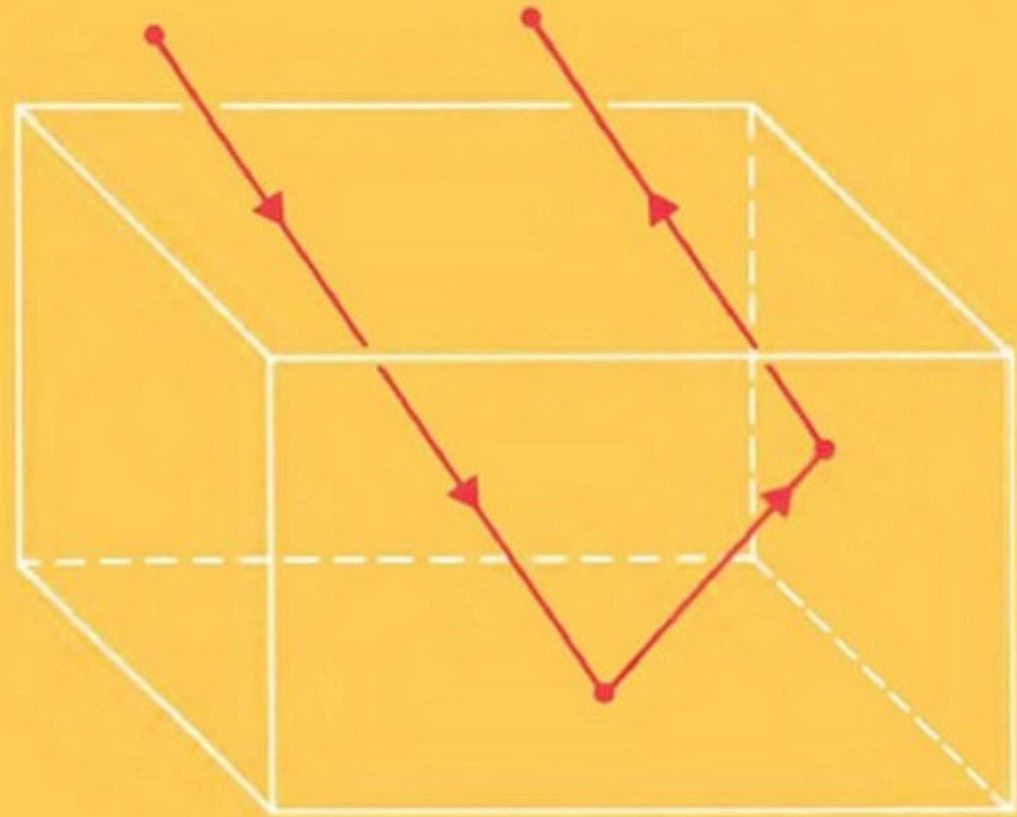
Philip Carlson

Solutions Manual for GEOMETRY

A High School Course

by S. Lang and G. Murrow



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Solutions Manual for

Geometry: A High School Course

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Philip Carlson

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With 100 Figures

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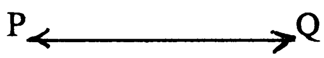
CHAPTER 1

Distance and Angles

§ 1. Exercises

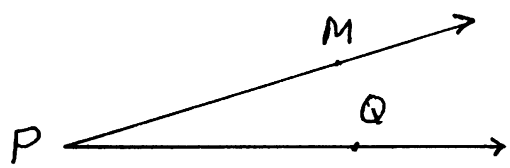
(Pages 5,6)

1. a. 

   b. 

   c. 

   d. 

2. a. 

   b. The two rays, R_{PM} and R_{PQ}, form a whole line if M, P and Q are collinear and P lies between M and Q.

3. d(P,Q) + d(Q,M) = d(P,M)

4. $\overline{AB}$ is <u>not</u> parallel to $\overline{PQ}$ because L_{AB} is not parallel to L_{PQ}.

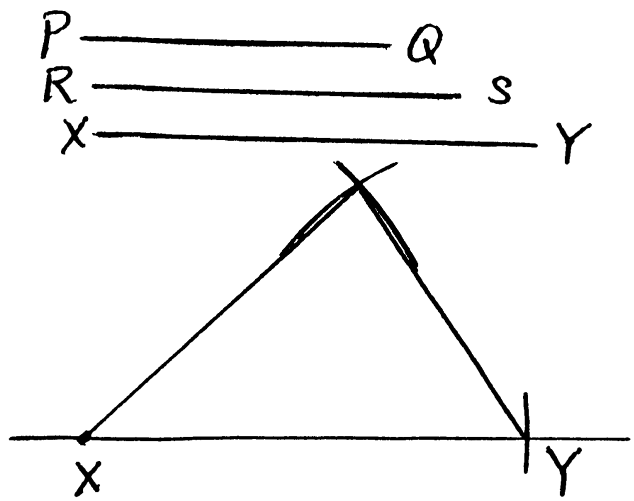
5. Line segments connecting any three points in the plane do not necessarily form a triangle since the three points may be collinear.

6. If line L were parallel to line U there would be 2 lines parallel to line U passing through the point P. This contradicts PAR 2. Therefore line L is not parallel to line U and must intersect it.

7. By the definition of parallelism, two lines, K and L, are parallel if K = L or K is not equal to L and does not intersect L. If P is on line L and K = L, then K and L are parallel lines.

Experiment 1-1
(Pages 7,8)

1.



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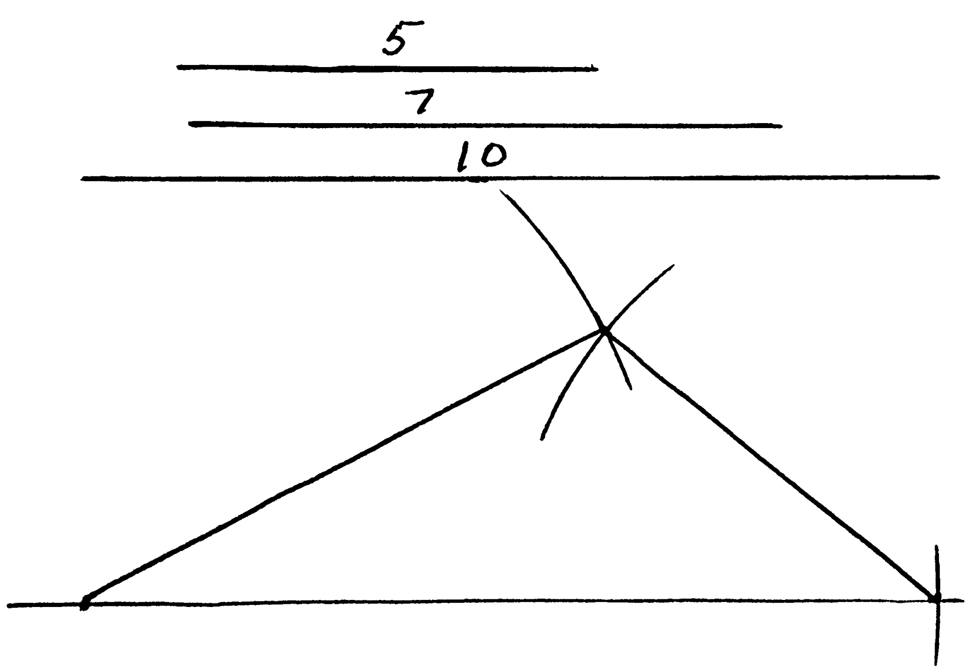

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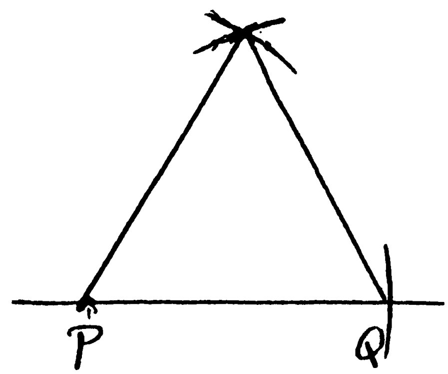

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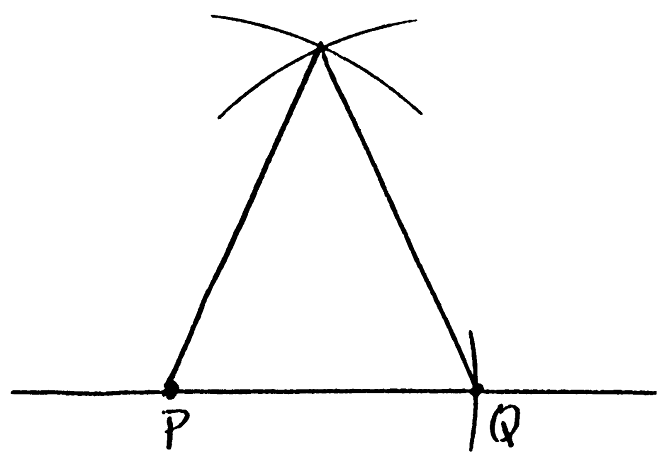

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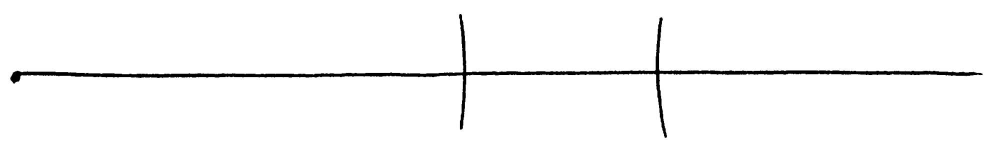
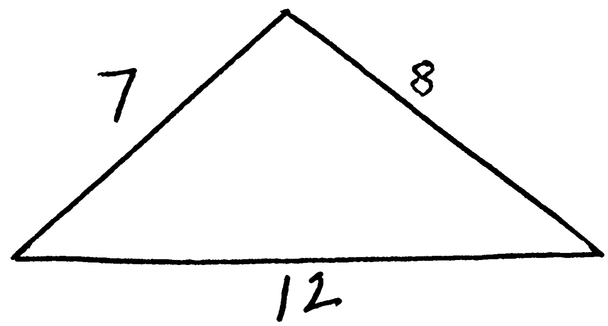


6. The arcs with radii 7 cm and 5 cm do not intersect since $5+7 < 15$.

7. a.



b. For a triangle, the sum of the lengths of any two sides is greater than the length of the third side.

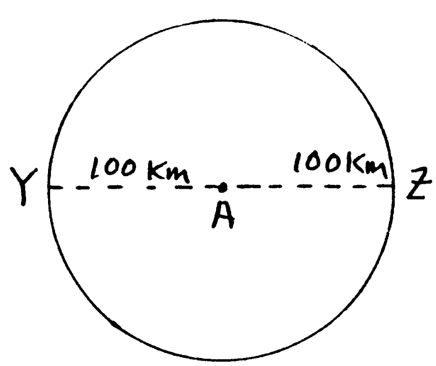
8. You cannot construct a triangle with sides 5 cm, 10 cm and 15 cm since the segments would be collinear.

9. If P, Q and M are points in the plane and d(P,Q) + d(Q,M) = d(P,M) the points P, Q and M are collinear.

§ 2. Exercises
(Page 11)

1. a. Both Ygleph and Zyzzx are, at most, 100 km from the antenna.

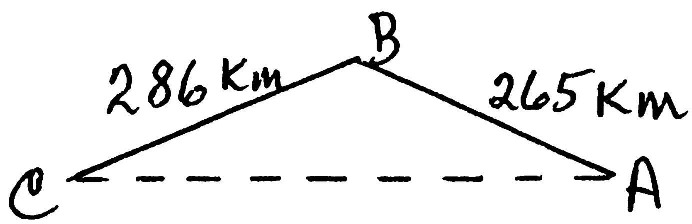
b. The messenger travelling from Ygleph to the antenna and then to Zyzzx would travel at most 200 km.



c. The maximum possible distance between Ygleph and Zyzzx is 200 km. This distance would occur if Ygleph and Zyzzx were at opposite ends of a diameter of a circle with radius 100 km. (see diagram in 1b.)

Proof:
d(Y,Z) $\le$ d(Y,A) + d(A,Z) by the Triangle Inequality
$\le$ 100 + 100 since Y and Z receive the signal
$\le$ 200

2. d(A,C) $\le$ d(A,B) + d(B,C) by the Triangle inequality
d(A,C) $\le$ 265 + 286 = 551 by the charts
Also 286 $\le$ d(A,C) + d(A,B) by the Triangle inequality
$\le$ d(A,C) + 265, since d(A,B) = 265.
Hence, d(A,C) $\ge$ 286-265=21.
Therefore, 21 $\le$ d(A,C) $\le$ 551

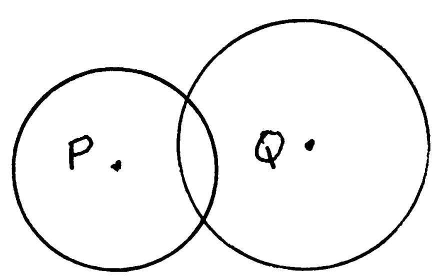


3. (a) yes
(b) yes
(c) no, 5 + 2 < 8
(d) yes
(e) no, 1 1/2 + 3 1/2 = 5, collinear
(f) yes

4. If two sides of a triangle are 12 cm and 20 cm, the third side must be larger than _8_ cm, and smaller than _32_ cm.

20 - 12 < x < 12 + 20, where x is the length of the third side

5. d(P,Q) < r_1 + r_2



6. X 1½ Z ______________ Y
5

$d(X,Z) + d(Z,Y) = d(X,Y)$, by SEG postulate.
Hence, $1\ 1/2 + d(Z,Y) = 5$ and $d(Z,Y) = 3\ 1/2$.

7. Check your own work.

8. Since X and Y are contained in the disc, $d(P,X) \leq r$ and $d(P,Y) \leq r$.
$d(X,Y) \leq d(P,X) + d(P,Y)$ by the Triangle inequality
Therefore, $d(X,Y) \leq r + r = 2r$

Experiment 1-2

(Pages 12, 13)

1. a. If a number is even, then it is divisible by 2.
If a number is divisible by 2, then it is even.

b. If $6x = 18$, then $x = 3$.
If $x = 3$, then $6x = 18$.

c. If a car is registered in California, then it has California license plates.
If a car has California license plates, then it is registered in California.

d. If all of the angles of a triangle have equal measure, then the triangle is equilateral.
If the triangle is equilateral, then all of its angles have equal measure.

e. If two distinct lines are parallel, then they do not intersect.
If two distinct lines do not intersect, then they are parallel.

2. a. <u>False</u>, If the square of a number is 9, then the number is not necessarily 3 because $(-3)^2 = 9$ also.

b. <u>False</u>, If a man lives in the U.S., then he does not necessarily live in California. He may live in any U.S. state.

c. <u>False</u>, If $a^2 = b^2$, then a is not necessarily equal to b. For example, $a = -3$ and $b = 3$ and $a^2 = b^2$.

d. True

e. False, The statement If the sum of integers x, y and z is 3y, then x, y and z are consecutive integers is a false statement. For example,
let $x = 5$, $y = 10$ and $z = 15$. Then $x + y + z = 30 = 3y$; but 5, 10 and 15 are not consecutive integers.

3. The converse of a true "if-then" statement is not always true. Note #2 (a, b, c and d).

Make up five "if-then" statements and test them to determine whether they are true or not by testing the statement for any exceptions. If there are any exceptions, the "if-then" statement is false.

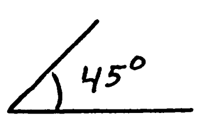
§ 3. Exercises

(Pages 22-25)

1. a. $\angle ABC$ or $\angle CBA$ or $\angle B$
   b. $\angle RPQ$ or $\angle QPR$
   c. $\angle YOX$ or $\angle XOY$
   d. $\angle BOC$ or $\angle COB$

2. a. $x = 180 - 40 = 140°$
   b. $x = 360 - 68 = 292°$
   c. $x = 146 - 35 = 111°$

3. a.



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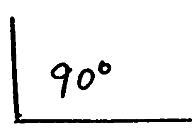

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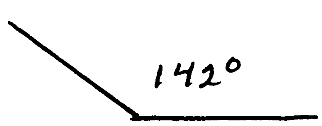

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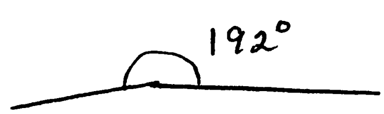

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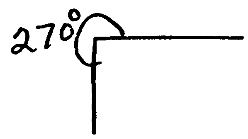


4. (1) 45° (2) 60° (3) 135°
(4) 90° (5) 20° (6) 225°

5. 90/360 = 1/4 The length of arc is (1/4) (36) = 9.

6. a. 45/360 = 1/8 The length of arc is (1/8)(36) = 9/2
b. 180/360 = 1/2 The length of arc is (1/2)(36) = 18
c. 60/360 = 1/6 The length of arc is (1/6)(36) = 6

d. (x/360)(36) = x /10

7. (1/360)(40,000) = 40,000/360 = 1000/9 km

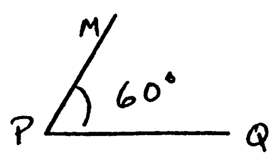
8. a. Distance to the equator = (43/360)(40,000) = 172000/360 km = 43000/9 km
b. (47/360)(40,000) = 47000/9 km to North Pole since 90°-43° = 47°

9. If l is the latitude of your home city or town, take [(90 - l)/360](40,000) = distance to N.P.

10. All of the angles $\angle AXB$ have the same measure.

11. a. 30° b. 220° c. 110°

12. a.



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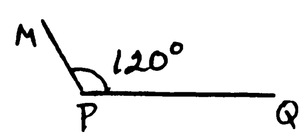

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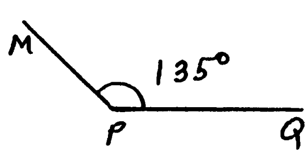

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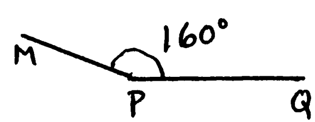

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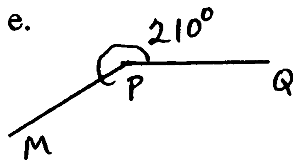

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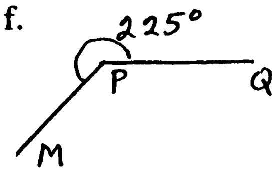
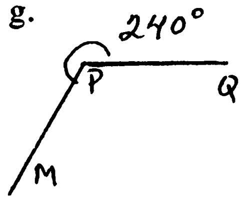


g.



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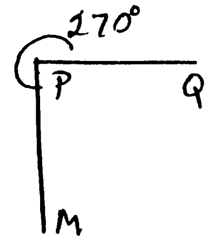
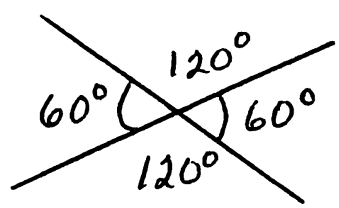


13. Converse: If points Q, P and M lie on the same line, then m(∠OPM) = 0°. It is false as shown by

<———————180°———————>
Q P M

Experiment 1-3
(Pages 27-29)

1. For example,



a. m(∠A) = m(∠A') = 120°,
b. m(∠B) = m(∠B') = 60°

c. Measure the angles as above.

d. Conclusion: Opposite angles have the same measure.

2. a. Produce a figure similar to fig. 1.51

b. The measure of the angle formed by the bisectors is 90°.

Exercise 1.

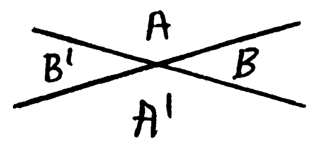
a. m(∠A) + m(∠B) = 180° since they form a straight angle.
b. For the same reason, m(∠A') + m(∠B) = 180°.
c. m(∠A) = m(∠A')

Exercise 2.

a. m($\angle$QOP) + m($\angle$ROP) = 180° since these adjacent angles form a straight angle.

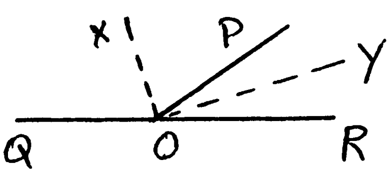
b. Let m($\angle$QOP) = x°. Then m($\angle$ROP) = (180-x)°
m(angle between bisectors) = (1/2)m($\angle$QOP) + (1/2)m($\angle$ROP)
= (1/2)(m($\angle$QOP) + m($\angle$ROP))
= (1/2)(x + (180 - x)) = (1/2)(180) = 90°.

Proof of 1.



Because $\angle$A and $\angle$B form a straight angle, we know m($\angle$A) + m($\angle$B) = 180. Similarly,
m($\angle$A') + m($\angle$B) = 180. Therefore,
m($\angle$A) + m($\angle$B) = m($\angle$A') + m($\angle$B).
m($\angle$A) = m($\angle$A') subtracting m($\angle$B) from each side.
When two lines intersect, opposite angles have the same measure.

Proof of 2



Let R_{OX} bisect $\angle$QOP and R_{OY} bisect $\angle$ROP.
Since $\angle$QOP and $\angle$ROP form a straight angle we know
m($\angle$QOP) = m($\angle$ROP) = 180. If we let m($\angle$QOP) = x and substitute in the first equation, we obtain
x + m($\angle$ROP) = 180. Thus
m($\angle$ROP) = 180 -x.
The angle between the bisectors, $\angle$XOY, has measure
m($\angle$XOY)= (1/2) m($\angle$QOP) + (1/2) m($\angle$ROP)
= (1/2) (m($\angle$QOP) + m($\angle$ROP))
= (1/2) (x + (180 - x))
= (1/2) (180) = 90°.
Thus, we have that the bisectors of a pair of linear angles form an angle whose measure is 90°.

§ 4. Exercise

(Pages 33-36)

1. A <u>postulate</u> is a statement accepted as true without proof.
   a. Acceptable answers include all postulates given in the text. The Triangle Inequality Postulate is one example.
   b. e.g. Given real numbers x, y and a, if $x=y$ then $x+a=y+a$.

2. a. $m(\angle NOM) = m(\angle POQ) = 90 - 55 = 35°$
   b. $m(\angle SOQ) = m(\angle RON)$, Opposite angles

3. Given $m(\angle WPX) = 2m(\angle WPY) = 2x°$
   and $m(\angle WPY) = x°$. (see figure 1.57)
   $m(\angle YPV) = m(\angle WPX) = 2x°$, opposite angles
   $x + 2x = 180$, since $\angle WPX$ and $\angle WPY$ form a straight angle.
   Thus $3x = 180$ and $x = 60$. Therefore, $m(\angle YPV) = 120°$.

4. Assume R_{OB} bisects $\angle AOC$ and R_{OC} bisects $\angle BOD$.
   Prove: $m(\angle AOB) = m(\angle COD)$(see fig.1.58)

   Let $m(\angle AOB) = x$, $m(\angle BOC) = y$ and $m(\angle COD = z$.
   Since R_{OB} bisects $\angle AOC$, $x = y$.
   Since R_{OC} bisects $\angle BOD$, $y = z$.
   Therefore $x = z$ and $m(\angle AOB) = m(\angle COD)$.

5. Assume lines $\overleftrightarrow{PV}$, $\overleftrightarrow{QT}$ and RS meet in point O and line $\overleftrightarrow{QT}$ bisects $\angle POR$. Prove: Line $\overleftrightarrow{QT}$ bisects $\angle SOV$. see fig.1.59)

   $m(\angle POQ) = m(\angle QOR)$ since R_{OT} bisects $\angle POR$
   $m(\angle POQ) = m(\angle TOV)$, opposite angles
   $m(\angle QOR) = m\angle SOT)$, opposite angles
   Therefore, $m(\angle TOV) = m(\angle SOT)$.
   Thus R_{OT} bisects $\angle SOV$ and line QT bisects $\angle SOV$.

6. Assume: $d(P,Q) = d(R,S)$
   Prove: $d(P,R) = d(Q,S)$ (see fig. 1.60)

   Since $d(P,Q) = d(R,S)$ we can add $d(Q,R)$ to both sides of this equation and obtain
   $d(Q,R) = d(P,Q) + d(Q,R)$ by SEG postulate
   $= d(R,S) + d(Q,R)$ by assumption
   $= d(Q.S)$ by SEG postulate

7. Assume points X and Y are contained in a disc with radius r around a point P. Prove: $d(X,Y) \leq 2r$.

$d(X,Y) \leq d(P,X) + d(P,Y)$, by the Triangle Inequality
$\leq r + r = 2r$ because X, Y are in the disc centered at P, radius r

8. Assume line L intersects lines K and U so that $\angle 1$ is supplementary to $\angle 2$. Prove: $m(\angle 3) = m(\angle 4)$ (see fig.1.61)

$m(\angle 1)+m(\angle 2) = 180$, $\angle 1$ supplementary to $\angle 2$
$m(\angle 1)+m(\angle 3) = 180$, $\angle 1$, $\angle 3$ are linear angles Therefore,
$m(\angle 1)+m(\angle 2) = m(\angle 1)+m(\angle 3)$. Thus
$(\angle 2) = m(\angle 3)$, by subtraction and
$m(\angle 2) = m(\angle 4)$ since they are opposite angles.
$m(\angle 3) = m(\angle 4)$ by substitution.

9. Assume in triangle ABC, $m(\angle CAB) = m(\angle CBA)$ and in triangle ABD, $m(\angle DAB) = m(\angle DBA)$.
Prove: $m(\angle CAD) = m(\angle\ CBD)$ (see fig.1.62)

Since $\angle DAB$ and $\angle CAD$ are adjacent angles, we obtain
$m(\angle DAB) + m(\angle CAD) = m(\angle CAB)$.
$m(\angle CAD) = m(\angle CAB)\text{-}m(\angle DAB)$ by subtraction and
$= m(\angle CBA)\text{-}m(\angle DBA)$, assumption and substitution
$= m(\angle CBD)$ $\angle CBA$ and $\angle DBA$ are adjacent angles.
Therefore, $m(\angle CAD) = m(\angle CBD)$

10. Assume $m(\angle b) = m(\angle c)$
Prove: $m(\angle a) = m(\angle d)$ (see fig. 1.63)

$\angle a$ and $\angle b$ are opposite angles as are $\angle c$ and $\angle d$. By theorem 1.1
$m(\angle a) = m(\angle b)$ and
$m(\angle c) = m(\angle d)$. Then
$m(\angle b) = m(\angle c)$ by assumption, and
$m(\angle a) = m(\angle d)$ by substitution.

11. Assume points P, B, C and Q are collinear, $m(\angle x) = m(\angle y)$, segment $\overline{BK}$ bisects $\angle ABC$ and segment $\overline{CK}$ bisects $\angle ACB$.
Prove: $m(\angle KBC) = m(\angle KCB)$ (see fig. 1.64)

Since $\angle PBA$ and $\angle ABC$ are linear angles we have
$m(\angle x)+m(\angle ABC) = 180$
$m(\angle ABC) = 180 - m(\angle x)$. Similarly, since

m($\angle$ACB) = 180 - m($\angle$y) since $\angle$y and $\angle$ACB are linear angles.
Segment $\overline{BK}$ bisects $\angle$ABC so
m($\angle$KCB) = 1/2 m($\angle$ABC) and
= 1/2(180 - m($\angle$x)). Thus
= 1/2(180 - m($\angle$y), by assumption.
Also, segment $\overline{CK}$ bisects $\angle$ACB and
m($\angle$KCB) = 1/2 m($\angle$ACB) Thus
= 1/2(180 - m($\angle$y) by substitution.
m($\angle$KBC) = m($\angle$KCB) since both equal 1/2(180 - m($\angle$y).

§ 5. Exercise

(Pages 41-43)

1. Since K $\perp$ V and L $\perp$ V we have K parallel to L by theorem 1.2. Two lines perpendicular to the same line in a plane are parallel.

2. Given that $K \perp L_1$ and L_1 is parallel to L_2 we have $K \perp L_2$ by PERP 2. Given two parallel lines, L_1 and L_2, if $K \perp L_1$, then $K \perp L_2$.

3. Assume $\overline{PR} \perp \overline{PT}$ and $\overline{PQ} \perp \overline{PS}$
Prove: m($\angle$a) = m($\angle$b) (see fig. 1.76)

   By assumption, $\overline{PR} \perp \overline{PT}$ thus $\angle$TRP is a right angle and m($\angle$b)+m($\angle$x)=90. Since $\overline{PQ} \perp \overline{PS}$ we also have m($\angle$a)+m($\angle$x)=90. Since both sums equal 90 we obtain m($\angle$a)+m($\angle$x)=m($\angle$b)+m($\angle$x). By subtraction m($\angle$a)=m($\angle$b).

4. For each example, the assumption should lead to a contradiction of some accepted statement (as in the example).

5. Assume: ABCD is a parallelogram with $\angle$A a right angle (see fig. 1.77) Prove: $\angle$B, $\angle$C and $\angle$D are right angles.

   Since $\angle$A is a right angle, $\overline{AB} \perp \overline{AD}$. Since ABCD is a parallelogram $\overline{AD}$ is parallel to $\overline{AB}$. By Perp 2 we conclude that $\overline{AB} \perp \overline{BC}$ and $\angle$B is a right angle. Also, $\overline{AB}$ is parallel to $\overline{CD}$ and BC $\perp$ AB so Perp 2 implies that $\overline{BC} \perp \overline{CD}$ and $\angle$C is a right angle. Finally, since $\overline{CD} \perp \overline{BC}$ and $\overline{BC}$ is parallel to $\overline{AD}$ we obtain $\overline{CD} \perp \overline{AD}$ and $\angle$D is a right angle by Perp 2. Therefore, $\angle$B, $\angle$C and $\angle$D are right angles and ABCD is a rectangle.