## Solutions Manual

# Fundamentals of Engineering Electromagnetics


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ADDISON-WESLEY PUBLISHING COMPANY

Reading, Massachusetts • Menlo Park, California • New York
Don Mills, Ontario • Wokingham, England • Amsterdam • Bonn
Sydney • Singapore • Tokyo • Madrid • San Juan • Milan • Paris

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ISBN 0-201-90916-2
7 8 9 10-DM-9695949392

# PREFACE

This solutions manual is prepared for the convenience of those professors who assign my Fundamentals of Engineering Electromagnetics as the textbook for their classes. All problems in the book are solved in sufficient detail so that no trouble should be encountered in arriving at the final results†. To lend confidence to the students who are assigned to do the problems, answers to odd-numbered problems are given at the end of the book. I have asked my publisher, the Addison-Wesley Publishing Company, to exercise strict control in sending out this solutions manual to prevent it from getting into the hands of students.

I realize that, no matter how careful I have endeavored to be, occasional errors may still exist. I should be grateful if you would be kind enough to notify me as you discover them either in the book or in this manual.

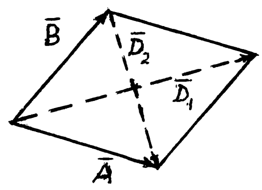
D.K.C.

†In this manual letters with an overbar represent vector quantities which are printed with a boldface in the book. A vector from point $P_1$ to point $P_2$ is indicated by $\overrightarrow{P_1P_2}$.

# Chapter 2

## Vector Analysis

**P. 2-1** Denoting the diagonals of the rhombus by $\bar{D}_1$ and $\bar{D}_2$, we have:



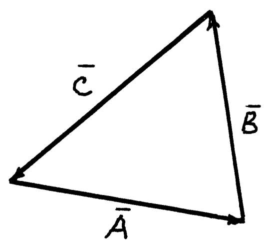

(a) $\bar{D}_1 = \bar{A} + \bar{B}$,
$\bar{D}_2 = \bar{A} - \bar{B}$.

(b) $\bar{D}_1 \cdot \bar{D}_2 = (\bar{A} + \bar{B}) \cdot (\bar{A} - \bar{B})$
$= \bar{A} \cdot \bar{A} - \bar{B} \cdot \bar{B} = 0$,

since $|\bar{A}| = |\bar{B}|$.

Thus, $\bar{D}_1 \perp \bar{D}_2$.

**P. 2-2**




$$\bar{A} + \bar{B} + \bar{C} = 0.$$

$\bar{A} \times$ : $\quad \bar{A} \times \bar{B} = \bar{C} \times \bar{A}$.

$\bar{C} \times$ : $\quad \bar{C} \times \bar{A} = \bar{B} \times \bar{C}$.

$\bar{B} \times$ : $\quad \bar{B} \times \bar{C} = \bar{A} \times \bar{B}$.

Magnitude relations:

$AB \sin\theta_{AB} = CA \sin\theta_{CA} = BC \sin\theta_{BC}$.

Hence,
$$\frac{A}{\sin\theta_{BC}} = \frac{B}{\sin\theta_{CA}} = \frac{C}{\sin\theta_{AB}}. \quad \text{(Law of sines.)}$$

**P. 2-3** a) $\bar{a}_B = \dfrac{\bar{a}_x 4 - \bar{a}_y 6 + \bar{a}_z 12}{\sqrt{4^2 + 6^2 + 12^2}} = \bar{a}_x \dfrac{2}{7} - \bar{a}_y \dfrac{3}{7} + \bar{a}_z \dfrac{6}{7}$.

b) $\bar{B} - \bar{A} = -\bar{a}_x 2 - \bar{a}_y 8 + \bar{a}_z 15$, $\quad |\bar{B} - \bar{A}| = \sqrt{2^2 + 8^2 + 15^2} = 17.1$.

c) $\bar{A} \cdot \bar{a}_B = 6 \times \dfrac{2}{7} - 2 \times \dfrac{3}{7} - 3 \times \dfrac{6}{7} = -17.1$.

d) $\bar{B} \cdot \bar{A} = 24 - 12 - 36 = -24$.

e) $\bar{B} \cdot \bar{a}_A = \dfrac{\bar{B} \cdot \bar{A}}{|\bar{A}|} = \dfrac{-24}{\sqrt{6^2 + 2^2 + 3^2}} = -\dfrac{24}{7} = -3.43$.

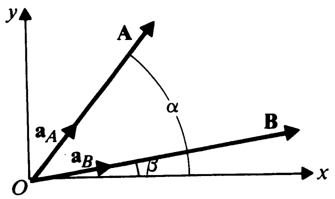
f) $\cos\theta_{AB} = \dfrac{\bar{B} \cdot \bar{A}}{BA} = \dfrac{-24}{14 \times 7} = -0.245$, $\quad \theta_{AB} = 180° - 75.8° = 104.2°$

g)

$$\bar{A}\times\bar{C} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 6 & 2 & -3 \\ 5 & 0 & -2 \end{vmatrix} = -\bar{a}_x 4 - \bar{a}_y 3 - \bar{a}_z 10$$

h) $\bar{A}\cdot(\bar{B}\times\bar{C}) = (\bar{A}\times\bar{B})\cdot\bar{C} = -(\bar{A}\times\bar{C})\cdot\bar{B} = -\left[(-4)4+(-3)(-6)+(-10)12\right] = -118.$

P.2-4




$$\bar{a}_A = \bar{a}_x\cos\alpha + \bar{a}_y\sin\alpha,$$
$$\bar{a}_B = \bar{a}_x\cos\beta + \bar{a}_y\sin\beta.$$

a) $\bar{a}_A\cdot\bar{a}_B = \cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta.$

b)

$$\bar{a}_B\times\bar{a}_A = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \cos\beta & \sin\beta & 0 \\ \cos\alpha & \sin\alpha & 0 \end{vmatrix} = \bar{a}_z(\sin\alpha\cos\beta - \cos\alpha\sin\beta)$$

$$= \bar{a}_z\sin(\alpha-\beta).$$

$$\therefore\ \sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta.$$

P.2-5 a) $\overrightarrow{P_1P_2} = \overrightarrow{OP_2} - \overrightarrow{OP_1} = -\bar{a}_x 4 + \bar{a}_y + \bar{a}_z 3,$

$\overrightarrow{P_2P_3} = \overrightarrow{OP_3} - \overrightarrow{OP_2} = \bar{a}_x 6 - \bar{a}_y 5 + \bar{a}_z,$

$\overrightarrow{P_1P_3} = \overrightarrow{OP_3} - \overrightarrow{OP_1} = \bar{a}_x 2 - \bar{a}_y 4 + \bar{a}_z 4.$

$\overrightarrow{P_1P_2}\cdot\overrightarrow{P_1P_3} = 0. \longrightarrow$ *Right angle at corner* $P_1$.

b) *Area of triangle* $= \frac{1}{2}\left|\overrightarrow{P_1P_2}\times\overrightarrow{P_1P_3}\right| = \frac{1}{2}\left|\overrightarrow{P_1P_2}\right|\left|\overrightarrow{P_1P_3}\right| = 15.3.$

P.2-6 a) $\overrightarrow{P_1P_2} = \bar{a}_x 2 + \bar{a}_y 4 - \bar{a}_z 4, \quad \left|\overrightarrow{P_1P_2}\right| = \sqrt{2^2+4^2+4^2} = 6.$

b) *Perpendicular distance from* $P_3$ *to the line*

$= \left|\overrightarrow{P_3P_1}\times\bar{a}_{P_1P_2}\right| = \left|(\overrightarrow{OP_1}-\overrightarrow{OP_3})\times\frac{1}{6}\overrightarrow{P_1P_2}\right|$

$= \left|(-\bar{a}_x 5 - \bar{a}_y)\times\frac{1}{6}(\bar{a}_x 2 + \bar{a}_y 4 - \bar{a}_z 4)\right| = \frac{1}{6}\left|\bar{a}_x 4 - \bar{a}_y 20 - \bar{a}_z 18\right| = 4.53.$

**P.2-7** Given: $\bar{A} = \bar{a}_x 5 - \bar{a}_y 2 + \bar{a}_z$.

a) Let $\bar{a}_B = \bar{a}_x B_x + \bar{a}_y B_y + \bar{a}_z B_z$,
where $(B_x^2 + B_y^2 + B_z^2)^{1/2} = 1$. (1)

$$\bar{a}_B \,//\, \bar{A} \text{ requires } \bar{a}_B \times \bar{A} = 0 = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ B_x & B_y & B_z \\ 5 & -2 & 1 \end{vmatrix},$$

where yields:

$$B_y + 2B_z = 0, \quad (2a)$$
$$-B_x + 5B_z = 0. \quad (2b)$$
$$-2B_x - 5B_y = 0. \quad (2c)$$

Equations (2a), (2b), and (2c) are not all independent.
Solving Eqs. (1) and (2), we obtain

$$B_x = \frac{5}{\sqrt{30}}, \quad B_y = -\frac{2}{\sqrt{30}}, \quad \text{and } B_z = \frac{1}{\sqrt{30}}$$

$$\therefore \quad \bar{a}_B = \frac{1}{\sqrt{30}}(\bar{a}_x 5 - \bar{a}_y 2 + \bar{a}_z).$$

b) Let $\bar{a}_C = \bar{a}_x C_x + \bar{a}_y C_y + \bar{a}_z C_z$, where $C_z = 0$,
and $C_x^2 + C_y^2 = 1$. (3)

$\bar{a}_C \perp \bar{A}$ requires $\bar{a}_C \cdot \bar{A} = 0$, or

$$5C_x - 2C_y = 0. \quad (4)$$

Solution of Eqs. (3) and (4) yields

$$C_x = \frac{2}{\sqrt{29}}, \quad \text{and} \quad C_y = \frac{5}{\sqrt{29}}.$$

$$\therefore \quad \bar{a}_C = \frac{1}{\sqrt{29}}(\bar{a}_x 2 + \bar{a}_y 5).$$

**P.2-8** Given: $\bar{A} = \bar{A}_1 + \bar{A}_2 = \bar{a}_x 2 - \bar{a}_y 5 + \bar{a}_z 3$,

$$\bar{B} = -\bar{a}_x + \bar{a}_y 4,$$
$$\bar{A}_1 \perp \bar{B} \longrightarrow \bar{A}_1 \cdot \bar{B} = 0,$$
$$\bar{A}_2 \,//\, \bar{B} \longrightarrow \bar{A}_2 \times \bar{B} = 0.$$

Solving, we have

$$\bar{A}_1 = \frac{3}{17}(\bar{a}_x 4 + \bar{a}_y + \bar{a}_z 17) \quad \text{and} \quad \bar{A}_2 = \frac{22}{17}(\bar{a}_x - \bar{a}_y 4).$$

P.2-10 $\overrightarrow{OP_1} = -\bar{a}_x - \bar{a}_z 2,$

$$\overrightarrow{OP_2} = \bar{a}_x(r\cos\phi) + \bar{a}_y(r\sin\phi) + \bar{a}_z z$$

$$= \bar{a}_x\left(-\frac{3}{2}\right) + \bar{a}_y\frac{\sqrt{3}}{2} + \bar{a}_z\,.$$

$$\overrightarrow{P_1P_2} = \overrightarrow{OP_2} - \overrightarrow{OP_1} = -\bar{a}_x\frac{1}{2} + \bar{a}_y\frac{\sqrt{3}}{2} + \bar{a}_z 3, \qquad |\overrightarrow{P_1P_2}| = \sqrt{10}.$$

At $P_1(-1, 0, -2)$, $\bar{A}_{P_1} = -\bar{a}_x 2 + \bar{a}_z$.

$$\bar{A}_{P_1}\cdot\bar{a}_{P_1P_2} = \bar{A}_{P_1}\cdot\frac{\overrightarrow{P_1P_2}}{|\overrightarrow{P_1P_2}|} = \frac{4}{\sqrt{10}} = 1.265$$

P.2-11 a)

$$\left.\begin{aligned} x &= r\cos\phi = 3\cos 240^\circ = -\frac{3}{2},\\ y &= r\sin\phi = 3\sin 240^\circ = -3\sqrt{3}/2,\\ z &= -4 \end{aligned}\right\}\ \left(-\frac{3}{2}, -\frac{3\sqrt{3}}{2}, -4\right)$$

b)

$$\left.\begin{aligned} R &= (r^2+z^2)^{1/2} = (3^2+4^2)^{1/2} = 5,\\ \theta &= \tan^{-1}(r/z) = \tan^{-1}\left(\frac{3}{-4}\right) = 143.1^\circ,\\ \phi &= 4\pi/3 = 240^\circ. \end{aligned}\right\}\ (5, 143.1^\circ, 240^\circ)$$

P.2-12 a) $-\sin\phi$, b) $\sin\theta\sin\phi$, c) $\cos\theta$,

d) $-\bar{a}_z\cos\phi$, e) $-\bar{a}_\phi\cos\theta$, f) $-\bar{a}_\phi\cos\theta$.

P.2-13 a) In Cartesian coordinates, $\bar{A} = \bar{a}_x A_x + \bar{a}_y A_y + \bar{a}_z A_z$.

$$A_r = \bar{a}_r\cdot\bar{A} = (\bar{a}_r\cdot\bar{a}_x)A_x + (\bar{a}_r\cdot\bar{a}_y)A_y + (\bar{a}_r\cdot\bar{a}_z)A_z$$

$$= A_x\cos\phi_1 + A_y\sin\phi_1$$

b) In spherical coordinates, $\bar{A} = \bar{a}_R A_R + \bar{a}_\theta A_\theta + \bar{a}_\phi A_\phi$.

$$A_r = \bar{a}_r\cdot\bar{A} = (\bar{a}_r\cdot\bar{a}_R)A_R + (\bar{a}_r\cdot\bar{a}_\theta)A_\theta + (\bar{a}_r\cdot\bar{a}_\phi)A_\phi$$

$$= A_R\sin\theta_1 + A_\theta\cos\theta_1$$

$$= \frac{A_R r_1}{\sqrt{r_1^2+z_1^2}} + \frac{A_\theta z_1}{\sqrt{r_1^2+z_1^2}}.$$

**P. 2-14** a) In Cartesian coordinates, $\bar{E}=\bar{a}_x E_x+\bar{a}_y E_y+\bar{a}_z E_z$.

$$E_\theta=\bar{a}_\theta\cdot\bar{E}=(\bar{a}_\theta\cdot\bar{a}_x)E_x+(\bar{a}_\theta\cdot\bar{a}_y)E_y+(\bar{a}_\theta\cdot\bar{a}_z)E_z$$
$$=E_x\cos\theta_1\cos\phi_1+E_y\cos\theta_1\sin\phi_1-E_z\sin\theta_1.$$

b) In cylindrical coordinates, $\bar{E}=\bar{a}_r E_r+\bar{a}_\phi E_\phi+\bar{a}_z E_z$.

$$E_\theta=\bar{a}_\theta\cdot\bar{E}=(\bar{a}_\theta\cdot\bar{a}_r)E_r+(\bar{a}_\theta\cdot\bar{a}_\phi)E_\phi+(\bar{a}_\theta\cdot\bar{a}_z)E_z$$
$$=E_r\cos\theta_1-E_z\sin\theta_1.$$

**P. 2-15** a) $\bar{F}_p=\bar{a}_R\dfrac{12}{\sqrt{(-2)^2+(-4)^2+4^2}}=\bar{a}_R\dfrac{12}{6}=\bar{a}_R 2.$

$$(F_p)_y=2\left(\frac{-4}{\sqrt{(-2)^2+(-4)^2+4^2}}\right)=-\frac{4}{3}$$

b) $\bar{a}_F=\dfrac{1}{6}(-\bar{a}_x 2-\bar{a}_y 4+\bar{a}_z 4)=\dfrac{1}{3}(-\bar{a}_x-\bar{a}_y 2+\bar{a}_z 2).$

$$\bar{a}_A=\frac{1}{\sqrt{2^2+(-3)^2+(-6)^2}}(\bar{a}_x 2-\bar{a}_y 3-\bar{a}_z 6)=\frac{1}{7}(\bar{a}_x 2-\bar{a}_y 3-\bar{a}_z 6).$$

$$\theta_{FA}=\cos^{-1}(\bar{a}_F\cdot\bar{a}_A)=\cos^{-1}\frac{1}{21}(-2+6-12)=\cos^{-1}\left(\frac{-8}{21}\right)$$
$$=\cos^{-1}(-0.381)=180^\circ-67.6^\circ=112.4^\circ.$$

**P. 2-16** $\displaystyle\int_{P_1}^{P_2}\bar{E}\cdot d\bar{l}=\int_{P_1}^{P_2}(y\,dx+x\,dy).$

a) $x=2y^2$, $dx=4y\,dy$; $\displaystyle\int_{P_1}^{P_2}\bar{E}\cdot d\bar{l}=\int_1^2(4y^2dy+2y^2dy)=14.$

b) $x=6y-4$, $dx=6\,dy$; $\displaystyle\int_{P_1}^{P_2}\bar{E}\cdot d\bar{l}=\int_1^2[6y\,dy+(6y-4)]dy=14.$

Equal line integrals along two specific paths do not necessarily imply a conservative field. $\bar{E}$ is a conservative field in this case because $\bar{E}=\bar{\nabla}(xy+c)$.

**P. 2-17** a) $\bar{R}=\bar{a}_x x+\bar{a}_y y+\bar{a}_z z,\quad \dfrac{1}{R}=(x^2+y^2+z^2)^{-1/2}.$

$$\bar{\nabla}\left(\frac{1}{R}\right)=\bar{a}_x\frac{\partial}{\partial x}\left(\frac{1}{R}\right)+\bar{a}_y\frac{\partial}{\partial y}\left(\frac{1}{R}\right)+\bar{a}_z\frac{\partial}{\partial z}\left(\frac{1}{R}\right)$$
$$=-\frac{1}{R^3}(\bar{a}_x x+\bar{a}_y y+\bar{a}_z z)=-\bar{R}/R^3.$$

b) $\bar{R}=\bar{a}_R R,\ \bar{\nabla}\left(\dfrac{1}{R}\right)=\bar{a}_R\dfrac{\partial}{\partial R}\left(\dfrac{1}{R}\right)=-\bar{a}_R\left(\dfrac{1}{R^2}\right)=-\bar{R}/R^3.$

P.2-18 a) $\bar{\nabla} V = \bar{a}_x(2y+z) + \bar{a}_y(2x-z) + \bar{a}_z(x-y)$

$= \bar{a}_x(-2) + \bar{a}_y 4 + \bar{a}_z 3$ ; Magnitude $= \sqrt{29}$.

b) $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = \bar{a}_x(-2) + \bar{a}_y 3 + \bar{a}_z 6$,

$$\bar{a}_{PQ} = \frac{\overrightarrow{PQ}}{\sqrt{(-2)^2+3^2+6^2}} = \frac{1}{7}(-\bar{a}_x 2 + \bar{a}_y 3 + \bar{a}_z 6).$$

Rate of increase of V from P toward Q $= (\bar{\nabla} V)\cdot \bar{a}_{PQ}$

$$= \frac{1}{7}(4+12+18) = \frac{34}{7}.$$

P.2-19 a) $\dfrac{\partial \bar{a}_r}{\partial \phi} = \bar{a}_\phi$ ; $\quad \dfrac{\partial \bar{a}_\phi}{\partial \phi} = -\bar{a}_r$.

b)
$$\bar{\nabla}\cdot\bar{A} = \left(\bar{a}_r\frac{\partial}{\partial r} + \bar{a}_\phi\frac{\partial}{r\partial\phi} + \bar{a}_z\frac{\partial}{\partial z}\right)\cdot(\bar{a}_r A_r + \bar{a}_\phi A_\phi + \bar{a}_z A_z)$$
$$= \frac{\partial A_r}{\partial r} + \bar{a}_\phi\frac{1}{r}\cdot\frac{\partial}{\partial\phi}(\bar{a}_r A_r) + \frac{\partial A_\phi}{r\partial\phi} + \frac{\partial A_z}{\partial z}$$
$$= \frac{\partial A_r}{\partial r} + \bar{a}_\phi\frac{1}{r}\cdot\left(\bar{a}_r\frac{\partial A_r}{\partial\phi} + A_r\frac{\partial \bar{a}_r}{\partial\phi}\right) + \frac{\partial A_\phi}{r\partial\phi} + \frac{\partial A_z}{\partial z}$$
$$= \frac{\partial A_r}{\partial r} + \frac{A_r}{r} + \frac{\partial A_\phi}{r\partial\phi} + \frac{\partial A_z}{\partial z}$$
$$= \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{\partial A_\phi}{r\partial\phi} + \frac{\partial A_z}{\partial z}.$$

P.2-20 In spherical coordinates,

$$\bar{\nabla}\cdot\bar{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R), \quad \text{if } \bar{A} = \bar{a}_R A_R.$$

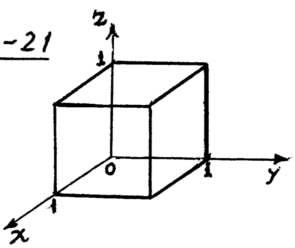
a) $\bar{A} = f_1(\bar{R}) = \bar{a}_R R^n, \quad A_R = R^n.$

$$\bar{\nabla}\cdot\bar{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^{n+2}) = (n+2)R^{n-1}.$$

b) $\bar{A} = f_2(\bar{R}) = \bar{a}_R\dfrac{k}{R^2}, \quad A_R = kR^{-2}.$

$$\bar{\nabla}\cdot\bar{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(k) = 0.$$

P.2-21




$\bar{F} = \bar{a}_x xy + \bar{a}_y yz + \bar{a}_z zx$. To find $\oint \bar{F}\cdot d\bar{s}$;

a) Left face: $y=0$, $d\bar{s} = -\bar{a}_y\,dx\,dz$.

$$\int_0^1\int_0^1 -yz\,dx\,dz = 0. \qquad (1)$$

Right face: $y=1$, $d\bar{s}=\bar{a}_y\,dx\,dz$.

$$\int_0^1\int_0^1 z\,dx\,dz=\frac{1}{2}. \qquad (2)$$

Top face: $z=1$, $d\bar{s}=\bar{a}_z\,dx\,dy$.

$$\int_0^1\int_0^1 dx\,dy=\frac{1}{2}. \qquad (3)$$

Bottom face: $z=0$, $d\bar{s}=-\bar{a}_z\,dx\,dy$, $\int\bar{F}\cdot d\bar{s}=0$. (4)

Front face: $x=1$, $d\bar{s}=\bar{a}_x\,dy\,dz$.

$$\int_0^1\int_0^1 y\,dy\,dz=\frac{1}{2}. \qquad (5)$$

Back face: $x=0$, $d\bar{s}=-\bar{a}_x\,dy\,dz$, $\int\bar{F}\cdot d\bar{s}=0$. (6)

Adding the results in (1), (2), (3), (4), (5), and (6):

$$\oint\bar{F}\cdot d\bar{s}=\frac{3}{2}.$$

b) $\bar{\nabla}\cdot\bar{F}=y+z+x$, $dv=dx\,dy\,dz$.

$$\int\bar{\nabla}\cdot\bar{F}\,dv=\int_0^1\int_0^1\int_0^1(x+y+z)\,dx\,dy\,dz=\frac{3}{2}.$$

P. 2-22 $\quad\bar{A}=\bar{a}_r r^2+\bar{a}_z 2z$.

$$\oint_S\bar{A}\cdot d\bar{s}=\left(\int_{\substack{top\\face}}+\int_{\substack{bottom\\face}}+\int_{walls}\right)\bar{A}\cdot d\bar{s}.$$

Top face ($z=4$): $\bar{A}=\bar{a}_r r^2+\bar{a}_z 8$, $d\bar{s}=\bar{a}_z\,ds$.

$$\int_{\substack{top\\face}}\bar{A}\cdot d\bar{s}=\int_{\substack{top\\face}}8\,ds=8(\pi 5^2)=200\pi.$$

Bottom face ($z=0$): $\bar{A}=\bar{a}_r r^2$, $d\bar{s}=-\bar{a}_z\,ds$, $\int_{\substack{bottom\\face}}\bar{A}\cdot d\bar{s}=0$.

Walls ($r=5$): $\bar{A}=\bar{a}_r 25+\bar{a}_z 2z$, $d\bar{s}=\bar{a}_r\,ds$.

$$\int_{walls}\bar{A}\cdot d\bar{s}=25\int_{walls}ds=25(2\pi 5\times 4)=1000\pi.$$

$$\therefore\ \oint\bar{A}\cdot d\bar{s}=200\pi+0+1000\pi=1{,}200\pi.$$

$$\bar{\nabla}\cdot\bar{A}=3r+2,\quad \int_V\bar{\nabla}\cdot\bar{A}\,dv=\int_0^4\int_0^{2\pi}\int_0^5(3r+2)r\,dr\,d\phi\,dz=1{,}200\pi.$$
$$=\oint\bar{A}\cdot d\bar{s}.$$

P. 2-23 $\bar{A} = \bar{a}_z z = \bar{a}_z R\cos\theta$.

a) Over the hemispherical surface: $d\bar{s} = \bar{a}_R R^2 \sin\theta\, d\theta\, d\phi$.

$$\int \bar{A}\cdot d\bar{s} = \int_0^{\pi/2}\int_0^{2\pi} \bar{a}_z (R\cos\theta)\cdot \bar{a}_R R^2 \sin\theta\, d\theta\, d\phi$$

$$= R^3 2\pi \int_0^{\pi/2} \cos^2\theta \sin\theta\, d\theta = \frac{2}{3}\pi R^3.$$

Over the flat base: $z=0,\ \bar{A}=0,\ \int \bar{A}\cdot d\bar{s} = 0$.

$$\therefore \oint \bar{A}\cdot d\bar{s} = \frac{2}{3}\pi R^3$$

b) $\bar{\nabla}\cdot\bar{A} = \dfrac{\partial A_z}{\partial z} = \dfrac{\partial z}{\partial z} = 1$.

c) $\int \bar{\nabla}\cdot\bar{A}\, dv = 1 \times (\text{volume of hemispherical region}) = \frac{2}{3}\pi R^3$

$= \oint \bar{A}\cdot d\bar{s} \longrightarrow$ Divergence theorem is proved.

P. 2-24 $\bar{D} = \bar{a}_R \dfrac{\cos^2\phi}{R^3}$. $\quad d\bar{s} = \begin{cases} \bar{a}_R R^2 \sin\theta\, d\theta\, d\phi, & \text{at } R=3. \\ -\bar{a}_R R^2 \sin\theta\, d\theta\, d\phi, & \text{at } R=2. \end{cases}$

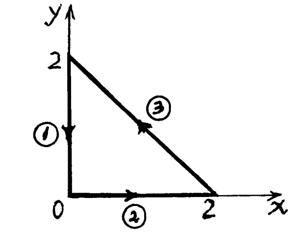
a) $$\oint \bar{D}\cdot d\bar{s} = \int_0^{2\pi}\int_0^{\pi} \left(\frac{1}{3}-\frac{1}{2}\right)\sin\theta\, d\theta\cdot\cos^2\phi\, d\phi$$

$$= -\frac{1}{6}\int_0^{\pi}\sin\theta\, d\theta\int_0^{2\pi}\cos^2\phi\, d\phi = -\frac{1}{6}(2)\pi = -\frac{\pi}{3}.$$

b) $\bar{\nabla}\cdot\bar{D} = -\dfrac{\cos^2\phi}{R^4}$, $\quad dv = R^2 \sin\theta\, dR\, d\theta\, d\phi$.

$$\int \bar{\nabla}\cdot\bar{D}\, dv = \int_0^{2\pi}\int_0^{\pi}\int_2^{3}\left(-\frac{\cos^2\phi}{R^2}\right)\sin\theta\, dR\, d\theta\, d\phi = -\frac{\pi}{3}.$$

P. 2-26




a) $d\bar{\ell} = \bar{a}_x dx + \bar{a}_y dy$,

$\bar{A}\cdot d\bar{\ell} = (2x^2+y^2)dx + (xy-y^2)dy$.

Path ①: $x=0,\ dx=0,\ \int\bar{A}\cdot d\bar{\ell} = -\int_2^0 y^2 dy = 8/3$.

Path ②: $y=0,\ dy=0,\ \int\bar{A}\cdot d\bar{\ell} = \int_0^2 2x^2 dx = 16/3$.

Path ③: $y=2-x,\ dy=-dx,\ \int\bar{A}\cdot d\bar{\ell} = -28/3$.

$$\oint\bar{A}\cdot d\bar{\ell} = \frac{8}{3} + \frac{16}{3} - \frac{28}{3} = -\frac{4}{3}.$$

b) $\bar{\nabla}\times\bar{A} = -\bar{a}_z y,\ d\bar{s} = \bar{a}_z dx\, dy,\ \int(\bar{\nabla}\times\bar{A})\cdot d\bar{s} = -\int_0^2\left[\int_0^{2-x} y\, dy\right]dx = -\frac{4}{3}$.

c) No. $\bar{\nabla}\times\bar{A} \neq 0$.

P.2-27 $\bar{F} = \bar{a}_r 5r\sin\phi + \bar{a}_\phi r^2\cos\phi$.

a) Path AB: $r=1$, $\bar{F} = \bar{a}_r 5\sin\phi + \bar{a}_\phi \cos\phi$; $d\bar{\ell} = \bar{a}_\phi d\phi$.

$$\int_{AB} \bar{F}\cdot d\bar{\ell} = \int_0^{\pi/2} \cos\phi\, d\phi = 1.$$

Path BC: $\phi = \pi/2$, $\bar{F} = \bar{a}_r 5r$; $d\bar{\ell} = \bar{a}_r dr$.

$$\int_{BC} \bar{F}\cdot d\bar{\ell} = \int_1^2 5r\, dr = 15/2.$$

Path CD: $r=2$, $\bar{F} = \bar{a}_r 10\sin\phi + \bar{a}_\phi 4\cos\phi$; $d\bar{\ell} = \bar{a}_\phi 2d\phi$.

$$\int_{CD} \bar{F}\cdot d\bar{\ell} = \int_{\pi/2}^{0} 8\cos\phi\, d\phi = -8.$$

Path DA: $\phi = 0$, $\bar{F} = \bar{a}_\phi r^2$; $d\bar{\ell} = \bar{a}_r dr$.

$$\int_{DA} \bar{F}\cdot d\bar{\ell} = 0.$$

$$\therefore \oint_{ABCDA} \bar{F}\cdot d\bar{\ell} = 1 + \frac{15}{2} - 8 = \frac{1}{2}.$$

b) $\bar{\nabla}\times\bar{F} = \bar{a}_z \frac{1}{r}\left[\frac{\partial}{\partial r}(rF_\phi) - \frac{\partial F_r}{\partial\phi}\right] = \bar{a}_z (3r-5)\cos\phi.$

c) $d\bar{s} = -\bar{a}_z\, r\, dr\, d\phi$, $\quad (\bar{\nabla}\times\bar{F})\cdot d\bar{s} = -r(3r-5)dr\cos\phi\, d\phi.$

$$\int (\bar{\nabla}\times\bar{F})\cdot d\bar{s} = -\int_1^2 r(3r-5)dr \int_0^{\pi/2}\cos\phi\, d\phi = \frac{1}{2}.$$

P.2-28 $\bar{A} = \bar{a}_\phi 3\sin(\phi/2)$.

$$\bar{\nabla}\times\bar{A} = \frac{3}{R\sin\theta}\left(\bar{a}_R \cos\theta\sin\frac{\phi}{2} - \bar{a}_\theta \sin\theta\sin\frac{\phi}{2}\right).$$

Assume the hemispherical bowl to be located in the lower half of the xy-plane and its circular rim coincident with the xy-plane. Tracing the rim in a counterclockwise direction, we have

$$d\bar{\ell} = \bar{a}_\phi 4\, d\phi, \quad d\bar{s} = -\bar{a}_R 4^2 \sin\theta\, d\theta\, d\phi.$$

$$\oint_C \bar{A}\cdot d\bar{\ell} = \int_0^{2\pi} (\bar{A})_{\substack{R=4\\ \theta=\pi/2}} \cdot (\bar{a}_\phi 4\, d\phi) = \int_0^{2\pi} 12\sin\left(\frac{\phi}{2}\right)d\phi = 48.$$

$$\int_S (\bar{\nabla}\times\bar{A})\cdot d\bar{s} = -12\int_0^{2\pi}\int_{\pi/2}^{\pi} \cos\theta\sin\frac{\phi}{2}\, d\theta\, d\phi = 48.$$

$$= \oint_C \bar{A}\cdot d\bar{\ell}.$$

P.2-30 $\bar{F} = \bar{a}_x(x+3y-c_1 z) + \bar{a}_y(c_2 x + 5z) + \bar{a}_z(2x - c_3 y + c_4 z)$.

a) $\bar{F}$ is irrotational:

$$\bar{\nabla}\times\bar{F} = \bar{a}_x\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right) + \bar{a}_y\left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right) + \bar{a}_z\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right) = 0.$$

Each component must vanish.

$$\frac{\partial}{\partial y}(2x - c_3 y + c_4 z) - \frac{\partial}{\partial z}(c_2 x + 5z) = 0 \longrightarrow c_3 = -5.$$

$$\frac{\partial}{\partial z}(x + 3y - c_1 z) - \frac{\partial}{\partial x}(2x - c_3 y + c_4 z) = 0 \longrightarrow c_1 = -2.$$

$$\frac{\partial}{\partial x}(c_2 x + 5z) - \frac{\partial}{\partial y}(x + 3y - c_1 z) = 0 \longrightarrow c_2 = 3.$$

b) $F$ is also solenoidal:

$$\bar{\nabla}\cdot\bar{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} = 0.$$

$$\frac{\partial}{\partial x}(x + 3y - c_1 z) + \frac{\partial}{\partial y}(c_2 x + 5z) + \frac{\partial}{\partial z}(2x - c_3 y + c_4 z) = 0.$$

$$\longrightarrow c_4 = -1.$$