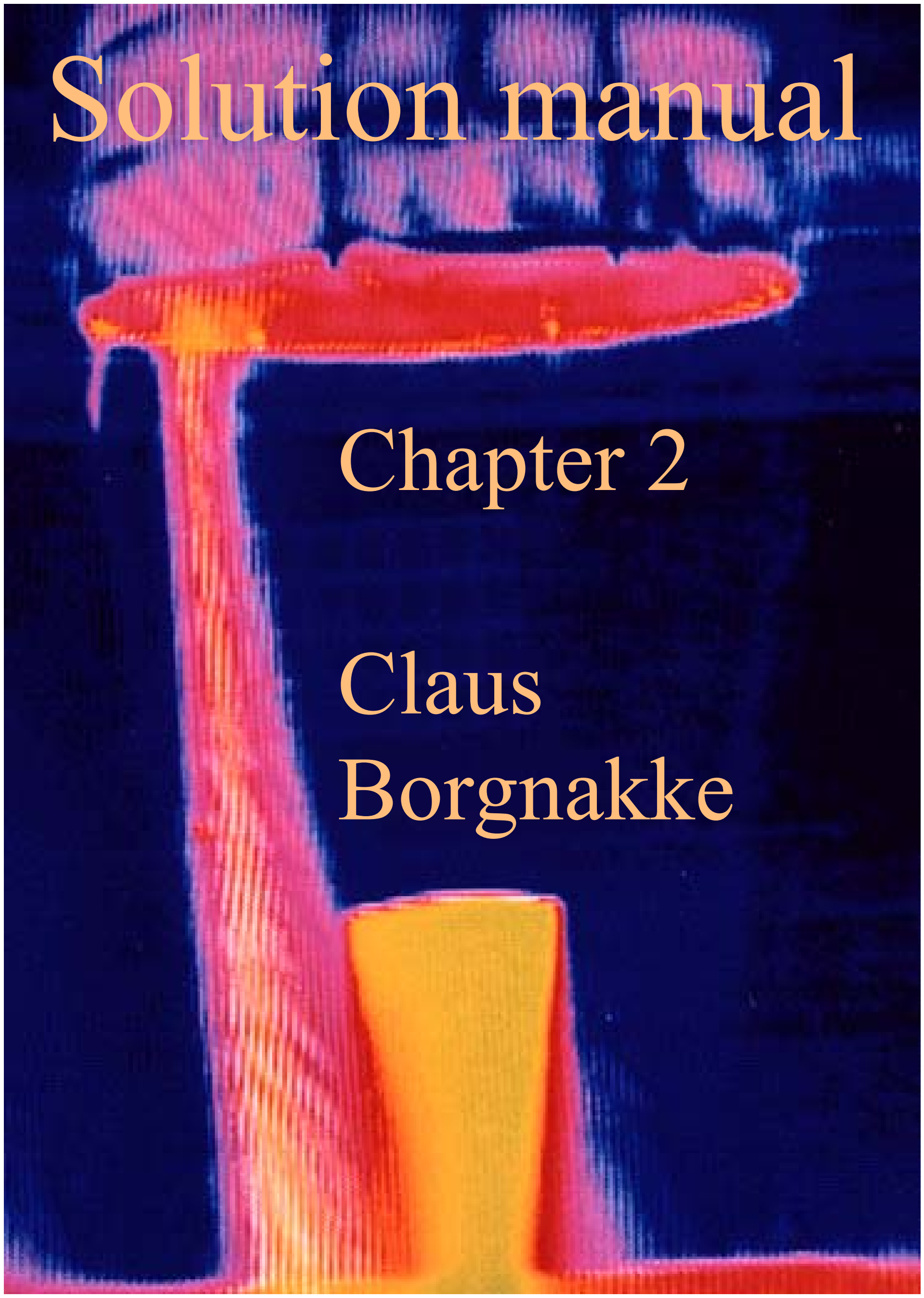
Introduction to Engineering thermodynamics
2^{nd} Edition, Sonntag and Borgnakke



The picture is a false color thermal image of the space shuttle's main engine. The sheet in the lower middle is after a normal shock across which you have changes in P, T and density. Courtesy of NASA.

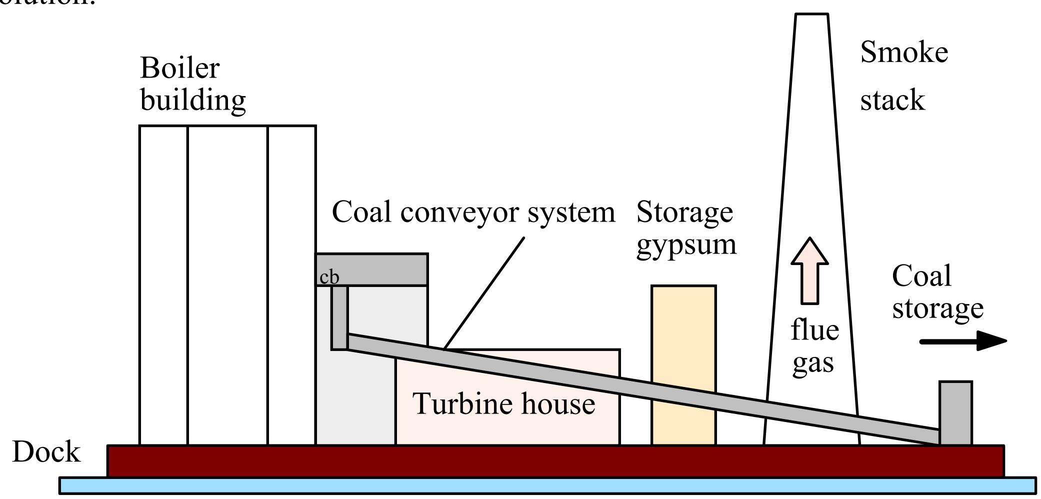
CONTENT

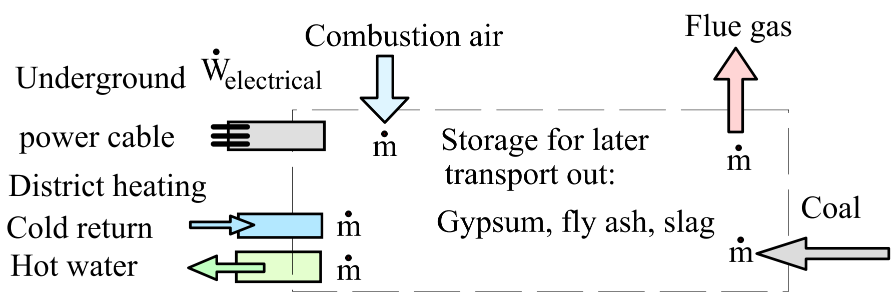
Concept Problems

2.1

Make a control volume around the whole power plant in Fig. 1.2 and with the help of Fig. 1.1 list what flows of mass and energy are in or out and any storage of energy. Make sure you know what is inside and what is outside your chosen C.V.

Solution:

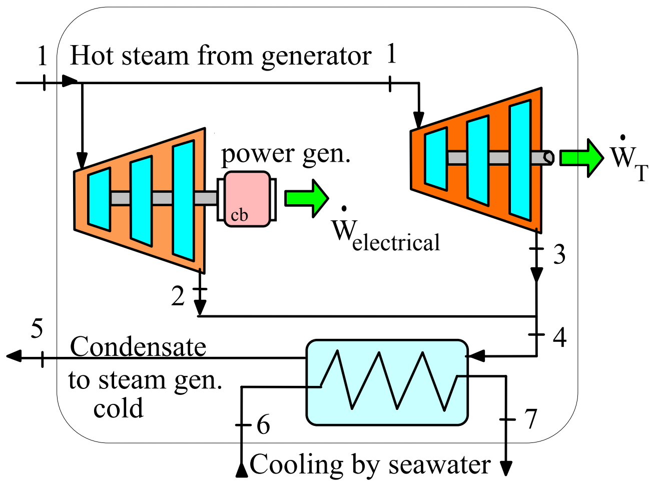




2.2

Make a control volume that includes the steam flow around in the main turbine loop in the nuclear propulsion system in Fig.1.3. Identify mass flows (hot or cold) and energy transfers that enter or leave the C.V.

Solution:



The electrical power also leaves the C.V. to be used for lights, instruments and to charge the batteries.

2.3

Separate the list P, F, V, v, ρ, T, a, m, L, t, and **V** into intensive, extensive, and non-properties.

Solution:

Intensive properties are independent upon mass: P, v, ρ, T
Extensive properties scales with mass: V, m
Non-properties: F, a, L, t, **V**

Comment: You could claim that acceleration a and velocity **V** are physical properties for the dynamic motion of the mass, but not thermal properties.

2.4

Water in nature exist in different phases such as solid, liquid and vapor (gas). Indicate the relative magnitude of density and specific volume for the three phases.

Solution:

Values are indicated in Figure 2.7 as density for common substances. More accurate values are found in Tables A.3, A.4 and A.5

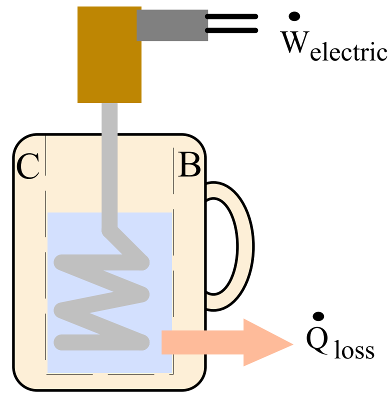
Water as solid (ice) has density of around 900 kg/m^3
Water as liquid has density of around 1000 kg/m^3
Water as vapor has density of around 1 kg/m^3 (sensitive to P and T)

2.5

An electric dip heater is put into a cup of water and heats it from 20^{o}C to 80^{o}C. Show the energy flow(s) and storage and explain what changes.

Solution:

Electric power is converted in the heater element (an electric resistor) so it becomes hot and gives energy by heat transfer to the water. The water heats up and thus stores energy and as it is warmer than the cup material it heats the cup which also stores some energy. The cup being warmer than the air gives a smaller amount of energy (a rate) to the air as a heat loss.

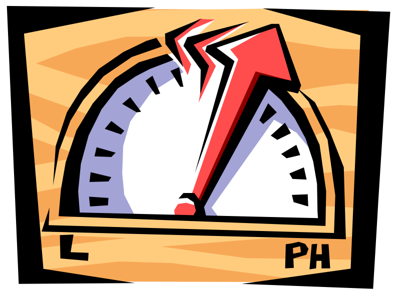


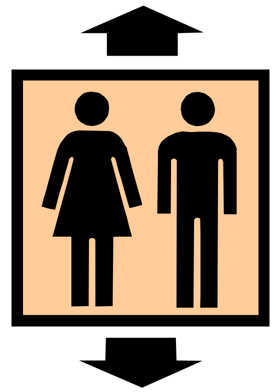
2.6

An escalator brings four people of total 300 kg, 25 m up in a building. Explain what happens with respect to energy transfer and stored energy.

Solution:

The four people (300 kg) have their potential energy raised, which is how the energy is stored. The energy is supplied as electrical power to the motor that pulls the escalator with a cable.





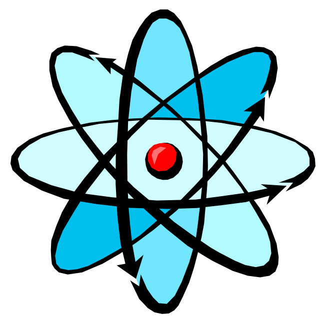
2.7

Is density a unique measure of mass distribution in a volume? Does it vary? If so, on what kind of scale (distance)?

Solution:

Density is an average of mass per unit volume and we sense if it is not evenly distributed by holding a mass that is more heavy in one side than the other. Through the volume of the same substance (say air in a room) density varies only little from one location to another on scales of meter, cm or mm. If the volume you look at has different substances (air and the furniture in the room) then it can change abruptly as you look at a small volume of air next to a volume of hardwood.

Finally if we look at very small scales on the order of the size of atoms the density can vary infinitely, since the mass (electrons, neutrons and positrons) occupy very little volume relative to all the empty space between them.



2.8

Can you carry 1 m^3 of liquid water?

Solution:

The density of liquid water is about 1000 kg/m^3 from Figure 2.7, see also Table A.3. Therefore the mass in one cubic meter is

$$m = \rho V = 1000 \text{ kg/m}^3 \times 1 \text{ m}^3 = 1000 \text{ kg}$$

and we can not carry that in the standard gravitational field.

2.9

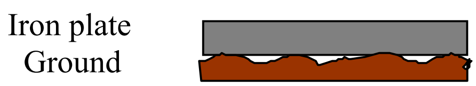
The pressure at the bottom of a swimming pool is evenly distributed. Suppose we look at a cast iron plate of 7272 kg lying on the ground with an area of 100 m^2. What is the average pressure below that? Is it just as evenly distributed as the pressure at the bottom of the pool?

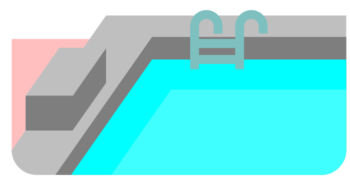
Solution:
The pressure is force per unit area from page 25:

$$P = F/A = mg/A = 7272 \text{ kg} \times (9.81 \text{ m/s}^2) / 100 \text{ m}^2 = \mathbf{713.4\ Pa}$$

The iron plate being cast can be reasonable plane and flat, but it is stiff and rigid. However, the ground is usually uneven so the contact between the plate and the ground is made over an area much smaller than the 100 m^2. Thus the local pressure at the contact locations is much larger than the quoted value above.

The pressure at the bottom of the swimming pool is very even due to the ability of the fluid (water) to have full contact with the bottom by deforming itself. This is the main difference between a fluid behavior and a solid behavior.





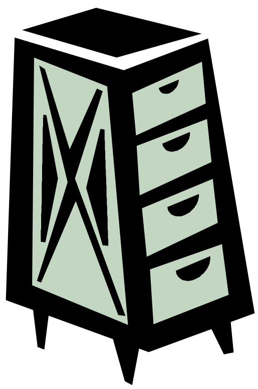
2.10

I put four adjustable feet on a heavy cabinet. What feature of the feet will ensure that the cabinet does not make marks in the floor?

Answer:
The area that is in contact with the floor supports the total mass in the gravitational field.

$$F = PA = mg$$

so for a given mass the smaller the area is the larger the pressure becomes.



2.11

A manometer with water shows a ΔP of $P_o/10$; what is the column height difference?

Solution:

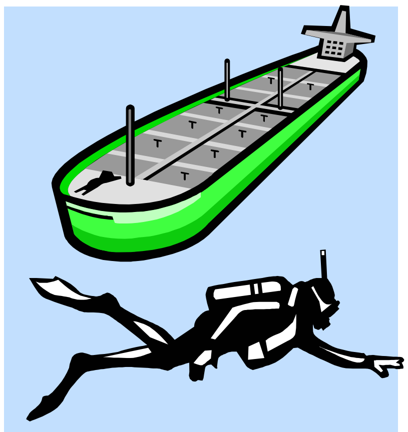
$$\Delta P = P_o/10 = \rho H g$$

$$H = P_o/(10\ \rho\ g) = \frac{101.3 \times 1000 \text{ Pa}}{10 \times 997 \text{ kg/m}^3 \times 9.80665 \text{ m/s}^2}$$

$= \mathbf{1.036\ m}$

2.12

Two divers swim at 20 m depth. One of them swims right in under a supertanker; the other stays away from the tanker. Who feels a greater pressure?



Solution:

Each one feels the local pressure which is the static pressure only a function of depth.

$$P_{ocean} = P_0 + \Delta P = P_0 + \rho g H$$

So they feel exactly the same pressure.

2.13

A water skier does not sink too far down in the water if his speed is high enough. What makes that situation different from our static pressure calculations?

The water pressure right under the ski is not a static pressure but a static plus dynamic pressure that pushes the water away from the ski. The faster you go, the smaller amount of water is displaced but at a higher velocity.

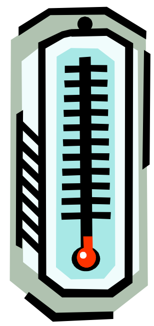
2.14

What is the smallest temperature in degrees Celsuis you can have? Kelvin?

Solution:

The lowest temperature is absolute zero which is at zero degrees Kelvin at which point the temperature in Celsius is negative

$$T_K = 0 \text{ K} = -273.15 \,^{o}\text{C}$$



2.15

Convert the formula for water density in In-text Concept Question “e” to be for T in degrees Kelvin.

Solution:

$$\rho = 1008 - T_C/2 \quad [\mathrm{kg/m^3}]$$

We need to express degrees Celsius in degrees Kelvin

$$T_C = T_K - 273.15$$

and substitute into formula

$$\rho = 1008 - T_C/2 = 1008 - (T_K - 273.15)/2 = 1144.6 - T_K/2$$

Properties and units

2.16

A steel cylinder of mass 2 kg contains 4 L of liquid water at 25^{o}C at 200 kPa. Find the total mass and volume of the system. List two extensive and three intensive properties of the water

Solution:

Density of steel in Table A.3: $\rho = 7820 \text{ kg/m}^3$

Volume of steel: $V = m/\rho = \dfrac{2 \text{ kg}}{7820 \text{ kg/m}^3} = 0.000\ 256 \text{ m}^3$

Density of water in Table A.4: $\rho = 997 \text{ kg/m}^3$

Mass of water: $m = \rho V = 997 \text{ kg/m}^3 \times 0.004 \text{ m}^3 = 3.988 \text{ kg}$

Total mass: $m = m_{steel} + m_{water} = 2 + 3.988 = \mathbf{5.988\ kg}$

Total volume: $V = V_{steel} + V_{water} = 0.000\ 256 + 0.004$

$= \mathbf{0.004\ 256\ \ m^3 = 4.26\ L}$

Extensive properties: m, V
Intensive properties: ρ (or $v = 1/\rho$), T, P

2.17

An apple "weighs" 80 g and has a volume of 100 cm^3 in a refrigerator at 8^oC. What is the apple density? List three intensive and two extensive properties of the apple.

Solution:

$$\rho = \frac{m}{V} = \frac{0.08}{0.0001} \ \frac{kg}{m^3} = 800 \ \frac{kg}{m^3}$$

Intensive

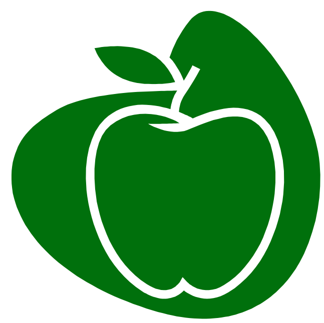
$$\rho = 800 \ \frac{kg}{m^3}\ ; \qquad v = \frac{1}{\rho} = 0.001\ 25 \ \frac{m^3}{kg}$$

$$T = \ 8°C; \qquad P = 101 \text{ kPa}$$

Extensive

$$m = 80 \text{ g} = 0.08 \text{ kg}$$

$$V = 100 \text{ cm}^3 = 0.1 \text{ L} = 0.0001 \text{ m}^3$$



2.18

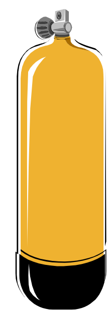
A pressurized steel bottle is charged with 5 kg of oxygen gas and 7 kg of nitrogen gas. How many kmoles are in the bottle?

Table A2 : $M_{O2} = 31.999$; $M_{N2} = 28.013$

$$n_{O2} = m_{O2} / M_{O2} = \frac{5}{31.999} = 0.15625 \text{ kmol}$$

$$n_{O2} = m_{N2} / M_{N2} = \frac{7}{28.013} = 0.24988 \text{ kmol}$$

$$n_{tot} = n_{O2} + n_{N2} = 0.15625 + 0.24988 = \mathbf{0.406\ kmol}$$



Force and Energy

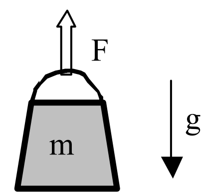
2.19

The "standard" acceleration (at sea level and 45° latitude) due to gravity is 9.80665 m/s^2. What is the force needed to hold a mass of 2 kg at rest in this gravitational field? How much mass can a force of 1 N support?

Solution:

$$ma = 0 = \sum F = F - mg$$

$$F = mg = 2 \text{ kg} \times 9.80665 \text{ m/s}^2 = \mathbf{19.613\ N}$$



$$F = mg \qquad \Rightarrow$$

$$m = \frac{F}{g} = \frac{1 \text{ N}}{9.80665 \text{ m/s}^2} = \mathbf{0.102\ kg}$$

2.20

When you move up from the surface of the earth the gravitation is reduced as g = $9.807 - 3.32 \times 10^{-6} z$, with z as the elevation in meters. How many percent is the weight of an airplane reduced when it cruises at 11 000 m?

Solution:

$g_o = 9.807 \text{ ms}^{-2}$

$g_H = 9.807 - 3.32 \times 10^{-6} \times 11\,000 = 9.7705 \text{ ms}^{-2}$

$W_o = m\, g_o \;;\; W_H = m\, g_H$

$$W_H/W_o = g_H/g_o = \frac{9.7705}{9.807} = 0.9963$$

Reduction = 1 – 0.9963 = 0.0037 or **0.37%**

i.e. we can neglect that for most applications.

2.21

A car drives at 60 km/h and is brought to a full stop with constant deceleration in 5 seconds. If the total car and driver mass is 1075 kg find the necessary force.

Solution:

Acceleration is the time rate of change of velocity.

$$a = \frac{d\mathbf{V}}{dt} = \frac{60 \times 1000}{3600 \times 5} = 3.333 \text{ m/s}^2$$

$ma = \sum F$;

$F_{net} = ma = 1075 \text{ kg} \times 3.333 \text{ m/s}^2 =$ **3583 N**

2.22

A car of mass 1775 kg travels with a velocity of 100 km/h. Find the kinetic energy. How high should it be lifted in the standard gravitational field to have a potential energy that equals the kinetic energy?

Solution:

Standard kinetic energy of the mass is

$$\text{KIN} = \tfrac{1}{2}\, m\, \mathbf{V}^2 = \tfrac{1}{2} \times\ 1775\ \text{kg} \times \left(\frac{100 \times 1000}{3600}\right)^2\ \text{m}^2/\text{s}^2$$

$$= \tfrac{1}{2} \times 1775 \times 27.778\ \text{Nm} = 684\ 800\ \text{J}$$

$$= \mathbf{684.8\ kJ}$$

Standard potential energy is

$$\text{POT} = \text{mgh}$$

$$h = \tfrac{1}{2}\, m\, \mathbf{V}^2 / mg\ = \frac{684\ 800\ \text{Nm}}{1775\ \text{kg} \times 9.807\ \text{m/s}^2} = \mathbf{39.3\ m}$$

2.23

A steel plate of 950 kg accelerates from rest with 3 m/s^2 for a period of 10s. What force is needed and what is the final velocity?

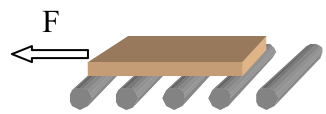
Solution:

Constant acceleration can be integrated to get velocity.

$$a = \frac{d\mathbf{V}}{dt} \quad \Rightarrow \quad \int d\mathbf{V} = \int a\ dt \quad \Rightarrow \quad \Delta\mathbf{V} = a\ \Delta t$$

$$\Delta\mathbf{V} = a\ \Delta t = 3\ m/s^2 \times 10\ s = 30\ m/s$$

$$\Rightarrow \quad \mathbf{V = 30\ m/s}$$



$$F = ma = 950\ kg \times 3\ m/s^2 = \mathbf{2850\ N}$$

2.24

A 15 kg steel container has 1.75 kilomoles of liquid propane inside. A force of 2 kN now accelerates this system. What is the acceleration?

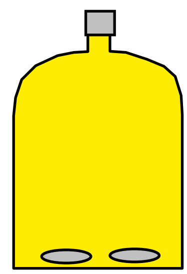
Solution:

The molecular weight for propane is $M = 44.094$ from Table A.2. The force must accelerate both the container mass and the propane mass.

$$m = m_{steel} + m_{propane} = 15 + (1.75 \times 44.094) = 92.165 \text{ kg}$$

$$ma = \sum F \quad \Rightarrow \quad a = \sum F / m$$

$$a = \frac{2000 \text{ N}}{92.165 \text{ kg}} = \mathbf{21.7\ m/s^2}$$



Specific Volume

2.25

A tank has two rooms separated by a membrane. Room A has 1 kg air and volume 0.5 m^3, room B has 0.75 m^3 air with density 0.8 kg/m^3. The membrane is broken and the air comes to a uniform state. Find the final density of the air.

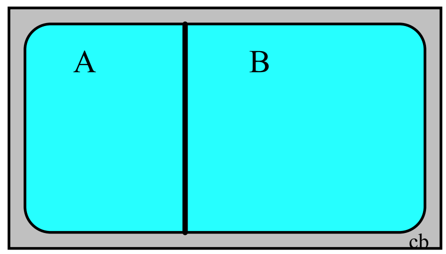
Solution:

Density is mass per unit volume

$$m = m_A + m_B = m_A + \rho_B V_B = 1 + 0.8 \times 0.75 = 1.6 \text{ kg}$$

$$V = V_A + V_B = 0.5 + 0.75 = 1.25 \text{ m}^3$$

$$\rho = \frac{m}{V} = \frac{1.6}{1.25} = \mathbf{1.28\ kg/m^3}$$



2.26

A 1 m^3 container is filled with 400 kg of granite stone, 200 kg dry sand and 0.2 m^3 of liquid 25°C water. Use properties from tables A.3 and A.4. Find the average specific volume and density of the masses when you exclude air mass and volume.

Solution:

Specific volume and density are ratios of total mass and total volume.

$$m_{liq} = V_{liq}/v_{liq} = V_{liq}\,\rho_{liq} = 0.2\ m^3 \times 997\ kg/m^3 = 199.4\ kg$$

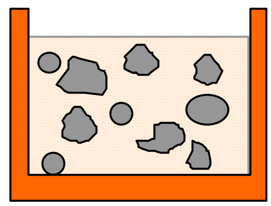
$$m_{TOT} = m_{stone} + m_{sand} + m_{liq} = 400 + 200 + 199.4 = 799.4\ kg$$

$$V_{stone} = mv = m/\rho = 400\ kg/\ 2750\ kg/m^3 = 0.1455\ m^3$$

$$V_{sand} = mv = m/\rho = 200/\ 1500 = 0.1333\ m^3$$

$$V_{TOT} = V_{stone} + V_{sand} + V_{liq}$$

$$= 0.1455 + 0.1333 + 0.2 = 0.4788\ m^3$$



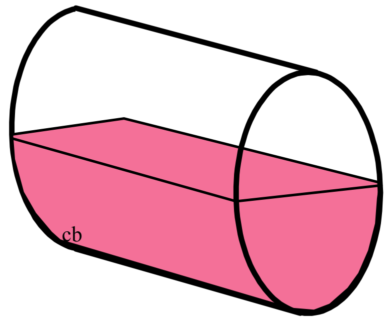
$$v = V_{TOT} / m_{TOT} = 0.4788/799.4 = \mathbf{0.000599\ m^3/kg}$$

$$\rho = 1/v = m_{TOT}/V_{TOT} = 799.4/0.4788 = \mathbf{1669.6\ kg/m^3}$$

2.27

A 15-kg steel gas tank holds 300 L of liquid gasoline, having a density of 800 kg/m^3. If the system is decelerated with 6 m/s^2 what is the needed force?

Solution:



$$m = m_{tank} + m_{gasoline}$$
$$= 15 \text{ kg} + 0.3 \text{ m}^3 \times 800 \text{ kg/m}^3$$
$$= 255 \text{ kg}$$
$$F = ma = 255 \text{ kg} \times 6 \text{ m/s}^2$$
$$= \mathbf{1530\ N}$$

2.28

How much mass is there approximately in 1 L of mercury (Hg)? Atmospheric air?

Solution:

A volume of 1 L equals 0.001 m^3, see Table A.1. From Figure 2.7 the density is in the range of 10 000 kg/m^3 so we get

$$m = \rho V = 10\ 000\ kg/m^3 \times 0.001\ m^3 = \mathbf{10\ kg}$$

A more accurate value from Table A.4 is $\rho = 13\ 580\ kg/m^3$.

For the air we see in Figure 2.7 that density is about 1 kg/m^3 so we get

$$m = \rho V = 1\ kg/m^3 \times 0.001\ m^3 = \mathbf{0.001\ kg}$$

A more accurate value from Table A.5 is $\rho = 1.17\ kg/m^3$ at 100 kPa, 25^{o}C.

Pressure

2.29

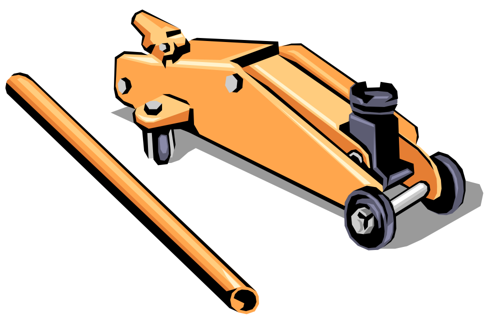
A hydraulic lift has a maximum fluid pressure of 500 kPa. What should the piston-cylinder diameter be so it can lift a mass of 850 kg?

Solution:

With the piston at rest the static force balance is

$$F\uparrow = P\,A = F\downarrow = mg$$
$$A = \pi\, r^2 = \pi\, D^2/4$$
$$PA = P\,\pi\, D^2/4 = mg \quad \Rightarrow \quad D^2 = \frac{4mg}{P\,\pi}$$

$$D = 2\sqrt{\frac{mg}{P\pi}} = 2\sqrt{\frac{850\text{ kg} \times 9.807\text{ m/s}^2}{500\text{ kPa} \times \pi \times 1000\text{ (Pa/kPa)}}} = \mathbf{0.146\ m}$$



2.30

A piston/cylinder with cross sectional area of 0.01 m^2 has a piston mass of 100 kg resting on the stops, as shown in Fig. P2.45. With an outside atmospheric pressure of 100 kPa, what should the water pressure be to lift the piston?

Solution:

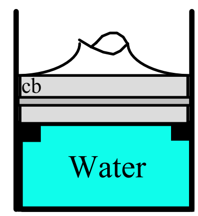
The force acting down on the piston comes from gravitation and the outside atmospheric pressure acting over the top surface.

Force balance: $F\uparrow = F\downarrow = PA = m_p g + P_0 A$

Now solve for P (divide by 1000 to convert to kPa for 2nd term)

$$P = P_0 + \frac{m_p g}{A} = 100 \text{ kPa} + \frac{100 \times 9.80665}{0.01 \times 1000} \text{ kPa}$$

$$= 100 \text{ kPa} + 98.07 \text{ kPa} = \mathbf{198\ kPa}$$



2.31

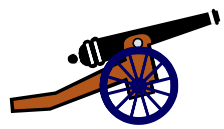
A cannon-ball of 5 kg acts as a piston in a cylinder of 0.15 m diameter. As the gun-powder is burned a pressure of 7 MPa is created in the gas behind the ball. What is the acceleration of the ball if the cylinder (cannon) is pointing horizontally?

Solution:

The cannon ball has 101 kPa on the side facing the atmosphere.

$$ma = F = P_1 \times A - P_0 \times A = (P_1 - P_0\ ) \times A$$
$$= (7000 - 101)\ \text{kPa} \times\ \pi\ (\ 0.15^2\ /4\ )\ \text{m}^2 = 121.9\ \text{kN}$$

$$a = \frac{F}{m} = \frac{121.9\ \text{kN}}{5\ \text{kg}} = \mathbf{24\ 380\ m/s^2}$$



2.32

A vertical hydraulic cylinder has a 125-mm diameter piston with hydraulic fluid inside the cylinder and an ambient pressure of 1 bar. Assuming standard gravity, find the piston mass that will create a pressure inside of 1500 kPa.

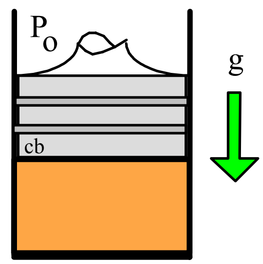
Solution:

Force balance:

$$F\uparrow = PA = F\downarrow = P_0A + m_pg;$$

$$P_0 = 1 \text{ bar} = 100 \text{ kPa}$$

$$A = (\pi/4)\, D^2 = (\pi/4) \times 0.125^2 = 0.01227 \text{ m}^2$$



$$m_p = (P - P_0)\,\frac{A}{g} = (\ 1500 - 100\ ) \times 1000 \times \frac{0.01227}{9.80665} = \mathbf{1752\ kg}$$

2.33

A valve in a cylinder has a cross sectional area of 11 cm^2 with a pressure of 735 kPa inside the cylinder and 99 kPa outside. How large a force is needed to open the valve?

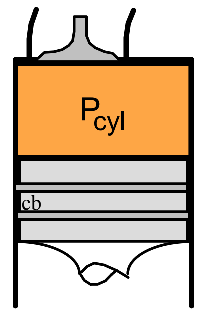
$$F_{net} = P_{in}A - P_{out}A$$

$$= (735 - 99)\ \text{kPa} \times 11\ \text{cm}^2$$

$$= 6996\ \text{kPa cm}^2$$

$$= 6996 \times \frac{\text{kN}}{\text{m}^2} \times 10^{-4}\ \text{m}^2$$

$$= \mathbf{700\ N}$$



2.34

You dive 5 m down in the ocean. What is the absolute pressure there?

Solution:

The pressure difference for a column is from Eq.2.2 and the density of water is from Table A.4.

$$\Delta P = \rho g H$$
$$= 997 \ \text{kg/m}^3 \times 9.81 \ \text{m/s}^2 \times 5 \ \text{m}$$
$$= 48\,903 \ \text{Pa} = 48.903 \ \text{kPa}$$

$$P_{ocean} = P_0 + \Delta P$$
$$= 101.325 + 48.903$$
$$= \mathbf{150 \ kPa}$$

2.35

A large exhaust fan in a laboratory room keeps the pressure inside at 10 cm water relative vacuum to the hallway. What is the net force on the door measuring 1.9 m by 1.1 m?

Solution:

The net force on the door is the difference between the forces on the two sides as the pressure times the area

$$F = P_{outside}\,A - P_{inside}\,A = \Delta P \times A$$
$$= 10 \text{ cm } H_2O \times 1.9 \text{ m} \times 1.1 \text{ m}$$
$$= 0.10 \times 9.80638 \text{ kPa} \times 2.09 \text{ m}^2$$
$$= \mathbf{2049\ N}$$

Table A.1: 1 m H_2O is 9.80638 kPa and kPa is kN/m^2.

2.36

The hydraulic lift in an auto-repair shop has a cylinder diameter of 0.2 m. To what pressure should the hydraulic fluid be pumped to lift 40 kg of piston/arms and 700 kg of a car?

Solution:

Force acting on the mass by the gravitational field

$$F\downarrow = ma = mg = 740 \times 9.80665 = 7256.9 \text{ N}$$

Force balance: $F\uparrow = (P - P_0)\, A = F\downarrow \quad => \quad P = P_0 + F\downarrow / A$

$$A = \pi D^2 (1 / 4) = 0.031416 \text{ m}^2$$

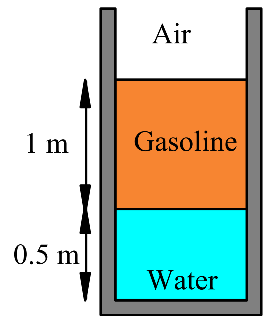
$$P = 101 + 7256.9 / (0.031416 \times 1000) = \mathbf{332\ kPa}$$

2.37

A 2.5 m tall steel cylinder has a cross sectional area of 1.5 m^2. At the bottom with a height of 0.5 m is liquid water on top of which is a 1 m high layer of gasoline. The gasoline surface is exposed to atmospheric air at 101 kPa. What is the highest pressure in the water?

Solution:

The pressure in the fluid goes up with the depth as



$$P = P_{top} + \Delta P = P_{top} + \rho g h$$

and since we have two fluid layers we get

$$P = P_{top} + [(\rho h)_{gasoline} + (\rho h)_{water}]\ g$$

The densities from Table A.4 are:

$$\rho_{gasoline} = 750\ \text{kg/m}^3; \quad \rho_{water} = 997\ \text{kg/m}^3$$

$$P = 101 + [750 \times 1 + 997 \times 0.5]\ \frac{9.807}{1000} = \mathbf{113.2\ kPa}$$

2.38

At the beach, atmospheric pressure is 1025 mbar. You dive 15 m down in the ocean and you later climb a hill up to 250 m elevation. Assume the density of water is about 1000 kg/m^3 and the density of air is 1.18 kg/m^3. What pressure do you feel at each place?

Solution:

$$\Delta P = \rho g h$$

$$P_{ocean} = P_0 + \Delta P = 1025 \times 100 + 1000 \times 9.81 \times 15$$

$$= 2.4965 \times 10^5 \text{ Pa} = \mathbf{250\ kPa}$$

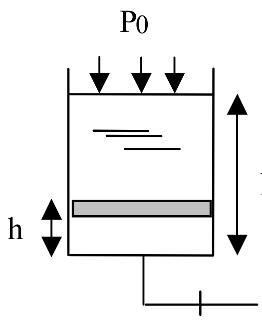
$$P_{hill} = P_0 - \Delta P = 1025 \times 100 - 1.18 \times 9.81 \times 250$$

$$= 0.99606 \times 10^5 \text{ Pa} = \mathbf{99.61\ kPa}$$

2.39

Liquid water with density ρ is filled on top of a thin piston in a cylinder with cross-sectional area *A* and total height *H*. Air is let in under the piston so it pushes up, spilling the water over the edge. Deduce the formula for the air pressure as a function of the piston elevation from the bottom, *h*.

Solution:



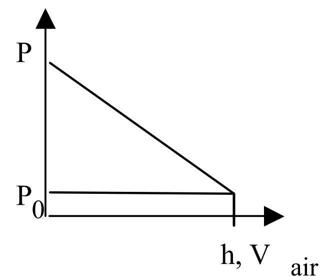
Force balance

Piston: F↑ = F↓

$$PA = P_0A + m_{H_2O}g$$

$$P = P_0 + m_{H_2O}g/A$$

$$\mathbf{P = P_0 + (H - h)\rho g}$$



2.40

A tornado rips off a 100 m^2 roof with a mass of 1000 kg. What is the minimum vacuum pressure needed to do that if we neglect the anchoring forces?

Solution:

The net force on the roof is the difference between the forces on the two sides as the pressure times the area

$$F = P_{inside} A - P_{outside} A = \Delta P\ A$$

That force must overcome the gravitation mg, so the balance is

$$\Delta P\ A = mg$$

$$\Delta P = mg/A = (1000 \text{ kg} \times 9.807 \text{ m/s}^2)/100 \text{ m}^2 = \mathbf{98\ Pa = 0.098\ kPa}$$

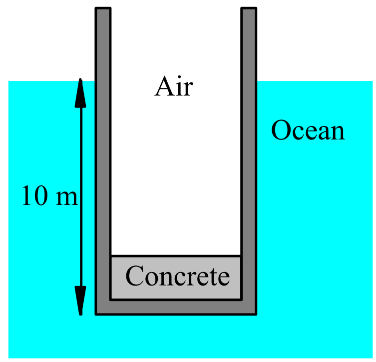
Remember that kPa is kN/m^2.

2.41

A steel tank of cross sectional area 3 m^2 and 16 m tall weighs 10 000 kg and it is open at the top. We want to float it in the ocean so it sticks 10 m straight down by pouring concrete into the bottom of it. How much concrete should I put in?

Solution:

The force up on the tank is from the water pressure at the bottom times its area. The force down is the gravitation times mass and the atmospheric pressure.



$$F\uparrow = PA = (\rho_{ocean}gh + P_0)A$$

$$F\downarrow = (m_{tank} + m_{concrete})g + P_0A$$

The force balance becomes

$$F\uparrow = F\downarrow = (\rho_{ocean}gh + P_0)A = (m_{tank} + m_{concrete})g + P_0A$$

Solve for the mass of concrete

$$m_{concrete} = (\rho_{ocean}hA - m_{tank}) = 997 \times 10 \times 3 - 10\ 000 = \mathbf{19\ 910\ kg}$$

Notice: The first term is the mass of the displaced ocean water. The net force up is the weight (mg) of this mass called bouyancy, P_0 cancel.

Manometers and Barometers

2.42

What pressure difference does a 10 m column of atmospheric air show?

Solution:

The pressure difference for a column is from Eq.2.2

$$\Delta P = \rho g H$$

So we need density of air from Fig.2.7, $\rho = 1.2\ kg/m^3$

$$\Delta P = 1.2\ kg/m^3 \times 9.81\ ms^{-2} \times 10\ m = 117.7\ Pa = \mathbf{0.12\ kPa}$$

2.43

The density of atmospheric air is about 1.15 kg/m^3, which we assume is constant. How large an absolute pressure will a pilot see when flying 1500 m above ground level where the pressure is 101 kPa.

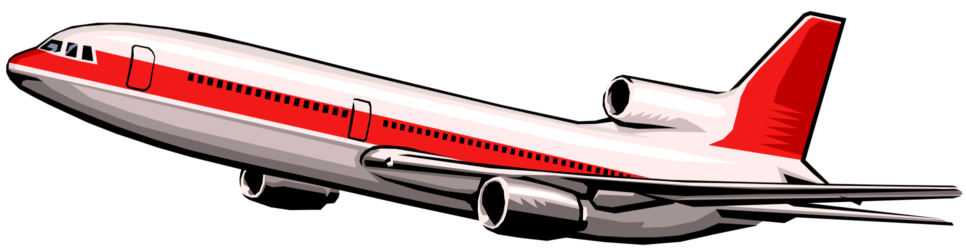
Solution:

Assume g and ρ are constant then the pressure difference to carry a column of height 1500 m is from Fig.2.10

$$\Delta P = \rho g h = 1.15 \text{ kg/m}^3 \times 9.807 \text{ ms}^{-2} \times 1500 \text{ m}$$
$$= 16\,917 \text{ Pa} = 16.9 \text{ kPa}$$

The pressure on top of the column of air is then

$$P = P_0 - \Delta P = 101 - 16.9 = \mathbf{84.1\ kPa}$$



2.44

A manometer shows a pressure difference of 1 m of liquid mercury. Find ΔP in kPa.

Solution:

Hg : $L = 1$ m; $\rho = 13\ 580\ kg/m^3$ from Table A.4 (or read Fig 2.7)

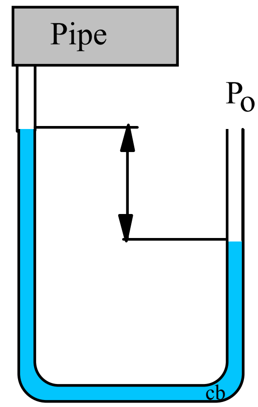
The pressure difference ΔP balances the column of height L so from Eq.2.2

$$\Delta P = \rho\, g\, L = 13\ 580\ kg/m^3 \times 9.80665\ m/s^2 \times 1.0\ m \times 10^{-3}\ kPa/Pa$$
$$= \mathbf{133.2\ kPa}$$

2.45

Blue manometer fluid of density 925 kg/m^3 shows a column height difference of 3 cm vacuum with one end attached to a pipe and the other open to P_0 = 101 kPa. What is the absolute pressure in the pipe?

Solution:



Since the manometer shows a vacuum we have

$P_{PIPE} = P_0 - \Delta P$

$\Delta P = \rho gh = 925 \times 9.807 \times 0.03$

$= 272.1 \text{ Pa} = 0.272 \text{ kPa}$

$P_{PIPE} = 101 - 0.272 =$ **100.73 kPa**

2.46

The pressure gauge on an air tank shows 75 kPa when the diver is 10 m down in the ocean. At what depth will the gauge pressure be zero? What does that mean?

Ocean H_20 pressure at 10 m depth is

$$P_{water} = P_0 + \rho Lg = 101.3 + \frac{997 \times 10 \times 9.80665}{1000} = 199 \text{ kPa}$$

Air Pressure (absolute) in tank

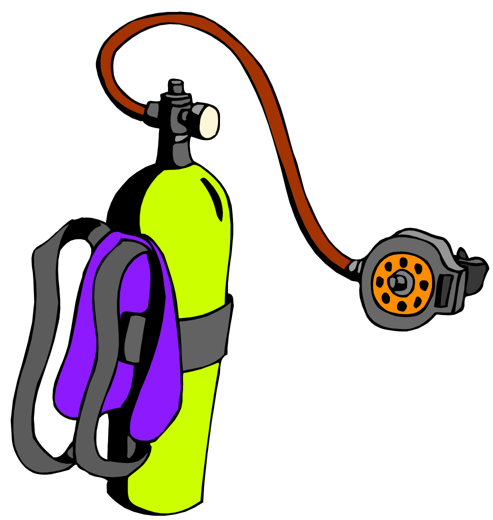
$$P_{tank} = 199 + 75 = 274 \text{ kPa}$$

Tank Pressure (gauge) reads zero at H_20 local pressure

$$274 = 101.3 + \frac{997 \times 9.80665}{1000} L$$

$$L = \mathbf{17.66\ m}$$

At this depth you will have to suck the air in, it can no longer push itself through a valve.



2.47

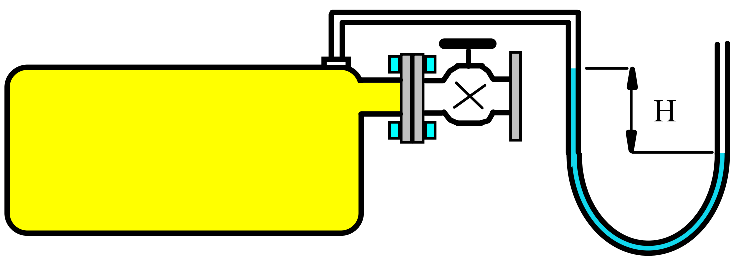
The absolute pressure in a tank is 85 kPa and the local ambient absolute pressure is 97 kPa. If a U-tube with mercury, density 13550 kg/m^3, is attached to the tank to measure the vacuum, what column height difference would it show?

Solution:

$$\Delta P = P_0 - P_{tank} = \rho g \, H$$

$$H = (P_0 - P_{tank}) / \rho g = [(97 - 85) \times 1000] / (13550 \times 9.80665)$$

$$= \mathbf{0.090\ m = 90\ mm}$$



2.48

The difference in height between the columns of a manometer is 200 mm with a fluid of density 900 kg/m^3. What is the pressure difference? What is the height difference if the same pressure difference is measured using mercury, density 13600 kg/ m^3, as manometer fluid?

Solution:

$$\Delta P = \rho_1 g h_1 = 900 \text{ kg/m}^3 \times 9.807 \text{ m/s}^2 \times 0.2 \text{ m} = 1765.26 \text{ Pa} = \mathbf{1.77\ kPa}$$

$$h_{Hg} = \Delta P/ (\rho_{hg}\, g) = (\rho_1\, g h_1) / (\rho_{hg}\, g) = \frac{900}{13600} \times 0.2$$

$$= \mathbf{0.0132\ m = 13.2\ mm}$$

2.49

A barometer to measure absolute pressure shows a mercury column height of 725 mm. The temperature is such that the density of the mercury is 13 550 kg/m^3. Find the ambient pressure.

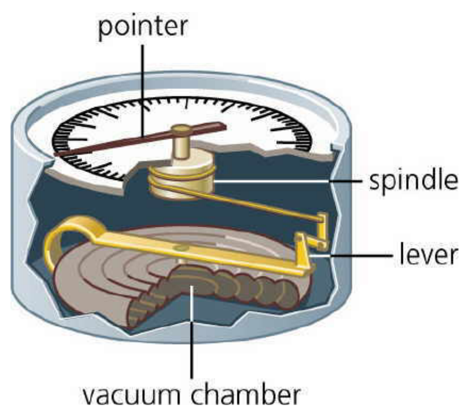
Solution:

Hg : $L = 725$ mm = 0.725 m; $\rho = 13\ 550\ kg/m^3$

The external pressure P balances the column of height L so from Fig.2.10

$$P = \rho L g = 13\ 550\ kg/m^3 \times 9.80665\ m/s^2 \times 0.725\ m \times 10^{-3}\ kPa/Pa$$

$= \mathbf{96.34\ kPa}$



2.50

An absolute pressure gauge attached to a steel cylinder shows 135 kPa. We want to attach a manometer using liquid water a day that P_{atm} = 101 kPa. How high a fluid level difference must we plan for?

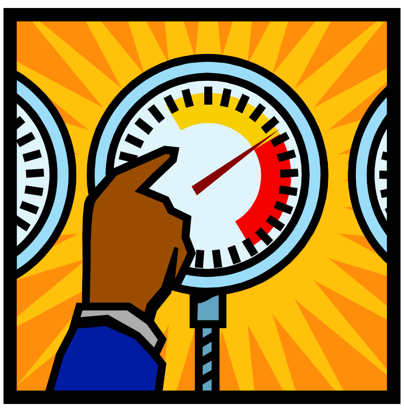
Solution:

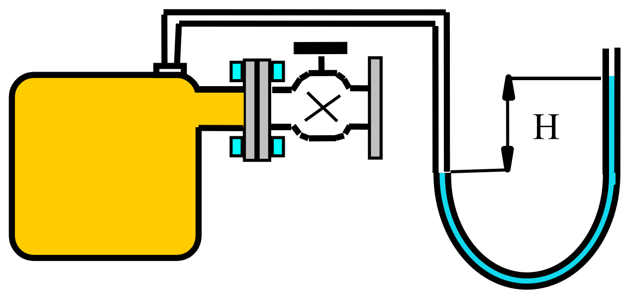
Since the manometer shows a pressure difference we have

$$\Delta P = P_{CYL} - P_{atm} = \rho L g$$

$$L = \Delta P / \rho g = \frac{(135 - 101)\ \text{kPa}}{997\ \text{kg m}^{-3} \times 10 \times 9.807\ \text{m/s}^2} \frac{1000\ \text{Pa}}{\text{kPa}}$$

$$= \mathbf{3.467\ m}$$





2.51

A differential pressure gauge mounted on a vessel shows 1.25 MPa and a local barometer gives atmospheric pressure as 0.96 bar. Find the absolute pressure inside the vessel.

Solution:

Convert all pressures to units of kPa.

$P_{gauge} = 1.25 \text{ MPa} = 1250 \text{ kPa};$

$P_0 = 0.96 \text{ bar} = 96 \text{ kPa}$

$P = P_{gauge} + P_0 = 1250 + 96 = \mathbf{1346\ kPa}$

2.52

A submarine maintains 101 kPa inside it and it dives 240 m down in the ocean having an average density of 1030 kg/m^3. What is the pressure difference between the inside and the outside of the submarine hull?

Solution:

Assume the atmosphere over the ocean is at 101 kPa, then ΔP is from the 240 m column water.

$$\Delta P = \rho L g$$
$$= (1030 \text{ kg/m}^3 \times 240 \text{ m} \times 9.807 \text{ m/s}^2) / 1000 = \mathbf{2424 \text{ kPa}}$$

2.53

A barometer measures 760 mmHg at street level and 735 mmHg on top of a building. How tall is the building if we assume air density of 1.15 kg/m^3?

Solution:

$$\Delta P = \rho g H$$

$$H = \Delta P/\rho g = \frac{760 - 735}{1.15 \times 9.807} \frac{\text{mmHg}}{\text{kg/m}^2\text{s}^2} \frac{133.32 \text{ Pa}}{\text{mmHg}} = \mathbf{295\ m}$$

2.54

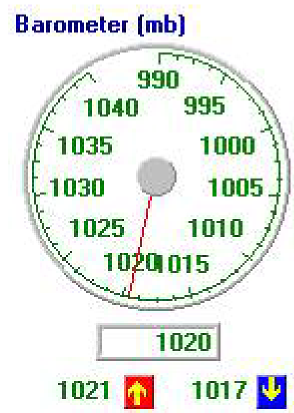
Assume we use a pressure gauge to measure the air pressure at street level and at the roof of a tall building. If the pressure difference can be determined with an accuracy of 1 mbar (0.001 bar) what uncertainty in the height estimate does that corresponds to?

Solution:

$\rho_{air} = 1.169 \text{ kg/m}^3$ from Table A.5

$\Delta P = 0.001 \text{ bar} = 100 \text{ Pa}$

$$L = \frac{\Delta P}{\rho g} = \frac{100}{1.169 \times 9.807} = \mathbf{8.72\ m}$$



2.55

A pipe flowing light oil has a manometer attached as shown in Fig. P2.55. What is the absolute pressure in the pipe flow?

Solution:

Table A.3: $\rho_{oil} = 910 \text{ kg/m}^3$; $\rho_{water} = 997 \text{ kg/m}^3$

$$P_{BOT} = P_0 + \rho_{water}\, g\, H_{tot} = P_0 + 997 \times 9.807 \times 0.8$$
$$= P_o + 7822 \text{ Pa}$$

$$P_{PIPE} = P_{BOT} - \rho_{water}\, g\, H_1 - \rho_{oil}\, g\, H_2$$
$$= P_{BOT} - 997 \times 9.807 \times 0.1 - 910 \times 9.807 \times 0.2$$
$$= P_{BOT} - 977.7 \text{ Pa} - 1784.9 \text{ Pa}$$

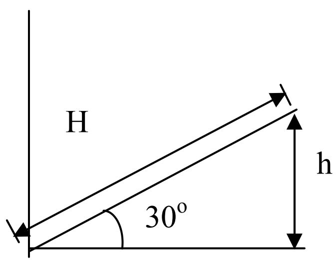
$$P_{PIPE} = P_o + (7822 - 977.7 - 1784.9) \text{ Pa}$$
$$= P_o + 5059.4 \text{ Pa} = 101.325 + 5.06 = \mathbf{106.4\ kPa}$$

2.56

A U-tube manometer filled with water, density 1000 kg/m^3, shows a height difference of 25 cm. What is the gauge pressure? If the right branch is tilted to make an angle of 30° with the horizontal, as shown in Fig. P2.56, what should the length of the column in the tilted tube be relative to the U-tube?

Solution:

Same height in the two sides in the direction of g.



$\Delta P = F/A = mg/A = V\rho g/A = h\rho g$

$= 0.25 \times 1000 \times 9.807 = 2452.5$ Pa

$=$ **2.45 kPa**

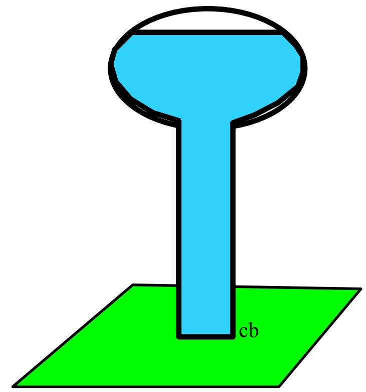
$h = H \times \sin 30°$

$\Rightarrow H = h/\sin 30° = 2h =$ **50 cm**

2.57

In the city water tower, water is pumped up to a level 25 m above ground in a pressurized tank with air at 125 kPa over the water surface. This is illustrated in Fig. P2.57. Assuming the water density is 1000 kg/m^3 and standard gravity, find the pressure required to pump more water in at ground level.

Solution:



$\Delta P = \rho \, L \, g$

$= 1000 \text{ kg/m}^3 \times 25 \text{ m} \times 9.807 \text{ m/s}^2$

$= 245\ 175 \text{ Pa} = 245.2 \text{ kPa}$

$P_{bottom} = P_{top} + \Delta P$

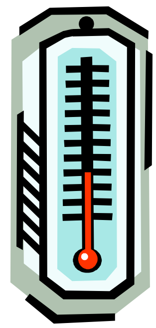
$= 125 + 245.2$

$=$ **370 kPa**

2.58

What is a temperature of $-5^{o}C$ in degrees Kelvin?

Solution:



The offset from Celsius to Kelvin is 273.15 K, so we get

$$T_K = T_C + 273.15 = -5 + 273.15$$
$$= \mathbf{268.15\ K}$$

2.59

Density of liquid water is $\rho = 1008 - T/2$ [kg/m^3] with T in ^{o}C. If the temperature increases 10^{o}C how much deeper does a 1 m layer of water become?

Solution:

The density change for a change in temperature of 10^{o}C becomes

$$\Delta\rho = -\Delta T/2 = -5 \text{ kg/m}^3$$

from an ambient density of

$$\rho = 1008 - T/2 = 1008 - 25/2 = 995.5 \text{ kg/m}^3$$

Assume the area is the same and the mass is the same $m = \rho V = \rho A H$, then we have

$$\Delta m = 0 = V\Delta\rho + \rho\Delta V \quad \Rightarrow \quad \Delta V = - V\Delta\rho/\rho$$

and the change in the height is

$$\Delta H = \frac{\Delta V}{A} = \frac{H\Delta V}{V} = \frac{-H\Delta\rho}{\rho} = \frac{-1 \times (-5)}{995.5} = \mathbf{0.005\ m}$$

barely measurable.

Temperature

2.60

The density of mercury changes approximately linearly with temperature as

$$\rho_{Hg} = 13595 - 2.5\,T \;\; \text{kg/ m}^3 \qquad T \;\; \text{in Celsius}$$

so the same pressure difference will result in a manometer reading that is influenced by temperature. If a pressure difference of 100 kPa is measured in the summer at 35°C and in the winter at −15°C, what is the difference in column height between the two measurements?

Solution:

The manometer reading h relates to the pressure difference as

$$\Delta P = \rho\, L\, g \quad \Rightarrow \quad L = \frac{\Delta P}{\rho g}$$

The manometer fluid density from the given formula gives

$$\rho_{su} = 13595 - 2.5 \times 35 = 13507.5 \text{ kg/m}^3$$

$$\rho_{W} = 13595 - 2.5 \times (-15) = 13632.5 \;\text{ kg/m}^3$$

The two different heights that we will measure become

$$L_{su} = \frac{100 \times 10^3}{13507.5 \times 9.807} \frac{\text{kPa (Pa/kPa)}}{(\text{kg/m}^3)\text{ m/s}^2} = 0.7549 \text{ m}$$

$$L_{W} = \frac{100 \times 10^3}{13632.5 \times 9.807} \frac{\text{kPa (Pa/kPa)}}{(\text{kg/m}^3)\text{ m/s}^2} = 0.7480 \text{ m}$$

$$\Delta L = L_{su} - L_{W} = \mathbf{0.0069 \text{ m} = 6.9 \text{ mm}}$$

2.61

A mercury thermometer measures temperature by measuring the volume expansion of a fixed mass of liquid Hg due to a change in the density, see problem 2.35. Find the relative change (%) in volume for a change in temperature from 10°C to 20°C.

Solution:

From 10°C to 20°C

At 10°C : $\rho_{Hg} = 13595 - 2.5 \times 10 = 13570$ kg/m^3

At 20°C : $\rho_{Hg} = 13595 - 2.5 \times 20 = 13545$ kg/m^3

The volume from the mass and density is: $V = m/\rho$

$$\text{Relative Change} = \frac{V_{20} - V_{10}}{V_{10}} = \frac{(m/\rho_{20}) - (m/\rho_{10})}{m/\rho_{10}}$$

$$= \frac{\rho_{10}}{\rho_{20}} - 1 = \frac{13570}{13545} - 1 = \mathbf{0.0018\ (0.18\%)}$$

2.62

The atmosphere becomes colder at higher elevation. As an average the standard atmospheric absolute temperature can be expressed as $T_{atm} = 288 - 6.5 \times 10^{-3}\, z$, where z is the elevation in meters. How cold is it outside an airplane cruising at 12 000 m expressed in Kelvin and in Celsius?

Solution:

For an elevation of $z = 12\,000$ m we get

$$T_{atm} = 288 - 6.5 \times 10^{-3}\, z = \mathbf{210\ K}$$

To express that in degrees Celsius we get

$$T_C = T - 273.15 = \mathbf{-63.15^{o}C}$$