C H A P T E R P
Preparation for Calculus

CHAPTER P
Preparation for Calculus

Section P.1 Graphs and Models

1. $y = -\frac{3}{2}x + 3$

   x-intercept: $(2, 0)$

   y-intercept: $(0, 3)$

   Matches graph (b).

2. $y = \sqrt{9 - x^2}$

   x-intercepts: $(-3, 0), (3, 0)$

   y-intercept: $(0, 3)$

   Matches graph (d).

3. $y = 3 - x^2$

   x-intercepts: $(\sqrt{3}, 0), (-\sqrt{3}, 0)$

   y-intercept: $(0, 3)$

   Matches graph (a).

4. $y = x^3 - x$

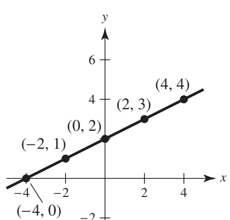
   x-intercepts: $(0, 0), (-1, 0), (1, 0)$

   y-intercept: $(0, 0)$

   Matches graph (c).

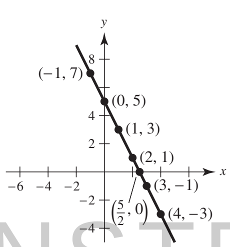
5. $y = \frac{1}{2}x + 2$

x	-4	-2	0	2	4
y	0	1	2	3	4



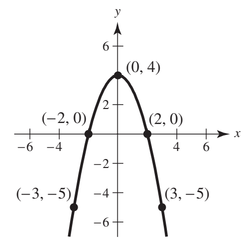
6. $y = 5 - 2x$

x	-1	0	1	2	$\frac{5}{2}$	3	4
y	7	5	3	1	0	-1	-3



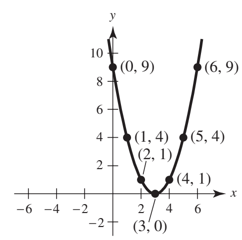
7. $y = 4 - x^2$

x	-3	-2	0	2	3
y	-5	0	4	0	-5



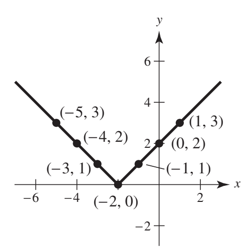
8. $y = (x - 3)^2$

x	0	1	2	3	4	5	6
y	9	4	1	0	1	4	9



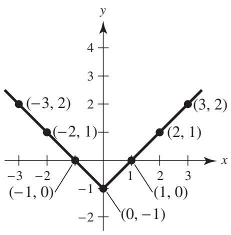
9. $y = |x + 2|$

x	-5	-4	-3	-2	-1	0	1
y	3	2	1	0	1	2	3



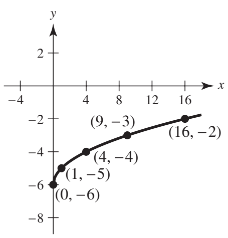
10. $y = |x| - 1$

x	-3	-2	-1	0	1	2	3
y	2	1	0	-1	0	1	2



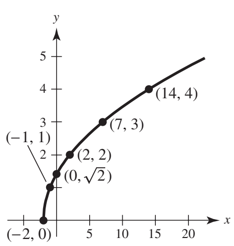
11. $y = \sqrt{x} - 6$

x	0	1	4	9	16
y	-6	-5	-4	-3	-2



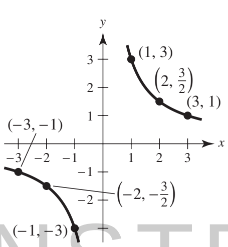
12. $y = \sqrt{x + 2}$

x	-2	-1	0	2	7	14
y	0	1	$\sqrt{2}$	2	3	4



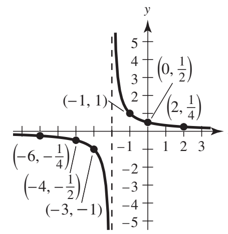
13. $y = \dfrac{3}{x}$

x	-3	-2	-1	0	1	2	3
y	-1	$-\frac{3}{2}$	-3	Undef.	3	$\frac{3}{2}$	1

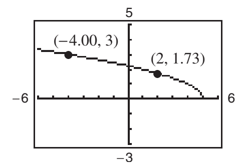


14. $y = \dfrac{1}{x + 2}$

x	-6	-4	-3	-2	-1	0	2
y	$-\frac{1}{4}$	$-\frac{1}{2}$	-1	Undef.	1	$\frac{1}{2}$	$\frac{1}{4}$



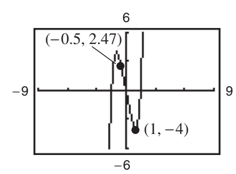
15. $y = \sqrt{5 - x}$



(a) $(2, y) = (2, 1.73)$ $\left(y = \sqrt{5 - 2} = \sqrt{3} \approx 1.73\right)$

(b) $(x, 3) = (-4, 3)$ $\left(3 = \sqrt{5 - (-4)}\right)$

16. $y = x^5 - 5x$



(a) $(-0.5, y) = (-0.5, 2.47)$

(b) $(x, -4) = (-1.65, -4)$ and $(x, -4) = (1, -4)$

17. $y = 2x - 5$

y-intercept: $y = 2(0) - 5 = -5$; $(0, -5)$

x-intercept: $0 = 2x - 5$

$5 = 2x$

$x = \frac{5}{2}$; $\left(\frac{5}{2}, 0\right)$

18. $y = 4x^2 + 3$

y-intercept: $y = 4(0)^2 + 3 = 3$; $(0, 3)$

x-intercept: $0 = 4x^2 + 3$

$-3 = 4x^2$

None. y cannot equal 0.

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19. $y = x^2 + x - 2$

y-intercept: $y = 0^2 + 0 - 2$

$y = -2;\ (0, -2)$

x-intercepts: $0 = x^2 + x - 2$

$0 = (x + 2)(x - 1)$

$x = -2, 1;\ (-2, 0), (1, 0)$

20. $y^2 = x^3 - 4x$

y-intercept: $y^2 = 0^3 - 4(0)$

$y = 0;\ (0, 0)$

x-intercepts: $0 = x^3 - 4x$

$0 = x(x - 2)(x + 2)$

$x = 0, \pm 2;\ (0, 0), (\pm 2, 0)$

21. $y = x\sqrt{16 - x^2}$

y-intercept: $y = 0\sqrt{16 - 0^2} = 0;\ (0, 0)$

x-intercepts: $0 = x\sqrt{16 - x^2}$

$0 = x\sqrt{(4 - x)(4 + x)}$

$x = 0, 4, -4;\ (0, 0), (4, 0), (-4, 0)$

22. $y = (x - 1)\sqrt{x^2 + 1}$

y-intercept: $y = (0 - 1)\sqrt{0^2 + 1}$

$y = -1;\ (0, -1)$

x-intercept: $0 = (x - 1)\sqrt{x^2 + 1}$

$x = 1;\ (1, 0)$

23. $y = \dfrac{2 - \sqrt{x}}{5x + 1}$

y-intercept: $y = \dfrac{2 - \sqrt{0}}{5(0) + 1} = 2;\ (0, 2)$

x-intercept: $0 = \dfrac{2 - \sqrt{x}}{5x + 1}$

$0 = 2 - \sqrt{x}$

$x = 4;\ (4, 0)$

24. $y = \dfrac{x^2 + 3x}{(3x + 1)^2}$

y-intercept: $y = \dfrac{0^2 + 3(0)}{[3(0) + 1]^2}$

$y = 0;\ (0, 0)$

x-intercepts: $0 = \dfrac{x^2 + 3x}{(3x + 1)^2}$

$0 = \dfrac{x(x + 3)}{(3x + 1)^2}$

$x = 0, -3;\ (0, 0), (-3, 0)$

25. $x^2y - x^2 + 4y = 0$

y-intercept: $0^2(y) - 0^2 + 4y = 0$

$y = 0;\ (0, 0)$

x-intercept: $x^2(0) - x^2 + 4(0) = 0$

$x = 0;\ (0, 0)$

26. $y = 2x - \sqrt{x^2 + 1}$

y-intercept: $y = 2(0) - \sqrt{0^2 + 1}$

$y = -1;\ (0, -1)$

x-intercept: $0 = 2x - \sqrt{x^2 + 1}$

$2x = \sqrt{x^2 + 1}$

$4x^2 = x^2 + 1$

$3x^2 = 1$

$x^2 = \dfrac{1}{3}$

$x = \pm\dfrac{\sqrt{3}}{3}$

$x = \dfrac{\sqrt{3}}{3};\ \left(\dfrac{\sqrt{3}}{3}, 0\right)$

Note: $x = -\sqrt{3}/3$ is an extraneous solution.

27. Symmetric with respect to the y-axis because $y = (-x)^2 - 6 = x^2 - 6$.

28. $y = x^2 - x$

No symmetry with respect to either axis or the origin.

29. Symmetric with respect to the x-axis because $(-y)^2 = y^2 = x^3 - 8x$.

30. Symmetric with respect to the origin because

$$(-y) = (-x)^3 + (-x)$$
$$-y = -x^3 - x$$
$$y = x^3 + x.$$

31. Symmetric with respect to the origin because $(-x)(-y) = xy = 4.$

32. Symmetric with respect to the x-axis because $x(-y)^2 = xy^2 = -10.$

33. $y = 4 - \sqrt{x + 3}$

No symmetry with respect to either axis or the origin.

34. Symmetric with respect to the origin because

$$(-x)(-y) - \sqrt{4 - (-x)^2} = 0$$
$$xy - \sqrt{4 - x^2} = 0.$$

35. Symmetric with respect to the origin because

$$-y = \frac{-x}{(-x)^2 + 1}$$
$$y = \frac{x}{x^2 + 1}.$$

36. $y = \dfrac{x^2}{x^2 + 1}$ is symmetric with respect to the y-axis because $y = \dfrac{(-x)^2}{(-x)^2 + 1} = \dfrac{x^2}{x^2 + 1}.$

37. $y = \left|x^3 + x\right|$ is symmetric with respect to the y-axis because $y = \left|(-x)^3 + (-x)\right| = \left|-(x^3 + x)\right| = \left|x^3 + x\right|.$

38. $|y| - x = 3$ is symmetric with respect to the x-axis because

$$|-y| - x = 3$$
$$|y| - x = 3.$$

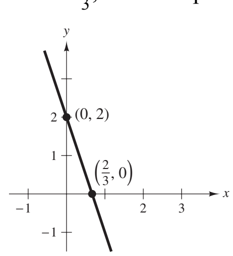
39. $y = 2 - 3x$

$y = 2 - 3(0) = 2$, y-intercept

$0 = 2 - 3(x) \Rightarrow 3x = 2 \Rightarrow x = \frac{2}{3}$, x-intercept

Intercepts: $(0, 2)$, $\left(\frac{2}{3}, 0\right)$

Symmetry: none



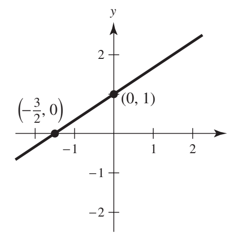
40. $y = \frac{2}{3}x + 1$

$y = \frac{2}{3}(0) + 1 = 1$, y-intercept

$0 = \frac{2}{3}x + 1 \Rightarrow -\frac{2}{3}x = 1 \Rightarrow x = -\frac{3}{2}$, x-intercept

Intercepts: $(0, 1)$, $\left(-\frac{3}{2}, 0\right)$

Symmetry: none



41. $y = 9 - x^2$

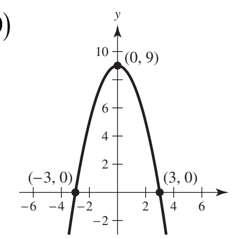
$y = 9 - (0)^2 = 9$, y-intercept

$0 = 9 - x^2 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$, x-intercepts

Intercepts: $(0, 9)$, $(3, 0)$, $(-3, 0)$

$y = 9 - (-x)^2 = 9 - x^2$

Symmetry: y-axis



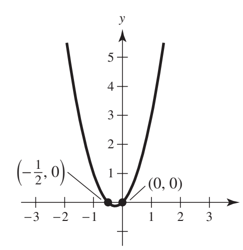
42. $y = 2x^2 + x = x(2x + 1)$

$y = 0(2(0) + 1) = 0$, y-intercept

$0 = x(2x + 1) \Rightarrow x = 0, -\frac{1}{2}$, x-intercepts

Intercepts: $(0, 0)$, $\left(-\frac{1}{2}, 0\right)$

Symmetry: none



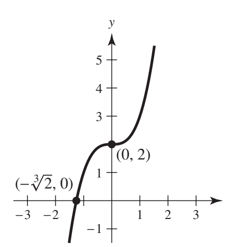
43. $y = x^3 + 2$

$y = 0^3 + 2 = 2$, y-intercept

$0 = x^3 + 2 \Rightarrow x^3 = -2 \Rightarrow x = -\sqrt[3]{2}$, x-intercept

Intercepts: $\left(-\sqrt[3]{2}, 0\right)$, $(0, 2)$

Symmetry: none



44. $y = x^3 - 4x$

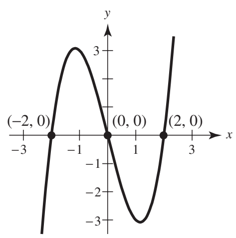
$y = 0^3 - 4(0) = 0$, y-intercept

$$x^3 - 4x = 0$$
$$x(x^2 - 4) = 0$$
$$x(x + 2)(x - 2) = 0$$
$$x = 0, \pm 2, \ x\text{-intercepts}$$

Intercepts: $(0, 0), (2, 0), (-2, 0)$

$y = (-x)^3 - 4(-x) = -x^3 + 4x = -(x^3 - 4x)$

Symmetry: origin



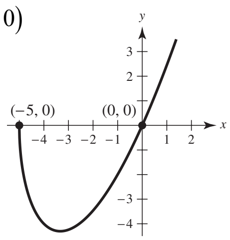
45. $y = x\sqrt{x + 5}$

$y = 0\sqrt{0 + 5} = 0$, y-intercept

$x\sqrt{x + 5} = 0 \Rightarrow x = 0, -5$, x-intercepts

Intercepts: $(0, 0), (-5, 0)$

Symmetry: none



46. $y = \sqrt{25 - x^2}$

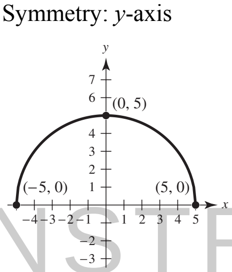
$y = \sqrt{25 - 0^2} = \sqrt{25} = 5$, y-intercept

$$\sqrt{25 - x^2} = 0$$
$$25 - x^2 = 0$$
$$(5 + x)(5 - x) = 0$$
$$x = \pm 5, \ x\text{-intercept}$$

Intercepts: $(0, 5), (5, 0), (-5, 0)$

$y = \sqrt{25 - (-x)^2} = \sqrt{25 - x^2}$

Symmetry: y-axis



47. $x = y^3$

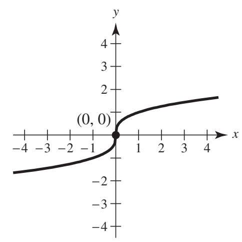
$y^3 = 0 \Rightarrow y = 0$, y-intercept

$x = 0$, x-intercept

Intercept: $(0, 0)$

$-x = (-y)^3 \Rightarrow -x = -y^3$

Symmetry: origin



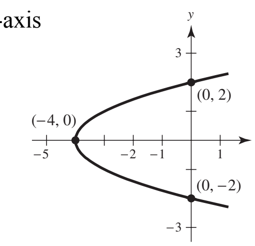
48. $x = y^2 - 4$

$$y^2 - 4 = 0$$
$$(y + 2)(y - 2) = 0$$
$$y = \pm 2, \ y\text{-intercepts}$$
$$x = 0^2 - 4 = -4, \ x\text{-intercept}$$

Intercepts: $(0, 2), (0, -2), (-4, 0)$

$x = (-y)^2 - 4 = y^2 - 4$

Symmetry: x-axis



49. $y = \dfrac{8}{x}$

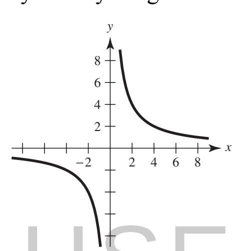
$y = \dfrac{8}{0} \Rightarrow$ Undefined $\Rightarrow$ no y-intercept

$\dfrac{8}{x} = 0 \Rightarrow$ No solution $\Rightarrow$ no x-intercept

Intercepts: none

$-y = \dfrac{8}{-x} \Rightarrow y = \dfrac{8}{x}$

Symmetry: origin



50. $y = \dfrac{10}{x^2 + 1}$

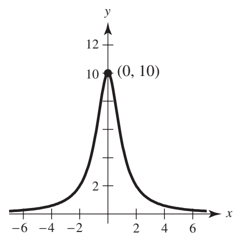
$y = \dfrac{10}{0^2 + 1} = 10$, y-intercept

$\dfrac{10}{x^2 + 1} = 0 \Rightarrow$ No solution $\Rightarrow$ no x-intercepts

Intercept: $(0, 10)$

$y = \dfrac{10}{(-x)^2 + 1} = \dfrac{10}{x^2 + 1}$

Symmetry: y-axis



51. $y = 6 - |x|$

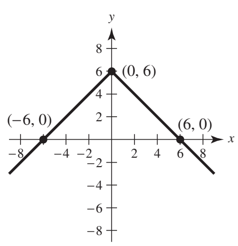
$y = 6 - |0| = 6$, y-intercept

$$\begin{aligned} 6 - |x| &= 0 \\ 6 &= |x| \\ x &= \pm 6,\ x\text{-intercepts} \end{aligned}$$

Intercepts: $(0, 6)$, $(-6, 0)$, $(6, 0)$

$y = 6 - |-x| = 6 - |x|$

Symmetry: y-axis



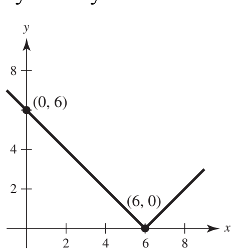
52. $y = |6 - x|$

$y = |6 - 0| = |6| = 6$, y-intercept

$$\begin{aligned} |6 - x| &= 0 \\ 6 - x &= 0 \\ 6 &= x,\ x\text{-intercept} \end{aligned}$$

Intercepts: $(0, 6)$, $(6, 0)$

Symmetry: none



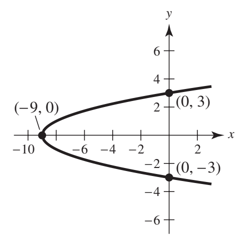
53. $y^2 - x = 9$

$$\begin{aligned} y^2 &= x + 9 \\ y &= \pm\sqrt{x + 9} \\ y &= \pm\sqrt{0 + 9} = \pm\sqrt{9} = \pm 3,\ y\text{-intercepts} \\ \pm\sqrt{x + 9} &= 0 \\ x + 9 &= 0 \\ x &= -9,\ x\text{-intercept} \end{aligned}$$

Intercepts: $(0, 3)$, $(0, -3)$, $(-9, 0)$

$(-y)^2 - x = 9 \Rightarrow y^2 - x = 9$

Symmetry: x-axis



54. $x^2 + 4y^2 = 4 \Rightarrow y = \pm\dfrac{\sqrt{4 - x^2}}{2}$

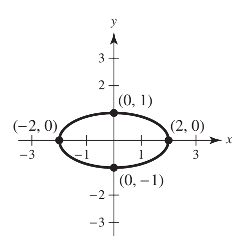
$y = \pm\dfrac{\sqrt{4 - 0^2}}{2} = \pm\dfrac{\sqrt{4}}{2} = \pm 1$, y-intercepts

$$\begin{aligned} x^2 + 4(0)^2 &= 4 \\ x^2 &= 4 \\ x &= \pm 2,\ x\text{-intercepts} \end{aligned}$$

Intercepts: $(-2, 0)$, $(2, 0)$, $(0, -1)$, $(0, 1)$

$(-x)^2 + 4(-y)^2 = 4 \Rightarrow x^2 + 4y^2 = 4$

Symmetry: origin and both axes



55. $x + 3y^2 = 6$

$3y^2 = 6 - x$

$y = \pm\sqrt{\dfrac{6-x}{3}}$

$y = \pm\sqrt{\dfrac{6-0}{3}} = \pm\sqrt{2}$, y-intercepts

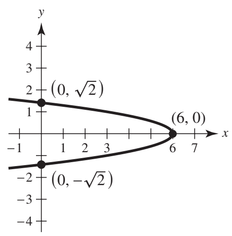
$x + 3(0)^2 = 6$

$x = 6$, x-intercept

Intercepts: $(6, 0), (0, \sqrt{2}), (0, -\sqrt{2})$

$x + 3(-y)^2 = 6 \Rightarrow x + 3y^2 = 6$

Symmetry: x-axis



56. $3x - 4y^2 = 8$

$4y^2 = 3x - 8$

$y = \pm\sqrt{\frac{3}{4}x - 2}$

$y = \pm\sqrt{\dfrac{3}{4}(0) - 2} = \pm\sqrt{-2}$

$\Rightarrow$ no solution $\Rightarrow$ no y-intercepts

$3x - 4(0)^2 = 8$

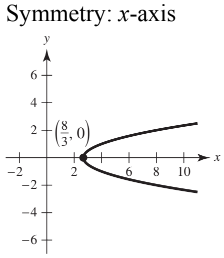
$3x = 8$

$x = \dfrac{8}{3}$, x-intercept

Intercept: $\left(\frac{8}{3}, 0\right)$

$3x - 4(-y)^2 = 8 \Rightarrow 3x - 4y^2 = 8$

Symmetry: x-axis



57. $x + y = 8 \Rightarrow y = 8 - x$

$4x - y = 7 \Rightarrow y = 4x - 7$

$8 - x = 4x - 7$

$15 = 5x$

$3 = x$

The corresponding y-value is $y = 5$.

Point of intersection: $(3, 5)$

58. $3x - 2y = -4 \Rightarrow y = \dfrac{3x+4}{2}$

$4x + 2y = -10 \Rightarrow y = \dfrac{-4x-10}{2}$

$\dfrac{3x+4}{2} = \dfrac{-4x-10}{2}$

$3x + 4 = -4x - 10$

$7x = -14$

$x = -2$

The corresponding y-value is $y = -1$.

Point of intersection: $(-2, -1)$

59. $x^2 + y = 6 \Rightarrow y = 6 - x^2$

$x + y = 4 \Rightarrow y = 4 - x$

$6 - x^2 = 4 - x$

$0 = x^2 - x - 2$

$0 = (x - 2)(x + 1)$

$x = 2, -1$

The corresponding y-values are $y = 2$ (for $x = 2$) and $y = 5$ (for $x = -1$).

Points of intersection: $(2, 2), (-1, 5)$

60. $x = 3 - y^2 \Rightarrow y^2 = 3 - x$

$y = x - 1$

$3 - x = (x - 1)^2$

$3 - x = x^2 - 2x + 1$

$0 = x^2 - x - 2 = (x + 1)(x - 2)$

$x = -1$ or $x = 2$

The corresponding y-values are $y = -2$ (for $x = -1$) and $y = 1$ (for $x = 2$).

Points of intersection: $(-1, -2), (2, 1)$

61. $x^2 + y^2 = 5 \Rightarrow y^2 = 5 - x^2$

$x - y = 1 \Rightarrow y = x - 1$

$$5 - x^2 = (x - 1)^2$$
$$5 - x^2 = x^2 - 2x + 1$$
$$0 = 2x^2 - 2x - 4 = 2(x + 1)(x - 2)$$
$$x = -1 \text{ or } x = 2$$

The corresponding y-values are $y = -2$ (for $x = -1$) and $y = 1$ (for $x = 2$).

Points of intersection: $(-1, -2), (2, 1)$

62. $x^2 + y^2 = 25 \Rightarrow y^2 = 25 - x^2$

$-3x + y = 15 \Rightarrow y = 3x + 15$

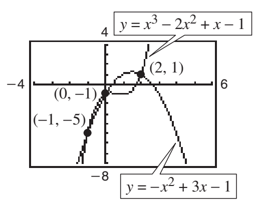
$$25 - x^2 = (3x + 15)^2$$
$$25 - x^2 = 9x^2 + 90x + 225$$
$$0 = 10x^2 + 90x + 200$$
$$0 = x^2 + 9x + 20$$
$$0 = (x + 5)(x + 4)$$
$$x = -4 \text{ or } x = -5$$

The corresponding y-values are $y = 3$ (for $x = -4$) and $y = 0$ (for $x = -5$).

Points of intersection: $(-4, 3), (-5, 0)$

63. $y = x^3 - 2x^2 + x - 1$

$y = -x^2 + 3x - 1$



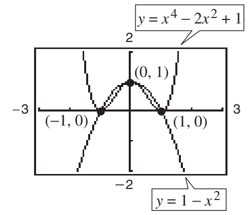
Points of intersection: $(-1, -5), (0, -1), (2, 1)$

Analytically, $x^3 - 2x^2 + x - 1 = -x^2 + 3x - 1$

$$x^3 - x^2 - 2x = 0$$
$$x(x - 2)(x + 1) = 0$$
$$x = -1, 0, 2.$$

64. $y = x^4 - 2x^2 + 1$

$y = 1 - x^2$



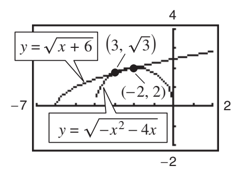
Points of intersection: $(-1, 0), (0, 1), (1, 0)$

Analytically, $1 - x^2 = x^4 - 2x^2 + 1$

$$0 = x^4 - x^2$$
$$0 = x^2(x + 1)(x - 1)$$
$$x = -1, 0, 1.$$

65. $y = \sqrt{x + 6}$

$y = \sqrt{-x^2 - 4x}$



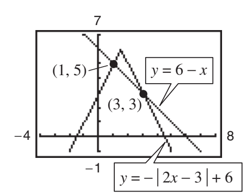
Points of intersection: $(-2, 2), (-3, \sqrt{3}) \approx (-3, 1.732)$

Analytically, $\sqrt{x + 6} = \sqrt{-x^2 - 4x}$

$$x + 6 = -x^2 - 4x$$
$$x^2 + 5x + 6 = 0$$
$$(x + 3)(x + 2) = 0$$
$$x = -3, -2.$$

66. $y = -|2x - 3| + 6$

$y = 6 - x$



Points of intersection: (3, 3), (1, 5)

Analytically, $-|2x - 3| + 6 = 6 - x$

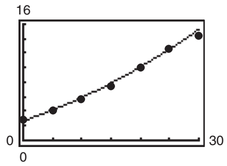
$$|2x - 3| = x$$

$2x - 3 = x$ or $2x - 3 = -x$

$x = 3$ or $x = 1$.

67. (a) Using a graphing utility, you obtain
$y = 0.005t^2 + 0.27t + 2.7$.

(b)



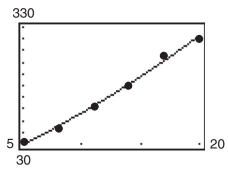
(c) For 2020, $t = 40$.

$$y = 0.005(40)^2 + 0.27(40) + 2.7$$
$$= 21.5$$

The GDP in 2020 will be \$21.5 trillion.

68. (a) Using a graphing utility, you obtain
$y = 0.24t^2 + 12.6t - 40$.

(b)



The model is a good fit for the data.

(c) For 2020, $t = 30$.

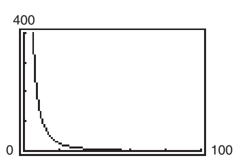
$$y = 0.24(30)^2 + 12.6(30) - 40$$
$$= 554$$

The number of cellular phone subscribers in 2020 will be 554 million.

69.
$$\begin{aligned} C &= R \\ 2.04x + 5600 &= 3.29x \\ 5600 &= 3.29x - 2.04x \\ 5600 &= 1.25x \\ x &= \frac{5600}{1.25} = 4480 \end{aligned}$$

To break even, 4480 units must be sold.

70. $y = \dfrac{10{,}770}{x^2} - 0.37$



If the diameter is doubled, the resistance is changed by approximately a factor of $\frac{1}{4}$. For instance, $y(20) \approx 26.555$ and $y(40) \approx 6.36125$.

71. $y = kx^3$

(a) $(1, 4)$: $4 = k(1)^3 \Rightarrow k = 4$

(b) $(-2, 1)$: $1 = k(-2)^3 = -8k \Rightarrow k = -\frac{1}{8}$

(c) $(0, 0)$: $0 = k(0)^3 \Rightarrow k$ can be any real number.

(d) $(-1, -1)$: $-1 = k(-1)^3 = -k \Rightarrow k = 1$

72. $y^2 = 4kx$

(a) $(1, 1)$:
$$1^2 = 4k(1)$$
$$1 = 4k$$
$$k = \tfrac{1}{4}$$

(b) $(2, 4)$:
$$(4)^2 = 4k(2)$$
$$16 = 8k$$
$$k = 2$$

(c) $(0, 0)$: $0^2 = 4k(0)$
k can be any real number.

(d) $(3, 3)$:
$$(3)^2 = 4k(3)$$
$$9 = 12k$$
$$k = \tfrac{9}{12} = \tfrac{3}{4}$$

73. Answers may vary. *Sample answer*:
$y = (x + 4)(x - 3)(x - 8)$ has intercepts at $x = -4$, $x = 3$, and $x = 8$.

74. Answers may vary. *Sample answer*:
$y = \left(x + \frac{3}{2}\right)(x - 4)\left(x - \frac{5}{2}\right)$ has intercepts at $x = -\frac{3}{2}$, $x = 4$, and $x = \frac{5}{2}$.

75. (a) If (x, y) is on the graph, then so is $(-x, y)$ by y-axis symmetry. Because $(-x, y)$ is on the graph, then so is $(-x, -y)$ by x-axis symmetry. So, the graph is symmetric with respect to the origin. The converse is not true. For example, $y = x^3$ has origin symmetry but is not symmetric with respect to either the x-axis or the y-axis.

(b) Assume that the graph has x-axis and origin symmetry. If (x, y) is on the graph, so is $(x, -y)$ by x-axis symmetry. Because $(x, -y)$ is on the graph, then so is $(-x, -(-y)) = (-x, y)$ by origin symmetry. Therefore, the graph is symmetric with respect to the y-axis. The argument is similar for y-axis and origin symmetry.

76. (a) Intercepts for $y = x^3 - x$:

y-intercept: $y = 0^3 - 0 = 0$; $(0, 0)$

x-intercepts: $0 = x^3 - x = x(x^2 - 1) = x(x - 1)(x + 1)$;

$(0, 0), (1, 0)\ (-1, 0)$

Intercepts for $y = x^2 + 2$:

y-intercept: $y = 0 + 2 = 2$; $(0, 2)$

x-intercepts: $0 = x^2 + 2$

None. y cannot equal 0.

(b) Symmetry with respect to the origin for $y = x^3 - x$ because

$-y = (-x)^3 - (-x) = -x^3 + x.$

Symmetry with respect to the y-axis for $y = x^2 + 2$ because

$y = (-x)^2 + 2 = x^2 + 2.$

(c)
$$x^3 - x = x^2 + 2$$
$$x^3 - x^2 - x - 2 = 0$$
$$(x - 2)(x^2 + x + 1) = 0$$
$$x = 2 \Rightarrow y = 6$$

Point of intersection : (2, 6)

Note: The polynomial $x^2 + x + 1$ has no real roots.

77. False. x-axis symmetry means that if $(-4, -5)$ is on the graph, then $(-4, 5)$ is also on the graph. For example, $(4, -5)$ is not on the graph of $x = y^2 - 29$, whereas $(-4, -5)$ is on the graph.

78. True. $f(4) = f(-4)$.

79. True. The x-intercepts are $\left(\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}, 0\right)$.

80. True. The x-intercept is $\left(-\dfrac{b}{2a}, 0\right)$.

Section P.2 Linear Models and Rates of Change

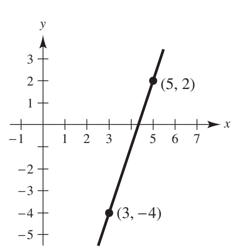
1. $m = 2$

2. $m = 0$

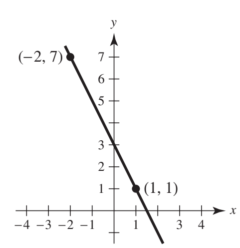
3. $m = -1$

4. $m = -12$

5. $m = \dfrac{2 - (-4)}{5 - 3} = \dfrac{6}{2} = 3$

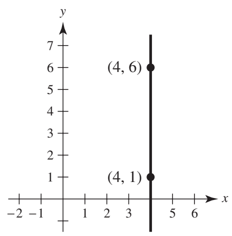


6. $m = \dfrac{7 - 1}{-2 - 1} = \dfrac{6}{-3} = -2$



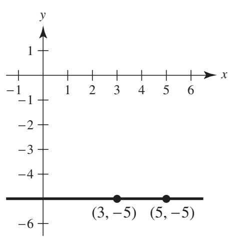
7. $m = \dfrac{1-6}{4-4} = \dfrac{-5}{0}$, undefined.

The line is vertical.

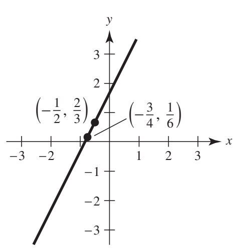


8. $m = \dfrac{-5-(-5)}{5-3} = \dfrac{0}{2} = 0$

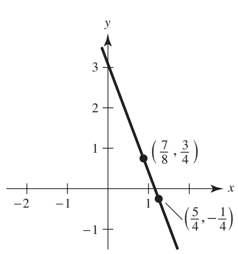
The line is horizontal.



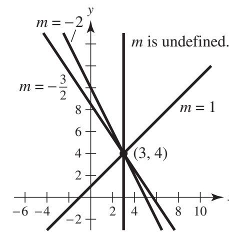
9. $m = \dfrac{\dfrac{2}{3}-\dfrac{1}{6}}{-\dfrac{1}{2}-\left(-\dfrac{3}{4}\right)} = \dfrac{\dfrac{1}{2}}{\dfrac{1}{4}} = 2$



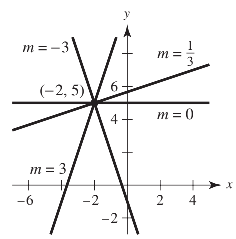
10. $m = \dfrac{\left(\dfrac{3}{4}\right)-\left(-\dfrac{1}{4}\right)}{\left(\dfrac{7}{8}\right)-\left(\dfrac{5}{4}\right)} = \dfrac{1}{-\dfrac{3}{8}} = -\dfrac{8}{3}$



11.



12.



13. Because the slope is 0, the line is horizontal and its equation is $y = 2$. Therefore, three additional points are $(0, 2)$, $(1, 2)$, $(5, 2)$.

14. Because the slope is undefined, the line is vertical and its equation is $x = -4$. Therefore, three additional points are $(-4, 0)$, $(-4, 1)$, $(-4, 2)$.

15. The equation of this line is

$$y - 7 = -3(x - 1)$$
$$y = -3x + 10.$$

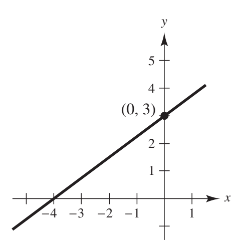
Therefore, three additional points are (0, 10), (2, 4), and (3, 1).

16. The equation of this line is

$$y + 2 = 2(x + 2)$$
$$y = 2x + 2.$$

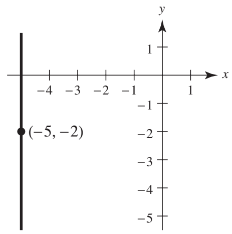
Therefore, three additional points are $(-3, -4)$, $(-1, 0)$, and (0, 2).

17.
$$y = \tfrac{3}{4}x + 3$$
$$4y = 3x + 12$$
$$0 = 3x - 4y + 12$$

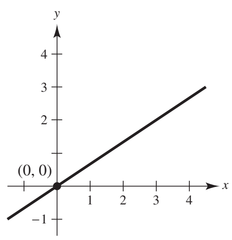


18. The slope is undefined so the line is vertical.

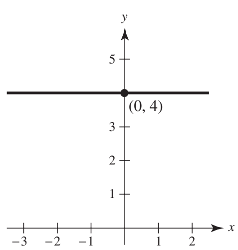
$$x = -5$$
$$x + 5 = 0$$



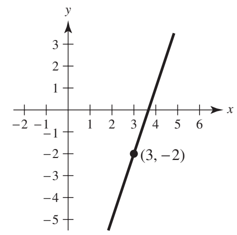
19.
$$y = \tfrac{2}{3}x$$
$$3y = 2x$$
$$0 = 2x - 3y$$



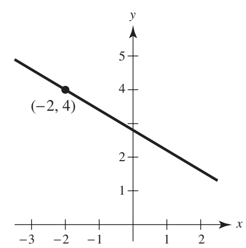
20.
$$y = 4$$
$$y - 4 = 0$$



21.
$$y + 2 = 3(x - 3)$$
$$y + 2 = 3x - 9$$
$$y = 3x - 11$$
$$0 = 3x - y - 11$$

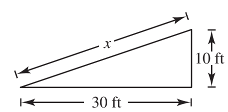


22.
$$y - 4 = -\tfrac{3}{5}(x + 2)$$
$$5y - 20 = -3x - 6$$
$$3x + 5y - 14 = 0$$



23. (a) Slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{1}{3}$

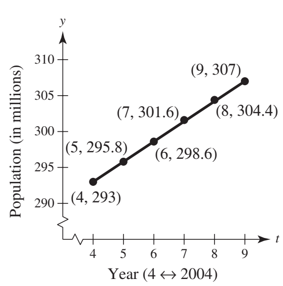
(b)



By the Pythagorean Theorem,

$$x^2 = 30^2 + 10^2 = 1000$$
$$x = 10\sqrt{10} \approx 31.623 \text{ feet.}$$

24. (a)



(b) The slopes are:
$$\frac{295.8 - 293.0}{5 - 4} = 2.8$$
$$\frac{298.6 - 295.8}{6 - 5} = 2.8$$
$$\frac{301.6 - 298.6}{7 - 6} = 3.0$$
$$\frac{304.4 - 301.6}{8 - 7} = 2.8$$
$$\frac{307.0 - 304.4}{9 - 8} = 2.6$$

The population increased least rapidly from 2008 to 2009.

(c) Average rate of change from 2004 to 2009:

$$\frac{307.0 - 293.0}{9 - 4} = \frac{14}{5}$$
$$= 2.8 \text{ million per yr}$$

(d) For 2020, $t = 20$ and $y \approx 16(2.8) + 293.0 = 337.8$ million.

$\left[\text{Equivalently, } y \approx 11(2.8) + 307.0 = 337.8.\right]$

25. $y = 4x - 3$

The slope is $m = 4$ and the y-intercept is $(0, -3)$.

26. $-x + y = 1$

$y = x + 1$

The slope is $m = 1$ and the y-intercept is $(0, 1)$.

27. $x + 5y = 20$

$y = -\frac{1}{5}x + 4$

Therefore, the slope is $m = -\frac{1}{5}$ and the y-intercept is $(0, 4)$.

28. $6x - 5y = 15$

$y = \frac{6}{5}x - 3$

Therefore, the slope is $m = \frac{6}{5}$ and the y-intercept is $(0, -3)$.

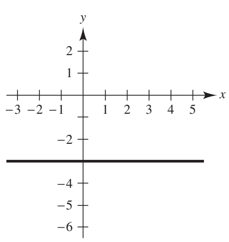
29. $x = 4$

The line is vertical. Therefore, the slope is undefined and there is no y-intercept.

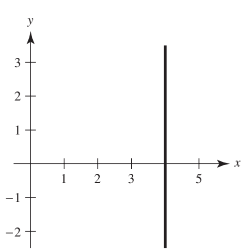
30. $y = -1$

The line is horizontal. Therefore, the slope is $m = 0$ and the y-intercept is $(0, -1)$.

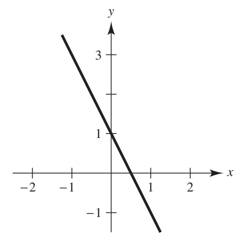
31. $y = -3$



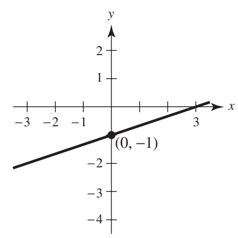
32. $x = 4$



33. $y = -2x + 1$

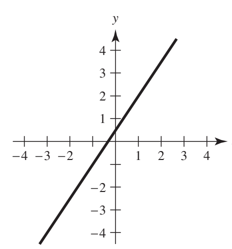


34. $y = \frac{1}{3}x - 1$



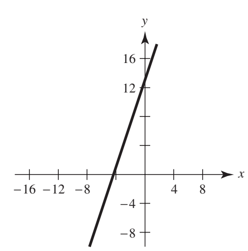
35. $y - 2 = \frac{3}{2}(x - 1)$

$y = \frac{3}{2}x + \frac{1}{2}$



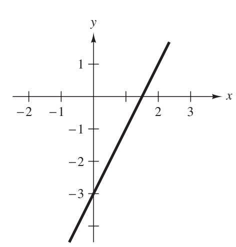
36. $y - 1 = 3(x + 4)$

$y = 3x + 13$



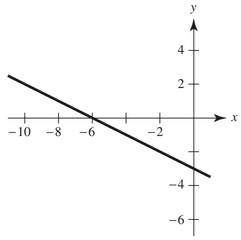
37. $2x - y - 3 = 0$

$y = 2x - 3$

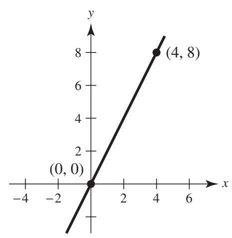


38. $x + 2y + 6 = 0$

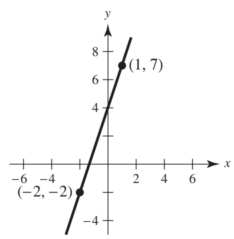
$$y = -\tfrac{1}{2}x - 3$$



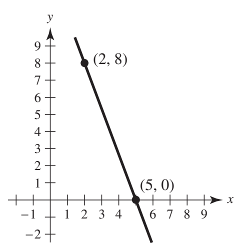
39.
$$m = \frac{8 - 0}{4 - 0} = 2$$
$$y - 0 = 2(x - 0)$$
$$y = 2x$$
$$0 = 2x - y$$



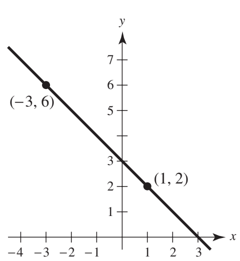
40.
$$m = \frac{7 - (-2)}{1 - (-2)} = \frac{9}{3} = 3$$
$$y - (-2) = 3(x - (-2))$$
$$y + 2 = 3(x + 2)$$
$$y = 3x + 4$$
$$0 = 3x - y + 4$$



41.
$$m = \frac{8 - 0}{2 - 5} = -\frac{8}{3}$$
$$y - 0 = -\frac{8}{3}(x - 5)$$
$$y = -\frac{8}{3}x + \frac{40}{3}$$
$$8x + 3y - 40 = 0$$



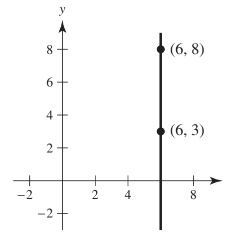
42.
$$m = \frac{6 - 2}{-3 - 1} = \frac{4}{-4} = -1$$
$$y - 2 = -1(x - 1)$$
$$y - 2 = -x + 1$$
$$x + y - 3 = 0$$



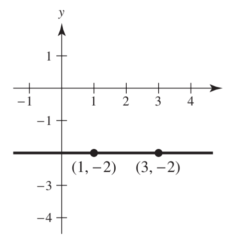
43.
$$m = \frac{8 - 3}{6 - 6} = \frac{5}{0}, \text{ undefined}$$

The line is horizontal.

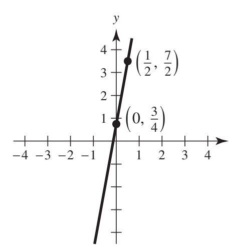
$$x = 6$$
$$x - 6 = 0$$



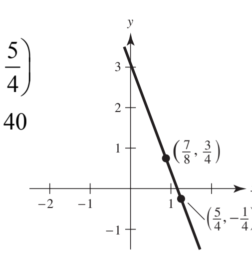
44.
$$m = \frac{-2 - (-2)}{3 - 1} = \frac{0}{2} = 0$$
$$y = -2$$
$$y + 2 = 0$$



45.
$$m = \frac{\frac{7}{2} - \frac{3}{4}}{\frac{1}{2} - 0} = \frac{\frac{11}{4}}{\frac{1}{2}} = \frac{11}{2}$$
$$y - \frac{3}{4} = \frac{11}{2}(x - 0)$$
$$y = \frac{11}{2}x + \frac{3}{4}$$
$$0 = 22x - 4y + 3$$

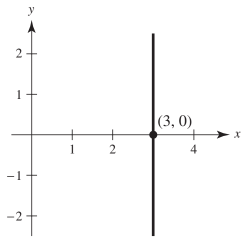


46.
$$m = \frac{\left(\frac{3}{4}\right) - \left(-\frac{1}{4}\right)}{\left(\frac{7}{8}\right) - \left(\frac{5}{4}\right)} = \frac{1}{-\frac{3}{8}} = -\frac{8}{3}$$
$$y + \frac{1}{4} = \frac{-8}{3}\left(x - \frac{5}{4}\right)$$
$$12y + 3 = -32x + 40$$
$$32x + 12y - 37 = 0$$



47. $x = 3$

$x - 3 = 0$

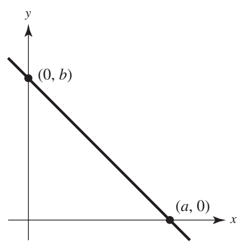


48. $m = -\dfrac{b}{a}$

$$y = \frac{-b}{a}x + b$$

$$\frac{b}{a}x + y = b$$

$$\frac{x}{a} + \frac{y}{b} = 1$$



49. $\dfrac{x}{2} + \dfrac{y}{3} = 1$

$3x + 2y - 6 = 0$

50. $\dfrac{x}{-\frac{2}{3}} + \dfrac{y}{-2} = 1$

$$\frac{-3x}{2} - \frac{y}{2} = 1$$

$$3x + y = -2$$

$$3x + y + 2 = 0$$

51. $\dfrac{x}{a} + \dfrac{y}{a} = 1$

$$\frac{1}{a} + \frac{2}{a} = 1$$

$$\frac{3}{a} = 1$$

$$a = 3 \Rightarrow x + y = 3$$

$$x + y - 3 = 0$$

52. $\dfrac{x}{a} + \dfrac{y}{a} = 1$

$$\frac{-3}{a} + \frac{4}{a} = 1$$

$$\frac{1}{a} = 1$$

$$a = 1 \Rightarrow x + y = 1$$

$$x + y - 1 = 0$$

53. $\dfrac{x}{2a} + \dfrac{y}{a} = 1$

$$\frac{9}{2a} + \frac{-2}{a} = 1$$

$$\frac{9 - 4}{2a} = 1$$

$$5 = 2a$$

$$a = \frac{5}{2}$$

$$\frac{x}{2\left(\frac{5}{2}\right)} + \frac{y}{\left(\frac{5}{2}\right)} = 1$$

$$\frac{x}{5} + \frac{2y}{5} = 1$$

$$x + 2y = 5$$

$$x + 2y - 5 = 0$$

54. $\dfrac{x}{a} + \dfrac{y}{-a} = 1$

$$\frac{\left(-\frac{2}{3}\right)}{a} + \frac{(-2)}{-a} = 1$$

$$-\frac{2}{3} + 2 = a$$

$$a = \frac{4}{3}$$

$$\frac{x}{\left(\frac{4}{3}\right)} + \frac{y}{\left(-\frac{4}{3}\right)} = 1$$

$$x - y = \frac{4}{3}$$

$$3x - 3y - 4 = 0$$

55. The given line is vertical.

(a) $x = -7$, or $x + 7 = 0$

(b) $y = -2$, or $y + 2 = 0$

56. The given line is horizontal.

(a) $y = 0$

(b) $x = -1$, or $x + 1 = 0$

57. $x - y = -2$

$y = x + 2$

$m = 1$

(a) $y - 5 = 1(x - 2)$

$y - 5 = x - 2$

$x - y + 3 = 0$

(b) $y - 5 = -1(x - 2)$

$y - 5 = -x + 2$

$x + y - 7 = 0$

58. $x + y = 7$

$y = -x + 7$

$m = -1$

(a) $y - 2 = -1(x + 3)$

$y - 2 = -x - 3$

$x + y + 1 = 0$

(b) $y - 2 = 1(x + 3)$

$y - 2 = x + 3$

$0 = x - y + 5$

59. $4x - 2y = 3$

$y = 2x - \frac{3}{2}$

$m = 2$

(a) $y - 1 = 2(x - 2)$

$y - 1 = 2x - 4$

$0 = 2x - y - 3$

(b) $y - 1 = -\frac{1}{2}(x - 2)$

$2y - 2 = -x + 2$

$x + 2y - 4 = 0$

60. $7x + 4y = 8$

$4y = -7x + 8$

$y = \frac{-7}{4}x + 2$

$m = -\frac{7}{4}$

(a) $y + \frac{1}{2} = \frac{-7}{4}\left(x - \frac{5}{6}\right)$

$y + \frac{1}{2} = \frac{-7}{4}x + \frac{35}{24}$

$24y + 12 = -42x + 35$

$42x + 24y - 23 = 0$

(b) $y + \frac{1}{2} = \frac{4}{7}\left(x - \frac{5}{6}\right)$

$42y + 21 = 24x - 20$

$24x - 42y - 41 = 0$

61. $5x - 3y = 0$

$y = \frac{5}{3}x$

$m = \frac{5}{3}$

(a) $y - \frac{7}{8} = \frac{5}{3}\left(x - \frac{3}{4}\right)$

$24y - 21 = 40x - 30$

$0 = 40x - 24y - 9$

(b) $y - \frac{7}{8} = -\frac{3}{5}\left(x - \frac{3}{4}\right)$

$40y - 35 = -24x + 18$

$24x + 40y - 53 = 0$

62. $3x + 4y = 7$

$4y = -3x + 7$

$y = -\frac{3}{4}x + \frac{7}{4}$

$m = -\frac{3}{4}$

(a) $y - (-5) = -\frac{3}{4}(x - 4)$

$y + 5 = -\frac{3}{4}x + 3$

$4y + 20 = -3x + 12$

$3x + 4y + 8 = 0$

(b) $y - (-5) = \frac{4}{3}(x - 4)$

$y + 5 = \frac{4}{3}x - \frac{16}{3}$

$3y + 15 = 4x - 16$

$0 = 4x - 3y - 31$

63. The slope is 250.

$V = 1850$ when $t = 2$.

$V = 250(t - 2) + 1850$

$= 250t + 1350$

64. The slope is 4.50.

$V = 156$ when $t = 2$.

$V = 4.5(t - 2) + 156$

$= 4.5t + 147$

65. The slope is -1600.

$V = 17{,}200$ when $t = 2$.

$V = -1600(t - 2) + 17{,}200$

$= -1600t + 20{,}400$

66. The slope is -5600.

$V = 245{,}000$ when $t = 2$.

$V = -5600(t - 2) + 245{,}000$

$= -5600t + 256{,}200$

67. $m_1 = \dfrac{1-0}{-2-(-1)} = -1$

$m_2 = \dfrac{-2-0}{2-(-1)} = -\dfrac{2}{3}$

$m_1 \neq m_2$

The points are not collinear.

68. $m_1 = \dfrac{-6-4}{7-0} = -\dfrac{10}{7}$

$m_2 = \dfrac{11-4}{-5-0} = -\dfrac{7}{5}$

$m_1 \neq m_2$

The points are not collinear.

69. Equations of perpendicular bisectors:

$y - \dfrac{c}{2} = \dfrac{a-b}{c}\left(x - \dfrac{a+b}{2}\right)$

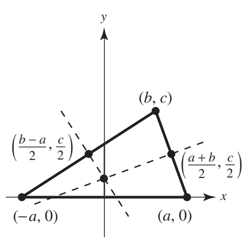
$y - \dfrac{c}{2} = \dfrac{a+b}{-c}\left(x - \dfrac{b-a}{2}\right)$

Setting the right-hand sides of the two equations equal and solving for x yields $x = 0$.

Letting $x = 0$ in either equation gives the point of intersection:

$\left(0, \dfrac{-a^2+b^2+c^2}{2c}\right)$.

This point lies on the third perpendicular bisector, $x = 0$.

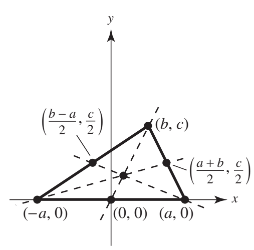


70. Equations of medians:

$y = \dfrac{c}{b}x$

$y = \dfrac{c}{3a+b}(x+a)$

$y = \dfrac{c}{-3a+b}(x-a)$



Solving simultaneously, the point of intersection is $\left(\dfrac{b}{3}, \dfrac{c}{3}\right)$.

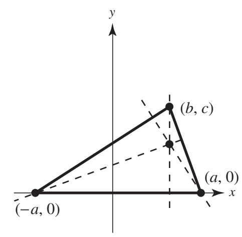
71. Equations of altitudes:

$y = \dfrac{a-b}{c}(x+a)$

$x = b$

$y = -\dfrac{a+b}{c}(x-a)$

Solving simultaneously, the point of intersection is $\left(b, \dfrac{a^2-b^2}{c}\right)$.



72. The slope of the line segment from $\left(\dfrac{b}{3}, \dfrac{c}{3}\right)$ to $\left(b, \dfrac{a^2-b^2}{c}\right)$ is:

$$m_1 = \frac{\left[\left(a^2-b^2\right)/c\right] - (c/3)}{b - (b/3)} = \frac{\left(3a^2-3b^2-c^2\right)/(3c)}{(2b)/3} = \frac{3a^2-3b^2-c^2}{2bc}$$

The slope of the line segment from $\left(\dfrac{b}{3}, \dfrac{c}{3}\right)$ to $\left(0, \dfrac{-a^2+b^2+c^2}{2c}\right)$ is:

$$m_2 = \frac{\left[\left(-a^2+b^2+c^2\right)/(2c)\right] - (c/3)}{0 - (b/3)} = \frac{\left(-3a^2+3b^2+3c^2-2c^2\right)/(6c)}{-b/3} = \frac{3a^2-3b^2-c^2}{2bc}$$

$m_1 = m_2$

Therefore, the points are collinear.

73. $ax + by = 4$

(a) The line is parallel to the x-axis if $a = 0$ and $b \neq 0$.

(b) The line is parallel to the y-axis if $b = 0$ and $a \neq 0$.

(c) Answers will vary. *Sample answer*: $a = -5$ and $b = 8$.

$$-5x + 8y = 4$$
$$y = \frac{1}{8}(5x + 4) = \frac{5}{8}x + \frac{1}{2}$$

(d) The slope must be $-\frac{5}{2}$.

Answers will vary. *Sample answer*: $a = 5$ and $b = 2$.

$$5x + 2y = 4$$
$$y = \frac{1}{2}(-5x + 4) = -\frac{5}{2}x + 2$$

(e) $a = \frac{5}{2}$ and $b = 3$.

$$\frac{5}{2}x + 3y = 4$$
$$5x + 6y = 8$$

74. (a) Lines c, d, e and f have positive slopes.

(b) Lines a and b have negative slopes.

(c) Lines c and e appear parallel.
Lines d and f appear parallel.

(d) Lines b and f appear perpendicular.
Lines b and d appear perpendicular.

75. Find the equation of the line through the points (0, 32) and (100, 212).

$$m = \frac{180}{100} = \frac{9}{5}$$
$$F - 32 = \frac{9}{5}(C - 0)$$
$$F = \frac{9}{5}C + 32$$

or

$$C = \frac{1}{9}(5F - 160)$$
$$5F - 9C - 160 = 0$$

For $F = 72°$, $C \approx 22.2°$.

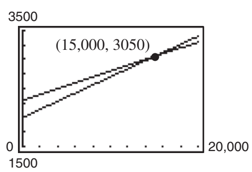
76. $C = 0.51x + 200$

For $x = 137$, $C = 0.51(137) + 200 = \$269.87$.

77. (a) Current job: $W_1 = 0.07s + 2000$

New job offer: $W_2 = 0.05s + 2300$

(b)



Using a graphing utility, the point of intersection is (15,000, 3050).

Analytically, $W_1 = W_2$

$$0.07s + 2000 = 0.05s + 2300$$
$$0.02s = 300$$
$$s = 15{,}000$$

So, $W_1 = W_2 = 0.07(15{,}000) + 2000 = 3050$.

When sales exceed \$15,000, the current job pays more.

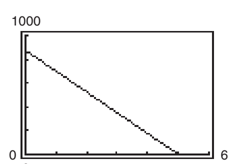
(c) No, if you can sell \$20,000 worth of goods, then $W_1 > W_2$.

(**Note:** $W_1 = 3400$ and $W_2 = 3300$ when $s = 20{,}000$.)

78. (a) Depreciation per year:

$$\frac{875}{5} = \$175$$
$$y = 875 - 175x$$

where $0 \leq x \leq 5$.



(b) $y = 875 - 175(2) = \$525$

(c)
$$200 = 875 - 175x$$
$$175x = 675$$
$$x \approx 3.86 \text{ years}$$

79. (a) Two points are (50, 780) and (47, 825).

The slope is

$$m = \frac{825 - 780}{47 - 50} = \frac{45}{-3} = -15.$$

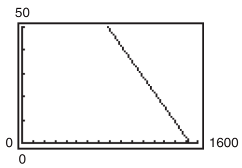
$$p - 780 = -15(x - 50)$$

$$p = -15x + 750 + 780 = -15x + 1530$$

or

$$x = \frac{1}{15}(1530 - p)$$

(b)



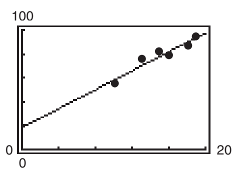
If $p = 855$, then $x = 45$ units.

(c) If $p = 795$, then $x = \frac{1}{15}(1530 - 795) = 49$ units

80. (a) $y = 18.91 + 3.97x$

$(x = \text{quiz score}, \ y = \text{test score})$

(b)

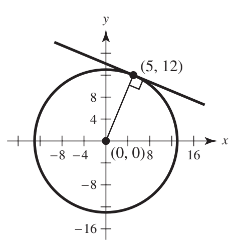


(c) If $x = 17$, $y = 18.91 + 3.97(17) = 86.4$.

(d) The slope shows the average increase in exam score for each unit increase in quiz score.

(e) The points would shift vertically upward 4 units. The new regression line would have a y-intercept 4 greater than before: $y = 22.91 + 3.97x$.

81. The tangent line is perpendicular to the line joining the point (5, 12) and the center (0, 0).



Slope of the line joining (5, 12) and (0, 0) is $\frac{12}{5}$.

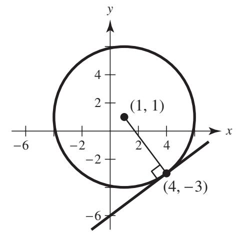
The equation of the tangent line is

$$y - 12 = \frac{-5}{12}(x - 5)$$

$$y = \frac{-5}{12}x + \frac{169}{12}$$

$$5x + 12y - 169 = 0.$$

82. The tangent line is perpendicular to the line joining the point $(4, -3)$ and the center of the circle, (1, 1).



Slope of the line joining (1, 1) and $(4, -3)$ is

$$\frac{1 + 3}{1 - 4} = \frac{-4}{3}.$$

Tangent line:

$$y + 3 = \frac{3}{4}(x - 4)$$

$$y = \frac{3}{4}x - 6$$

$$0 = 3x - 4y - 24$$

83. $x - y - 2 = 0 \Rightarrow d = \dfrac{|1(-2) + (-1)(1) - 2|}{\sqrt{1^2 + 1^2}} = \dfrac{5}{\sqrt{2}} = \dfrac{5\sqrt{2}}{2}$

84. $4x + 3y - 10 = 0 \Rightarrow d = \dfrac{|4(2) + 3(3) - 10|}{\sqrt{4^2 + 3^2}} = \dfrac{7}{5}$

85. A point on the line $x + y = 1$ is (0, 1). The distance from the point (0, 1) to $x + y - 5 = 0$ is

$$d = \frac{|1(0) + 1(1) - 5|}{\sqrt{1^2 + 1^2}} = \frac{|1 - 5|}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}.$$

86. A point on the line $3x - 4y = 1$ is $(-1, -1)$. The distance from the point $(-1, -1)$ to $3x - 4y - 10 = 0$ is

$$d = \frac{|3(-1) - 4(-1) - 10|}{\sqrt{3^2 + (-4)^2}} = \frac{|-3 + 4 - 10|}{5} = \frac{9}{5}.$$

87. If $A = 0$, then $By + C = 0$ is the horizontal line $y = -C/B$. The distance to (x_1, y_1) is

$$d = \left|y_1 - \left(\frac{-C}{B}\right)\right| = \frac{|By_1 + C|}{|B|} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

If $B = 0$, then $Ax + C = 0$ is the vertical line $x = -C/A$. The distance to (x_1, y_1) is

$$d = \left|x_1 - \left(\frac{-C}{A}\right)\right| = \frac{|Ax_1 + C|}{|A|} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

(Note that A and B cannot both be zero.) The slope of the line $Ax + By + C = 0$ is $-A/B$.

The equation of the line through (x_1, y_1) perpendicular to $Ax + By + C = 0$ is:

$$\begin{aligned} y - y_1 &= \frac{B}{A}(x - x_1) \\ Ay - Ay_1 &= Bx - Bx_1 \\ Bx_1 - Ay_1 &= Bx - Ay \end{aligned}$$

The point of intersection of these two lines is:

$$\begin{aligned} Ax + By &= -C &&\Rightarrow A^2x + ABy = -AC && (1) \\ Bx - Ay &= Bx_1 - Ay_1 &&\Rightarrow \underline{B^2x - ABy} = \underline{B^2x_1 - ABy_1} && (2) \\ & && \quad (A^2 + B^2)x = -AC + B^2x_1 - ABy_1 && \text{(By adding equations (1) and (2))} \\ & && \quad x = \frac{-AC + B^2x_1 - ABy_1}{A^2 + B^2} \end{aligned}$$

$$\begin{aligned} Ax + By &= -C &&\Rightarrow ABx + B^2y = -BC && (3) \\ Bx - Ay &= Bx_1 - Ay_1 &&\Rightarrow \underline{-ABx + A^2y} = \underline{-ABx_1 + A^2y_1} && (4) \\ & && \quad (A^2 + B^2)y = -BC - ABx_1 + A^2y_1 && \text{(By adding equations (3) and (4))} \\ & && \quad y = \frac{-BC - ABx_1 + A^2y_1}{A^2 + B^2} \end{aligned}$$

$$\left(\frac{-AC + B^2x_1 - ABy_1}{A^2 + B^2}, \frac{-BC - ABx_1 + A^2y_1}{A^2 + B^2}\right) \text{ point of intersection}$$

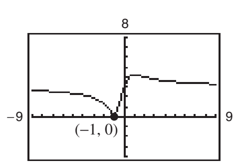
The distance between (x_1, y_1) and this point gives you the distance between (x_1, y_1) and the line $Ax + By + C = 0$.

$$\begin{aligned} d &= \sqrt{\left[\frac{-AC + B^2x_1 - ABy_1}{A^2 + B^2} - x_1\right]^2 + \left[\frac{-BC - ABx_1 + A^2y_1}{A^2 + B^2} - y_1\right]^2} \\ &= \sqrt{\left[\frac{-AC - ABy_1 - A^2x_1}{A^2 + B^2}\right]^2 + \left[\frac{-BC - ABx_1 - B^2y_1}{A^2 + B^2}\right]^2} \\ &= \sqrt{\left[\frac{-A(C + By_1 + Ax_1)}{A^2 + B^2}\right]^2 + \left[\frac{-B(C + Ax_1 + By_1)}{A^2 + B^2}\right]^2} = \sqrt{\frac{(A^2 + B^2)(C + Ax_1 + By_1)^2}{(A^2 + B^2)^2}} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}} \end{aligned}$$

88. $y = mx + 4 \Rightarrow mx + (-1)y + 4 = 0$

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}} = \frac{|m3 + (-1)(1) + 4|}{\sqrt{m^2 + (-1)^2}} = \frac{|3m + 3|}{\sqrt{m^2 + 1}}$$

The distance is 0 when $m = -1$. In this case, the line $y = -x + 4$ contains the point (3, 1).

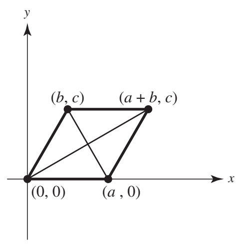


89. For simplicity, let the vertices of the rhombus be (0, 0), $(a, 0)$, (b, c), and $(a + b, c)$, as shown in the figure.

The slopes of the diagonals are then $m_1 = \dfrac{c}{a + b}$ and $m_2 = \dfrac{c}{b - a}$. Because the sides of the rhombus are equal, $a^2 = b^2 + c^2$, and you have

$$m_1 m_2 = \frac{c}{a + b} \cdot \frac{c}{b - a} = \frac{c^2}{b^2 - a^2} = \frac{c^2}{-c^2} = -1.$$

Therefore, the diagonals are perpendicular.



90. For simplicity, let the vertices of the quadrilateral be (0, 0), (a, 0), (b, c), and (d, e), as shown in the figure. The midpoints of the sides are

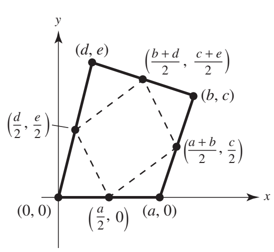
$$\left(\frac{a}{2}, 0\right), \left(\frac{a + b}{2}, \frac{c}{2}\right), \left(\frac{b + d}{2}, \frac{c + e}{2}\right), \text{ and } \left(\frac{d}{2}, \frac{e}{2}\right).$$

The slope of the opposite sides are equal:

$$\frac{\frac{c}{2} - 0}{\frac{a + b}{2} - \frac{a}{2}} = \frac{\frac{c + e}{2} - \frac{e}{2}}{\frac{b + d}{2} - \frac{d}{2}} = \frac{c}{b}$$

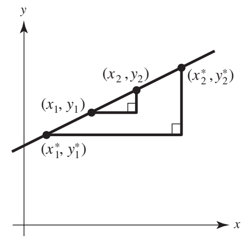
$$\frac{0 - \frac{e}{2}}{\frac{a}{2} - \frac{d}{2}} = \frac{\frac{c}{2} - \frac{c + e}{2}}{\frac{a + b}{2} - \frac{b + d}{2}} = -\frac{e}{a - d}$$

Therefore, the figure is a parallelogram.



91. Consider the figure below in which the four points are collinear. Because the triangles are similar, the result immediately follows.

$$\frac{y_2^* - y_1^*}{x_2^* - x_1^*} = \frac{y_2 - y_1}{x_2 - x_1}$$



92. If $m_1 = -1/m_2$, then $m_1 m_2 = -1$. Let L_3 be a line with slope m_3 that is perpendicular to L_1. Then $m_1 m_3 = -1$. So, $m_2 = m_3 \Rightarrow L_2$ and L_3 are parallel. Therefore, L_2 and L_1 are also perpendicular.

93. True.

$$ax + by = c_1 \Rightarrow y = -\frac{a}{b}x + \frac{c_1}{b} \Rightarrow m_1 = -\frac{a}{b}$$

$$bx - ay = c_2 \Rightarrow y = \frac{b}{a}x - \frac{c_2}{a} \Rightarrow m_2 = \frac{b}{a}$$

$$m_2 = -\frac{1}{m_1}$$

94. False; if m_1 is positive, then $m_2 = -1/m_1$ is negative.

95. True. The slope must be positive.

96. True. The general form $Ax + By + C = 0$ includes both horizontal and vertical lines.

Section P.3 Functions and Their Graphs

1. (a) $f(0) = 7(0) - 4 = -4$

(b) $f(-3) = 7(-3) - 4 = -25$

(c) $f(b) = 7(b) - 4 = 7b - 4$

(d) $f(x - 1) = 7(x - 1) - 4 = 7x - 11$

2. (a) $f(-4) = \sqrt{-4 + 5} = \sqrt{1} = 1$

(b) $f(11) = \sqrt{11 + 5} = \sqrt{16} = 4$

(c) $f(4) = \sqrt{4 + 5} = \sqrt{9} = 3$

(d) $f(x + \Delta x) = \sqrt{x + \Delta x + 5}$

3. (a) $g(0) = 5 - 0^2 = 5$

(b) $g(\sqrt{5}) = 5 - (\sqrt{5})^2 = 5 - 5 = 0$

(c) $g(-2) = 5 - (-2)^2 = 5 - 4 = 1$

(d) $g(t - 1) = 5 - (t - 1)^2 = 5 - (t^2 - 2t + 1)$
$= 4 + 2t - t^2$

4. (a) $g(4) = 4^2(4 - 4) = 0$

(b) $g\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^2\left(\frac{3}{2} - 4\right) = \frac{9}{4}\left(-\frac{5}{2}\right) = -\frac{45}{8}$

(c) $g(c) = c^2(c - 4) = c^3 - 4c^2$

(d) $g(t + 4) = (t + 4)^2(t + 4 - 4)$
$= (t + 4)^2 t = t^3 + 8t^2 + 16t$

5. (a) $f(0) = \cos(2(0)) = \cos 0 = 1$

(b) $f\left(-\frac{\pi}{4}\right) = \cos\left(2\left(-\frac{\pi}{4}\right)\right) = \cos\left(-\frac{\pi}{2}\right) = 0$

(c) $f\left(\frac{\pi}{3}\right) = \cos\left(2\left(\frac{\pi}{3}\right)\right) = \cos\frac{2\pi}{3} = -\frac{1}{2}$

(d) $f(\pi) = \cos(2(\pi)) = 1$

6. (a) $f(\pi) = \sin \pi = 0$

(b) $f\left(\frac{5\pi}{4}\right) = \sin\left(\frac{5\pi}{4}\right) = \frac{-\sqrt{2}}{2}$

(c) $f\left(\frac{2\pi}{3}\right) = \sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$

(d) $f\left(-\frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$

7. $\frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{(x + \Delta x)^3 - x^3}{\Delta x} = \frac{x^3 + 3x^2\Delta x + 3x^2(\Delta x)^2 + (\Delta x)^3 - x^3}{\Delta x} = 3x^2 + 3x\Delta x + (\Delta x)^2, \ \Delta x \neq 0$

8. $\frac{f(x) - f(1)}{x - 1} = \frac{3x - 1 - (3 - 1)}{x - 1} = \frac{3(x - 1)}{x - 1} = 3, \ x \neq 1$

9. $\frac{f(x) - f(2)}{x - 2} = \frac{\left(1/\sqrt{x - 1} - 1\right)}{x - 2}$

$= \frac{1 - \sqrt{x - 1}}{(x - 2)\sqrt{x - 1}} \cdot \frac{1 + \sqrt{x - 1}}{1 + \sqrt{x - 1}} = \frac{2 - x}{(x - 2)\sqrt{x - 1}\left(1 + \sqrt{x - 1}\right)} = \frac{-1}{\sqrt{x - 1}\left(1 + \sqrt{x - 1}\right)}, \ x \neq 2$

10. $\frac{f(x) - f(1)}{x - 1} = \frac{x^3 - x - 0}{x - 1} = \frac{x(x + 1)(x - 1)}{x - 1} = x(x + 1), \ x \neq 1$

11. $f(x) = 4x^2$

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

12. $g(x) = x^2 - 5$

Domain: $(-\infty, \infty)$

Range: $[-5, \infty)$

13. $f(x) = x^3$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

14. $h(x) = 4 - x^2$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 4]$

15. $g(x) = \sqrt{6x}$

Domain: $6x \geq 0$
$x \geq 0 \Rightarrow [0, \infty)$

Range: $[0, \infty)$

16. $h(x) = -\sqrt{x + 3}$

Domain: $x + 3 \geq 0 \Rightarrow [-3, \infty)$

Range: $(-\infty, 0]$

17. $f(x) = \sqrt{16 - x^2}$

$16 - x^2 \geq 0 \Rightarrow x^2 \leq 16$

Domain: $[-4, 4]$

Range: $[0, 4]$

Note: $y = \sqrt{16 - x^2}$ is a semicircle of radius 4.

18. $f(x) = |x - 3|$

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

19. $f(t) = \sec\dfrac{\pi t}{4}$

$\dfrac{\pi t}{4} \neq \dfrac{(2n + 1)\pi}{2} \Rightarrow t \neq 4n + 2$

Domain: all $t \neq 4n + 2$, n an integer

Range: $(-\infty, -1] \cup [1, \infty)$

20. $h(t) = \cot t$

Domain: all $t = n\pi$, n an integer

Range: $(-\infty, \infty)$

21. $f(x) = \dfrac{3}{x}$

Domain: all $x \neq 0 \Rightarrow (-\infty, 0) \cup (0, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$

22. $f(x) = \dfrac{x - 2}{x + 4}$

Domain: all $x \neq -4$

Range: all $y \neq 1$

[**Note:** You can see that the range is all $y \neq 1$ by graphing f.]

23. $f(x) = \sqrt{x} + \sqrt{1 - x}$

$x \geq 0$ and $1 - x \geq 0$

$x \geq 0$ and $x \leq 1$

Domain: $0 \leq x \leq 1 \Rightarrow [0, 1]$

24. $f(x) = \sqrt{x^2 - 3x + 2}$

$x^2 - 3x + 2 \geq 0$

$(x - 2)(x - 1) \geq 0$

Domain: $x \geq 2$ or $x \leq 1$

Domain: $(-\infty, 1] \cup [2, \infty)$

25. $g(x) = \dfrac{2}{1 - \cos x}$

$1 - \cos x \neq 0$

$\cos x \neq 1$

Domain: all $x \neq 2n\pi$, n an integer

26. $h(x) = \dfrac{1}{\sin x - (1/2)}$

$\sin x - \dfrac{1}{2} \neq 0$

$\sin x \neq \dfrac{1}{2}$

Domain: all $x \neq \dfrac{\pi}{6} + 2n\pi, \dfrac{5\pi}{6} + 2n\pi$, n integer

27. $f(x) = \dfrac{1}{|x + 3|}$

$|x + 3| \neq 0$

$x + 3 \neq 0$

Domain: all $x \neq -3$

Domain: $(-\infty, -3) \cup (-3, \infty)$

28. $g(x) = \dfrac{1}{|x^2 - 4|}$

$|x^2 - 4| \neq 0$

$(x - 2)(x + 2) \neq 0$

Domain: all $x \neq \pm 2$

Domain: $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

29. $f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \geq 0 \end{cases}$

(a) $f(-1) = 2(-1) + 1 = -1$

(b) $f(0) = 2(0) + 2 = 2$

(c) $f(2) = 2(2) + 2 = 6$

(d) $f(t^2 + 1) = 2(t^2 + 1) + 2 = 2t^2 + 4$

(**Note:** $t^2 + 1 \geq 0$ for all t.)

Domain: $(-\infty, \infty)$

Range: $(-\infty, 1) \cup [2, \infty)$

30. $f(x) = \begin{cases} x^2 + 2, & x \leq 1 \\ 2x^2 + 2, & x > 1 \end{cases}$

(a) $f(-2) = (-2)^2 + 2 = 6$

(b) $f(0) = 0^2 + 2 = 2$

(c) $f(1) = 1^2 + 2 = 3$

(d) $f(s^2 + 2) = 2(s^2 + 2)^2 + 2 = 2s^4 + 8s^2 + 10$

(**Note:** $s^2 + 2 > 1$ for all s.)

Domain: $(-\infty, \infty)$

Range: $[2, \infty)$

31. $f(x) = \begin{cases} |x| + 1, & x < 1 \\ -x + 1, & x \ge 1 \end{cases}$

(a) $f(-3) = |-3| + 1 = 4$

(b) $f(1) = -1 + 1 = 0$

(c) $f(3) = -3 + 1 = -2$

(d) $f(b^2 + 1) = -(b^2 + 1) + 1 = -b^2$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 0] \cup [1, \infty)$

32. $f(x) = \begin{cases} \sqrt{x + 4}, & x \le 5 \\ (x - 5)^2, & x > 5 \end{cases}$

(a) $f(-3) = \sqrt{-3 + 4} = \sqrt{1} = 1$

(b) $f(0) = \sqrt{0 + 4} = 2$

(c) $f(5) = \sqrt{5 + 4} = 3$

(d) $f(10) = (10 - 5)^2 = 25$

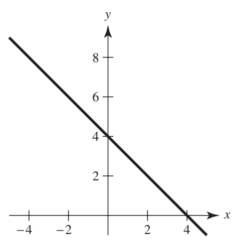
Domain: $[-4, \infty)$

Range: $[0, \infty)$

33. $f(x) = 4 - x$

Domain: $(-\infty, \infty)$

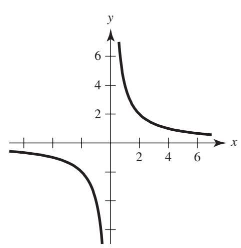
Range: $(-\infty, \infty)$



34. $g(x) = \dfrac{4}{x}$

Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$



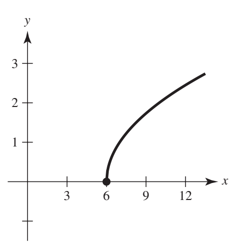
35. $h(x) = \sqrt{x - 6}$

Domain:

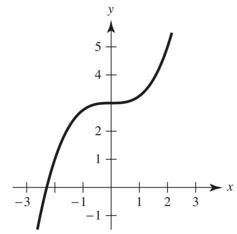
$x - 6 \ge 0$

$x \ge 6 \Rightarrow [6, \infty)$

Range: $[0, \infty)$



36. $f(x) = \frac{1}{4}x^3 + 3$



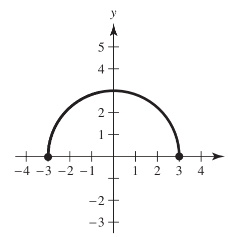
Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

37. $f(x) = \sqrt{9 - x^2}$

Domain: $[-3, 3]$

Range: $[0, 3]$



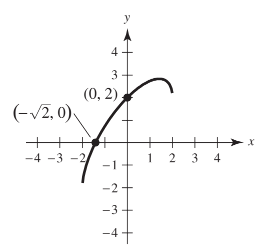
38. $f(x) = x + \sqrt{4 - x^2}$

Domain: $[-2, 2]$

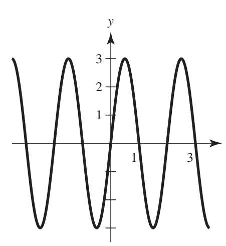
Range: $\left[-2, 2\sqrt{2}\right] \approx [-2, 2.83]$

y-intercept: $(0, 2)$

x-intercept: $\left(-\sqrt{2}, 0\right)$



39. $g(t) = 3 \sin \pi t$



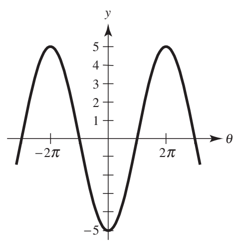
Domain: $(-\infty, \infty)$

Range: $[-3, 3]$

40. $h(\theta) = -5\cos\dfrac{\theta}{2}$

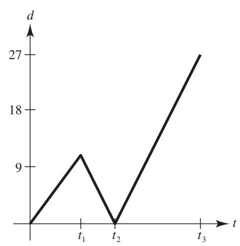
Domain: $(-\infty, \infty)$

Range: $[-5, 5]$



41. The student travels $\dfrac{2-0}{4-0} = \dfrac{1}{2}$ mi/min during the first 4 minutes. The student is stationary for the next 2 minutes. Finally, the student travels $\dfrac{6-2}{10-6} = 1$ mi/min during the final 4 minutes.

42.



43. $x - y^2 = 0 \Rightarrow y = \pm\sqrt{x}$

y is not a function of x. Some vertical lines intersect the graph twice.

44. $\sqrt{x^2 - 4} - y = 0 \Rightarrow y = \sqrt{x^2 - 4}$

y is a function of x. Vertical lines intersect the graph at most once.

45. y is a function of x. Vertical lines intersect the graph at most once.

46. $x^2 + y^2 = 4$

$$y = \pm\sqrt{4 - x^2}$$

y is not a function of x. Some vertical lines intersect the graph twice.

47. $x^2 + y^2 = 16 \Rightarrow y = \pm\sqrt{16 - x^2}$

y is not a function of x because there are two values of y for some x.

48. $x^2 + y = 16 \Rightarrow y = 16 - x^2$

y is a function of x because there is one value of y for each x.

49. $y^2 = x^2 - 1 \Rightarrow y = \pm\sqrt{x^2 - 1}$

y is not a function of x because there are two values of y for some x.

50. $x^2y - x^2 + 4y = 0 \Rightarrow y = \dfrac{x^2}{x^2 + 4}$

y is a function of x because there is one value of y for each x.

51. The transformation is a horizontal shift two units to the right.

Shifted function: $y = \sqrt{x - 2}$

52. The transformation is a vertical shift 4 units upward.

Shifted function: $y = \sin x + 4$

53. The transformation is a horizontal shift 2 units to the right and a vertical shift 1 unit downward.

Shifted function: $y = (x - 2)^2 - 1$

54. The transformation is a horizontal shift 1 unit to the left and a vertical shift 2 units upward.

Shifted function: $y = (x + 1)^3 + 2$

55. $y = f(x + 5)$ is a horizontal shift 5 units to the left. Matches d.

56. $y = f(x) - 5$ is a vertical shift 5 units downward. Matches b.

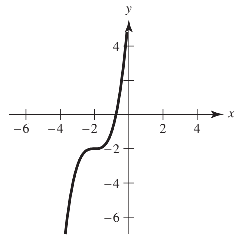
57. $y = -f(-x) - 2$ is a reflection in the y-axis, a reflection in the x-axis, and a vertical shift downward 2 units. Matches c.

58. $y = -f(x - 4)$ is a horizontal shift 4 units to the right, followed by a reflection in the x-axis. Matches a.

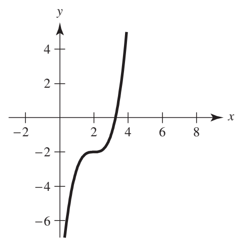
59. $y = f(x + 6) + 2$ is a horizontal shift to the left 6 units, and a vertical shift upward 2 units. Matches e.

60. $y = f(x - 1) + 3$ is a horizontal shift to the right 1 unit, and a vertical shift upward 3 units. Matches g.

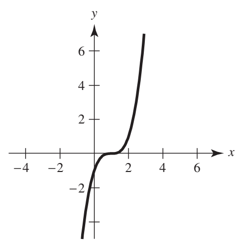
61. (a) The graph is shifted 3 units to the left.



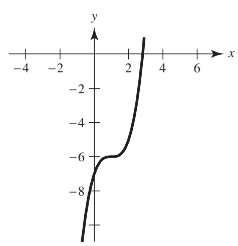
(b) The graph is shifted 1 unit to the right.



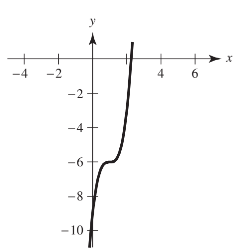
(c) The graph is shifted 2 units upward.



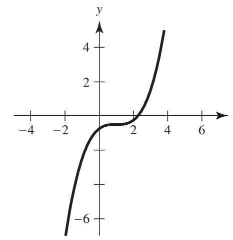
(d) The graph is shifted 4 units downward.



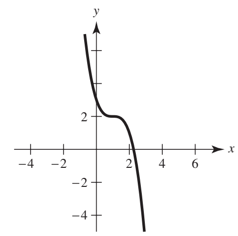
(e) The graph is stretched vertically by a factor of 3.



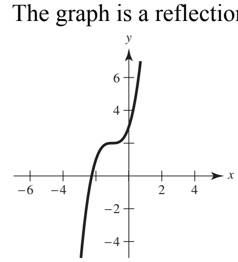
(f) The graph is stretched vertically by a factor of $\frac{1}{4}$.



(g) The graph is a reflection in the x-axis.



(h) The graph is a reflection about the origin.

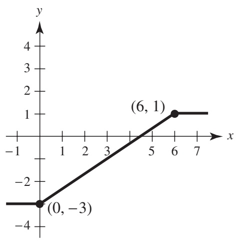


62. (a) $g(x) = f(x - 4)$

$g(6) = f(2) = 1$

$g(0) = f(-4) = -3$

The graph is shifted 4 units to the right.

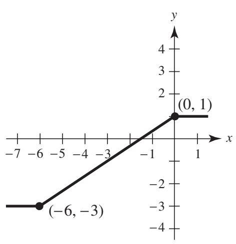


(b) $g(x) = f(x + 2)$

$g(0) = f(2) = 1$

$g(-6) = f(-4) = -3$

The graph is shifted 2 units to the left.

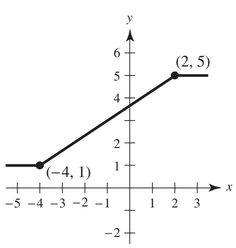


(c) $g(x) = f(x) + 4$

$g(2) = f(2) + 4 = 5$

$g(-4) = f(-4) + 4 = 1$

The graph is shifted 4 units upward.

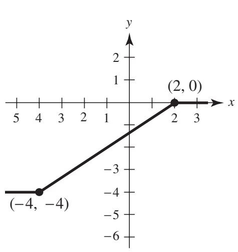


(d) $g(x) = f(x) - 1$

$g(2) = f(2) - 1 = 0$

$g(-4) = f(-4) - 1 = -4$

The graph is shifted 1 unit downward.

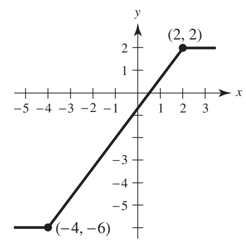


(e) $g(x) = 2f(x)$

$g(2) = 2f(2) = 2$

$g(-4) = 2f(-4) = -6$

The graph is stretched vertically by a factor of 2.

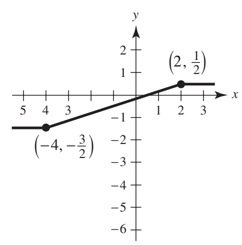


(f) $g(x) = \frac{1}{2}f(x)$

$g(2) = \frac{1}{2}f(2) = \frac{1}{2}$

$g(-4) = \frac{1}{2}f(-4) = -\frac{3}{2}$

The graph is stretched vertically by a factor of $\frac{1}{2}$.

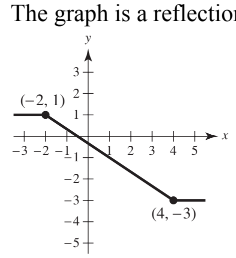


(g) $g(x) = f(-x)$

$g(-2) = f(2) = 1$

$g(4) = f(-4) = -3$

The graph is a reflection in the y-axis.

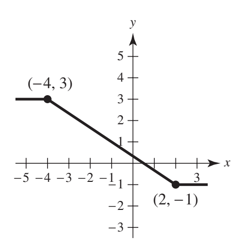


(h) $g(x) = -f(x)$

$g(2) = f(2) = -1$

$g(-4) = f(-4) = 3$

The graph is a reflection in the x-axis.



63. $f(x) = 3x - 4, \quad g(x) = 4$

(a) $f(x) + g(x) = (3x - 4) + 4 = 3x$

(b) $f(x) - g(x) = (3x - 4) - 4 = 3x - 8$

(c) $f(x) \cdot g(x) = (3x - 4)(4) = 12x - 16$

(d) $f(x)/g(x) = \dfrac{3x - 4}{4} = \dfrac{3}{4}x - 1$

64. $f(x) = x^2 + 5x + 4, \quad g(x) = x + 1$

(a) $f(x) + g(x) = (x^2 + 5x + 4) + (x + 1) = x^2 + 6x + 5$

(b) $f(x) - g(x) = (x^2 + 5x + 4) - (x + 1) = x^2 + 4x + 3$

(c) $f(x) \cdot g(x) = (x^2 + 5x + 4)(x + 1)$

$= x^3 + 5x^2 + 4x + x^2 + 5x + 4$

$= x^3 + 6x^2 + 9x + 4$

(d) $f(x)/g(x) = \dfrac{x^2 + 5x + 4}{x + 1} = \dfrac{(x + 4)(x + 1)}{x + 1} = x + 4, x \neq -1$

65. (a) $f(g(1)) = f(0) = 0$

(b) $g(f(1)) = g(1) = 0$

(c) $g(f(0)) = g(0) = -1$

(d) $f(g(-4)) = f(15) = \sqrt{15}$

(e) $f(g(x)) = f(x^2 - 1) = \sqrt{x^2 - 1}$

(f) $g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^2 - 1 = x - 1, \ (x \geq 0)$

66. $f(x) = \sin x, g(x) = \pi x$

(a) $f(g(2)) = f(2\pi) = \sin(2\pi) = 0$

(b) $f\left(g\left(\dfrac{1}{2}\right)\right) = f\left(\dfrac{\pi}{2}\right) = \sin\left(\dfrac{\pi}{2}\right) = 1$

(c) $g(f(0)) = g(0) = 0$

(d) $g\left(f\left(\dfrac{\pi}{4}\right)\right) = g\left(\sin\left(\dfrac{\pi}{4}\right)\right)$

$= g\left(\dfrac{\sqrt{2}}{2}\right) = \pi\left(\dfrac{\sqrt{2}}{2}\right) = \dfrac{\pi\sqrt{2}}{2}$

(e) $f(g(x)) = f(\pi x) = \sin(\pi x)$

(f) $g(f(x)) = g(\sin x) = \pi \sin x$

67. $f(x) = x^2, g(x) = \sqrt{x}$

$(f \circ g)(x) = f(g(x))$

$= f(\sqrt{x}) = (\sqrt{x})^2 = x, x \geq 0$

Domain: $[0, \infty)$

$(g \circ f)(x) = g(f(x)) = g(x^2) = \sqrt{x^2} = |x|$

Domain: $(-\infty, \infty)$

No. Their domains are different. $(f \circ g) = (g \circ f)$ for $x \geq 0$.

68. $f(x) = x^2 - 1, g(x) = \cos x$

$(f \circ g)(x) = f(g(x)) = f(\cos x) = \cos^2 x - 1$

Domain: $(-\infty, \infty)$

$(g \circ f)(x) = g(x^2 - 1) = \cos(x^2 - 1)$

Domain: $(-\infty, \infty)$

No, $f \circ g \neq g \circ f$.

69. $f(x) = \dfrac{3}{x},\ g(x) = x^2 - 1$

$(f \circ g)(x) = f(g(x)) = f(x^2 - 1) = \dfrac{3}{x^2 - 1}$

Domain: all $x \neq \pm 1 \Rightarrow (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

$(g \circ f)(x) = g(f(x))$

$$= g\left(\frac{3}{x}\right) = \left(\frac{3}{x}\right)^2 - 1 = \frac{9}{x^2} - 1 = \frac{9 - x^2}{x^2}$$

Domain: all $x \neq 0 \Rightarrow (-\infty, 0) \cup (0, \infty)$

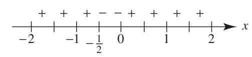
No, $f \circ g \neq g \circ f$.

70. $(f \circ g)(x) = f\left(\sqrt{x + 2}\right) = \dfrac{1}{\sqrt{x + 2}}$

Domain: $(-2, \infty)$

$(g \circ f)(x) = g\left(\dfrac{1}{x}\right) = \sqrt{\dfrac{1}{x} + 2} = \sqrt{\dfrac{1 + 2x}{x}}$

You can find the domain of $g \circ f$ by determining the intervals where $(1 + 2x)$ and x are both positive, or both negative.



Domain: $\left(-\infty, -\frac{1}{2}\right], (0, \infty)$

71. (a) $(f \circ g)(3) = f(g(3)) = f(-1) = 4$

(b) $g(f(2)) = g(1) = -2$

(c) $g(f(5)) = g(-5)$, which is undefined

(d) $(f \circ g)(-3) = f(g(-3)) = f(-2) = 3$

(e) $(g \circ f)(-1) = g(f(-1)) = g(4) = 2$

(f) $f(g(-1)) = f(-4)$, which is undefined

72. $(A \circ r)(t) = A(r(t)) = A(0.6t) = \pi(0.6t)^2 = 0.36\pi t^2$

$(A \circ r)(t)$ represents the area of the circle at time t.

73. $F(x) = \sqrt{2x - 2}$

Let $h(x) = 2x$, $g(x) = x - 2$ and $f(x) = \sqrt{x}$.

Then, $(f \circ g \circ h)(x) = f(g(2x)) = f((2x) - 2) = \sqrt{(2x) - 2} = \sqrt{2x - 2} = F(x)$.

[Other answers possible]

74. $F(x) = -4\sin(1 - x)$

Let $f(x) = -4x$, $g(x) = \sin x$ and $h(x) = 1 - x$. Then,

$(f \circ g \circ h)(x) = f(g(1 - x)) = f(\sin(1 - x)) = -4\sin(1 - x) = F(x)$.

[Other answers possible]

75. (a) If f is even, then $\left(\frac{3}{2}, 4\right)$ is on the graph.

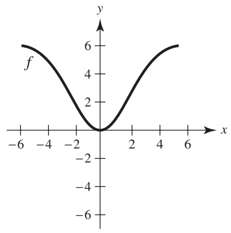
(b) If f is odd, then $\left(\frac{3}{2}, -4\right)$ is on the graph.

76. (a) If f is even, then $(-4, 9)$ is on the graph.

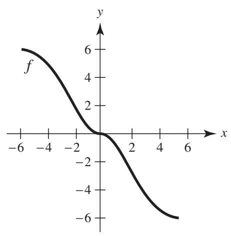
(b) If f is odd, then $(-4, -9)$ is on the graph.

77. f is even because the graph is symmetric about the y-axis. g is neither even nor odd. h is odd because the graph is symmetric about the origin.

78. (a) If f is even, then the graph is symmetric about the y-axis.



(b) If f is odd, then the graph is symmetric about the origin.



79. $f(x) = x^2(4 - x^2)$

$f(-x) = (-x)^2(4 - (-x)^2) = x^2(4 - x^2) = f(x)$

f is even.

$f(x) = x^2(4 - x^2) = 0$

$x^2(2 - x)(2 + x) = 0$

Zeros: $x = 0, -2, 2$

80. $f(x) = \sqrt[3]{x}$

$f(-x) = \sqrt[3]{(-x)} = -\sqrt[3]{x} = -f(x)$

f is odd.

$f(x) = \sqrt[3]{x} = 0 \Rightarrow x = 0$ is the zero.

81. $f(x) = x \cos x$

$f(-x) = (-x)\cos(-x) = -x \cos x = -f(x)$

f is odd.

$f(x) = x \cos x = 0$

Zeros: $x = 0, \dfrac{\pi}{2} + n\pi$, where n is an integer

82. $f(x) = \sin^2 x$

$f(-x) = \sin^2(-x) = \sin(-x)\sin(-x) = (-\sin x)(-\sin x) = \sin^2 x$

f is even.

$\sin^2 x = 0 \Rightarrow \sin x = 0$

Zeros: $x = n\pi$, where n is an integer

83. Slope $= \dfrac{4 - (-6)}{-2 - 0} = \dfrac{10}{-2} = -5$

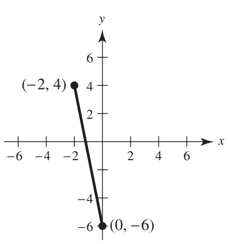
$y - 4 = -5(x - (-2))$

$y - 4 = -5x - 10$

$y = -5x - 6$

For the line segment, you must restrict the domain.

$f(x) = -5x - 6, \ -2 \le x \le 0$



84. Slope $= \dfrac{8 - 1}{5 - 3} = \dfrac{7}{2}$

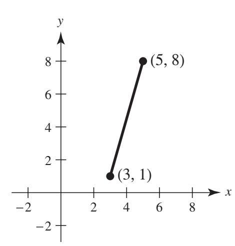
$y - 1 = \dfrac{7}{2}(x - 3)$

$y - 1 = \dfrac{7}{2}x - \dfrac{21}{2}$

$y = \dfrac{7}{2}x - \dfrac{19}{2}$

For the line segment, you must restrict the domain.

$f(x) = \dfrac{7}{2}x - \dfrac{19}{2}, \ 3 \le x \le 5$

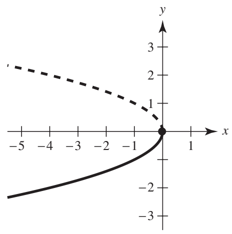


85. $x + y^2 = 0$

$$y^2 = -x$$

$$y = -\sqrt{-x}$$

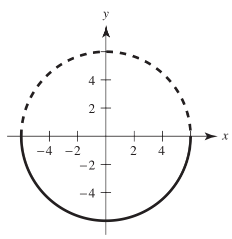
$$f(x) = -\sqrt{-x},\ x \le 0$$



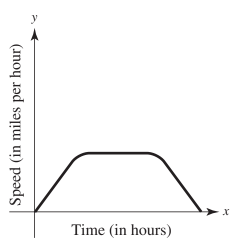
86. $x^2 + y^2 = 36$

$$y^2 = 36 - x^2$$

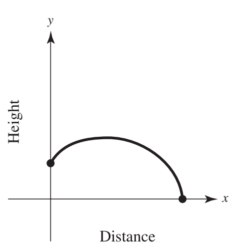
$$y = -\sqrt{36 - x^2},\ -6 \le x \le 6$$



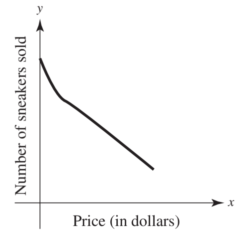
87. Answers will vary. *Sample answer*: Speed begins and ends at 0. The speed might be constant in the middle:



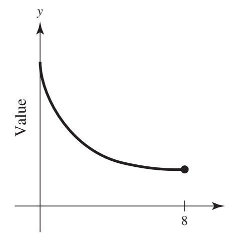
88. Answers will vary. *Sample answer*: Height begins a few feet above 0, and ends at 0.



89. Answers will vary. *Sample answer*: In general, as the price decreases, the store will sell more.



90. Answers will vary. *Sample answer*: As time goes on, the value of the car will decrease



91. $y = \sqrt{c - x^2}$

$$y^2 = c - x^2$$

$$x^2 + y^2 = c, \text{ a circle.}$$

For the domain to be $[-5, 5]$, $c = 25$.

92. For the domain to be the set of all real numbers, you must require that $x^2 + 3cx + 6 \ne 0$. So, the discriminant must be less than zero:

$$(3c)^2 - 4(6) < 0$$

$$9c^2 < 24$$

$$c^2 < \tfrac{8}{3}$$

$$-\sqrt{\tfrac{8}{3}} < c < \sqrt{\tfrac{8}{3}}$$

$$-\tfrac{2}{3}\sqrt{6} < c < \tfrac{2}{3}\sqrt{6}$$

93. (a) $T(4) = 16°$, $T(15) \approx 23°$

(b) If $H(t) = T(t - 1)$, then the changes in temperature will occur 1 hour later.

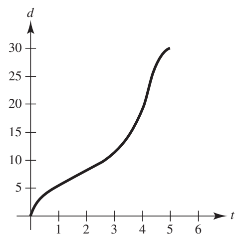
(c) If $H(t) = T(t) - 1$, then the overall temperature would be 1 degree lower.

94. (a) For each time t, there corresponds a depth d.

(b) Domain: $0 \le t \le 5$

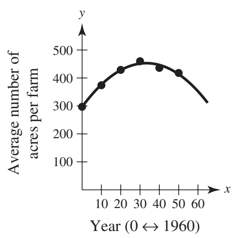
Range: $0 \le d \le 30$

(c)



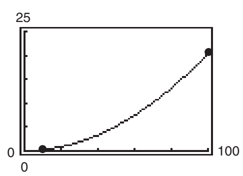
(d) $d(4) \approx 18$. At time 4 seconds, the depth is approximately 18 cm.

95. (a)



(b) $A(25) \approx 445$ (Answers will vary.)

96. (a)



(b) $$H\left(\frac{x}{1.6}\right) = 0.002\left(\frac{x}{1.6}\right)^2 + 0.005\left(\frac{x}{1.6}\right) - 0.029$$
$$= 0.00078125x^2 + 0.003125x - 0.029$$

97. $f(x) = |x| + |x - 2|$

If $x < 0$, then $f(x) = -x - (x - 2) = -2x + 2$.

If $0 \le x < 2$, then $f(x) = x - (x - 2) = 2$.

If $x \ge 2$, then $f(x) = x + (x - 2) = 2x - 2$.

So,

$$f(x) = \begin{cases} -2x + 2, & x \le 0 \\ 2, & 0 < x < 2. \\ 2x - 2, & x \ge 2 \end{cases}$$

98. $p_1(x) = x^3 - x + 1$ has one zero. $p_2(x) = x^3 - x$ has three zeros. Every cubic polynomial has at least one zero. Given $p(x) = Ax^3 + Bx^2 + Cx + D$, you have $p \to -\infty$ as $x \to -\infty$ and $p \to \infty$ as $x \to \infty$ if $A > 0$. Furthermore, $p \to \infty$ as $x \to -\infty$ and $p \to -\infty$ as $x \to \infty$ if $A < 0$. Because the graph has no breaks, the graph must cross the x-axis at least one time.

99. $$f(-x) = a_{2n+1}(-x)^{2n+1} + \cdots + a_3(-x)^3 + a_1(-x)$$
$$= -\left[a_{2n+1}x^{2n+1} + \cdots + a_3x^3 + a_1x\right]$$
$$= -f(x)$$

Odd

100. $$f(-x) = a_{2n}(-x)^{2n} + a_{2n-2}(-x)^{2n-2} + \cdots + a_2(-x)^2 + a_0$$
$$= a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0$$
$$= f(x)$$

Even

101. Let $F(x) = f(x)g(x)$ where f and g are even. Then $F(-x) = f(-x)g(-x) = f(x)g(x) = F(x)$.

So, $F(x)$ is even. Let $F(x) = f(x)g(x)$ where f and g are odd. Then

$F(-x) = f(-x)g(-x) = [-f(x)][-g(x)] = f(x)g(x) = F(x)$.

So, $F(x)$ is even.

102. Let $F(x) = f(x)g(x)$ where f is even and g is odd. Then

$F(-x) = f(-x)g(-x) = f(x)[-g(x)] = -f(x)g(x) = -F(x)$.

So, $F(x)$ is odd.

103. By equating slopes, $\dfrac{y-2}{0-3} = \dfrac{0-2}{x-3}$

$$y - 2 = \frac{6}{x-3}$$

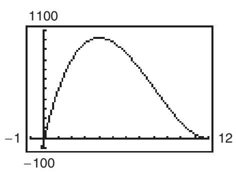
$$y = \frac{6}{x-3} + 2 = \frac{2x}{x-3},$$

$$L = \sqrt{x^2 + y^2} = \sqrt{x^2 + \left(\frac{2x}{x-3}\right)^2}.$$

104. (a) $V = x(24 - 2x)^2$

Domain: $0 < x < 12$

(b)



Maximum volume occurs at $x = 4$. So, the dimensions of the box would be $4 \times 16 \times 16$ cm.

(c)

x	*length and width*	*volume*
1	$24 - 2(1)$	$1[24 - 2(1)]^2 = 484$
2	$24 - 2(2)$	$2[24 - 2(2)]^2 = 800$
3	$24 - 2(3)$	$3[24 - 2(3)]^2 = 972$
4	$24 - 2(4)$	$4[24 - 2(4)]^2 = 1024$
5	$24 - 2(5)$	$5[24 - 2(5)]^2 = 980$
6	$24 - 2(6)$	$6[24 - 2(6)]^2 = 864$

The dimensions of the box that yield a maximum volume appear to be $4 \times 16 \times 16$ cm.

105. False. If $f(x) = x^2$, then $f(-3) = f(3) = 9$, but $-3 \neq 3$.

106. True

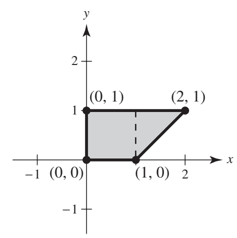
107. True. The function is even.

108. False. If $f(x) = x^2$ then, $f(3x) = (3x)^2 = 9x^2$ and $3f(x) = 3x^2$. So, $3f(x) \neq f(3x)$.

109. False. The constant function $f(x) = 0$ has symmetry with respect to the x-axis.

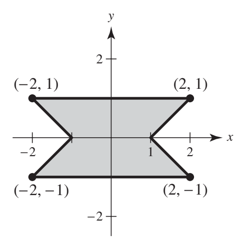
110. True. If the domain is $\{a\}$, then the range is $\{f(a)\}$.

111. First consider the portion of R in the first quadrant: $x \geq 0$, $0 \leq y \leq 1$ and $x - y \leq 1$; shown below.



The area of this region is $1 + \frac{1}{2} = \frac{3}{2}$.

By symmetry, you obtain the entire region R:



The area of R is $4\left(\frac{3}{2}\right) = 6$.

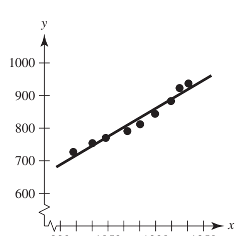
112. Let $g(x) = c$ be constant polynomial.

Then $f(g(x)) = f(c)$ and $g(f(x)) = c$.

So, $f(c) = c$. Because this is true for all real numbers c, f is the identity function: $f(x) = x$.

Section P.4 Fitting Models to Data

1. (a) and (b)

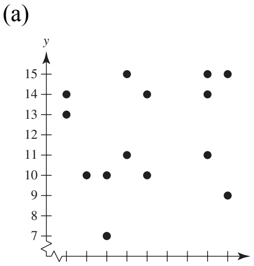


Yes, the data appear to be approximately linear.

The data can be modeled by equation $y = 0.6x + 150$. (Answers will vary).

(c) When $x = 1075$, $y = 0.6(1075) + 150 = 795$.

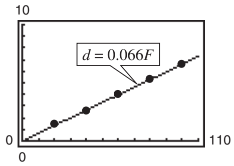
2. (a)



The data do not appear to be linear.

(b) Quiz scores are dependent on several variables such as study time, class attendance, and so on. These variables may change from one quiz to the next.

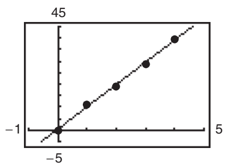
3. (a) $d = 0.066F$

(b) 

The model fits the data well.

(c) If $F = 55$, then $d \approx 0.066(55) = 3.63$ cm.

4. (a) $s = 9.7t + 0.4$

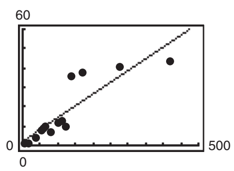
(b) 

The model fits the data well.

(c) If $t = 2.5$, $s = 24.65$ meters/second.

5. (a) Using a graphing utility, $y = 0.122x + 2.07$

The correlation coefficient is $r \approx 0.87$.

(b) 

(c) Greater per capita energy consumption by a country tends to correspond to greater per capita gross national income. The three countries that most differ from the linear model are Canada, Japan, and Italy.

(d) Using a graphing utility, the new model is $y = 0.142x - 1.66$.

The correlation coefficient is $r \approx 0.97$.

6. (a) Trigonometric function

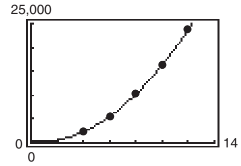
(b) Quadratic function

(c) No relationship

(d) Linear function

7. (a) Using graphing utility,

$S = 180.89x^2 - 205.79x + 272$.

(b) 

(c) When $x = 2$, $S \approx 583.98$ pounds.

(d) $\dfrac{2370}{584} \approx 4.06$

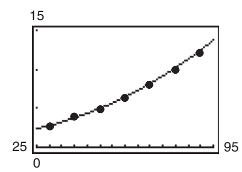
The breaking strength is approximately 4 times greater.

(e) $\dfrac{23{,}860}{5460} \approx 4.37$

When the height is doubled, the breaking strength increases approximately by a factor of 4.

8. (a) Using a graphing utility

$t = 0.0013s^2 + 0.005s + 1.48$.

(b) 

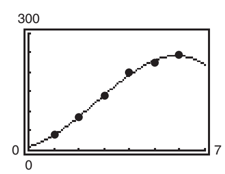
(c) According to the model, the times required to attain speeds of less than 20 miles per hour are all about the same. Furthermore, it takes 1.48 seconds to reach 0 miles per hour, which does not make sense.

(d) Adding $(0, 0)$ to the data produces

$t = 0.0009s^2 + 0.053s + 0.10$.

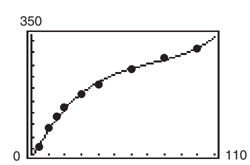
(e) Yes. Now the car starts at rest.

9. (a) $y = -1.806x^3 + 14.58x^2 + 16.4x + 10$

(b) 

(c) If $x = 4.5$, $y \approx 214$ horsepower.

10. (a) $T = 2.9856 \times 10^{-4} p^3 - 0.0641p^2 + 5.282p + 143.1$

(b) 

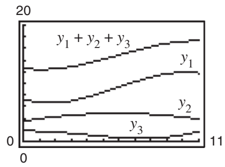
(c) For $T = 300°F$, $p \approx 68.29$ lb/in.^{2}.

(d) The model is based on data up to 100 pounds per square inch.

11. (a) $y_1 = -0.0172t^3 + 0.305t^2 - 0.87t + 7.3$

$y_2 = -0.038t^2 + 0.45t + 3.5$

$y_3 = 0.0063t^3 + -0.072t^2 + 0.02t + 1.8$

(b)


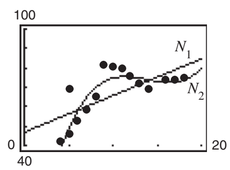
$y_1 + y_2 + y_3 = -0.0109t^3 + 0.195t^2 - 0.40t + 12.6$

For 2014, $t = 14$. So,

$$y_1 + y_2 + y_3 = -0.0109(14)^3 + 0.195(14)^2 - 0.40(14) + 12.6$$
$$\approx 15.31 \text{ cents/mile}$$

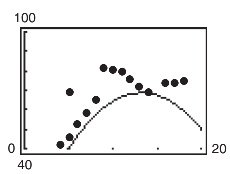
12. (a) $N_1 = 1.89t + 46.8$ Linear model

$N_2 = 0.0485t^3 - 2.015t^2 + 27.00t - 42.3$ Cubic model

(b)


(c) The cubic model is the better model.

(d) $N_3 = -0.414t^2 + 11.00t + 4.4$ Quadratic model



The model does not fit the data well.

(e) For 2014, $t = 24$ and

$N_1 \approx 92.16$ million

$N_2 \approx 115.524$ million

The linear model seems too high. The cubic model is better.

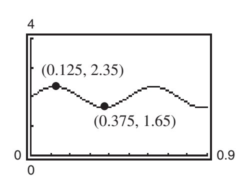
(f) Answers will vary.

13. (a) Yes, y is a function of t. At each time t, there is one and only one displacement y.

(b) The amplitude is approximately $(2.35 - 1.65)/2 = 0.35$.

The period is approximately $2(0.375 - 0.125) = 0.5$.

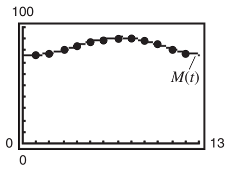
(c) One model is $y = 0.35 \sin(4\pi t) + 2$.

(d)


The model appears to fit the data.

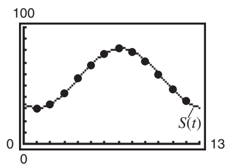
14. (a) $S(t) = 56.37 + 25.47\sin(0.5080t - 2.07)$

(b)



The model is a good fit.

(c)



The model is a good fit.

(d) The average is the constant term in each model. $83.70°\text{F}$ for Miami and $56.37°\text{F}$ for Syracuse.

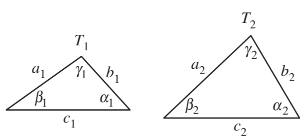
(e) The period for Miami is $2\pi/0.4912 \approx 12.8$. The period for Syracuse is $2\pi/0.5080 \approx 12.4$. In both cases the period is approximately 12, or one year.

(f) Syracuse has greater variability because $25.47 > 7.46$.

15. Answers will vary.

16. Answers will vary.

17. Yes, $A_1 \le A_2$. To see this, consider the two triangles of areas A_1 and A_2:



For $i = 1, 2$, the angles satisfy $\alpha i + \beta i + \gamma i = \pi$. At least one of $\alpha_1 \le \alpha_2$, $\beta_1 \le \beta_2$, $\gamma_1 \le \gamma_2$ must hold. Assume $\alpha_1 \le \alpha_2$. Because $\alpha_2 \le \pi/2$ (acute triangle), and the sine function increases on $[0, \pi/2]$, you have

$$A_1 = \tfrac{1}{2}b_1c_1 \sin\alpha_1 \le \tfrac{1}{2}b_2c_2 \sin\alpha_1$$
$$\le \tfrac{1}{2}b_2c_2 \sin\alpha_2 = A_2$$

Review Exercises for Chapter P

1. $y = 5x - 8$

$x = 0$: $y = 5(0) - 8 = -8 \Rightarrow (0, -8)$, y-intercept

$y = 0$: $0 = 5x - 8 \Rightarrow x = \frac{8}{5} \Rightarrow \left(\frac{8}{5}, 0\right)$, x-intercept

2. $y = x^2 - 8x + 12$

$x = 0$: $y = (0)^2 - 8(0) + 12 = 12 \Rightarrow (0, 12)$, y-intercept

$y = 0$: $x^2 - 8x + 12 = (x - 6)(x - 2) = 0 \Rightarrow x = 2, 6 \Rightarrow (2, 0), (6, 0)$, x-intercepts

3. $y = \dfrac{x - 3}{x - 4}$

$x = 0$: $y = \dfrac{0 - 3}{0 - 4} = \dfrac{3}{4} \Rightarrow \left(0, \dfrac{3}{4}\right)$, y-intercept

$y = 0$: $0 = \dfrac{x - 3}{x - 4} \Rightarrow x = 3 \Rightarrow (3, 0)$, x-intercept

4. $y = (x - 3)\sqrt{x + 4}$

$x = 0$: $y = (0 - 3)\sqrt{0 + 4} = -3\sqrt{4} = -3(2) = -6 \Rightarrow (0, -6)$, y-intercept

$y = 0$: $(x - 3)\sqrt{x + 4} = 0 \Rightarrow x = 3, -4 \Rightarrow (3, 0), (-4, 0)$, x-intercepts

5. $y = x^2 + 4x$ does not have symmetry with respect to either axis or the origin.

6. Symmetric with respect to y-axis because

$$y = (-x)^4 - (-x)^2 + 3$$
$$y = x^4 - x^2 + 3.$$

7. Symmetric with respect to both axes and the origin because:

$$y^2 = (-x^2) - 5 \qquad (-y)^2 = x^2 - 5 \qquad (-y)^2 = (-x)^2 - 5$$
$$y^2 = x^2 - 5 \qquad y^2 = x^2 - 5 \qquad y^2 = x^2 - 5$$

8. Symmetric with respect to the origin because:

$$(-x)(-y) = -2$$
$$xy = -2.$$

9. $y = -\frac{1}{2}x + 3$

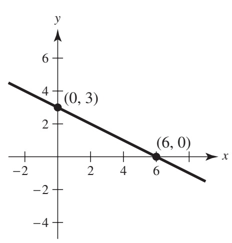
y-intercept: $y = -\frac{1}{2}(0) + 3 = 3$

$(0, 3)$

x-intercept: $-\frac{1}{2}x + 3 = 0$

$$-\frac{1}{2}x = -3$$
$$x = 6$$
$$(6, 0)$$

Symmetry: none



10. $y = -x^2 + 4$

y-intercept: $y = -(0)^2 + 4 = 4$

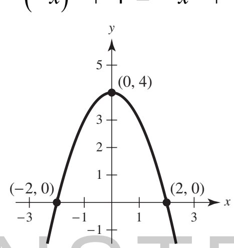
$(0, 4)$

x-intercepts: $-x^2 + 4 = 0$

$$(2 - x)(2 + x) = 0$$
$$x = \pm 2$$
$$(2, 0), (-2, 0)$$

Symmetric with respect to the y-axis because

$-(-x)^2 + 4 = -x^2 + 4.$



11. $y = x^3 - 4x$

y-intercept: $y = 0^3 - 4(0) = 0$

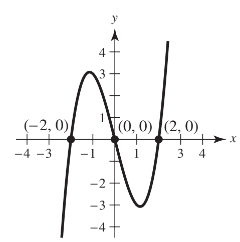
$(0, 0)$

x-intercepts: $x^3 - 4x = 0$

$$x(x^2 - 4) = 0$$
$$x(x - 2)(x + 2) = 0$$
$$x = 0, 2, -2$$
$$(0, 0), (2, 0), (-2, 0)$$

Symmetric with respect to the origin because

$(-x)^3 - 4(-x) = -x^3 + 4x = -(x^3 - 4x).$



12. $y^2 = 9 - x$

$$y^2 + x - 9 = 0$$

y-intercept: $y^2 = 9 - 0 = 9 \Rightarrow y = \pm 3$

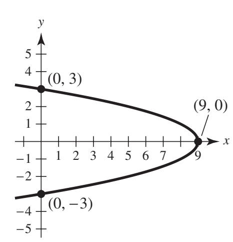
$(0, 3), (0, -3)$

x-intercept: $0^2 = 9 - x \Rightarrow x = 9$

$(9, 0)$

Symmetric with respect to the x-axis because

$(-y)^2 + x - 9 = y^2 + x - 9 = 0.$



13. $y = 2\sqrt{4 - x}$

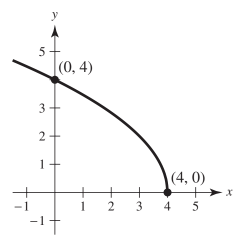
y-intercept: $y = 2\sqrt{4 - 0} = 2\sqrt{4} = 4$

$(0, 4)$

x-intercept:
$$\begin{aligned} 2\sqrt{4 - x} &= 0 \\ \sqrt{4 - x} &= 0 \\ 4 - x &= 0 \\ x &= 4 \end{aligned}$$

$(4, 0)$

Symmetry: none



14. $y = |x - 4| - 4$

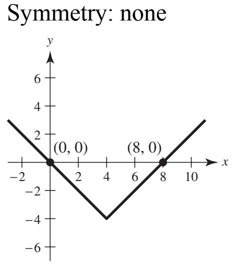
y-intercept: $y = |0 - 4| - 4 = |-4| - 4 = 4 - 4 = 0$

$(0, 0)$

x-intercepts:
$$\begin{aligned} |x - 4| - 4 &= 0 \\ |x - 4| &= 4 \\ x - 4 = 4 \text{ or } x - 4 &= -4 \\ x = 8 \quad x &= 0 \end{aligned}$$

$(0, 0), (8, 0)$

Symmetry: none



15. $5x + 3y = -1 \Rightarrow y = \frac{1}{3}(-5x - 1)$

$x - y = -5 \Rightarrow y = x + 5$

$$\begin{aligned} \tfrac{1}{3}(-5x - 1) &= x + 5 \\ -5x - 1 &= 3x + 15 \\ -16 &= 8x \\ -2 &= x \end{aligned}$$

For $x = -2$, $y = x + 5 = -2 + 5 = 3$.

Point of intersection is: $(-2, 3)$

16. $2x + 4y = 9 \Rightarrow y = \dfrac{-2x + 9}{4}$

$6x - 4y = 7 \Rightarrow y = \dfrac{6x - 7}{4}$

$$\begin{aligned} \frac{-2x + 9}{4} &= \frac{6x - 7}{4} \\ -2x + 9 &= 6x - 7 \\ -8x &= -16 \\ x &= 2 \end{aligned}$$

For $x = 2$, $y = \dfrac{6(2) - 7}{4} = \dfrac{5}{4}$

Point of intersection: $\left(2, \dfrac{5}{4}\right)$

17. $x - y = -5 \Rightarrow y = x + 5$

$x^2 - y = 1 \Rightarrow y = x^2 - 1$

$$\begin{aligned} x + 5 &= x^2 - 1 \\ 0 &= x^2 - x - 6 \\ 0 &= (x - 3)(x + 2) \\ x = 3 &\text{ or } x = -2 \end{aligned}$$

For $x = 3$, $y = 3 + 5 = 8$.

For $x = -2$, $y = -2 + 5 = 3$.

Points of intersection: $(3, 8)$, $(-2, 3)$

18. $x^2 + y^2 = 1 \Rightarrow y^2 = 1 - x^2$

$-x + y = 1 \Rightarrow y = x + 1$

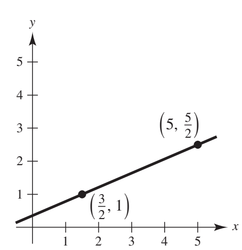
$$\begin{aligned} 1 - x^2 &= (x + 1)^2 \\ 1 - x^2 &= x^2 + 2x + 1 \\ 0 &= 2x^2 + 2x \\ 0 &= 2x(x + 1) \\ x = 0 &\text{ or } x = -1 \end{aligned}$$

For $x = 0$, $y = 0 + 1 = 1$.

For $x = -1$, $y = -1 + 1 = 0$.

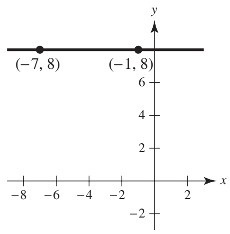
Points of intersection: $(0, 1)$, $(-1, 0)$

19.

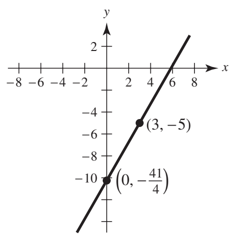


$$\text{Slope} = \frac{\left(\frac{5}{2}\right) - 1}{5 - \left(\frac{3}{2}\right)} = \frac{\frac{3}{2}}{\frac{7}{2}} = \frac{3}{7}$$

20. The line is horizontal and has slope 0.

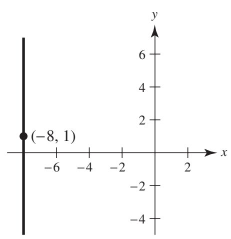


21.
$$y - (-5) = \tfrac{7}{4}(x - 3)$$
$$y + 5 = \tfrac{7}{4}x - \tfrac{21}{4}$$
$$4y + 20 = 7x - 21$$
$$0 = 7x - 4y - 41$$

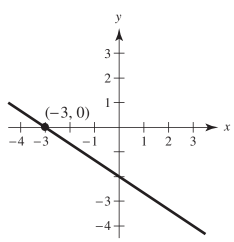


22. Because m is undefined the line is vertical.

$x = -8$ or $x + 8 = 0$



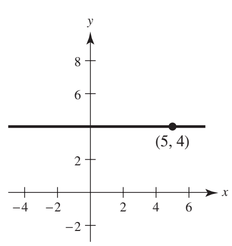
23.
$$y - 0 = -\tfrac{2}{3}(x - (-3))$$
$$y = -\tfrac{2}{3}x - 2$$
$$2x + 3y + 6 = 0$$



24. Because $m = 0$, the line is horizontal.

$y - 4 = 0(x - 5)$

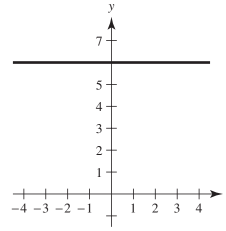
$y = 4$ or $y - 4 = 0$



25. $y = 6$

Slope: 0

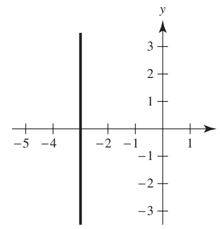
y-intercept: $(0, 6)$



26. $x = -3$

Slope: undefined

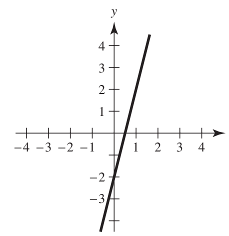
Line is vertical.



27. $y = 4x - 2$

Slope: 4

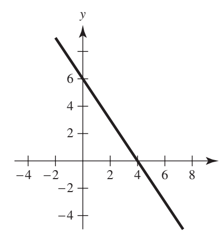
y-intercept: $(0, -2)$



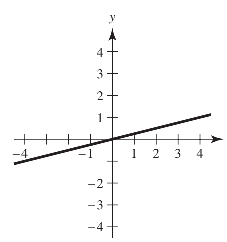
28.
$$3x + 2y = 12$$
$$2y = -3x + 12$$
$$y = \frac{-3}{2}x + 6$$

Slope : $-\dfrac{3}{2}$

y-intercept: $(0, 6)$



29.
$$m = \frac{2 - 0}{8 - 0} = \frac{1}{4}$$
$$y - 0 = \frac{1}{4}(x - 0)$$
$$y = \frac{1}{4}x$$
$$4y - x = 0$$



30.
$$m = \frac{-1 - 5}{10 - (-5)} = \frac{-6}{15} = -\frac{2}{5}$$
$$y - 5 = \frac{-2}{5}(x - (-5))$$
$$5y - 25 = -2x - 10$$
$$5y + 2x - 15 = 0$$