Introduction to Robotics
Analysis, Control, Applications

Solution Manual

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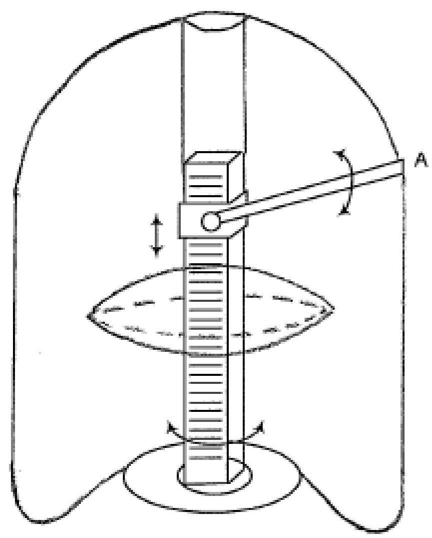
CHAPTER ONE

Problem 1.1
Draw the approximate workspace for the following robot. Assume the dimensions of the base and other parts of the structure of the robot are as shown.

Estimated student time to complete: 15-25 minutes
Prerequisite knowledge required: Text Section(s) 1.14

Solution:

The workspace shown is approximate.



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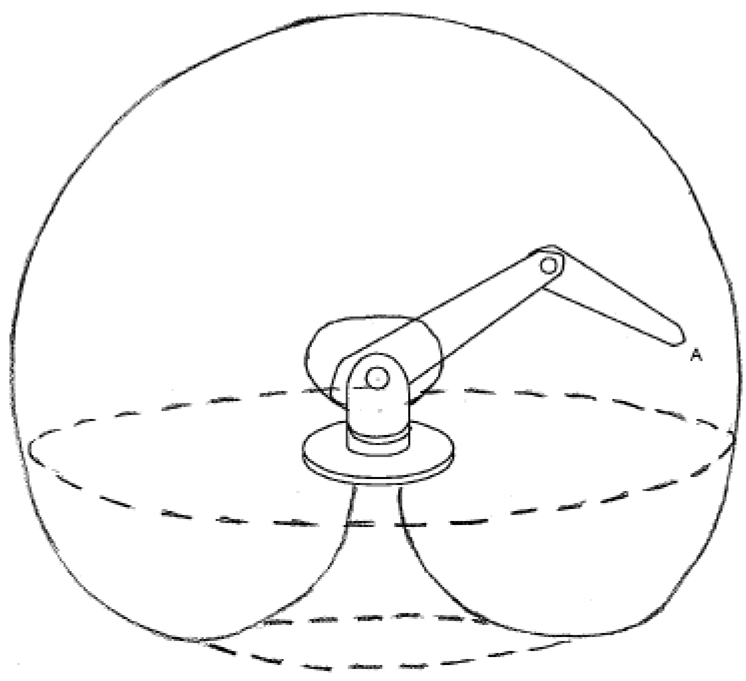
Problem 1.2
Draw the approximate workspace for the following robot. Assume the dimensions of the base and other parts of the structure of the robot are as shown.

Estimated student time to complete: 20-30 minutes
Prerequisite knowledge required: Text Section(s) 1.14

Solution:

The workspace shown is approximate.



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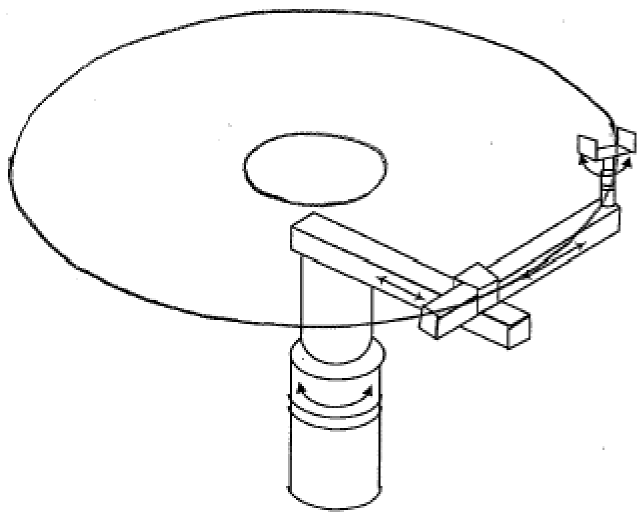
Problem 1.3
Draw the approximate workspace for the following robot. Assume the dimensions of the base and other parts of the structure of the robot are as shown.

Estimated student time to complete: 10-15 minutes
Prerequisite knowledge required: Text Section(s) 1.14

Solution:

The workspace shown is approximate.



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CHAPTER TWO

Problem 2.1
Write a unit vector in matrix form that describes the direction of the cross product of $\mathbf{p} = 5\mathbf{i} + 3\mathbf{k}$ and $\mathbf{q} = 3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$.

Estimated student time to complete: 5-10 minutes
Prerequisite knowledge required: Text Section(s) 2.4

Solution:

$$\mathbf{r} = \mathbf{p} \times \mathbf{q} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 0 & 3 \\ 3 & 4 & 5 \end{bmatrix} = \mathbf{i}\left(0 - 12\right) - \mathbf{j}\left(25 - 9\right) + \mathbf{k}\left(20 - 0\right) = -12\mathbf{i} - 16\mathbf{j} + 20\mathbf{k}$$

$$\lambda = \sqrt{r_x^2 + r_y^2 + r_z^2} = \sqrt{144 + 256 + 400} = 28.28$$

$$\mathbf{r} = \begin{bmatrix} \dfrac{-12}{28.28} \\ \dfrac{-16}{28.28} \\ \dfrac{20}{28.28} \end{bmatrix} = \begin{bmatrix} -0.424 \\ -0.566 \\ 0.707 \end{bmatrix}$$

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Problem 2.2

A vector **p** is 8 units long and is perpendicular to vectors **q** and **r** described below. Express the vector in matrix form.

$$\mathbf{q}_{unit} = \begin{bmatrix} 0.3 \\ q_y \\ 0.4 \\ 0 \end{bmatrix} \qquad \mathbf{r}_{unit} = \begin{bmatrix} r_x \\ 0.5 \\ 0.4 \\ 0 \end{bmatrix}$$

Estimated student time to complete: 15-20 minutes
Prerequisite knowledge required: Text Section(s) 2.4

Solution:

The two vectors given are unit vectors. Therefore, each missing component can be found as:

$$q_y = \sqrt{1 - 0.09 - 0.16} = 0.866$$
$$r_x = \sqrt{1 - 0.25 - 0.16} = 0.768$$

Since **p** is perpendicular to the other two vectors, it is in the direction of the cross product of the two. Therefore:

$$\lambda_p = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.3 & 0.866 & 0.4 \\ 0.768 & 0.5 & 0.4 \end{bmatrix} = \mathbf{i}(0.346 - 0.2) - \mathbf{j}(0.12 - 0.307) + \mathbf{k}(0.15 - 0.665)$$
$$= \mathbf{i}(0.146) + \mathbf{j}(0.187) - \mathbf{k}(0.515)$$

Since **q** and **r** are not perpendicular to each other, the resulting **p** is not a unit vector. Vector **p** can be found as:

$$\lambda_p = \mathbf{i}(0.146) + \mathbf{j}(0.187) - \mathbf{k}(0.515)$$
$$|\lambda_p| = \sqrt{(0.146)^2 + (0.187)^2 + (0.515)^2} = 0.567$$
$$w = \frac{8}{0.567} = 14.1$$
$$\mathbf{p} = w\left(\mathbf{i}(0.146) + \mathbf{j}(0.187) - \mathbf{k}(0.515)\right)$$
$$\mathbf{p} = \mathbf{i}(2.06) + \mathbf{j}(2.64) - \mathbf{k}(7.27)$$

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Problem 2.3

Will the three vectors **p**, **q**, and **r** in Problem 2.2 form a traditional frame? If not, find the necessary unit vector **s** to form a frame between **p**, **q**, and **s**.

Estimated student time to complete: 15-20 minutes
Prerequisite knowledge required: Text Section(s) 2.4

Solution:

As we saw in Problem 2.2, since $\mathbf{q}\times\mathbf{r}$ is not a unit vector, it means that **q** and **r** and not perpendicular to each other, and therefore, they cannot form a frame. However, **p** and **q** are perpendicular to each other, and we can select **s** to be perpendicular to those two. Of course, **p** is not a unit length, therefore we use the unit vector representing it.

$$\left|\lambda_p\right| = \sqrt{(0.146)^2 + (0.187)^2 + (0.515)^2} = 0.567$$

$$w = \frac{1}{0.567} = 1.764$$

$$\mathbf{p} = w\left(\mathbf{i}(0.146) + \mathbf{j}(0.187) - \mathbf{k}(0.515)\right)$$

$$\mathbf{p} = \mathbf{i}(0.257) + \mathbf{j}(0.33) - \mathbf{k}(0.908)$$

$$\mathbf{p} = \mathbf{i}(0.257) + \mathbf{j}(0.33) - \mathbf{k}(0.908)$$

$$\mathbf{s} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.257 & 0.33 & -0.908 \\ 0.3 & 0.866 & 0.4 \end{bmatrix} = \mathbf{i}(0.918) - \mathbf{j}(0.375) + \mathbf{k}(0.124)$$

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Problem 2.4

Suppose that instead of a frame, a point $P = (3,5,7)^T$ in space was translated a distance of $d = (2,3,4)^T$. Find the new location of the point relative to the reference frame.

Estimated student time to complete: 5 minutes
Prerequisite knowledge required: Text Section(s) 2.6

Solution:

As for a frame,

$$P_{new} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 7 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 11 \\ 1 \end{bmatrix}$$

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Problem 2.5

The following frame B was moved a distance of $d = (5, 2, 6)^T$. Find the new location of the frame relative to the reference frame.

$$B = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 1 & 0 & 0 & 4 \\ 0 & 0 & -1 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Estimated student time to complete: 5-10 minutes
Prerequisite knowledge required: Text Section(s) 2.6

Solution:

The transformation matrix representing the translation is used to find the new location as:

$$B_{new} = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 2 \\ 1 & 0 & 0 & 4 \\ 0 & 0 & -1 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 7 \\ 1 & 0 & 0 & 6 \\ 0 & 0 & -1 & 12 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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Problem 2.6
For frame *F*, find the values of the missing elements and complete the matrix representation of the frame.

$$F = \begin{bmatrix} ? & 0 & -1 & 5 \\ ? & 0 & 0 & 3 \\ ? & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Estimated student time to complete: 10 minutes
Prerequisite knowledge required: Text Section(s) 2.4

Solution:

$$F = \begin{bmatrix} n_x & 0 & -1 & 5 \\ n_y & 0 & 0 & 3 \\ n_z & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

From $\mathbf{n} \times \mathbf{o} = \mathbf{a}$ $\quad \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ n_x & n_y & n_z \\ 0 & 0 & -1 \end{vmatrix} = -\mathbf{i}$

Or: $\mathbf{i}(-n_y) - \mathbf{j}(-n_x) + \mathbf{k}(0) = -\mathbf{i}$, and therefore: $n_y = 1, \quad n_x = 0, \quad n_z = 0$

$$F = \begin{bmatrix} 0 & 0 & -1 & 5 \\ 1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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Problem 2.7
Find the values of the missing elements of frame B and complete the matrix representation of the frame.

$$B=\begin{bmatrix} 0.707 & ? & 0 & 2 \\ ? & 0 & 1 & 4 \\ ? & -0.707 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Estimated student time to complete: 15-20 minutes
Prerequisite knowledge required: Text Section(s) 2.4

Solution:

$$B=\begin{bmatrix} 0.707 & o_x & 0 & 2 \\ n_y & 0 & 1 & 4 \\ n_z & -0.707 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

From $\mathbf{n}\times\mathbf{o}=\mathbf{a}$ $\quad \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.707 & n_y & n_z \\ o_x & 0 & 0.707 \end{bmatrix}=\mathbf{j}$

Therefore: $\begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.707 & n_y & n_z \\ o_x & 0 & 0.707 \end{bmatrix}=\mathbf{j}$

And $\quad \mathbf{i}\left(0.707n_y\right)-\mathbf{j}\left(0.5-n_z o_x\right)+\mathbf{k}\left(-n_y o_x\right)=\mathbf{j} \quad\rightarrow\quad n_y=0$

From length equations: $|\mathbf{n}|=1$ or $\quad \begin{array}{l} 0.707^2+n_y^2+n_z^2=1 \quad\rightarrow\quad n_z=\pm 0.707 \\ o_x^2+0.5=1 \quad\rightarrow\quad o_x=\pm 0.707 \end{array}$

Therefore, there are two possible acceptable solutions:

$$B=\begin{bmatrix} 0.707 & 0.707 & 0 & 2 \\ 0 & 0 & 1 & 4 \\ 0.707 & -0.707 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad B=\begin{bmatrix} 0.707 & -0.707 & 0 & 2 \\ 0 & 0 & 1 & 4 \\ -0.707 & -0.707 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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Problem 2.8
Derive the matrix that represents a pure rotation about the *y*-axis of the reference frame.

Estimated student time to complete: 10 minutes
Prerequisite knowledge required: Text Section(s) 2.6.2.

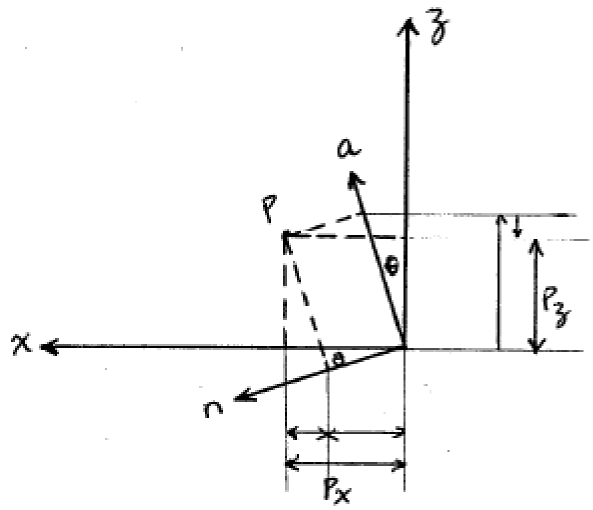
Solution:

From the figure:

$$p_x = p_n \cos\theta + p_a \sin\theta$$
$$p_y = p_o$$
$$p_z = -p_n \sin\theta + p_a \cos\theta$$

and

$$\begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} C & 0 & S \\ 0 & 1 & 0 \\ -S & 0 & C \end{bmatrix} \begin{bmatrix} p_n \\ p_o \\ p_a \end{bmatrix}$$



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Problem 2.9
Derive the matrix that represents a pure rotation about the *z*-axis of the reference frame.

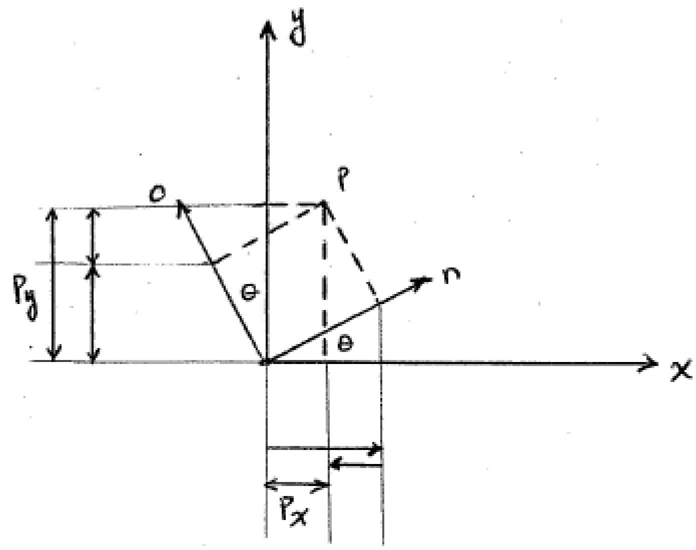
Estimated student time to complete: 10 minutes
Prerequisite knowledge required: Text Section(s) 2.6.2.

Solution:
From the Figure:

$$p_x = p_n \cos\theta - p_o \sin\theta$$
$$p_y = p_n \sin\theta + p_o \cos\theta$$
$$p_z = p_a$$

and

$$\begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} C & -S & 0 \\ S & C & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_n \\ p_o \\ p_a \end{bmatrix}$$



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Problem 2.10
Verify that the rotation matrices about the reference frame axes follow the required constraint equations set by orthogonality and length requirements of directional unit vectors.

Estimated student time to complete: 10 minutes.
Prerequisite knowledge required: Text Section(s) 2.4.5 and 2.6.2

Solution:

For $Rot(x,\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C & -S \\ 0 & S & C \end{bmatrix}$ we have:

$\mathbf{n} \bullet \mathbf{o} = 0$ or $n_x\, o_x + n_y\, o_y + n_z\, o_z = 0$
$\mathbf{n} \bullet \mathbf{a} = 0$
$\mathbf{a} \bullet \mathbf{o} = 0$
$|\mathbf{n}| = 1$
$|\mathbf{o}| = \sqrt{S^2 + C^2} = 1$
$|\mathbf{a}| = \sqrt{(-S)^2 + C^2} = 1$

$Rot(y,\theta)$ and $Rot(z,\theta)$ will be the same.

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Problem 2.11

Find the coordinates of point $P(2,3,4)^T$ relative to the reference frame after a rotation of 45° about the *x*-axis.

Estimated student time to complete: 10 minutes
Prerequisite knowledge required: Text Section(s) 2.6.3.

Solution:

$$^{U}P = Rot(x,45)\begin{bmatrix}2\\3\\4\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\0 & 0.707 & -0.707\\0 & 0.707 & 0.707\end{bmatrix}\begin{bmatrix}2\\3\\4\end{bmatrix} = \begin{bmatrix}2\\-0.707\\4.95\end{bmatrix}$$

Note that the rotation is written in a 3×3, not homogeneous form, because we are only concerned about the rotation part.

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