Solutions Manual for

Development Economics: Theory, Empirical Research, and Policy Analysis

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This solutions manual provides answers for selected discussion questions and for all problems in the text. Please do not post these answers on websites available to the general public.

Please send comments, questions and suggestions regarding the text, solutions manual, or PowerPoint presentations to Julie.Schaffner@tufts.edu. I welcome your feedback!

Chapter 1: Introduction

Discussion Question 1: Many development actors have rallied around the United Nations' Millennium Development Goals (MDGs), which are listed in Table 1.4 (see text).

a. What do the MDGs indicate about the relative emphasis placed by supporters on the following:
   - Income versus nonincome indicators of well-being
   - Well-being improvements for the poor versus the nonpoor
   - Immediate versus longer-term improvements

b. What might explain the emphasis in the MDGs on defining measureable targets?

c. The MDGs have little to say about the process or policies through which the targets might be achieved. What are the potential benefits of remaining silent about the processes that will deliver MDG success and the policies development actors should employ in their efforts to achieve the MDGs? Do you see any potential costs? See Collier and Dercon (2006).

[Discussion of the MDGs may be used to get students thinking about the many dimensions of development performance that development objectives might emphasize, and the difference between development objectives (i.e. values and priorities) and development methods (i.e. policies and approaches that might be used to achieve the objectives).]

a. The MDGs seems to place strong emphasis on income, education and health as important for well-being, and to place strong emphasis on improvements for people living on less than $1.25/day relative to people who are less poor (but still very poor by developed country standards) and the non-poor. The goals seemed to emphasize short- and medium-run improvements over longer-term improvement, because they set targets for 2015.

b. An emphasis on measurable targets might have several purposes. It might help focus efforts on successful outputs rather than on quantities of "inputs" to development efforts, thereby increasing interest in monitoring, evaluation, effectiveness, midcourse corrections, and re-design. It might also help focus diverse actors' attention on similar objectives, possibly aiding cooperation.

c. Focusing primarily on objectives rather than methods has the advantage of leaving the development community free to search for the best ways to achieve the objectives (perhaps acknowledging that there is no consensus about how best to do this). A possible cost of saying little about methods, pointed out by Collier and Dercon (2006), is that it might lead some development actors to pursue the objectives in the most direct and obvious ways, which need not, ultimately, be the most effective ways. For example, development actors might attempt to achieve the first goal only in the most direct way – by giving cash to poor households – instead of also trying to raise the incomes of the poor indirectly by, for example, strengthening property rights (thereby possibly encouraging investment and increasing the demand for low-skill labor in a long-lasting way).

Notice also that the quantitative targets (right column of Table 1.4) are neither pure statements of objective nor precise and complete statements about policy. For example, the third target is

to ensure that all boys and girls complete a full course of primary education. This reflects the value that everyone should have a real opportunity for primary education, and perhaps the belief that education is useful for sustained improvements in income and well-being, but it also implies the belief that policymakers should work toward the goal of expanding education by concentrating on efforts to get all children into school and to get them to remain in school through the official number of years of primary school. Unfortunately, the experience of the last 15 years is that even great success in getting all kids into and through primary school doesn't mean they obtain real primary education. The quality of teaching and learning has plummeted and many children leave primary school without even becoming literate.

The Collier and Dercon (2006) piece raises other provocative discussion questions, such as: Does the international community's push to focus on absolute poverty reduction in developing countries have normative justification, given that it seems to override the social choices of democratically elected governments in developing countries?

Chapter 2: Well-Being

Discussion Question 3: Consider two approaches for assessing household living standards and well-being. The first involves selecting a random sample of households within a region and using long, detailed questionnaires to elicit comprehensive information about income, consumption, and living standards more generally. The second involves a very short questionnaire that is administered to every household in a community, which includes only questions that are easy to answer and may be used to construct simple indices of households' living standards (e.g., questions about how many rooms respondents' homes have and whether the household head is literate). For what purposes is each method best suited? (Purposes might include identification of regions that merit priority in poverty reduction efforts, academic research on poverty, and assessment of eligibility for an emergency cash transfer program.) How could analysis of the results of the first approach be used to give practical guidance regarding the design of the second approach?

Long questionnaires administered to random samples of the population could be useful for identifying which regions are poorer than others. The long questionnaires allow reasonably accurate measurement of good well-being indicators (e.g. consumption expenditure per capita) and the random samples might allow good inferences about regional poverty rates without the expense of a full census. Data from long questionnaires and random samples might also allow economists to study the determinants of poverty and the effectiveness of various policies for reducing poverty.

Short questionnaires administered to everyone in a community, by contrast, might be useful as part of a proxy means test when implementing a targeted poverty reduction program.

Analysis of the first kind of data might allow researchers to construct a good short questionnaire to use in proxy means testing. With a random sample of answers to a long questionnaire that includes both good measures of consumption expenditure per capita and a variety of shorter questions, researchers could identify a set of simple questions that together are good predictors of per capita consumption expenditure and poverty levels. They could also produce an equation or rule for taking the answers to the simple questions and using them to determine whether a household is probably poor or not by a more accurate measure. Practitioners could then collect data only on the easier questions, and use the rule or equation to determine who is poor (and thereby eligible for the program by the proxy means test).

Problem 1: Suppose we know that a policy did not produce any change in a household's real per capita consumption expenditure. List at least five ways the policy might nonetheless have improved the household's well-being. That is, suggest at least five stories regarding how the household's circumstances might have changed, and how the household responded to those changes, that are consistent with the household's well-being rising even while its per capita consumption expenditure remains constant.

Good answers to this question reflect the use of the analytical framework of Chapter 2, and point clearly to changes that would raise well-being even **while not raising consumption expenditure.** Answers such as "receiving access to a better agricultural technology" (without

some sort of qualification) are off track, because the most obvious way through which such a change would raise well-being is by increasing income and consumption expenditure.

Here are some possible answers:

- The policy may have increased income (by providing a cash transfer or information about a new agricultural technology, or through many other types of intervention), but the additional income was put into saving rather than consumption expenditure.
- It may have improved the profitability of income generating opportunities, but the household took advantage of the opportunity to work less and earn the same income – enjoying more non-work time. We might see this in:
    - more leisure
    - children going to school rather than working
- It may have improved the household's current well-being along non-income dimensions such as
    - reduced pollution
    - better health
- It may have reduced the household's exposure to future risk or fluctuations, or improved the household's ability to cope with risk and fluctuations (without changing current income), by creating
    - infrastructure that reduces flood risk
    - a public works program that households can access in the future if they need it
    - improved access to credit that households could use to smooth consumption in the future
    - new opportunities to purchase insurance
- It may have improved the household's investment opportunities or ability to take up investment opportunities, for example through
    - improved access to school for children

Problem 2: Suppose you are attempting to choose a measure of living standards for use in determining which households most need assistance. Discuss the relative merits of the following possible measures of living standards:

- Real income per capita within the household over the last two weeks
- Real income per capita within the household over the last 12 months
- Real consumption expenditure per capita over the last month
- Per capita meat consumption over the last month
- Indicators of whether a household has a dirt floor, uses water from an improved source, and sends children to school
- Individual measures of height (for age), weight (for age), and recent illness

Measure	Strengths	Weaknesses
Real income per capita within the household over the last two weeks	• This is a summary measure of a household's ability to purchase goods and services that is adjusted at least crudely for variation in need across	• It is not sensitive to variation in households' capacity to obtain goods and services that are not sold in well-functioning markets (e.g.

	households.	health care). • It does not account for the hours of work required to obtain the given level of income. • It adjusts for differences in need only imperfectly. It adjusts for differences in numbers of household members but not, for example, in their health-related needs. • When measured over just two weeks, it may provide a poor measure of the household's usual capacity to purchase goods and services, because income fluctuates and households may be able to smooth consumption. • It is insensitive to differences in households' prospects regarding future income and consumption. • It is a household-level measure that does not allow study of the distribution of well-being within the household. • It is costly and difficult to measure.
Real income per capita within the household over last 12 months	• If measured well, it provides an even better measure of per capita capacity to purchase goods and services than the previous measure, because it is less subject to fluctuations across months or seasons.	• It has the same weaknesses as above. • It may also fail to identify households that suffer severe deprivation for short periods within a year. • It is difficult to measure accurately, because people have limited recall capacities.
Real consumption expenditure over the last month	• Like income per capita, it is a good summary measure of a household's capacity to purchase goods and services. • It is even better than income per capita measured over a short recall period if people can smooth consumption, because it may fluctuate much less than income from month to month. • Often it is thought to be measured more accurately than	• It has similar weaknesses as for the first measure. • It fails to register improvement when households use rising income to increase saving and investment rather than consumption.

	income.	
Per capita meat consumption over the last month	• If meat consumption is a steady fraction of income or consumption expenditure, then it would have comparable strengths to those measures. • It also has the merit of measuring a living standard of direct interest to policymakers concerned about nutrition. • It is easier to measure than total consumption expenditure.	• Because meat is a luxury, meat consumption may fluctuate more than total consumption expenditure. Meat consumption over a short period may, therefore, give a poor indication of usual living standards. • Some households may choose not to eat meat for religious or cultural reasons; a meat consumption measure might understate their level of well-being.
Indicators of whether a household has a dirt floor, uses water from an improved source, and sends children to school	• These measures may do a better job than income or consumption expenditure at measuring households' living standards along very important dimensions. • To the extent they reflect assets rather than income, they may also have more to say about likely future well-being than a current income measure. • They are easier to measure than income or consumption expenditure.	• They are hard to aggregate into a single index for identifying who is deprived. • Again, people with similar capacity to obtain goods and services may choose not to acquire some of these things because of differences in preferences.
Individual measures of height (for age), weight (for age) and recent illness.	• These measures shed light on health, which is of direct interest in the assessment of well-being. • They allow study of the comparative well-being of men/women, young/old within households (unlike all the other measures mentioned above). • Because they reflect health assets, they shed light on future prospects as well as the current well-being.	• They may not vary even when non-health dimensions of living standards vary a great deal.

Chapter 3: Economic Growth

Discussion Question 1: Read Collier (2007), Chapter 1. What does the author mean by "the bottom billion"? How does the author make his argument that achieving faster rates of economic growth must be the priority in development for the countries where the world's "bottom billion" live? What do you think of this argument?

Collier's "bottom billion" includes the billion people living in a set of very poor countries that have not been growing and that he believes are stuck in one of four poverty traps: the conflict trap, the natural resource trap, the landlocked with bad neighbors trap, and the bad governance trap. (Notice that they are not the poorest billion people in the world; they are the billion people living in the countries with lowest average income.) He argues that we should be more concerned about *growth* in these poorest countries—and less concerned about *immediate poverty reduction* there – for perhaps two reasons. First, without growth the size of the pie is extremely small in these countries; so growth will be necessary for raising them out of widespread poverty. Second, he asserts that growth is more important than immediate poverty reduction for giving people hope, and hope encourages good people to stay (rather than emigrate) and try to contribute. With a reference to Cuba, he also seems to assert that if too much attention is paid to poverty and inequality, these countries will get stuck being countries with equal but very low incomes (perhaps because the incentive effects of the poverty reduction policies will cause investment and growth to stagnate).

One might respond that these arguments for de-emphasizing immediate poverty reduction are not quite complete or fair, for several reasons. First, it is not clear that concentrated growth, which is not accompanied by poverty reduction, would inspire the kind of hope Collier believes necessary for many people. Second, even if we agree that growth is a necessary condition for development in the short and medium run, we may disagree about the relative (un)importance of getting today's children into school or protecting them from waterborne diseases. That is, we might be willing to slow down the attainment of middle class status down the road (by accepting a lower growth rate) in exchange for preventing more children now from dying unnecessarily.

On another subject: It is also useful to question his assertion that, among the 5 billion people in the developing world, only his "bottom billion" really merit international concern. Three quarters of today's global poor (defined using the $2 per day poverty line) in fact now live in middle income countries, including India and China, and are not included in his bottom billion. These poor families continue to live in deep poverty, and helping to eliminate their poverty will continue to be a major challenge for decades.

Discussion Question 2: As poor economies grow, the share of production that passes through formal markets rises as subsistence farmers become more integrated into markets and improved law enforcement reduces black market activity. Would this process tend to raise or lower the measured rate of economic growth? Would the measured rate of economic growth tend to understate or overstate the true rate of economic growth?

If countries measured GDP only by valuing goods and services that pass through markets, then if growth is accompanied by an increase in the share of goods and services passing through markets, the measured rate of growth would exceed the actual rate of growth. In practice it is harder to say, because countries use diverse methods for adjusting their GDP statistics for the existence of production that does not pass through markets.

Problem 1: If GDP per capita grows from an initial level of G_0 to the level G_t after t years have passed, then the average annually compounded rate of economic growth over the period is the growth rate r (expressed as a percentage) that solves the equation $G_0\left(1+\frac{r}{100}\right)^t = G_t$ Rearranging this expression, we find that

$$r = \left[\left(\frac{G_t}{G_0}\right)^{\frac{1}{t}} - 1\right] * 100$$

The rule of 72 says that if a country grows at an annually compounded rate of r, then we can approximate the number of years it will take for the country's GDP per capita to double (D) using the calculation: $D = 72/r$. To calculate doubling time exactly, notice that the number of years D that it takes to double an initial income per capita of G_0 for a country growing at rate r solves the equation $G_0\left(1+\frac{r}{100}\right)^D = 2G_0$. Dividing both sides by G_0, taking the natural logarithm of both sides, and rearranging, we derive this formula for determining doubling time exactly:

$$D = \frac{\ln(2)}{\ln(1+r/100)}$$

It just so happens that for growth rates in the relevant ranges for studies of economic growth, the right-hand side of this equation is a function of r that is well approximated by the function $72/r$. The following table lists real per capita GDP for selected countries in 1960 and 2000 (in U.S. dollars).

a. Calculate the average annually compounded rates of economic growth for each country to fill in column 3 in the table.

b. Use the rule of 72 to calculate the approximate number of years it would take for GDP per capita to double in each country, assuming it continues to grow steadily at the rate you reported for part a. Record your answers in column 4.

c. Use the formula presented above to calculate more exactly the number of years it would take for GDP per capita to double in each country, assuming it continues to grow steadily at the rate you reported in part a. Record your answers in column 5.

a., b., and c.

Country	(1) GDP per capita 1960 (U.S. Dollars)	(2) GDP per capita 2000 (U.S. Dollars)	(3) Average Annually Compounded Growth Rate 1960-2000 (Percent)	(4) Doubling time using "Rule of 72"	(5) Doubling time using exact calculation
Bolivia	2431.39	2929.19	0.47	153	148
China	448.13	891.39	1.73	42	40
Ghana	411.86	1392.20	3.09	23	23
Taiwan	1443.61	19183.93	6.68	11	11

Notice that the "rule of 72" approximation does a pretty good job for growth rates in this range.

Problem 2: Suppose a firm's production function is given by $F(L, H, K;A)$,where L,H, and K are the current quantities of labor, human capital, and physical capital employed in production, and A is an index of the current level of technology. For each of the following changes, indicate whether it would raise, lower, or leave unchanged: (a) the average product of labor (APL) in the firm and (b) total factor productivity (TFP) within the firm.

- an increase in K, holding L, H, and A constant, while the firm continues to operate on its production function
  - (a) raises APL (b) leaves TFP unaffected
- an increase in L, holding H, K and A constant, while the firm continues to operate on its production function
  - (a) reduces APL (b) leaves TFP unaffected
- an increase in A, holding L, H and K constant, while the firm continues to operate on its production function
  - (a) raises APL (b) raises TFP
- an increase in output that represents a movement toward operation on its production function, while holding L, H, K and A constant
  - (a) raise APL (b) raises TFP

Problem 3: According to the growth accounting equation discussed in Box 3.1, $g_y = g_A + \alpha g_k + (1-\alpha)g_h$, where g_y, g_k, and g_h, are growth rates of GDP per capita, capital per worker, and human capital per worker, and α is the share of capital income in total GDP. The first four columns of the following table give values for g_y, g_k, g_h, and α.

a. Fill in the fifth and sixth columns of the table with the growth attributed to physical and human capital accumulation. These may be calculated as αg_k and $(1-\alpha)g_h$, respectively.
b. Fill in the seventh column of the table, plugging the values of g_y, g_k, g_h, and αinto the growth accounting equation and backing out g_A.
c. Fill in the final column of the table by calculating the fraction of overall growth (g_y) that is attributed to TFP by the growth accounting framework. (That is, divide g_A by g_y and multiply by 100.)

a., b., and c.

g_y	g_k	g_h	α	Growth	Growth	TFP	TFP

				attributable to Physical capital	attributable to Human capital	growth	share in growth
5.2	4.0	2.0	.3	1.2	1.4	2.6	50
5.2	4.0	3.0	.3	1.2	2.1	1.9	37
5.2	4.0	2.0	.4	1.6	1.2	2.4	46

d. Discuss the potential for inaccurate estimates of g_h and α to render misleading estimates of the importance of TFP growth.

Estimates of TFP growth rates are calculated as a residual, after accounting for growth in physical and human capital. Thus, any inaccuracy in the measurement of these two parameters, which help quantify the growth that is "explained" by growth in physical and human capital, leads to inaccuracy in the estimate of TFP growth. Comparing rows 1 and 2, we see that if growth in human capital is overestimated, TFP will be underestimated. Comparing rows 1 and 3, we see that if the physical capital share is overestimated, and if physical capital has been growing more rapidly than human capital, then again TFP growth will be underestimated.

Problem 4: In this problem you will derive the growth accounting equation discussed in Box 3.1. Assume that the aggregate production function takes the form

$$y(t) = A(t)k(t)^{\alpha}h(t)^{1-\alpha}$$

where y, k and h represent GDP per capita, physical capital per worker and human capital per worker, and α is a technological parameter. We assume that A, k and h are changing over time for unspecified reasons, and use the functions $A(t)$, $k(t)$ and $h(t)$ to describe their levels at any point in time t. Derivatives of these functions with respect to time, $\frac{dy}{dt}$, $\frac{dk}{dt}$ and $\frac{dh}{dt}$, describe how fast they are growing (in absolute terms) at any point in time. Their percentage growth rates are $g_A = \frac{dA}{dt} / A$, $g_k = \frac{dk}{dt}$/k and $g_h = \frac{dh}{dt} / h$. Because Y is a function of A, k and h, it, too, is a function of time, with percentage growth rate $g_y = \frac{dy}{dt}$/y.

a. Take the derivative with respect to time t of both sides of the aggregate production function equation.

$$\frac{dy(t)}{dt} = \frac{dA(t)}{dt}k(t)^{\alpha}h(t)^{1-\alpha} + \alpha\frac{dk(t)}{dt}A(t)k(t)^{\alpha-1}h(t)^{1-\alpha} + (1-\alpha)\frac{dh(t)}{dt}A(t)k(t)^{\alpha}h(t)^{-\alpha}$$

b. Divide both sides of this new equation by y, so that the left hand side becomes g_y.

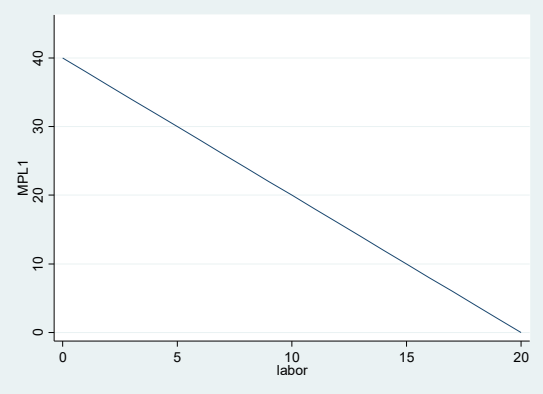
$$g_y = \frac{\frac{dA(t)}{dt}}{y}k(t)\alpha h(t)1-\alpha + \alpha\frac{\frac{dk(t)}{dt}}{y}A(t)k(t)\alpha-1h(t)1-\alpha + (1-\alpha)\frac{\frac{dh(t)}{dt}}{y}A(t)k(t)\alpha h(t)-\alpha$$

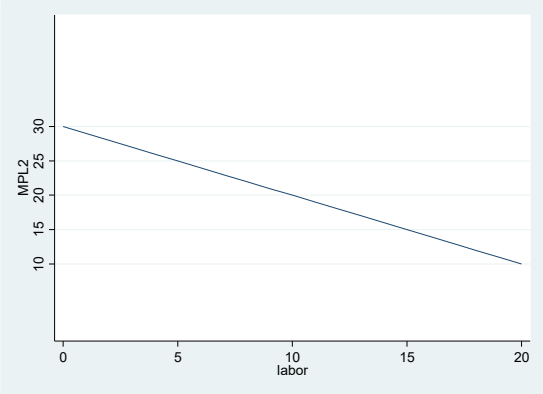
c. Show how to transform the equation you just derived into the following:
$g_y = g_A + \alpha g_k + (1-\alpha)g_h$

Substitute in $A(t)k(t)^{\alpha}h(t)^{1-\alpha}$ for $y(t)$ in the denominator of each right hand side term. In each of the three terms you can then cancel like terms in numerator and denominator to get the indicated relationship.

Problem 5: Consider two firms that produce the same output. The marginal product of labor in each firm is a declining function of the quantity of labor employed there. In Firm 1, the marginal product of labor MPL_1 is described by the function $MPL_1 = 40 - 2L_1$, where L_1 is the quantity of labor employed in Firm 1. In Firm 2, the marginal product of labor is described by $MPL_2 = 30 - L_2$, where L_2 is the quantity of labor employed in Firm 2.

a. Graph these functions in two graphs, side by side. Let your horizontal axes measure units of labor in the range of 0 to 20, and let your vertical axes measure the marginal product of labor in the range of 0 to 45 units of output.





b. Suppose $L_1 = 6$ and $L_2 = 8$. What is the marginal product of labor in Firm 1? What is the marginal product of labor in Firm 2? Explain why the total quantity of output produced by the two firms together would rise if one unit of labor was moved from Firm 2 to Firm 1.

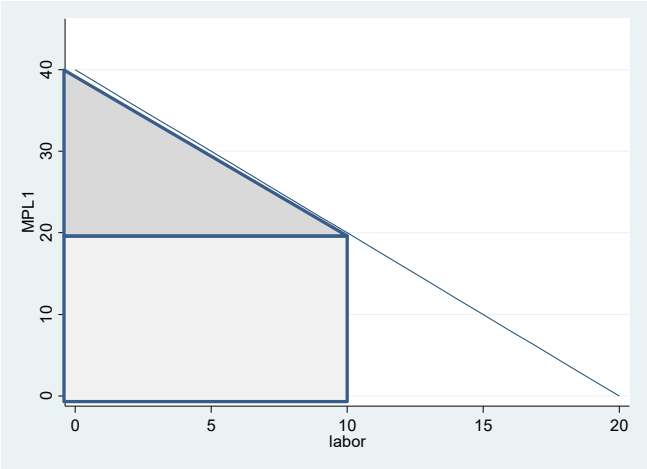
MPL_1=40-2*6=28

MPL_2=30-8=22

Taking away one unit from firm 2 would reduce output by 22, while adding it to firm 1 would increase output by 28.

c. At what levels of L_1 and L_2 do the marginal products of labor equal 20 in both firms? If these are in fact the quantities of labor employed in the two firms, what is the average product of labor in each firm? (Hint: The average product of labor is just the total product or total output divided by the quantity of labor employed. The total product is equal to the area under the marginal product of labor curve.) You have just shown that it is possible for the average products of labor to differ across firms (or sectors) even when the marginal products of labor are equal.

MPL_1 equals 20 when L_1=10
MPL_2 equals 20 when L_2=10
The shaded rectangle in the below graph has area 10*20=200. The shaded triangle has area .5*10*20=100. The total product in firm 1 is 300, and the average product is 300/10=30. Comparable calculations find that for firm 2 the area under the curve is 10*20+.5*10*10=250, and the average product is 25.



Chapter 4: Economic Growth Theory in Historical Perspective

Discussion Question 1: The Harrod–Domar model exhibits the knife-edge property that growth is consistent with continuously full employment only if (s/k)-d just happens to equal n. This knife-edge property is eliminated in the simple neoclassical model, which is characterized by continuously full employment. Discuss the importance of assuming variable rather than fixed proportions production technology for working this change.

With a fixed proportions production technology, producers with a given quantity of capital can employ only so many workers. This number of jobs may fall short of the number of workers (as assumed in the Harrod-Domar model). Even if the excess supply of workers drove down the wage, employers would have no way to adjust their production choices to hire more workers (using their current capital stocks). They might be encouraged to expand production by investing, but their ability to do so is constrained by the saving rate. With a variable proportions production technology, by contrast, employers can respond to lower wages by increasing their use of labor relative to capital. Even while holding the capital stock constant, they will hire labor until the value of the marginal product of labor equals the wage. Whoever is willing to work at that wage will find employment.

Discussion Question 2: For each of the bodies of growth theory described in this chapter, state whether production technologies are characterized by constant or increasing returns to scale. Where production technologies are characterized by increasing returns to scale, state whether the increasing returns to scale are present only at the aggregate level (while individual producers continue to work under the assumption that their technology is characterized by constant returns to scale), or whether increasing returns to scale are also present and important at the level of the individual firm. What roles do these assumptions play in shaping the predictions of the models?

Harrod-Domar model: Constant returns to scale. This, together with the assumption of abundant labor, means that the growth rate of output is the same as the growth rate of capital.

Lewis model: No potential to increase scale in the traditional sector and in fact no reduction in output if labor input is reduced. Constant returns to scale in the modern the sector. This implies that as labor is drawn from the traditional to the modern sector, total output rises.

Solow models: Constant returns to scale. This, together with diminishing marginal returns to individual inputs, means that as the ratio of capital to labor rises, the impact of further increases on output per person falls. This is crucial to the result that there is no growth in steady state.

Growth as by-product of human capital investment: Increasing returns to scale at the level of the entire economy. This creates the potential for investment that increases the capital stock (per worker) to be less productive in poor countries (where capital-labor ratios are lower) than in rich countries. Poor countries might tend toward a steady state with low levels of capital and income per worker, while rich countries tend toward the better steady state. The

increasing returns also come about through an externality, meaning that government might have a role to play in encouraging more of the relevant investments.

Poverty trap models: In at least some versions, increasing returns at micro and macro levels are important. Increasing returns at the micro level might imply that investment is not profitable unless producers expect to face a large enough market. This, in turn, might mean that any one investment is profitable only if many other producers also invest.

Discussion Question 3: Discuss the role of empirical research (including simple empirical observations) in driving the evolution of growth theory.

Observations of success in the USSR and of the depression in the U.S. encouraged early capital fundamentalism and structuralism. New data on the economic behavior of poor households, new studies of human capital and earnings, new growth accounting studies, and new observations about growth encouraged a shift to models that also incorporated human capital and technology as proximate sources of growth and to models in which decision-makers are responsive to changing incentives and in which markets work well. Observations of diverse growth experiences among poor countries, and the attempt to make models that treat technical change in more realistic ways, led to models in which growth does not necessarily happen ideally in the absence of intervention.

Problem 1: As discussed in the text, the assumptions of the Harrod– Domar model may be summarized by the equations

$$Y = \left(\frac{1}{v}\right) K \qquad (4.A)$$

$$\dot{Y} = \left(\frac{1}{v}\right) \dot{K} \qquad (4.B)$$

and

$$\dot{K} = sY - dK \qquad (4.C)$$

where the notation is as defined in the text. (Equations 4.A and 4.B are two ways of stating the same assumption, but both expressions are useful to remember in the derivations you will be required to do below.)

a. Demonstrate that equations 4.B and 4.C together imply the following result regarding the growth rate of GDP. (Notice that the text offers guidance about how to derive this equation.)

$$\frac{\dot{Y}}{Y} = \frac{s}{v} - d$$

Plugging equation 4.C into equation 4.B, we get

$$\dot{Y} = \left(\frac{1}{v}\right)(sY - dK)$$

Dividing by Y we get

$$\frac{\dot{Y}}{Y} = \frac{s}{v} - \left(\frac{d}{v}\right)\left(\frac{K}{Y}\right)$$

Recognizing that $1/v = Y/K$, we can do some cancelling in the second term and get:

$$\frac{\dot{Y}}{Y} = \frac{s}{v} - d$$

b. Show that equations 4.A and 4.C together imply the following result regarding the growth rate of K.

$$\frac{\dot{K}}{K} = \frac{s}{v} - d$$

From part a we know that

$$\frac{\dot{Y}}{Y} = \frac{s}{v} - d$$

From equation 4.A we know that

$$K = vY$$

and

$$\dot{K} = v\dot{Y}$$

Dividing by K, we get

$$\frac{\dot{K}}{K} = \frac{v}{K}\dot{Y}$$

Recognizing that $v=K/Y$, we get

$$\frac{\dot{K}}{K} = \frac{K}{Y}\frac{1}{K}\dot{Y} = \frac{\dot{Y}}{Y}$$

Plugging in the result from part a, we conclude that

$$\frac{\dot{K}}{K} = \frac{s}{v} - d$$

Problem 2: Consider the neoclassical growth model with technical change, and its diagrammatic summary in Figure 4.3. Suppose the rate of population growth n increased. Which element of the graph (i.e., the $k^*(n+d+g)$ line or the $sf(k^*)$ curve) would change and in what way? Draw such a change into a graph like the one in Figure 4.3. When the rate of population growth increases like this, what happens to the steady-state level of income per capita? What happens to the steady-state rate of growth in income per effective worker? What

is the immediate impact on the rate of growth in k^*? What is the immediate impact on the rate of growth of GDP per capita? Using intuitive, plain language, explain why the increase in the population growth rate has the short-run impact on growth that you just described and why that short-run impact eventually fades away.

If n increased, the straight line in Figure 4.3 would rotate up, still starting at the origin but having a steeper slope. The steady-state level of capital per worker and GDP per capita falls. The steady state growth rate is still the rate of technical change. Initially, if the economy was in steady state equilibrium at the higher level of GDP per capita, the increase in the population growth rate world slow the rate of capital accumulation, causing the rate of capital growth to fall below the rate required to keep everyone equipped at the initial level of capital per worker. Growth would slow below the rate of technical change, but as the ratio of capital to effective labor falls, the growth rate would pick up, until the economy reaches the new steady-state equilibrium. Intuitively, the increased rate of population growth makes the economy fall short of equipping all new workers with the initial level of capital per effective worker (while also replacing depreciated capital). Capital per effective worker falls, tending to reduce per capita income. Growth probably remains positive even so, because technology is also improving.

Problem 3: Suppose the aggregate production function takes the form

$$Y = A(K)\,F(K,L)$$

where $A(K)=K^{\beta}$ describes an external, economy-wide effect of K on A, $F(K,L)=L^{\alpha}K^{1-\alpha}$ and $0<\alpha<1$.

a. Demonstrate that if you double both K and L while holding the initial value of A constant (i.e., ignoring the external effect of K on A), Y doubles.

If $Y(L,K)=AL^{\alpha}K^{1-\alpha}$, then $Y(2L,2K)=A(2L)^{\alpha}(2K)^{1-\alpha} = 2^{\alpha+1-\alpha}\,AL^{\alpha}K^{1-\alpha} = 2Y(L,K)$

b. Demonstrate that if you double both K and L, taking into account the external effect of K on A, Y more than doubles.

If $Y(L,K)=K^{\beta}L^{\alpha}K^{1-\alpha}$, then $Y(2L,2K)= (2K)^{\beta}(2L)^{\alpha}(2K)^{1-\alpha} = 2^{1+\beta}\,K^{\beta}L^{\alpha}K^{1-\alpha} = 2^{1+\beta}Y(L,K)>2Y(L,K)$

c. Derive an expression for the marginal product of capital while ignoring the external effect of K on A. That is, holding A constant (rather than treating it as a function of K), take the derivative of the aggregate production function with respect to K. Show that if K increases while L holds constant, this marginal product of capital falls.

If $Y(L,K)=AL^{\alpha}K^{1-\alpha}$, then $\partial Y/\partial K = (1-\alpha)AL^{\alpha}K^{-\alpha}$, and because K is raised to a negative power, this derivative gets smaller as K rises.

d. Derive an expression for the marginal product of capital, taking into account the external effect of K on A. Show that if K increases while L holds constant, the marginal product of capital can fall or rise, depending on the values of α and β.

If $Y(L,K)=K^{\beta}L^{\alpha}K^{1-\alpha}$, then $\partial Y/\partial K = (1-\alpha+\beta)L^{\alpha}K^{-\alpha+\beta}$. If $\beta>\alpha$, then K is raised to a positive power, and this derivative is increasing in K.

Problem 4: Critical to the construction of some models of macro poverty traps is the assumption that the profitability of setting up a modern, high-productivity establishment in any one sector depends positively on the size of the market the establishment will face (which in turn is taken to depend positively on the number of other sectors in which modern, high-productivity establishments have set up). In this problem you will examine a very simple technology for modern production, involving a fixed cost of setup, in which profitability of setting up indeed depends positively on the number of units of output the firm anticipates being able to sell. Suppose that modern production can take place only after incurring a fixed cost of F units of labor. Once that cost is incurred, each unit of additional labor produces $\alpha>1$ units of output. The price of a unit of labor is 1. Suppose the price of a unit of output is 1 also. Let Q be the quantity of output the potential investor anticipates selling.

a. Derive an expression for the producer's profits (i.e., revenue minus labor costs) as a function of Q, F, and α.

Profit is equal to revenue minus cost. Revenue is equal to price times quantity sold. With price equal to 1, this is just Q. Cost is the fixed cost of F plus the variable cost of 1 for each unit of labor required. If each unit of labor produces α units of output, then each unit of output requires $1/\alpha$ units of labor, and the variable cost is $(1/\alpha)Q$. Thus profit is $Q-F-(1/\alpha)Q = (1-1/\alpha)Q - F$.

b. Making use of this expression, show that if $F = 0$ (meaning that there are no fixed costs) then setting up is profitable regardless of the level of Q.

If F is 0, then profit is $(1-1/\alpha)Q = [(\alpha-1)/\alpha]Q$, and this is positive, no matter what positive value Q takes, because $\alpha>1$.

c. Now assume $F>0$. Derive an expression for the minimum level of Q at which production is profitable. How does this minimum profitable scale change as F increases? As α increases?

If profit is $(1-1/\alpha)Q - F$, then to find the level of Q at which profits are zero (just turning from negative to positive), we set that expression to zero and re-arrange. When $(1-1/\alpha)Q - F=0$, $Q = F/[1-1/\alpha]$. As F increases, this minimum profitable scale increases (because $1> 1/\alpha$). As α increases, $1-1/\alpha$ increases and the break-even level of Q falls.

Chapter 5: Poverty, Inequality, and Vulnerability

Discussion Question 1: Consider giving one dollar to a poor person, keeping in mind that among a country's poor people, some have much lower incomes than others. Consider each of the aggregate poverty measures defined in the text, and assume that per capita household income is the measure of individual-level well-being they summarize. For each measure, discuss how the impact on the measure would differ depending on whether the additional dollar were given to a person who is just barely poor (with income just below the poverty line) or to a person who is very poor.

For the headcount ratio: If you gave one dollar to the least poor person (i.e. the person with the highest income that qualifies as poor), this indicator might show improvement, because a dollar might raise that person over the poverty line. If you gave the dollar to the poorest person, though, this indicator would not show improvement, because that would not raise the income of the poorest person above the poverty line.

For the total income gap: This measure would change the same way, whether you gave the dollar to the most poor or least poor person.

For the average proportional income gap: If you gave a dollar to the poorest person, this would go down. If you gave a dollar to the least poor person, the dollar might raise the person over the poverty line, and thus take the person out of the calculation of the average. It is possible that this measure would get worse.

For the poverty gap index: If you gave a dollar to the poorest person this would go down. Whether you give a dollar to the most poor or least poor person, this would reduce the measure by the same amount.

For the P_2 measure: If you gave a dollar to the poorest person, this would fall by more than if you gave the dollar to the least poor person.

Problem 1: This problem provides a brief review of summation notation, using an example related to the distribution of incomes in a population. Order the individuals in the population from 1 to *N*, with individual 1 being the poorest person and individual *N* being the richest. An individual's index is his rank number in this ordering. For example, the fifth-poorest person has person index 5. Let Y_i be the income of person *i*. In summation notation, the Greek letter Σ (capital sigma) denotes a sum. More specifically, the expression $\sum_{i=1}^{N} Y_i$, which is read as "the sum from $i = 1$ to N of Y-sub-i," can be defined as follows:

$$\sum_{i=1}^{N} Y_i = Y_1 + Y_2 + \ldots + Y_N.$$

a. Using summation notation, write down a formula for the mean (or simple average) of income in this population.

The mean is equal to the total income in the population, $\sum_{i=1}^{N} Y_i$, divided by the number of people in the population, *N*. Thus we could write the formula this way: $(1/N) \sum_{i=1}^{N} Y_i$.

b. Consider the expression

$$\frac{1}{q}\sum_{i=1}^{q}\frac{(z-Y_i)}{z}$$

where *z* is the income poverty line and *q* is the index of the individual with the highest income who remains under the poverty line. State in plain language the calculation this expression describes and offer an intuitive interpretation of the statistic that results from this calculation.

The expression to the right of the summation sign is person *i*'s proportional income gap. When we sum this from 1 to *q*, we are summing up these proportional income gaps only among the poor. When we then divide by *q*, which is the number of the poor, we are taking the average, among the poor, of the proportional income gaps.

Problem 2: The following table lists the incomes for all individuals in each of three very small countries (just 10 people each). Incomes are listed in currency units (CUs) per week. The official poverty line is 10 CUs per week.

Incomes in Currency Units Per Week

Individual	Country 1	Country 2	Country 3
1	8	3	6
2	8	3	6
3	8	9	6
4	8	9	6
5	8	12	6
6	8	12	12
7	12	12	12
8	12	12	12
9	12	12	12
10	12	12	12

a. Fill in the following table.

Poverty Measure	Country 1	Country 2	Country 3
P_0 (Headcount Ratio)	.6	.4	.5
P_1 (Poverty Gap Index)	.12	.16	.20
P_2	.024	.10	.08