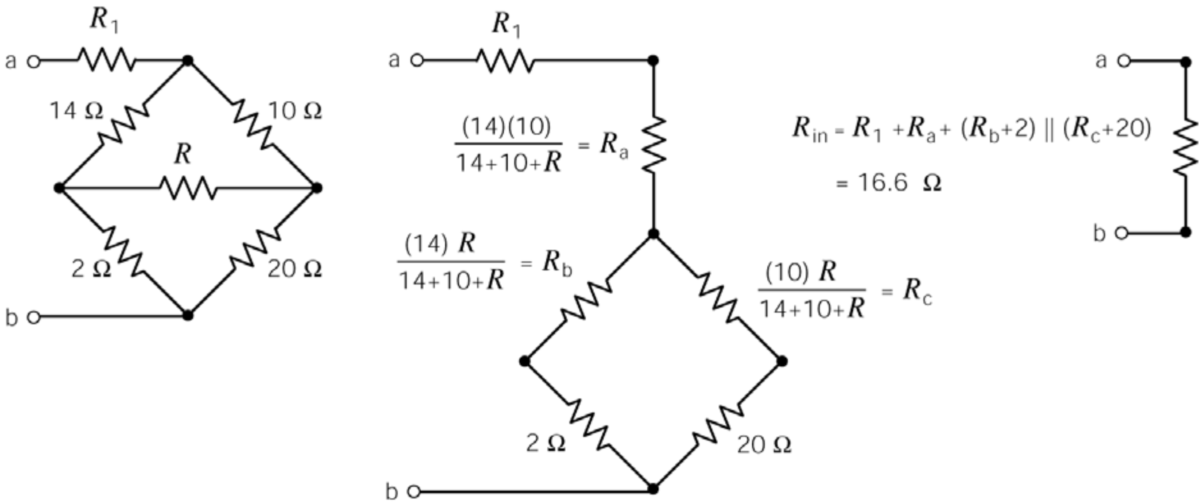
Design Problems

DP 17-1



We will need to find R and R_1 by trial and error. A Mathcad spreadsheet will help with the calculations. Given the restrictions $R \le 10\ \Omega$ and $R_1 \le 10\ \Omega$ we will start with $R = 10\ \Omega$ and $R_1 = 10\ \Omega$:

$R1 := 10 \qquad R := 10$

$Ra := \frac{14 \cdot 10}{14 + 10 + R} \qquad Rb := \frac{14 \cdot R}{14 + 10 + R} \qquad Rc := \frac{R \cdot 10}{14 + 10 + R}$

$Rin := R1 + Ra + \frac{(Rb + 2)\cdot(Rc + 20)}{Rb + 2 + Rc + 20} \qquad Rin = 18.947$

The specifications can be satisfied by reducing R_1:

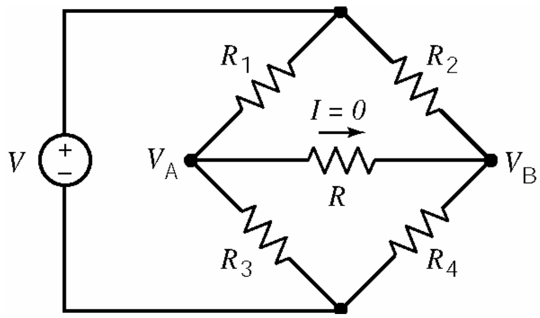
$R1 := 7.653 \qquad R := 10$

$Ra := \frac{14 \cdot 10}{14 + 10 + R} \qquad Rb := \frac{14 \cdot R}{14 + 10 + R} \qquad Rc := \frac{R \cdot 10}{14 + 10 + R}$

$Rin := R1 + Ra + \frac{(Rb + 2)\cdot(Rc + 20)}{Rb + 2 + Rc + 20} \qquad Rin = 16.6$

One solution is $R = 7.653\ \Omega$ and $R_1 = 10\ \Omega$.

DP 17-2



Need $V_A + V_B$ for balance

$$\frac{R_1\,V}{R_1+R_3}=\frac{R_2\,V}{R_2+R_4} \qquad (1)$$

$$\frac{R_3\,V}{R_1+R_3}=\frac{R_4\,V}{R_2+R_4} \qquad (2)$$

Dividing (1) by (2) yields: $\underline{\dfrac{R_1}{R_3}=\dfrac{R_2}{R_4}}$.

DP 17-3

$$\begin{aligned} V_1 &= h_{11}\,I_1 + h_{12}\,V_2 \\ I_2 &= h_{21}\,I_1 + h_{22}\,V_2 \end{aligned} \qquad \text{and} \quad V_2 = -I_2\,R_{\mathrm{L}} \quad \Rightarrow \quad I_2 = h_{21}\,I_1 - h_{22}\,R_{\mathrm{L}}\,I_2$$

Next

$$\frac{I_2}{I_1}=h_{21}\left(\frac{1}{1+h_{22}\,R_{\mathrm{L}}}\right) \quad \Rightarrow \quad A_{\mathrm{i}}=\frac{I_{\mathrm{L}}}{I_1}=-\frac{I_2}{I_1}=-h_{21}\left(\frac{1}{1+h_{22}\,R_{\mathrm{L}}}\right)$$

We require

$$79=80\left(\frac{1}{1+h_{22}R_{\mathrm{L}}}\right) \quad \Rightarrow \quad \frac{79}{80}\left(1+\frac{R_{\mathrm{L}}}{80\times 10^3}\right)=1 \quad \Rightarrow \quad R_{\mathrm{L}}=1.013\ \mathrm{k\Omega}\cong 1\ \mathrm{k\Omega}$$

Next

$$I_2=-\frac{V_2}{R_L}=h_{21}\,I_1+h_{22}\,V_2 \quad \Rightarrow \quad V_2\,(h_{22}+1/R_{\mathrm{L}})=-h_{21}\,I_1$$

Substituting this expression into the second hybrid equation gives:

$$V_1=h_{11}I_1+\frac{h_{12}(-h_{21})}{(h_{22}+{}^{1}\!/_{R_{\mathrm{L}}})}I_1$$

The input resistance can be approximated as

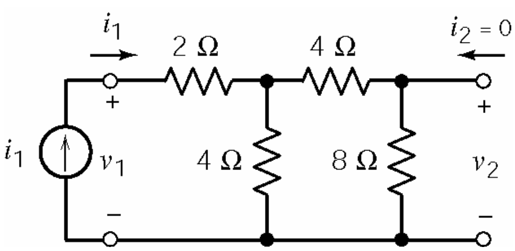
$$R_{\mathrm{in}}\approx h_{11}-h_{12}\,R_{\mathrm{L}}\,h_{21} \quad (\text{since } h_{22} << {}^{1}\!/_{R_{\mathrm{L}}})$$

Finally

$$R_{\mathrm{in}}=45-(5\times 10^{-4})(10^3)(80)=5\ \Omega\ <\ 10\ \Omega$$

DP 17-4

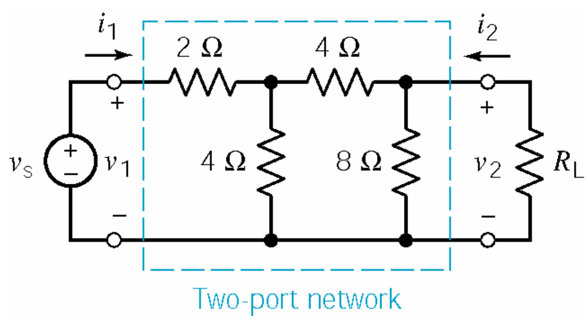
$$Z_{11} = 2 + \frac{4(12)}{4+12} = 5\ \Omega \quad \text{and} \quad Z_{22} = \frac{8(8)}{8+8} = 4\ \Omega$$



$$V_2 = 8\left[\frac{4}{4+12} I_1\right] = 2 I_1 \quad \Rightarrow \quad Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = 2\ \Omega$$

Similarly $Z_{12} = 2\ \Omega$

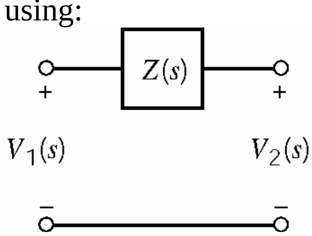
Thèvenin: $Z_T = Z_{22} = 4\ \Omega$ so for maximum power transfer, use R_L =4 Ω



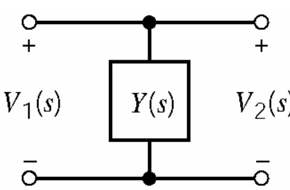
$$P_{RL} = \frac{\left(\frac{V_s}{2}\right)^2}{4} = 89.3\ \text{W} \Rightarrow V_s = 37.8\ \text{V}$$

DP 17-5

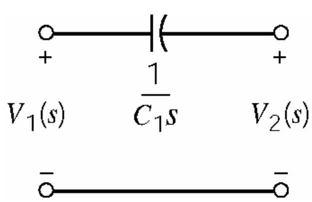
The circuit consists of 4 cascaded stages. Represent each stage by a transmission matrix using:



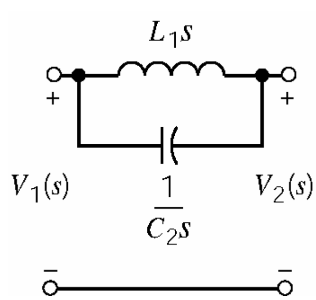
$$\mathbf{T} = \begin{bmatrix} 1 & Z(s) \\ 0 & 1 \end{bmatrix}$$



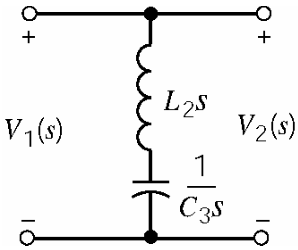
$$\mathbf{T} = \begin{bmatrix} 1 & 0 \\ Y(s) & 1 \end{bmatrix}$$



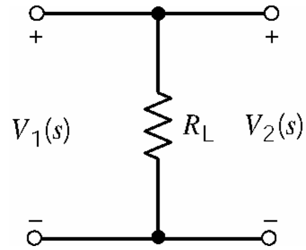
$$\mathbf{T}_a = \begin{bmatrix} 1 & \frac{1}{C_1 s} \\ 0 & 1 \end{bmatrix}$$



$$\mathbf{T}_b = \begin{bmatrix} 1 & \frac{L_1 s}{L_1 C_2 s^2 + 1} \\ 0 & 1 \end{bmatrix}$$



$$\mathbf{T} = \begin{bmatrix} 1 & 0 \\ \dfrac{C_3\, s}{L_2\, C_3\, s^2 + 1} & 1 \end{bmatrix}$$



$$\mathbf{T}_{\mathrm{d}} = \begin{bmatrix} 1 & 0 \\ \dfrac{1}{R_{\mathrm{L}}} & 1 \end{bmatrix}$$

$$\mathbf{T} = \mathbf{T}_{\mathrm{a}}\, \mathbf{T}_{\mathrm{b}}\, \mathbf{T}_{\mathrm{c}}\, \mathbf{T}_{\mathrm{d}} = \begin{bmatrix} 1 + \dfrac{L_1\left(C_1 + C_2\right)s^2 + 1}{\left(L_1\, C_2\, s^2 + 1\right)C_1\, s} \times \dfrac{L_2\, C_3\, s^2 + R_{\mathrm{L}} C_3\, s + 1}{R_{\mathrm{L}}\left(L_2\, C_3\, s^2 + 1\right)} & \dfrac{L_1\left(C_1 + C_2\right)s^2 + 1}{\left(L_1\, C_2\, s^2 + 1\right)C_1\, s} \\ \dfrac{L_2\, C_3\, s^2 + R_{\mathrm{L}} C_3\, s + 1}{R_{\mathrm{L}}\left(L_2\, C_3\, s^2 + 1\right)} & 1 \end{bmatrix}$$