1.1 $$f = f(x,t) = f(x(x',t'), t(t'))$$

$$\frac{\partial f}{\partial t'} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t'} + \frac{\partial f}{\partial t}\frac{\partial t}{\partial t'}$$

$$\frac{\partial f}{\partial x'} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial x'}$$

$$\frac{D\rho'}{Dt'} \equiv \frac{\partial \rho'}{\partial t'} + v'\frac{\partial \rho'}{\partial x'} = \frac{\partial \rho}{\partial t'} + v'\frac{\partial \rho}{\partial x'}$$

$$= \frac{\partial \rho}{\partial x}\frac{\partial x}{\partial t'} + \frac{\partial \rho}{\partial t}\frac{\partial t}{\partial t'} + v'\left[\frac{\partial \rho}{\partial x}\frac{\partial x}{\partial x'}\right]$$

$$= V\frac{\partial \rho}{\partial x} + \frac{\partial \rho}{\partial t} + v'\frac{\partial \rho}{\partial x} = (V+v')\frac{\partial \rho}{\partial x} + \frac{\partial \rho}{\partial t}$$

$$= \frac{\partial \rho}{\partial t} + v\frac{\partial \rho}{\partial x}$$ Same form as before

$$\frac{Dv'}{Dt} \equiv \frac{\partial v'}{\partial t'} + v'\frac{\partial v'}{\partial x'} = \frac{\partial}{\partial t'}(v-V) + (v-V)\frac{\partial}{\partial x'}(v-V)$$

$$= \frac{\partial v}{\partial t'} + (v-V)\frac{\partial v}{\partial x'}$$

$$= \frac{\partial v}{\partial x}\frac{\partial x}{\partial t'} + \frac{\partial v}{\partial t}\frac{\partial t}{\partial t'} + (v-V)\left[\frac{\partial v}{\partial x}\frac{\partial x}{\partial x'}\right]$$

$$= V\frac{\partial v}{\partial x} + \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} - V\frac{\partial v}{\partial x}$$

$$= \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}$$ Same form as before

1.2 Arbitrary Region. Vapor crosses the surface and the volume decreases.

1.3 The velocity of a material region is the velocity of the interface plus the circulation velocity of the liquid (and gas) along the interface.

1.4 Yes If $\underline{w}$ is the normal velocity of the fluid at the control surface, $\underline{w} \equiv (\underline{n} \cdot \underline{v})\, \underline{n}$, then fluid does not cross the surface. It can, however, slide along the surface

1.5
$$\underline{v}_i' \equiv \underline{v}_i - \underline{v}$$

$$m_i \underline{v}_i' = m_i \underline{v}_i - m_i \underline{v}$$

$$\sum m_i \underline{v}_i' = \sum m_i \underline{v}_i - \sum m_i \underline{v}$$

$$\sum m_i v_i' = \sum m_i \underline{v}_i - \underline{v} \sum m_i$$

$$\frac{\sum m_i v_i'}{\sum m_i} = \frac{\sum m_i \underline{v}_i}{\sum m_i} - \underline{v}$$

$$\frac{\sum m_i v_i'}{\sum m_i} = \underline{v} - \underline{v} = 0$$

1.6

$$\underline{v}_i = \underline{v}_i' + \underline{v}$$

$$\underline{v}_i \cdot \underline{v}_i = \underline{v}_i' \cdot \underline{v}_i' + 2\,\underline{v} \cdot \underline{v}_i' + \underline{v} \cdot \underline{v}$$

$$\sum m_i \tfrac{1}{2} \underline{v}_i \cdot \underline{v}_i = \sum m_i \tfrac{1}{2} \underline{v}_i' \cdot \underline{v}_i' + \underbrace{\tfrac{2}{2}\underline{v} \cdot \sum m_i \underline{v}_i'}_{=0 \text{ from } 1.5} + \sum m_i \tfrac{1}{2} \underline{v} \cdot \underline{v}$$

1.8

$$\frac{\text{momentum}}{\text{unit vol}} = \underline{P} = \lim \frac{\sum m_i \underline{v}_i}{V} = \lim \frac{\sum m_i}{V} \frac{\sum m_i \underline{v}_i}{\sum m_i}$$

$$= \lim \frac{\sum m_i}{V} \lim \frac{\sum m_i \underline{v}_i}{\sum m_i}$$

$$= \rho\, \underline{v}$$

1.7 Yes. Unsteady processes must maintain equilibrium among the particles. A time could be formed using the typical particle velocity v_p and the continuum length say $l_{\text{mean free path}}$

$$t_{\text{collision}} \sim \frac{l_{mfp}}{v_p}$$

1.8

1.8

$$\rho = \lim_{L \to 0} \frac{\sum m_i}{V}$$

$$\underline{v} = \lim_{L \to 0} \frac{\sum m_i v_i}{\sum m_i}$$

$$\rho \underline{v} = \lim_{L \to 0} \frac{\sum m_i}{V} \frac{\sum m_i v_i}{\sum m_i}$$

$$= \lim_{L \to 0} \frac{\sum m_i v_i}{V} = \lim_{L \to 0} \frac{\sum \underline{p}_i}{V}$$

Chapt 2 Problems

2.1 1 (a) no, shear forces present

2 (b) no, system not homogeneous

3 (d) Could be, neglect contraction & assume normal force only.
No, if true stress state with compression, tension and shear stresses are accounted for

2.2 Yes, S/E is intensive; $S_2 = \lambda S_1$; $E_2 = \lambda E_1$

$$\frac{S_2}{E_2} = \frac{S_1}{E_1}$$

2.3 $$S = R_0 N \left(\frac{E}{E_0}\right)^{1/2}\left(\frac{V}{V_0}\right)\left(\frac{N}{N_0}\right)^{-3/2}$$

$$\frac{1}{T} = \left.\frac{\partial S}{\partial E}\right)_{V,N} = R_0 N \frac{1}{2}\left(\frac{1}{E E_0}\right)^{1/2}\left(\frac{V}{V_0}\right)\left(\frac{N}{N_0}\right)^{-3/2}$$

$$\frac{P}{T} = \left.\frac{\partial S}{\partial V}\right)_{E,N} = \frac{R_0 N}{V_0}\left(\frac{E}{E_0}\right)^{1/2}\left(\frac{N}{N_0}\right)^{-3/2}$$

$$\frac{\mu}{T} = \left.\frac{\partial S}{\partial N}\right)_{E,V} = -\frac{1}{2} R_0 N^{-3/2}\left(\frac{E}{E_0}\right)^{1/2} N_0^{3/2}$$

Eulers Eq $$S = \frac{1}{T}E + \frac{P}{T}V + \frac{\mu}{T}N$$

$$S = \frac{1}{2}R_0 N\left(\frac{E}{E_0}\right)^{1/2}\left(\frac{V}{V_0}\right)\left(\frac{N}{N_0}\right)^{-3/2} + R_0 N\frac{V}{V_0}\left(\frac{E}{E_0}\right)^{1/2}\left(\frac{N}{N_0}\right)^{-3/2} - \frac{1}{2}R_0 N\left(\frac{E}{E_0}\right)^{\frac{1}{2}}\left(\frac{N_0}{N}\right)^{3/2}$$

$$S = R_0 N\frac{V}{V_0}\left(\frac{E}{E_0}\right)^{1/2}\left(\frac{N}{N_0}\right)^{-3/2}$$ which check the original equation

2.4 $$dE = TdS - pdV - \mu dN$$

so $$dS = \frac{1}{T}dE + \frac{p}{T}dV + \frac{\mu}{T}dN$$

$$\frac{1}{T} \equiv \left.\frac{\partial S}{\partial E}\right)_{V,N} \quad \text{or} \quad T = \frac{1}{\left.\frac{\partial S}{\partial E}\right|_{V,N}}$$

$$\frac{p}{T} \equiv \left.\frac{\partial S}{\partial V}\right)_{E,N} \quad \text{or} \quad p = \frac{\partial S/\partial V|_{E,N}}{\partial S/\partial E|_{V,N}}$$

$$\frac{\mu}{T} = \left.\frac{\partial S}{\partial N}\right)_{E,V} \quad \text{OR} \quad \mu = \frac{\partial S/\partial N|_{E,V}}{\partial S/\partial E|_{V,N}}$$

2.5 Divide 2.7.2b by m to get the fundamental equation in a unit mass basis

$$de = T\,ds - p\,dv$$

From the definition of enthalpy 2.9.4 and $v = \frac{1}{\rho}$

$$h = e + pv$$

$$dh = de + p\,dv + v\,dp \quad \text{OR}$$

$$= T\,ds + p\,dv$$

Thus since $dh = \left.\frac{\partial h}{\partial s}\right)_v ds + \left.\frac{\partial h}{\partial v}\right)_s dv$

$$T = \left.\frac{\partial h}{\partial s}\right|_v \quad \& \quad v = \left.\frac{\partial h}{\partial v}\right)_s$$

At $p = 30$ psia, $h = 1050\,s - 650$ Btu/lbm

$s = 1.8$ Btu/(lbm °R)

$\therefore$ $T = 1050\ °R$

$$v = \left.\frac{\partial h}{\partial v}\right)_s \approx \left.\frac{\Delta h}{\Delta v}\right|_s = \frac{-630-(-650)}{(35-30)\cdot 144}$$

$$= .0275\ \frac{\text{Btu}}{\text{lbm}}\ \frac{\text{ft}^2}{\text{lb}_f}\ 778\ \frac{\text{ft-lb}_f}{\text{Btu}}$$

$$= 21.40\ \frac{\text{ft}^3}{\text{lbm}}$$

$$\rho = \frac{1}{v} = 0.0463\ \frac{\text{lbm}}{\text{ft}^3}$$

$$h = 1050\,s - 650 = 1240\ \frac{\text{Btu}}{\text{lbm}}$$

$$e = h - pv = 1240 - \frac{30\cdot 144\cdot 21.4}{778}$$

$$= 1240 - 118.8$$

$$= 1121\ \text{Btu/lbm}$$

2.6 $$G \equiv H - TS = E + PV - TS$$

but from 2.4.4

$$dE = TdS - pdV - \mu dN$$

so $$dG = dE + PdV - VdP - TdS - SdT$$

$$= -\mu dN - VdP - SdT$$

This is the fundamental equation of thermodynamics for G and the independent variables are:

$$N, T, \& P$$

or $$G = G(T, P, N)$$

gives all thermodynamic information about a substance.

$\boxed{2.7}$ $p = \rho R T \qquad \rho = \frac{p}{RT}$

$$\alpha = \frac{1}{\rho}\frac{\partial \rho}{\partial p}\Big)_T$$

$$= \frac{1}{\rho}\frac{1}{RT} = \frac{1}{p}$$

$$\beta = -\frac{1}{\rho}\frac{\partial \rho}{\partial T}\Big)_p$$

$$= -\frac{1}{\rho}\frac{p}{R}(-T^{-2}) = \frac{p}{\rho R T^2} = \frac{1}{T}$$

$\boxed{2.8}$

$$T ds = de + p\, d\rho^{-1} \qquad (2.7.2 \div M)$$

$$(2.9.3) \qquad de = c_v\, dT + \rho^{-2}\left[p - T\frac{\partial p}{\partial T}\Big)_\rho\right] d\rho$$

$$T ds = c_v dT + (-p\rho^{-2})d\rho + \rho^{-2}\left[p - T\frac{\partial p}{\partial T}\Big)_\rho\right] d\rho$$

$$= c_v dT - \frac{T}{\rho^2}\frac{\partial p}{\partial T}\Big)_\rho d\rho$$

$$ds = c_v \frac{dT}{T} - \frac{1}{\rho^2}\frac{\partial p}{\partial T}\Big)_\rho d\rho$$

$\boxed{2.9}$ $p = \rho R T \qquad \frac{\partial p}{\partial T}\Big)_\rho = \rho R$

$$ds = \int_{T_0, \rho_0}^{T, \rho_0} \frac{c_v(T, \rho_0 = \text{const})}{T} dT - R\int_{T, \rho_0}^{T, \rho} \frac{d\rho}{\rho}$$

$$s - s_0 = c_v \ln\frac{T}{T_0} - R \ln\frac{\rho}{\rho_0}$$

2.4 ~~$TdS = dE + pdV + \mu dN$~~ 1^{st} ed

or

~~$dE = +TdS - pdV - \mu dN$~~

equations of state are $T = \left.\frac{\partial E}{\partial S}\right)_{V,N}$ $\quad -p = \left.\frac{\partial E}{\partial V}\right)_{S,N}$ & $\mu = \left.\frac{\partial E}{\partial N}\right)_{S,V}$

2.5 see page 2-3

2.6 ~~skipped~~ see next page

~~2.6~~ Consider fixed-adiabatic-impermeable is changed to

old 1^{st} ed fixed-diathermal-permeable

$S = S_A + S_B$ from state ① to state ② ΔE, ΔN change

$$\Delta S = \Delta S_A + \Delta S_B = \frac{1}{T_A}\Delta E_A + \frac{1}{T_B}\Delta E_B + \frac{\mu_A \Delta N_A}{T_A} + \frac{\mu_B}{T_B}\Delta N_B$$

for the manifold we find the max state by $\Delta S = 0$

Now $\Delta E_A = -\Delta E_B$ $\qquad \Delta N_A = -\Delta N_B$

$$\Delta S = 0 = \left(\frac{1}{T_A} - \frac{1}{T_B}\right)\Delta E_A + \left(\frac{\mu_A}{T_A} - \frac{\mu_B}{T_B}\right)\Delta N_A$$

for arbitrary small changes in ΔE_A & ΔN_A. Hence

$$T_A = T_B$$

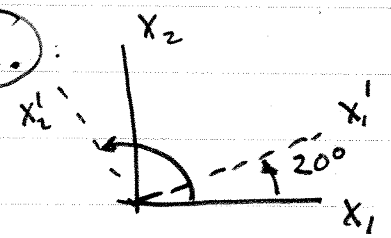
and

$$\mu_A = \mu_B$$

for the member of the manifold with maximum entropy.

Chapter 3 Solutions

1.



In new system coordinates of P are x_1', x_2', x_3'. They are

$$x_j' = c_{ij'} x_i$$

Now

$c_{11'} = \cos 20° = .940$ $\quad c_{12'} = \cos 110° = -.342$ $\quad c_{13'} = \cos 90 = 0$

$c_{21'} = \cos 70 = .342$ $\quad c_{22'} = \cos 20 = .940$ $\quad c_{23'} = 0$

$c_{31'} = \cos 90 = 0$ $\quad c_{32'} = 0$ $\quad c_{33'} = \cos 0 = 1$

$$x_1' = c_{i1'} x_i = c_{11'} x_1 + c_{21'} x_2 + c_{31'} x_3$$
$$= .94(5) + .342(4) + 0(0)$$
$$= \underline{6.07}$$

$$x_2' = c_{i2'} x_i = c_{12'} x_1 + c_{22'} x_2 + c_{32'} x_2$$
$$= -.342(5) + .940(4) + 0(0)$$
$$= \underline{2.05}$$

$$x_3' = c_{13'} x_1 + c_{23'} x_2 + c_{33'} x_3$$
$$= 0 + 0 + 1 \cdot 0$$
$$= \underline{0}$$

2.

$a = b_i c_{ij} d_j$ OK

$a = b_i c_i + d_j$ No free index j occurs in only one term

$a_i = \delta_{ij} b_i + c_i$ No 〃 〃 〃 〃

$a_k = b_i c_{ki}$ OK

$a_k = b_k c + d_i e_{ih}$ OK

Chapter 3

$a_i = b_i + c_{ij} d_{ji} e_i$ No i appears 3 times in last term

$a_l = \varepsilon_{ijk} b_j c_k$ No free index on left is l but on right ~~$\cdot$~~ i

$a_{ij} = b_{ji}$ OK

$a_{ij} = b_i c_j + e_{jk}$ No free index k in last term is not the same as i in other terms

$a_{kl} = b_i c_{ki} d_e + e_{ki}$ No free index i in last term is not same as l in others

2. (a) if u & v $\perp$ then $\underline{u} \cdot \underline{v} = 0$

$u_i v_i = u_1 v_1 + u_2 v_2 + u_3 v_3 = 3 \cdot 4 + 2 \cdot 1 + (-7) \cdot 2$

$= 12 + 2 - 14 = 0 \Rightarrow \perp$

(b)

$v = \sqrt{4^2 + 1^2 + 2^2} = 4.583$

$w = \sqrt{6^2 + 4^2 + (-5)^2} = 8.775$

(c)

$\underset{\sim}{v} \cdot \underset{\sim}{w} = vw \cos\theta = v_1 w_1 + v_2 w_2 + v_3 w_3$

$= 4 \cdot 6 + 1 \cdot 4 + 2(-5)$

$= 18$

$\cos\theta = \frac{18}{4.883 \quad 8.775} = .488$

$\theta = 63.4^\circ$

(d) $\alpha_i = \frac{w_i}{w}$ $\quad \alpha_1 = \frac{6}{8.775} = .684$

$\alpha_2 = \frac{4}{8.775} = .456$ $\quad \alpha_3 = \frac{-5}{8.775} = -.570$

(e) $\alpha \cdot u = \alpha_1 u_1 + \alpha_2 u_2 + \alpha_3 u_3 = .684(3) + .456(2) - .570(-7)$

6.95

Chapter 3

(4.) No; the cosines are items which cannot be measured with a single coordinate system

(5.) δ_{ij} is defined as $\delta_{11}=1$ $\delta_{12}=0$, $\delta_{13}=0$ etc
In a new coordinate system, it would be

$$\delta'_{ij} = C_{ki}' C_{lj}' \delta_{kl}$$

By definition as the substitution tensor we set $l=k$

$$\delta'_{ij} = C_{ki}' C_{kj}'$$

But the RHS, by the law of cosines 3.1.16, is δ_{ij} hence

$$\delta'_{ij} = \delta_{ij}$$

The components are the same in any coordinate system

(6.) $(a \times b)\cdot c = a\cdot(b\times c) = (c\times a)\cdot b$

$$\varepsilon_{ijk} a_j b_k c_i = \varepsilon_{jki} a_j b_k c_i \quad \text{since } \varepsilon_{jki} = \varepsilon_{ijk}$$
$$= a\cdot(b\times c)$$

$$\varepsilon_{ijk} a_j b_k c_i = \varepsilon_{kij} c_i a_j b_k \quad \text{since } \varepsilon_{kij} = \varepsilon_{ijk}$$
$$= (c\cdot a)\times b$$

See next page for the other parts of problem (6)

(7.) $T_{(ij)} \equiv \frac{1}{2} T_{ij} + \frac{1}{2} T_{ji}$ $\qquad T_{[ij]} \equiv \frac{1}{2} T_{ij} - \frac{1}{2} T_{ji}$

$$\begin{bmatrix} 6 & 3 & 1 \\ 4 & 0 & 5 \\ 1 & 3 & 2 \end{bmatrix} = \begin{bmatrix} 6 & \frac{4+3}{2} & \frac{1+1}{2} \\ \frac{7}{2} & 0 & \frac{5+3}{2} \\ 1 & 4 & 2 \end{bmatrix} + \begin{bmatrix} 0 & \frac{3-4}{2} & \frac{1-1}{2} \\ +\frac{1}{2} & 0 & \frac{5-3}{2} \\ 0 & -1 & 0 \end{bmatrix}$$

(7.) continued

$$\begin{array}{c} T_{ij} \\ \begin{bmatrix} 6 & 3 & 1 \\ 4 & 0 & 5 \\ 1 & 3 & 2 \end{bmatrix} \end{array} = \begin{array}{c} T_{(ij)} \\ \begin{bmatrix} 6 & \frac{7}{2} & 1 \\ \frac{7}{2} & 0 & 4 \\ 1 & 4 & 2 \end{bmatrix} \end{array} + \begin{array}{c} T_{[ij]} \\ \begin{bmatrix} 0 & -\frac{1}{2} & 0 \\ +\frac{1}{2} & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \end{array}$$

Dual vector

$d_i = \varepsilon_{ijk} T_{jk}$

$$d_1 = \varepsilon_{1jk} T_{jk} = \varepsilon_{123} T_{23} + \varepsilon_{132} T_{32}$$
$$= -3 + 5 = \underset{\sim}{+2}$$

$$d_2 = \varepsilon_{2jk} T_{jk} = \varepsilon_{231} T_{31} + \varepsilon_{213} T_{13}$$
$$= 1 - 1 = \underset{\sim}{0}$$

$$d_3 = \varepsilon_{3jk} T_{jk} = \varepsilon_{312} T_{12} + \varepsilon_{321} T_{21}$$
$$= 3 - 4 = \underset{\sim}{-1}$$

$\underset{\sim}{d} = (-2, 0, -1)$

---

continued

(6.) $t \times (u \times v) = \underline{u}(t \cdot v) - \underline{v}(\underline{t} \cdot \underline{u})$

$$\varepsilon_{pqi} t_q \varepsilon_{ijk} u_j v_k = \varepsilon_{ipq} \varepsilon_{ijk} t_q u_j v_k$$
$$= (\delta_{pj}\delta_{qk} - \delta_{pk}\delta_{qj}) t_q u_j v_k$$
$$= v_k t_k u_p - v_p t_j u_j$$

which is the left side of the equation above

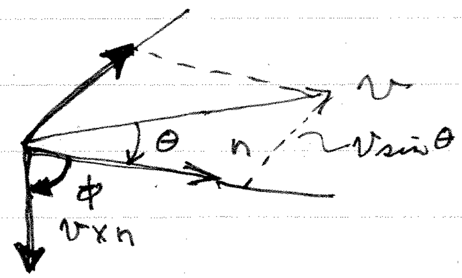
$$\underline{u} \times \underline{v} = -\underline{v} \times \underline{u}$$
$$\varepsilon_{ijk} u_j v_k = -\varepsilon_{ikj} v_k u_j$$

(8.) $S_{ij}T_{ji}$ since $S_{ij} = S_{ji}$ & $T_{ji} = -T_{ij}$

$$S_{ij}T_{ji} = S_{ji}(-T_{ij}) = -S_{ji}T_{ij} = -S_{ij}T_{ji}$$

a quantity equal to its negative must be zero

(9.)



$$\underset{\sim}{v} \times \underset{\sim}{n} = 1 v \sin\theta$$

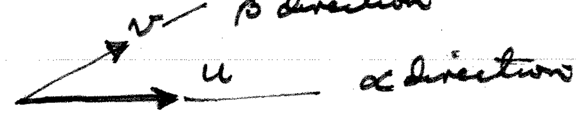
$$n \times (v \times n) = 1 \cdot v \sin\theta \cdot \sin\phi$$

$$\sin\phi = 1$$

$$n \times (v \times n) = v \sin\theta$$

(10.) $w_i = \varepsilon_{ijk} u_j v_k$

$w = uv \sin(\alpha,\beta)$



choose x_i system so u_i ; $u_1 = u, u_2 = 0, u_3 = 0$

choose x_i' system so v_i' ; $v_1' = v, v_2' = 0, v_3' = 0$

Now $$w^2 = w_i w_i = \varepsilon_{ijk} u_j v_k \varepsilon_{ipq} u_p v_q = \varepsilon_{ijk}\varepsilon_{ipq} u_j v_k u_p v_q$$

$$= (\delta_{jp}\delta_{kq} - \delta_{jq}\delta_{kp}) u_j v_k u_p v_q$$

$$= u_j u_j v_k v_k - u_j v_j u_k v_k$$

$$= u^2 v^2 - u_j v_j u_k v_k$$

Next since $v_k = c_{\ell k} v_\ell'$, $u_k v_k = u_k c_{\ell k} v_\ell' = u_1 c_{11} v_1'$

$$u_k v_k = uv\cos(\alpha,\beta)$$

$\therefore$

$$w^2 = u^2v^2 - u^2v^2\cos^2(\alpha,\beta) = u^2v^2(1-\cos^2(\alpha,\beta))$$

$$= u^2v^2\sin^2(\alpha,\beta)$$

(11.) $\partial_i \delta_j = \partial_i x_j$ $\partial_1 x_2 = 0$, $\partial_1 x_1 = 1$ etc therefore

$$\partial_i x_j = \delta_{ij}$$

12

$$\mathrm{div}(\phi \underline{v}) = \phi\, \mathrm{div}\, \underline{v} + \underline{v} \cdot \mathrm{grad}\, \phi$$

$$\nabla\cdot(\phi \underline{v}) = \phi\, \nabla\cdot\underline{v} + \underline{v}\cdot\nabla\phi$$

$$\partial_i(\phi v_i) = \phi\, \partial_i v_i + v_i \partial_i \phi$$

$$\mathrm{div}(\underline{u}\times\underline{v}) = \underline{v}\cdot \mathrm{curl}\, \underline{u} - \underline{u}\cdot \mathrm{curl}\, \underline{v}$$

$$\nabla\cdot(\underline{u}\times\underline{v}) = \underline{v}\cdot(\nabla\times\underline{u}) - \underline{u}\cdot(\nabla\times\underline{v})$$

$$\partial_i(\varepsilon_{ijk} u_j v_k) = \varepsilon_{ijk}\, \partial_i u_j v_k = \varepsilon_{ijk} u_j \partial_i v_k + \varepsilon_{ijk} v_k \partial_i u_j$$

$$= u_j(-\varepsilon_{jik}\partial_i v_k) + v_k\, \varepsilon_{kij}\partial_i u_j$$

$$= -\underline{u}\cdot(\nabla\times\underline{v}) + \underline{v}\cdot(\nabla\times\underline{u})$$

$$\mathrm{curl}(\underline{u}\times\underline{v}) = \underline{v}\cdot \mathrm{grad}\,\underline{u} - \underline{u}\cdot \mathrm{grad}\,\underline{v} + \underline{u}\, \mathrm{div}\,\underline{v} - \underline{v}\, \mathrm{div}\,\underline{u}$$

$$\nabla\times(\underline{u}\times\underline{v}) = (\underline{v}\cdot\nabla)\underline{u} - (\underline{u}\cdot\nabla)\underline{v} + \underline{u}\, \nabla\cdot\underline{v} - \underline{v}\, \nabla\cdot\underline{u}$$

$$\varepsilon_{lmi}\partial_m(\varepsilon_{ijk} u_j v_k) = \varepsilon_{ilm}\varepsilon_{ijk}\, \partial_m(u_j v_k)$$

$$= (\delta_{lj}\delta_{mk} - \delta_{lk}\delta_{mj})\, \partial_m(u_j v_k)$$

$$= \partial_k(u_l v_k) - \partial_j(u_j v_l)$$

$$= u_l \partial_k v_k + v_k \partial_k u_l - u_j \partial_j v_l - v_l \partial_j u_j$$

which is $\underline{u}\, \nabla\cdot\underline{v} + (\underline{v}\cdot\nabla)\underline{u} - (\underline{u}\cdot\nabla)\underline{v} - (\underline{v}\ \nabla)\cdot\underline{u}$

Chapter 3

(13) $\partial_i\partial_j(\;)$ is symmetric because the order of two differentiation processes may be interchanged.

$$\text{curl grad } \phi \;, \quad \varepsilon_{kji}\,\partial_j\partial_i\phi = 0$$

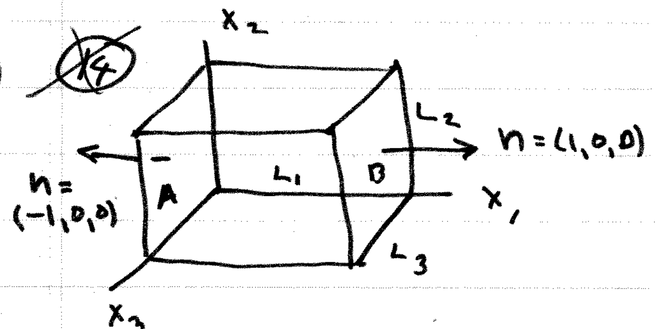
This is always zero because ε_{kji} is antisymmetric and $\partial_j\partial_i$ is symmetric so the product is zero as in problem 3.8

$$\text{div curl } v \;, \quad \partial_i(\varepsilon_{ijk}\,\partial_j v_k)$$

$$= \varepsilon_{ijk}\,\partial_i\partial_j v_k = 0$$

1st ed

~~14~~



$$\int_R \partial_i(T_{jk})\,dV = \int_S n_i T_{jk}\,dS$$

$$T_{jk\ldots} = \phi = x_1$$

$$\int_R \partial_i(x_1)\,dV = \int_S n_i x_1\,dS$$

for $i = 1$, $\partial_1 x_1 = 1$

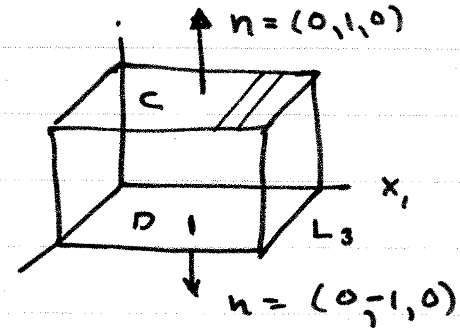
$$\int_R dV = Vol = L_1 \cdot L_2 \cdot L_3$$

$$\int_S n_1 x_1\,dS = \int_B 1 \cdot L_1\,dS + \int_A (-1)\cdot(0)\,dS \qquad n_1 = 0 \text{ on all other sides}$$

$$= L_1 \int_B dS = L_1 \cdot L_2 \cdot L_3$$

Chapter 3

(14) continued 1st ed



for $i = 2$ $\quad \partial_2 x_1 = 0$

$$\int_R 0 \, dV = 0$$

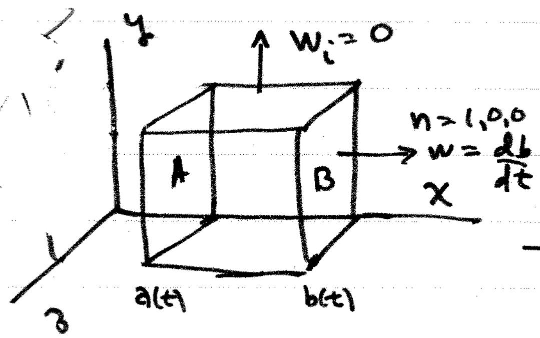
$$\int_S n_2 x_1 \, dS = \int_C 1 \, x_1 \, dS + \int_D (-1) x_1 \, dS$$

$$= \int x_1 L_3 \, dx_1 - \int x_1 L_3 \, dx_1$$

$$= 0$$

for $i = 3$ $\quad \partial_3 x_1 = 0$ and both integrals are again zero

(14) 2nd 3rd



$$\frac{d}{dt}\int_R f(x,t)\, dV = \int_R \frac{\partial f}{\partial t} dV + \int_S n_i w_i f \, dS$$

Take $T_{ij..}(x_i,t) = f(x,t)$. Let R be $a(t), b(t)$ on x axis and one unit on y & z axes

$$\frac{d}{dt}\int_0^1\int_0^1\int_a^b f(x,t)\, dx\, dy\, dz = \frac{d}{dt}\int_a^b f \, dx$$

$$\int_0^1\int_0^1\int_a^b \frac{\partial f}{\partial t} dV = \int_a^b \partial f/\partial t \, dx$$

$$\int_S n_i w_i f \, dS = \int_B (1) \frac{db}{dt} f(x,t)\, dy\, dz + \int_A (-1) \frac{da}{dt} f(x,t)\, dy\, dz$$

$$= \frac{db}{dt} f(b,t) - \frac{da}{dt} f(a,t)$$

$$T_{ij} = -\frac{2}{3}\mu\,\delta_{ij}\,\partial_k v_k + 2\mu\,\partial_{(i}v_{j)}$$

1st ed

$$\partial_i T_{ij} = -\frac{2}{3}\mu\,\delta_{ij}\,\partial_i\underbrace{\partial_k v_k}_{=0} + 2\mu\,\partial_i\left[\frac{1}{2}\partial_i v_j + \frac{1}{2}\partial_j v_i\right]$$

$$= \mu\,\partial_i\partial_i v_j + \mu\,\partial_i\partial_j v_i$$

$$= \mu\,\partial_i\partial_i v_j + \mu\,\partial_j\underbrace{\partial_i v_i}_{=0}$$

2nd & 3rd edition

(15)

$$v_j\partial_j v_i = \partial_i\left(\frac{1}{2}v^2\right) - \varepsilon_{ijk}\,v_j\,\omega_k$$

$$\omega_k = \varepsilon_{k\ell m}\,\partial_\ell v_m$$

$$v_j\partial_j v_i \overset{?}{=} \partial_i\left(\frac{1}{2}v^2\right) - \varepsilon_{ijk}\,\varepsilon_{k\ell m}\,v_j\,\partial_\ell v_m$$

$$\overset{?}{=} \partial_i\left(\frac{1}{2}v^2\right) - \varepsilon_{kij}\,\varepsilon_{k\ell m}\,v_j\,\partial_\ell v_m$$

$$\overset{?}{=} \partial_i\left(\frac{1}{2}v^2\right) - \left[\delta_{i\ell}\delta_{jm} - \delta_{im}\delta_{j\ell}\right]v_j\,\partial_\ell v_m$$

$$\overset{?}{=} \frac{1}{2}v_k\partial_i v_k + \frac{1}{2}v_k\partial_i v_k - v_j\partial_i v_j + v_j\partial_j v_i$$

$$= v_j\partial_j v_i \qquad \text{OK}$$

Chapter 3

3.16 $\varepsilon_{ijk}\varepsilon_{ije} \stackrel{?}{=} 2\delta_{ke}$

$$\varepsilon_{ijk}\varepsilon_{ipl} = \delta_{jp}\delta_{kl} - \delta_{jl}\delta_{kp}$$

now set $p = j$

$$\varepsilon_{ijk}\varepsilon_{ipl} = \delta_{jj}\delta_{kl} - \delta_{jl}\delta_{kj}$$

$$\delta_{jj} = \delta_{11} + \delta_{22} + \delta_{33} = 3$$

$$\delta_{jl}\delta_{kj} = \delta_{1l}\delta_{k1} + \delta_{2l}\delta_{k2} + \delta_{3l}\delta_{k3}$$

$$= \delta_{kl}$$

3.17 NEXT PAGE

$$\varepsilon_{ijk}\varepsilon_{ipl} = 3\delta_{kl} - \delta_{kl} = 2\delta_{kl}$$

3.18

$$-\nabla\times\nabla\times\underline{v} \stackrel{?}{=} \nabla^2\underline{v} = \nabla\cdot\nabla(\underline{v})$$

$$-\varepsilon_{lmi}\partial_m\varepsilon_{ijk}\partial_j v_k \stackrel{?}{=} \partial_j\partial_j v_l$$

$$-\varepsilon_{ilm}\varepsilon_{ijk}\partial_m\partial_j v_k =$$

$$-\left[\delta_{lj}\delta_{mk} - \delta_{lk}\delta_{mj}\right]\partial_m\partial_j v_k =$$

$$-\{\partial_k\partial_l v_k - \partial_j\partial_j v_l\} =$$

$$\partial_j\partial_j v_l - \partial_l\underbrace{\partial_k v_k}_{=0} = \partial_j\partial_j v_l$$