Instructor's Resource Manual

Discrete Mathematics: Mathematical Reasoning and Proof with Puzzles, Patterns, and Games

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Introduction

As noted in the preface to the text, our aim in this instructor's guide is to provide additional information on how to read the text. The material in the guide generally falls into one of several categories.

- The guide organizes information from the text itself. For example, the chapter summaries provide useful information for the instructor. In this guide we recreate those summaries, along with annotations where appropriate.
- In the preface, we indicate four possible courses that might use this text. In this guide, we provide further details on how to approach various sections depending on the nature of your course.
- We give practical advice based on our own use of the text.
- Where there are several possible approaches to a topic, we explain the rationale for our approach. In some cases, we offer ideas on how to use the section(s) if you prefer a different approach.
- We provide "forward references." That is, we indicate which future sections are particularly dependent on the section at hand. This may include advice on specific examples or exercises to highlight if you plan to cover that future section.

We begin by highlighting some important information from the preface.

Four possible courses

Discrete math courses vary a great deal, but there are at least four courses for which this text is well-suited. We briefly outline the core sections that an instructor should consider for each of these courses. Later sections of this guide provide further details and recommendations for emphasis within each section. In those sections we will refer to these four courses as "**mixed**", "**computer science**", "**transition**", and "**high school**".

1. A mixed math, computer science, and math education audience requires breadth among applications and conceptual topics. The following sections would provide this experience: • 1.1 – 1.5 • 2.1 – 2.3, 2.5, 2.6 • 3.1 – 3.5 • 4.1, 4.4, 4.5, 4.7 • 5.1 – 5.4 • 6.1 – 6.3 • 7.1, 7.3, 7.5	2. A first-year course for computer science students would want to cover specific computer science applications as well as the logic and proof techniques recommended by ACM: • 1.1 – 1.5 • 2.1 – 2.3, 2.5, 2.6 • 3.1 – 3.5 • 4.1, 4.2, 4.4 – 4.6, 4.8 • 5.1 – 5.3, 5.5, 5.6 • 7.1, 7.3, 7.6, 7.7
3. A sophomore or junior level math major transition course might focus on mathematical proof and classical structures, hence the following core sections would be appropriate: • 1.1 – 1.6 • 2.1 – 2.7 • 3.1 – 3.3 • 4.1 – 4.5 • 7.1 – 7.7	4. Advanced high school students might have applications of discrete math as their key interest, so the following sections could provide the core of a course for this audience: • 1.1 – 1.4 • 2.1 – 2.3, 2.6, 2.7 • 3.1 – 3.5 • 4.1, 4.2, 4.6, 4.7 • 5.1 – 5.4, 5.6 • 6.1 – 6.3, 6.6 • 7.1, 7.3, 7.5, 7.7

Contents and Organization

A list of mathematical topics that are addressed within the framework of this book is given below. This list covers most of the content and learning objectives recommended by the ACM/IEEE (for a one-semester course for computer science majors), the MAA (for math majors), and the CBMS document on the Mathematical Education of Teachers (for future middle and secondary level teachers).

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Number sequences Truth tables Propositional logic Predicate logic Mathematical proof Proof by induction Pigeonhole principle Modular arithmetic Representing numbers Sets Boolean algebra Logic circuits	Functions Binary relations Equivalence relations Numerical functions Iterated function systems Counting techniques RSA cryptography Recurrence relations Growth of functions Discrete probability Expected value Matrix arithmetic	Markov chains Eulerian graphs Planar graphs Hamiltonian graphs Traveling salesperson Directed graphs Adjacency matrices Spanning trees Solving puzzles Analyzing games Binary trees

These topics are broadly organized into four parts:

1. Thinking and writing about mathematics – Chapters 1 and 2. The *content* of the book is discrete mathematics, but the *theme* is learning to think and write mathematically.
2. Sets, functions and relations – Chapters 3 and 4. Students are encouraged to apply their problem-solving and proof-writing skills with this new subject matter.
3. Combinatorics and probability – Chapters 5 and 6. Although mathematical proof does not play as prominent a role in this portion, mathematical reasoning is still paramount.
4. Graph theory – Chapter 7. This portion develops graph theory, emphasizing a closer look at puzzles and games. The topic is motivated by the famous Bridges of Königsberg puzzle.

Threads

The book uses several major threads to help weave the core topics into a cohesive whole.

- Throughout the book, we emphasize the application of **mathematical reasoning** to the solution of problems.
- An important *content* thread is the focus on (mathematical) **puzzles, games, or magic tricks**.
- In addition, we have included a variety of "real world **excursions**" to show students the utility of discrete mathematics.
- Another important feature is the recurrent (so to speak) use of **recursive modeling** and its companion, **mathematical induction**.

Aids to Teaching and Learning

- **Practice problems.** Each section contains a number of practice problems distributed throughout. The intent is that students use them to test their understanding as they read the section. Detailed solutions are given at the end of the section.
- **Section reviews**. These are placed at the end of each chapter. This guide includes a copy, with additional information where appropriate.
- **Exercises**. The book has over 1,100 exercises, ranging from routine reinforcement to conceptual challenges. Those numbered in **blue** contain an answer in the back of the book, and a more detailed solution in the *Student Solutions Manual*.

Other resources

- The *Instructor's Solution Manual* ISBN 0-471-488011 provides complete solutions to all the exercises in the book.
- There are a number of online applications available for students, using URL either http://www.wiley.com/college/ensley. These are indicated in the text using a lightning-bolt icon.

Thinking and Writing about Mathematics
Chapters 1 and 2

Here is an abbreviated Table of Contents for the first two chapters. Detailed versions are given below, with the discussion of the individual chapters.

Note: Sections 1.3–1.6 introduce formal logic. How much you choose to emphasize those sections can be a matter of personal taste. The construction of Chapter 2 allows you to move directly into mathematical writing and proofs without spending a large amount of time on formal logic. (See the notes for Chapters 1 and 2.) ***In general, we use formal logic to supplement the discussion of proof-writing, but not as the primary motivation for how it is done.***

Chapter 1 Puzzles, Patterns, and Mathematical Language

This chapter establishes first examples of the puzzles and games that will be studied throughout the course. Formal concepts, such as truth tables and expressing patterns in sequences using recurrence relations and analytic expressions, are introduced. Predicates and quantifiers are introduced, along with a first look at implications. The Excursion section continues the discussion of the language of implication, and considers the difficulties of analyzing arguments in ordinary discourse using symbolic logic as a tool.

Coverage in sample courses

1.1 First Examples – all courses
This section is important not so much for the specific content, but for setting the tone for the course.

1.2 Number puzzles and sequences – all courses
This section is crucial to the development in Chapter 2, and begins one of the important themes of the book – recursive reasoning.

1.3 Truth-tellers, liars, and propositional logic – all courses
This section introduces some notation and terminology that is used later. The amount of emphasis can be a matter of your personal taste.