Review and Applications of Algebra 2

Exercise 2.1

Basic Problems

1.
$$(-p) + (-3p) + 4p = -p - 3p + 4p = 0$$

2.
$$(5s-2t)-(2s-4t)=5s-2t-2s+4t=3s+2t$$

3.
$$4x^2y + (-3x^2y) - (-5x^2y) = 4x^2y - 3x^2y + 5x^2y = 6x^2y$$

4.
$$1 - (7e^2 - 5 + 3e - e^3) = 1 - 7e^2 + 5 - 3e + e^3 = e^3 - 7e^2 - 3e + 6$$

5.
$$(6x^2 - 3xy + 4y^2) - (8y^2 - 10xy - x^2) = 6x^2 - 3xy + 4y^2 - 8y^2 + 10xy + x^2$$

= $\frac{7x^2 - 4y^2 + 7xy}{}$

6.
$$6a - 3a - 2(2b - a) = 6a - 3a - 4b + 2a = 5a - 4b$$

7.
$$\frac{3y}{1.2}$$
 + 6.42y - 4y + 7 = 2.5y + 6.42y - 4y + 7 = $\underline{4.92y + 7}$

8.
$$13.2 + 7.4t - 3.6 + \frac{2.8t}{0.4} = 13.2 + 7.4t - 3.6 + 7t = \underline{14.4t + 9.6}$$

Intermediate Problems

9.
$$4a(3ab - 5a + 6b) = 12a^2b - 20a^2 + 24ab$$

10.
$$9k(4 - 8k + 7k^2) = 36k - 72k^2 + 63k^3$$

11.
$$-5xy(2x^2 - xy - 3y^2) = -10x^3y + 5x^2y^2 + 15xy^3$$

12.
$$(3p^2 - 5p)(-4p + 2) = -12p^3 + 6p^2 + 20p^2 - 10p = -12p^3 + 26p^2 - 10p$$

13.
$$3(a-2)(4a+1) - 5(2a+3)(a-7) = 3(4a^2 + a - 8a - 2) - 5(2a^2 - 14a + 3a - 21)$$

= $12a^2 - 21a - 6 - 10a^2 + 55a + 105$
= $2a^2 + 34a + 99$

14.
$$5(2x - y)(y + 3x) - 6x(x - 5y) = 5(2xy + 6x^2 - y^2 - 3xy) - 6x^2 + 30xy$$

= $-5xy + 30x^2 - 5y^2 - 6x^2 + 30xy$
= $24x^2 + 25xy - 5y^2$

$$\frac{18x^2}{3x} = \underline{6x}$$

$$\frac{6 a^2 b}{-2ab^2} = \frac{-3 \frac{a}{b}}{-3 \frac{a}{b}}$$

$$\frac{x^2y-xy^2}{}$$

$$\frac{x^{2}y - xy^{2}}{xy} =$$
17.
$$\frac{\underline{x - y}}{-4x + 10x^{2} - 6x^{3}}$$
18.
$$= \underline{8 - 20x + 12x^{2}}$$

18.
$$-0.5x = 8 - 20x + 12x$$

$$\frac{12x^3 - 24x^2 + 36x}{48x} = \frac{x^2 - 2x + 3}{4}$$

19.

Exercise 2.1 (continued)

21.
$$3d^2 - 4d + 15 = 3(2.5)^2 - 4(2.5) + 15$$

= $18.75 - 10 + 15$
= 23.75

22.
$$15g - 9h + 3 = 15(14) - 9(15) + 3 = 78$$

23.
$$7x(4y-8) = 7(3.2)(4 \times 1.5 - 8) = 22.4(6 - 8) = -44.8$$

24.
$$(1+i)^m - 1 = (1 + 0.0225)^4 - 1 = 0.093083$$

25.
$$I \div Pr = \frac{\$13.75}{\$500 \times 0.11} = \underline{0.250}$$

$$\frac{N}{1-d} = \frac{\$89.10}{1-0.10} = \$99.00$$

27.
$$P(1+rt) = \$770 \left(1+0.013 \times \frac{223}{365}\right) = \$770(1.0079425) = \frac{\$776.12}{5}$$

 $S = \$2500 = \2500

28.
$$\frac{S}{1+rt} = \frac{\$2500}{1+0.085 \times \frac{123}{365}} = \frac{\$2500}{1.028644} = \frac{\$2430.38}{1.028644}$$

29.
$$P(1+i)^n = $1280(1+0.025)^3 = $1378.42$$

$$\frac{S}{30.} = \frac{\$850}{(1+i)^n} = \frac{\$850}{(1+0.0075)^6} = \frac{\$850}{1.045852} = \frac{\$812.73}{1.045852}$$

Advanced Problems

31.
$$\frac{2x+9}{4} - 1.2(x-1) = 0.5x + 2.25 - 1.2x + 1.2 = -0.7x + 3.45$$

32.
$$\frac{x}{2} - x^2 + \frac{4}{5} - 0.2x^2 - \frac{4}{5}x + \frac{1}{2} = 0.5x - x^2 + 0.8 - 0.2x^2 - 0.8x + 0.5$$
$$= -1.2x^2 - 0.3x + 1.3$$

33.
$$\frac{8x}{0.5} + \frac{5.5x}{11} + 0.5(4.6x - 17)$$
 = $16x + 0.5x + 2.3x - 8.5 = 18.8x - 8.5$

34.
$$\frac{2x}{1.045} - \frac{2.016x}{3} + \frac{x}{2} = 1.9139x - 0.6720x + 0.5x = 1.7419x$$

35.
$$R\left[\frac{(1+i)^n-1}{i}\right] = \$550\left(\frac{1.085^3-1}{0.085}\right) = \$550\left(\frac{0.2772891}{0.085}\right) = \frac{\$1794.22}{0.085}$$

36.
$$R\left[\frac{(1+i)^{n}-1}{i}\right](1+i) = \$910\left(\frac{1.1038129^{4}-1}{0.1038129}\right)(1.1038129)$$
$$= \$910\left(\frac{0.4845057}{0.1038129}\right)(1.1038129)$$
$$= $4687.97$$

$$\frac{R}{i} \left[1 - \frac{1}{(1+i)^n} \right] = \frac{\$630}{0.115} \left(1 - \frac{1}{1.115^2} \right) = \frac{\$1071.77}{1.115^2}$$

Exercise 2.2

Basic Problems

1.
$$I = Prt$$

\$6.25 = $P(0.05)0.25$
\$6.25 = $0.0125P$
 $\frac{$6.25}{0.0125} = \frac{$500.00}{0}$
 $PV = \frac{PMT}{i}$
2. $$150,000 = \frac{$900}{i}$
\$150,000 = $\frac{$900}{i}$
\$150,000 = $\frac{$900}{i}$

L = 0.90 = \$990.00

5.
$$N = L(1 - d)$$

\$410.85 = \$498(1 - d)
 $\frac{$410.85}{$498}$
= 1 - d
0.825 = 1 - d
 $d = 1 - 0.825 = 0.175$

6.
$$S = P(1 + rt)$$

$$\$5100 = \$5000(1 + 0.0025t)$$

$$\$5100 = \$5000 + \$12.5t$$

$$\$5100 - \$5000 = \$12.5t$$

$$\frac{\$100}{t = \$12.5} = \underline{8.00}$$

7.
$$NI = (CM)X - FC$$

$$\$15,000 = CM(5000) - \$60,000$$

$$\$15,000 + \$60,000 = 5000CM$$

$$\frac{\$75,000}{5000} = \$15.00$$

8.
$$NI = (CM)X - FC$$

$$-\$542.50 = (\$13.50)X - \$18,970$$

$$\$18,970 - \$542.50 = (\$13.50)X$$

$$X = \frac{\$18,427.50}{\$13.50} = \underline{1365}$$

9.
$$N = L (1-d_1)(1-d_2)(1-d_3)$$

$$\$1468.80 = L(1-0.20)(1-0.15)(1-0.10)$$

$$\$1468.80 = L(0.80)(0.85)(0.90)$$

$$\frac{\$1468.80}{0.6120}$$

$$L = = \$2400.00$$

10.
$$c = \frac{V_f - V_i}{V_i}$$

$$0.12 = \frac{V_f - \$6700}{\$6700}$$

$$0.12(\$6700) = V_f - \$6700$$

$$$804 + $6700 = V_f$$

$$V_f = $7504.00$$

$$c = \frac{V_f - V_i}{V_i}$$

11.

$$0.07 = \frac{\$1850 - V_i}{V_i}$$

$$0.07V_i = \$1850 - V_i$$

$$0.07V_i + V_i = \$1850$$

$$1.07V_i = \$1850$$

$$V_i = \frac{\$1850}{1.07}$$

$$V_i = \$1728.97$$

Intermediate Problems

12.
$$a^2 \times a^3 = \underline{a^5}$$

13.
$$(x^6)(x^{-4}) = \underline{x^2}$$

14.
$$b^{10} \div b^6 = b^{10-6} = \underline{b^4}$$

15.
$$h^7 \div h^{-4} = h^{7-(-4)} = \underline{h^{11}}$$

16.
$$(1+i)^4 \times (1+i)^9 = (1+i)^{13}$$

17.
$$(1+i) \times (1+i)^n = (1+i)^{n+1}$$

18.
$$(x^4)^7 = x^{4x7} = \underline{x}^{28}$$

19.
$$(y^3)^3 = \underline{y^9}$$

20.
$$(t^6)^{\frac{1}{3}} = \underline{t^2}$$

21.
$$(n^{0.5})^8 = \underline{n^4}$$

22.
$$\frac{(x^5)(x^6)}{x^9} = x^{5+6-9} = \underline{x^2}$$

$$\frac{(x^5)^6}{x^9} = x^{5 \times 6 - 9} = \underline{x^{21}}$$

24.
$$\left[2(1+i)\right]^2 = 4(1+i)^2$$

$$\left(\frac{1+i}{3i}\right)^3 = \frac{\left(1+i\right)^3}{27i^3}$$

$$8^{\frac{4}{3}} = \left(8^{\frac{1}{3}}\right)^4 = 2^4 = \underline{16}$$

$$27. \quad -27^{\frac{2}{3}} = -\left(27^{\frac{1}{3}}\right)^2 = \underline{-9}$$

28.
$$\left(\frac{2}{5}\right)^3 = 0.4^3 = \underline{0.064}$$

29.
$$5^{\frac{-3}{4}} = 5^{-0.75} = 0.299070$$

30.
$$(0.001)^{-2} = 1,000,000$$

31.
$$0.893^{-\frac{1}{2}} = 0.893^{-0.5} = 1.05822$$

32.
$$(1.0085)^5(1.0085)^3 = 1.0085^8 = 1.07006$$

33.
$$(1.005)^3(1.005)^{-6} = 1.005^{-3} = 0.985149$$

$$34$$
 $\sqrt[3]{1.03} = 1.03^{0.\overline{3}} = 1.00990$

35.
$$\sqrt[6]{1.05} = 1.00816$$

Advanced Problems

36.
$$\frac{4r^5t^6}{(2r^2t)^3} = \frac{4r^5t^6}{8r^6t^3} = \frac{r^{5-6}t^{6-3}}{2} = \frac{t^3}{\underline{2r}}$$

37.
$$\frac{(-r^3)(2r)^4}{(2r^{-2})^2} = \frac{-r^3(16r^4)}{4r^{-4}} = -4r^{3+4-(-4)} = \underline{-4r^{11}}$$

$$\frac{(3x^2y^3)^5}{(xy^2)^3} = \frac{243x^{10}y^{15}}{x^3y^6} = 243x^{10-3}y^{15-6} = \underline{243x^7y^9}$$

39.
$$\frac{6(-3xy)^4}{(-3x^{-3})^2} = \frac{6(81x^4y^4)}{9x^{-6}} = \frac{486x^4y^4}{9x^{-6}} = 54x^{4-(-6)}y^4 = \underline{54x^{10}y^4}$$

$$\left(4^{4}\right)\left(3^{-3}\right)\left(-\frac{3}{4}\right)^{3} = \frac{4^{4}}{3^{3}}\left(-\frac{3^{3}}{4^{3}}\right) = \underline{-4}$$

40.

$$\left[\left(-\frac{3}{4} \right)^2 \right]^{-2} = \left(-\frac{3}{4} \right)^{-4} = \left(-\frac{4}{3} \right)^4 = \frac{256}{81} = \underline{3.16049}$$

$$\left(\frac{2}{3}\right)^3 \left(-\frac{3}{2}\right)^2 \left(-\frac{3}{2}\right)^{-3} = \left(\frac{2}{3}\right)^3 \left(\frac{3}{2}\right)^2 \left(-\frac{2}{3}\right)^3 = \frac{2}{3} \left(-\frac{2}{3}\right)^3 = -\frac{16}{81} = \underline{-0.197531}$$

$$\frac{1.03^{16} - 1}{0.03} = \underbrace{\frac{20.1569}{0.03}}$$

$$\frac{1 - 1.0225^{-20}}{0.0225} = \frac{0.3591835}{0.0225} = \frac{15.9637}{0.0225}$$

45.
$$(1+0.055)^{1/6} - 1 = 0.00896339$$

Exercise 2.3

Basic Problems

1.
$$10a + 10 = 12 + 9a$$

 $10a - 9a = 12 - 10$
 $a = \underline{2}$

Exercise 2.3 (continued)

2.
$$29 - 4y = 2y - 7$$

 $36 = 6y$
 $y = \underline{6}$

3.
$$0.5 (x-3) = 20$$

 $x-3 = 40$
 $x = \underline{43}$

4.
$$\frac{1}{3}(x-2)=4$$

 $x-2=12$
 $x=14$

5.
$$y = 192 + 0.04y$$

 $y - 0.04y = 192$
 $y = \frac{192}{0.96} = \underline{200}$

6.
$$x - 0.025x = 341.25$$

 $0.975x = 341.25$
 $x = \frac{341.25}{0.975} = \underline{350}$

7.
$$12x - 4(2x - 1) = 6(x + 1) - 3$$

 $12x - 8x + 4 = 6x + 6 - 3$
 $-2x = -1$
 $x = 0.5$

8.
$$3y-4 = 3(y+6) - 2(y+3)$$

= $3y + 18 - 2y - 6$
 $2y = 16$
 $y = 8$

9.
$$8 - 0.5(x + 3) = 0.25(x - 1)$$

 $8 - 0.5x - 1.5 = 0.25x - 0.25$

$$-0.75x = -6.75$$

$$x = \underline{9}$$
10.
$$5(2 - c) = 10(2c - 4) - 6(3c + 1)$$

$$10 - 5c = 20c - 40 - 18c - 6$$

$$-7c = -56$$

$$c = 8$$

Intermediate Problems

11. x - y = 2 ① 3x + 4y = 20 ②

①
$$\stackrel{\cdot}{\leftarrow}$$
 3: $3x - 3y = \frac{6}{12}$
Subtract: $7y = 14$
 $y = 2$

Substitute into equation ①:

$$x-2 = 2$$

 $x = 4$
 $(x, y) = (4, 2)$

Check: LHS of @ = 3(4) + 4(2) = 20 = RHS of @

Substitute into equation ①:

$$y - 3(-2) = 11$$

 $y = 11 - 6 = 5$
 $(x, y) = (-2, 5)$

Check: LHS of @ = -4(5) + 5(-2) = -30 = RHS of @

13.
$$7p - 3q = 23$$
 ① $\frac{-2p - 3q}{9p} = \frac{5}{18}$ ② Subtract: $9p = 2$

Substitute into equation ①:

$$7(2) - 3q = 23$$

 $3q = -23 + 14$
 $q = -3$
 $(p, q) = (2, -3)$

Check: LHS of ② = -2(2) - 3(-3) = 5 = RHS of ②

14.
$$y = 2x$$
 ①
$$\frac{7x - y}{7x} = \frac{35}{2x + 35}$$
 ②
$$5x = 35$$

$$x = 7$$

Substitute into ①:

$$y = 2(7) = 14$$

(x, y) = $(7, 14)$

Check: LHS of @ = 7(7) - 14 = 49 - 14 = 35 = RHS of <math>@

15.
$$-3c + d = -500$$
 ① $0.7c + 0.2d = 550$ ②

To eliminate d,

①
$$\stackrel{\checkmark}{\bullet}$$
 0.2: $-0.6c + 0.2d = -100$

②:
$$0.7c + 0.2d = 550$$

 $-1.3c + 0 = -650$

Subtract:
$$-1.3c + 0 = -650$$

 $c = 500$

Substitute into ①:
$$d = 3(500) - 500 = 1000$$

$$(c, d) = (500, 1000)$$

Check: LHS of
$$② = 0.7(500) + 0.2(1000) = 550 = RHS$$
 of $②$

To eliminate y,

①
$$\stackrel{\checkmark}{\sim}$$
 0.7: 0.021x + 0.035y = 35.7

②
$$\stackrel{?}{\sim}$$
 0.05: $0.04x - 0.035y = 7$
Add: $0.061x + 0 = 42.7$
 $x = 700$

Substitute into ②:

$$0.8(700) - 0.7y = 140$$

$$-0.7y = -420$$

$$y = 600$$

$$(x, y) = (700, 600)$$

Check: LHS of \bigcirc = 0.03(700) + 0.05(600) = 51 = RHS of \bigcirc

17.
$$2v + 6w = 1$$
 ① $10v - 9w = 18$ ②

To eliminate v,

②
$$\stackrel{?}{\checkmark}$$
 2: $20v - 18w = 36$
act: $0 + 78w = -26$

Subtract:

$$\mathbf{w} = -\frac{1}{3}$$

Substitute into ①:

$$2v + 6^{\left(-\frac{1}{3}\right)} = 1$$

$$2v = 1 + 2$$

$$v = \frac{3}{2}$$

$$(v, w) = \left(\frac{3}{2}, -\frac{1}{3}\right)$$

Check: LHS of ② =
$$10(\frac{3}{2}) - 9(-\frac{1}{3})$$
 = 18 = RHS of ②

Advanced Problems

21.
$$\frac{x}{1.1^{2}} + 2x(1.1)^{3} = \$1000$$

$$0.8264463x + 2.622x = \$1000$$

$$3.488446x = \$1000$$

$$x = \underline{\$286.66}$$

22.
$$\frac{3x}{1.025^{6}} + x(1.025)^{8} = \$2641.35$$

$$2.586891x + 1.218403x = \$2641.35$$

$$x = \frac{\$694.13}{694.13}$$
23.
$$\frac{2x}{1.03^{7}} + x + x(1.03^{10}) = \$1000 + \frac{\$2000}{1.03^{4}}$$

$$1.626183x + x + 1.343916x = \$1000 + \$1776.974$$

$$3.970099x = \$2776.974$$

$$x = \frac{\$699.47}{1.05^{7}}$$
24.
$$x(1.05)^{3} + \$1000 + \frac{x}{1.05^{7}} = \frac{\$5000}{1.05^{2}}$$

$$1.157625x + 0.7106813x = \$4535.147 - \$1000$$

$$x = \frac{\$1892.17}{1.05}$$
25.
$$x = \frac{\$1892.17}{1.021863x + 1.945318x = \$1160.20}$$

$$2.967181x = \$1160.20$$

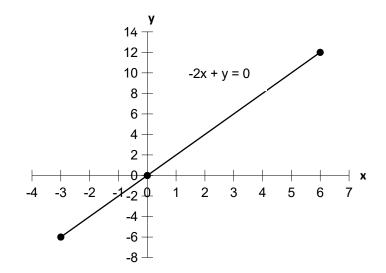
x = \$391.01

Exercise 2.4

Basic Problems

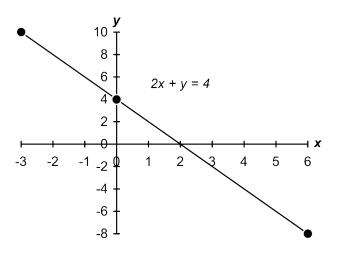
1.

X:	3	0	6
у:	-6	0	12



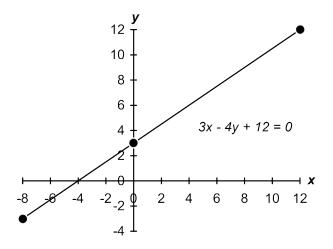
2.

X:	-3	0	6
y:	10	4	-8



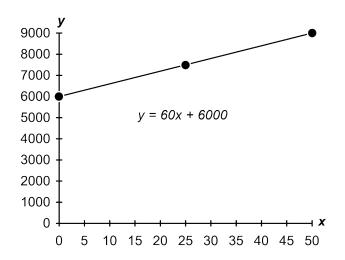
3.

	_	_	
X:	-8	0	12
у:	-3	3	12



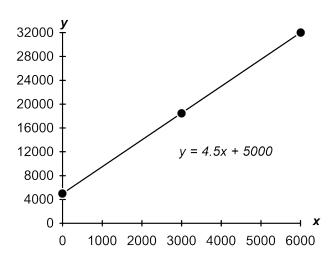
4.

X:	0	25	50
у:	6000	7500	9000



5.

X:	0	3000	6000
у:	5000	18,500	32,000



6. In each part, rearrange the equation to render it in the form y = mx + b

a.
$$2x = 3y + 4$$

$$3y = 2x - 4$$

$$y = \frac{2}{3} x - \frac{4}{3}$$

The slope is $m = \frac{2}{3}$ and the <u>y-intercept is</u> $b = \frac{4}{3}$

b.
$$8 - 3x = 2y$$

$$2y = -3x + 8$$

$$y = -\frac{3}{2}x + 4$$

The slope is $m = -\frac{3}{2}$ and the <u>y-intercept is</u> $b = \underline{4}$

6.
$$c$$
. $8x - 2y - 3 = 0$

$$-2y = -8x + 3$$

$$y = 4x - \frac{3}{2}$$

The <u>slope is</u> $m = \underline{4}$ and the <u>y-intercept is</u> $b = \frac{-\frac{3}{2}}{2}$.

$$d. \qquad 6x = 9y$$

$$y = \frac{6}{9} x = \frac{2}{3} x$$

$$y = \frac{2}{3} x$$

The slope is $m = \frac{2}{3}$ and the <u>y-intercept is</u> b = 0.

Intermediate Problems

7. The plumber charges a \$100 service charge plus 4(\$20) = \$80 per hour

Then
$$C = $100 + $80H$$

Expressing this equation in the form y = mx + b

$$C = $80H + $100$$

On a plot of C vs. H, slope = \$80 and C-intercept = \$100.

8. Ehud earns \$1500 per month plus 5% of sales. Then gross earnings

$$E = $1500 + 0.05R$$

Expressing this equation in the form y = mx + b

$$E = 0.05R + $1500$$

On a plot of E vs. R, $\underline{\text{slope} = 0.05}$ and $\underline{\text{E-intercept} = \$1500}$.

9. a. Comparing the equation $F = \frac{9}{5}C + 32$ to y = mx + b,

we can conclude that a plot of F vs. C will have

$$\underline{\text{slope}} = \frac{9}{5}$$
 and $\underline{F\text{-intercept}} = 32$.

Slope =
$$\frac{\text{Change in F}}{\text{Change in C}}$$

Therefore, (Change in F) = Slope(Change in C) = $\frac{9}{5}$ (10 Celsius) = 18 Fahrenheit

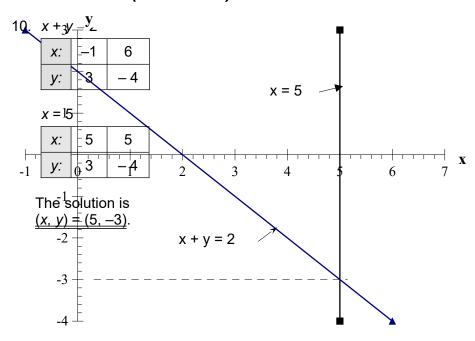
c.
$$F = \frac{9}{5}C + 32$$

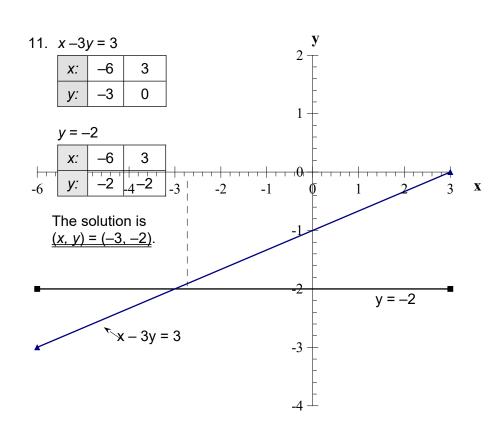
$$\frac{9}{5}$$
 C = F - 32

$$C = \frac{5}{9} F - \frac{5}{9} (32) = \frac{5}{9} F - \frac{1779}{9}$$

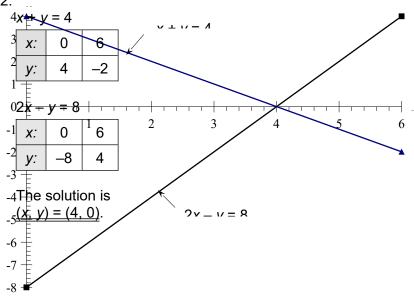
On a plot of *C* vs. *F*, slope = $\frac{5}{9}$ and C-intercept = $\frac{-17\frac{7}{9}}{9}$.

Exercise 2.4 (continued)

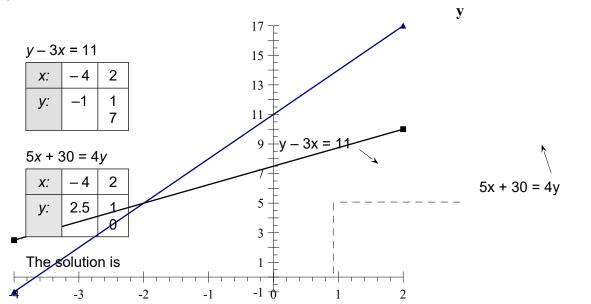








13.



X

Advanced Problem

14. a. Given: TR = \$6X

On a plot of TR vs. X, slope = \$6 and TR-intercept = \$0.

b. TC = \$2X + \$80,000

On a plot of TC vs. X, slope = \$2 and TC-intercept = \$80,000.

c. NI = \$4X - \$80,000

On a plot of NI vs. X, slope = \$4 and NI-intercept = -\$80,000.

d. The steepest line is the one with the largest slope.

Therefore, the *TR* line is steepest.

- e. The increase in NI per pair of sunglasses sold is the "change in NI" divided by the "change in X". This is just the slope of the NI vs. X line. Therefore, NI increases by $\underline{\$4}$ for each pair of sunglasses sold.
- f. The coefficient of X in the TR equation is the unit selling price, which is unchanged.

Therefore, the slope remains unchanged.

The coefficient of *X* in the *TC* equation is the unit cost.

Therefore, the slope decreases (from \$2 to \$1.75).

The coefficient of X in the NI equation equals

(Unit selling price) – (Unit cost)

Therefore, the slope increases (from \$4 to \$4.25).

Exercise 2.5

Basic Problems

1. Step 2: Hits last month = 2655 after the $\frac{2}{7}$ increase. Let the number of hits 1 year ago be n.

Step 3: Hits last month = Hits 1 year ago +
$$\frac{2}{7}$$
 (Hits 1 year ago)

Step 4: 2655 = n +
$$\frac{2}{7}$$
 n

Step 5: 2655 =
$$\frac{9}{7}$$
 n

Multiply both sides by $\frac{7}{9}$.

$$n = 2655 \times \frac{7}{9} = 2065$$

The Web site had 2065 hits in the same month 1 year ago.

2. Step 2: Retail price = \$712; Markup = 60% of wholesale of cost.

Let the wholesale cost be C.

Step 4:
$$$712 = C + 0.6C$$

$$\frac{$712}{1.6} = $445.00$$

C =

The wholesale cost is \$445.00.

- 3. Step 2: Tag price = \$39.95 (including 13% HST). Let the plant's pretax price be P.
 - Step 3: Tag price = Pretax price + HST

Step 4:
$$$39.95 = P + 0.13P$$

The amount of HST is \$39.95 - \$35.35 = \$4.60

- 4. Step 2: Commission rate = 2.5% on the first \$5000 and 1.5% on the remainder Commission amount = \$227. Let the transaction amount be x.
 - Step 3: Commission amount = 0.025(\$5000) + 0.015(Remainder)

Step 4:
$$$227 = $125.00 + 0.015(x - $5000)$$

Step 5:
$$$102 = 0.015x - $75.00$$

$$$102 + $75 = 0.015x$$

$$x = \overline{0.015} = $11,800.00$$

The amount of the transaction was \$11.800.00.

5. Step 2: Let the basic price be P. First 20 meals at P.

Next 20 meals at P - \$2. Additional meals at P - \$3.

Step 4:
$$20P + 20(P - \$2) + (73 - 40)(P - \$3) = \$1686$$

Step 5:
$$20P + 20P - $40 + 33P - $99 = $1686$$

$$73P = $1686 + $99 + $40$$

$$P = 73 = $25.00$$

The basic price per meal is \$25.00.

6. Step 2: Rental Plan 1: \$295 per week + \$0.15 × (Distance in excess of 1000 km)

Let *d* represent the distance at which the costs of both plans are equal.

Step 4:
$$$295 + $0.15(d - 1000) = $389$$

Step 5:
$$$295 + $0.15d - $150 = $389$$

$$$0.15d = $244$$

$$d = 1627 \text{ km}$$

To the nearest kilometre, the unlimited driving plan will be cheaper if you drive more than 1627 km in the one-week interval.

7. Step 2: Tax rate = 38%; Overtime hourly rate = 1.5(\$23.50) = \$35.25

Cost of canoe = \$2750

Let *h* represent the hours of overtime Alicia must work.

Step 3: Gross overtime earnings – Income tax = Cost of the canoe

Step 4: \$35.25h - 0.38(\$35.25h) = \$2750

Step 5: \$21.855*h* = \$2750

h = 125.83 hours

Alicia must work 125\% hours of overtime to earn enough money to buy the canoe.

8. Let x represent the number of units of product X and v represent the number of units of product Y. Then

> v = 93(1)

0.5x + 0.75y = 60.5

① 4 0.5: 0.5x + 0.5v = 46.5

0 + 0.25v = 14Subtract:

v = 56

Substitute into ①: x + 56 = 93

x = 37

Therefore, <u>37 units of X</u> and <u>56 units of Y</u> were produced last week.

9. Let the price per litre of milk be m and the price per dozen eggs be e. Then

(1) 5m + 4e = \$19.51

9m + 3e = \$22.98(2)

To eliminate e.

15m + 12e = \$58.53 ①**¿** 3:

②**¿**4: 36m + 12e = \$91.92

-21 m + 0 = -\$33.39Subtract:

m = \$1.59

Substitute into ①: 5(\$1.59) + 4e = \$19.51

e = \$2.89

Milk costs \$1.59 per litre and eggs cost \$2.89 per dozen.

10. Let M be the number of litres of milk and J be the number of cans of orange juice per week.

> \$1.50M + \$1.30J = \$57.00(1)

> \$1.60M + \$1.39J = \$60.85(2)

To eliminate M.

①**4** 1.60: 2.40M + 2.08J = 91.20

② 4.50: 2.40M + 2.085J = 91.2750 - 0.005J = -\$0.075

Subtract: J = 15

Substitution of J = 15 into either equation will give M = 25. Hence 25 litres of milk and 15 cans of orange juice are purchased each week.

Intermediate Problems

11. Step 2: Number of two-bedroom homes = 0.4(Number of three-bedroom homes)

Number of two-bedroom homes = 2(Number of four-bedroom homes)

Total number of homes = 96

Let *h* represent the number of two-bedroom homes

Step 3: # 2-bedroom homes + # 3-bedroom homes + # 4-bedroom homes = 96

Step 4:
$$h + \frac{h}{0.4} + \frac{h}{2} = 96$$

Step 5: $h + 2.5h + 0.5h = 96$
 $4h = 96$
 $h = 24$

There should be $\underline{24 \text{ two-bedroom homes}}$, $2.5(24) = \underline{60 \text{ three-bedroom homes}}$, and $0.5(24) = \underline{12 \text{ four-bedroom homes}}$.

12. Step 2: Cost of radio advertising = 0.5(Cost of internet advertising)

Cost of TV advertising = 0.6(Cost of radio advertising)

Total advertising budget = \$160,000

Let r represent the amount allocated to radio advertising

Step 3: Radio advertising + TV advertising + Internet advertising = \$160,000

Step 4:
$$r+0.6r+\frac{r}{0.5}=\$160,000$$

Step 5: $3.6r=\$160,000$
 $r=\$44,444.44$

The advertising budget allocations should be:

\$44,444 to radio advertising,

0.6(\$44,444.44) = \$26,667 to TV advertising, and 2(\$44,444.44) = \$88,889 to internet advertising.

13. Step 2: By-laws require: 5 parking spaces per 100 square meters,

4% of spaces for customers with physical disabilities

In remaining 96%, # regular spaces = 1.4(# small car spaces)

Total area = 27,500 square meters

Let *s* represent the number of small car spaces.

Step 3: Total # spaces = # spaces for customers with physical disabilities + # regular spaces + # small spaces

Step 4:
$$\frac{27,500}{100} \times 5 = 0.04 \times \frac{27,500}{100} \times 5 + s + 1.4s$$

Step 5: $1375 = 55 + 2.4s$
 $s = 550$

The shopping centre must have <u>55 parking spaces for customers with physical disabilities</u>, <u>550 small-car spaces</u>, and <u>770 regular parking spaces</u>.

14. Step 2: Overall portfolio's rate of return = 1.1%, equity fund's rate of return = -3.3%, bond fund's rate of return = 7.7%.

Let *e* represent the fraction of the portfolio initially invested in the equity fund.

Step 3: Overall rate of return = Weighted average rate of return

= (Equity fraction)(Equity return) + (Bond fraction)(Bond return)

Step 4: 1.1% = e(-3.3%) + (1 - e)(7.7%)

Step 5: 1.1 = -3.3e + 7.7 - 7.7e

-6.6 = -11.0ee = 0.600

Therefore, <u>60.0%</u> of Erin's original portfolio was invested in the equity fund.

15. Step 2: Pile A steel is 5.25% nickel; pile B steel is 2.84% nickel.

We want a 32.5-tonne mixture from A and B averaging 4.15% nickel.

Let A represent the tonnes of steel required from pile A.

Step 3: Wt. of nickel in 32.5 tonnes of mixture

= Wt. of nickel in steel from pile A + Wt. of nickel in steel from pile B

= (% nickel in pile A)(Amount from A) + (% nickel in pile B)(Amount from B)

Step 4: 0.0415(32.5) = 0.0525A + 0.0284(32.5 - A)

Step 5: 1.34875 = 0.0525A + 0.9230 - 0.0284A

0.42575 = 0.0241*A*

A = 17.67 tonnes

The recycling company should mix 17.67 tonnes from pile A with 14.83 tonnes from pile B.

16. Step 2: Total options = 100,000

of options to an executive = 2000 + # of options to an engineer

of options to an engineer = 1.5(# of options to a technician)

There are 3 executives, 8 engineers, and 14 technicians.

Let *t* represent the number of options to each technician.

Step 3: Total options = Total options to engineers

+ Total options to technicians + Total options to executives

Step 4: 100,000 = 8(1.5t) + 14t + 3(2000 + 1.5t)

Step 5: = 12t + 14t + 6000 + 4.5t

94,000 = 30.5t

t = 3082 options

Each technician will receive 3082 options,

each engineer will receive $1.\overline{5(3082)} = 4623$ options,

and each executive will receive $2000 + \overline{4623} = 6623$ options.

17. Step 2: Plan A: 20 cents/minute for local calls and 40 cents/minute for long distance calls Plan B: 35 cents/minute any time

Let *d* represent the fraction of long-distance usage at which costs are equal.

- Step 3: Cost of Plan A = Cost of plan B
- Step 4: Pick any amount of usage in a month—say 1000 minutes.

d(1000)\$0.40 + (1 - d)(1000)\$0.20 = 1000(\$0.35)

Step 5: 400d + 200 - 200d = 350200d = 150

d = 0.75

If long distance usage exceeds <u>75%</u> of overall usage, plan B will be cheaper.

18. Step 2: Raisins cost \$3.75 per kg; peanuts cost \$2.89 per kg.

Cost per kg of ingredients in 50 kg of "trail mix" is to be \$3.20.

Let *p* represent the weight of peanuts in the mixture.

Step 3: Cost of 50 kg of trail mix = Cost of p kg peanuts + Cost of (50 - p) kg of raisins

Step 4: 50(\$3.20) = p(\$2.89) + (50 - p)(\$3.75)

Step 5: \$160.00 = \$2.89p + \$187.50 - \$3.75p -\$27.50 = -\$0.86pp = 31.98 kg

32.0 kg of peanuts should be mixed with 18.0 kg of raisins.

19. Step 2: Total bill = \$3310. Total hours = 41.

Hourly rate = \$120 for CGA

= \$50 for clerk.

Let *x* represent the CGA's hours.

Step 3: Total bill = (CGA hours x CGA rate) + (Clerk hours x Clerk rate)

Step 4: \$3310 = x(\$120) + (41 - x)\$50

Step 5: \$3310 = \$120x + \$2050 - \$50x

1260 = 70xx = 18

The <u>CGA worked 18 hours</u> and the <u>clerk worked 41 – 18 = $\underline{23}$ hours</u>.

20. Step 2: Total investment = \$32,760

Sue's investment = 1.2(Joan's investment)

Joan's investment = 1.2(Stella's investment)

Let L represent Stella's investment.

Step 3: Sue's investment + Joan's investment + Stella's investment = Total investment

Step 4: Joan's investment = 1.2L

Sue's investment = 1.2(1.2L) = 1.44L

1.44L + 1.2L + L = \$32,760

Step 5: 3.64L = \$32,760

$$\frac{\$32,760}{3.64} = \$9000$$

(continued)

Stella will contribute \$9000, Joan will contribute 1.2(\$9000) = \$10,800, and Sue will contribute 1.2(\$10,800) = \$12,960

21. Step 2: Sven receives 30% less than George (or 70% of George's share).

Robert receives 25% more than George (or 1.25 times George's share).

Net income = \$88,880

Let G represent George's share.

Step 3: George's share + Robert's share + Sven's share = Net income

G = \$30,128.81

George's share is \$30,128.81, Robert's share is 1.25(\$30,128.81) = \$37,661.02, and Sven's share is 0.7(\$30,128.81) = \$21,090.17.

22. Step 2: Time to make X is 20 minutes.

Time to make Y is 30 minutes.

Total time is 47 hours. Total units = 120. Let Y represent the number of units of Y.

Step 3: Total time = (Number of X)
$$\times$$
 (Time for X) + (Number of Y) \times (Time for Y)

Step 4:
$$47 \times 60 = (120 - Y)20 + Y(30)$$

Step 5:
$$2820 = 2400 - 20Y + 30Y$$

$$420 = 10Y$$

Forty-two units of product Y were manufactured.

23. Step 2: Price of blue ticket = \$19.00. Price of red ticket = \$25.50.

Total tickets = 4460. Total revenue = \$93.450.

Let the number of tickets in the red section be R.

Step 3: Total revenue = (Number of red × Price of red) + (Number of blue × Price of blue)

Step 4:
$$$93,450 = R($25.50) + (4460 - R)$19.00$$

$$6.5R = 8710$$

$$R = 1340$$

1340 seats were sold in the red section and 4460 – 1340 = 3120 seats were sold in the blue section.

24. Step 2: Regal owns a 58% interest in a mineral claim. Yukon owns the remainder (42%). Regal sells one fifth of its interest for \$1.2 million.

Let the V represent the implied value of the entire mineral claim.

- Step 3: \(\sqrt{5} \) (or 20\%) of a 58\% interest is worth \$1.2 million
- Step 4: 0.20(0.58)V = \$1,200,000

Step 5: $V = 0.20 \times 0.58 = $10,344,828$

The implied value of Yukon's interest is

$$0.42V = 0.42 \times \$10,344,828 = \$4,344,828$$

Step 2: ⁵/₇ of entrants complete Level 1. ²/₉ of Level 1 completers fail Level 2.

587 students completed Level 2 last year.

Let the N represent the original number who began Level 1.

Step 3: $\frac{7}{9}$ of $\frac{5}{7}$ of entrants will complete Level 2.

Step 4:
$$\frac{\frac{7}{9} \times \frac{5}{7}}{\frac{5}{7}}$$
 N = 587
9×7

Step 5: N = $\frac{7 \times 5}{7 \times 5}$ x 587 = 1056.6

1057 students began Level 1.

26. Step 2: ⁴/₇ of inventory was sold at cost.

7 inventory was sold to liquidators at 45% of cost, yielding \$6700.

Let C represent the original cost of the entire inventory.

Step 3: 7 of inventory was sold to liquidators at 45% of cost, yielding \$6700.

Step 4:
$$\frac{3}{7}$$
 (0.45C) = \$6700 $\frac{7 \times $6700}{}$

Step 5: $C = 3 \times 0.45 = $34,740.74$

a. The cost of inventory sold to liquidators was

$$\frac{3}{7}$$
 (\$34,740.74) = $\frac{$14,888.89}{}$

b. The cost of the remaining inventory sold in the bankruptcy sale was

\$34,740.74 - \$14,888.89 = \$19.851.85

27. Let r represent the number of regular members and s the number of student members.

Then
$$r + s = 583$$

Total revenue: \$2140r + \$856s = \$942,028

⊕ \$856: \$856r + \$856s = \$499,048\$1284r + 0 = \$442.980Subtract:

r = 345

Substitute into ①: 345 + s = 583

s = 238

The club had 238 student members and 345 regular members.

28. Let a represent the adult airfare and c represent the child airfare.

Mrs. Ramsev's cost: 2c = \$610a +Chudnowskis' cost:

3c = \$10502a +

2a + 4c = \$1220①**¿** 2: Subtract: -c = -\$170

Substitute c = \$170 into ①:a + 2(\$170) = \$610

a = \$610 - \$340 = \$270

2

48

The airfare is \$270 per adult and \$170 per child.

29. Let h represent the rate per hour and k represent the rate per km.

Vratislav's cost: 2h + 47k = \$54.45 ① Bryn's cost: 5h + 93k = \$127.55 ②

To eliminate h,

① 5: 10h + 235k = \$272.25 ① ② 2: 10h + 186k = \$255.10 ② Subtract: 0 + 49k = \$17.15 k = \$0.35 per km

Substitute into ①:

$$2h + 47(\$0.35) = \$54.45$$

 $2h = \$54.45 - \16.45
 $h = \$19.00$ per hour

Budget Truck Rentals charged \$19.00 per hour plus \$0.35 per km.

Advanced Problems

30. Step 2: Each of 4 children receive 0.5(Wife's share).

Each of 13 grandchildren receive $0.\overline{3}$ (Child's share). Total distribution = \$759,000. Let w represent the wife's share.

Step 3: Total amount = Wife's share + 4(Child's share) + 13(Grandchild's share)

Step 4: \$759,000 = w + 4(0.5w) +
$$13^{(0.\overline{3})}$$
(0.5w)

Step 5: \$759,000 = w + 2w +
$$2.1\overline{6}w$$

= $5.1\overline{6}w$
w = \$146,903.226

Each child will receive 0.5(\$146,903.226) = \$73,451.61

and each grandchild will receive $0.\overline{3}$ (\$73,451.61) = \$24,483.87.

31. Step 2: Stage B workers = 1.6(Stage A workers)

Stage C workers = 0.75(Stage B workers)

Total workers = 114. Let A represent the number of Stage A workers.

Step 3: Total workers = A workers + B workers + C workers

Step 4:
$$114 = A + 1.6A + 0.75(1.6A)$$

$$A = 30$$

 $\underline{30}$ workers should be allocated $\underline{\text{to Stage A}}$, 1.6(30) = $\underline{48}$ workers $\underline{\text{to Stage B}}$, and 114 – 30 – 48 = $\underline{36}$ workers to $\underline{\text{Stage C}}$.

32. Step 2: Hillside charge = 2(Barnett charge) – \$1000

Westside charge = Hillside charge + \$2000

Total charges = \$27,600. Let B represent the Barnett charge.

Step 3: Total charges = Barnett charge + Hillside charge + Westside charge

Step 4:
$$$27,600 = B + 2B - $1000 + 2B - $1000 + $2000$$

Step 5:
$$$27,600 = 5B$$

$$B = $5520$$

Exercise 2.6

Basic Problems

$$c = \frac{V_f - V_i}{V_i} \times 100 = \frac{\$100 - \$95}{\$95} \times 100 = \frac{5.26\%}{5.26\%}$$

1.

$$c = \frac{V_f - V_i}{V_i} \times 100 = \frac{35 \text{ kg} - 135 \text{ kg}}{135 \text{ kg}} \times 100 = \frac{-74.07\%}{135 \text{ kg}}$$

2

$$c = \frac{V_f - V_i}{V_i} \times 100 = \frac{0.13 - 0.11}{0.11} \times 100 = \frac{18.18\%}{0.11}$$

3

$$c = \frac{V_f - V_i}{V_i} \times 100 = \frac{0.085 - 0.095}{0.095} \times 100 = \underline{-10.53\%}$$

4

5.
$$V_f = V_i$$
 $(1+c) = $134.39[1+(-0.12)]$ $= $134.39(0.88) = 118.26

6.
$$V_f = V_i$$
 $(1+c) = 112g(1+1.12) = \underline{237.44g}$

$$V_i = \frac{V_f}{1+c} = \frac{\$75}{1+2.00} = \frac{\$25.00}{1}$$

$$V_i = \frac{V_f}{1+c} = \frac{\$75}{1+(-0.50)} = \frac{\$150.00}{1}$$

8.

9. Given:
$$V_i$$
 = \$90, V_f = \$100
$$c = \frac{\$100 - \$90}{\$90} \times 100 = \underline{11.11\%}$$

\$100 is 11.11% more than \$90.

10. Given: V_i = \$110, V_f = \$100

$$c = \frac{V_f - V_i}{V_i} \times 100 = \frac{\$100 - \$110}{\$110} \times 100 = \frac{-9.09\%}{100}$$

\$100 is 9.09% less than \$110.

11. Given: c = 25%, $V_f = 100

$$V_i = \frac{V_f}{1+c} = \frac{\$100}{1+0.25} = \frac{\$80.00}{1}$$

\$80.00 increased by 25% equals \$100.00.

12. Given: V_f = \$75, c = 75%

$$V_i = \frac{V_f}{1+c} = \frac{\$75}{1+0.75} = \frac{\$42.86}{1}$$

\$75 is 75% more than \$42.86.

13. Given: V_i = \$759.00, V_f = \$754.30

$$c = \frac{V_f - V_i}{V_i} \times 100 = \frac{\$754.30 - \$759.00}{\$759.00} \times 100 = \underline{-0.62\%}$$

\$754.30 is 0.62% less than \$759.00.

Exercise 2.6 (continued)

14. Given: V_i = \$75, c = 75% $V_f = V_i(1 + c) = $75(1 + 0.75) = 131.25

\$75.00 becomes \$131.25 after an increase of 75%.

15. Given: $V_f = $100, c = -10\%$

$$V_i = \frac{V_f}{1+c} = \frac{\$100}{1+(-0.10)} = \frac{\$111.11}{1}$$

\$100.00 is 10% less than \$111.11

16. Given: V_f = \$100, c = -20%

$$V_i = \frac{V_f}{1+c} = \frac{\$100}{1+(-0.20)} = \frac{\$125.00}{1}$$

\$125 after a reduction of 20% equals \$100.

17. Given: V_i = \$900, c = -90%

$$V_f = V_i(1 + c) = $900[1 + (-0.9)] = $90.00$$

\$900 after a decrease of 90% is \$90.00.

18. Given: c = 0.75%, $V_i = $10,000$

$$V_f = V_i (1 + c) = $10,000(1 + 0.0075) = $10,075.00$$

\$10,000 after an increase of $\frac{3}{4}$ % is \$10,075.00.

19. Given: c = 210%, $V_f = 465

$$V_i = \frac{V_f}{1+c} = \frac{\$465}{1+2.1} = \frac{\$150.00}{1}$$

\$150.00 after being increased by 210% equals \$465.

Intermediate Problems

20. Let the retail price be p. Then

$$p + 0.13 p = $281.37$$

 $\frac{$281.37}{p = 1.13} = \underline{$249.00}$
The jacket's retail price was \$249.00.

21. Let the number of students enrolled in September, 2014 be s. Then

$$s + 0.0526 s = 1200$$

$$1.0526 s = 1200$$

$$\frac{1200}{s = 1.0526} \approx \frac{1140}{s}$$

Rounded to the nearest person, the number of students enrolled in September, 2014 was 1140.

22. Let next year's sales be n. Then

$$n = \$18,400(1+0.12)$$

$$n = \$20,608$$

Nykita is expecting next year's sales to be \$20,608.

Exercise 2.6 (continued)

23. Given: V_i = \$285,000, V_f = \$334,000

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$334,000 - \$285,000}{\$285,000} \times 100\%$$

The value of Amir's real estate investment grew by 17.19%.

24. Let Jamal's earnings this year be e. Then

$$e = $87,650(1 - 0.065)$$

$$e = $81,952.75$$

Rounded to the nearest dollar, Jamal's earnings this year were \$81,953.

25. Let the population figure on July 1, 2011 be p. Then

$$p + 0.0439p = 35,851,800$$

$$p = \frac{35,851,800}{1.0439} \approx 34,344,094$$

Rounded to the nearest 1000, the population on July 1, 2011 was 34,344,000.

26. *a.* Given: $V_i = 32,400$, $V_f = 27,450$

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{27,450 - 32,400}{32,400} \times 100\% = \frac{-15.28\%}{200\%}$$

The number of hammers sold declined by 15.28%.

b. Given: $V_i = 15.10 , $V_f = 15.50

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$15.50 - \$15.10}{\$15.10} \times 100\% = \frac{2.65\%}{15.10}$$

The average selling price increased by 2.65%.

c. Year 1 revenue = 32,400(\$15.10) = \$489,240

Year 2 revenue = 27,450(\$15.50) = \$425,475

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$425,475 - \$489,240}{\$489,240} \times 100\% = \frac{-13.03\%}{100\%}$$

The revenue decreased by 13.03%.

27. *a.* Given: $V_i = \$0.55$, $V_f = \$1.55$

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$1.55 - \$0.55}{\$0.55} \times 100\% = \frac{181.82\%}{\$0.55}$$

The share price rose by 181.82% in the first year.

b. Given: $V_i = \$1.55$, $V_f = \$0.75$

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$0.75 - \$1.55}{\$1.55} \times 100\% = \frac{-51.61\%}{\$1.55}$$

The share price declined by 51.61% in the second year.

c. Given: $V_i = \$0.55$, $V_f = \$0.75$

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$0.75 - \$0.55}{\$0.55} \times 100\% = \underline{36.36\%}$$

The share price rose by 36.36% over 2 years.

28. Initial unit price = $\frac{\$5.49}{1.65 \text{ l}}$ = \$3.327 per litre

Final unit price = 2.2 l = \$3.627 per litre

The percent increase in the unit price is

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$3.627 - \$3.327}{\$3.327} \times 100\% = \frac{9.02\%}{\$3.327}$$

29. Initial unit price = $\frac{700 \text{ g}}{}$ = 1.5686 cents per g

Final unit price = 600 g = 1.6633 cents per q

The percent increase in unit price is

$$c = \frac{V_f - V_i}{V_i} \times 100 = \frac{1.6633 - 1.5686}{1.5686} \times 100 = \underline{6.04\%}$$

30. Given: V_f = \$348,535, c = -1.8%

$$V_i = \frac{V_f}{1+c} = \frac{\$348,535}{0.982} \approx \frac{\$354,900}{0.982}$$

Rounded to the nearest \$100, the average price one month ago was \$354,900.

31. Given: V_f = \$348.60, c = -0.30

$$V_i = \frac{V_f}{1+c} = \frac{\$348.60}{1+(-0.30)} = \frac{\$348.60}{0.70} = \frac{\$498.00}{0.60}$$

The regular price of the boots is \$498.00

32. Given: V_f = 231,200,000, c = 3.66%

$$V_i = \frac{V_f}{1+c} = \frac{231,200,000}{1+0.0366} = \frac{231,200,000}{1.0366} \approx \frac{223,037,000}{1.0366}$$

Rounded to the nearest 1000 units, Apple sold 223,037,000 iPhones in 2014.

Given: V_f = \$2,030,000,000, c = 241% 33.

Solution:
$$V_f = \$2,030,000,000, c = 241\%$$

$$c = \frac{V_f - V_i}{V_i} \times 100$$

$$241 = \frac{\$2,030,000,000 - V_i}{V_i} \times 100$$

$$2.41 = \frac{\$2,030,000,000 - V_i}{V_i}$$

$$2.41V_i = \$2,030,000,000 - V_i$$

$$3.41V_i = $2,030,000,000$$

$$V_i = \frac{\$2,030,000,000}{3,41} \approx \$595,308,000$$

34. The fees to Fund A will be

$$\frac{(Fees \ to \ Fund \ A) - (Fees \ to \ Fund \ B)}{(Fees \ to \ Fund \ B)} \times 100\% = \frac{2.38\% - 1.65\%}{1.65\%} \times 100\% = \frac{44.24\%}{1.65\%}$$

more than the fees to Fund B.

35. Percent change in the GST rate

$$=\!\!\frac{(\textit{Final GST rate}) - (\textit{Initial GST rate})}{(\textit{Initial GST rate})} \times 100\,\% = \!\!\frac{5\,\% - 6\,\%}{6\,\%} \times 100\,\% = \!\!\frac{-16.67\,\%}{6\,\%}$$

The GST paid by consumers was reduced by 16.67%.

36. Given: $V_f = \$0.45$, c = 76%

$$V_i = \frac{V_f}{1+c} = \frac{\$0.45}{1+(-0.76)} = \$1.88$$

Price decline = $V_i - V_f$ = \$1.88 - \$0.45 = \$1.43

The share price dropped by \$1.43.

37. If the Canadian dollar is worth 1.5% less than the US dollar,

Canadian dollar =
$$(1 - 0.015)(US dollar) = 0.985(US dollar)$$

Hence, US dollar = 0.985 = 1.0152(Canadian dollar)

Therefore, the US dollar is worth 1.52% more than the Canadian dollar.

38. Current unit price = 100 g = 1.15 cents per g

New unit price = 1.075(1.15 cents per g) = 1.23625 cents per g

Price of an 80-g bar = $(80 \text{ g}) \times (1.23625 \text{ cents per g}) = 98.9 \text{ cents} = 0.99

39. Canada's exports to US exceeded imports from the US by 14.1%.

That is, Exports = 1.141(Imports)

Therefore, Imports = 1.141 = 0.8764(Exports)

That is, Canada's imports from US (= US exports to Canada) were

$$1 - 0.8764 = 0.1236 = 12.36\%$$

less than Canada's exports to US (= US imports from Canada.)

40. Given: 2014 sales revenues were 14.2% less than 2013 sales revenues

Hence, (Sales for 2014) =
$$(1 - 0.142)$$
(Sales for 2013) = 0.858(Sales for 2013)

Therefore, (Sales for 2013) = 0.858 = 1.1655(Sales for 2014)

That is, sales revenues for 2013 were 116.55% of sales revenues for 2014.

Advanced Problems

Given: For the appreciation, V_i = Purchase price, c = 140%, V_f = List price For the price reduction, V_i = List price, c = -10%, $V_f = $172,800$

List price =
$$\frac{V_f}{1+c} = \frac{\$172,800}{1+(-0.1)} = \$192,000$$

Original purchase price =
$$\frac{V_f}{1+c} = \frac{\$192,000}{1+1.4} = \$80,000$$

The owner originally paid \$80,000 for the property.

42. Given: For the markup, V_i = Cost, c = 22%, V_f = List price

For the markdown, V_i = List price, c = -10%, V_f = \$17.568

$$\frac{V_f}{1+c} = \frac{\$17,568}{1+(-0.10)} = \$19,520$$
 List price =

Cost (to dealer) =
$$\frac{V_f}{1+c} = \frac{\$19,520}{1+0.22} = \$ \frac{16,000}{1}$$

The dealer paid \$16,000 for the car.

43. Next year there must be 15% fewer students per teacher.

With the same number of students,

$$\frac{\text{Students}}{\text{Teachers next year}} = 0.85 \left(\frac{\text{Students}}{\text{Teachers now}} \right)$$

Teachers next year = 0.85 = 1.1765(Teachers now) Therefore.

That is, if the number of students does not change, the number of teachers must be increased by 17.65%.

44. Use ppm as the abbreviation for "pages per minute".

Given: Lightning printer prints 30% more ppm than the Reliable printer.

That is, the Lightning's printing speed is 1.30 times the Reliable's printing speed.

Therefore, the Reliable's printing speed is

$$\frac{1}{1.3}$$
 = 0.7692 = 76.92% of the Lightning's printing speed

Therefore, the Reliable's printing speed is

100% - 76.92% = 23.08% less than the Lighting's speed.

The Lightning printer will require 23.08% less time than the Reliable for a long printing job.

45. Given: Euro is worth 32% more than the Canadian dollar.

Canadian dollar = 1.32 = 0.7576(Euro) = 75.76% of a Euro. Therefore,

That is, the Canadian dollar is worth 100% - 75.76% = 24.24% less than the Euro.

46. Let us use OT as an abbreviation for "overtime".

The number of OT hours permitted by this year's budget is

OT hours (this year) = $\overline{OT \text{ hourly rate (this year)}}$

The number of overtime hours permitted by next year's budget is

OT hours (next year) =
$$\frac{OT \ budget \ (next \ year)}{OT \ hourly \ rate \ (next \ year)} = \frac{1.03[OT \ budget \ (this \ year)]}{1.05[OT \ hourly \ rate \ (this \ year)]}$$

OT budget (this year)

= 0.98095OT hourly rate (this year)

= 98.10% of this year's OT hours

The number of OT hours must be reduced by 100% - 98.10% = 1.90%.

Review Problems

Basic Problems

1.
$$a.$$
 $2(7x - 3y) - 3(2x - 3y) = 14x - 6y - 6x + 9y = $8x + 3y$$

b.
$$15x - (4 - 10x + 12) = 15x - 4 + 10x - 12 = 25x - 16$$

2. Given:
$$NI = $200,000$$
, $CM = 8 , $X = 40,000$

$$NI = (CM)X - FC$$

$$$200,000 = $8(40,000) - FC$$

$$$200,000 - $320,000 = -FC$$

$$-$120,000 = -FC$$

$$FC = $120,000$$

3. Given:
$$S = $1243.75, P = $1200, t = \frac{7}{12}$$

$$S = P(1 + rt)$$

$$$1243.75 = $1200 \left[1 + r \left(\frac{7}{12} \right) \right]$$

$$\frac{\$1243.75}{\$1200} = 1 + r\left(\frac{7}{12}\right)$$

$$1.0365 - 1 = r\left(\frac{7}{12}\right)$$

$$0.0365 = 0.58\overline{3} r$$

$$r = \frac{0.0365}{0.58\,\overline{3}}$$

$$r = 0.0626 \times 100\% = 6.26\%$$

4.
$$a. 3.1t + 145 = 10 + 7.6t$$

$$3.1t - 7.6t = 10 - 145$$

 $-4.5t = -135$

$$t = 30$$

b.
$$1.25y - 20.5 = 0.5y - 11.5$$

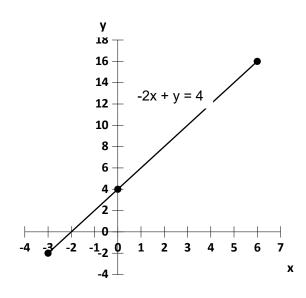
$$1.25\dot{y} - 0.5\dot{y} = -11.5 + 20.5$$

$$0.75y = 9$$

 $y = 12$

5.

x:	-3	0	6
<i>y</i> :	-2	4	16



6. In each part, rearrange the equation to render it in the form y = (slope)x + (intercept)

a.
$$2b + 3 = 5a$$

$$2b = 5a - 3$$

$$b = \frac{5}{2}a - \frac{3}{2}$$

The slope is $\frac{5}{2}$ and the <u>b-intercept is $\frac{-\frac{3}{2}}{2}$ </u>

b.
$$3a - 4b = 12$$

$$-4b = -3a + 12$$

$$b = \frac{3}{4}a - 3$$

The slope is $\frac{3}{4}$ and the <u>b-intercept is</u> $\underline{-3}$.

c.
$$7a = -8b$$

$$8b = -7a$$

$$b = -\frac{7}{8}a$$

The slope is $\frac{-\frac{7}{8}}{8}$ and the <u>b-intercept is 0</u>.

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7. Step 2: Total revenue for the afternoon: \$240.75

Total number of swimmers for the afternoon: 126

Adult price: \$3.50 Child price: \$1.25

Let A represent the number of adults and C represent the number of children.

Step 3: Total number of swimmers = Number of adults + Number of children
Total revenue = Revenue from adults + Revenue from children

Step 4: 126 = A + C

- \$240.75 = \$3.50*A* + \$1.25*C*
- Step 5: Rearrange ①: A = 126 C

Substitute into ②: \$240.75 = \$3.50(126 - C) + \$1.25C

Sove: \$240.75 = \$441 - \$3.50*C* + \$1.25*C* \$240.75 = \$441 - \$2.25*C* \$240.75 - \$441 = -\$2.25*C* -\$200.25 = -\$2.25*C C* = -\$200.25/-\$2.25 = 89

There were 89 children and 126 - 89 = 37 adults who swam during the afternoon.

8. Step 2: Total kilometres paved = 11.5.

There were 4.25 more kilometres paved on day two than on day one.

Let the number of kilometres paved on day one be X.

Then the number of kilometres paved on day two is (X + 4.25)

Step 3: Total Kms paved = Kms paved on day one + Kms paved on day two

Step 4: 11.5 = X + (X + 4.25)

 $\underline{3.625 \text{ kilometres}}$ were paved $\underline{\text{on day one}}$ and $3.625 + 4.25 = \underline{7.875 \text{ kilometres}}$ were paved $\underline{\text{on day two}}$.

9. *a.* Given: c = 17.5%, $V_i = 29.43

$$V_f = V_i (1 + c) = $29.43(1.175) = $34.58$$

\$34.58 is 17.5% more than \$29.43.

b. Given: V_f = \$100, c = -80%

$$V_i = \frac{V_f}{1+c} = \frac{\$100}{1-0.80} = \frac{\$500.00}{1}$$

80% off \$500 leaves \$100.

c. Given: V_f = \$100, c = -15%

$$V_i = \frac{V_f}{1+c} = \frac{\$100}{1-0.15} = \frac{\$117.65}{1}$$

\$117.65 reduced by 15% equals \$100.

(continued)

Review Problems (continued)

- 9. *d.* Given: V_i = \$47.50, c = 320% $V_f = V_i$ (1 + c) = \$47.50(1 + 3.2) = \$199.50 \$47.50 after an increase of 320% is \$199.50.
 - e. Given: c = -62%, $V_f = \$213.56$ $V_i = \frac{V_f}{1+c} = \frac{\$213.56}{1-0.62} = \frac{\$562.00}{1}$

\$562 decreased by 62% equals \$213.56.

f. Given: c = 125%, $V_f = 787.50 $V_i = \frac{V_f}{1+c} = \frac{$787.50}{1+1.25} = \frac{$350.00}{1+1.25}$

\$350 increased by 125% equals \$787.50.

g. Given: c = -30%, $V_i = \$300$ $V_f = V_i$ (1+c) = \$300 (1-0.30) = \$210.00 \$210 is 30% less than \$300.

Intermediate Problems

$$\frac{9y-7}{3} - 2.3(y-2) = 3y-2.\overline{3} - 2.3y+4.6 = \underline{0.7y+2.2\overline{6}}$$

11.
$$4(3a + 2b)(2b - a) - 5a(2a - b) = 4(6ab - 3a^2 + 4b^2 - 2ab) - 10a^2 + 5ab$$

= $\frac{-22a^2 + 21ab + 16b^2}{-22a^2 + 21ab + 16b^2}$

12. a.
$$L(1-d_1)(1-d_2)(1-d_3) = \$340(1-0.15)(1-0.08)(1-0.05) = \underline{\$252.59}$$

$$\frac{R}{i} \left[1 - \frac{1}{(1+i)^n} \right] = \frac{\$575}{0.085} \left[1 - \frac{1}{(1+0.085)^3} \right] = \$6764.706(1-0.7829081) = \underline{\$1468.56}$$
b.

13.
$$N = L (1-d_1)(1-d_2)(1-d_3)$$

$$\$324.30 = \$498(1-0.20)(1-d_2)(1-0.075)$$

$$\$324.30 = \$368.52(1-d_2)$$

$$\frac{\$324.30}{\$368.52} = (1-d_2)$$

$$d_2 = 1 - 0.8800 = 0.120 = 12.0\%$$

14. a.
$$6(4y-3)(2-3y) - 3(5-y)(1+4y) = 6(8y-12y^2-6+9y) - 3(5+20y-y-4y^2)$$

= $\frac{-60y^2+45y-51}{}$

b.
$$\frac{5b-4}{4} - \frac{25-b}{1.25} + \frac{7}{8}b = 1.25b - 1 - 20 + 0.8b + 0.875b = \underline{2.925b - 21}$$
$$\frac{96\text{nm}^2 - 72\text{n}^2 m^2}{48\text{n}^2 m} = \frac{4\text{m} - 3\text{nm}}{2\text{n}} = \frac{4\text{m}}{2\text{n}} - \frac{3\text{nm}}{2\text{n}} = 2\frac{m}{n} - 1.5\text{m}$$

$$\frac{(-3x^2)^3(2x^{-2})}{6x^5} = \frac{(-27x^6)(2x^{-2})}{6x^5} = \frac{9}{x}$$

16. a.
$$1.0075^{24} = \underline{1.19641}$$

b. $(1.05)^{1/6} - 1 = \underline{0.00816485}$
 $\underline{(1+0.0075)^{36} - 1} = \underline{41.1527}$

17. a.
$$4a - 5b = 30$$
 ① ② $2a - 6b = 22$ ② To eliminate a, ① \times 1: $4a - 5b = 30$

② × 2:
$$4a - 12b = 44$$

Subtract: $7b = -14$
 $b = -2$

Substitute into ①:
$$4a - 5(-2) = 30$$

 $4a = 30 - 10$
 $a = 5$

Hence,
$$(a, b) = (5, -2)$$

b.
$$76x - 29y = 1050$$
 ① $-13x - 63y = 250$ ②

To eliminate x,

$$\textcircled{1} \times 13$$
: $988x - 377y = 13,650$ $\textcircled{2} \times 76$: $-988x - 4788y = 19,000$ Add: $-5165y = 32,650$ $y = -6.321$

Substitute into ①: 76x - 29(-6.321) = 1050 76x = 1050 - 183.31x = 11.40

Hence, (x, y) = (11.40, -6.32)

18.
$$3x + 5y = 11$$
 ① $2x - y = 16$ ②

To eliminate y,

Add:

①:
$$3x + 5y = 11$$

② $\times 5$: $10x - 5y = 80$
 $13x + 0 = 91$
 $x = 7$

Substitute into equation ②: 2(7) - y = 16y = -2

Hence,
$$(x, y) = (7, -2)$$

19. The homeowner pays \$28 per month plus \$2.75 per cubic metre of water used.

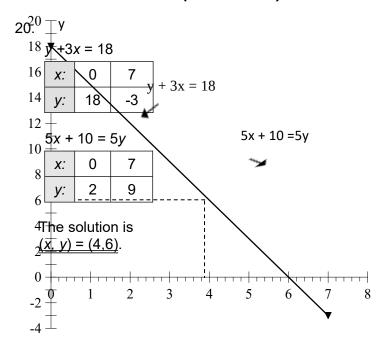
Then B = \$28 + \$2.75C

Expressing this equation in the form y = mx + b

$$B = $2.75C + $28$$

On a plot of B vs. C, slope = \$2.75 and B-intercept = \$28.

Review Problems (continued)



21. Given: Grace's share = 1.2(Kajsa's share); Mary Anne's share = $\frac{5}{8}$ (Grace's share)

Total allocated = \$36,000

Let K represent Kajsa's share.

(Kajsa's share) + (Grace's share) + (Mary Anne's share) = \$36,000

K + 1.2K +
$$\frac{5}{8}$$
(1.2K) = \$36,000
2.95 K = \$36,000
K = \$12,203.39

<u>Kajsa should receive \$12,203.39</u>. <u>Grace should receive</u> $1.2K = \frac{$14,644.07}{5}$.

Mary Anne should receive (\$14,644.07) = \$9152.54.

22. Given: Total initial investment = \$7800; Value 1 year later = \$9310

Percent change in ABC portion = 15%

Percent change in XYZ portion = 25%

Let X represent the amount invested in XYZ Inc.

The solution "idea" is:

(Amount invested in ABC)1.15 + (Amount invested in XYZ)1.25 = \$9310

Hence,

(\$7800 - X)1.15 + (X)1.25 = \$9310

\$8970 - 1.15X + 1.25X = \$9310 0.10X = \$9310 - \$8970 X = \$3400Rory invested \$3400 in XYZ Inc. and \$7800 - \$3400 = \$4400 in ABC Ltd.

Let R represent the price per kg for red snapper and let L represent the price per kg for lingcod. Then

To eliminate R,

Subtract:
$$-0.47865L = -$1.6992$$

$$L = $3.55$$

$$R = $4.10$$

Nguyen was paid \$3.55 per kg for lingcod and \$4.10 per kg for red snapper.

24. Given:

Year 1 value
$$(V_i)$$
 Year 2 value (V_f)

Average price: \$1160 \$1280
$$\frac{23,750-34,300}{34,300} \times 100\% = \underline{-30.76\%}$$
a. Percent change in gold production = $\frac{34,300}{1280-\$1160} \times 100\% = 10.34\%$

$$\frac{\$1280 - \$1160}{\$1160} \times 100\% = \underline{10.34\%}$$

- b. Percent change in price =
- c. Year 1 revenue, $V_i = 34,300(\$1160) = \39.788 million Year 2 revenue, $V_b = 23,750(\$1280) = \30.400 million

$$\frac{\$30.400 - \$39.788}{\$39.788} \times 100\% = \underline{-23.60\%}$$

Percent change in revenue =

25. Given: For the first year, V_i = \$3.40, V_f = \$11.50. For the second year, $V_i = 11.50 , c = -35%.

$$c = \frac{V_f - V_i}{V_i} \times 100\% = \frac{\$11.50 - \$3.40}{\$3.40} \times 100\% = \frac{238.24\%}{\$3.40}$$

a.

The share price increased by 238.24% in the first year.

- b. Current share price, $V_f = V_i (1 + c) = $11.50(1 0.35) = 7.48 .
- 26. Given: For the first year, c = 150%

For the second year,
$$c = -40\%$$
, $V_f = 24

The price at the beginning of the second year was

$$V_i = \frac{V_f}{1+c} = \frac{\$24}{1-0.40} = \$40.00 = V_f$$
 for the first year.

The price at the beginning of the first year was

$$V_i = \frac{V_f}{1+c} = \frac{\$40.00}{1+1.50} = \frac{\$16.00}{1+1.50}$$

Barry bought the stock for \$16.00 per share.

- 27. Given: Last year's revenue = \$2,347,000 Last year's expenses = \$2,189,000
 - a. Given: Percent change in revenue = 10%; Percent change in expenses = 5%

Anticipated revenues, $V_f = V_i(1 + c) = \$2,347,000(1.1) = \$2,581,700$

Anticipated expenses = \$2,189,000(1.05) = \$2,298,450 \$283,250

Anticipated profit 2,347,000 - 2,189,000 = 158,000Last year's profit

$$\frac{$283,250 - $158,000}{$150,000} \times 100\% = \frac{79.27\%}{}$$

Percent increase in profit = \$158,000

b. Given: c(revenue) = -10%; c(expenses) = -5%

Anticipated revenues = \$2,347,000(1-0.10) = \$2,112,300Anticipated expenses = \$2,189,000(1-0.05) = \$2,079,550

Anticipated profit

Anticipated profit \$32,750
$$\frac{$32,750-$158,000}{$158,000} \times 100\% = \frac{-79.27\%}{$158,000}$$
Percent change in profit will decline by 79.27%

The operating profit will decline by 79.27%.

28. a.
$$\frac{(1.00\,\overline{6})^{240}-1}{0.00\,\overline{6}} = \frac{4.926802-1}{0.00\,\overline{6}} = \underline{589.020}$$

$$(1+0.025)^{1/3}-1=\underline{0.00826484}$$

Advanced Problems

29.
$$\left(-\frac{2x^2}{3}\right)^{-2} \left(\frac{5^2}{6x^3}\right) \left(-\frac{15}{x^5}\right)^{-1} = \left(\frac{3}{2x^2}\right)^2 \left(\frac{25}{6x^3}\right) \left(-\frac{x^5}{15}\right) = -\frac{5}{\underline{8x^2}}$$

30. a.
$$\frac{x}{1.08^3} + \frac{x}{2}(1.08)^4 = $850$$

0.793832x + 0.680245x = \$850

$$x = $576.6$$

$$32x + 0.680245x = $850$$

$$x = $576.63$$

$$\frac{$576.63}{1.08^{3}} + \frac{$576.63}{2}(1.08)^{4} = $457.749 + $392.250 = $850.00$$

Check:

$$2x\left(1+0.085\times\frac{77}{365}\right)+\frac{x}{1+0.085\times\frac{132}{365}}=\$1565.70$$

b. 2.03586x + 0.97018x = \$1565.70x = \$520.85

Check:

$$(1+0.085 \times \frac{77}{365}) + \frac{\$520.85}{1+0.085 \times \frac{132}{365}} = \$1060.38 + \$505.32 = \$1565.70$$

$$\frac{S}{1+rt} = \$2500(1.1025)^2 + \frac{\$1500}{1+0.09 \times \frac{93}{265}}$$

 $\frac{S}{1+rt} = \$2500(1.1025)^2 + \frac{\$1500}{1+0.09 \times \frac{93}{365}} = \$3038.766 + \$1466.374 = \underline{\$4505.14}$

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32. a.
$$\frac{2x}{1+0.13 \times \frac{92}{365}} + x \left(1+0.13 \times \frac{59}{365}\right) = \$831$$

$$1.93655x + 1.02101x = \$831$$

$$2.95756x = \$831$$

$$x = \frac{\$280.97}{2}$$
b.
$$3x(1.03^5) + \frac{x}{1.03^3} + x = \frac{\$2500}{1.03^2}$$

$$3.47782x + 0.915142x + x = \$2356.49$$

$$x = \frac{\$436.96}{2}$$

33. 60% of a $\frac{1}{8}$ interest was purchased for \$65,000. Let the V represent the implied value of the entire partnership.

Then
$$0.60 \times \frac{3}{8} \text{ V} = \$65,000$$

$$V = \frac{8 \times \$65,000}{0.60 \times 3} = \$288,889$$

The implied value of the chalet was \$288,889.

34. Let b represent the base salary and r represent the commission rate. Then

$$r(\$27,000) + b = \$2815.00$$
 ① ② $r(\$35,500) + b = \3197.50 ② ② $-\$8500r = -\382.50 $r = 0.045$

Substitute into ①: 0.045(\$27,000) + b = \$2815

Subtract:

b = \$1600

Deanna's base salary is \$1600 per month and her commission rate is 4.5%.

35. Let the regular season ticket prices be R for the red section and B for the blue section. Then

$$2500R + 4500B = \$50,250 \ \textcircled{1}$$

$$2500(1.3R) + 4500(1.2B) = \$62,400 \ \textcircled{2}$$

$$\textcircled{1} \times 1.2: \quad \underline{2500(1.2R) + 4500(1.2B)} = \underline{\$60,300}$$
Subtract:
$$2500(0.1R) + 0 = \$2100$$

$$R = \$8.40$$
Substitute into \textcircled{1}:
$$2500(\$8.40) + 4500B = \$50,250$$

$$B = \$6.50$$

The ticket prices for the playoffs cost

$$1.3 \times \$8.40 = $10.92$$
 in the "reds" and $1.2 \times \$6.50 = 7.80 in the "blues".