

Complete Solutions Guide

for

FUNCTIONS AND CHANGE:
A Modeling Approach
to College Algebra

Sixth Edition

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Solution Guide for Prologue: Calculator Arithmetic

CALCULATOR ARITHMETIC

1. **Valentine's Day:** To find the percentage we first calculate

$$\frac{\text{Average female expenditure}}{\text{Average male expenditure}} = \frac{\$96.58}{\$190.53} = 0.5069.$$

Thus the average female expenditure was 50.69% of the average male expenditure.

2. **Pet owners:** First we find the number of households that owned at least one pet. Because 65% of the 117 million households owned at least one pet, this number is

$$65\% \times 117 = 0.65 \times 117 = 76.05 \text{ million.}$$

Now 42% of those households owned at least two pets, so the number owning at least two pets is

$$42\% \times 76.05 = 0.42 \times 76.05 = 31.94 \text{ million.}$$

Therefore, the number of households that owned at least two pets is 31.94 million.

3. **A billion dollars:** A stack of a billion one-dollar bills would be $0.0043 \times 1,000,000,000 = 4,300,000$ inches high. In miles this height is

$$4,300,000 \text{ inches} \times \frac{1 \text{ foot}}{12 \text{ inches}} \times \frac{1 \text{ mile}}{5280 \text{ feet}} = 67.87 \text{ miles.}$$

So the stack would be 67.87 miles high.

4. **National debt:** Each American owed $\frac{\$18,151,998 \text{ million}}{321 \text{ million}} = \$56,548.28$ or about 57 thousand dollars.

5. **10% discount and 10% tax:** The sales price is 10% off of the original price of \$75.00, so the sales price is $75.00 - 0.10 \times 75.00 = 67.50$ dollars. Adding in the sales tax of 10% on this sales price, we'll need to pay $67.50 + 0.10 \times 67.50 = 74.25$ dollars.

6. **A good investment:** The total value of your investment today is:

$$\text{Original investment} + 13\% \text{ increase} = 850 + 0.13 \times 850 = \$960.50.$$

7. **A bad investment:** The total value of your investment today is:

$$\text{Original investment} - 7\% \text{ loss} = 720 - 0.07 \times 720 = \$669.60.$$

8. **An uncertain investment:** At the end of the first year the investment was worth

$$\text{Original investment} + 12\% \text{ increase} = 1300 + 0.12 \times 1300 = \$1456.$$

Since we lost money the second year, our investment at the end of the second year was worth

$$\text{Value at end of first year} - 12\% \text{ loss} = 1456 - 0.12 \times 1456 = \$1281.28.$$

Consequently we have lost \$18.72 of our original investment.

9. **Pay raise:** The percent pay raise is obtained from

$$\frac{\text{Amount of raise}}{\text{Original hourly pay}}.$$

The raise was $9.50 - 9.25 = 0.25$ dollar while the original hourly pay is \$9.25, so the fraction is $\frac{0.25}{9.25} = 0.0270$. Thus we have received a raise of 2.70%.

10. **Heart disease:** The percent decrease is obtained from

$$\frac{\text{Amount of decrease}}{\text{Original amount}}.$$

Since the number of deaths decreased from 235 to 221, the amount of decrease is 14 and so the fraction is $\frac{14}{235} = 0.0596$. The percent decrease due to heart disease is 5.96%.

11. **Trade discount:**

(a) The cost price is $9.99 - 40\% \times 9.99 = 5.99$ dollars.

(b) The difference between the suggested retail price and the cost price is $65.00 - 37.00 = 28.00$ dollars. We want to determine what percentage of \$65 this difference represents. We find the percentage by division: $\frac{28.00}{65.00} = 0.4308$ or 43.08%. This is the trade discount used.

12. **Series discount:**

(a) Applying the first discount gives a price of $80.00 - 25\% \times 80.00 = 60.00$ dollars. Applying the second discount to this gives $60.00 - 10\% \times 60.00 = 54.00$ dollars. The retailer's cost price is \$54.

- (b) Applying the first discount gives a price of $100.00 - 35\% \times 100.00 = 65.00$ dollars. Applying the second discount to this gives a price of $65.00 - 10\% \times 65.00 = 58.50$ dollars. Applying the third discount gives $58.50 - 5\% \times 58.50 = 55.575$. The retailer's cost price is \$55.58.
- (c) Examining the calculations in Part (b), we see that the actual discount resulting from this series is $100 - 55.575 = 44.425$. This represents a single discount of about 44.43% off of the original retail price of \$100.
- (d) Again, we examine the calculations in Part (b). In the first step we subtracted 35% of 100 from 100. This is the same as computing 65% of 100, so it is 100×0.65 . In the second step we took 10% of that result and subtracted it from that result; this is the same as multiplying 100×0.65 by 90%, or 0.90, so the result of the second step is $100 \times 0.65 \times 0.90$. Continuing in this way, we see that the result of the third step is $100 \times 0.65 \times 0.90 \times 0.95$. Here the factor 0.65 indicates that after the first discount the price is 65% of retail, the factor 0.90 indicates that after the second discount the price is 90% of the previous price, and so on.

13. **Present value:** We are given that the future value is \$5000 and that $r = 0.12$. Thus the present value is

$$\frac{\text{Future value}}{1 + r} = \frac{5000}{1 + 0.12} = 4464.29 \text{ dollars.}$$

14. **Future value:**

- (a) A future value interest factor of 2 will make an investment double since an investment of P dollars yields a return of $P \times 2$ or $2P$ dollars. A future value interest factor of 3 will make an investment triple.
- (b) The future value interest factor for a 7 year investment earning 9% interest compounded annually is

$$(1 + \text{interest rate})^{\text{years}} = (1 + 0.09)^7 = 1.83.$$

- (c) The 7 year future value for a \$5000 investment is

$$\text{Investment} \times \text{future value interest factor} = 5000 \times 1.83 = \$9150.$$

Note: If the answer in Part (b) is not rounded, one gets \$9140.20, which is more accurate. Since the exercise asked you to "use the results from Part (b)..." and we normally round to two decimal places, \$9150 is a reasonable answer. This illustrates the effect of rounding and that care must be taken regarding rounding of intermediate-step calculations.

15. The Rule of 72:

- (a) The Rule of 72 says our investment should double in

$$\frac{72}{\% \text{ interest rate}} = \frac{72}{13} = 5.54 \text{ years.}$$

- (b) Using Part (a), the future value interest factor is

$$(1 + \text{interest rate})^{\text{years}} = (1 + 0.13)^{5.54} = 1.97.$$

This is less than the doubling future value interest factor of 2.

- (c) Using our value from Part (b), the future value of a \$5000 investment is

$$\text{Original investment} \times \text{future value interest factor} = 5000 \times 1.97 = \$9850.$$

So our investment did not exactly double using the Rule of 72.

16. The Truth in Lending Act:

- (a) The credit card company should report an APR of

$$12 \times \text{monthly interest rate} = 12 \times 1.9 = 22.8\%.$$

- (b) We would expect to owe

$$\begin{aligned} &\text{original debt} + 22.8\% \text{ of original debt} \\ &= 6000 + 6000 \times 0.228 = \$7368.00. \end{aligned}$$

- (c) The actual amount we would owe is
- $6000 \times 1.019^{12} = \7520.41
- .

17. The size of the Earth:

- (a) The equator is a circle with a radius of approximately 4000 miles. The distance around the equator is its circumference, which is

$$2\pi \times \text{radius} = 2\pi \times 4000 = 25,132.74 \text{ miles,}$$

or approximately 25,000 miles.

- (b) The volume of the Earth is

$$\frac{4}{3}\pi \times \text{radius}^3 = \frac{4}{3}\pi \times 4000^3 = 268,082,573,100 \text{ cubic miles.}$$

Note that the calculator gives 2.680825731E11, which is the way the calculator writes numbers in scientific notation. It means $2.680825731 \times 10^{11}$ and should be written as such. That is about 268 billion cubic miles or 2.68×10^{11} cubic miles.

(c) The surface area of the Earth is about

$$4\pi \times \text{radius}^2 = 4\pi \times 4000^2 = 201,061,929.8 \text{ square miles,}$$

or approximately 201,000,000 square miles.

18. When the radius increases:

(a) To wrap around a wheel of radius 2 feet, the length of the rope needs to be the circumference of the circle, which is

$$2\pi \times \text{radius} = 2\pi \times 2 = 12.57 \text{ feet.}$$

If the radius changes to 3 feet, we need

$$2\pi \times \text{radius} = 2\pi \times 3 = 18.85 \text{ feet.}$$

That is an additional 6.28 feet of rope.

(b) This is similar to Part (a), but this time the radius changes from 21,120,000 feet to 21,120,001 feet. To go around the equator, we need

$$2\pi \times \text{radius} = 2\pi \times 21,120,000 = 132,700,873.7 \text{ feet.}$$

If the radius is increased by one, then we need

$$2\pi \times \text{radius} = 2\pi \times 21,120,001 = 132,700,880 \text{ feet.}$$

Thus we need 6.3 additional feet of rope.

It is perhaps counter-intuitive, but whenever a circle (of any size) has its radius increased by 1, the circumference will be increased by 2π , or about 6.28 feet. (The small error in Part (b) is due to rounding.) This is an example of ideas we will explore in a great deal more depth as the course progresses, namely, that the circumference is a *linear* function of the radius, and a linear function has a constant rate of change.

19. The length of Earth's orbit:

(a) If the orbit is a circle then its circumference is the distance traveled. That circumference is

$$2\pi \times \text{radius} = 2\pi \times 93 = 584.34 \text{ million miles,}$$

or about 584 million miles. This can also be calculated as

$$2\pi \times \text{radius} = 2\pi \times 93,000,000 = 584,336,233.6 \text{ miles.}$$

(b) Velocity is distance traveled divided by time elapsed. The velocity is given by

$$\frac{\text{Distance traveled}}{\text{Time elapsed}} = \frac{584.34 \text{ million miles}}{1 \text{ year}} = 584.34 \text{ million miles per year,}$$

or about 584 million miles per year. This can also be calculated as

$$\frac{584,336,233.6 \text{ miles}}{1 \text{ year}} = 584,336,233.6 \text{ miles per year.}$$

(c) There are 24 hours per day and 365 days per year. So there are $24 \times 365 = 8760$ hours per year.

(d) The velocity in miles per hour is

$$\frac{\text{Miles traveled}}{\text{Hours elapsed}} = \frac{584.34}{8760} = 0.0667 \text{ million miles per hour.}$$

This is approximately 67,000 miles per hour. This can also be calculated as

$$\frac{\text{Miles traveled}}{\text{Hours elapsed}} = \frac{584,336,233.6}{8760} = 66,705.05 \text{ miles per hour.}$$

20. **A population of bacteria:** Using the formula we expect

$$2000 \times 1.07^{\text{hours}} = 2000 \times 1.07^8 = 3436.37 \text{ bacteria.}$$

Since we don't expect to see fractional parts of bacteria, it would be appropriate to report that there are about 3436 bacteria after 8 hours.

There are 48 hours in 2 days, so we expect

$$2000 \times 1.07^{\text{hours}} = 2000 \times 1.07^{48} = 51,457.81 \text{ bacteria.}$$

As above, we would report this as 51,458 bacteria after 2 days.

21. **Newton's second law of motion:** A man with a mass of 75 kilograms weighs $75 \times 9.8 = 735$ newtons. In pounds this is 735×0.225 , or about 165.38.

22. **Weight on the moon:** On the moon a man with a mass of 75 kilograms weighs $75 \times 1.67 = 125.25$ newtons. In pounds this is 125.25×0.225 , or about 28.18.

23. **Frequency of musical notes:** The frequency of the next higher note than middle C is $261.63 \times 2^{1/12}$, or about 277.19 cycles per second. The D note is one note higher, so its frequency in cycles per second is

$$(261.63 \times 2^{1/12}) \times 2^{1/12},$$

or about 293.67.

24. **Lean body weight in males:** The lean body weight of a young adult male who weighs 188 pounds and has an abdominal circumference of 35 inches is

$$98.42 + 1.08 \times 188 - 4.14 \times 35 = 156.56 \text{ pounds.}$$

It follows that his body fat weighs $188 - 156.56 = 31.44$ pounds. To compute the body fat percent we calculate $\frac{31.44}{188}$ and find 16.72%.

25. **Lean body weight in females:** The lean body weight of a young adult female who weighs 132 pounds and has wrist diameter of 2 inches, abdominal circumference of 27 inches, hip circumference of 37 inches, and forearm circumference of 7 inches is

$$19.81 + 0.73 \times 132 + 21.2 \times 2 - 0.88 \times 27 - 1.39 \times 37 + 2.43 \times 7 = 100.39 \text{ pounds.}$$

It follows that her body fat weighs $132 - 100.39 = 31.61$ pounds. To compute the body fat percent we calculate $\frac{31.61}{132}$ and find 23.95%.

26. **Manning's equation:** The hydraulic radius R is $\frac{1}{4} \times 3 = 0.75$ foot. Because $S = 0.2$ and $n = 0.012$, the formula gives

$$v = \frac{1.486}{n} R^{2/3} S^{1/2} = \frac{1.486}{0.012} 0.75^{2/3} 0.2^{1/2} = 45.72.$$

The velocity is 45.72 feet per second.

27. **Relativistic length:** The apparent length of the rocket ship is given by the formula $200\sqrt{1 - r^2}$, where r is the ratio of the ship's velocity to the speed of light. Since the ship is travelling at 99% of the speed of light, this means that $r = 0.99$. Plugging this into the formula yields that the 200 meter spaceship will appear to be only $200\sqrt{1 - 0.99^2} = 200\sqrt{(1 - 0.99 \wedge 2)} = 28.21$ meters long.

28. **Equity in a home:** The formula for your equity after k monthly payments is

$$350,000 \times \frac{1.007^k - 1}{1.007^{360} - 1}$$

dollars. After 10 years, you will have made $10 \times 12 = 120$ payments, so using $k = 120$ yields

$$350,000 \times \frac{1.007^{120} - 1}{1.007^{360} - 1},$$

which can be calculated as $350000 \times (1.007 \wedge 120 - 1) \div (1.007 \wedge 360 - 1) = 40,491.25$ dollars in equity.

29. **Advantage Cash card:**

- (a) The Advantage Cash card gives a discount of 5% and you pay no sales tax, so you pay \$1.00 less 5%, which is \$0.95.

(b) If you pay cash, you must also pay sales tax of 7.375%, so you pay a total of \$1.00 plus 7.375%, which is $\$1.07375$ to five decimal places, or \$1.07.

(c) If you open an Advantage Cash card for \$300, you get a bonus of 5%. Now 5% of \$300 is $300 \times .05 = 15$ dollars, so your card balance is \$315. You also get a discount of 5% off the retail price and pay no sales tax, so you can purchase a total retail value such that 95% of it equals \$315, that is

$$\text{Retail value} \times 0.95 = 315,$$

so the retail value is $315/0.95 = 331.58$ dollars.

(d) If you have \$300 cash, then you can buy a retail value such that when you add to it 7.375% for sales tax, you get \$300. So

$$\text{Retail value} \times 1.07375 = 300,$$

and thus the retail value you can buy is $300/1.07375 = 279.39$ dollars.

(e) From Part (d) to Part (c), the increase is $331.58 - 279.39 = 52.19$, so the percentage increase is $52.19/279.39 \times 100\% = 18.68\%$. In practical terms, this means that using the Advantage Cash card allows you to buy 18.68% more food than using cash.

Skill Building Exercises

S-1. **Basic calculations:** In typewriter notation, $\frac{2.6 \times 5.9}{6.3}$ is $(2.6 \times 5.9) \div 6.3$, which equals 2.434... and so is rounded to two decimal places as 2.43.

S-2. **Basic calculations:** In typewriter notation, $3^{3.2} - 2^{2.3}$ is $3 \wedge 3.2 - 2 \wedge 2.3$, which equals 28.710... and so is rounded to two decimal places as 28.71.

S-3. **Basic calculations:** In typewriter notation, $\frac{e}{\sqrt{\pi}}$ is $e \div (\sqrt{\pi})$, which equals 1.533... and so is rounded to two decimal places as 1.53.

S-4. **Basic calculations:** In typewriter notation, $\frac{7.6^{1.7}}{9.2}$ is $(7.6 \wedge 1.7) \div 9.2$, which equals 3.416... and so is rounded to two decimal places as 3.42.

S-5. **Parentheses and grouping:** When we add parentheses, $\frac{7.3 - 6.8}{2.5 + 1.8}$ becomes $\frac{(7.3 - 6.8)}{(2.5 + 1.8)}$, which, in typewriter notation, becomes $(7.3 - 6.8) \div (2.5 + 1.8)$. This equals 0.116... and so is rounded to two decimal places as 0.12.

S-6. **Parentheses and grouping:** When we add parentheses, $3^{2.4 \times 1.8 - 2}$ becomes $3^{(2.4 \times 1.8 - 2)}$, which, in typewriter notation, becomes $3 \wedge (2.4 \times 1.8 - 2)$. This equals 12.791... and so is rounded to two decimal places as 12.79.

S-7. **Parentheses and grouping:** When we add parentheses, $\frac{\sqrt{6+e}+1}{3}$ becomes $\frac{(\sqrt{(6+e)}+1)}{3}$, which, in typewriter notation, becomes $(\sqrt{(6+e)}+1) \div 3$. This equals 1.317... and so is rounded to two decimal places as 1.32.

S-8. **Parentheses and grouping:** When we add parentheses, $\frac{\pi-e}{\pi+e}$ becomes $\frac{(\pi-e)}{(\pi+e)}$. which, in typewriter notation, becomes $(\pi-e) \div (\pi+e)$. This equals 0.072... and so is rounded to two decimal places as 0.07.

S-9. **Subtraction versus sign:** Noting which are negative signs and which are subtraction signs, we see that $\frac{-3}{4-9}$ means $\frac{\textit{negative } 3}{4 \textit{ subtract } 9}$. Adding parentheses and putting it into typewriter notation yields $\textit{negative } 3 \div (4 \textit{ subtract } 9)$, which equals 0.6.

S-10. **Subtraction versus sign:** Noting which are negative signs and which are subtraction signs, we see that -2^{-3} means $\textit{negative } 2 \textit{ subtract } 4 \textit{ negative } 3$. In typewriter notation this is $\textit{negative } 2 \textit{ subtract } 4 \wedge \textit{negative } 3$, which equals $-2.015...$ and so is rounded to two decimal places as -2.02 .

S-11. **Subtraction versus sign:** Noting which are negative signs and which are subtraction signs, we see that $-\sqrt{8.6-3.9}$ means $\textit{negative } \sqrt{8.6 \textit{ subtract } 3.9}$. In typewriter notation this is

$$\textit{negative } \sqrt{(8.6 \textit{ subtract } 3.9)},$$

which equals $-2.167...$ and so is rounded to two decimal places as -2.17 .

S-12. **Subtraction versus sign:** Noting which are negative signs and which are subtraction signs, we see that $\frac{-\sqrt{10}+5^{-0.3}}{17-6.6}$ means $\frac{\textit{negative } \sqrt{10}+5 \textit{ negative } 0.3}{17 \textit{ subtract } 6.6}$. In typewriter notation this is $(\textit{negative } \sqrt{(10)}+5 \wedge (\textit{negative } 0.3)) \div (17 \textit{ subtract } 6.6)$, which equals $-0.244...$ and so is rounded to two decimal places as -0.24 .

S-13. **Chain calculations:**

(a) To do this as a chain calculation, we first calculate $\frac{3}{7.2+5.9}$ and then complete the calculation by adding the second fraction to this first answer. In typewriter notation $\frac{3}{7.2+5.9}$ is $3 \div (7.2+5.9)$, which is calculated as 0.2290076336; this is used as *Ans* in the next part of the calculation. Turning to the full expression, we calculate it as $\textit{Ans} + \frac{7}{6.4 \times 2.8}$ which is, in typewriter notation, $\textit{Ans} + 7 \div (6.4 \times 2.8)$. This is 0.619..., which rounds to 0.62.

- (b) To do this as a chain calculation, we first calculate the exponent, $1 - \frac{1}{36}$, and then the full expression becomes

$$\left(1 + \frac{1}{36}\right)^{Ans}.$$

In typewriter notation, the first calculation is $1 - 1 \div 36$, and the second is $(1 + 1 \div 36) \wedge Ans$. This equals 1.026... and so is rounded to two decimal places as 1.03.

- S-14. **Evaluate expression:** In typewriter notation, $e^{-3} - \pi^2$ is $e \wedge (\textit{negative } 3) - \pi \wedge 2$, which equals $-9.819\dots$ and so is rounded to two decimal places as -9.82 .
- S-15. **Evaluate expression:** In typewriter notation, $\frac{5.2}{7.3 + 0.2 \wedge 4.5}$ is $5.2 \div (7.3 + 0.2 \wedge 4.5)$, which equals 0.712... and so is rounded to two decimal places as 0.71.
- S-16. **Arithmetic:** Writing in typewriter notation, we have $(4.3 + 8.6)(8.4 - 3.5) = 63.21$, rounded to two decimal places.
- S-17. **Arithmetic:** Writing in typewriter notation, we have $(2 \wedge 3.2 - 1) \div (\sqrt{(3)} + 4) = 1.43$, rounded to two decimal places.
- S-18. **Arithmetic:** Writing in typewriter notation, we have $\sqrt{(2 \wedge \textit{negative } 3 + e)} = 1.69$, rounded to two decimal places, where *negative* means to use a minus sign.
- S-19. **Arithmetic:** Writing in typewriter notation, we have $(2 \wedge \textit{negative } 3 + \sqrt{(7)} + \pi)(e \wedge 2 + 7.6 \div 6.7) = 50.39$, rounded to two decimal places, where *negative* means to use a minus sign.
- S-20. **Arithmetic:** Writing in typewriter notation, we have $(17 \times 3.6) \div (13 + 12 \div 3.2) = 3.65$, rounded to two decimal places.
- S-21. **Evaluating formulas:** To evaluate the formula $\frac{A - B}{A + B}$ we plug in the values for A and B to yield $\frac{4.7 - 2.3}{4.7 + 2.3} = (4.7 - 2.3) \div (4.7 + 2.3) = 0.34$, rounded to two decimal places.
- S-22. **Evaluating formulas:** To evaluate the formula $\frac{p(1 + r)}{\sqrt{r}}$ we plug in the values for p and r to yield $\frac{144(1 + 0.13)}{\sqrt{0.13}} = (144(1 + 0.13)) \div (\sqrt{(0.13)}) = 451.30$, rounded to two decimal places.
- S-23. **Evaluating formulas:** To evaluate the formula $\sqrt{x^2 + y^2}$ we plug in the values for x and y to yield $\sqrt{1.7^2 + 3.2^2} = \sqrt{(1.7 \wedge 2 + 3.2 \wedge 2)} = 3.62$, rounded to two decimal places.
- S-24. **Evaluating formulas:** To evaluate the formula $p^{1+1/q}$ we plug in the values for p and q to yield $4^{1+1/0.3} = 4 \wedge (1 + 1 \div 0.3) = 406.37$, rounded to two decimal places.

S-25. **Evaluating formulas:** To evaluate the formula $(1 - \sqrt{A})(1 + \sqrt{B})$ we plug in the values for A and B to yield $(1 - \sqrt{3})(1 + \sqrt{5}) = (1 - \sqrt{(3)})(1 + \sqrt{(5)}) = -2.37$, rounded to two decimal places.

S-26. **Evaluating formulas:** To evaluate the formula $\left(1 + \frac{1}{x}\right)^2$ we plug in the value for x to yield $\left(1 + \frac{1}{20}\right)^2 = (1 + 1 \div 20) \wedge 2 = 1.10$, rounded to two decimal places.

S-27. **Evaluating formulas:** To evaluate the formula $\sqrt{b^2 - 4ac}$ we plug in the values for b , a , and c to yield $\sqrt{7^2 - 4 \times 2 \times 0.07} = \sqrt{(7 \wedge 2 - 4 \times 2 \times 0.07)} = 6.96$, rounded to two decimal places.

S-28. **Evaluating formulas:** To evaluate the formula $\frac{1}{1 + \frac{1}{x}}$ we plug in the value for x to yield $\frac{1}{1 + \frac{1}{0.7}} = 1 \div (1 + 1 \div 0.7) = 0.41$, rounded to two decimal places.

S-29. **Evaluating formulas:** To evaluate the formula $(x + y)^{-x}$ we plug in the values for x and y to yield $(3 + 4)^{-3} = (3 + 4) \wedge (\text{negative } 3) = 0.0029$, rounded to four decimal places.

S-30. **Evaluating formulas:** To evaluate the formula $\frac{A}{\sqrt{A} + \sqrt{B}}$ we plug in the values for A and B to yield $\frac{5}{\sqrt{5} + \sqrt{6}} = 5 \div (\sqrt{(5)} + \sqrt{(6)}) = 1.07$, rounded to two decimal places.

S-31. **Lending money:** The interest due is $I = Prt$ where $P = 5000$, $r = 0.05$, which is 5% as a decimal, and $t = 3$, so $I = 5000 \times 0.05 \times 3 = 750$ dollars.

S-32. **Monthly payment:** The monthly payment is given by the formula

$$M = \frac{Pr(1+r)^t}{(1+r)^t - 1},$$

where $P = 12,000$, $r = 0.05$, and $t = 36$, so plugging in these values yields

$$M = \frac{12,000 \times 0.05 \times (1 + 0.05)^{36}}{(1 + 0.05)^{36} - 1},$$

which in typewriter notation is $M = (12000 \times 0.05 \times ((1 + 0.05) \wedge 36)) \div ((1 + 0.05) \wedge 36 - 1) = 725.21$ dollars.

S-33. **Temperature:** Since $F = \frac{9}{5}C + 32$, then when $C = 32$, $F = \frac{9}{5}32 + 32 = (9 \div 5) \times 32 + 32 = 89.60$ degrees Fahrenheit.

S-34. **A skydiver:** The velocity is given by $v = 176(1 - 0.834^t)$ and $t = 5$, so the velocity after 5 seconds is $v = 176(1 - 0.834 \wedge 5) = 104.99$ feet per second.

S-35. **Future value:** The future value is given by $F = P(1 + r)^t$, where $P = 1000$, $r = 0.06$, and $t = 5$, so plugging these in gives a future value of $F = 1000(1 + 0.06)^5 = 1000(1 + 0.06) \wedge 5 = 1338.23$ dollars.

S-36. **A population of deer:** The number of deer is given by

$$N = \frac{12.36}{0.03 + 0.55^t},$$

so when $t = 10$, the number of deer is

$$N = \frac{12.36}{0.03 + 0.55^{10}} = 12.36 \div (0.03 + 0.55 \wedge 10) = 379.922490\dots,$$

which should be reported as about 380 deer (and not to two decimal places, since the number of deer must be a whole number).

S-37. **Carbon 14:** The amount of carbon 14 is given by $C = 5 \times 0.5^{t/5730}$, so for $t = 5000$, the amount of carbon 14 remaining is $C = 5 \times 0.5^{5000/5730} = 5 \times 0.5 \wedge (5000 \div 5730) = 2.73$ grams.

S-38. **Getting three sixes:** The probability of rolling exactly 3 sixes is

$$p = \frac{n(n-1)(n-2)}{750} \left(\frac{5}{6}\right)^n,$$

so when $n = 7$, the probability is

$$p = \frac{7(7-1)(7-2)}{750} \left(\frac{5}{6}\right)^7 = ((7 \times (7-1) \times (7-2)) \div 750) \times (5 \div 6) \wedge 7 = 0.08.$$

Prologue Review Exercises

- 1. Parentheses and grouping:** In typewriter notation, $\frac{5.7 + 8.3}{5.2 - 9.4}$ is $(5.7 + 8.3) \div (5.2 - 9.4)$, which equals $-3.333\dots$ and so is rounded to two decimal places as -3.33 .
- 2. Evaluate expression:** In typewriter notation, $\frac{8.4}{3.5 + e^{-6.2}}$ is $8.4 \div (3.5 + e \wedge (\textit{negative } 6.2))$, which equals $2.398\dots$ and so is rounded to two decimal places as 2.40 .
- 3. Evaluate expression:** In typewriter notation, $\left(7 + \frac{1}{e}\right)^{\left(\frac{5}{2+\pi}\right)}$ is $(7 + 1 \div e) \wedge (5 \div (2 + \pi))$, which equals $6.973\dots$ and so is rounded to two decimal places as 6.97 . This can also be done as a chain calculation.
- 4. Gas mileage:** The number of gallons required to travel 27 miles is

$$g = \frac{27}{15} = 1.8 \text{ gallons.}$$

The number of gallons required to travel 250 miles is

$$g = \frac{250}{15} = 16.67 \text{ gallons.}$$

5. **Kepler's third law:** The mean distance from Pluto to the sun is

$$D = 93 \times 249^{2/3} = 3680.86 \text{ million miles,}$$

or about 3681 million miles. For Earth we have $P = 1$ year, and the mean distance is

$$D = 93 \times 1^{2/3} = 93 \text{ million miles.}$$

6. **Traffic signal:** If the approach speed is 80 feet per second then the length of the yellow light should be

$$n = 1 + \frac{80}{30} + \frac{100}{80} = 4.92 \text{ seconds.}$$

Solution Guide for Chapter 1: Functions

1.1 FUNCTIONS GIVEN BY FORMULAS

1. Movie tickets:

- (a) Because 2009 is 9 years after the year 2000, the expression $C(9)$ is the average cost, in dollars, of a movie ticket in 2009.
- (b) Because 2012 is 12 years after the year 2000, the average cost of a movie ticket in 2012 expressed in functional notation is $C(12)$.
- (c) The average cost of a movie ticket in 2012 is

$$C(12) = 5.40 + 0.22 \times 12 = 8.04 \text{ dollars.}$$

Therefore, the average cost of a movie ticket in 2012 is \$8.04.

2. McDonald's:

- (a) Because 2005 is 5 years after the year 2000, the expression $M(5)$ is the total revenue for McDonald's, in billions of dollars, in 2005.
- (b) Because 2010 is 10 years after the year 2000, the total revenue, in billions of dollars, in 2010 expressed in functional notation is $M(10)$.
- (c) The total revenue in 2010 is

$$M(10) = 1.19 \times 10 + 13.22 = 25.12 \text{ billion dollars.}$$

Therefore, the total revenue in 2010 is \$25.12 billion.

3. Speed from skid marks:

- (a) In functional notation the speed for a 60-foot-long skid mark is $S(60)$. The value is

$$S(60) = 5.05\sqrt{60} = 39.12 \text{ miles per hour.}$$

Therefore, the speed at which the skid mark will be 60 feet long is 39.12 miles per hour.

(b) The expression $S(100)$ represents the speed, in miles per hour, at which an emergency stop will on dry pavement leave a skid mark that is 100 feet long.

4. **Harris-Benedict formula:** We give an example. Assume that a male weighs 180 pounds, is 70 inches tall, and is 40 years old. In functional notation his basal metabolic rate is $M(180, 70, 40)$. The value is

$$M(180, 70, 40) = 66 + 6.3 \times 180 + 12.7 \times 70 - 6.8 \times 40 = 1817 \text{ calories.}$$

Therefore, the basal metabolic rate for this man is 1817 calories.

5. **Adult weight from puppy weight:**

(a) In functional notation the adult weight of a puppy that weighs 6 pounds at 14 weeks is $W(14, 6)$.

(b) The predicted adult weight of a puppy that weighs 6 pounds at 14 weeks is

$$W(14, 6) = 52 \times \frac{6}{14} = 22.29 \text{ pounds.}$$

Therefore, the predicted adult weight for this puppy is 22.29 pounds.

6. **Gross profit margin:**

(a) In functional notation the gross profit margin for a company that has a gross profit of \$335,000 and a total revenue of \$540,000 is $M(335,000, 540,000)$.

(b) The gross profit margin for a company that has a gross profit of \$335,000 and a total revenue of \$540,000 is

$$M(335,000, 540,000) = \frac{335,000}{540,000} = 0.62.$$

Therefore, the gross profit margin for this company is 0.62 or 62%.

(c) If the gross profit stays the same but total revenue increases, in the fraction $M = \frac{G}{T}$ the numerator G stays the same and the denominator T increases. In this situation the fraction decreases (we are dividing by a larger number), so the gross profit margin decreases. There are other ways to see the same result: pick different numbers to plug in, or think about the meaning of the gross profit margin and how it would change for fixed gross profit and increasing total revenue.

7. **Tax owed:**

(a) In functional notation the tax owed on a taxable income of \$13,000 is $T(13,000)$. The value is

$$T(13,000) = 0.11 \times 13,000 - 500 = 930 \text{ dollars.}$$

- (b) The tax owed on a taxable income of \$14,000 is

$$T(14,000) = 0.11 \times 14,000 - 500 = 1040 \text{ dollars.}$$

Using the answer to Part (a), we see that the tax increases by $1040 - 930 = 110$ dollars.

- (c) The tax owed on a taxable income of \$15,000 is

$$T(15,000) = 0.11 \times 15,000 - 500 = 1150 \text{ dollars.}$$

Thus the tax increases by $1150 - 1040 = 110$ dollars again.

8. Pole vault:

- (a) The value of $H(4)$ is

$$H(4) = 0.05 \times 4 + 3.3 = 3.5 \text{ meters.}$$

This means that, according to this model, the height of the winning pole vault in 1904 was 3.5 meters.

- (b) According to this model, the height of the winning pole vault in 1900 was

$$H(0) = 0.05 \times 0 + 3.3 = 3.3 \text{ meters.}$$

Using the answer to Part (a), we see that from 1900 to 1904 the winning height increased by $3.5 - 3.3 = 0.2$ meter.

According to this model, the height of the winning pole vault in 1908 was

$$H(8) = 0.05 \times 8 + 3.3 = 3.7 \text{ meters.}$$

Thus from 1904 to 1908 the winning height increased by $3.7 - 3.5 = 0.2$ meter again.

9. Flying ball:

- (a) In functional notation the velocity 1 second after the ball is thrown is $V(1)$. The value is

$$V(1) = 40 - 32 \times 1 = 8 \text{ feet per second.}$$

Because the upward velocity is positive, the ball is rising.

- (b) The velocity 2 seconds after the ball is thrown is

$$V(2) = 40 - 32 \times 2 = -24 \text{ feet per second.}$$

Because the upward velocity is negative, the ball is falling.

- (c) The velocity 1.25 seconds after the ball is thrown is

$$V(1.25) = 40 - 32 \times 1.25 = 0 \text{ feet per second.}$$

Because the velocity is 0, we surmise from Parts (a) and (b) that the ball is at the peak of its flight.

- (d) Using the answers to Parts (a) and (b), we see that from 1 second to 2 seconds the velocity changes by

$$V(2) - V(1) = -24 - 8 = -32 \text{ feet per second.}$$

Because

$$V(3) = 40 - 32 \times 3 = -56 \text{ feet per second,}$$

from 2 seconds to 3 seconds the velocity changes by

$$V(3) - V(2) = -56 - (-24) = -32 \text{ feet per second.}$$

Because

$$V(4) = 40 - 32 \times 4 = -88 \text{ feet per second,}$$

from 3 seconds to 4 seconds the velocity changes by

$$V(4) - V(3) = -88 - (-56) = -32 \text{ feet per second.}$$

Over each of these 1-second intervals the velocity changes by -32 feet per second. In practical terms, this means that the velocity decreases by 32 feet per second for each second that passes. This indicates that the *downward* acceleration of the ball is constant at 32 feet per second per second, which makes sense because the acceleration due to gravity is constant near the surface of Earth.

10. Flushing chlorine:

- (a) The initial concentration in the tank is found at time $t = 0$, and so by calculating $C(0)$:

$$C(0) = 0.1 + 2.78e^{-0.37 \times 0} = 2.88 \text{ milligrams per gallon.}$$

- (b) The concentration of chlorine in the tank after 3 hours is represented by $C(3)$ in functional notation. Its value is

$$C(3) = 0.1 + 2.78e^{-0.37 \times 3} = 1.02 \text{ milligrams per gallon.}$$

11. A population of deer:

- (a) Now
- $N(0)$
- represents the number of deer initially on the reserve and

$$N(0) = \frac{12.36}{0.03 + 0.55^0} = 12 \text{ deer.}$$

So there were 12 deer in the initial herd.

- (b) We calculate using

$$N(10) = \frac{12.36}{0.03 + 0.55^{10}} = 379.92 \text{ deer.}$$

This says that after 10 years there should be about 380 deer in the reserve.

- (c) The number of deer in the herd after 15 years is represented by
- $N(15)$
- , and this value is

$$N(15) = \frac{0.36}{0.03 + 0.55^{15}} = 410.26 \text{ deer.}$$

This says that there should be about 410 deer in the reserve after 15 years.

- (d) The difference in the deer population from the tenth to the fifteenth year is given by
- $N(15) - N(10) = 410.26 - 379.92 = 30.34$
- . Thus the population increased by about 30 deer from the tenth to the fifteenth year.

12. A car that gets 32 miles per gallon:

- (a) The price,
- g
- , is in dollars per gallon. Also, 98 cents per gallon is the same as 0.98 dollar per gallon, so
- $g = 0.98$
- . Since the distance is
- $d = 230$
- , the cost is expressed in functional notation as
- $C(0.98, 230)$
- . This is calculated as

$$C(0.98, 230) = \frac{0.98 \times 230}{32} = \$7.04.$$

- (b) We have that

$$C(3.53, 172) = \frac{3.53 \times 172}{32} = \$18.97.$$

This represents the cost of driving 172 miles if gas costs \$3.53 per gallon.

13. Radioactive substances:

- (a) The amount of carbon 14 left after 800 years is expressed in functional notation as
- $C(800)$
- . This is calculated as

$$C(800) = 5 \times 0.5^{800/5730} = 4.54 \text{ grams.}$$

- (b) There are many ways to do this part of the exercise. The simplest is to note that half the amount is left when the exponent of 0.5 is 1 since then the 5 is multiplied by
- $0.5 = \frac{1}{2}$
- . The exponent in the formula is 1 when
- $t = 5730$
- years.

Another way to do this part is to experiment with various values for t , increasing the value when the answer is less than 2.5 and decreasing it when the answer comes out more than 2.5. Students are in fact discovering and executing a crude version of the bisection method.

14. A roast:

- (a) Since the roast has been in the refrigerator for a while, we can expect its initial temperature to be the same as that of the refrigerator. That is, the temperature of the refrigerator is given by $R(0)$, and

$$R(0) = 325 - 280e^{-0.005 \times 0} = 45 \text{ degrees Fahrenheit.}$$

- (b) The temperature of the roast 30 minutes after being put in the oven is expressed in functional notation as $R(30)$. This is calculated as

$$R(30) = 325 - 280e^{-0.005 \times 30} = 84 \text{ degrees Fahrenheit.}$$

- (c) The initial temperature of the roast was calculated in Part (a) and found to be 45 degrees. After 10 minutes, its temperature is

$$R(10) = 325 - 280e^{-0.005 \times 10} = 58.66 \text{ degrees Fahrenheit.}$$

Thus the temperature increased by $58.66 - 45 = 13.66$ degrees in the first 10 minutes of cooking.

- (d) The temperature of the roast at the end of the first hour is

$$R(60) = 325 - 280e^{-0.005 \times 60} = 117.57 \text{ degrees.}$$

After an hour and ten minutes, its temperature is

$$R(70) = 325 - 280e^{-0.005 \times 70} = 127.69 \text{ degrees.}$$

The difference is only 10.12 degrees Fahrenheit.

15. What if interest is compounded more often than monthly?

- (a) We would expect our monthly payment to be higher if the interest is compounded daily since additional interest is charged on interest which has been compounded.
- (b) Continuous compounding should result in a larger monthly payment since the interest is compounded at an even faster rate than with daily compounding.

- (c) We are given that $P = 7800$ and $t = 48$. Because the APR is 8.04% or 0.0804, we compute that

$$r = \frac{\text{APR}}{12} = \frac{0.0804}{12} = 0.0067.$$

Thus the monthly payment is

$$M(7800, 0.0067, 48) = \frac{7800(e^{0.0067} - 1)}{1 - e^{-0.0067 \times 48}} = 190.67 \text{ dollars.}$$

Our monthly payment here is 10 cents higher than if interest is compounded monthly as in Example 1.2 (where the payment was \$190.57).

16. Present value:

- (a) The function tells you how much to invest now if you want to get back a future value of F after t years with an interest rate of r .

- (b) The discount rate here is

$$\frac{1}{(1 + 0.09)^{18}} = 0.21.$$

- (c) We use the discount rate calculated in Part (b):

$$100,000 \times 0.21 = \$21,000.$$

This is an example of where rounding at an intermediate step can cause difficulties. If the discount rate is not rounded, one gets a final answer of \$21,199.37.

17. How much can I borrow?

- (a) Since we will be paying \$350 per month for 4 years, then we will be making 48 payments, or $t = 48$. Also, r is the monthly interest rate of 0.75%, or 0.0075 as a decimal. The amount of money we can afford to borrow in this case is given in functional notation by $P(350, 0.0075, 48)$. It is calculated as

$$P(350, 0.0075, 48) = 350 \times \frac{1}{0.0075} \times \left(1 - \frac{1}{(1 + 0.0075)^{48}}\right) = \$14,064.67.$$

- (b) If the monthly interest rate is 0.25% then we can afford to borrow

$$P(350, 0.0025, 48) = 350 \times \frac{1}{0.0025} \times \left(1 - \frac{1}{(1 + 0.0025)^{48}}\right) = \$15,812.54.$$

- (c) If we make monthly payments over 5 years then we will make 60 payments in all. So now we can afford to borrow

$$P(350, 0.0025, 60) = 350 \times \frac{1}{0.0025} \times \left(1 - \frac{1}{(1 + 0.0025)^{60}}\right) = \$19,478.33.$$

18. **Financing a new car:** If we take 3.9% APR, then since interest is compounded monthly, we use $r = \frac{0.039}{12} = 0.00325$. We are borrowing $P = 14,000$ dollars over a 48 month period, and so our monthly payment will be

$$\frac{14000 \times 0.00325 \times (1 + 0.00325)^{48}}{(1 + 0.00325)^{48} - 1} = \$315.48.$$

If we take the rebate, we borrow at an APR of 8.85%. In this case we use $r = \frac{0.0885}{12} = 0.007375$. This time, we borrow $P = 12,000$ dollars over a 48 month period, and so our monthly payment will be

$$\frac{12000 \times 0.007375 \times (1 + 0.007375)^{48}}{(1 + 0.007375)^{48} - 1} = \$297.77.$$

Since the rebate results in a lower monthly payment, that is the best option to choose. Over the life of the loan, we would under the 3.9% APR option pay $48 \times 315.48 = \$15,143.04$. Under the rebate option, we would pay $48 \times 297.77 = \$14,292.96$. Thus, by choosing the rebate option, we save a total of $15,143.04 - 14,292.96 = 850.08$ dollars over the life of the loan.

19. **Brightness of stars:** Here we have $m_1 = -1.45$ and $m_2 = 2.04$. Thus

$$t = 2.512^{m_2 - m_1} = 2.512^{2.04 - (-1.45)} = 2.512^{3.49} = 24.89.$$

Hence Sirius appears 24.89 times brighter than Polaris.

20. **Stellar distances:**

- (a) Here $d(2.2, 0.7)$ represents the distance from Earth (in light-years) of a star with apparent magnitude 2.2 and absolute magnitude 0.7.
- (b) We are given that $m = -1.45$ and $M = 1.45$. Thus the distance from Sirius to Earth is

$$d(-1.45, 1.45) = 3.26 \times 10^{(-1.45 - 1.45 + 5)/5} = 8.57 \text{ light-years.}$$

21. **Parallax:** We are given that $p = 0.751$. Thus the distance from Alpha Centauri to the sun is about

$$d(0.751) = \frac{3.26}{0.751} = 4.34 \text{ light-years.}$$

22. **Sound pressure and decibels:**

- (a) We are given that $D = 65$. Thus the pressure exerted is

$$P(65) = 0.0002 \times 1.122^{65} = 0.36 \text{ dyne per square centimeter.}$$

(b) We are given that $D = 120$. Thus the corresponding pressure level is

$$P(120) = 0.0002 \times 1.122^{120} = 199.61 \text{ dynes per square centimeter.}$$

23. Mitscherlich's equation:

(a) We are given that $b = 1$. Thus the percentage (as a decimal) of maximum yield is

$$Y(1) = 1 - 0.5^1 = 0.5.$$

Hence 50% of maximum yield is produced if 1 baule is applied.

(b) In functional notation the percentage (as a decimal) of maximum yield produced by 3 baules is $Y(3)$. The value is

$$Y(3) = 1 - 0.5^3 = 0.875,$$

or about 0.88. This is 88% of maximum yield.

(c) Now 500 pounds of nitrogen per acre corresponds to $\frac{500}{223}$ baules, so the percentage (as a decimal) of maximum yield is $1 - 0.5^{500/223}$, or about 0.79. This is 79% of maximum yield.

24. Yield response to several growth factors:

(a) We are given that $b = 1$, $c = 2$, and $d = 3$, so in functional notation the percentage (as a decimal) of maximum yield produced is $Y(1, 2, 3)$. The value is

$$Y(1, 2, 3) = (1 - 0.5^1)(1 - 0.5^2)(1 - 0.5^3),$$

or about 0.33. This is 33% of maximum yield.

(b) Now 200 pounds of nitrogen per acre corresponds to $\frac{200}{223}$ baule, 100 pounds of phosphorus per acre corresponds to $\frac{100}{45}$ baules, and 150 pounds of potassium per acre corresponds to $\frac{150}{76}$ baules. Thus the percentage (as a decimal) of maximum yield is

$$(1 - 0.5^{200/223})(1 - 0.5^{100/45})(1 - 0.5^{150/76}),$$

or about 0.27. This is 27% of maximum yield.

25. Thermal conductivity: We are given that $k = 0.85$ for glass and that $t_1 = 24$, $t_2 = 5$.

(a) Because $d = 0.007$, the heat flow is

$$Q = \frac{0.85(24 - 5)}{0.007} = 2307.14 \text{ watts per square meter.}$$

(b) The total heat loss is

$$\text{Heat flow} \times \text{Area of window} = 2307.14 \times 2.5,$$

or about 5767.85 watts.

26. **Reynolds number:** We are given that $d = 0.05$ and $v = 0.2$.

(a) Because $\mu = 0.00059$ and $D = 867$, the Reynolds number is

$$R = \frac{0.2 \times 0.05 \times 867}{0.00059} = 14,694.92.$$

Because this is greater than 2000, the flow is turbulent.

(b) Because $\mu = 1.49$ and $D = 1216.3$, the Reynolds number is

$$R = \frac{0.2 \times 0.05 \times 1216.3}{1.49} = 8.16.$$

Because this is less than 2000, the flow is streamline.

27. **Fault rupture length:** Here we have $M = 6.5$, so the expected length is

$$L(6.5) = 0.0000017 \times 10.47^{6.5} = 7.25 \text{ kilometers.}$$

28. **Tubeworm:** Here we have $L = 2$, so the time required (in years) is

$$T(2) = 14e^{1.4 \times 2} - 20,$$

or about 210. Thus the model estimates that the age is about 210 years.

29. **Equity in a home:**

(a) The monthly interest rate as a decimal is $r = \text{APR}/12 = 0.06/12 = 0.005$.

(b) The mortgage is for $P = 400,000$, the term is 30 years, so in months $t = 30 \times 12 = 360$, $r = 0.005$, and after 20 years of payments, $k = 20 \times 12 = 240$, so in functional notation it is $E(240)$. The value of $E(240)$ is

$$400,000 \times \frac{(1 + 0.005)^{240} - 1}{(1 + 0.005)^{360} - 1},$$

which equals 183,985.66 dollars.

(c) To find the equity after y years, use $k = 12y$ months, so the formula for E in terms of y is

$$E(y) = 400,000 \times \frac{(1 + 0.005)^{12y} - 1}{(1 + 0.005)^{360} - 1}.$$

30. Adjustable rate mortgage—approximating payments:

- (a) The mortgage is $P = 325,000$, the monthly interest rate is $r = \text{APR}/12 = 0.045/12 = 0.00375$, and $t = 30 \times 12 = 360$ since the term is 30 years. Using the monthly payment formula, we have

$$M = \frac{Pr(1+r)^t}{(1+r)^t - 1} = \frac{325,000 \times 0.00375 \times (1 + 0.00375)^{360}}{(1 + 0.00375)^{360} - 1} = 1646.73 \text{ dollars.}$$

- (b) The ratio is $\frac{1646.73}{6000} = 0.27$, so the mortgage payment is 27% of your income.

- (c) Now the mortgage is still $P = 325,000$, the monthly interest rate is $r = \text{APR}/12 = 0.07/12 = 0.00583$, and $t = 28 \times 12 = 336$ since the term is 28 years. Using the monthly payment formula, we have

$$M = \frac{Pr(1+r)^t}{(1+r)^t - 1} = \frac{325,000 \times 0.00583 \times (1 + 0.00583)^{336}}{(1 + 0.00583)^{336} - 1} = 2207.87 \text{ dollars,}$$

or about \$2208 each month.

- (d) Now the percentage is $\frac{2207.87}{6000} = 0.37$, so the mortgage payment is 37% of your income.

31. Adjustable rate mortgage—exact payments:

- (a) To find the equity after 24 months, we use $k = 24$. Here $P = 325,000$, $r = \text{APR}/12 = 0.045/12 = 0.00375$, and $t = 30 \times 12 = 360$, so the equity accrued after 24 months is

$$E = 325,000 \times \frac{(1 + 0.00375)^{24} - 1}{(1 + 0.00375)^{360} - 1} = 10,726.84 \text{ dollars.}$$

- (b) The new mortgage amount is $P = 325,000 - 10,726.84 = 314,273.16$, the new interest rate is $r = \text{APR}/12 = 0.07/12 = 0.00583$, the new term is $t = 28 \times 12 = 336$, and so the new monthly payment is

$$M = \frac{Pr(1+r)^t}{(1+r)^t - 1} = \frac{314,273.16 \times 0.00583 \times (1 + 0.00583)^{336}}{(1 + 0.00583)^{336} - 1} = 2135.00 \text{ dollars.}$$

32. Research project: Answers will vary.**Skill Building Exercises**

S-1. **Evaluating formulas:** To evaluate $f(x) = \frac{\sqrt{x+1}}{x^2+1}$ at $x = 2$, simply substitute 2 for x .

Thus the value of f at 2 is $\frac{\sqrt{2+1}}{2^2+1}$, which equals 0.346... and so is rounded to 0.35.

- S-2. **Evaluating formulas:** To evaluate $f(x) = (3 + x^{1.2})^{x+3.8}$ at $x = 4.3$, simply substitute 4.3 for x . Thus the value of f at 4.3 is $(3 + 4.3^{1.2})^{4.3+3.8}$, which equals 42943441.08, which is 42,943,441.08.
- S-3. **Evaluating formulas:** To evaluate $g(x, y) = \frac{x^3 + y^3}{x^2 + y^2}$ at $x = 4.1$, $y = 2.6$, simply substitute 4.1 for x and 2.6 for y . Thus the value of g when $x = 4.1$ and $y = 2.6$ is $\frac{4.1^3 + 2.6^3}{4.1^2 + 2.6^2}$, which equals 3.669... and so is rounded to 3.67.
- S-4. **Evaluating formulas:** To get the function value $f(1.3)$, substitute 1.3 for t in the formula $f(t) = 87.2 - e^{4t}$. Thus $f(1.3) = 87.2 - e^{4 \times 1.3}$, which equals $-94.172...$ and so is rounded to -94.17 .
- S-5. **Evaluating formulas:** To get the function value $f(6.1)$, substitute 6.1 for s in the formula $f(s) = \frac{s^2 + 1}{s^2 - 1}$. Thus $f(6.1) = \frac{6.1^2 + 1}{6.1^2 - 1}$, which equals 1.055... and so is rounded to 1.06.
- S-6. **Evaluating formulas:** To get the function value $f(2, 5, 7)$, substitute 2 for r , 5 for s , and 7 for t in the formula $f(r, s, t) = \sqrt{r + \sqrt{s + \sqrt{t}}}$. Thus $f(2, 5, 7) = \sqrt{2 + \sqrt{5 + \sqrt{7}}}$, which equals 2.182... and so is rounded to 2.18.
- S-7. **Evaluating formulas:** To get the function value $h(3, 2.2, 9.7)$, substitute 3 for x , 2.2 for y , and 9.7 for z in the formula $h(x, y, z) = \frac{x^y}{z}$. Thus $h(3, 2.2, 9.7) = \frac{3^{2.2}}{9.7}$, which equals 1.155... and so is rounded to 1.16.
- S-8. **Evaluating formulas:** To get the function value $H(3, 4, 0.7)$, substitute 3 for p , 4 for q , and 0.7 for r in the formula $H(p, q, r) = \frac{2 + 2^{-p}}{q + r^2}$. Thus $H(3, 4, 0.7) = \frac{2 + 2^{-3}}{4 + 0.7^2}$, which equals 0.473... and so is rounded to 0.47.
- S-9. **Evaluating formulas:** To evaluate $f(x, y) = \frac{1.2^x + 1.3^y}{\sqrt{x + y}}$ at $x = 3$ and $y = 4$, simply substitute 3 for x and 4 for y . Thus the value of f when $x = 3$ and $y = 4$ is $\frac{1.2^3 + 1.3^4}{\sqrt{3 + 4}}$, which equals 1.732... and so is rounded to 1.73.
- S-10. **Evaluating formulas:** To get the function value $g(2, 3, 4)$, substitute 2 for s , 3 for t , and 4 for u in the formula $g(s, t, u) = (1 + s/t)^u$. Thus $g(2, 3, 4) = (1 + 2/3)^4$, which equals 7.716... and so is rounded to 7.72.
- S-11. **Evaluating formulas:** To get the function value $W(2.2, 3.3, 4.4)$, substitute 2.2 for a , 3.3 for b , and 4.4 for c in the formula $W(a, b, c) = \frac{a^b - b^a}{c^a - a^c}$. Thus $H(2.2, 3.3, 4.4) = \frac{2.2^{3.3} - 3.3^{2.2}}{4.4^{2.2} - 2.2^{4.4}}$, which equals 0.0555... and so is rounded to 0.06.
- S-12. **Evaluating formulas:** We simply substitute 3 for x : $f(3) = 3 \times 3 + \frac{1}{3}$, which equals 9.333... and so is rounded to 9.33.

- S-13. **Evaluating formulas:** We simply substitute 3 for x : $f(3) = 3^{-3} - \frac{3^2}{3+1}$, which equals $-2.212\dots$ and so is rounded to -2.21 .
- S-14. **Evaluating formulas:** We simply substitute 3 for x : $f(3) = \sqrt{2 \times 3 + 5}$, which equals $3.316\dots$ and so is rounded to 3.32 .
- S-15. **Evaluating formulas:** We simply substitute 0 for t to obtain $C(0) = 0.1 + 2.78e^{-0.37 \times 0}$, which equals 2.88; we substitute 10 for t to obtain $C(10) = 0.1 + 2.78e^{-0.37 \times 10}$, which equals 0.17.
- S-16. **Evaluating formulas:** We simply substitute 0 for t to obtain $N(0) = \frac{12.36}{0.03 + 0.55^0}$, which equals 12; we substitute 10 for t to obtain $N(10) = \frac{12.36}{0.03 + 0.55^{10}}$, which equals 379.92.
- S-17. **Evaluating formulas:** To evaluate $C(g, d) = \frac{gd}{32}$ at $g = 52.3$ and $d = 13.5$, simply substitute 52.3 for g and 13.5 for d . Thus the value of $C(52.3, 13.5)$ is $\frac{52.3 \times 13.5}{32}$, which equals $22.064\dots$ and so is rounded to 22.06.
- S-18. **Evaluating formulas:** We simply substitute 0 for t to obtain $C(0) = 5 \times 0.5^{0/5730}$, which equals 5; we substitute 3000 for t to obtain $C(3000) = 5 \times 0.5^{3000/5730}$, which equals 3.48.
- S-19. **Evaluating formulas:** We simply substitute 0 for t to obtain $R(0) = 325 - 280e^{-0.005 \times 0}$, which equals 45; we substitute 30 for t to obtain $R(30) = 325 - 280e^{-0.005 \times 30}$, which equals 84.00.
- S-20. **Evaluating formulas:** To get the function value $M(5000, 0.04, 36)$, substitute 5000 for P , 0.04 for r , and 36 for t in the formula $M(P, r, t) = \frac{P(e^r - 1)}{1 - e^{-rt}}$. Thus $M(5000, 0.04, 36) = \frac{5000(e^{0.04} - 1)}{1 - e^{-0.04 \times 36}}$, which equals 267.4109... and so is rounded to 267.41.
- S-21. **Evaluating formulas:** To get the function value $P(500, 0.06, 30)$, substitute 500 for F , 0.06 for r , and 30 for t in the formula $P(F, r, t) = \frac{F}{(1+r)^t}$. Thus $P(500, 0.06, 30) = \frac{500}{(1+0.06)^{30}}$, which equals 87.055... and so is rounded to 87.06.
- S-22. **Evaluating formulas:** To get the function value $P(300, 0.005, 40)$, substitute 300 for M , 0.005 for r , and 40 for t in the formula $P(M, r, t) = \frac{M}{r} \left(1 - \frac{1}{(1+r)^t}\right)$. Thus $P(300, 0.005, 40) = \frac{300}{0.005} \left(1 - \frac{1}{(1+0.005)^{40}}\right)$, which equals 10,851.668... and so is rounded to 10,851.67.
- S-23. **Evaluating formulas:** To get the function value $d(3.1, 0.5)$, substitute 3.1 for m and 0.5 for M , in the formula $d(m, M) = 3.26 \times 10^{(m-M+5)/5}$. Thus $d(3.1, 0.5) = 3.26 \times 10^{(3.1-0.5+5)/5}$, which equals 107.948... and so is rounded to 107.95.

- S-24. **Evaluating formulas:** To get the function value $A(5.1, 3.2)$, substitute 5.1 for u and 3.2 for v , in the formula $A(u, v) = \frac{\sqrt{u} + \sqrt{v}}{\sqrt{u} - \sqrt{v}}$. Thus $A(5.1, 3.2) = \frac{\sqrt{5.1} + \sqrt{3.2}}{\sqrt{5.1} - \sqrt{3.2}}$, which equals 8.6208... and so is rounded to 8.62.
- S-25. **Profit:** To express the expected profit, note that the variable t is measured in years, so 2 years and 6 months corresponds to $t = 2.5$, and so the expected profit is expressed by $p(2.5)$.
- S-26. **Grocery bill:** To express the cost of buying 2 bags of potato chips, 3 sodas, and 5 hot dogs, note that these values correspond to $p = 2$, $s = 3$, and $h = 5$, and so the cost is expressed by $c(2, 3, 5)$.
- S-27. **Speed of fish:** Since $s(L)$ is the top speed of a fish L inches long, $s(13)$ is the top speed of a fish 13 inches long.
- S-28. **Time to pay off a loan:** To indicate the time required to pay off the loan, we use $P = 12,000$, $r = 0.05$ (which is the APR of 5% as a decimal), and $m = 450$, so the time required to pay off the loan is $T(12,000, 0.05, 450)$ in functional notation.
- S-29. **Doubling time:** In practical terms, $D(5000, 0.06)$ is the time required for an investment of $P = 5000$ dollars at an APR of $r = 0.06$, so 6%, to double in value.
- S-30. **Monthly payment:** If you borrow $P = 23,000$ dollars at an APR of 5%, so $r = 0.05$, to be paid off in 4 years, so $t = 4 \times 12 = 48$ months, then $M(23,000, 0.05, 48)$ expresses in functional notation the monthly payment for such a loan.
- S-31. **A bird population:** In practical terms $N(7)$ is the number of birds in the population 7 years after observation began.
- S-32. **Inflation:** Because February is the second month of the year, in practical terms $I(2005, 2)$ is the inflation rate in February of 2005.
- S-33. **Dow Jones Industrial Average:** Because 2 p.m. is 120 minutes after noon, in practical terms $D(120)$ is the Dow Jones Industrial Average at 2 p.m. today.

1.2 FUNCTIONS GIVEN BY TABLES

1. Minimum wage:

- (a) According to the table, $m(1990)$ is \$6.66.
- (b) In functional notation the minimum wage in 1985 is $m(1985)$.
- (c) Because 1985 is halfway between 1980 and 1990, we can estimate $m(1985)$ by averaging:

$$\frac{m(1980) + m(1990)}{2} = \frac{8.46 + 6.66}{2} = 7.56.$$

Hence we estimate that the minimum wage in 1985 was about \$7.56.

2. Mortgage rates:

- (a) In practical terms $r(2003)$ is the rate for 30-year fixed-rate mortgages in 2003.
- (b) First we find the average rate of change per year in r from 2000 to 2005:

$$\frac{r(2005) - r(2000)}{2005 - 2000} = \frac{5.87 - 8.05}{5} = -0.436 \text{ percentage point per year.}$$

Thus we estimate $r(2003)$ as

$$r(2000) + 3 \times \text{Yearly change} = 8.05 + 3 \times (-0.436) = 6.742 \text{ percent.}$$

Hence we estimate that $r(2003)$ is about 6.74%.

3. Box office hits:

- (a) According to the table, $M(2014)$ is *American Sniper*, and $B(2014)$ is 350.13 million dollars.
- (b) In functional notation the amount for the movie with the highest gross in 2013 is $B(2013)$.

4. Mobile phone sales: The average rate of change per year in $M(t)$ from 2000 to 2005 is

$$\frac{M(2005) - M(2000)}{2005 - 2000} = \frac{778.75 - 414.99}{5} = 72.75 \text{ million dollars per year.}$$

5. Choosing a bat:

- (a) Here $B(55)$ is the recommended bat length for a man weighing between 161 and 170 pounds if his height is 55 inches. According to the table, that length is 31 inches.

(b) In functional notation the recommended bat length for a man weighing between 161 and 170 pounds if his height is 63 inches is $B(63)$.

6. **Freight on Class I railroads:** The average rate of change per year in F from 2005 to 2007 is

$$\frac{F(2007) - F(2005)}{2007 - 2005} = \frac{52.9 - 44.5}{2} = 4.2 \text{ billion dollars per year.}$$

This result means that, on average over this two-year period, freight revenue for Class I railroads increased by 4.2 billion dollars each year.

7. **The American food dollar:**

(a) Here $P(1989) = 30\%$. This means that in 1989 Americans spent 30% of their food dollars eating out.

(b) The expression $P(1999)$ is the percent of the American food dollar spent eating away from home in 1999. Since 1999 falls halfway between 1989 and 2009, our estimate for $P(1999)$ is the average of $P(1989)$ and $P(2009)$, or

$$\frac{P(1989) + P(2009)}{2} = \frac{30 + 34}{2} = 32.$$

Approximately 32% of the American food dollar in 1999 was spent eating out.

(c) The average rate of change per year in percentage of the food dollar spent away from home from 1989 to 2009 is

$$\frac{P(2009) - P(1989)}{2009 - 1989} = \frac{34 - 30}{20} = 0.2,$$

or 0.2 percentage point per year.

(d) The expression $P(2004)$ is the percent of the American food dollar spent eating away from home in 2004. We estimate it as

$$P(2004) = P(1989) + 15 \times \text{Yearly change} = 30 + 15 \times 0.2 = 33,$$

or 33%.

(e) Assuming the increase in P continues at the same rate of about 0.2 percentage point per year as we calculated in Part (c), then we estimate

$$P(2014) = P(2009) + 5 \times \text{Yearly change} = 34 + 5 \times 0.2 = 35,$$

or 35%.

8. Gross domestic product:

(a) Here $G(2004)$ is the U.S. gross domestic product (in trillions of dollars) in the year 2004. According to the table, its value is 11.87 trillion dollars.

(b) In functional notation the gross domestic product in the year 2007 is $G(2007)$. Because 2007 is halfway between 2004 and 2010, we can estimate the value by averaging:

$$\frac{G(2004) + G(2010)}{2} = \frac{11.87 + 14.66}{2} = 13.265.$$

Hence we estimate that the gross domestic product in 2007 was about 13.27 trillion dollars.

(c) The average yearly rate of change is

$$\frac{G(2014) - G(2010)}{2014 - 2010} = \frac{17.42 - 14.66}{4} = 0.69$$

or about 0.69 trillion dollars per year.

(d) We predict $G(2022)$ using the average rate of change from Part (c):

$$G(2014) + 8 \times 0.69 = 17.42 + 8 \times 0.69 = 22.94.$$

Hence we predict the gross domestic product in 2022 to be about 22.94 trillion dollars.

9. Internet access:

(a) Here $I(2000) = 113$ million. This means that 113 million adult Americans had internet access in 2000.

(b) The average rate of change per year from 2003 to 2009 is

$$\frac{\text{Change in } I}{\text{Time elapsed}} = \frac{I(2009) - I(2003)}{6} = \frac{196 - 166}{6} = 5 \text{ million per year.}$$

(c) Since the average rate of change per year from 2003 to 2009 is 5 million per year, we estimate

$$I(2007) = I(2003) + 4 \times \text{Yearly change} = 166 + 4 \times 5 = 186 \text{ million adult Americans.}$$

10. A cold front:

(a) Since h is hours since 4 p.m., the time 5:30 p.m. is represented by $h = 1.5$. The temperature in Stillwater at that time is given in functional notation by $T(1.5)$.

To estimate the value of $T(1.5)$, we average $T(1)$ and $T(2)$:

$$\frac{T(1) + T(2)}{2} = \frac{59 + 38}{2} = 48.5 \text{ degrees.}$$

- (b) The average rate of change per minute in temperature between 5 p.m. and 6 p.m. is

$$\frac{\text{Change in temperature}}{\text{Number of minutes}} = \frac{38 - 59}{60} = \frac{-21}{60} = -0.35 \text{ degree per minute.}$$

The average decrease per minute in temperature between 5 p.m. and 6 p.m. is

$$\frac{\text{Decrease in temperature}}{\text{Number of minutes}} = \frac{59 - 38}{60} = \frac{21}{60} = 0.35 \text{ degree per minute.}$$

Note that this is the magnitude of the average rate of change.

- (c) Now 5:12 p.m. is 12 minutes after 5 p.m. Since 5 p.m. corresponds to $h = 1$, we estimate the temperature at 5:12 as

$$T(1) + 12 \times \text{temperature change per minute} = 59 + 12 \times -0.35 = 54.8 \text{ degrees.}$$

- (d) Freezing is at 32 degrees Fahrenheit, so we want to know the value of h for which $T(h) = 32$. Since 32 is midway between $T(2) = 38$ and $T(3) = 26$, a good estimate is $h = 2.5$ or at 6:30 p.m.

Another way to do this is to use the average decrease between $h = 2$ and $h = 3$, which is $\frac{38 - 26}{60} = 0.2$ degree per minute. To decrease from 38 degrees to 32 degrees is a total decrease of 6 degrees. It requires 30 decreases of 0.2 degree to make a 6 degree decrease. Thus freezing is reached at 30 minutes after 6:00 p.m., so at 6:30 p.m.

11. **A troublesome snowball:** Here $W(t)$ is the volume of dirty water soaked into the carpet, so its limiting value is the total volume of water frozen in the snowball. The limiting value is reached when the snowball has completely melted.

12. **Falling with a parachute:**

- (a) Increasing velocity causes increased air resistance, and eventually this increases to match the downward pull of gravity. When this happens, velocity does not change.
- (b) A reasonable estimate is 19.97 feet per second, or anything slightly larger.

13. **Carbon 14:**

- (a) The average yearly rate of change for the first 5000 years is

$$\frac{\text{Amount of change}}{\text{Years elapsed}} = \frac{C(5) - C(0)}{5000} = \frac{2.73 - 5}{5000} = -4.54 \times 10^{-4} \text{ gram per year.}$$

That is -0.000454 gram per year.

(b) We use the average yearly rate of change from Part (a):

$$C(1.236) = C(0) + 1236 \times \text{yearly rate of change} = 5 + 1236 \times -0.000454 = 4.44 \text{ grams.}$$

(c) The limiting value is zero since all of the carbon 14 will eventually decay.

14. Newton's law of cooling:

(a) The expression $A(75)$ is the temperature (in degrees Fahrenheit) of the piece of aluminum 75 minutes after it is removed from the oven.

Since 75 falls in the middle of 60 and 90, we use the average of $A(60)$ and $A(90)$ to estimate $A(75)$. This average is

$$A(75) = \frac{A(60) + A(90)}{2} = \frac{100 + 81}{2} = 90.5 \text{ degrees Fahrenheit.}$$

(b) The average decrease per minute of the temperature in the first half hour is

$$\frac{\text{Temperature drop}}{\text{Minutes}} = \frac{A(0) - A(30)}{30} = \frac{302 - 152}{30} = 5 \text{ degrees per minute.}$$

(c) The average decrease per minute in the first half of the second hour is

$$\frac{\text{Temperature drop}}{\text{Minutes}} = \frac{A(60) - A(90)}{30} = \frac{100 - 81}{30} = 0.63 \text{ degree per minute.}$$

(d) We can see from Parts (c) and (d) that the aluminum cools at a faster rate early and then cools more slowly later. This shows that the object cools more rapidly when the difference between the object's temperature and the surrounding temperature is large.

(e) The temperature of the aluminum after one hour and 13 minutes is $A(73)$. We already know from Part (c) that the average rate of decrease in A from 60 to 90 minutes is 0.63 degree per minute. We estimate

$$A(73) = A(60) - 13 \times \text{temperature drop per minute} = 100 - 13 \times 0.63 = 91.81 \text{ degrees.}$$

(f) The temperature of the oven is the same as the initial temperature of the aluminum (assuming the aluminum was left in the oven long enough for the temperature to stabilize). Thus the oven temperature is $A(0) = 302$ degrees Fahrenheit.

(g) The temperature of the aluminum approaches room temperature eventually, so we expect that room temperature will be a limiting value of A .

(h) The room temperature is 72 degrees Fahrenheit since the table indicates that is where the aluminum stops cooling.

15. **Effective percentage rate for various compounding periods:**

- (a) We have that $n = 1$ represents compounding yearly, $n = 2$ represents compounding semiannually, $n = 12$ represents compounding monthly, $n = 365$ represents compounding daily, $n = 8760$ represents compounding hourly, and $n = 525,600$ represents compounding every minute.
- (b) We have that $E(12)$ is the EAR when compounding monthly, and $E(12) = 12.683\%$.
- (c) If interest is compounded daily then the EAR is $E(365)$. So the interest accrued in one year is

$$8000 \times E(365) = 8000 \times 0.12747 = \$1019.76.$$

- (d) If interest were compounded continuously then the EAR would probably be about 12.750%. As the length of the compounding period decreases, the EAR given in the table appears to stabilize at this value.

16. **New construction:**

- (a) The average yearly rate of change from 2000 to 2003 is

$$\frac{B(2003) - B(2000)}{2003 - 2000} = \frac{891.5 - 831.1}{3} = 20.13 \text{ billion dollars per year.}$$

Continuing in this way, we get the following table, where the rate of change is measured in billions of dollars per year.

Period	2000 to 2003	2003 to 2006	2006 to 2009
Rate of change	20.13	92.03	-77.33

- (b) Here $B(2008)$ is the value (in billions of dollars) of new construction put in place in the United States in 2008. We estimate it using the entry from the table for the average rate of change over the last period:

$$B(2006) + 2 \times -77.33 = 1167.6 + 2 \times -77.33 = 1012.94.$$

Hence we estimate that $B(2006)$ is about 1012.94 (in billions of dollars).

- (c) By looking at the table we made for Part (a), we see that the largest average yearly rate of change occurred over the period from 2003 to 2006. This was the period of greatest growth.
- (d) The value of new construction was greatest in the year 2006.

17. Growth in height:

- (a) In functional notation, the height of the man at age 13 is given by
- $H(13)$
- .

From ages 10 to 15, the average yearly growth rate in height is

$$\frac{\text{Inches increased}}{\text{Years elapsed}} = \frac{67.0 - 55.0}{5} = 2.4 \text{ inches per year.}$$

Since age 13 is 3 years after age 10, we can estimate $H(13)$ as

$$H(10) + 3 \times \text{yearly growth} = 55.0 + 3 \times 2.4 = 62.2 \text{ inches.}$$

- (b) i. We calculate the average yearly growth rate for each 5-year period just as we calculated 2.4 inches per year as the average yearly growth rate from ages 10 to 15 in Part (a). The average yearly growth rate is measured in inches per year.

Age change	0 to 5	5 to 10	10 to 15	15 to 20	20 to 25
Average yearly growth rate	4.2	2.5	2.4	1.3	0.1

- ii. The man grew the most from age 0 to age 5.
 iii. The trend is that as the man gets older, he grows more slowly.
- (c) It is reasonable to guess that 74 or 75 inches is the limiting value for the height of this man. He grew only 0.5 inches from ages 20 to 25, so it is reasonable to expect little or no further growth from age 25 on.

18. Growth in weight:

- (a) We calculate the average yearly rate of change in weight for each four-year period by calculating

$$\frac{\text{Change in weight}}{\text{Elapsed time}}$$

for each four-year period. The average rate of growth is measured in pounds per year.

Age change	4 to 8	8 to 12	12 to 16	16 to 20	20 to 24
Growth rate	4.5	6.75	11.75	7	1.75

- (b) The man gained, on average, more weight per year from age 4 through age 16, after which the man continued to gain weight, but more slowly. The man gained the most in weight from age 12 to age 16.
- (c) Assuming the man continued to gain 1.75 pounds per year after age 24, then at age 30 (six years later), he would weigh

$$W(24) + 6 \times \text{yearly increase} = 163 + 6 \times 1.75 = 173.5 \text{ pounds.}$$

- (d) Using the average rate of change to estimate his weight at birth gives

$$W(4) - 4 \times \text{yearly increase} = 36 - 4 \times 4.5 = 18 \text{ pounds.}$$

This is quite an unreasonable answer, and we must conclude that the assumption that the child grew 4.5 pounds per year from ages 0 through 4 is incorrect. Based on experience, we expect that the man would weigh at most 12 pounds at birth.

19. Tax owed:

- (a) The average rate of change over the first interval is

$$\frac{T(16,200) - T(16,000)}{16,200 - 16,000} = \frac{888 - 870}{200} = 0.09 \text{ dollar per dollar.}$$

Continuing in this way, we get the following table, where the rate of change is measured in dollars per dollar.

Interval	16,000 to 16,200	16,200 to 16,400	16,400 to 16,600
Rate of change	0.09	0.09	0.09

- (b) The average rate of change has a constant value of 0.09 dollar per dollar. This suggests that, at every income level in the table, for every increase of \$1 in taxable income the tax owed increases by \$0.09, or 9 cents.
- (c) Because the average rate of change is a nonzero constant and thus does not tend to 0, we would expect T not to have a limiting value but rather to increase at a constant rate as I increases.

20. Sales income:

- (a) The average rate of change over the first interval is

$$\frac{N(500,000) - N(450,000)}{500,000 - 450,000} = \frac{5500 - 4000}{50,000} = 0.03 \text{ dollar per dollar.}$$

Continuing in this way, we get the following table, where the rate of change is measured in dollars per dollar.

Interval	450,000 to 500,000	500,000 to 550,000	550,000 to 600,000
Rate of change	0.03	0.03	0.03

The average rate of change has a constant value of 0.03 dollar per dollar.

- (b) To estimate the monthly income for sales of \$520,000 we use the entry from the table for the average rate of change over the second interval:

$$N(500,000) + 20,000 \times 0.03 = 5500 + 20,000 \times 0.03 = 6100 \text{ dollars.}$$

Hence the monthly income for sales of \$520,000 is about \$6100. Because the average rate of change is constant, this should be an accurate representation of the actual income.

- (c) Because the average rate of change is a nonzero constant and thus does not tend to 0, we would expect N not to have a limiting value but rather to increase at a constant rate as s increases.

21. **Yellowfin tuna:**

- (a) The average rate of change in weight is

$$\frac{W(110) - W(100)}{110 - 100} = \frac{56.8 - 42.5}{10} = 1.43 \text{ pounds per centimeter.}$$

- (b) The average rate of change in weight is

$$\frac{W(180) - W(160)}{180 - 160} = \frac{256 - 179}{20} = 3.85 \text{ pounds per centimeter.}$$

- (c) Examining the table shows that the rate of change in weight is smaller for small tuna than it is for large tuna. Hence an extra centimeter of length makes more difference in weight for a large tuna.
- (d) To estimate the weight of a yellowfin tuna that is 167 centimeters long we use the average rate of change we found in Part (b):

$$W(160) + 7 \times 3.85 = 179 + 7 \times 3.85 = 205.95 \text{ pounds.}$$

Hence the weight of a yellowfin tuna that is 167 centimeters long is 205.95, or about 206.0, pounds.

- (e) Here we are thinking of the weight as the variable and the length as a function of the weight. The average rate of change in length is

$$\frac{\text{Length at 256 pounds} - \text{Length at 179 pounds}}{256 - 179} = \frac{180 - 160}{256 - 179} = 0.26,$$

so the average rate of change is 0.26 centimeter per pound. Note that this number is the reciprocal of the answer from Part (b).

- (f) To estimate the length of a yellowfin tuna that weighs 225 pounds we use the average rate of change we found in Part (e):

$$\text{Length at 179 pounds} + (225 - 179) \times 0.26 = 160 + 46 \times 0.26 = 171.96 \text{ centimeters.}$$

Hence the length of a yellowfin tuna that weighs 225 pounds is 171.96, or about 172, centimeters.

22. Arterial blood flow:

- (a) The average rate of change from 10% to 15% is

$$\frac{1.75 - 1.46}{5} = 0.058 \text{ per percentage point.}$$

Then for a 12% increase in radius we estimate that the blood flow rate will be

$$1.46 + 2 \times 0.058 = 1.576,$$

or about 1.58, times greater.

- (b) If we start with a radius of, say, 1 unit, an increase of 12% will give a radius of $1 + 0.12 = 1.12$ units. Another increase of 12% will give a radius of $1.12 + 0.12 \times 1.12 = 1.2544$ units. The combined effect is a relative increase of $\frac{1.2544 - 1}{1} = 0.2544$, or 25.44%.
- (c) From Part (b) we know that an increase of 25.44% corresponds to 2 successive increases of 12%, and by Part (a) this corresponds to multiplying the flow rate by 1.58 twice. Thus the flow rate is 1.58×1.58 , or about 2.50, times greater in an artery that is 25.44% larger.
- (d) The average rate of change from 15% to 20% is

$$\frac{2.07 - 1.75}{5} = 0.064 \text{ per percentage point.}$$

Then for a 25.44% increase in radius we estimate that the blood flow rate will be

$$\text{Factor for 20\% increase} + 5.44 \times 0.064 = 2.07 + 5.44 \times 0.064,$$

or about 2.42, times greater.

23. Widget production:

- (a) The average rate of change over the first interval is

$$\frac{W(20) - W(10)}{10} = \frac{37.5 - 25.0}{10} = 1.25 \text{ thousand widgets per worker.}$$

Continuing in this way, we get the following table, where the rate of change is measured in thousands of widgets per worker.

Interval	10 to 20	20 to 30	30 to 40	40 to 50
Rate of change	1.25	0.63	0.31	0.15

- (b) The average rate of change decreases and approaches 0 as we go across the table. This means that the increase in production gained from adding another worker gets smaller and smaller as the level of workers employed moves higher and higher. Eventually there is very little benefit in employing an extra worker.

- (c) To estimate how many widgets will be produced if there are 55 full-time workers we use the entry from the table for the average rate of change over the last interval:

$$W(50) + 5 \times 0.15 = 48.4 + 5 \times 0.15 = 49.15 \text{ thousand widgets.}$$

Hence the number of widgets produced by 55 full-time workers is about 49.2 thousand, or 49,200.

- (d) Because the average rate of change is decreasing, the actual increase in production in going from 50 to 55 workers is likely to be less than what the average rate of change from 40 to 50 suggests. Thus our estimate is likely to be too high.

24. Timber stumpage prices:

- (a) The average rate of change per year in Mid-Atlantic prices from 2002 to 2005 is

$$\frac{6.70 - 4.00}{3} = 0.90 \text{ dollar per ton per year.}$$

- (b) We estimate the Mid-Atlantic price in 2004 using the price in 2002, as follows:

$$4.00 + 2 \times 0.90 = 5.80 \text{ dollars per ton.}$$

Hence the Mid-Atlantic price in 2002 is estimated at about 5.80 dollars per ton.

- (c) The average rate of change per year in Southeast prices from 2005 to 2007 is

$$\frac{7.00 - 7.50}{2} = -0.25 \text{ dollars per ton per year.}$$

- (d) We estimate the price in the Southeast in 2008 using the price in 2007, as follows:

$$7.00 + 1 \times -0.25 = 6.75 \text{ dollars per ton.}$$

Hence the Southeast price in 2008 is estimated at about 6.75 dollars per ton.

- (e) The relative increase in Southeast prices from 2002 to 2007 was $\frac{7.00 - 6.50}{6.50} = 0.077$, or about 7.7%. The relative increase in Mid-Atlantic prices over this period was $\frac{10.00 - 4.00}{4.00} = 1.50$, or 150%.

- (f) The percentage increase in Mid-Atlantic prices was greater, so the Mid Atlantic was a better investment.

25. The Margaria-Kalamen test:

- (a) The average rate of change per year in excellence level from 25 years to 35 years old is

$$\frac{168 - 210}{35 - 25} = -4.2 \text{ points per year.}$$

- (b) We estimate the power score needed for a 27-year-old man using the score for a 25-year-old man and the average rate of change from Part (a):

$$210 + 2 \times -4.2 = 201.6 \text{ points.}$$

Hence the power score that would merit an excellent rating for a 27-year-old man is 201.6, or about 202, points.

- (c) The decrease in power score for excellent rating over these three periods is the greatest in the second period (35 years to 45 years), so we would expect to see the greatest decrease in leg power from 35 to 45 years old.

26. ACRS and MACRS:

- (a) Under ACRS, the total depreciation allowed over the life of the truck is

$$25 + 38 + 37 = 100 \text{ percent.}$$

Under MACRS, the total depreciation allowed is

$$20 + 32 + 19.2 + 11.52 + 11.52 + 5.76 = 100 \text{ percent.}$$

In both systems, the total amount of the original expense is recovered eventually.

- (b) Under ACRS, the tax deduction allowed in each year is: first, 25% of \$14,500, or \$3625; second, 38% of \$14,500, or \$5510; third, 37% of \$14,500, or \$5365.
- (c) Under MACRS, the tax deduction allowed in each of the first 3 years is: first, 20% of \$14,500, or \$2900; second, 32% of \$14,500, or \$4640; third, 19.2% of \$14,500, or \$2784.
- (d) From Part (b), under ACRS the tax deduction during the second year is \$5510. Hence the total tax savings is 28% of \$5510, or \$1542.80.

27. Home equity:

- (a) $E(10)$ is the equity accrued after 10 years of payments. Its value, according to the table, is \$27,734.
- (b) We calculate the average rate of change for each 5-year period below:

5-year interval	0 to 5	5 to 10	10 to 15	15 to 20	20 to 25	25 to 30
Average rate of change	2361.60	3185.20	4296.60	5795.40	7817.00	10,544.20

- (c) The equity increases more rapidly late in the life of the mortgage.

- (d) To estimate the equity accrued after 17 years, we add 2 years average rate of change (from 15 to 20 years) to the equity accrued after 15 years:

$$E(17) = E(15) + 2 \times 5795.40 = 49,217 + 2 \times 5795.40 = 60,807.80 \text{ dollars.}$$

- (e) No, it wouldn't make sense to estimate past 30 years, since the mortgage has a term of 30 years.

28. Defense spending:

- (a) The average yearly rate of change in defense spending from 1990 to 1995 is

$$\frac{\text{Change from 1990 to 1995}}{\text{Years elapsed}} = \frac{D(5) - D(0)}{5 - 0} = \frac{310.0 - 328.4}{5} = -3.68 \text{ billion dollars per year.}$$

- (b) We estimate $D(3)$ by starting from $D(0)$ and adding three years' average rate of change:

$$D(3) = D(0) + 3 \times -3.68 = 328.4 + 3 \times -3.68 = 317.36 \text{ billion dollars.}$$

This means that federal defense spending was about 317.4 billion dollars in 1993.

- (c) The average yearly rate of change in defense spending from 2005 to 2010 is

$$\frac{\text{Change from 2005 to 2010}}{\text{Years elapsed}} = \frac{D(20) - D(15)}{20 - 15} = \frac{843.8 - 565.5}{5} = 55.66 \text{ billion dollars per year.}$$

- (d) We estimate $D(22)$ by starting from $D(20)$ and adding two years' average rate of change:

$$D(22) = D(20) + 2 \times 55.66 = 843.8 + 2 \times 55.66 = 955.12 \text{ billion dollars.}$$

- 29. A home experiment:** Answers will vary greatly. In general, there will be initially a small percentage of bread surface covered with mold. That percentage will quickly rise as the mold covers much of the bread surface. There are usually a few small patches which the mold covers more slowly. Ultimately all the bread surface is covered with mold. Here is a typical data table:

Time	8 am	4 pm	12 am	8 am	4 pm	12 am
Mold	10%	25%	60%	98%	100%	100%

- 30. Research project:** Answers will vary.

Skill Building Exercises

- S-1. **Function values:** According to the table, when $t = 10$, then $N = 17.6$ and so $N(10) = 17.6$.
- S-2. **Function values:** According to the table, when $t = 20$, then $N = 23.8$ and so $N(20) = 23.8$.
- S-3. **Function values:** According to the table, when $t = 30$, then $N = 44.6$ and so $N(30) = 44.6$.
- S-4. **Function values:** According to the table, when $t = 40$, then $N = 51.3$ and so $N(40) = 51.3$.
- S-5. **Function values:** According to the table, when $t = 50$, then $N = 53.2$ and so $N(50) = 53.2$.
- S-6. **Function values:** According to the table, when $t = 60$, then $N = 53.7$ and so $N(60) = 53.7$.
- S-7. **Function values:** According to the table, when $t = 70$, then $N = 53.9$ and so $N(70) = 53.9$.
- S-8. **Averaging:** We can estimate the value of $N(15)$ by finding the average of $N(10)$ and $N(20)$ since 15 is the average of 10 and 20. The average is $\frac{N(10) + N(20)}{2} = \frac{17.6 + 23.8}{2}$, which equals 20.7.
- S-9. **Averaging:** We can estimate the value of $N(25)$ by finding the average of $N(20)$ and $N(30)$ since 25 is the average of 20 and 30. The average is $\frac{N(20) + N(30)}{2} = \frac{23.8 + 44.6}{2}$, which equals 34.2.
- S-10. **Averaging:** We can estimate the value of $N(35)$ by finding the average of $N(30)$ and $N(40)$ since 35 is the average of 30 and 40. The average is $\frac{N(30) + N(40)}{2} = \frac{44.6 + 51.3}{2}$, which equals 47.95. We round to 48.0 since the table has one decimal place of accuracy.
- S-11. **Averaging:** We can estimate the value of $N(45)$ by finding the average of $N(40)$ and $N(50)$ since 45 is the average of 40 and 50. The average is $\frac{N(40) + N(50)}{2} = \frac{51.3 + 53.2}{2}$, which equals 52.25. We round to 52.3 since the table has one decimal place of accuracy.
- S-12. **Averaging:** We can estimate the value of $N(55)$ by finding the average of $N(50)$ and $N(60)$ since 55 is the average of 50 and 60. The average is $\frac{N(50) + N(60)}{2} = \frac{53.2 + 53.7}{2}$, which equals 53.45. We round to 53.5 since the table has one decimal place of accuracy.

S-13. **Averaging:** We can estimate the value of $N(65)$ by finding the average of $N(60)$ and $N(70)$ since 65 is the average of 60 and 70. The average is $\frac{N(60) + N(70)}{2} = \frac{53.7 + 53.9}{2}$, which equals 53.8.

S-14. **Average rate of change:** The average rate of change in N from $t = 10$ to $t = 20$ is given by the change in N divided by the change in t :

$$\frac{N(20) - N(10)}{20 - 10} = \frac{23.8 - 17.6}{10} = 0.62.$$

Thus the average rate of change in N is 0.62. To estimate the value of $N(13)$, we begin at $N(10)$ and add 3 times the average rate of change:

$$N(13) = N(10) + 3 \times 0.62 = 17.6 + 3 \times 0.62 = 19.46,$$

which we round to 19.5, since the table has one decimal place accuracy.

S-15. **Average rate of change:** The average rate of change in N from $t = 20$ to $t = 30$ is given by the change in N divided by the change in t :

$$\frac{N(30) - N(20)}{30 - 20} = \frac{44.6 - 23.8}{10} = 2.08.$$

Thus the average rate of change in N is 2.08. To estimate the value of $N(27)$, we begin at $N(20)$ and add 7 times the average rate of change:

$$N(27) = N(20) + 7 \times 2.08 = 23.8 + 7 \times 2.08 = 38.36,$$

which we round to 38.4, since the table has one decimal place of accuracy.

S-16. **Average rate of change:** The average rate of change in N from $t = 30$ to $t = 40$ is given by the change in N divided by the change in t :

$$\frac{N(40) - N(30)}{40 - 30} = \frac{51.3 - 44.6}{10} = 0.67.$$

Thus the average rate of change in N is 0.67. To estimate the value of $N(36)$, we begin at $N(30)$ and add 6 times the average rate of change:

$$N(36) = N(30) + 6 \times 0.67 = 44.6 + 6 \times 0.67 = 48.62,$$

which we round to 48.6, since the table has one decimal place of accuracy.

S-17. **Average rate of change:** The average rate of change in N from $t = 40$ to $t = 50$ is given by the change in N divided by the change in t :

$$\frac{N(50) - N(40)}{50 - 40} = \frac{53.2 - 51.3}{10} = 0.19.$$

Thus the average rate of change in N is 0.19. To estimate the value of $N(42)$, we begin at $N(40)$ and add 2 times the average rate of change:

$$N(42) = N(40) + 2 \times 0.19 = 51.3 + 2 \times 0.19 = 51.68,$$

which we round to 51.7, since the table has one decimal place of accuracy.

S-18. **Average rate of change:** The average rate of change in N from $t = 50$ to $t = 60$ is given by the change in N divided by the change in t :

$$\frac{N(60) - N(50)}{60 - 50} = \frac{53.7 - 53.2}{10} = 0.05.$$

Thus the average rate of change in N is 0.05. To estimate the value of $N(58)$, we begin at $N(50)$ and add 8 times the average rate of change:

$$N(58) = N(50) + 8 \times 0.05 = 53.2 + 8 \times 0.05 = 53.6.$$

S-19. **Average rate of change:** The average rate of change in N from $t = 60$ to $t = 70$ is given by the change in N divided by the change in t :

$$\frac{N(70) - N(60)}{70 - 60} = \frac{53.9 - 53.7}{10} = 0.02.$$

Thus the average rate of change in N is 0.02. To estimate the value of $N(64)$, we begin at $N(60)$ and add 4 times the average rate of change:

$$N(64) = N(60) + 4 \times 0.02 = 53.7 + 4 \times 0.02 = 53.78,$$

which we round to 53.8, since the table has one decimal place of accuracy.

S-20. **Limiting values:** Assuming that a limiting value is expected, we could reasonably estimate the limiting value of N as about 54 since the rate of change of N is getting very small as N goes from 53.2 to 53.7 and to 53.9.

S-21. **Function values:** Here $f(0)$ is 5.7 since that is the corresponding value in the table.

S-22. **Function values:** Here $f(5)$ is 4.3 since that is the corresponding value in the table.

S-23. **Function values:** Here $f(10)$ is 1.1 since that is the corresponding value in the table.

S-24. **Function values:** Here $f(15)$ is -3.6 since that is the corresponding value in the table.

S-25. **Function values:** Here $f(20)$ is -7.9 since that is the corresponding value in the table.

S-26. **Average rate of change:** The average rate of change in f from $x = 0$ to $x = 5$ is given by the change in f divided by the change in x :

$$\frac{f(5) - f(0)}{5 - 0} = \frac{4.3 - 5.7}{5} = -0.28.$$

Thus the average rate of change in f is -0.28 . To estimate the value of $f(3)$, we calculate $f(3)$ as $f(0)$ plus 3 times the average rate of change:

$$f(3) = f(0) + 3 \times -0.28 = 5.7 + 3 \times -0.28,$$

which equals 4.86, or about 4.9.

S-27. **Average rate of change:** The average rate of change in f from $x = 5$ to $x = 10$ is given by the change in f divided by the change in x :

$$\frac{f(10) - f(5)}{10 - 5} = \frac{1.1 - 4.3}{5} = -0.64.$$

Thus the average rate of change in f is -0.64 . To estimate the value of $f(7)$, we calculate $f(7)$ as $f(5)$ plus 2 times the average rate of change:

$$f(7) = f(5) + 2 \times -0.64 = 4.3 + 2 \times -0.64,$$

which equals 3.02, or about 3.0.

S-28. **Average rate of change:** The average rate of change in f from $x = 10$ to $x = 15$ is given by the change in f divided by the change in x :

$$\frac{f(15) - f(10)}{15 - 10} = \frac{-3.6 - 1.1}{5} = -0.94.$$

Thus the average rate of change in f is -0.94 . To estimate the value of $f(13)$, we calculate $f(13)$ as $f(10)$ plus 3 times the average rate of change:

$$f(13) = f(10) + 3 \times -0.94 = 1.1 + 3 \times -0.94,$$

which equals -1.72 , or about -1.7 .

S-29. **Average rate of change:** The average rate of change in f from $x = 15$ to $x = 20$ is given by the change in f divided by the change in x :

$$\frac{f(20) - f(15)}{20 - 15} = \frac{-7.9 - (-3.6)}{5} = -0.86.$$

Thus the average rate of change in f is -0.86 . To estimate the value of $f(19)$, we calculate $f(19)$ as $f(15)$ plus 4 times the average rate of change:

$$f(19) = f(15) + 4 \times -0.86 = -3.6 + 4 \times -0.86,$$

which equals -7.04 , or about -7.0 .

S-30. **Average rate of change:** To estimate the value of $f(25)$, we calculate $f(25)$ as $f(20)$ plus 5 times the average rate of change:

$$f(25) = f(20) + 5 \times -0.86 = -7.9 + 5 \times -0.86,$$

which equals -12.2 .

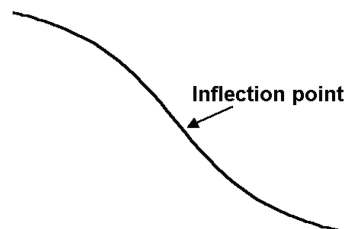
S-31. **When limiting values occur:** We expect S to have a limiting value of 0. This is because the average speed S gets closer and closer to 0 as the time t required to travel 100 miles increases.

S-32. **Does a limiting value occur?** We do not expect D to have a limiting value since the rocket ship travels indefinitely at a constant velocity; there is no limit to the distance it will travel.

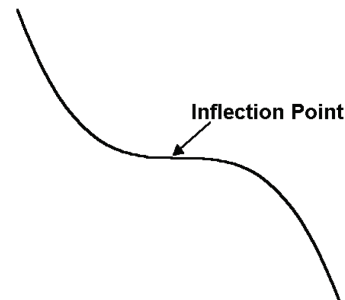
1.3 FUNCTIONS GIVEN BY GRAPHS

1. Sketching a graph with given concavity:

(a)



(b)



2. An investment:

- (a) The original investment is given by $v(2010)$, and from the graph this value is about \$5000.
- (b) The graph is concave up. This means that the investment is increasing at an increasing rate; it increases slowly at first and then more rapidly later.
- (c) The investment will reach \$55,000 somewhere around the year 2040.
- (d) (Answers will vary here because function values are estimated from the graph.) We estimate that $v(2050)$ is about \$105,000 and that $v(2060)$ is about \$280,000. Thus the average yearly increase from 2050 to 2060 is

$$\frac{\text{Change in } v}{\text{Years elapsed}} = \frac{280,000 - 105,000}{10} = \$17,500 \text{ per year.}$$

- (e) In Part (a) we saw that $v(2010)$ is about \$5000, and we estimate that $v(2020)$ is about \$15,000. Thus the average yearly increase from 2010 to 2020 is

$$\frac{\text{Change in } v}{\text{Years elapsed}} = \frac{15,000 - 5000}{10} = \$1000 \text{ per year.}$$

This is smaller than the increase found in Part (d), so the larger increase is from 2050 to 2060. This reflects the fact that the graph is concave up (see Part (b)).

3. Household debt:

- (a) In practical terms $h(1975)$ is the average American household debt as a percentage of disposable income in 1975. From the graph, the value is about 60%.
- (b) The maximum value of about 130% occurs in about 2006.

4. Auto loan rates: Loan rates fluctuated quite a bit, but generally speaking they declined from a maximum of about 16% to a minimum of about 4%.

5. Supply and demand curves:

- (a) The supply increases as the price increases.
- (b) The demand decreases as the price increases.

6. Equilibrium price: The equilibrium price is about \$2.50 per item. There are about 200 items supplied at this price.

7. Skirt length:

- (a) As the ratio increases the hem get closer to the ankle, so a larger ratio indicates a longer skirt.

- (b) By the answer to Part (a), skirt lengths were the shortest when the ratio was the smallest. Locating the minimum point of the solid graph, we see that the ratio was the smallest in about 1969.
- (c) When skirt lengths reach to the ankle, the ratio is 1. According to the graph, the ratio never reached 1. So skirt lengths never reached to the ankle during this period.

8. Number of weddings:

- (a) The maximum value of W occurs at the highest point of the graph, and that was reached in 1990.
- (b) The minimum value of W occurs at the lowest point of the graph, and that value is 1.52 million weddings.
- (c) The number of weddings increased at the fastest rate when the rise in the graph was steepest, and that occurred from 1965 to 1970. (The period from 1960 to 1965 saw a similar, but smaller, rise.)

9. Unemployment:

- (a) Here $U(1990)$ is the U.S. unemployment rate, as a percentage, in the year 1990. According to the graph, that rate is 5%.
- (b) The decade of highest unemployment occurred where the graph was at its highest over a 10-year period, and that occurred from about 1930 to 1940. That was the decade of the Great Depression.
- (c) The most recent date before 2010 when the graph reached the same height as in 2010 was around 1982 or 1983. Then employment was as high as in 2010.

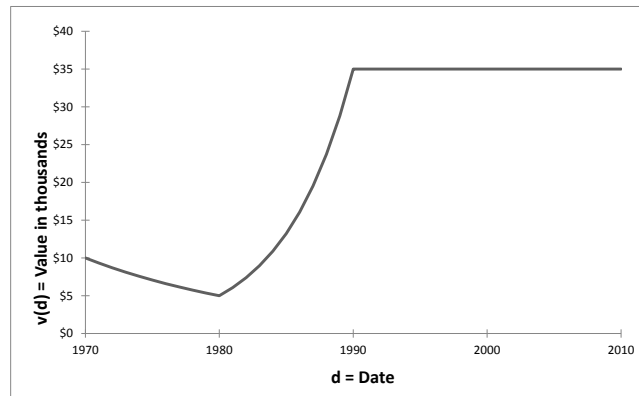
10. Cancer mortality rates:

- (a) The graph rising the most from 1970 through 1994 is the graph for black males. That group showed the largest rate of increase in cancer mortality from 1970 through 1994.
- (b) Cancer mortality for white males was at a maximum where that graph was at its highest, and that occurred at the interval 1985–89.

11. A stock market investment

- (a) According to the exercise our original investment made in 1970 was \$10,000, so $v(1970) = \$10,000$. Our investment lost half its value in the 70's, so $v(1980) = \$5,000$. The investment was worth \$35,000 in 1990, so $v(1990) = \$35,000$. Since the stock remained stable after 1990, the value of the investment stays at the same level of \$35,000. So $v(2010) = \$35,000$.

(b)



The graph should reflect the function values from Part (a), and it should increase from 1980 to 1990 and remain flat after that. There remains room for interpretation, and many graphs similar to the one above are acceptable.

- (c) The stock was most rapidly increasing in the 1980's. For the graph above, 1985 is a good guess. (The answer will depend on what the graph in Part (b) looks like.)

12. The value of the Canadian dollar: (Answers will vary due to estimates in reading the graph.)

- (a) In 1990, the Canadian dollar value was about \$0.86 American. It increased a little in value until it reached about \$0.87 (roughly) in 1992 and then declined until 2002, when the value was about \$0.63.
- (b) The Canadian dollar was worth \$0.80 around 1993.
- (c) The value of the Canadian dollar was about \$0.87 in 1992 and about \$0.76 in 1994. So the average yearly decrease was

$$\frac{\text{Decrease in value}}{\text{Years elapsed}} = \frac{0.87 - 0.76}{2} = 0.06 \text{ dollar per year.}$$

Thus the Canadian dollar decreased in value by about 6 cents per year from 1992 to 1994.

- (d) An investor would want to buy low and sell high. So the investor should buy in 1990 and sell in 1991 approximately.

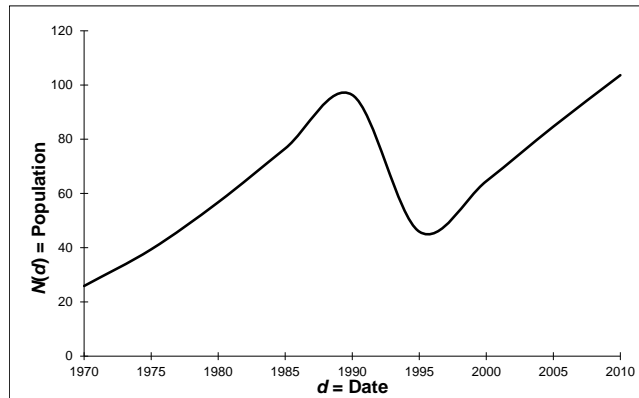
13. River flow:

- (a) Because the end of July is 7 months since the start of the year, the flow at that time is $F(7)$ in functional notation. According to the graph, the value is about 1500 cubic feet per second.
- (b) The flow is at its greatest at $t = 6$, which corresponds to the end of June.
- (c) The flow is increasing the fastest at $t = 5$, corresponding to an inflection point on the graph. The time is the end of May.
- (d) The graph is practically level from $t = 0$ to $t = 2$, so the function is nearly constant there, and the average rate of change over this interval is about 0 cubic feet per second.
- (e) Because the flow is measured near the river's headwaters in the Rocky Mountains, we expect any change in flow to come from melting snow primarily. This is consistent with almost no change in flow during the first two months of the year (as seen in Part (d)), a maximum increase in flow at the end of May (as seen in Part (c)), and a peak flow one month later (as seen in Part (b)).

14. Logistic population growth:

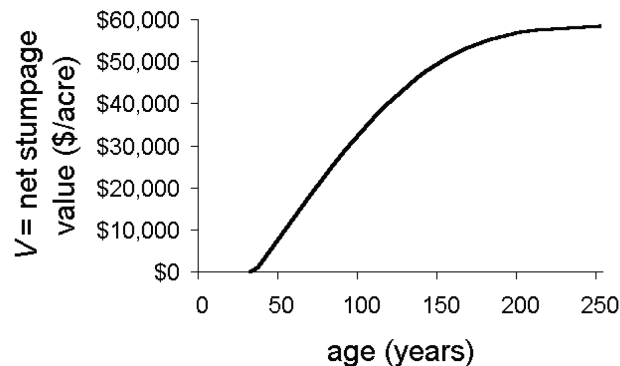
- (a) The population grows rapidly at first, and then the growth slows and eventually levels off.
- (b) The population reached 80 thousand around 1985.
- (c) The equation is $N(d) = 80$ thousand individuals.
- (d) The graph is concave up from 1970 to about 1985. During this period the population increased at an increasing rate; it increased slowly near 1970 and more rapidly near 1985.
- (e) The graph is concave down from about 1985 to 2010. During this period the population increased at a decreasing rate; it grew rapidly near 1985 and much more slowly near 2010.
- (f) There is a point of inflection at about 1985. This means that the population grew the fastest at about 1985.
- (g) Since the graph begins to level off near 140, the environmental carrying capacity is around 140,000 animals.

(h)



15. Cutting trees:

- (a) The graph shows the net stumpage value of a 60-year-old Douglas fir stand to be about \$14,000 per acre.
- (b) The graph shows a 110-year-old Douglas fir stand has net stumpage value of \$40,000 per acre.
- (c) When the costs involved in harvesting equal the commercial value, the net stumpage value is 0. Thus we need to know when V is 0. The graph shows that a 30-year-old Douglas fir stand has a net stumpage value of \$0 per acre.
- (d) The net stumpage value seems to be increasing the fastest in trees about 60 years old.
- (e) We would expect the trees to reach an age where they don't grow as much. When this happens the net stumpage value should level out. Here is one possible extended graph for the stumpage value of the Douglas fir.



16. Wind chill:

- (a) The effective temperature is 0 at a wind speed of about 27 (between 25 and 30) miles per hour.

- (b) The equation is $T(v) = 0$.
- (c) From the graph we can see that the effective temperature drops off rapidly near 7 miles per hour, so a small increase in wind speed would have the greatest effect there.
- (d) The shape of the graph indicates that the effective temperature is leveling off near 45 miles per hour, so an increase in wind speed should not have a large impact on the temperature adjusted for wind chill.

17. Tornadoes in Oklahoma:

- (a) The most tornadoes were reported in 2011. There were about 74 tornadoes reported that year.
- (b) The fewest tornadoes were reported in 2014. There were about 4 tornadoes reported then.
- (c) In 2011 there were about 74 tornadoes reported, and in 2012 there were about 26 tornadoes reported. Hence the average rate of decrease was

$$\frac{\text{Decrease in } T}{\text{Years elapsed}} = \frac{74 - 26}{1} = 48 \text{ tornadoes per year.}$$

- (d) In 2009 the number of tornadoes reported was about 25, and in 2011 the number of tornadoes reported was about 74. Hence the average rate of increase was

$$\frac{\text{Increase in } T}{\text{Years elapsed}} = \frac{74 - 25}{2} = 24.5 \text{ tornadoes per year.}$$

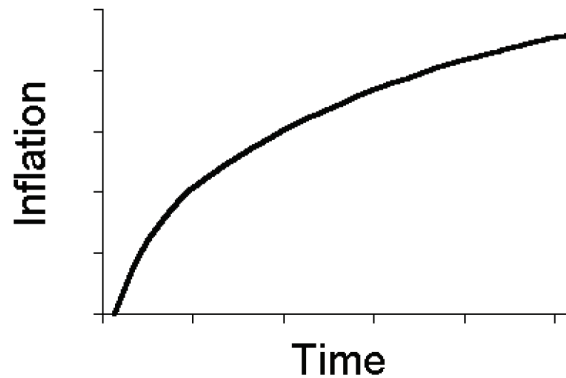
So there is an increase of about 24 to 25 tornadoes per year—the answer may vary depending on how the graph is interpreted.

- (e) The number of tornadoes reported in 2009 was about 25, and the number of tornadoes reported in 2012 was about 26. Hence the average rate of change was

$$\frac{\text{Change in } T}{\text{Years elapsed}} = \frac{26 - 25}{3} = \frac{1}{3} \text{ tornado per year,}$$

which is close to 0. The answer may vary depending on how the graph is interpreted.

18. **Inflation:** The graph should show an increasing function which is concave down, so increasing, but at a decreasing rate. For example:



19. **Driving a car:** There are many possible stories. Common elements include that you start 6 miles from home and drive towards home, arriving there in about 12 minutes, staying at home for about 4 minutes, then driving about $2\frac{1}{2}$ miles away from home and staying there.

Additional features might be the velocity (rate of change) towards home as about 30 mph and the velocity away from home as about 15 mph.

20. **Walking to school:**

- (a) The farthest distance is about 1000 yards and the farthest distance, according to the story, corresponds to the distance between home and the school.
- (b) This distance (to school) is first reached in about 10 minutes.
- (c) According to the graph, you are first 900 yards from home at about 9 minutes and then again at about 16 minutes.
- (d) The average rate of change from $t = 0$ to $t = 3$ is

$$\frac{\text{Change in distance}}{\text{Minutes elapsed}} = \frac{D(3) - D(0)}{3} = \frac{300}{3} = 100 \text{ yards per minute.}$$

From $t = 3$ to $t = 6$, the average rate of change is $\frac{600 - 300}{3} = 100$ yards per minute. From $t = 6$ to $t = 9$, the average rate of change is $\frac{900 - 600}{3} = 100$ yards per minute.

- (e) All three rates of change from Part (d) are the same, namely 100 yards per minute, so that is how fast you are walking to school. This part of the graph is a straight line, and we expect the rate of change for a straight line to be the same everywhere.

- (f) You are walking back west the fastest when the graph is the steepest downward, which is at about 16 or 17 minutes.

21. **Photosynthesis:** In Parts (a) and (b) answers may vary somewhat.

- (a) We want to find where the graph corresponding to 80 degrees crosses the horizontal axis. The crossing point is at about 0.7 thousand foot-candles, or about 700 foot-candles.
- (b) We want to find where the graph corresponding to 80 degrees crosses the graph corresponding to 40 degrees. The crossing point is at about 0.8 thousand foot-candles, or about 800 foot-candles.
- (c) Among the three graphs, the graph corresponding to 80 degrees meets the vertical axis at the lowest point, so a temperature of 80 degrees will result in the largest emission of carbon dioxide in the dark.
- (d) The graph corresponding to 40 degrees is the most level, so that temperature gives the net exchange that is least sensitive to light.

22. **Protein content of wheat grain:**

- (a) The minimum of the graph corresponding to 49% moisture occurs at a nitrogen level of about 50 pounds per acre.
- (b) We want to find where the graph corresponding to 34% moisture crosses the graph corresponding to 1% moisture. The crossing point is at about 25 pounds of nitrogen per acre.
- (c) In this situation a protein content of 13% is reached when the nitrogen level (in pounds per acre) is 15 and when it is 100. The less expensive choice is to apply 15 pounds of nitrogen per acre.
- (d) To the right of 45 on the horizontal axis, the highest graph is the one corresponding to irrigating when soil moisture dropped to 1%. Hence, for nitrogen levels at 45 pounds per acre or higher, increased protein content in wheat grain is associated with lower soil moisture.

23. **Carbon dioxide concentrations:**

- (a) The minimum of the graph occurs at about 2 p.m.
- (b) The maximum concentration is attained over the entire interval from about 6 a.m. to about 9 a.m.

- (c) Net absorption is indicated by the interval where the graph is decreasing. This is the period from about 9 a.m. to about 2 p.m.
- (d) From 6 a.m. to 9 a.m. the graph is level, so the net carbon dioxide exchange is zero during that period.

24. Hydrographs:

- (a) The maximum value of runoff occurs at about 10 on the horizontal axis. The units on this axis are hours since rainfall excess began, so the time to peak is about 10 hours.
- (b) The inflection point after peak runoff corresponds to about 15 on the horizontal axis. If the end of rainfall excess occurs at 5 on that axis, the time of concentration is about $15 - 5 = 10$ hours.
- (c) The end of runoff occurs when the graph reaches the horizontal axis, and this is at about 45 on that axis. From Part (a) we know that peak runoff occurs at about 10 on the axis, so the recession time is about $45 - 10 = 35$ hours.
- (d) As we noted in Part (c), the end of runoff occurs at about 45 on the horizontal axis. This means that the time base is about 45 hours.

25. Profit from fertilizer:

- (a) The maximum value of yield occurs at about 100 on the horizontal axis, so about 100 pounds of nitrogen per acre should be applied to produce maximum crop yield.
- (b) The profit can be read by measuring the difference between the yield and cost graphs.
- (c) The maximum difference between the graphs occurs at about 85 on the horizontal axis, so about 85 pounds of nitrogen per acre should be applied to produce maximum profit. (In this part answers may vary somewhat.)

26. H-R diagrams:

- (a) A star which is 10,000 times as luminous as the sun has a relative luminosity of 10,000, and examining the graph for main sequence stars shows that this level occurs at about 22 on the horizontal axis. This means that the surface temperature is about 22,000 kelvins.
- (b) The relative luminosity of the sun is 1 because it is 1 time as luminous as itself.

- (c) We know that the sun has a relative luminosity of 1, and examining the graph for main sequence stars shows that this level occurs at about 5.5 on the horizontal axis. This means that the surface temperature is about 5500 kelvins.
- (d) Because the graph of relative luminosity for white dwarfs lies below that for main sequence stars, a white dwarf would be less luminous than a main sequence star having the same surface temperature.
- (e) We know from Part (c) that the surface temperature of the sun is about 5500 kelvins. Examining the graph for supergiants shows that a supergiant star with this surface temperature has a relative luminosity of about 100,000. This means that the surface area of such a supergiant is about 100,000 times larger than the surface area of the sun.

27. Home equity:

- (a) The graph is concave up.
- (b) Since the graph is increasing and concave up, this means that equity is increasing faster and faster, that is, increasing at an increasing rate.
- (c) Since the mortgage is for 30 years, the amount of the mortgage is the home equity after 30 years, and this is about \$335,000.

28. Relativistic time:

- (a) According to the graph, $T(0.6)$ is about 1.3. In practical terms this means that on a rocket ship going at 60% of the speed of light, an hourglass will empty in about 1.3 hours, as it appears to a stationary observer.
- (b) The graph is concave up.
- (c) A small change in velocity causes a greater change in apparent time at faster speeds since the graph is concave up.

29. Grain beetles:

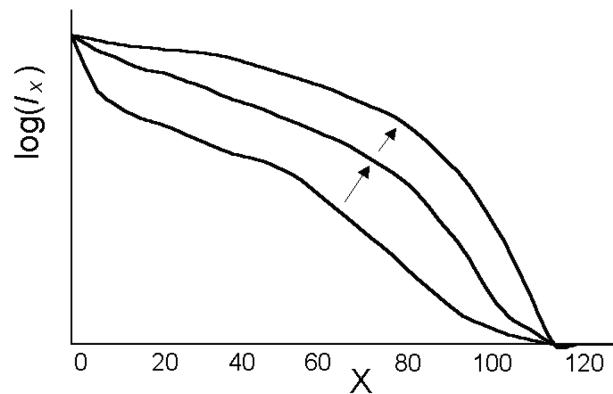
- (a) The graph indicates that the beetles in the wheat stored at 32.3 degrees had a higher mortality rate than the other group. Consequently, it would be better to store the wheat at the higher temperature so that the beetles would not live as long.
- (b) The graph indicates that the beetles feeding on the wheat had a higher mortality rate. Consequently, the beetles would be more likely to infest the maize.

30. **Life insurance rates:** The general principle here is that we expect higher insurance rates for higher mortality rates.

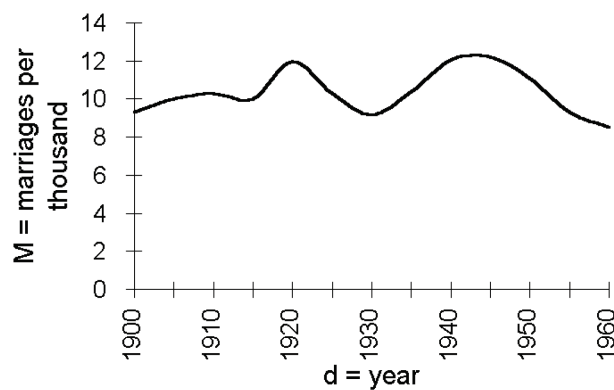
(a) Figure 1.49 indicates that the mortality rates for males are generally higher than those for females. That explains why life insurance rates for men are higher than those for women.

(b) The curves indicate that the mortality rates for older persons increase greatly as they age. This explains why insurance rates for the elderly increase so rapidly.

31. **Span of life:** The following graph shows a sequence of survivorship curves which illustrates an increasing life span with a fixed maximum life span of 120 years.



32. **A research project:** This graph was obtained by using all the data points from 1900 to 1960:



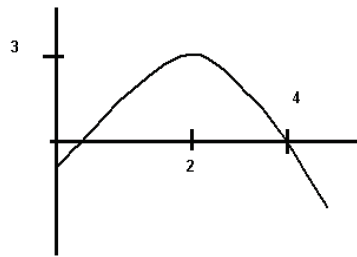
This is different from all the graphs presented in the text.

Skill Building Exercises

- S-1. **A function given by a graph:** The value of $f(1.8)$ is obtained from the graph by locating 1.8 on the horizontal axis, then moving vertically up to the point of the graph over 1.8, and then moving horizontally to locate the corresponding value on the vertical axis, Thus $f(1.8)$ is a little below 3.5, so about 3.3.
- S-2. **A function given by a graph:** The value of $f(2.4)$ is obtained from the graph by locating 2.4 on the horizontal axis, then moving vertically up to the point of the graph over 2.4, and then moving horizontally to locate the corresponding value on the vertical axis, Thus $f(2.4)$ is about 4.0.
- S-3. **A function given by a graph:** The value of $f(4.2)$ is obtained from the graph by locating 4.2 on the horizontal axis, then moving vertically up to the point of the graph over 4.2, and then moving horizontally to locate the corresponding value on the vertical axis, Thus $f(4.2)$ is about 0.5.
- S-4. **A function given by a graph:** The smallest value of x for which $f(x) = 1.5$ is obtained from the graph by locating 1.5 on the vertical axis, then moving horizontally to the first point on the graph, and then moving vertically down to locate the corresponding value on the horizontal axis. Thus the smallest value of x for which $f(x) = 1.5$ is about $x = 1.0$.
- S-5. **A function given by a graph:** The largest value of x for which $f(x) = 1.5$ is obtained from the graph by locating 1.5 on the vertical axis, then moving horizontally to the furthest point on the graph, and then moving vertically down to locate the corresponding value on the horizontal axis. Thus the smallest value of x for which $f(x) = 1.5$ is about $x = 4.0$.
- S-6. **A function given by a graph:** The smallest value of x for which $f(x) = 3$ is obtained from the graph by locating 3 on the vertical axis, then moving horizontally to the first point on the graph, and then moving vertically down to locate the corresponding value on the horizontal axis. Thus the smallest value of x for which $f(x) = 3$ is about $x = 1.7$.
- S-7. **A function given by a graph:** The graph reaches a maximum at its highest point, which is where $x = 2.4$ and $f(x)$ is about 4.0.
- S-8. **A function given by a graph:** The graph is increasing for values of x from $x = 0$ to $x = 2.4$.
- S-9. **A function given by a graph:** The graph is decreasing for values of x from $x = 2.4$ to $x = 4.8$.

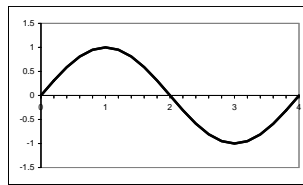
- S-10. **A function given by a graph:** Between $x = 1.8$ and $x = 2.4$, the graph is concave down.
- S-11. **A function given by a graph:** Between $x = 0$ and $x = 1.8$, the graph is concave up.
- S-12. **A function given by a graph:** Between $x = 2.4$ and $x = 3.0$, the graph is concave down.
- S-13. **A function given by a graph:** Between $x = 3.0$ and $x = 4.8$, the graph is concave up.
- S-14. **A function given by a graph:** Points of inflection are where the graph changes concavity. These are approximately at $x = 1.8$ and $x = 3.0$.
- S-15. **A function given by a graph:** The graph achieves its maximum value at about $x = 0.5$ and $x = 2.5$.
- S-16. **A function given by a graph:** The maximum value of $f(x)$ is 1.
- S-17. **A function given by a graph:** The graph achieves its minimum value at about $x = 1.5$ and $x = 3.5$.
- S-18. **A function given by a graph:** The minimum value of $f(x)$ is -1 .
- S-19. **A function given by a graph:** The inflection points of $f(x)$ occur at about $x = 1$, $x = 2$, and $x = 3$.
- S-20. **A function given by a graph:** Between $x = 1$ and $x = 2$, the graph is concave up.
- S-21. **A function given by a graph:** Between $x = 2$ and $x = 3$, the graph is concave down.
- S-22. **A function given by a graph:** The graph is both decreasing and concave up between $x = 1$ and $x = 1.5$ and between $x = 3$ and $x = 3.5$.
- S-23. **A function given by a graph:** The graph is both increasing and concave up between $x = 1.5$ and $x = 2$ and between $x = 3.5$ and $x = 4$.
- S-24. **Concavity again:** If a graph is increasing, but at a decreasing rate, then it will look like the portion of the graph at the beginning of the Skill Building Exercises for x between 1.8 and 2.4; in particular the graph is concave down.
- S-25. **Special points:** Points where concavity changes are called points of inflection.

S-26. **Sketching graphs:** Answers will vary. One solution is

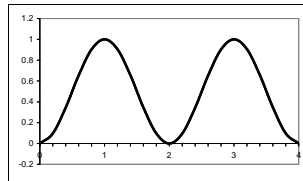


S-27. **Sketching graphs:** Answers will vary. One solution has the same shape as the graph at the beginning of the Skill Building Exercises, but with the maximum at $x = 3$.

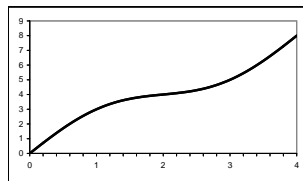
S-28. **Sketching graphs:** Answers will vary. One solution is



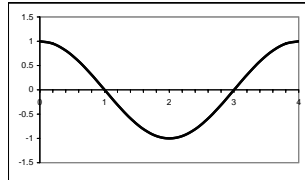
S-29. **Sketching graphs:** Answers will vary. One solution is



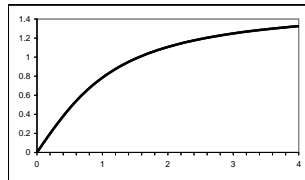
S-30. **Sketching graphs:** Answers will vary. One solution is



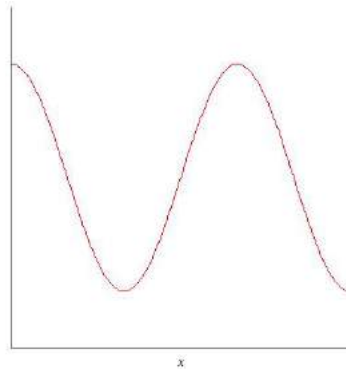
S-31. **Sketching graphs:** Answers will vary. One solution is



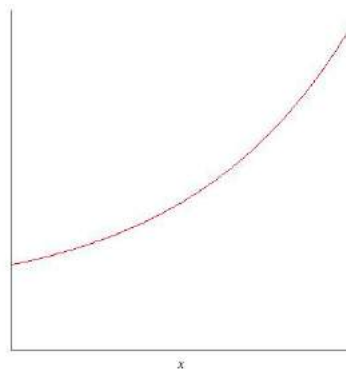
S-32. **Sketching graphs:** Answers will vary. One solution is



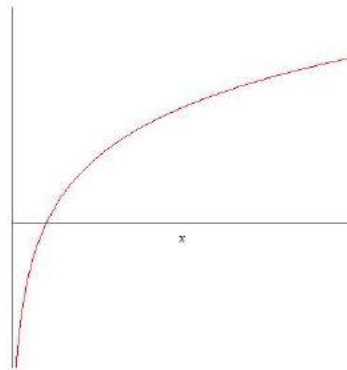
S-33. Answers will vary. One solution is



S-34. Answers will vary. One solution is



S-35. Answers will vary. One solution is



1.4 FUNCTIONS GIVEN BY WORDS

1. **Fixed and variable cost:** Now

$$C(0) = \text{Fixed cost} = 1500 \text{ dollars,}$$

and

$$\begin{aligned} C(1) &= \text{Fixed cost} + \text{Variable costs due to 1 item} \\ &= 1500 + 3 \times 1 = 1503 \text{ dollars,} \end{aligned}$$

$$\begin{aligned} C(2) &= \text{Fixed cost} + \text{Variable costs due to 2 items} \\ &= 1500 + 3 \times 2 = 1506 \text{ dollars,} \end{aligned}$$

$$\begin{aligned} C(3) &= \text{Fixed cost} + \text{Variable costs due to 3 items} \\ &= 1500 + 3 \times 3 = 1509 \text{ dollars,} \end{aligned}$$

and so on. This suggests the formula $C = 1500 + 3n$.

2. **Depreciation:** Now

$$T(0) = \text{Taxable value initially} = 23,000 \text{ dollars,}$$

and

$$\begin{aligned} T(1) &= \text{Taxable value initially} - \text{Decrease over 1 yr} \\ &= 23,000 - 2000 \times 1 = 21,000 \text{ dollars,} \end{aligned}$$

$$\begin{aligned} T(2) &= \text{Taxable value initially} - \text{Decrease over 2 yr} \\ &= 23,000 - 2000 \times 2 = 19,000 \text{ dollars,} \end{aligned}$$

and so on. This suggests the formula $T = 23,000 - 2000n$.

3. Fat in fast food:

- (a) There are 24 grams of fat in each hamburger, so for 3 hamburgers the total fat is $24 \times 3 = 72$ grams. Altogether, the amount of fat in an order of 3 hamburgers, 1 chicken sandwich, 2 orders of fries, and 2 orders of onion rings is

$$24 \times 3 + 13 \times 1 + 30 \times 2 + 25 \times 2 = 195 \text{ grams.}$$

- (b) To find the total grams of fat F in an order, as in part a we add the fat due to hamburgers, chicken sandwiches, and so forth. For example, h hamburgers contribute $24h$ grams of fat. The resulting formula is

$$F = 24h + 13c + 30f + 25o.$$

4. Thanksgiving dinner:

- (a) We start with an example. If the turkey weighs 12 pounds, because the cooking time for a turkey is about 15 minutes per pound we will need $15 \times 12 = 180$ minutes to bake the turkey after the oven is preheated. It takes 10 minutes to preheat the oven, so the total time T needed to preheat and then bake a turkey weighing 12 pounds is $T = 15 \times 12 + 10$ minutes or 190 minutes. This result suggests the formula

$$T = 15 \text{ minutes per pound} \times \text{Weight in pounds} + 10 \text{ minutes to preheat} = 15 \times p + 10,$$

$$\text{or } T = 15p + 10.$$

- (b) To find the approximate time required to prepare a turkey weighing 18 pounds, we put $p = 18$ in the formula from part a: $T(18) = 15 \times 18 + 10 = 280$. Thus the time required is 280 minutes or 4 hours and 40 minutes.

5. United States population growth:

- (a) Since t is the number of years since 1960, the year 1963 is represented by $t = 3$. In functional notation, the population of the U.S. in 1963 is given by $N(3)$. To calculate its value, we use the fact that the population increases by 1.2% per year. Since $N(0) = 180$, in millions,

$$N(1) = \text{Population in 1960} + 1.2\% \text{ growth}$$

$$= 180 + 0.012 \times 180 = 182.16 \text{ million}$$

$$N(2) = \text{Population in 1961} + 1.2\% \text{ increase}$$

$$= 182.16 + 0.012 \times 182.16 = 184.35 \text{ million}$$

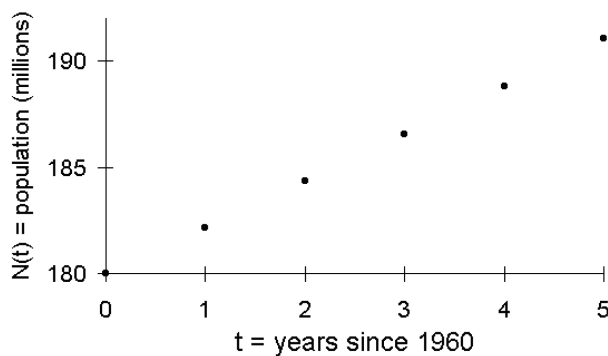
$$N(3) = \text{Population in 1962} + 1.2\% \text{ growth}$$

$$= 184.35 + 0.012 \times 184.35 = 186.56 \text{ million.}$$

(b) The table is given below, calculating $N(4)$ and $N(5)$ as in Part (a) above.

Year	t	$N(t)$
1960	0	180.00
1961	1	182.16
1962	2	184.35
1963	3	186.56
1964	4	188.80
1965	5	191.06

(c) A graph is shown below.



(d) The formula 180×1.012^t can be shown to give the same values as those found in Part (b) by substituting $t = 0, t = 1, \dots$ into the formula:

t	180×1.012^t
0	$180 \times 1.012^0 = 180.00$
1	$180 \times 1.012^1 = 182.16$
2	$180 \times 1.012^2 = 184.35$
3	$180 \times 1.012^3 = 186.56$
4	$180 \times 1.012^4 = 188.80$
5	$180 \times 1.012^5 = 191.06$

(e) Using the formula and the fact that 2000 corresponds to $t = 40$, we find that the population in 2000 from this prediction is

$$N(40) = 180 \times 1.012^{40} = 290.06 \text{ million people.}$$

This estimate is about 9 million higher than the actual population.

6. Education and income:

- (a) The median income of an individual with two years of college is, in functional notation, $I(2)$. The median income increases by 12% per year of college, and we begin with $I(0) = 35,400$. Thus

$$I(1) = \text{Income for no college} + 12\% \text{ increase} = 35,400 + 0.12 \times 35,400 = 39,648,$$

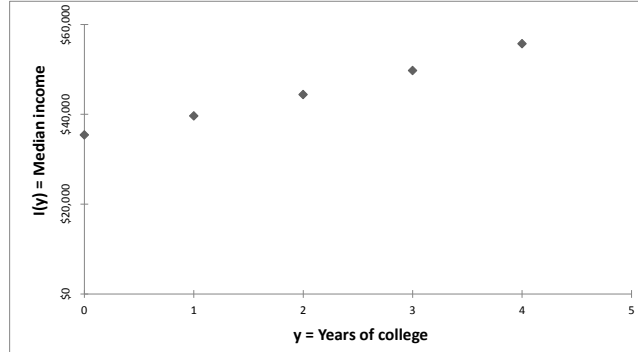
$$I(2) = \text{Income for 1 year} + 12\% \text{ increase} = 39,648 + 0.12 \times 39,648 = 44,405.76.$$

The median income of an individual with two years of college in 2011 was \$44,405.76, or about \$44,410.

- (b) Calculating $I(3)$ and $I(4)$ as above in Part (a), we get the following table.

Years of college	Median income (in 2011)
0	\$ 35,400.00
1	\$ 39,648.00
2	\$ 44,405.76
3	\$ 49,734.45
4	\$ 55,702.58

- (c) The graph is:



- (d) The formula $I = 35.4 \times 1.12^y$ in thousands of dollars yields the values for I shown below.

$y = \text{Years of college}$	$I = 35.4 \times 1.12^y$ in thousands of dollars
0	$35.4 \times 1.12^0 = 35.40000$
1	$35.4 \times 1.12^1 = 39.64800$
2	$35.4 \times 1.12^2 = 44.40576$
3	$35.4 \times 1.12^3 = 49.73445$
4	$35.4 \times 1.12^4 = 55.70259$

This is identical to the calculation in Part (b), except for a small difference in $I(4)$.

- (e) A master's degree means two additional years after the bachelor's degree, so $y = 4 + 2 = 6$. Using the formula, we have

$$I(6) = 35.4 \times 1.12^6 = 69.87332 \text{ thousand dollars.}$$

The median income of a person with a master's degree in 2011 would be \$69,873.32, or about \$69,870.

- (f) Three years past the master's degree means that $y = 6 + 3 = 9$. Using the formula, we have

$$I(9) = 35.4 \times 1.12^9 = 98.16699 \text{ thousand dollars.}$$

Assuming the formula applies, we see that the median income of a person with a doctoral degree in 2011 would be \$98,166.99, or about \$98,170.

- (g) Since the actual median income of a person with a doctoral degree in 2011 was \$91,000, which is lower than the amount predicted assuming 12% increases per year, the actual increase in median income for the years spent earning a doctoral degree is lower than 12%.

7. Altitude:

- (a) Let t be the time in minutes since takeoff and A the altitude in feet. Then

$$A(0) = \text{Initial altitude} = 200 \text{ feet,}$$

and

$$\begin{aligned} A(1) &= \text{Initial altitude} + \text{Increase over 1 minute} \\ &= 200 + 150 \times 1 = 350 \text{ feet,} \end{aligned}$$

$$\begin{aligned} A(2) &= \text{Initial altitude} + \text{Increase over 2 minutes} \\ &= 200 + 150 \times 2 = 500 \text{ feet,} \end{aligned}$$

$$\begin{aligned} A(3) &= \text{Initial altitude} + \text{Increase over 3 minutes} \\ &= 200 + 150 \times 3 = 650 \text{ feet,} \end{aligned}$$

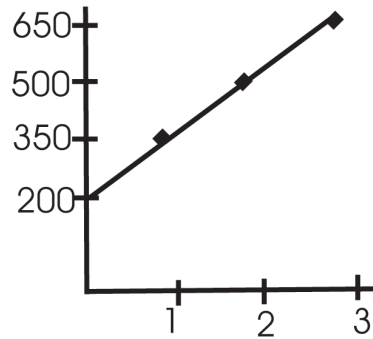
and so on. This suggests the formula

$$A = \text{Initial altitude} + \text{Increase over } t \text{ minutes} = 200 + 150 \times t,$$

or $A = 200 + 150t$.

- (b) In functional notation the altitude 90 seconds after takeoff is $A(1.5)$ because 90 seconds corresponds to 1.5 minutes. The value is $200 + 150 \times 1.5 = 425$ feet.

(c) Here is the graph.



The altitude changes at a constant rate, and this is reflected in the fact that the graph is a straight line.

8. Swimming records:

(a) Let t be the time in years since 1950 and R the record time in seconds. Then

$$R(0) = \text{Record time in 1950} = 63.2 \text{ seconds,}$$

and

$$\begin{aligned} R(1) &= \text{Record time in 1950} - \text{Decrease over 1 yr} \\ &= 63.2 - 0.4 \times 1 = 62.8 \text{ seconds,} \end{aligned}$$

$$\begin{aligned} R(2) &= \text{Record time in 1950} - \text{Decrease over 2 yr} \\ &= 63.2 - 0.4 \times 2 = 62.4 \text{ seconds,} \end{aligned}$$

and so on. This suggests the formula

$$R = \text{Record time in 1950} - \text{Decrease over } t \text{ yr} = 63.2 - 0.4 \times t,$$

or $R = 63.2 - 0.4t$.

(b) In functional notation the world record time in 1955 is $R(5)$ because 1955 is 5 years after 1950. The value is $63.2 - 0.4 \times 5 = 61.2$ seconds.

(c) We would not expect the formula to be valid indefinitely because the record time could not continue to decrease at the same rate over a long period. In fact, according to the formula the record time will eventually be 0, which is nonsense.

9. A rental:

(a) If we rent the car for 2 days and drive 100 miles, it will cost us

$$2 \text{ days rental} + \text{charge for 100 miles} = 49 \times 2 + 0.25 \times 100 = \$123.$$

- (b) Let d be the number of days we rent the car, m the number of miles we drive the car, and C the cost in dollars of renting the car. Then the formula that gives us the cost of renting a car is

$$C(d, m) = 49 \times \text{days} + 0.25 \times \text{miles} = 49d + 0.25m.$$

- (c) Since we drove from Dallas to Austin and back then we traveled a total of 500 miles. We kept the car for one week, or 7 days. Hence in functional notation the cost of the rental car is $C(7, 500)$. This is calculated as

$$C(7, 500) = 49 \times 7 + 0.25 \times 500 = \$468.$$

10. Preparing a letter:

- (a) The secretarial cost is 2×9.25 , the paper cost is 3×0.03 , and the envelope cost is 0.50. In total, the letter costs $2 \times 9.25 + 3 \times 0.03 + 0.50 = \19.09 .
- (b) Let h be the number of hours spent typing the letter, let p be the number of pages of the letter, and let C be the cost of preparing and mailing the letter (measured in dollars). Then

$$C(h, p) = \text{Secretarial cost} + \text{Cost for paper} + \text{Envelope} = 9.25h + 0.03p + 0.50.$$

- (c) Here $h = \frac{25}{60}$ and $p = 2$, so the cost is

$$C\left(\frac{25}{60}, 2\right) = 9.25 \times \frac{25}{60} + 0.03 \times 2 + 0.50 = \$4.41.$$

11. Preparing a letter, continued:

- (a) There are 2 pages of regular stationery and 1 of fancy letterhead stationery, so the total cost is $2 \times 0.03 + 1 \times 0.16 = 0.22$ dollar, or 22 cents.
- (b) We know from Part (a) that the cost for stationery is 0.22 dollar. The secretarial cost is 2×9.25 dollars, and the cost for the envelope is 0.50 dollar. In total, the letter costs $0.22 + 2 \times 9.25 + 0.50 = 19.22$ dollars.
- (c) Let c be the cost in dollars of the stationery and p the number of pages. There are $p - 1$ pages of regular stationery and 1 of fancy letterhead stationery. Then

$$\begin{aligned} c(p) &= \text{Cost for fancy letterhead stationery} + \text{Cost for regular stationery} \\ &= 0.16 + 0.03(p - 1). \end{aligned}$$

- (d) Let h be the number of hours spent typing the letter, let p be the number of pages of the letter, and let C be the cost of preparing and mailing the letter (measured in dollars). Then

$$\begin{aligned} C(h, p) &= \text{Secretarial cost} + \text{Cost for paper} + \text{Cost for envelope} \\ &= 9.25h + 0.16 + 0.03(p - 1) + 0.50. \end{aligned}$$

(This can also be written as $C(h, p) = 9.25h + 0.03(p - 1) + 0.66$ or as $C(h, p) = 9.25h + 0.03p + 0.63$.)

- (e) Here $h = \frac{25}{60}$ and $p = 2$, so the cost is

$$C\left(\frac{25}{60}, 2\right) = 9.25 \times \frac{25}{60} + 0.16 + 0.03(2 - 1) + 0.50 = 4.54 \text{ dollars.}$$

12. A car that gets m miles per gallon:

- (a) If we divide the number of miles traveled by how many miles per gallon the car gets, we get the number of gallons used. If we multiply that by the cost of gas per gallon we get the cost of operating the car. So in this case, the cost will be

$$\frac{\text{Miles}}{\text{Miles per gallon}} \times \text{Cost per gallon} = \frac{100}{25} \times \$3.49 = \$13.96.$$

Here we converted the cost of a gallon of gas from 349 cents to 3.49 dollars so that the cost of operating the car will be in dollars.

- (b) The cost is given by the formula

$$C(m, g, d) = \frac{\text{Miles}}{\text{Miles per gallon}} \times \text{Cost per gallon} = \frac{d}{m}g,$$

where C is measured in dollars, d is measured in miles, g is measured in dollars per gallon, and m is measured in miles per gallon.

- (c) The cost is expressed in functional notation as $C(28, 3.69, 138)$. This is calculated as

$$C(28, 3.69, 138) = \frac{138 \times 3.69}{28} = \$18.19.$$

13. Stock turnover rate:

- (a) If 350 shirts are sold annually then the number of orders of 50 shirts is $\frac{350}{50} = 7$.
- (b) The annual stock turnover rate is 7, the number computed in Part (a), because this is the number of times that the average inventory of 50 shirts needs to be replaced if 350 shirts are sold in a year.
- (c) If 500 shirts were sold in a year then the annual stock turnover rate would be $\frac{500}{50} = 10$.

(d) Let T be the annual stock turnover rate and S the number of shirts sold in a year.

Then

$$T = \frac{\text{Number of shirts sold}}{\text{Average inventory}} = \frac{S}{50}.$$

Thus the formula is $T = \frac{S}{50}$.

14. Stock turnover rate, continued:

(a) If the average inventory is 5 shovels then the annual stock turnover rate is $\frac{80}{5} = 16$.

This is the same as the yearly number of orders of 5 shovels.

(b) If the average inventory is 20 shovels then the annual stock turnover rate is $\frac{80}{20} = 4$.

(c) Let T be the annual stock turnover rate and I the average inventory of shovels.

Then

$$T = \frac{\text{Number of shovels sold}}{\text{Average inventory}} = \frac{80}{I}.$$

Thus the formula is $T = \frac{80}{I}$.

15. Total cost:

(a) Let N be the number of widgets produced in a month and C the total cost in dollars. Because

$$\text{Total cost} = \text{Variable cost} \times \text{Number of items} + \text{Fixed costs},$$

we have $C = 15N + 9000$.

(b) In functional notation the total cost is $C(250)$, and the value is $15 \times 250 + 9000 = 12,750$ dollars.

16. Total revenue and profit:

(a) Let N be the number of widgets produced in a month and R the total revenue in dollars. Because

$$\text{Total revenue} = \text{Selling price} \times \text{Number of items},$$

we have $R = 25N$.

(b) Let P be the profit in dollars. Because

$$\text{Profit} = \text{Total revenue} - \text{Total cost},$$

we have $P = R - C$. Using the formulas from Part (a) and from Exercise 15, we obtain

$$P = 25N - (15N + 9000).$$

This can be simplified to $P = 10N - 9000$.

(c) In functional notation the profit is $P(250)$, and the value is $10 \times 250 - 9000 = -6500$ dollars.

(d) We saw in Part (c) that the profit at a level of 250 widgets is negative, so the manufacturer has a loss at this level. At the production level of 1000 widgets, the profit is

$$P(1000) = 10 \times 1000 - 9000 = 1000 \text{ dollars};$$

this number is positive, so the manufacturer turns a profit at this level.

17. More on revenue:

(a) We compute

$$p(100) = 50 - 0.01 \times 100 = 49$$

$$p(200) = 50 - 0.01 \times 200 = 48$$

$$p(300) = 50 - 0.01 \times 300 = 47$$

$$p(400) = 50 - 0.01 \times 400 = 46$$

$$p(500) = 50 - 0.01 \times 500 = 45.$$

These values agree with those in the table.

(b) We have $R = pN$, so $R = (50 - 0.01N)N$ dollars.

(c) In functional notation the total revenue is $R(450)$, and the value is

$$(50 - 0.01 \times 450) \times 450 = 20,475 \text{ dollars}.$$

18. More on profit:

(a) We know that $P = R - C$, that $R = (50 - 0.01N)N$, and that $C = 15N + 9000$.

Thus we have

$$P = (50 - 0.01N)N - (15N + 9000).$$

(This can also be written as $P = -0.01N^2 + 35N - 9000$.)

(b) At a level of 200 widgets, the profit is

$$P(200) = (50 - 0.01 \times 200) \times 200 - (15 \times 200 + 9000) = -2400 \text{ dollars};$$

this number is negative, so the manufacturer has a loss at this level. At a level of 700 widgets, the profit is

$$P(700) = (50 - 0.01 \times 700) \times 700 - (15 \times 700 + 9000) = 10,600 \text{ dollars};$$

this number is positive, so the manufacturer turns a profit at this level. At a level of 1200 widgets, the profit is

$$P(1200) = (50 - 0.01 \times 1200) \times 1200 - (15 \times 1200 + 9000) = 18,600 \text{ dollars};$$

this number is positive, so the manufacturer turns a profit at this level.

19. Renting motel rooms:

(a) Since the group rents 2 extra rooms, we take off \$4 from the base price of each room, which means that we charge \$81 for each room. Since we rented 3 rooms altogether, we take in a total of $3 \times 81 = \$243$.

(b) The formula that tells us how much to charge for each room is

$$\text{Rate} = \text{Base price} - \$2 \text{ per extra room dollars.}$$

Now if n rooms are rented, then the number of extra rooms is $n - 1$. Since the base price is \$85, the formula can be written as

$$\text{Rate} = 85 - 2 \times \text{number of extra rooms} = 85 - 2(n - 1) \text{ dollars.}$$

This can also be written as $\text{Rate} = 87 - 2n$ dollars.

(c) The total revenue is the number of rooms times the rental per room:

$$R(n) = \text{Rooms rented} \times \text{Price per room} = n \times (85 - 2(n - 1)) \text{ dollars.}$$

This can also be written as $R(n) = n(87 - 2n)$ dollars.

(d) The total cost of renting 9 rooms is expressed in functional notation as $R(9)$. This is calculated as $R(9) = 9(85 - 2 \times (9 - 1)) = \621 .

20. A cattle pen: The formula for the area of a rectangle is:

$$\text{Area} = \text{Length} \times \text{Width}$$

and so

$$400 = \text{Length} \times \text{Width.}$$

(a) If one side of the pen is 40 feet long, then, since the area is 400 square feet, the other side must be 10 feet long. The picture should show a rectangle with two sides labeled 40 feet and two sides labeled 10 feet. The total amount of fence needed is two times 40 plus two times 10, or a total of 100 feet of fence.

- (b) If the pen were square, then each of the four sides would have the same length s . Since the area is 400 square feet, $400 = s \times s = s^2$ and so $s = \sqrt{400} = 20$ feet. So if the fence were a square, the total amount of fence needed would be four times 20, or a total of 80 feet of fence.
- (c) Yes. The pens in Parts (a) and (b) have the same area but different perimeters.
- (d) If one side of the rectangle has length l , then the width w can be calculated since $l \times w = 400$. Thus $w = \frac{400}{l}$. The total amount of fence needed is two times l plus two times w . Thus

$$\begin{aligned} \text{Amount of fence} &= 2l + 2w \\ F &= 2l + 2\left(\frac{400}{l}\right). \end{aligned}$$

21. Catering a dinner:

- (a) i. If 50 people attend, then your cost is the rental fee plus the caterer's fee for each of the 50 people, or a total of $150 + 50 \times 10 = 650$ dollars. If 50 people attend, then to break even, each ticket should cost $\frac{650}{50} = 13$ dollars.
- ii. Let n be the number of people attending and C the amount in dollars you should charge per person. Your total cost is the rental fee plus the caterer's fee for each of the n people, or a total of $150 + 10n$ dollars. Since n people attend, then to break even, each ticket should cost

$$C = \frac{150 + 10n}{n} \text{ dollars.}$$

This can also be written as $C = \frac{150}{n} + 10$, which can be thought of as each person's share of the \$150 rental fee plus the \$10 caterer's fee.

- iii. If 65 people attend, then the amount to charge each is expressed in functional notation as $C(65)$. This is calculated as

$$C(65) = \frac{150}{65} + 10 = \$12.31 \text{ per ticket.}$$

- (b) To make a profit of \$100, you should think of your cost as being \$100 more in determining your ticket price, so your new price would be

$$P = \frac{100 + 150 + 10n}{n} = \frac{250 + 10n}{n} \text{ dollars.}$$

This can also be written as $P = \frac{250}{n} + 10$ or as $P = \frac{100}{n} + \frac{150}{n} + 10$, which can be thought of as each person's share of the \$100 profit, plus each person's share of the \$150 rental fee, plus the \$10 caterer's fee.

22. A car:

(a) Since distance traveled is speed times time, the constant of proportionality is the time we traveled, namely 3 hours.

(b) We have

$$d = \text{Distance} = \text{Speed} \times \text{Time} = 3s \text{ miles.}$$

23. Production rate:

(a) Let k be the constant of proportionality. Then $t = kn$.

(b) Since t is total number of items produced and n is the number of employees, it follows that k is the number of items produced per employee.

24. Density:

(a) The total weight is $3 \times 2 = 6$ pounds.

(b) The relation is

$$\text{Weight} = \text{Size} \times \text{Density},$$

or $w = sd$. The constant of proportionality is the size s .

(c) We have $s = 14$ and $d = 0.3$, so $w = 14 \times 0.3 = 4.2$ pounds. Thus the total weight is 4.2 pounds.

25. Head and pressure:

(a) Because p is proportional to h with constant of proportionality 0.434, the equation is $p = 0.434h$.

(b) The head of water at the mouth of the nozzle is $8 \times 12 = 96$ feet. The back pressure is the value of p given by the equation in Part (a) with the head of $h = 96$ feet: $p = 0.434 \times 96 = 41.66$. Thus the back pressure is 41.66 pounds per square inch.

(c) The head is the height of the nozzle above the pumper, and in this case that value is $185 - 40 = 145$ feet. The back pressure is $p = 0.434 \times 145 = 62.93$ pounds per square inch.

26. Head and aquifers: We compare the pressure of the fluid in the sand with the pressure of the water in the rock. For the fluid in the sand, the head is 4.3 meters and the density is 1050 kilograms per cubic meter, so the pressure is

$$\text{Pressure} = \text{Density} \times 9.8 \times \text{Head} = 1050 \times 9.8 \times 4.3 = 44,247 \text{ newtons per square meter.}$$

For the water in the rock, the head is 4.4 meters and the density is 990 kilograms per cubic meter, so the pressure is

$$\text{Pressure} = 990 \times 9.8 \times 4.4 = 42,688.8, \text{ or about } 42,689, \text{ newtons per square meter.}$$

Because the pressure of the fluid in the sand is higher than that of the water in the rock, the city water supply is in danger of contamination.

27. Darcy's law:

- (a) Because V is proportional to S with constant of proportionality K , the equation is $V = KS$.
- (b) The constant K equals the permeability of sandstone, 0.041 meter per day. Also, S is given as 0.03. We compute the velocity of the water flow using the equation in Part (a): $V = 0.041 \times 0.03 = 0.00123$. The units are found by multiplying the units for K with those for S , and we have that the velocity is 0.00123 meter per day.
- (c) Now we take the constant K to be 41 meters per day, but S is still 0.03. The velocity of the water flow is $V = 41 \times 0.03 = 1.23$ meters per day.

28. Hubble's constant:

- (a) Because V is proportional to D with constant of proportionality H , the equation is $V = HD$.
- (b) We are given that $H = 70$ and that $D = 122.7$. We compute the velocity using the equation in Part (a): $V = 70 \times 122.7 = 8589$. Thus the velocity is 8589 kilometers per second.
- (c) The relation gives $y = \frac{10^{12}}{H} = \frac{10^{12}}{530}$, which is about 1.89×10^9 years, or about 1.9 billion years.
- (d) The relation gives $y = \frac{10^{12}}{70}$, which is about 1.43×10^{10} years, or about 14.3 billion years.

29. Loan origination fee:

- (a) The fees for securing a mortgage for \$322,000 are \$2500 plus 2% of the mortgage amount. In this case, that is $2500 + 0.02 \times 322,000 = \8940 .
- (b) The formula for the loan fee is $F = 2500 + 0.02M$.

30. **The $3x + 1$ problem:**

(a) We have

$$f(1) = 3(1) + 1 = 4$$

$$f(4) = \frac{4}{2} = 2$$

$$f(2) = \frac{2}{2} = 1.$$

The procedure repeats this cycle over and over.

(b) We have

$$f(5) = 3(5) + 1 = 16$$

$$f(16) = \frac{16}{2} = 8$$

$$f(8) = \frac{8}{2} = 4$$

$$f(4) = 2$$

$$f(2) = 1.$$

It took 5 steps to get to 1.

(c) We have

$$f(7) = 3(7) + 1 = 22$$

$$f(22) = 11$$

$$f(11) = 34$$

$$f(34) = 17$$

$$f(17) = 52$$

$$f(52) = 26$$

$$f(26) = 13$$

$$f(13) = 40$$

$$f(40) = 20$$

$$f(20) = 10$$

$$f(10) = 5.$$

We followed the rest of this trail in Part (b). So it takes 16 steps to get to 1 starting with 7.

- (d) These answers will vary. If a number is found that does not lead back to 1, the computation should be checked very carefully.

31. **Research project:** Answers will vary.

Skill Building Exercises

S-1. **A description:** If you have \$5000 and spend half the balance each month, then the new balance will be

$$\text{New balance after 1 month} = 5000 - \frac{1}{2} \times 5000 = 2500$$

$$\text{New balance after 2 months} = 2500 - \frac{1}{2} \times 2500 = 1250$$

$$\text{New balance after 3 months} = 1250 - \frac{1}{2} \times 1250 = 625$$

$$\text{New balance after 4 months} = 625 - \frac{1}{2} \times 625 = 312.5$$

and so the balance left after 4 months is \$312.50.

S-2. **Light:** If light travels 186,000 miles per second, then the amount of time it takes to travel 93,000,000 miles is $\frac{93,000,000}{186,000} = 500$ seconds, which is 8.33 minutes. Thus it takes 8.33 minutes for light to travel from the sun to Earth.

S-3. **A description:** We know that $f(0) = 5$ and that each time x increases by 1, the value of f triples, that is, it is three times its previous value. Therefore

$$f(1) = 3 \times f(0) = 3 \times 5 = 15$$

$$f(2) = 3 \times f(1) = 3 \times 15 = 45$$

$$f(3) = 3 \times f(2) = 3 \times 45 = 135$$

$$f(4) = 3 \times f(3) = 3 \times 135 = 405$$

so $f(4) = 405$. On the other hand, 5×3^4 is also 405.

S-4. **Getting a formula:** The profit P is the revenue you receive from each glass of lemonade sold less the cost of the ingredients. That is,

$$\begin{aligned} P &= \text{Revenue received} - \text{Cost of ingredients} \\ &= \text{Price per glass} \times \text{Glasses sold} - \text{Cost of ingredients} \\ &= 0.25 \times n - 2.00 \text{ dollars.} \end{aligned}$$

S-5. **Getting a formula:** If the man loses 67 strands of hair each time he showers, then if he showers s times, he will lose $67 \times s$ strands of hair. Thus $N = 67s$.

- S-6. **Getting a formula:** If you pay \$500 rental plus you pay \$10 per guest, then the total cost you pay is \$500 plus \$10 times the number of guests. Thus $C = 500 + 10n$.
- S-7. **Getting a formula:** If you start with \$500 and you add to that \$37 each month, then the balance will be \$500 plus \$37 times the number of months. Thus the balance B is given by $B = 500 + 37t$.
- S-8. **Getting a formula:** We measure all temperatures in degrees. The difference between the temperature of the object T and room temperature 75 is 325×0.07^t . Thus $T - 75 = 325 \times 0.07^t$, and therefore $T = 75 + 325 \times 0.07^t$.
- S-9. **Getting a formula:** You pay \$249 for the iPod and the cost of downloaded songs is \$0.99 per song, so the total cost you pay is \$249 plus \$0.99 times the number of songs. Thus $C = 249 + 0.99s$.
- S-10. **Getting a formula:** You pay $2700 + 622 + 850 + 322 = 4494$ dollars for the complete home video system. The cost of cable service is \$122 per month, so the total cost you pay is \$4494 plus \$122 times the number of months. Thus $C = 4494 + 122m$.
- S-11. **Getting a formula:** Your total cost is the sum of the cost of the staples, the paper, and the pens. The cost of the staples is \$1.46 times the number of boxes of staples, the cost of the paper is \$3.50 times the number of reams of paper, and the cost of the pens is \$2.40 times the number of boxes of pens. Thus $C = 1.46s + 3.50r + 2.40p$.
- S-12. **Getting a formula:** To find the total profit, we first find the circulation, which is 8000 plus 5 papers per dollar spent on advertising, a , so the total circulation is $8000 + 5a$. The profit on this circulation, taking into account the cost of advertising, is 7 cents, or \$0.07, per paper sold, so the profit is $0.07(8000 + 5a)$. Finally, we find the total profit by subtracting the \$400 cost of the delivery of the papers, so $P = 0.07(8000 + 5a) - 400$.
- S-13. **Getting a formula:** To find the net profit, we find the revenue from the jewelry and subtract the cost of the silver and the turquoise. The revenue from the jewelry is \$500 times the number of ounces, which is the total of the silver and turquoise ounces, so the revenue is $500(s + t)$. The cost of the materials is \$300 times the number of ounces of silver plus the \$21 times the number of ounces of turquoise, so the total cost is $300s + 21t$, and so $P = 500(s + t) - (300s + 21t)$, or $500(s + t) - 300s - 21t$.
- S-14. **Proportionality:** If f is proportional to x and the constant of proportionality is 8, then $f = 8x$.
- S-15. **Constant of proportionality:** If $g(t) = 16t$, then g is proportional to t with constant of proportionality 16.

- S-16. **Proportionality and initial value:** Since y is proportional to x , this means that y is a multiple of x . When x is 0, then y is a multiple of 0 and so therefore is also equal to 0.
- S-17. **Pizza:** For cheese pizzas of a fixed diameter, the weight is proportional to the thickness. For example, if you stack two pizzas of the same diameter the weight doubles, as does the thickness.
- S-18. **More pizza:** The weight of a pizza is not proportional to its diameter. For example if one pizza has twice the diameter of the other, then the larger has four times the area of the smaller (and so four times the weight), not twice the area .
- S-19. **A tire:** The circumference of a tire is proportional to its radius, since the circumference is 2π times the radius.
- S-20. **Snowfall:** Yes, the amount of snowfall is proportional to the time since the snow is falling at a constant rate (in fact, that rate is the constant of proportionality).
- S-21. **A square box:** Since the volume of a cube is the third power of the length of a side, it is not proportional to the length of a side. For example, if the length of the sides doubles, then the volume of the cube is multiplied by 8, not 2.
- S-22. **A man's height:** A man's height is not proportional to his age since a maximum height is usually reached by the mid-twenties at the latest, so after that the age increases while the height doesn't. For example, a 60-year-old is not twice the height of when he was 30 years old.
- S-23. **Wages:** Since the man makes \$16 per hour, the monthly salary is 16 times the number of hours worked, so the monthly salary is proportional to the number of hours worked. Here the constant of proportionality is 16 dollars per hour.
- S-24. **Sales tax:** The sales tax owed is a fixed percentage of the purchase price, so the tax is proportional to the price. Here the constant of proportionality is the sales tax rate (as a decimal).
- S-25. **Sodas:** Since the price per soda is fixed, the cost of buying sodas is proportional to the number of sodas bought. Here the constant of proportionality is the price per soda.

Chapter 1 Review Exercises

1. **Evaluating formulas:** To get the function value $M(9500, 0.01, 24)$, substitute $P = 9500$, $r = 0.01$, and $t = 24$ in the formula

$$M(P, r, t) = \frac{Pr(1+r)^t}{(1+r)^t - 1}.$$

The result is

$$\frac{9500 \times 0.01 \times (1 + 0.01)^{24}}{(1 + 0.01)^{24} - 1},$$

which equals 447.20.

2. **U.S. population:**

- (a) Because 1790 corresponds to $t = 0$, the population in 1790 was $3.93 \times 1.03^0 = 3.93$ million.
- (b) Because 1810 is 20 years after 1790, we take $t = 20$ and get that $N(20)$ is functional notation for the population in 1790.
- (c) To find the population in 1810 we put $t = 20$ in the formula. The result is $3.93 \times 1.03^{20} = 7.10$, so the population in 1810 was 7.10 million according to the formula.

3. **Averages and average rate of change:**

- (a) Because 5 is halfway between 4 and 6, we estimate $f(5)$ by

$$\frac{f(4) + f(6)}{2} = \frac{40.1 + 43.7}{2} = 41.9.$$

- (b) The average rate of change is the change in f divided by the change in x , and that is

$$\frac{f(6) - f(4)}{2} = \frac{43.7 - 40.1}{2} = 1.8.$$

4. **High school graduates:**

- (a) Here $N(1989)$ represents the number, in millions, graduating from high school in 1989. According to the table, its value is 2.47 million.
- (b) In functional notation the number of graduates in 1988 is $N(1988)$. We estimate its value by averaging:

$$\frac{N(1987) + N(1989)}{2} = \frac{2.65 + 2.47}{2} = 2.56.$$

Thus there were about 2.56 million graduating in 1988.

- (c) The average rate of change is the change in N divided by the change in t , and that is

$$\frac{N(1991) - N(1989)}{2} = \frac{2.29 - 2.47}{2} = -0.09 \text{ million per year.}$$

- (d) To estimate the value of $N(1994)$, we calculate $N(1991)$ plus 3 years of change at the average rate found in the previous part. So, $N(1994)$ is estimated to be

$$N(1991) + 3 \times -0.09 = 2.29 - 0.27 = 2.02.$$

Our estimate for $N(1994)$ is 2.02 million.

5. Increasing, decreasing, and concavity:

- (a) The function is increasing from 2006 to 2016.
- (b) It is concave down from 2012 to 2016 and concave up from 2006 to 2012.
- (c) There is an inflection point at $d = 2012$.

6. Logistic population growth:

- (a) The population grows rapidly at first and then the growth slows. Eventually it levels off.
- (b) The population reaches 300 in mid-2009.
- (c) The population is increasing most rapidly in 2008.
- (d) The point of most rapid population increase is an inflection point.

7. **Getting a formula:** The balance (in dollars) is the initial balance of \$780 minus \$39 times the number of withdrawals. Thus $B = 780 - 39t$.

8. Cell phone charges:

- (a) Let t denote the number of text messages and C the charge, in dollars.
- (b) The charge (in dollars) is the flat monthly rate of \$39.95 plus \$0.10 times the number of messages in excess of 100. That excess is $t - 100$, so the formula is $C = 39.95 + 0.1(t - 100)$.
- (c) In functional notation the cost if you have 450 messages is $C(450)$. The value is $39.95 + 0.1(450 - 100) = 74.95$ dollars.
- (d) If the number of text messages is less than 100 then the only charge is the flat monthly rate of \$39.95. So the formula is $C = 39.95$.

9. Cell phone charges again:

- (a) Let t denote the number of text messages, m the number of minutes, and C the charge, in dollars.
- (b) The charge (in dollars) is the flat monthly rate of \$34.95, plus \$0.35 times the number of minutes in excess of 4000, plus \$0.10 times the number of messages in excess of 100. Thus the formula is $C = 34.95 + 0.35(m - 4000) + 0.1(t - 100)$.
- (c) The charges are $34.95 + 0.35(6000 - 4000) + 0.1(450 - 100) = 769.95$ dollars.
- (d) If the number of text messages is less than 100 then the only charges are the flat monthly rate of \$34.95 plus \$0.35 times the number of minutes in excess of 4000. So the formula is $C = 34.95 + 0.35(m - 4000)$.
- (e) We use the formula from Part (d). The charges are $34.95 + 0.35(4200 - 4000) = 104.95$ dollars.

10. Practicing calculations:

- (a) We calculate that $C(0) = 0.2 + 2.77e^{-0.37 \times 0} = 2.97$.
- (b) We calculate that $C(0) = \frac{12.36}{0.03 + 0.55^0} = 12$.
- (c) We calculate that $C(0) = \frac{0 - 1}{\sqrt{0 + 1}} = -1$.
- (d) We calculate that $C(0) = 5 \times 0.5^{0/5730} = 5$.

11. Amortization:

- (a) We calculate that

$$M(5500, 0.01, 24) = \frac{5500 \times 0.01 \times (1 + 0.01)^{24}}{(1 + 0.01)^{24} - 1} = 258.90 \text{ dollars.}$$

Your monthly payment if you borrow \$5500 at a monthly rate of 1% for 24 months is \$258.90.

- (b) In functional notation the payment is $M(8000, 0.006, 36)$. The value is

$$\frac{8000 \times 0.006 \times (1 + 0.006)^{36}}{(1 + 0.006)^{36} - 1} = 247.75 \text{ dollars.}$$

12. Using average rate of change:

- (a) The average rate of change is the change in f divided by the change in x , and that is

$$\frac{f(3) - f(0)}{3} = \frac{55 - 50}{3} = 1.67.$$

- (b) Again, the average rate of change is the change in f divided by the change in x , and in this case that is

$$\frac{f(6) - f(3)}{3} = \frac{61 - 55}{3} = 2.$$

- (c) Because 4 is 1 unit more than 3 and the average rate of change is 2, we estimate $f(4)$ by

$$f(3) + 1 \times 2 = 55 + 2 = 57.$$

13. Timber values under Scribner scale:

- (a) The average rate of change from \$20 to \$24 is $\frac{81.60 - 68.00}{24 - 20} = 3.4$. The units here are dollar value per MBF Scribner divided by dollar value per cord. Continuing in this way, we get the following table, where the rate of change has the units just given.

Interval	20 to 24	24 to 28	28 to 36
Rate of change	3.4	3.4	3.4

Note that the change in the variable for the last interval is 8, not 4.

- (b) No, the value per MBF Scribner should not have a limiting value: Its rate of change is a nonzero constant, so we expect it to increase at a constant rate.
- (c) We use the average rate of change from \$24 to \$28 to estimate the value per MBF Scribner when the value per cord is \$25. That estimate is $81.60 + 1 \times 3.4 = 85$ dollars per MBF Scribner. Because \$85 is greater than \$71, if you are selling then \$25 per cord is a better value, but if you are buying then \$71 per MBF Scribner is a better value. Another way to do this is to note by inspecting the table that the value of \$25 per cord is greater than the value of \$71 per MBF Scribner: The value of \$25 per cord is greater than the value of \$24 per cord listed in the table, so it is higher than the equivalent value of \$81.60 per MBF Scribner listed in the table.
- 14. Concavity:** If a graph is decreasing at an increasing rate then it is concave down. If it is decreasing at a decreasing rate then it is concave up.

15. Longleaf pines:

- (a) The height of the tree increases quickly at first, but the growth rate decreases as the tree ages. It makes sense for a young tree to grow more quickly than an older tree.
- (b) According to the graph the tree height for a 60-year-old tree is about 132 feet.
- (c) Yes, there is a limiting value, since the graph eventually levels off.

- (d) The graph is concave down. This means that the height is increasing at a decreasing rate, so each year the amount of growth decreases.

16. Getting a formula:

- (a) If we rent 3 rooms we get a discount of $2 \times 2 = 4$ dollars, so each room will cost $56 - 4 = 52$ dollars.
- (b) Because each room will cost \$52 dollars, we will pay $3 \times 52 = 156$ dollars altogether.
- (c) If we rent n rooms we get a discount of $2(n-1) = 2n - 2$ dollars. If we let R denote the rental cost in dollars per room then $R = 56 - 2(n-1)$ or $R = 58 - 2n$ dollars.
- (d) Let C denote the total cost in dollars. Then $C = n \times (56 - 2(n-1))$ or $C = n \times (58 - 2n)$.

17. A wedding reception:

- (a) If you invite 100 guests then the cost is \$3200 for the venue plus \$31 times 50 (because 100 guests makes an excess of 50 over the number included). Thus the cost is $3200 + 31 \times 50 = 4750$ dollars.
- (b) If you invite n guests then the cost is \$3200 for the venue plus \$31 times the number in excess of 50. That excess is $n - 100$, so if we let C denote the cost in dollars then $C = 3200 + 31(n - 50)$ or $C = 1650 + 31n$.
- (c) We want to find n so that $C = 5500$. By the formula from Part (b) this says that $1650 + 31n = 5500$. By trial and error (or by solving for n using algebra) we find that we can invite 124 guests.

18. Limiting values:

- (a) No, not all tables show limiting values.
- (b) We can identify a limiting value from a table by checking whether the last few entries in the table show little change. If so, the limiting value is approximated by the trend established by the last few entries.
- (c) No, not all graphs show limiting values.
- (d) We can identify a limiting value from a graph by checking whether the last portion of the graph levels off. If so, the limiting value is approximated by that value where the graph is level.

A FURTHER LOOK: AVERAGE RATES OF CHANGE WITH FORMULAS

1. **Calculating rates of change:** The average rate of change is

$$\frac{f(4) - f(2)}{4 - 2} = \frac{\frac{1}{4} - \frac{1}{2}}{2} = -\frac{1}{8} \text{ or } -0.125.$$

Rounding to two decimal places gives -0.13 .

2. **Calculating rates of change:** The average rate of change is

$$\frac{f(2) - f(1)}{2 - 1} = \frac{5 - 2}{1} = 3.$$

3. **Calculating rates of change:** The average rate of change is

$$\frac{f(9) - f(4)}{9 - 4} = \frac{3 - 2}{5} = \frac{1}{5} \text{ or } 0.20.$$

4. **Average rates of change with variables:** The average rate of change is

$$\frac{f(3+h) - f(3)}{(3+h) - 3} = \frac{2(3+h) + 1 - (2 \times 3 + 1)}{h} = \frac{2h}{h} = 2.$$

5. **Average rates of change with variables:** The average rate of change is

$$\frac{f(h) - f(0)}{h - 0} = \frac{h^2 - 0^2}{h} = \frac{h^2}{h} = h.$$

6. **Difference quotients:** The average rate of change from x to $x+h$ is

$$\frac{f(x+h) - f(x)}{(x+h) - x} = \frac{3(x+h) + 1 - (3x + 1)}{h} = \frac{3x + 3h + 1 - 3x - 1}{h} = \frac{3h}{h} = 3.$$

7. **Difference quotients:** The average rate of change from x to $x+h$ is

$$\begin{aligned} \frac{f(x+h) - f(x)}{(x+h) - x} &= \frac{(x+h)^2 + (x+h) - (x^2 + x)}{h} = \frac{x^2 + 2xh + h^2 + x + h - x^2 - x}{h} \\ &= \frac{2xh + h^2 + h}{h} = 2x + h + 1. \end{aligned}$$

8. **Linear functions:** If $f(x) = mx + b$ then the average rate of change from p to q is

$$\frac{f(q) - f(p)}{q - p} = \frac{(mq + b) - (mp + b)}{q - p} = \frac{mq - mp}{q - p} = \frac{m(q - p)}{q - p} = m.$$

Thus the average rate of change is m , and this does not depend on either p or q .

9. **The effect of adding a constant:** The average rate of change for f is

$$\frac{f(b) - f(a)}{b - a},$$

and the average rate of change for g is

$$\frac{g(b) - g(a)}{b - a} = \frac{(f(b) + c) - (f(a) + c)}{b - a} = \frac{f(b) - f(a)}{b - a}.$$

Thus the two rates are the same. This makes sense because the rate of change of a constant is 0.

10. **A fish:** The average rate of growth over the first year is

$$\frac{L(1) - L(0)}{1 - 0} = \frac{(10 - \frac{1}{2}) - (10 - 1)}{1} = \frac{1}{2} \text{ or } 0.5 \text{ inch per year.}$$

11. **Radioactive decay:**

(a) The average rate of change from $t = 0$ to $t = 2$ is

$$\frac{A(2) - A(0)}{2 - 0} = \frac{\frac{20}{2^2} - \frac{20}{2^0}}{2} = \frac{5 - 20}{2} = \frac{-15}{2} \text{ or } -7.50 \text{ grams per minute.}$$

(b) Physically the meaning of a negative rate of change is that the amount of the radioactive substance is decreasing.

12. **A derivative:** When h is close to 0, the average rate of change $2x + h$ is close to $2x$. Thus the derivative of x^2 is $2x$.

13. **A derivative:** When h is close to 0, the average rate of change $3x^2 + 3xh + h^2$ is close to $3x^2$, because the two terms involving h get smaller and smaller as h gets smaller and smaller. Thus the derivative of x^3 is $3x^2$.

A FURTHER LOOK: AREAS ASSOCIATED WITH GRAPHS

1. **Rectangle:** The shaded region in the figure is a rectangle with base $6 - 3 = 3$. The height of the rectangle is the height of the graph of $f(x) = x^2$ at $x = 3$, so the height is $3^2 = 9$. Thus the area is

$$\text{Area of rectangle} = \text{Base} \times \text{Height} = 3 \times 9 = 27.$$

2. **Rectangle:** The shaded region in the figure is a rectangle with base $4 - 1 = 3$. The height of the rectangle is the height of the graph of $f(x) = x^2$ at $x = 4$, so the height is $4^2 = 16$. Thus the area is

$$\text{Area of rectangle} = \text{Base} \times \text{Height} = 3 \times 16 = 48.$$

3. **Triangle:** The shaded region in the figure is a triangle with base $8 - 2 = 6$. The height of the triangle is the height of the graph of $f(x) = x - 2$ at $x = 8$, so the height is $8 - 2 = 6$. Thus the area is

$$\text{Area of triangle} = \frac{1}{2} \text{Base} \times \text{Height} = \frac{1}{2} \times 6 \times 6 = 18.$$

4. **Triangle:** The shaded region in the figure is a triangle with base $5 - 1 = 4$. The height of the triangle is the height of the graph of $f(x) = 10 - 2x$ at $x = 1$, so the height is $10 - 2 \times 1 = 8$. Thus the area is

$$\text{Area of triangle} = \frac{1}{2} \text{Base} \times \text{Height} = \frac{1}{2} \times 4 \times 8 = 16.$$

5. **Lower sum:** The shaded region in the figure is made up of three rectangles. Each of the rectangles has base 1. The heights of the rectangles are determined by the graph. The left-hand rectangle has height $2^3 = 8$, so its area is $1 \times 8 = 8$. The middle rectangle has height $3^3 = 27$, so its area is $1 \times 27 = 27$. The right-hand rectangle has height $4^3 = 64$, so its area is $1 \times 64 = 64$. Therefore, the total area is $8 + 27 + 64 = 99$.

6. **Upper sum:** The shaded region in the figure is made up of three rectangles. Each of the rectangles has base 1. The heights of the rectangles are determined by the graph. The left-hand rectangle has height $3^3 = 27$, so its area is $1 \times 27 = 27$. The middle rectangle has height $4^3 = 64$, so its area is $1 \times 64 = 64$. The right-hand rectangle has height $5^3 = 125$, so its area is $1 \times 125 = 125$. Therefore, the total area is $27 + 64 + 125 = 216$.

7. **Trapezoid:** We think of the area of the shaded region as the difference between the areas of two triangles, one with base from $x = 0$ to $x = 3$ and the other with base from $x = 0$ to $x = 5$. The first of these triangles has base 3. Its height is the height of the graph of $f(x) = 4x$ at $x = 3$, so its height is $4 \times 3 = 12$. Thus the area of the first triangle is

$$\text{Area of triangle} = \frac{1}{2} \text{Base} \times \text{Height} = \frac{1}{2} \times 3 \times 12 = 18.$$

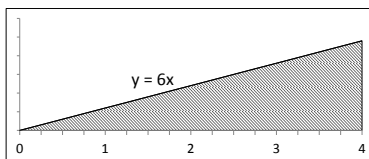
Similarly, the second triangle has base 5 and height $4 \times 5 = 20$, so its area is $\frac{1}{2} \times 5 \times 20 = 50$. The area of the shaded region is the difference between these two areas, which equals $50 - 18 = 32$. This area can be found in other ways, one of which is to use the formula for the area of a trapezoid.

8. **Trapezoid:** We think of the area of the shaded region as the difference between the areas of two triangles, one with base from $x = 4$ to $x = 6$ and the other with base from $x = 2$ to $x = 6$. The first of these triangles has base 2. Its height is the height of the graph of $f(x) = 18 - 3x$ at $x = 4$, so its height is $18 - 3 \times 4 = 6$. Thus the area of the first triangle is

$$\text{Area of triangle} = \frac{1}{2} \text{Base} \times \text{Height} = \frac{1}{2} \times 2 \times 6 = 6.$$

Similarly, the second triangle has base 4 and height $18 - 3 \times 2 = 12$, so its area is $\frac{1}{2} \times 4 \times 12 = 24$. The area of the shaded region is the difference between these two areas, which equals $24 - 6 = 18$. This area can be found in other ways, one of which is to use the formula for the area of a trapezoid.

9. **Describing an area:** The shaded region in the figure below represents the region indicated in the exercise. (The right-hand edge of the region is part of the line $x = 4$.)

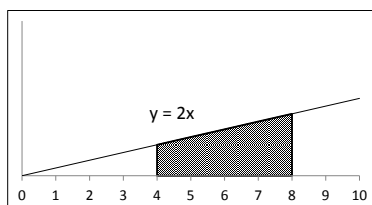


The region is a triangle with base 4. The height of the triangle is the height of the graph

of $f(x) = 6x$ at $x = 4$, so the height is $6 \times 4 = 24$. Thus the area is

$$\text{Area of triangle} = \frac{1}{2} \text{Base} \times \text{Height} = \frac{1}{2} \times 4 \times 24 = 48.$$

10. **Describing an area:** The shaded region in the figure below represents the region indicated in the exercise. (The left-hand edge of the region is part of the line $x = 4$, and the right-hand edge is part of the line $x = 8$.)



We think of the area of the region as the difference between the areas of two triangles, one with base from $x = 0$ to $x = 4$ and the other with base from $x = 0$ to $x = 8$. The first of these triangles has base 4. Its height is the height of the graph of $f(x) = 2x$ at $x = 4$, so its height is $2 \times 4 = 8$. Thus the area of the first triangle is

$$\text{Area of triangle} = \frac{1}{2} \text{Base} \times \text{Height} = \frac{1}{2} \times 4 \times 8 = 16.$$

Similarly, the second triangle has base 8 and height $2 \times 8 = 16$, so its area is $\frac{1}{2} \times 8 \times 16 = 64$. The area of the shaded region is the difference between these two areas, which equals $64 - 16 = 48$. This area can be found in other ways, one of which is to use the formula for the area of a trapezoid.

11. **An upper sum:** The shaded region in the figure is made up of two rectangles. Each of the rectangles has base 2. The heights of the rectangles are determined by the graph. The left-hand rectangle has height $40 - 1^2 = 39$, so its area is $2 \times 39 = 78$. The right-hand rectangle has height $40 - 3^2 = 31$, so its area is $2 \times 31 = 62$. Therefore, the total area is $78 + 62 = 140$.
12. **A lower sum:** The shaded region in the figure is made up of two rectangles. Each of the rectangles has base 2. The heights of the rectangles are determined by the graph. The left-hand rectangle has height $40 - 3^2 = 31$, so its area is $2 \times 31 = 62$. The right-hand rectangle has height $40 - 5^2 = 15$, so its area is $2 \times 15 = 30$. Therefore, the total area is $62 + 30 = 92$.

A FURTHER LOOK: DEFINITION OF A FUNCTION

1. We calculate $f(2) = \frac{2+3}{2-1} = 5$.
2. We calculate $f(9) = 9 + \sqrt{9} = 12$.
3. We replace x in the formula by $x+2$: $f(x+2) = (x+2) + 1 = x+3$.
4. We replace x in the formula by x^2 : $f(x^2) = x^2 - (x^2)^2 = x^2 - x^4$.
5. Because division by 0 is not allowed, the domain of f is all numbers for which the denominator is not 0. Thus, the domain is all real numbers except 1.
6. Because division by 0 is not allowed, the domain of f is all numbers for which the denominator is not 0. Thus, the domain is all real numbers except 0 and 3.
7. Because we cannot take the square root of a negative number, the domain of f is all numbers for which $x-4$ is nonnegative. Thus, the domain is all real numbers greater than or equal to 4.
8. Because we can solve the equation to get $y = x^{1/3}$, the equation does determine y as a function of x .
9. The equation does not determine y as a function of x . For example, if $x = 1$ there are two possible values for y : $y = 2^{1/4}$ and $y = -2^{1/4}$.
10. We calculate $f(0) = 5 \times 0.5^{0/4} = 5$ and $f(5) = 5 \times 0.5^{5/4} = 2.10$.