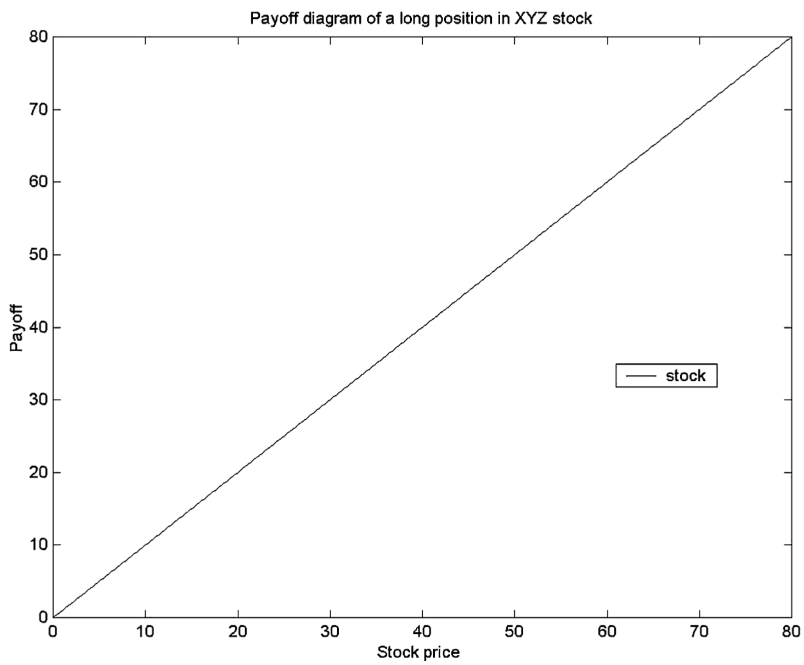
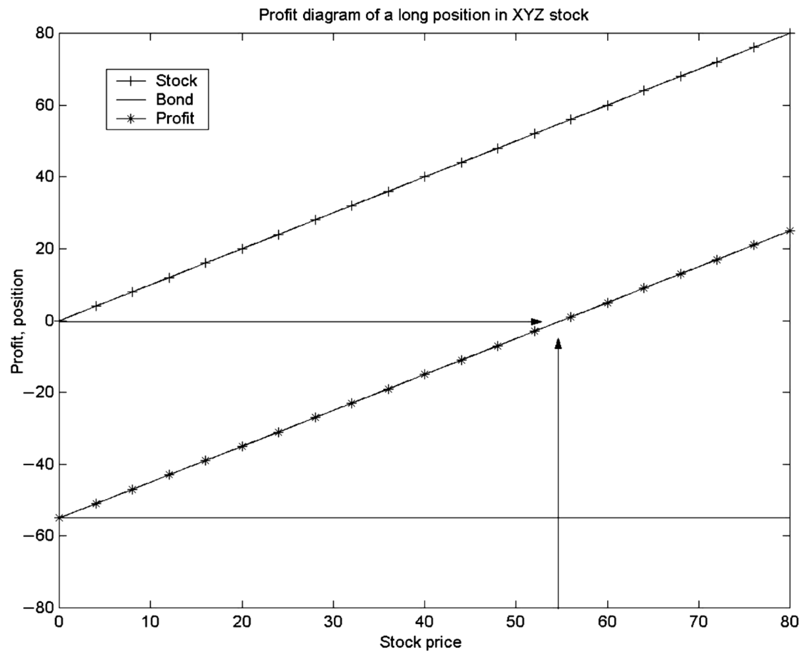
Chapter 2
An Introduction to Forwards and Options

■ Question 2.1

The payoff diagram of the stock is just a graph of the stock price as a function of the stock price:

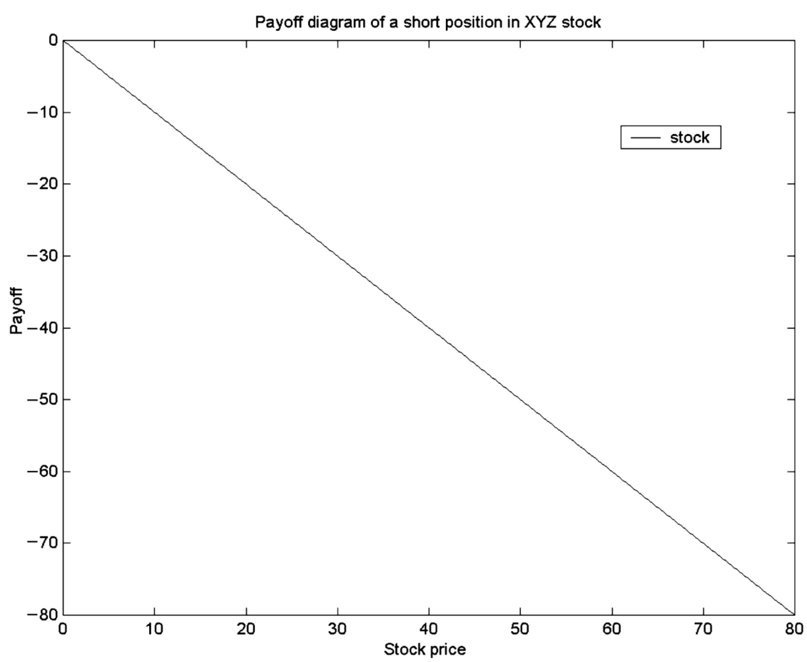


In order to obtain the profit diagram at expiration, we have to incorporate the initial costs of the stock, i.e., how we finance the initial investment. We do this by assuming we borrow \$50 at 10% interest. Note that even if we use our own funds to finance the purchase, we want to incorporate the opportunity costs of our investment (i.e., the interest we could have earned). By borrowing \$50, after one year we have to pay back: $\$50 \times (1 + 0.1) = \$55.$ The second figure shows the graph of the stock, the \$55 we have to pay back, and of the sum of the two positions, which is the profit graph. The arrows show that at a stock price of \$55, the profit at expiration is indeed zero. When the stock price is \$55, our stock is worth exactly what we owe and, hence, we break even.

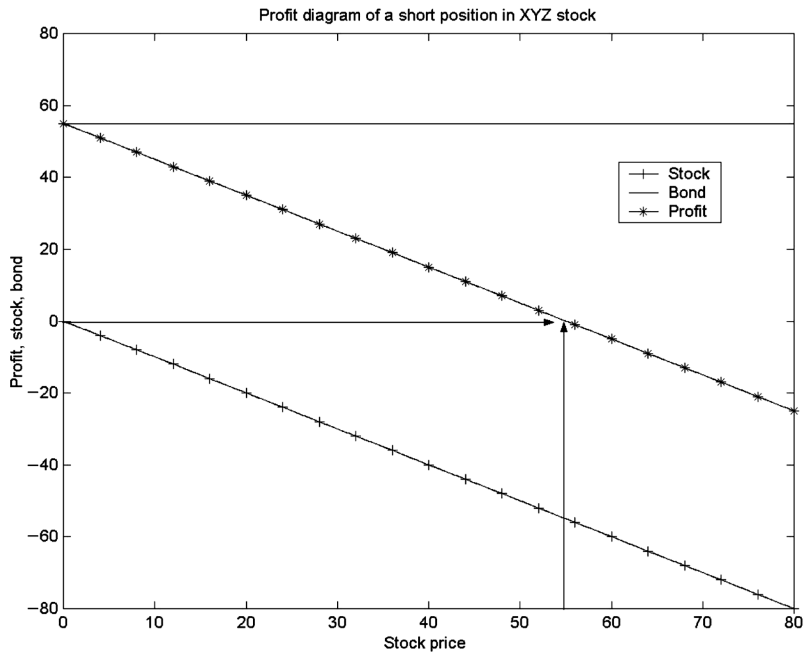


■ Question 2.2

Since we shorted the stock initially, our payoff at expiration is negative and equal—the stock price. This is the amount we have to spend in order to replace the share of stock we borrowed.



In order to obtain the profit diagram at expiration, we have to lend out the money we received from the short sale of the stock. We do so by buying a bond for $50. After one year we receive from the investment in the bond: $\$50 \times (1+0.1) = \$55.$ The second figure shows the graph of the payoff of the shorted stock, the money we receive from the investment in the bond, and the sum of the two positions, which is the profit graph. The arrows show that at a stock price of $55, the profit at expiration is indeed zero. At $55, the amount of money we receive from our bond investment is exactly offset by the amount of money we need to buy back the one share of stock we borrowed.



■ Question 2.3

The position that is the opposite of a purchased call is a written call. A seller of a call option is said to be the option writer, or to have a short position in the call option. The call option writer is the counterparty to the option buyer, and his payoffs and profits are just the opposite of those of the call option buyer.

Similarly, the position that is the opposite of a purchased put option is a written put option. Again, the payoff and profit for a written put are just the opposite of those of the purchased put.

It is important to note that the opposite of a purchased call is NOT the purchased put. If you do not see why, please draw a payoff diagram with a purchased call and a purchased put.

■ Question 2.4

1. The payoff to a long forward at expiration is equal to:

$$\text{Payoff to long forward} = \text{Spot price at expiration} - \text{forward price}$$

Therefore, we can construct the following table:

Price of asset in 6 months	Agreed forward price	Payoff to the long forward
40	50	–10
45	50	–5
50	50	0
55	50	5
60	50	10

2. The payoff to a purchased call option at expiration is:

$$\text{Payoff to call option} = \max[0, \text{spot price at expiration} - \text{strike price}]$$

The strike price given is equal to $50. Therefore, we can construct the following table:

Price of asset in 6 months	Strike price	Payoff to the call option
40	50	0
45	50	0
50	50	0
55	50	5
60	50	10

3. If we compare the two contracts, we immediately see that the call option has a protection for adverse movements in the price of the asset.

If the spot price is below $50, the buyer of the call option is able to walk away and does not incur a loss; whereas, the holder of the long forward position incurs a loss since he is obligated to buy the asset for $50.

If the spot price is above $50, the holder of the call option and the holder of the long forward position have identical payoffs. Therefore, the call option should be more expensive due to this attractive walk away feature.

■ Question 2.5

1. The payoff to a short forward at expiration is equal to:

$$\text{Payoff to short forward} = \text{forward price} - \text{spot price at expiration}$$

Therefore, we can construct the following table:

Price of asset in 6 months	Agreed forward price	Payoff to the short forward
40	50	10
45	50	5
50	50	0
55	50	–5
60	50	–10

2. The payoff to a purchased put option at expiration is:

$$\text{Payoff to put option} = \max[0, \text{strike price} - \text{spot price at expiration}]$$

The strike price given is equal to \$50. Therefore, we can construct the following table:

Price of asset in 6 months	Strike price	Payoff to the put option
40	50	10
45	50	5
50	50	0
55	50	0
60	50	0

3. The same logic as in Question 2.4(3) applies. If we compare the two contracts, we see that the put option holder is protected from increases in the price of the asset.

   If the spot price is above \$50, the buyer of the put option is able to walk away and does not incur a loss; whereas, the holder of the short forward position incurs a loss since he is obligated to sell the asset for \$50.

   If the spot price is above \$50, the holder of the put option and the holder of the short forward position have identical payoffs. Therefore, the put option should be more expensive due to this walk away feature.

■ Question 2.6

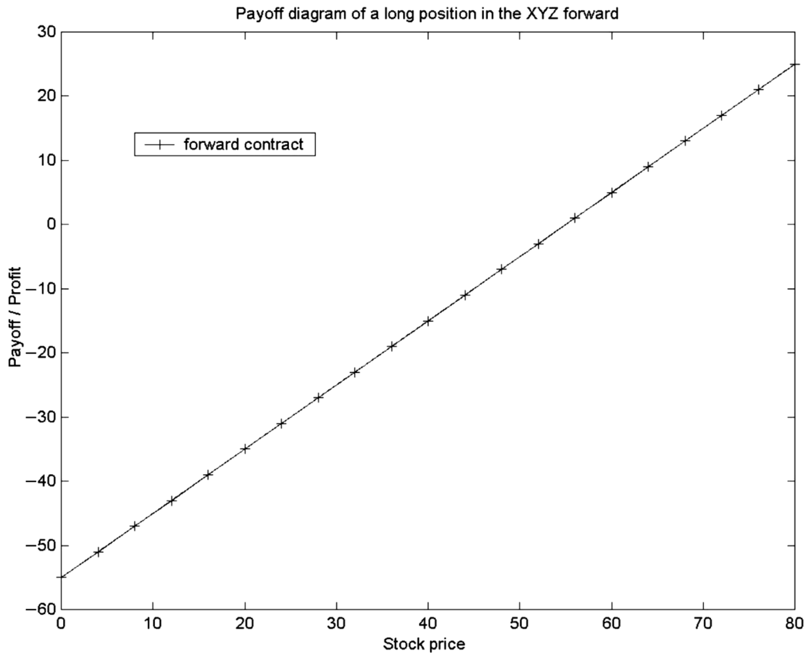
We need to solve the following equation to determine the effective annual interest rate: $\$91 \times (1 + r) = \$100.$ We obtain $r = 100/91 - 1 = 0.0989,$ which means that the effective annual interest rate is approximately 9.9%.

Remember that when we drew profit diagrams for the forward or call option, we drew the payoff on the vertical axis, and the index price at the expiration of the contract on the horizontal axis. Since the default-free zero-coupon bond pays \$100 regardless of the index price, the payoff diagram is just a horizontal line equal to \$100 (on the *y*-axis).

The textbook provides the answer to the question concerning the profit diagram in the section "Zero-Coupon Bonds in Payoff and Profit Diagrams." When we were calculating profits, we saw that we had to find the future value of the initial investment. In this case, our initial investment is \$91. How do we find the future value? We use the current risk-free interest rate and multiply the initial investment by it. However, as our bond is default-free, and does not bear coupons, the effective annual interest rate is exactly the 9.9% we have calculated before. Therefore, the future value of \$91 is $\$91 \times (1 + 0.0989) = \$100,$ and our profit in six months is zero! In general, by incorporating the time value of money in our profit diagrams, all risk free borrowing and lending has no affect on the profit diagrams for any investment strategy.

■ Question 2.7

1. It does not cost anything to enter into a forward contract—we do not pay a premium. Therefore, the payoff diagram of a forward contract coincides with the profit diagram:



2. We have seen in Question 2.1, that in order to obtain the profit diagram at expiration of a purchase of XYZ stock, we have to finance the initial investment. We did so by borrowing $50. After one year we had to pay back: $\$50 \times (1+0.1) = \$55.$ Therefore, our total profit at expiration from the purchase of a stock that was financed by a loan was: $\$S_T - \$55,$ where S_T is the value of one share of XYZ at expiration. This profit from buying the stock with borrowed funds is the same as the profit from our long forward contract. Both positions require zero initial cash and have the same payoff, hence there is no advantage in investing in either the stock or the forward contract.

3. The dividend is only paid to the owner of the stock. The owner of the long forward contract is not entitled to receive the dividend, because she only has a claim to buy the stock in the future for a given price, but she does not own it yet. Therefore, it does matter now whether we own the stock or the long forward contract. Because everything else is the same as in Parts (1) and (2), it is now beneficial to own the share. We can receive an additional payment in the form of the dividend if we own the stock at the ex-dividend date. This question hints at the very important fact that we have to be careful to take into account all the benefits and costs of an asset when we try to compare prices. We will encounter similar problems in later chapters.

■ Question 2.8

As in Question 2.7(2), there will be no advantage in either buying the stock or taking a long forward position if buying the stock with borrowed money has the same profit/payoff diagram as a long forward position. Note that the payoff and profit are the same since these strategies have zero initial cost. The payoff/profit of a long forward contract with a price for delivery of $53 is equal to $\$S_T - \$53,$ where S_T is the (unknown) value of one share of XYZ in one year. We must now determine what interest rate makes the buying stock with borrowed money strategy have the same profit. If we borrow $50 today to buy one share of XYZ stock (that costs $50), we have to repay in one year: $\$50 \times (1+r)$. Our total profit is therefore: $\$S_T - \$50 \times (1+r)$. Now we can equate the two profit equations and solve for the interest rate r:

$$\begin{aligned} \$S_T - \$53 = \$S_T - \$50 \times (1+r) &\Rightarrow \\ \$53 = \$50 \times (1+r) &\Rightarrow \\ \frac{\$53}{\$50} - 1 = r &\Rightarrow \\ r = 0.06 & \end{aligned}$$

Therefore, the 1-year effective interest rate that is consistent with no advantage to either buying the stock or forward contract is 6%.

■ Question 2.9

1. If the forward price is $1,100, then the long position of the one-year forward contract receives in one year a profit of: $\$S_T - \$1{,}100$, where S_T is the (unknown) value of the S&R index in one year. Remember that it costs nothing to enter the forward contract.

   Let us again follow our strategy of borrowing money to finance the purchase of the index today, so that there is zero initial cost. If we borrow $1,000 today to buy the S&R index (that costs $1,000), we have to repay in one year: $\$1{,}000 \times (1+0.10) = \$1{,}100$. Our total profit in one year from borrowing to buy the S&R index is therefore: $\$S_T - \$1{,}100$. The profits from the two strategies are identical.

2. Compared to the forward price of $1,110, the forward price of $1,200 is worse for our long position since we are now obligated to buy at a higher price. For example, suppose the index after one year is $1,150. We would have made $50 with our long position in Part (1), but now, with a forward price of $1,100, we are losing $50. As there was no advantage in buying either the index with borrowed money or going long forward at a price of $1,100, we now need to be “bribed” to enter into the off-market forward contract. We need to find an equation that makes all three strategies (i.e., the two long forward positions and the buying index with borrowed money) equivalent.

   As a reminder, the long forward position (at $1,100) and the buying the index with borrowed money has a profit of $\$S_T - \$1{,}100$. Let x be the amount of money we receive today for entering in the higher priced $1,200 off-market forward contract. Note that we should receive money. When invested at the 10% interest rate, we will receive $x \times 1.1$. Our profit from receiving the premium and entering into the off market long forward position at $1,200 is $S_T - 1{,}200 + 1.1x$. Equating this with the profit from the $1,100 forward contract (and/or buying the index with borrowed money) $\$S_T - \$1{,}100$ we have

$$\begin{aligned} S_T - 1{,}200 + 1.1x = S_T - 1{,}100 &\Rightarrow \\ 1.1x = 100 &\Rightarrow \\ x = 100/1.1 = 90.91 & \end{aligned}$$

   Hence we must receive $90.91 today to be willing to enter into a long forward contract at $1,200.

3. Similarly, the forward price of $1,000 is advantageous relative to a $1,100 forward price and we will have to pay a premium in order for there to be no advantage of one over the other. Proceeding as before, let *x* be the amount of money we pay for entering in the lower priced $1,200 off-market forward contract. To arrive at our profit, we will borrow the *x* dollars at 10%.

   Our profit from borrowing the premium amount and entering into the off market long forward position at $1,000 is $S_T - 1{,}000 - 1.1x.$ Equating this with the profit from the $1,100 forward contract (and/or buying the index with borrowed money) $\$S_T - \$1{,}100$ we have

$$\begin{aligned} S_T - 1{,}000 - 1.1x &= S_T - 1{,}100 && \Rightarrow \\ 100 &= 1.1x && \Rightarrow \\ x &= 100/1.1 = 90.91 \end{aligned}$$

   Hence we would be willing to pay $90.91 today to enter into a long forward contract at $1,000.

■ Question 2.10

1. Figure 2.6 depicts the profit from a long call option on the S&R index with 6 months to expiration and a strike price of $1,000 with the future price of the option premium equal to $95.68. The profit of the long call option is $\max[0, S_T - \$1{,}000] - \$95.68$ where S_T is the (unknown) value of the S&R index at expiration of the call option (six months).

   The S&R index price at which the call option profit diagram intersects the *x*-axis is the point of zero profit. Since the option payoff, $\max[0, S_T - \$1{,}000],$ must be greater than zero at zero profit (to cover the premium), zero profit is at the point where $S_T - 1{,}000 = 95.68$ which is $S_T = \$1{,}095.68.$

2. The profit of the 6 month forward contract with a forward price of 1,020 is $S_T - 1{,}020.$ In order to find the S&R index price at which the call option and the forward contract have the same profit, we need to find the value of S_T such that

$$S_T - 1{,}020 = \max[0, S_T - \$1{,}000] - \$95.68.$$

   Since the option must have a premium, one can see from the graph that the intersection of the two profit diagrams must occur to the left of the "kink" in the option profit function (i.e., the option is out-of-the-money). This makes sense, for if the option is in-the-money, the forward and the option have the same payoff which implies the option has a lower profit since it requires a premium. Mathematically, we can show this by noting if $S_T > 1{,}000$ then the left hand side of the above equation is $S_T - 1{,}020$ which is greater than the right hand side, $S_T - 1{,}095.68.$ Since S_T must be less than 1,000 for the two lines to intersect, we can solve $\$S_T - \$1{,}020 = -\$95.68 \Rightarrow S_T = \$924.32,$ which is the value we were asked to verify.

■ Question 2.11

1. Figure 2.8 depicts the profit from a long put option on the S&R index with 6 months to expiration and a strike price of $1,000 with the future value of the put premium equal to $75.68. The profit of the long put option is $\max[0, \$1{,}000 - S_T] - \$75.68$ where S_T is the (unknown) value of the S&R index at expiration of the put option (six months).

   The S&R index price at which the put option profit diagram intersects the *x*-axis is the point of zero profit. Since the option payoff, $\max[0, \$1{,}000 - S_T],$ must be greater than zero at zero profit (to cover the premium), zero profit is at the point where $1{,}000 - S_T = 75.68$ which is $S_T = \$924.32.$

2. In order for the profit of the put option to be equal to a short forward position, the option must be out-of-the-money (i.e., $S_T > 1,000$) for they have the same payoff if the option is in-the-money and there is a premium to the option. So we must solve for the point where the short forward position's profit $1,020 - S_T$ equals the profit of the out-of-the-money put –75.68. This implies $S_T = 1,020 + 75.68 = 1,095.68$ which was to be shown. Notice that we use the term "profit" even though it is a "loss"; i.e., negative profits are losses.

■ Question 2.12

1. **Long Forward.** The maximum loss occurs if the stock price at expiration is zero (the stock price cannot be less than zero, because companies have limited liability). The forward then pays 0—forward price. The maximum gain is unlimited. The stock price at expiration has no absolute bound, and the profit of the long forward position becomes arbitrarily as large as the stock price at expiration does.

2. **Short Forward.** The profit for a short forward contract is forward price—stock price at expiration. Since there is no absolute bound to the stock price, the short forward position has an unlimited potential for loss. The maximum gain occurs if the stock price is zero and we are able sell a worthless stock for the forward price.

3. **Long Call.** We will not exercise the call option we purchased if the stock price at expiration is less than the strike price (i.e., the option is out-of-the-money). Hence, if the call option is out-of-the-money, the option holder's maximum loss is the future value of the premium we paid initially to buy the option. As the stock price can grow without bound, so can the payoff to the option; hence the profit is unlimited as the stock price goes to infinity.

4. **Short Call.** We have no control over the exercise decision when we write a call. The buyer of the call option decides whether to exercise or not and will only exercise it when it is in-the-money (i.e., when the stock price is greater than the strike price). As we have the opposite side, we will never receive a positive payoff at expiration. Our profit is maximized when the option expires out-of-the-money, the profit being the future value of the premium. The maximum loss is unbounded since the stock price at expiration can be very large and has no bound. At very large stock prices, we are obligated to sell a very valuable stock for a relatively low strike price.

5. **Long Put.** We will not exercise the put option if the stock price at expiration is larger than the strike price. Consequently, the only thing we lose whenever the terminal stock price is larger than the strike is the future value of the premium we paid initially to buy the option. We will profit from a decline in the stock prices. However, stock prices cannot be smaller than zero, so our maximum gain is restricted to strike price less the future value of the premium and it occurs at a terminal stock price of zero.

6. **Short Put.** We have no control over the exercise decision when we write a put. The buyer of the put option decides whether to exercise or not, and he will only exercise it if the option is in-the-money (i.e., the stock price is lower than the strike price). As we have the opposite side, we never have a payoff at the expiration of the put option. Our profit is capped to the future value of the premium when the option is out-of-the-money (i.e., the stock price is higher than the strike price).

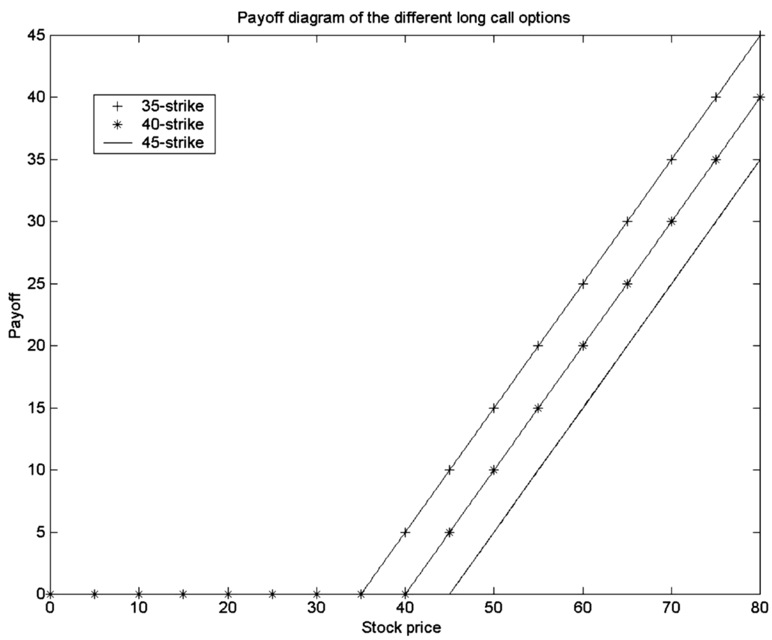
   The put option writer has a negative payoff whenever the stock price is smaller than the strike. When the stock price is as low as possible (equal to zero) we lose the strike price because somebody sells us a worthless asset for the strike price. This is the maximum loss as we are only compensated by the future value of the premium we received.

Question 2.13

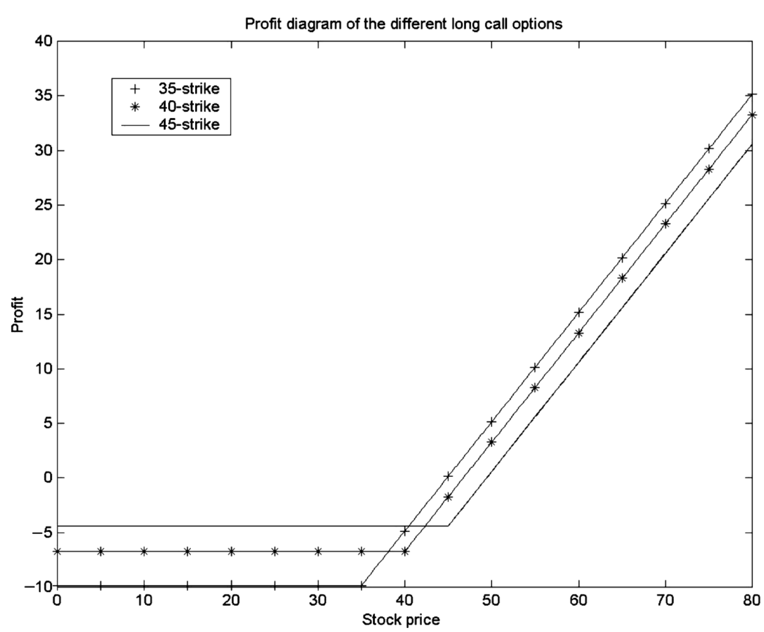
In order to be able to draw profit diagrams, we need to find the future values of the call premia. They are:

1. 35-strike call: $\$9.12 \times (1+0.08) = \$9.8496$
2. 40-strike call: $\$6.22 \times (1+0.08) = \$6.7176$
3. 45-strike call: $\$4.08 \times (1+0.08) = \$4.4064$

We can now graph the payoff and profit diagrams for the call options. The payoff diagram looks as follows:



We get the profit diagram by deducting the option premia from the payoff graphs. The profit diagram looks as follows:



Before considering the payoff diagrams, the call's strike price is the price at which you have the option to buy. The lower the buying price, the more valuable the call option (all else equal). As to the payoff diagram, the 35-strike call's payoff is equal to the other two's payoffs when the stock price expires below 35, and strictly above the others when the stock price is above 35. Since its payoff is greater than or equal to the other two options, it must be the most valuable contract. Similarly, the 40-strike call has a payoff larger than the 45-strike call when the stock price is above 40 (they both pay off zero when the stock price is below 40). Hence the 40-strike call is more valuable than the 45-strike call.

As an aside, if the option premium of the 35-strike call was less than the premium of the 40-strike call, the 35-strike profit would be above the 40-strike profit, regardless of the stock price at expiration. Since profits take into consideration the initial cash flows, the 35-strike would be a superior investment relative to the 40-strike call (we will see this implies an arbitrage opportunity).

■ Question 2.14

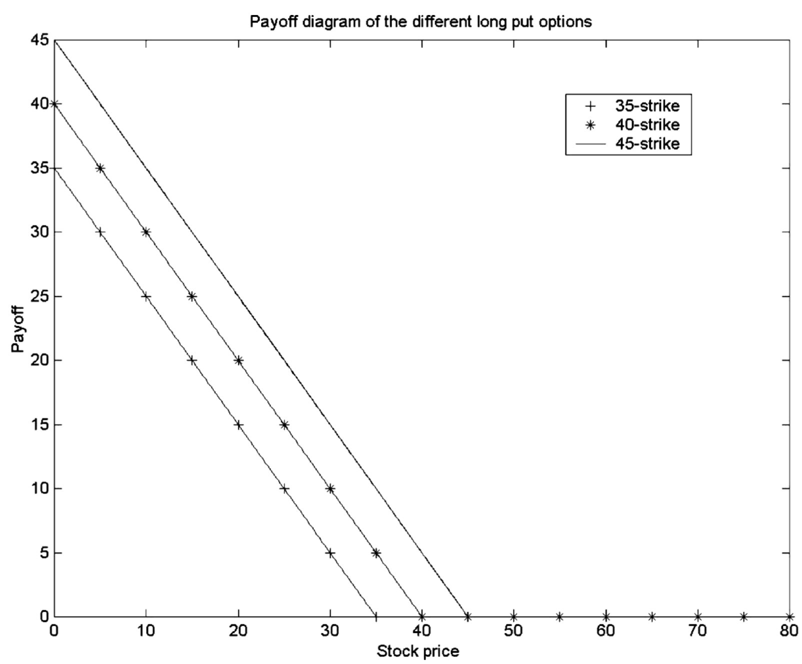
In order to be able to draw profit diagrams, we need to find the future values of the put premia. They are:

1. 35-strike put: $\$1.53 \times (1+0.08) = \$1.6524$

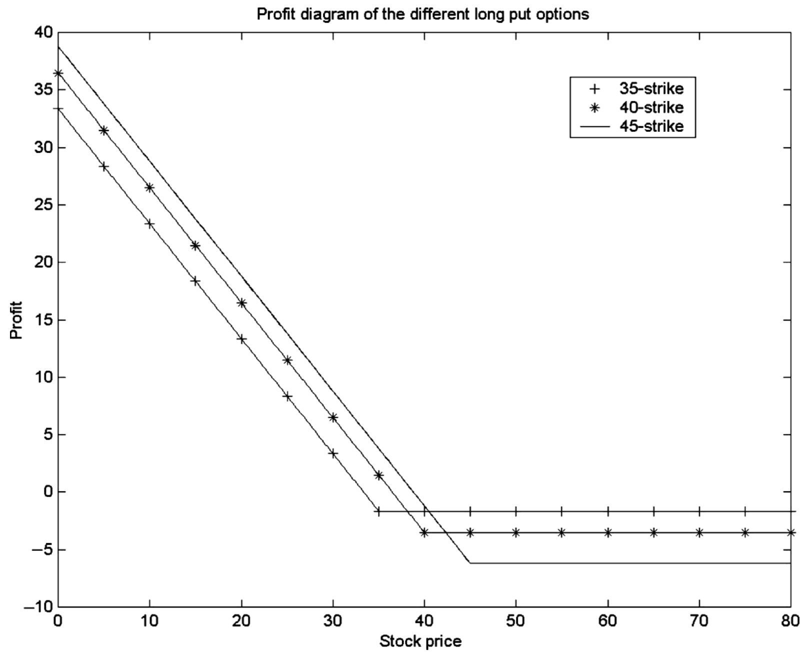
2. 40-strike put: $\$3.26 \times (1+0.08) = \$3.5208$

3. 45-strike put: $\$5.75 \times (1+0.08) = \$6.21$

We get the following payoff diagrams:



We get the profit diagram by deducting the option premia from the payoff graphs. The profit diagram looks as follows:



Before considering the payoff diagrams, the put's strike price is the price at which you have the option to sell. The higher the selling price, the more valuable the put option (all else equal). As to the payoff diagram, the 45-strike put's payoff is equal to the other two's payoffs when the stock price expires above 45, and strictly above the others when the stock price is below 45. Since its payoff is greater than or equal to the other two options it must be the most valuable contract. Similarly, the 40-strike put has a payoff larger than the 35-strike put when the stock price is below 40 (they both payoff zero when the stock price is above 40). Hence the 40-strike put is more valuable than the 35-strike put.

As an aside, if the option premium of the 45-strike put was less than the premium of the 40-strike put, the 45-strike profit would be above the 40-strike profit, regardless of the stock price at expiration. Since profits take into consideration the initial cash flows, the 45-strike put would be a superior investment relative to the 40-strike put (we will see this implies an arbitrage opportunity).

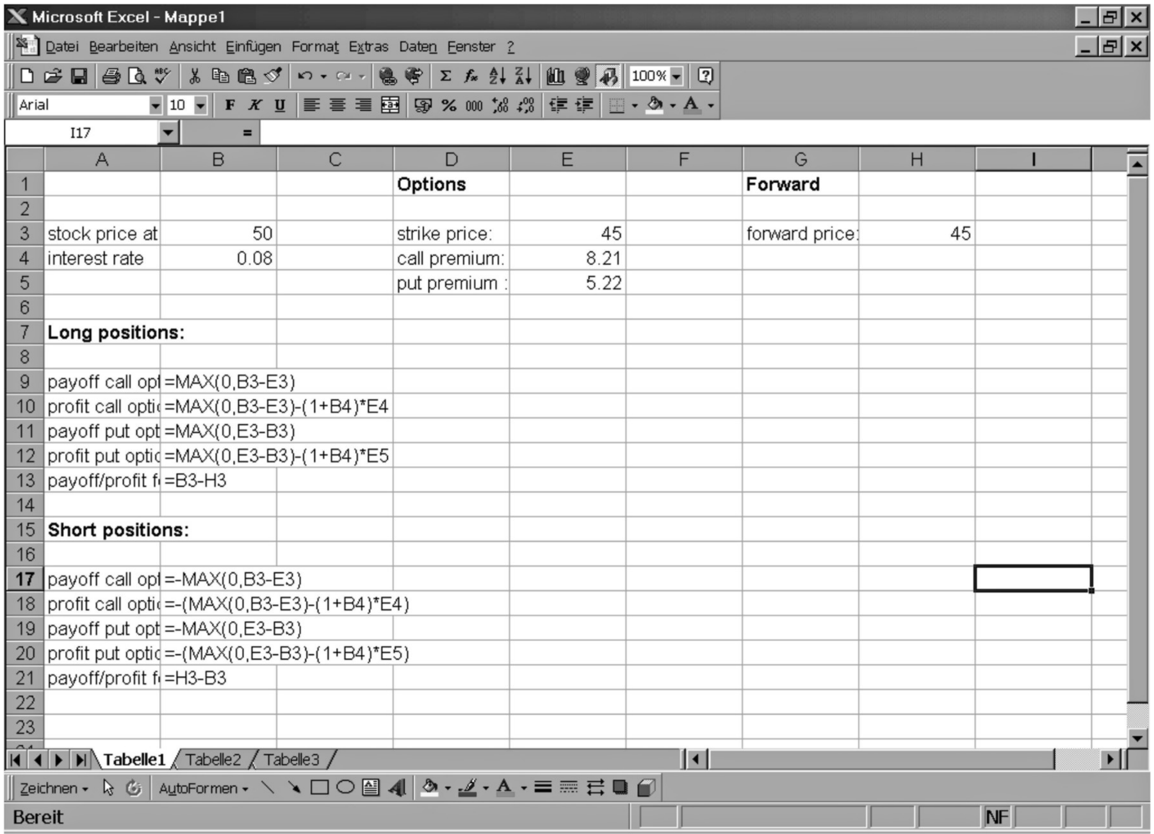
■ Question 2.15

When we found the equivalence between a long forward position and a loan-financed stock index purchase, we relied on knowing today what our interest on the loan is. There was no uncertainty about the payment to be made. If we were to finance the purchase of the index by short selling IBM stock, we would introduce uncertainty in our dollar amount being owed, because the future value of the IBM stock is unknown today. Therefore, we could not calculate today the amount to be repaid, and it would be impossible to establish an equivalence between the forward and loan-financed index purchase today.

The calculation of a profit diagram would only be possible if we assumed an arbitrary value for IBM at expiration of the futures. We could then draw many profit diagrams with different assumed values for IBM to get an idea of the many possible profits we could make; i.e., the profit lines would shift depending on different values of IBM which would imply different borrowing rates. Alternatively, we could create a 3-dimensional profit diagram, with two axes (the *x*-axis and *y*-axis) being the S&R value and the IBM value and the third axis (the *z*-axis) being the profit.

■ Question 2.16

The following is a copy of an Excel spreadsheet that solves the problem:



	A	B	C	D	E	F	G	H	I
1				**Options**			**Forward**		
2									
3	stock price at	50		strike price:	45		forward price:	45	
4	interest rate	0.08		call premium:	8.21				
5				put premium :	5.22				
6									
7	**Long positions:**								
8									
9	payoff call opt	=MAX(0,B3-E3)							
10	profit call optic	=MAX(0,B3-E3)-(1+B4)*E4							
11	payoff put opt	=MAX(0,E3-B3)							
12	profit put optic	=MAX(0,E3-B3)-(1+B4)*E5							
13	payoff/profit f	=B3-H3							
14									
15	**Short positions:**								
16									
17	payoff call opt	=-MAX(0,B3-E3)							
18	profit call optic	=-(MAX(0,B3-E3)-(1+B4)*E4)							
19	payoff put opt	=-MAX(0,E3-B3)							
20	profit put optic	=-(MAX(0,E3-B3)-(1+B4)*E5)							
21	payoff/profit f	=H3-B3							
22									
23									