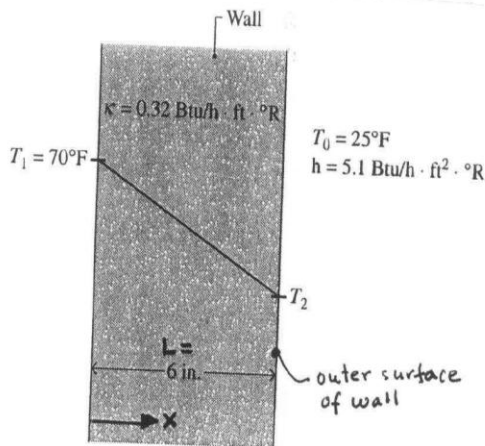


PROBLEM 2.47

KNOWN: Data are provided for an exterior wall of a building at steady state.

FIND: Determine the outer surface temperature and the heat transfer rate.

SCHEMATIC & GIVEN DATA:



ENGR. MODEL:

1. The wall is at steady state.
2. The temperature varies linearly through the wall.
3. Heat transfer at the outer wall surface is by convection only.

ANALYSIS:

Using Eq. 2.31 together with assumption 2

$$\begin{aligned}\dot{Q}_x &= -kA \frac{dT}{dx} \\ &= -kA \left[\frac{T_2 - T_1}{L} \right] \quad (*)\end{aligned}$$

At steady state, the rate of heat transfer by conduction to the outer surface of the wall equals the rate of heat transfer by convection from the outer surface, where convection is given by Eq. 2.34: $\dot{Q}_c = hA[T_2 - T_0]$. Thus, at $x = L$

$$\begin{aligned}\dot{Q}_x &= \dot{Q}_c \\ -kA \left[\frac{T_2 - T_1}{L} \right] &= hA[T_2 - T_0]\end{aligned}$$

Solving

$$\begin{aligned}T_2 &= \frac{hLT_0 + kT_1}{hL + k} = \frac{(5.1 \frac{\text{Btu}}{\text{h} \cdot \text{ft}^2 \cdot ^\circ\text{R}})(0.5 \text{ ft})(485^\circ\text{R}) + (0.32 \frac{\text{Btu}}{\text{h} \cdot \text{ft} \cdot ^\circ\text{R}})(530^\circ\text{R})}{(5.1 \frac{\text{Btu}}{\text{h} \cdot \text{ft}^2 \cdot ^\circ\text{R}})(0.5 \text{ ft}) + (0.32 \frac{\text{Btu}}{\text{h} \cdot \text{ft} \cdot ^\circ\text{R}})} \\ &= 490^\circ\text{R} \quad (30^\circ\text{F})\end{aligned}$$

Then, using Eq. (*) we get

$$\begin{aligned}\frac{\dot{Q}_x}{A} &= -k \left[\frac{T_2 - T_1}{L} \right] \\ &= -0.32 \frac{\text{Btu}}{\text{h} \cdot \text{ft} \cdot ^\circ\text{R}} \left[\frac{490^\circ\text{R} - 530^\circ\text{R}}{0.5 \text{ ft}} \right] = 25.6 \frac{\text{Btu}}{\text{h} \cdot \text{ft}^2} \quad \leftarrow\end{aligned}$$