

Chapter 1. Outcomes, Event, and Sample Spaces

1.1 Answers will vary; students could use (x, y) coordinates or polar coordinates. Using (x, y) coordinates, one possible outcome is $(0.25, 0.5)$; one event might be $\{(x, y) \mid 0.2 < x < 0.5, 0.36 < y < 0.42\}$. The sample space is $\{(x, y) \mid x^2 + y^2 = 1\}$.

1.2 Answers will vary. One possible outcome is $x = 20$ (he finds the first Q book on the twentieth book sorted). One event is $\{x \mid x < 100\}$ (he finds the first Q book somewhere in the first 100 books). The sample space is $\{0, 1, 2, 3, \dots\}$.

1.3 Answers will vary. One possible outcome is he grabs lemon-lime, lemon-lime, orange. One event might be $\{(x_1, x_2, x_3, x_4, \text{orange}) \mid \text{each of } x_1, x_2, x_3, x_4 \text{ is either lemon-lime or fruit punch}\}$. The sample space is $\{(x_1, \dots, x_j, \text{orange}) \mid 0 \leq j \leq 12 \text{ where the } x_j \text{ are lemon-lime or orange}\}$.

1.4 Answers will vary. One possible outcome is $\{A\heartsuit, A\clubsuit, A\spadesuit, A'', K\heartsuit\}$. One possible event is $\{x_1\heartsuit, x_2\heartsuit, x_3\heartsuit, x_4\heartsuit, x_5\heartsuit \mid x_j \text{ are different values}\}$. This would be a heart flush. The sample space is $\{x_1, x_2, x_3, x_4, x_5 \mid x_i \in \{A\clubsuit, \dots, K\clubsuit, A'', \dots, K'', A\heartsuit, \dots, K\heartsuit, A\spadesuit, \dots, K\spadesuit\}\}$.

1.5 Because listing the outcome of 75 calls is tiring, the following can represent any of an outcome, an event, or the sample space: $\{(x_1, x_2, \dots, x_{75}) \mid x_1 + x_2 + \dots + x_{75} \leq 400\}$.

1.6 Answers will vary. One possible outcome is that the pink crayon is the first one selected: $\{1\}$. One possible event is $\{1, 2, 3, 4, 5\}$ (the pink crayon is one of the first five selected). The sample space is $\{1, 2, 3, \dots, 24\}$.

1.7 a. 6; $S = \{(rrbb), (brrr), (rbrb), (brbr), (brrb), (rbbr)\}$. **b.** $2^6 = 64$. There are 2^n subsets in any set with n elements.

1.8 a. Each jar is used only once, and she stops with purple, so (assuming the other colors are red, blue, and green) $S = \{P, RP, BP, GP, RBP, RGP, BGP, BRP, GRP, GBP, RBGP, RGBP, BGRP, BRGP, GBRP, GRBP\}$. **b.** $2^{16} = 65,536$. **c.** Answers will vary. A few outcomes might be RGRGP, BBGP, and RGBRGP. **d.** $S = \{x_1, \dots, x_j, P \mid 0 \leq x_i \text{ and } x_i \text{ is one of the other three colors}\}$.

1.9 a. Because another text could come immediately after she answers one (an immeasurably short time), and there are three waiting times to consider. $S = \{(x_1, x_2, x_3) \mid x_i \in \mathbb{R}^{\geq 0}\}$. **b.** $S = \{(x_1, y_1, x_2, y_2, x_3) \mid x_i, y_i \in \mathbb{R}^{\geq 0}\}$. **c.** $S = \{(x_1, y_1, x_2, y_2, x_3) \mid x_1 < x_2 < x_3; y_1 > y_2 \in \mathbb{R}^{\geq 0}\}$.

1.10 a. Using the first player to roll followed by the second player, $S = \{11, 12, 13, 14, 15, 16, 21, 22, \dots, 65, 66\}$. **b.** Nothing changes – each player is still rolling a die. **c.** Nothing changes if we consider a (left, right) orientation to the final positions of the dice.

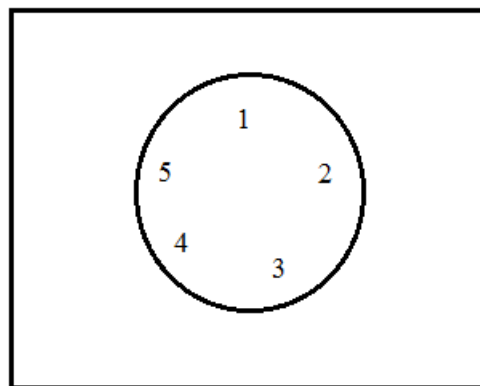
1.11 The diagonal line is $x + y = 2$, so $S = \{(x, y) \mid x, y \in \mathbb{R}^{\geq 0}, x, y > 0\}$.

1.12 The diagonal line is $y = 4 - x$, so $S = \{(x, y) \mid x, y \in \mathbb{R}^{\geq 0}, 0 \leq x \leq 2, 0 \leq y \leq 4 - x\}$.

1.13 If order does not matter, there are $\binom{8}{6} = 21$ ways to choose 6 holes to be filled.

Alternatively, there are $\binom{8}{2} = 21$ ways to select 2 holes to leave open.

1.14 Because rotations do not make a difference in circular seating, fix the position of Alice at 1. There are two couples to arrange in the remaining seats. Put a couple in positions 2 and 3 (there are two ways to do this). The other couple goes in positions 4 and 5. Within each couple, there are two possible arrangements. Suppose Bob and Catherine occupy seats 2 and 3. Is Bob in seat 2 or seat 3? The same applies to Doug and Edna. So, there are $2 \times 2 \times 2 = 8$ possible seating arrangements that allow the two couples to sit together.



1.15 The final table is given below. The text indicates only one possible outcome for 3 (1,1,1) and 18 (6,6,6). A few of the others are:

4: (1,1,2), (2,1,1), (1,2,1)

5: (1,2,2), (2,1,2), (2,2,1), (1,1,3), (1,3,1), and (3,1,1)

10: (2,3,5), (2,5,3), (3,2,5), (3,5,2), (5,2,3), (5,3,2), (6,2,2), (2,6,2), (2,2,6), (3,3,4), (3,4,3), (4,3,3), (1,3,6), (1,6,3), (3,1,6), (3,6,1), (6,3,1), (6,1,3), (2,4,4), (4,2,4), (4,4,2)

A_i	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
$ A_i $	1	3	6	9	12	15	18	21	24	18	15	12	9	6	3	1