.7 hr to rotate 362° relative to the stars. The length of the sidereal day, when the same star would cross the meridian, is then 360° × (48.7 hr / 362°) = 48.4 hr.

14. It is thought that the angle of Earth's axis varies by a few degrees over tens of thousands of years. If the angle were 20° instead of 23.5°, what would be the angle of the noontime Sun from the horizon at your own latitude on the solstices?

14. The Sun would be 3.5° lower in summer, 3.5° higher in winter. For example, in Lafayette, Colorado (exactly 40° N latitude), on the June solstice the Sun rises to 73.5° above the southern horizon; on the December solstice, it rises to just 26.5°. If the axial tilt changed to 20°, the Sun would get only 70° high as a June maximum, and 30° high for the December maximum.

15. What is your longitude, and what is the longitude of the center of your time zone? (This should be a multiple of 15°.) Calculate the time at which the Sun should, on average, cross your own meridian, assuming that the Sun crosses the meridian at exactly 12:00 at the center of your time zone.

15. Example: Dodge City, KS, is located at almost exactly 100° West longitude in the Central Time Zone. This time zone is defined for 90° West Latitude. Therefore, Dodge City is 10° West of the 90° West longitude. The Earth turns at about 15° an hour, so 10° represents about 10/15 of an hour, or 40 minutes. So if the Sun crossed the meridian at 12 noon at 90° West longitude, it would cross the meridian at about 12:40 p.m. in Dodge City. (Note this is more than the maximum 30 minute time difference that time zones were initially designed to encompass because the edges of the Central Time Zone have been adjusted to match political boundaries).

Unit 8

13. How long does someone have to wait from the time of a third-quarter moon until the next full moon?

13. (About 22 days). If the Moon is currently at third quarter (day 22 of the lunar month). You have to wait for ¾ of a lunar month (approximately 22 days) until the next full moon.
14. Calculate how many minutes later the Moon rises each day on average.

14. (About 49 minutes later). Since the Moon is in the same phase and rises at the same local time once every 29.5 days (on average), the Moon must rise 1/29.5 of a day later each day (on average). This results in a shift of 24 hr / 29.5 = 0.814 hr × 60 min/hr = 49 min later each day.

Note that this will not be exactly correct primarily because of two factors. First, the Moon changes declination each day, so just as the Sun rises earlier in summer than winter, moonrise will be earlier when the Moon is farther north (for a northern observer). Second, the Moon's orbit is quite elliptical, so it speeds up and slows down in its orbit over the course of the month (Unit 12.2).

15. Show that the ratio of the Moon's diameter to the Sun's diameter is very similar to the ratio of the Moon's distance from Earth to the Sun's distance from Earth. (Sizes and distances needed to solve this are in Appendix Tables 1 and 7.)

15. The ratio of the Moon's and Sun's diameters is (2 × 1738 km) / (2 × 6.96 × 10^5 km) = 0.00250. In other words, the Sun is about 400 times larger in diameter than the Moon. The ratio of Moon's and Sun's average distances: (384.4 × 103 km)/(1.496 × 108 km) = 0.00257. So the Moon is also about 400 times closer than the Sun. This coincidence means the Sun and Moon are seen to be about the same angular size in our sky.

16. A total solar eclipse occurs over Africa on August 2, 2027. Based on the 6585-day, 8-hour time between repetitions of similar eclipses, when and where will the next similar total eclipse occur?

16. (August 12, 2045 across the Gulf of Mexico and the western U.S.)
A very similar total solar eclipse to the August 2, 2027 eclipse occurs a *saros* later. Looking at the map of eclipse shadows, we see that this eclipse follows the northern coast of Africa, around 30 degrees latitude. First, we estimate the location of the eclipse. The added 8 hours means that the eclipse shadow falls 1/3 of a day, or 120°, further west than Africa on the Earth. This places it near the Gulf of Mexico and up into the USA. 18 years is 18*365 = 6570 days (not accounting for leap years), so 6585 days is 18 years, 15 days. Between 2027 and (2027 + 18 = 2045) are 5 leap years, (2028, 2032, 2036, 2040, 2044), so we need to subtract a day for each of these. That leaves us with 18 years, 10 days, which we add to August 2, 2027, giving us the date August 12, 2045. More information about this eclipse can be found at the NASA Goddard Spaceflight Center's Eclipse Web Site,
http://eclipse.gsfc.nasa.gov/SEgoogle/SEgoogle2001/SE2045Aug12Tgoogle.html

17. If Figure 8.12 A were expanded to include the Sun at the same scale, how big would the Sun be? How far away would it be? (Sizes and distances needed to solve this are in Appendix Tables 1 and 7.

17. (About 22-23 cm in diameter, and 24-25 m away). First, we need the scale of the diagram. In Figure 8.12B, the Earth's diameter is almost exactly 2 mm, and its radius therefore 1 mm. The actual radius of the Earth in Appendix Table 1 is 6378 km, so the scale of the image is 1 mm = 6378 km. You can also determine the scale by measuring the distance to the Moon, which measures about 63 mm; if we assume it's at its average distance of 384,400 km from Earth given in Appendix Table 7 this gives a scale of 384,400 km/63 mm = 6102 km/mm. (These numbers are very close, so the sizes and distances are about at the same scale in the drawing, but

there is some uncertainty given the size of the Figure, and we don't know for sure what the Moon's distance is in the drawing. An answer based on either number should be acceptable and both are about the same.)

From Appendix Table 1, the radius of the Sun is 6.96×10^8 m = 6.96×10^5 km, and it is at a distance of 1.5×10^{11} m = 1.5×10^8 km from the Earth. With the Earth-determined scale, for the drawing the Sun would be:

6.96×10^5 km × 1 mm/6378 km = 109 mm in radius or 218 mm (21.8 cm) in diameter,

and you would have to locate it 1.5×10^8 km × 1 mm/6378 km = 23,518 mm = 23.5 meters away! (For the Moon distance-determined scale, the Sun would be 228 mm in diameter and 24.6 m away.)

Unit 9

10. If the same year-numbering systems were used today as were used in Europe before 532 c.e., what year would we currently be in?

10. (The current year + 753.) The system in Europe before 532 C.E. was based on the founding of Rome in 753 B.C.E, and the year 532 corresponds to the year 753 + 532 = 1285 using that system. So the present year plus 753 will give the answer. The other system mentioned in the text is based on when Diocletian became emperor of Rome; in that system, 532 C.E. is year 248; so there is a difference of 532 – 248 = 284 years from the present C.E. year. For example, 2015 C.E. gives 2015 + 753 = 2768 since the founding of Rome, and 2015 – 248 = 1767 in the reign of Diocletian.

11. What year will 2020 c.e. be in the Chinese, Islamic, and Jewish calendar systems?

11. In September 2000, the Jewish calendar started year 5761 with Rosh Hashanah; in 2020 it was the year 5780 before Rosh Hashanah and 5781 after it.

From Figure 9.2, the Islamic year 1426 began on February 10, 2005. Since the Islamic year is 11 days shorter on average, after 15 years, the Islamic year 1441 began about 11 × 20 = 220 days (or about 7 months) earlier on the Gregorian calendar, placing the first day of 1441 at the end of June 2019. This means the year 2020 C.E. began with 1441 and ended in 1442 of the Islamic calendar. Note that the Islamic calendar picks up an extra year compared to the common calendar once every 33 or 34 years. The extra year here was gained in 2008, which saw the beginning of two Islamic years.

In the Chinese system, the 79^{th} sixty-year cycle began in early 1984 C.E. and will end in early 2044 C.E.. That means in early 1984 C.E., 78 cycles or 78 × 60 = 4680 years were completed, and the year 4681 began. There are 36 years between 2020 and 1984 C.E., which means that in early 2020 C.E. the Chinese year 4681 + 36 = 4717 began. Note that some authorities begin the count a year earlier (which would make it the Chinese year 4622), but in either case it is agreed to be the year of the Metal Rat.

12. Suppose a 12-month lunar calendar year (such as in the Islamic calendar) began on January 1. How many solar years would pass before the lunar calendar year would again begin within one week of January 1? How many lunar calendar years would have elapsed?

12. (32 solar years, or 33 lunar calendar years). A lunar calendar year is shorter than a solar year by 11 days, so each year the "lunar new year" begins 11 days earlier on average in the common solar calendar. For example, if the first day of the lunar calendar was Jan 1, then 12 lunar months would be completed around Dec 21 later that year, and by about Dec 10 the following year. We would have to wait a number of years until the missing time accumulated to a full year. Since 365.25/11 = 33.2, 33 lunar calendar years should accumulate the necessary difference.

Checking this, after 33 lunar calendar years the date will have shifted earlier by 33 × 11 = 363 days—this is less than 7 days short of 365.25 days, as expected. One fewer solar years has gone by in the same interval.

Note that in a more detailed calculation with a lunar month of 29.53 days, a lunar calendar year adds up to 354.36 days. Compared to a more precise value of the solar year of 365.24 days, the difference is actually 10.88 days. The ratio 365.24/10.88 = 33.57, and in fact depending on the number of leap years spanned in the common calendar over the intervening years, there may be 33 or 34 lunar calendar years corresponding to 32 or 33 solar years to get the closest match.

13. The first day of the month in the Islamic calendar begins when the crescent moon is first spotted. Usually, the earliest this is possible is about 16 hours after the new moon. How many degrees has the Moon moved since it was new at this time? Approximately how long after the Sun sets will the Moon set if it is just 16 hours past new?

13. (32 minutes). The Moon takes 29.5 days to move 360° relative to the Sun. Since 29.5 days × 24 hr/day = 708 hr, the Moon moves 360° / 708 hr ≈ 0.5°/hr relative to the Sun. Therefore 16 hours after the new moon, the Moon will have moved about 16 hr × 0.5°/hr = 8° from the Sun, and because the sky turns at 15°/hr, it will set 8°/(15°/hr) = 0.53 hr = 32 minutes later. (Note that these results are approximate since the Moon may be off the ecliptic by several degrees, and moonset timing depends on the angle between Sun and Moon relative to the horizon.)

14. Under the Julian calendar system, the year averages to 365.25 days after 4 years. In the Gregorian system, determine the average length of the year after 400 years. If a country needed to convert from the Julian to the Gregorian calendar now, how many days would it need to remove from the calendar?

14. (13 days). In the Julian calendar, there are 365.25 d/y × 400 y = 146100 days in 400 years. The Gregorian calendar removes leap years in centuries not divisible by 400. In 400 years, there are three centuries not divisible by 400, so the Gregorian calendar would have three fewer days, or 146097 days, and average 365.2425 days per year. To correct the calendar today, we would have to remove the 10 days needed in 1582, plus the three extra days that would have been in a Julian calendar since then for the leap years in 1700, 1800, and 1900, for 13 days total.

15. Suppose the year were 365.170 days long. How would you design a pattern of leap years to make the average year length exactly match this value within a span of 1000 years?

15. If a year had 365.170 days, over 1000 years there would be 365.170 × 1000 = 365,170 days. If all 1000 years had exactly 365 days, there would be 365,000 days leaving an extra 170 days. These can be made up with 170 leap years of 366 days each, and the other 830 (= 1000 – 170) would have 365 days. Since 1000/170 = 5.88, a leap year is needed every five or six years. The question is how to distribute these years with an extra day that is easy to remember. If there was a leap year every five years, there would be 200 in 1000 years, which is 30 too many. If every six years, there would only be 166 of them, four too few. One scheme that would work would be a leap year every five years, skipping 3 each century—for example, every five years except for years that end in 25, 50, and 75.

Unit 10

10. How large would Mars appear in angular size when it is closest to Earth? At its closest, the separation between Earth and Mars is about 55 million km, and Mars' diameter is
6740 km. Jupiter is about 588 million km from Earth at its closest. Given that Jupiter
has a physical diameter of 140,000 km, what is the largest possible angular size of Jupiter? (Refer back to Unit 10.4 for the angular size formula.)

10. (Mars: 0.007°; Jupiter: 0.014°). Use the angle-distance equation:

$\alpha = 57.3° \times$ diameter/distance $= 57.3° \times (6740\text{ km} / 55 \times 10^6\text{ km}) = 0.007° = 25$ arc seconds

At the smallest separation between Earth and Jupiter of 588 million km, when both are on the same side of the solar system and Jupiter is opposite the Sun, Jupiter (diameter = 140,000 km) has an angular diameter of:

$\alpha = 57.3° \times$ diameter/distance $= 57.3° \times 1.4 \times 10^5 / 5.88 \times 10^8 = 0.0136° = 50$ arc seconds

This means Jupiter would appear almost twice as large as Mars in a telescope, even though it is more than 10 times further away.

11. Suppose you are an alien living on the fictitious warlike planet Myrmidon and you want to measure its size. The Sun is shining directly down a missile silo 1000 miles to your south, while at your location the Sun is 36° from straight overhead. What is the circumference of Myrmidon? What is its radius?

11. (1592 miles). Use Eratosthenes' method to find the size of the planet Myrmidon. From the observations we know that 1000 miles on the surface spans an angle of 36°, so Myrmidon's circumference must satisfy the equation: circumference/1000 miles = 360°/36° = 10.
Hence, circumference = 10 × 1000 miles = 10,000 miles. And since circumference = $2\pi R$, we find its radius R = 10,000 miles / 2π = 1592 miles, or about ¾ the size of our Moon.

12. A cloud directly above you is about 10° across. From the weather report you know that the cloud is at an altitude of 2200 m. How wide is the cloud?

12. (380 m). The size of a cloud that is 10° across and 2200 m away is:

$$\ell = d \times \alpha/57.3° = 2200 \text{ m} \times 10°/57.3° = 380 \text{ m}.$$

13. The Andromeda Galaxy, M31, is in many ways similar to our own galaxy but slightly larger. The linear diameter of the Andromeda Galaxy along its longest axis is 140,000 lightyears, but from our perspective, the Andromeda Galaxy has a maximum angular diameter of 3.18°. How far away is the Andromeda Galaxy?

13. (2.5 Million ly). From the linear and angular diameters of the Andromeda galaxy, we can find its distance:

$$d = \ell \times (57.3°/\alpha) = (140{,}000 \text{ light-years}) \times (57.3°/3.18°) = 2.5 \times 10^6 \text{ light-years}$$

This “nearby” galaxy is about 2.5 million light-years distant.

14. Suppose you see a person whom you know is 1.8 m (6.0 ft) tall, standing at a distance where he appears to have an angular height of 2°. How far away is he?

14. (About 52 m away). A person 1.8 m tall who has an angular size of 2° must be standing at a distance:

$$d = \ell \times (57.3°/\alpha) = 1.8 \text{ m} \times 57.3°/2° = 51.6 \text{ m}$$

For most people, when held at arm’s length, the width of a thumb has an angular size of about 2 degrees. So if you see a person standing about 50 meters away, they would appear to you to be the same angular size as your thumb on your outstretched arm.

15. Suppose the Moon is exactly 0.500° wide when it is near the horizon. If later in the night you saw it straight overhead, what would its angular diameter be? (Assume Earth’s radius is 6378 km and the Moon’s distance at the first observation is 398,000 km.)

15. If the Moon has an angular size of 0.500° when it is near the horizon, it will be *larger* when it is overhead later that night because it will be slightly closer to you—by the size of Earth’s radius. The ratio of the distances is 398,000 km / (398,000 km–6378 km) = 398,000/391622 = 1.0163. Since you are closer, and angular size-distance is an inverse relationship, the Moon will appear bigger by that factor and have an angular size of 0.500° × 1.0163 = 0.508°. (Note that this is the opposite of what most people expect to be true because of the Moon illusion.)

16. Aristarchus’s critics argued that parallax was not observed and therefore Earth could not be in motion. In fact we know now that the stars are very far away and parallax motion is very small. Let’s examine this by estimating

the change in angular size of the angular separation between two stars 10 light-years away from the Sun. The two stars are 3 light-years apart. Calculate the angular separation of the two stars when Earth is closest to them in its orbit and when it is farthest away, on the opposite point on its orbit. If human vision (without the aid of a telescope) can at best distinguish a difference of about 0.02°, would the change in angular separation be detectable to the naked eye?

16. (No, the change is not detectable to an unaided eye). If two stars are 10 ly from the Sun, then their distance from the Earth can vary by plus or minus 1 AU as the Earth orbits the Sun. Expressing light years in AU: 1 ly = 63,240 AU. (from Appendix) so the distance varies from 632,401 AU to 632,399 AU. If the stars are 3 ly = 189,720 AU apart, we can calculate their angular separation from the formula:

Closest: $\alpha = 57.3° \times \ell / d = 57.3° \times 189{,}720 \text{ AU} / 632{,}399 \text{ AU} = 17.1900272°$
Farthest: $\alpha = 57.3° \times \ell / d = 57.3° \times 189{,}720 \text{ AU} / 632{,}401 \text{ AU} = 17.1899728°$

The difference is 0.000054°, much smaller than the 0.02° difference the eye can distinguish.

Unit 11

10. If we discover a new planet in the Solar System and observe that it takes about 366 days between oppositions, what could we say about it?

10. If a planet was in opposition once every 366 days, that would imply it shifts by less than about 1° in its orbit each year, indicating that it must take more than 360 years to complete an orbit. (If you use 365.25 days as the length of the year, you might estimate an orbital period as large as 487 years.) Given that this orbit is slower than any of the other planets, the new planet must lie in a very large, slow orbit, far from the Sun.

11. On average, Mars spends about 72 days in "retrograde motion" when observers on
Earth see it moving temporarily westward along the ecliptic. If Mars' average orbital speed
is 27.7 km/second, how far does Mars actually move in its orbit during that time? How far does Earth move in that time, if its average speed is 29.8 km/sec?

11. Use distance = velocity × time. In 72 days, Mars would move:

$$(27.7 \text{ km/sec}) \times 72 \text{ days} \times \frac{24 \text{ hours}}{\text{day}} \times \frac{3600 \text{ seconds}}{\text{hour}} = 1.72 \times 10^8 \text{ km}$$

In the same time, Earth would move farther since it is moving around the Sun faster:

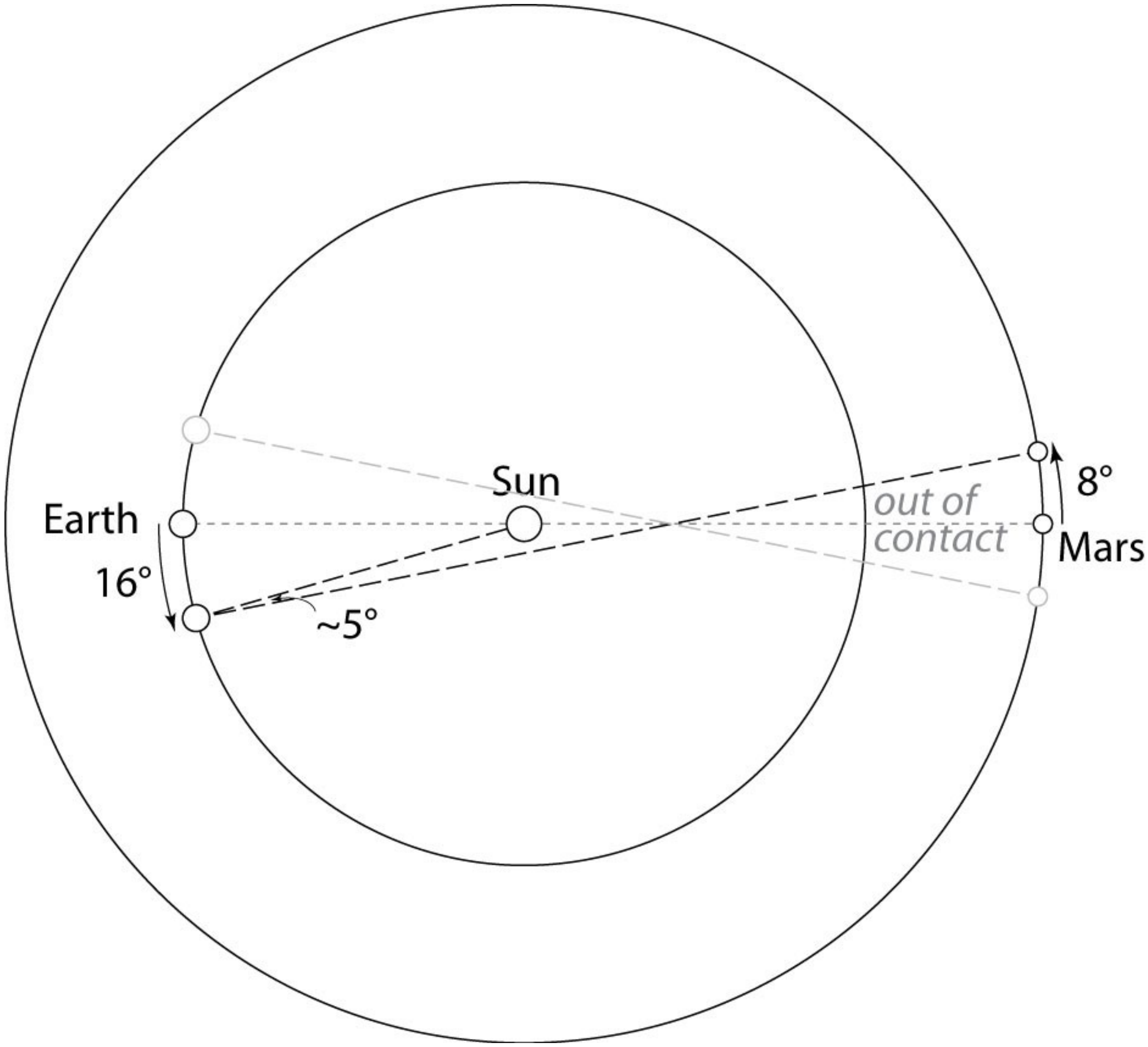
$$(29.8\ \text{km/sec}) \times 72\ \text{days} \times \frac{24\ \text{hours}}{\text{day}} \times \frac{3600\ \text{seconds}}{\text{hour}} = 1.85 \times 10^{8}\ \text{km}$$

Because Earth moves faster around the Sun, it “catches up and laps” the outer planets, and it is when that happens that we see those planets move in retrograde motion. And because Earth’s orbit is closer to the Sun, moving farther on a smaller orbit means Mars will appear to shift its location in front of the background stars even more.

12. When Mars is close to conjunction, within an angle of about 5° of the Sun, we lose radio contact with our spacecraft there. Draw a scale diagram of the orbits of Earth and Mars and, given that Earth moves about 1° per day in its orbit and Mars about 0.5° per day in its orbit, estimate approximately how long this period of communication problems lasts.

12. (About 32-36 days) Starting from when Mars is in conjunction, the Earth’s and Mars’s motions on opposite sides of the Sun partly cancel out, keeping Mars hidden longer than if only one of them was moving. Earth’s more rapid motion eventually overtakes Mars bringing it back into view. The diagram shows a scale construction of the two orbits (with Mars’s orbit 1.52 times larger than Earth’s and ignoring the ellipticity.)

By testing several angles with a protractor, shifting Earth by 1° per day and Mars ½° per day in their orbits around the Sun, it appears that Mars reaches an angle of ~5° from the Sun (as seen from Earth) about 16–18 days after conjunction. In total, then Mars is difficult to contact for about 32 to 36 days, counting the time before and after conjunction.
12. (cont.)



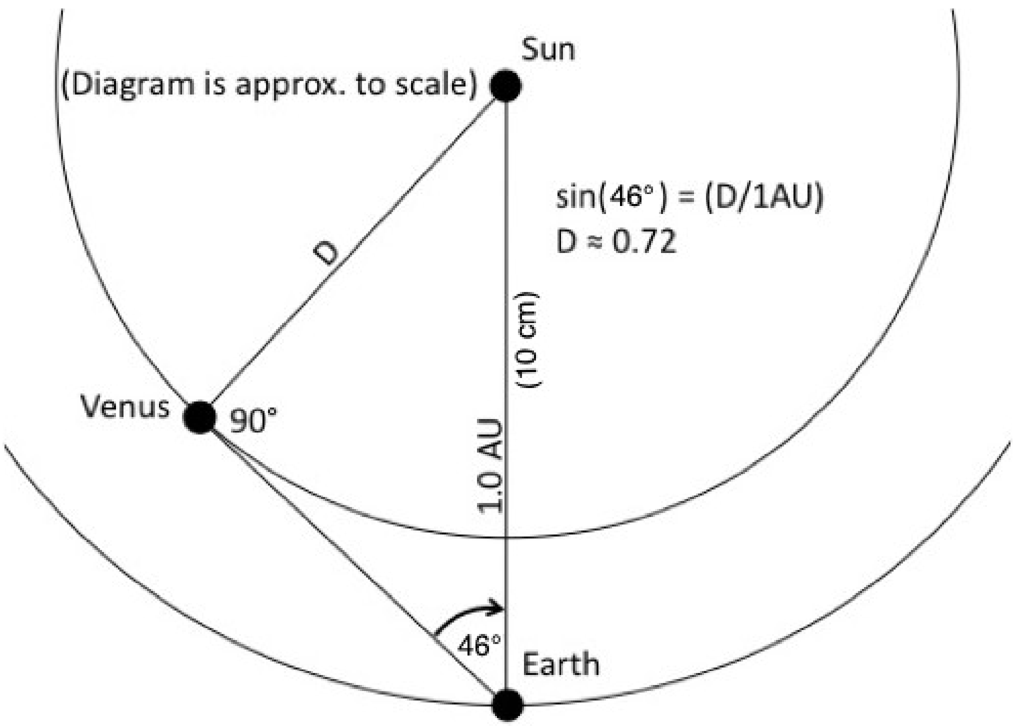
13. Using a protractor, draw two lines that make an angle of 46° relative to each other, matching Venus's average greatest elongation. At a point on one of the lines 10 cm from where the lines intersect, place a compass and draw the largest circle you can that just touches the other line. What is the radius of this circle? Explain how this geometric construction relates to the size of Venus's orbit.

13. (The radius is 0.72 AU, representing Venus' distance from the Sun).
First, we draw a line from Venus to Earth as shown in the diagram. Then 46° off of that, we draw a 10 cm line from Earth towards the Sun. Starting at the end of that line, we draw the circle that represents the orbit of Venus, so that it just touches our first line. We now have a line tangent to a circle, so the angle between our Venus-Earth line and the Venus-Sun line (D) is 90°.
13. (cont.) The length of D divided by the length of 1 AU measured in the figure gives the value of D, the orbital distance of Venus, in AU. Alternatively, we can use the law of sines and the right triangle we have just created to solve for D. The right angle is at Venus when Venus

appears to be farthest from the Sun (at greatest elongation). If the angle between Venus and the Sun is 46 as seen from Earth, then from trigonometry:

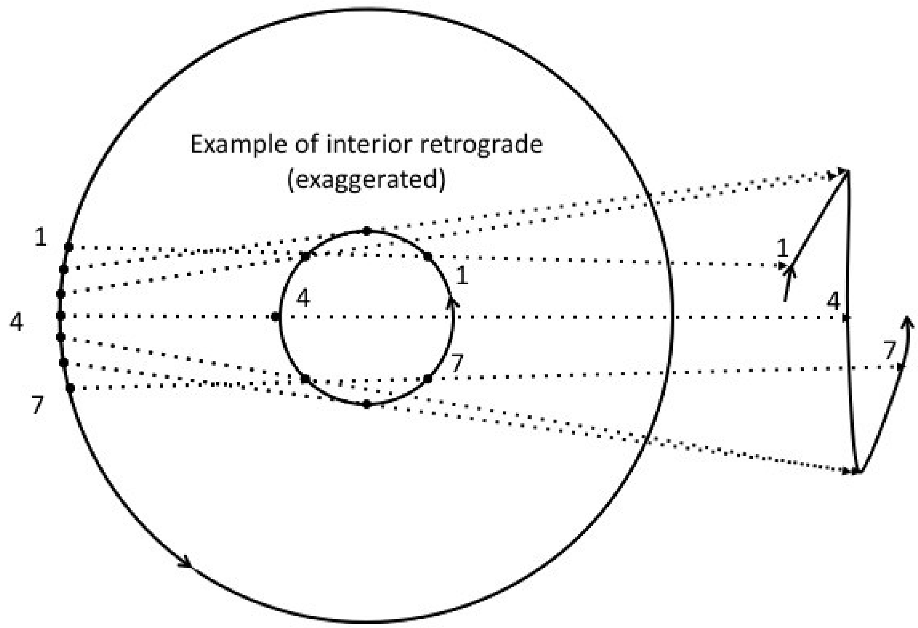
sin (46) = Venus Distance / 1.0 A = D / 1 AU.

So D = 1 AU × sin(46) = 0.72 AU.



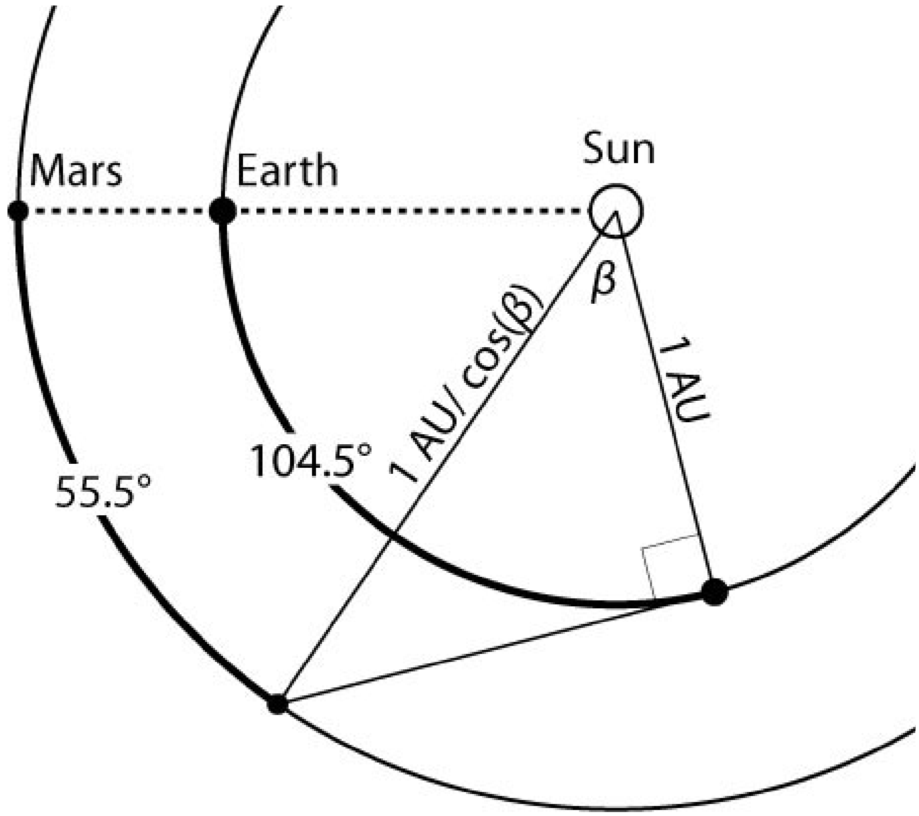
14. Make a diagram similar to Figure 11.7 B that shows retrograde motion for a planet closer to the Sun than Earth. Does retrograde motion occur near superior conjunction, inferior conjunction, or greatest elongation? What would an observer on this planet see of Earth's motion over this same time?

14. (Inferior conjunction; an observer would see Earth in retrograde). The retrograde motion for a planet closer to the Sun occurs around inferior conjunction as the planet passes Earth between the Earth and the Sun. An observer on this planet during this time would also see the Earth, near opposition, going through retrograde motion.



15. Mars spends, on average, about 106 days (15.4% of its orbit) moving from opposition to quadrature. Make a geometric construction like Figure 11.11 (or use trigonometry) to show that Mars's orbit is about 1.52 times the size of Earth's

15. Mars spends, on average, about 106 days (15.4% of its orbit) moving from opposition to quadrature. This is illustrated in the figure below. To draw this, we need to know how much of their orbits the planets move through. In the 106 days from opposition to quadrature, the Earth travels 106/365 × 360 = 104.5 along its orbit. During the same time, Mars travels 106/687 × 360 = 55.5 along its orbit. Mars must be 1.52 times farther from the Sun than the Earth along the line at 55.5° in order for Mars to be 90° away from the Sun when viewed from Earth. If Mars were anywhere else along that line from the Sun, the angle at Earth would be different.
15. (cont.)



Using trigonometry it is possible to solve for the size of Mars's orbit more directly. The difference between the angles the two planets move is β = 104.5°–55.5° = 49° as shown in the diagram. The distance from the Sun to Mars is the hypotenuse of the right triangle, therefore cos(β) = 1 AU / (Mars distance from Sun). Since β = 49°, we get Mars's distance from the Sun equal to 1 AU / cos(49°) = 1.52 AU.

The eccentricity of Mars's orbit is sufficient that the results of this calculation vary depending on the portion of the orbit where Mars is traveling—and neither Mars nor the Earth travels at constant speed, so the time between opposition and quadrature can vary significantly.

Unit 12

10. Ceres has an orbital period of 4.60 years. What is its semimajor axis? Where in the Solar System does this place Ceres?

10. (2.8 AU, in the asteroid belt). From Ceres' orbital period of 4.6 yr, use Kepler's 3rd Law: $a^3 = P^2$ to solve for the semimajor axis. For P= 4.6 yr, we find $a^3 = 4.6^2 = 21.2$, so taking the cube root, we find a = 2.8 AU. This is between the orbits of Mars and Jupiter, placing Ceres in the asteroid belt.

11. Sedna, an object in the outer Solar System, has a semimajor axis of 526 AU. What is its orbital period? How does this compare to Pluto's orbital period of 248.1 years?

11. From Sedna's semimajor axis of a = 526 AU, we can use Kepler's 3rd Law to solve for the period. $P^2 = a^3 = (526)^3 = 1.4553 \times 10^8$, so taking the square root we find P = 12,064 years. The ratio of Sedna's orbit to that of Pluto = 12,064/248.1 = 48.6, so the orbital period of Sedna is almost 50 times greater than Pluto's period.

12. Suppose an asteroid is discovered with an orbit that brings it as close as 0.5 AU from the Sun and as far away as 5.5 AU. What is the semimajor axis of its orbit? What is its orbital period?

12. (a= 3.0 AU and P = 5.2 years). The semimajor axis is half the sum of the closest and farthest distance from the Sun, so for this asteroid the semimajor axis a = (0.5 AU + 5.5 AU)/2 = 3.0 AU. We can solve for the asteroid's period using Kepler's 3rd Law $P^2 = a^3 = 3^3 = 27$, so taking the square root we find P = 5.2 years.

13. If a comet orbits the Sun and reaches 1 AU at its closest approach to the Sun, and its orbital period is 27 years, what is its maximum distance from the Sun?

13. A comet with an orbital period of 27 years has a semimajor axis that can be found from Kepler's 3rd Law: $a^3 = P^2 = 27^2 = 729$; taking the cube root, we find a = 9 AU. Since the semimajor axis is the average of the maximum and minimum distance from the Sun, we know that (max + min)/2 = a or (max + min) = 2a = 18 AU. Therefore, if the minimum distance to the Sun is 1 AU, the maximum distance from the Sun must be 18 AU – 1 AU = 17 AU.

14. Calculate the speed (in km/sec) of an asteroid in a circular orbit at 4 AU by dividing the orbit's circumference by its period.

14. (15 km/sec). For an asteroid orbit with a = 4 AU, the period is given by Kepler's 3rd Law as $P = 4^{3/2}$ = 8 yr. A circular orbit of R = 4 AU has a circumference = 2πR.

The speed of the asteroid is then found from:

v = circumference / period
= 2πR / P
= 2π(4 AU * 1.5 × 10^8 km/AU) / (8 yr * 60 sec/min * 60 min/hr * 24 hr/day * 365 day/yr)
= 15 km/sec.

15. A comet is discovered with an orbit that extends from Venus's orbit to Neptune's orbit. Use Kepler's second law to estimate how many times faster the comet is moving when it is closest to the Sun compared to when it is farthest from the Sun. (Hint: The area swept out over a short period of time when the planet is at these two extremes can be approximated by a triangle, and the area of a triangle equals one-half of its base times its height.

15. (41.5 times faster.) The ratio of distances at Neptune's versus Venus's distance is (30.0 AU/0.723 AU) = 41.5. When the comet is closest to the Sun, its motion over a fixed amount of time can be pictured as the base of a triangle with the height of the triangle equal to the distance to the Sun. When the comet is farthest from the Sun, we can imagine the same kind of triangle, but it now has a much greater "height" so the base of the triangle must be proportionately shorter. Thus, the ratio of speeds is inversely related to the ratio of the distances. Therefore, the comet travels 41.5 times faster when it is closest to the Sun than when it is farthest.

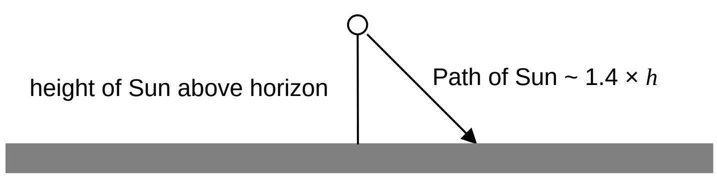
Unit 13

10. Use your hand and fingers to estimate the angular size of at least three constellations. Sketch the constellations to scale with your measurements.

10. The main part of Orion, for example, is about 20° tall and 10° wide. (Sizes can be estimated from sky charts, but there is a lot of distortion at the edges because of the spherical projection.)

11. Sailors have "handy" rules for estimating the time until sunset. Approximately how many minutes before sunset is the Sun "one finger" above the horizon at the equator? At 45°N or S latitude?

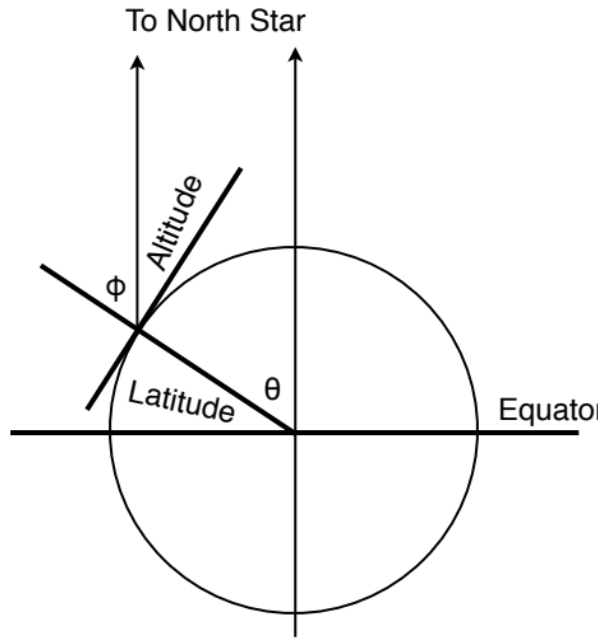
11. Typically, the width of a finger is similar to the width of a thumbnail, which is about 2° wide when held at arm's length. The Earth turns at about 1 degree every 4 minutes, so if the Sun were "one finger" above the western horizon at the equator, sunset would be in about 8 minutes. (The little finger might be a little less, the middle finger a little more.) At about 45° latitude, the Sun sets at a 45° angle, and hence its path to the horizon is longer. To figure out the length of the path, you can sketch the path and measure the difference:



The length of the Sun's path may also be recognized as the hypotenuse of a 45°-45°-90° triangle, which is $\sqrt{2}$ times the length of a side. (Or using trigonometry, we find that the longer distance is $h/\sin(45°) = h/0.707 = 1.41\ h$, where h is the vertical height of the Sun above the horizon.) A finger's width above the horizon translates to a motion of the Sun by about 2° × 1.4 = 2.8°, so the Sun will take about 8 min × 1.4 ≈ 11 minutes per finger before it sets at 45° latitude.

12. Demonstrate through the use of a drawing that the altitude of the North Star, as seen by anyone in the Northern Hemisphere, is equal to the observer's latitude.

12. The picture at right shows Earth as viewed in profile and above the equator. By inspection of the figure we note that the two arrows pointing toward the North Star are parallel lines. This implies that the angles θ and ϕ are equal. Moreover, since the latitude angle + θ = 90° = Altitude + ϕ it follows that the altitude must also equal the latitude angle.



As a side note, students are often reluctant to accept that lines toward the North Star from different parts of the Earth are effectively parallel.

12. (cont.) For some it is helpful to remind them that Polaris is far bigger than the entire Earth. In fact it is about 50 times larger than the Sun, so the difference in direction from different spots on the Earth is a tiny fraction of the point of light we see in the night sky.

13. Calculate a precise value for how many degrees the Moon moves each day, keeping in mind that to complete one lunar month of 29.53 days, the Moon must once again align with the Sun, which also has shifted along the ecliptic during that time.

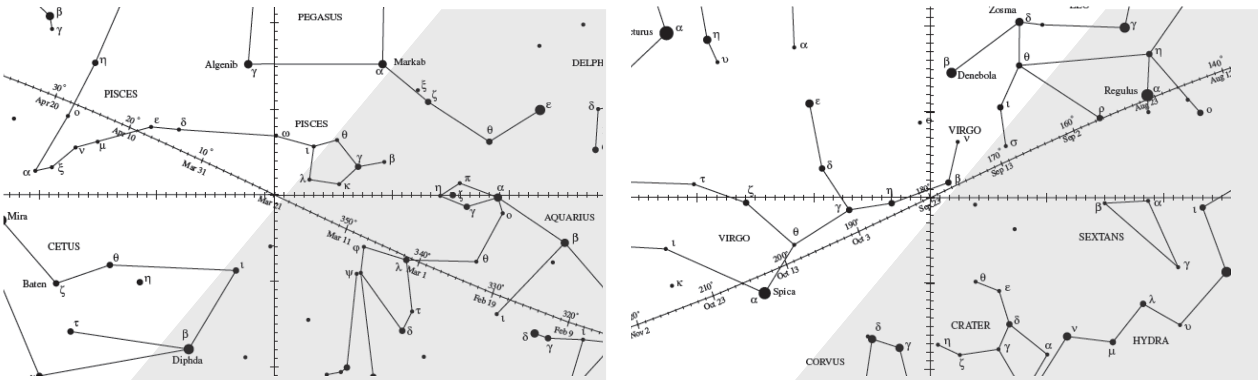
13. (13.18°/day). The Moon revolves (360°/29.53 days = 12.19°/day) around the Earth each day, and during that day the Earth has revolved (360°/365.25 days = 0.99°/day) around the Sun in the same direction, so the Moon revolves 12.19°/day + 0.99°/day = 13.18°/day relative to the stars. (Note this is the same answer as you get dividing 360° by the sidereal lunar month of 27.3 days).

14. Even at greatest elongation, Mercury will not always be visible high above the horizon after sunset or before sunrise. This is primarily because the angle between the ecliptic and the horizon is shallow sometimes during the year. In that case, Mercury is far from the Sun but still close to the horizon. When the ecliptic is more nearly perpendicular to the horizon in your viewing location, Mercury will have a higher altitude. By examining the foldout star chart, estimate at what times of year the ecliptic will be at the best angle for you to observe Mercury in the evening sky. Is there an upcoming greatest elongation that is favorable for you?

14. The angle of the horizon at sunset depends on the observer’s latitude. On the equator the horizon is perpendicular to the celestial equator (running north-south on the celestial sphere), so on the equator the ecliptic runs most nearly perpendicular to the horizon at the solstices.
At other latitudes, the horizon at sunset is tilted from north-south by an angle equal to the latitude. This is illustrated below for latitude 40°N, with the grayed-out area beneath the horizon for sunset on the March equinox (left) and the September equinox (right).

As can be seen in the illustrations, the path of the ecliptic is most nearly perpendicular to the horizon in the evening sky near the beginning of spring, which is when it would be easiest to see Mercury as an evening star. The pre-dawn horizon is tilted 40° in the other direction, so Mercury is easiest to see as a morning star near the beginning of autumn for northern observers. These answers would be reversed for an observer at mid-southern-hemisphere latitudes.

14. (cont.)



15. If you mark the position of the shadow cast by the top of a flagpole at the same clock time every day throughout the year, you will find that the marks trace out a figure-eight pattern. Explain why this happens by referring to the equation of time.

15. The Equation of Time shows the offset between sundial time and mean solar time. These are not the same because of the Earth's changing speed in its elliptical orbit and the tilt of the Earth's axis (as discussed in Unit 13.5). If we measure the shadow cast by the top of the flagpole once every 24 hours as measured by a standard clock, the Sun will be east of its mean position when the sundial is behind clock time, or west when the sundial is ahead as given by the Equation of Time. The shadow therefore reaches farthest to the west in February and July, and farthest to the east in May and November. Because the Sun also moves north and south of the celestial equator during the course of the year, so the east-west motions over each half year produce an S-shaped pattern that join to make a figure-8 shaped pattern.

Alternate answer for Unit 9 #11 if you change the manuscript question to ask about 2025:

11. What year will 2025 c.e. be in the Chinese, Islamic, and Jewish calendar systems?

11. In September 2000, the Jewish calendar started the year 5761 with Rosh Hashanah; in 2025 it will be the year 5785 before Rosh Hashanah and 5786 after it.

From Figure 9.2, the Islamic year 1426 began on February 10, 2005. Since the Islamic year is 11 days shorter on average, after 20 years, the Islamic year 1446 will begin about 11 × 20 = 220 days (or about 7 months) earlier on the Gregorian calendar, placing the first day of 1446 at the end of June 2024. This means the year 2025 C.E. will begin with 1446 and end in 1447 of the Islamic calendar. Note that the Islamic calendar picks up an extra year compared to the common calendar once every 33 or 34 years. The extra year here was gained in 2008, which saw the beginning of two Islamic years.

In the Chinese system, the 79^{th} sixty-year cycle began in early 1984 C.E. and will end in early 2044 C.E.. That means in early 1984 C.E., 78 cycles or 78 × 60 = 4680 years were completed, and the year 4681 began. There are 41 years between 2025 and 1984 C.E., which means that in early 2025 C.E. the Chinese year 4681 + 41 = 4722 will begin. Note that some authorities begin the count a year earlier (which would make it the Chinese year 4623), but in either case it is agreed to be the year of the Metal Rat.