

CHAPTER 2

- P2.1.** A W24×192 of A242 steel is to be used as a beam in a building structure. What are the values of the yield stress, F_y , ultimate tensile stress, F_u , and modulus of elasticity, E to be used in the design of this beam.

Solution

Enter LRFD Table 2-4, and note that a W24×192 shape belongs to Group 3. In LRFD Table 2-1, for W-shapes in A242 steel, the only alloy available for Group 3 shapes is of Grade 46 (see note k). Also, from this table, observe that A242 steel of Grade 46 has a yield stress F_y of 46 ksi and an ultimate tensile stress F_u of 67 ksi. Modulus of elasticity for all steels, $E = 29,000$ ksi.

- P2.2.** A PL3×8 of A514 steel is to be used as a tension member in a truss. What are the values of the yield stress, F_y , and ultimate tensile stress, F_u , to be used in the design of this member?

Solution

Enter LRFD Table 2-2, and note that a 3 in. thick A514 plate is available in Grade 90 only. Also, from this table, observe that A514 steel of Grade 90 has a yield stress F_y of 90 ksi and a minimum specified ultimate tensile stress F_u of 100 ksi.

- P2.3.** A chandelier weighing 2 kips hangs from the dome of a theater. The rod from which it hangs is 20 ft long and has a diameter of $\frac{1}{2}$ in. Calculate the stress and strain in the rod and its elongation. Neglect the weight of the rod itself in the calculations.

Solution

Tensile force, $T = 2$ kips; Length, $L = 20$ ft = 240 in.

Diameter, $d = \frac{1}{2}$ in.; Modulus of elasticity, $E = 29,000$ ksi

Cross-sectional area, $A = \pi d^2/4 = 0.196$ in.² (LRFD Table 1-21)

Stress, $f = T/A = 2.00 \div 0.196 = 10.2$ ksi (Ans.)

Strain, $\epsilon = f/E = 10.2 \div 29,000 = 0.000352$ in./in. (Ans.)

$$\text{Elongation, } e = \epsilon L = 0.000352 (240) = 0.0845 \text{ in.} \quad (\text{Ans.})$$

- P2.4.** The rod in Problem P2.3 is of A36 steel having a stress-strain diagram (see Figs. 2.6.1 and 2.6.2) with $\epsilon_{st} = 0.012$ and $\epsilon_u = 0.18$. Determine the load that causes the rod to yield; and the load that causes the rod to fracture. Also, determine the elongation of the bar corresponding to strains of ϵ_y , ϵ_{st} , and ϵ_u . Comment on your results.

Solution

For A36 steel: $F_y = 36$ ksi, and $F_u = 58$ ksi (from LRFD Table 2-1)

$$\text{Load that causes the rod to yield in tension, } T_y = A F_y = 0.196 (36.0) = 7.06 \text{ kips} \quad (\text{Ans.})$$

$$\text{Load that causes the rod to fracture in tension, } T_u = A F_u = 0.196 (58.0) = 11.4 \text{ kips} \quad (\text{Ans.})$$

$$\text{Elastic strain, } \epsilon_y = F_y \div E = 36.0 \div 29,000 = 0.00124 \text{ in./in.}$$

Elongation of the rod corresponding to ϵ_y (maximum elastic elongation),

$$\Delta_y = \epsilon_y L = 0.00124(240) = 0.298 \text{ in.} \quad (\text{Ans.})$$

Strain hardening strain, $\epsilon_{st} = 0.012$ in./in.

$$\text{Elongation of the rod at onset of strain hardening, } \Delta_{st} = \epsilon_{st} L = 0.012 (240) = 2.88 \text{ in.} \quad (\text{Ans.})$$

Fracture strain, $\epsilon_u = 0.18$ in./in.

$$\text{Elongation of the rod at fracture, } \Delta_f = \epsilon_u L = 0.18 (240) = 43.2 \text{ in.} \quad (\text{Ans.})$$

The maximum elastic elongation of the member (0.3 in.) is (tolerably) small. The large elongations (43 in.) that precede the fracture give a visual warning of the impending failure of the member.

- P2.5.** Locate the principal axes for the beam sections given in Fig. P2.5. Also, calculate the cross-sectional area, weight per linear foot, and moment of inertia, and section moduli about the x - and y -axes.

See Figure P2.5 of text book

Solution

$$\text{W24} \times 68: \quad A = 20.1 \text{ in.}^2; \quad d = 23.7 \text{ in.}; \quad I_x = 1830 \text{ in.}^4; \quad I_y = 70.4 \text{ in.}^4$$

$$\text{PL } \frac{1}{2} \times 12: \quad A = 6.00 \text{ in.}^2; \quad I_x = (1/12) b t^3 = (1/12) (12.0) (\frac{1}{2})^3 = 0.125 \text{ in.}^4$$

$$I_y = (1/12) t b^3 = (1/12) (\frac{1}{2}) (12.0)^3 = 72.0 \text{ in.}^4$$

Built-up section

$$A = A_1 + A_2 = 20.1 + 6.00 = 26.1 \text{ in.}^2 \quad (\text{Ans.})$$

$$\text{Weight of the built-up section} = 26.1(3.40) = 88.7 \text{ plf} \quad (\text{Ans.})$$

The centers of gravity G_1 , G_2 and G all lie on the vertical axis of symmetry.

Assume reference axis $x'-x'$ at top fiber of the built-up section.

$$\bar{y} = \frac{(20.1)(11.85) + (6.00)(23.7 + 0.25)}{26.1} = 14.63 \text{ in.}$$

$$d_1 = 14.63 - 11.85 = 2.78 \text{ in.}; \quad d_2 = 23.95 - 14.63 \text{ in.} = 9.32 \text{ in.}$$

$$I_x = [1830 + 20.1(2.78)^2] + [0 + 6.00(9.32)^2] = 2510 \text{ in.}^4 \quad (\text{Ans.})$$

$$c_t = 14.63 \text{ in.}; \quad c_b = 23.7 + 0.5 - 14.63 = 9.57 \text{ in.}$$

$$S_{xt} = 2510 \div 14.63 = 172 \text{ in.}^3; \quad S_{xb} = 2510 \div 9.57 = 262 \text{ in.}^3 \quad (\text{Ans.})$$

As G_1 , G_2 and G are all located on the symmetry line,

$$I_y = [70.4 + 0] + [72.0 + 0] = 142 \text{ in.}^4 \quad (\text{Ans.})$$

$$c = 6.00 \text{ in.}$$

$$S_y = 142 \div 6.00 = 23.7 \text{ in.}^3 \quad (\text{Ans.})$$

P2.6. Locate the principal axes for the beam sections given in Figs. P2.6. Also, calculate the cross-sectional area, weight per linear foot, and moment of inertia, and section moduli about the x - and y -axes.

See Figure P2.6 of text book.

Solution

The centers of gravity G_1 , G_2 , G_3 and G all lie on the vertical axis of symmetry. To locate G , take an arbitrary axis $x'-x'$ coinciding with the bottom fibers of the bottom glange plate.

$$A = A_1 + A_2 + A_3 = 2.00(8.00) + 1.00(16.0) + 2.00(12.0) = 56.0 \text{ in.}^2 \quad (\text{Ans.})$$

$$\text{Weight of the built-up section} = 56.0(3.40) = 190 \text{ plf} \quad (\text{Ans.})$$

$$\bar{y} = \frac{16.0(19.0) + 16.0(10.0) + 24.0(1.00)}{56.0} = 8.71 \text{ in.}$$

$$d_1 = 19.0 - 8.71 = 10.3 \text{ in.}; \quad d_2 = 10.0 - 8.71 = 1.29 \text{ in.}; \quad d_3 = 8.71 - 1.00 = -7.71 \text{ in.}$$

$$I_x = [(1/12)(8.00)(2.00)^3 + 16.0(10.3)^2] + [(1/12)(1.00)(16.0)^3 + 16.0(1.29)^2]$$

$$+ [(1/12)(12.0)(2.00)^3 + 24.0(7.71)^2] = 3510 \text{ in.}^4 \quad (\text{Ans.})$$

$$c_t = 20.0 - 8.71 = 11.3 \text{ in.}; \quad c_b = 8.71 \text{ in.}$$

$$S_{xt} = 3510 \div 11.3 = 311 \text{ in.}^3; \quad S_{xb} = 3510 \div 8.71 = 403 \text{ in.}^3 \quad (\text{Ans.})$$

$$I_y = (1/12)(2.00)(8.00)^3 + 0.0 + (1/12)(2.00)(12.0)^3 = 373 \text{ in.}^4 \quad (\text{Ans.})$$

$$S_y = 373 \div 6.00 = 62.2 \text{ in.}^3 \quad (\text{Ans.})$$

- P2.7.** Locate the principal axes for the beam sections given in Fig. P2.7. Also, calculate the cross-sectional area, weight per linear foot, and moment of inertia, and section moduli about the x - and y -axes.

See Figure P2.7 of text book.

Solution

As the built-up section has two axes of symmetry, the center of gravity G of the built-up section coincides with the point of intersection of these two axes.

$$A = 2(36.0)(4.00) + 2(2.00)(36.0) = 432 \text{ in.}^2 \quad (\text{Ans.})$$

$$\text{Weight} = 432(3.40) \div 1000 = 1.47 \text{ klf} \quad (\text{Ans.})$$

$$I_x = 2[(1/12)(36.0)(4.00)^3 + 144(20.0)^2] + 2[(1/12)(2.00)(36.0)^3 + 0] = 131,000 \text{ in.}^4 \quad (\text{Ans.})$$

$$I_y = 2[(1/12)(4.00)(36.0)^3 + 0] + 2[(1/12)(36.0)(2.00)^3 + 72.0(15.0 + 1.0)^2] \\ = 68,000 \text{ in.}^4 \quad (\text{Ans.})$$

$$S_x = 131,000 \div 22.0 = 5960 \text{ in.}^3; \quad S_y = 68,000 \div 18.0 = 3780 \text{ in.}^3 \quad (\text{Ans.})$$

- P2.8.** Locate the principal axes for the column section given in Fig P2.8. Also, calculate the cross-sectional area, weight per linear foot, and moment of inertia, and radius of gyration about the x - and y -axes.

See Figure P2.8 of text book.

Solution

As the built-up section has two axes of symmetry, the center of gravity G of the built-up section coincides with the point of intersection of these two axes.

$$A = 2[28.0(7.125)] + 5.0(16.5) = 482 \text{ in.}^2 \quad (\text{Ans.})$$

$$\text{Weight} = 482(3.40) \div 1000 = 1.64 \text{ klf} \quad (\text{Ans.})$$

$$I_x = 2[(1/12)(28.0)(7.125)^3 + 28.0(7.125)(8.25 + 0.5 \times 7.125)^2] + (1/12)(5.00)(16.5)^3$$

$$= 59,200 \text{ in.}^4 \quad (\text{Ans.})$$

$$I_y = 2[(1/12)(7.125)(28.0)^3] + (1/12)(16.5)(5.00)^3 = 26,200 \text{ in.}^4 \quad (\text{Ans.})$$

$$r_x = \sqrt{\frac{59,200}{482}} = 11.1 \text{ in.}; \quad r_y = \sqrt{\frac{26,200}{482}} = 7.37 \text{ in.} \quad (\text{Ans.})$$

P2.9. Locate the principal axes for the column section given in Fig P2.9. Also, calculate the cross-sectional area, weight per linear foot, and moment of inertia, and radius of gyration about the x - and y -axes.

See Figure P2.9 of text book.

Solution

As the built-up section has two axes of symmetry, the center of gravity G of the built-up section coincides with the point of intersection of these two axes.

Built-up section

$$A = (24.0)^2 - (15.0)^2 = 351 \text{ in.}^2 \quad (\text{Ans.})$$

$$\text{Weight} = 351(3.40) \div 1000 = 1.19 \text{ klf} \quad (\text{Ans.})$$

$$I_x = (1/12)(24.0)(24.0)^3 - (1/12)(15.0)(15.0)^3 = 23,400 \text{ in.}^4 = I_y \quad (\text{Ans.})$$

$$r_x = \sqrt{\frac{23,400}{351}} = 8.17 \text{ in.} = r_y \quad (\text{Ans.})$$

P2.10. Locate the principal axes for the column section given in Fig P2.10. Also, calculate the cross-sectional area, weight per linear foot, and moment of inertia, and radius of gyration about the x - and y -axes.

See Figure P2.10 of text book.

Solution

$$W14 \times 730: A = 215 \text{ in.}^2; \quad b_f = 17.9 \text{ in.}; \quad I_x = 14,300 \text{ in.}^4; \quad I_y = 4720 \text{ in.}^4$$

$$\text{PL } 3 \times 24: A = 3.00(24.0) = 72.0 \text{ in.}^2; \quad I_x = (1/12)(3.00)(24.0)^3 = 3456 \text{ in.}^4$$

$$I_y = (1/12)(24.0)(3.00)^3 = 54.0 \text{ in.}^4$$

Built-up section

As the built-up section has two axes of symmetry, the center of gravity G of the built-up section coincides with the point of intersection of these two axes.

$$A = 215 + 2(72.0) = 359 \text{ in.}^2 \quad (\text{Ans.})$$

$$\text{Weight} = 359(3.40) \div 1000 = 1.22 \text{ klf} \quad (\text{Ans.})$$

$$I_x = 14,300 + 2(3456) = 21,200 \text{ in.}^4 \quad (\text{Ans.})$$

$$I_y = 4720 + 2[54.0 + 72.0(0.5 \times 17.9 + 1.5)^2] = 20,600 \text{ in.}^4 \quad (\text{Ans.})$$

$$r_x = \sqrt{\frac{21,200}{359}} = 7.68 \text{ in.}; \quad r_y = \sqrt{\frac{20,600}{359}} = 7.58 \text{ in.} \quad (\text{Ans.})$$

- P2.11.** Locate the principal axes for the column section given in Fig P2.11. Also, calculate the cross-sectional area, weight per linear foot, and moment of inertia, and radius of gyration about the x - and y -axes.

See Figure P2.11 of text book.

Solution

$$\text{W14} \times 145: \quad A = 42.7 \text{ in.}^2; \quad b_f = 15.5 \text{ in.}; \quad d = 14.8 \text{ in.}$$

$$I_x = 1710 \text{ in.}^4; \quad I_y = 677 \text{ in.}^4$$

Built-up section

As the section has two axes of symmetry, the center of gravity G of the built-up section coincides with the point of intersection of these two axes.

$$A = 4(42.7) = 171 \text{ in.}^2 \quad (\text{Ans.})$$

$$\text{Weight} = 171(3.40) \div 1000 = 0.581 \text{ klf} \quad (\text{Ans.})$$

$$I_x = 2[1710 + 42.7(0.5 \times 15.5 + 0.5 \times 14.8)^2] + 2[677 + 0] = 24,400 \text{ in.}^4 = I_y \quad (\text{Ans.})$$

$$r_x = r_y = \sqrt{\frac{24,400}{171}} = 12.0 \text{ in.} \quad (\text{Ans.})$$

