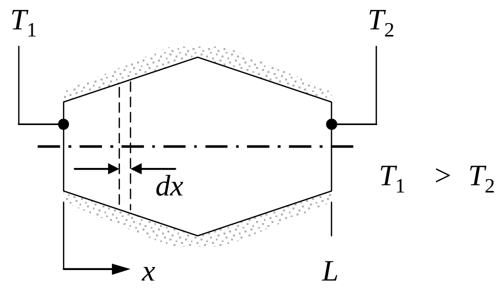
PROBLEM 2.2

KNOWN: Axisymmetric object with varying cross-sectional area and different temperatures at its two ends, insulated on its sides.

FIND: Shapes of heat flux distribution and temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction, (3) Constant properties, (4) Adiabatic sides, (5) No internal heat generation. (6) Surface temperatures T_1 and T_2 are fixed.

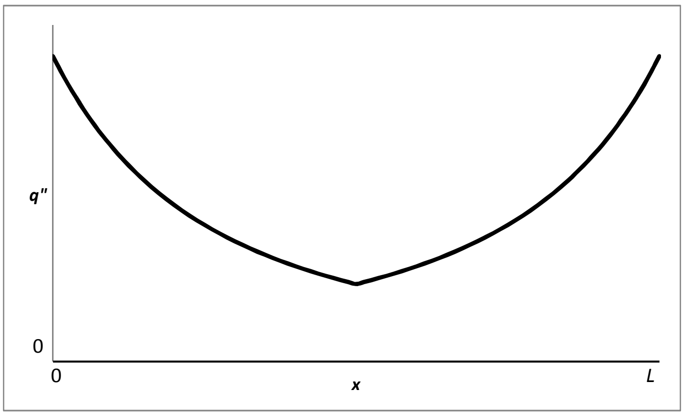
ANALYSIS: For the prescribed conditions, it follows from conservation of energy, Eq. 1.12c, that for a differential control volume, $\dot{E}_{in} = \dot{E}_{out}$ or $q_x = q_{x+dx}$. Hence

$$q_x \text{ is independent of x.}$$

Therefore

$$q_x = q_x'' A_c = \text{constant} \qquad (1)$$

where A_c is the cross-sectional area perpendicular to the x-direction. Therefore the heat flux must be inversely proportional to the cross-sectional area. The radius of the object first increases and then decreases linearly with x, so the cross-sectional area increases and then decreases as x^2. The resulting heat flux distribution is sketched below.



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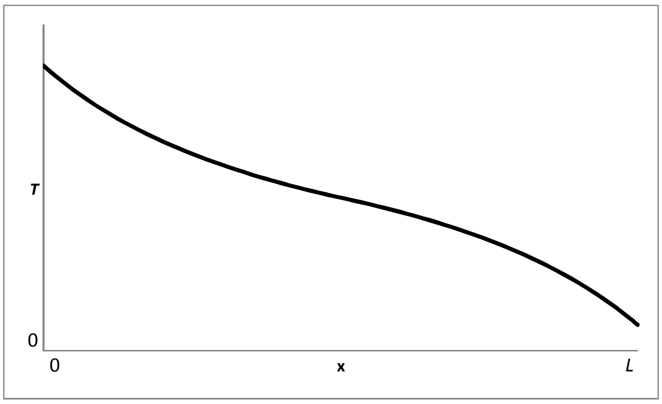
PROBLEM 2.2 (Cont.)

To find the temperature distribution, we can use Fourier's law:

$$q''_x = -k\frac{dT}{dx} \tag{2}$$

Therefore the temperature gradient is negative and its magnitude is proportional to the heat flux. The temperature decreases most rapidly where the heat flux is largest and more slowly where the heat flux is smaller.

Based on the heat flux plot above we can prepare the sketch of the temperature distribution below.



The temperature distribution is independent of the thermal conductivity. The heat rate and local heat fluxes are both proportional to the thermal conductivity of the material.

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COMMENTS: If the heat rate was fixed the temperature difference, $T_1 - T_2$, would be inversely proportional to the thermal conductivity. The temperature distribution would be of the same shape, but local temperatures T(x) would vary as the thermal conductivity is adjusted.