

**Industrial Organization: Markets and Strategies**

Paul Belleflamme and Martin Peitz  
published by Cambridge University Press

Part I. Getting started

**Exercises**

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**Exercise 1** *Free trade and competitive markets [included in 2nd edition of the book]*

Consider the market for shoes in country A. Demand is assumed to be  $100 - p$  where  $p$  is the final consumer price. Suppose that country A does not produce shoes and that there are two importers B and C. The export prices for shoes in both countries are  $p_B = 59.99$  and  $p_C$ , respectively. Furthermore suppose that country A is a small country so that its demand does not influence export prices. Suppose that, initially, country A levies a uniform import tariff of  $t = 10$  on each pair of imported shoes.

1. Assume  $p_C = 45$ . What is the effect on demand and welfare in country A if country A signs a free trade agreement with country B?
2. Assume  $p_C = 50$ . What is now the effect on demand and welfare in country A if country A signs a free trade agreement with country B?

**Solutions to Exercise 1** In the absence of a free-trade agreement with country B, the consumers in country A always buy shoes from country C as  $p_C + t < 59.99 + t$  whether  $p_C = 45$  or  $p_C = 50$ . If there is a free-trade agreement with country B, then consumers in country A compare  $p_C + t$  with  $p_B = 59.99$ . As  $t = 10$ , we have that consumers buy shoes from country C if  $p_C = 45$  but from country B if  $p_C = 50$ . Comparing the two situations, we see that the free-trade agreement with country B has no effect if  $p_C = 45$  (i.e., if country C is very inexpensive with respect to country B), as consumers in country A continue to buy shoes from country C. However, if  $p_C = 50$ , the free-trade agreement makes consumers buy shoes from country B rather than from country C; this allows them to pay a lower price, which increases their surplus.

**Exercise 2** *Monopoly problem [included in 2nd edition of the book]*

Consider a monopolist with a linear demand curve:  $q = a - bp$ , where  $a, b > 0$ . It produces at constant marginal cost  $c$  and has no fixed cost. Assume that  $0 < c < a/b$ .

1. Find the monopoly price, quantity, and profits.
2. Derive the inverse demand curve  $P(q)$ . Draw  $P(q)$ , the MR-curve, and the MC-curve in a diagram. Explain why we need the assumption  $c < a/b$ .

3. Does it matter that the monopolist sets price instead of quantity?
4. Calculate the deadweight loss of monopoly.
5. A change in  $b$  results in two opposing effects on the deadweight loss. Calculate the effect of a change in  $b$  on the deadweight loss.
6. Derive the price elasticity of demand  $\eta$  for any price. How does  $\eta$  change with  $p$ ?
7. Show mathematically as well as graphically that the price elasticity of demand  $\eta > 1$  at the monopoly price.

## Solutions to Exercise 2

1. The monopoly chooses  $p$  to maximize  $\pi = (p - c)(a - bp)$ . The first-order condition yields  $a - bp - bp + bc = 0$ , which is equivalent to  $p^m = (a + bc) / (2b)$ . We compute then  $q^m = a - bp^m = (a - bc) / 2$  and  $p^m - c = (a - bc) / (2b)$ . It follows that  $\pi^m = (a - bc)^2 / (4b)$ .
2. The inverse demand curve  $P(q)$  is obtained by inverting  $q = a - bp$ :  $bp = a - q \Leftrightarrow p = a/b - q/b$ . The intercept on the vertical axis (where price is measured) is  $a/b$ , and the intercept on the horizontal axis (where quantity is measured) is  $a$ . The MC curve is a horizontal line that cuts the vertical axis at  $c$ ; if  $c$  were larger than  $a/b$ , inverse demand would be everywhere below the MC and no production would be profitable. The MR curve is  $MR = a/b - 2q/b$ , which has the same vertical intercept as the inverse demand curve but cuts the horizontal axis at  $a/2$  instead of  $a$ .
3. No, because the monopoly controls the demand function, i.e., the relationship between  $p$  and  $q$ . To be sure, solve the monopoly problem in terms of quantity. That is, let the monopoly choose  $q$  to maximize  $\pi = (a/b - q/b)q - cq$ .
4. The first-best is achieved at marginal cost pricing:  $p^* = c$ ; the corresponding quantity is  $q^* = a - bc$ . Welfare is then equal to  $W^* = (1/2)(a/b - c)(a - bc) = (a - bc)^2 / (2b)$ . Under monopoly, the consumer surplus is equal to  $CS^m = (a - bc)^2 / (8b)$ ; adding the monopoly's profit, we compute welfare under monopoly as  $W^m = 3(a - bc)^2 / (8b)$ . Hence, the deadweight loss of monopoly is  $W^* - W^m = (a - bc)^2 / (8b)$ .
5. The price elasticity of demand is defined as

$$\eta(p) = -q'(p) \frac{p}{q(p)} = \frac{bp}{a - bp}.$$

We compute

$$\eta'(p) = \frac{ab}{(a - bp)^2} > 0,$$

meaning that the price elasticity of demand increases with  $p$ .

6. We have that

$$\eta(p^m) = \frac{bp^m}{a - bp^m} = \frac{a + bc}{a - bc} > 1.$$

**Exercise 3** *Two-period monopoly problem [included in 2nd edition of the book]*

Consider a monopolist that produces for two periods. The demand curves in both periods are  $q^t = 1 - p^t$  for  $t = 1, 2$ . The marginal costs are  $c$  in the first and  $c - \lambda q^1$  in the second period. Here,  $\lambda$  is a small and positive number. There is a discount factor of  $\delta$  between the periods.

1. Explain briefly how the monopolist's problem changes compared to a situation where the marginal cost is  $c$  in both periods.
2. Find the quantities  $q^1$  and  $q^2$  that the monopolist chooses in the two periods. Hint: Start by solving the monopolist's problem in the second period and then continue to the first period.
3. Derive the restriction on  $\lambda$  that ensures that the profit function is strictly concave.

### Solutions to Exercise 3

1. If the marginal cost is  $c$  in both periods, then the two periods are unrelated (as the monopolist's decisions in the first period do not affect the demand nor the cost in the second period). In contrast, when the second-period marginal cost is  $c - \lambda q^1$ , the monopolist lowers his cost in the second period by producing more in the first period (this can result from some learning economies, for instance).
2. In the second period, the monopolist chooses  $q^2$  to maximize  $\pi^2 = (1 - q^2) q^2 - (c - \lambda q^1) q^2$ . The optimum is easily found as  $q^2 = (1/2) (1 - c + \lambda q^1)$ , resulting in a profit of  $\pi^2 = (1/4) (1 - c + \lambda q^1)^2$ . In the first period, the monopolist chooses  $q^1$  to maximize  $\pi^1 + \delta \pi^2 = (1 - q^1) q^1 - c q^1 + (\delta + 4) (1 - c + \lambda q^1)^2$ . The first-order condition yields

$$(1 + 2(4 + \delta)\lambda)(1 - c) - 2(1 - (4 + \delta)\lambda^2)q^1 = 0,$$

and the second-order condition yields  $-2(1 - (4 + \delta)\lambda^2) < 0$ , which is satisfied as long as  $(4 + \delta)\lambda^2 < 1$  or  $\lambda < \sqrt{4 + \delta}$  (this is the answer to question 3). Then, the optimum is found as

$$q^1 = \frac{1 + 2(4 + \delta)\lambda}{1 - (4 + \delta)\lambda^2} \frac{1 - c}{2}.$$

It follows that

$$q^2 = \frac{2 + \lambda}{2(1 - (4 + \delta)\lambda^2)} \frac{1 - c}{2}.$$

We check that for  $\lambda = 0$ ,  $q^1 = q^2 = (1 - c)/2$ , while for  $0 < \lambda < \sqrt{4 + \delta}$ ,  $q^1 > q^2 > (1 - c)/2$ .

**Exercise 4** *Market structure for rating agencies*<sup>1</sup>

In the recent financial crisis, rating agencies have become a focus of attention. The market has traditionally been dominated by a few big agencies, currently Standard & Poor, Moody's and Fitch. In 2006, the Securities and Exchange Commission (SEC) introduced measures to speed up the approval process for rating agencies with the aim to increase competition. However, in response to the financial crisis, the Fed introduced lending programmes for which—this is what it said initially—it accepts only collateral that has been appraised by one of the big three. Discuss the likely consequences of such a decision on market structure.

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<sup>1</sup>see "the wages of sin", in: The Economist, April 25, 2009, page 76.