

Appendix C.1 Real Number and the Real Number Line

1. $0.7 = \frac{7}{10}$

Rational

2. $-3678 = \frac{-3678}{1}$

Rational

3. $\frac{3\pi}{2}$

Irrational
(since π is irrational)

4. $3\sqrt{2} - 1$

Irrational
(since $\sqrt{2}$ is irrational)

5. 4.3451451

Rational

6. $\frac{22}{7}$

Rational

7. $\sqrt[3]{64} = 4$

Rational

8. 0.81778177

Rational

9. $4\frac{5}{8} = \frac{45}{8}$

Rational

10. $(\sqrt{2})^3 = 2\sqrt{2}$

Irrational

11. Let $x = 0.363\overline{6}$.

$$100x = 36.36\overline{36}$$

$$\underline{-x = -0.363\overline{6}}$$

$$99x = 36$$

$$x = \frac{36}{99} = \frac{4}{11}$$

12. Let $x = 0.3181\overline{8}$.

$$1,000x = 318.181\overline{8}$$

$$\underline{-10x = -3.181\overline{8}}$$

$$990x = 315$$

$$x = \frac{315}{990} = \frac{7}{22}$$

13. Let $x = 0.29729\overline{7}$.

$$1,000x = 297.29729\overline{7}$$

$$\underline{-x = -0.29729\overline{7}}$$

$$999x = 297$$

$$x = \frac{297}{999} = \frac{11}{37}$$

14. Let $x = 0.990099\overline{00}$.

$$10,000x = 9900.990099\overline{00}$$

$$\underline{-x = -0.990099\overline{00}}$$

$$9,999x = 9,900$$

$$x = \frac{9,900}{9,999} = \frac{100}{101}$$

15. Given $a < b$:

(a) $a + 2 < b + 2$; True

(b) $5b < 5a$; False

(c) $5 - a > 5 - b$; True

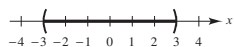
(d) $\frac{1}{a} < \frac{1}{b}$; False

(e) $(a - b)(b - a) > 0$; False

(f) $a^2 < b^2$; False

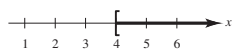
16.	Interval Notation	Set Notation	Graph
	$[-2, 0)$	$\{x: -2 \leq x < 0\}$	
	$(-\infty, -4]$	$\{x: x \leq -4\}$	
	$\left[3, \frac{11}{2}\right]$	$\left\{x: 3 \leq x \leq \frac{11}{2}\right\}$	
	$(-1, 7)$	$\{x: -1 < x < 7\}$	

- 17.
- x
- is greater than
- -3
- and less than
- 3
- .

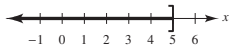


The interval is bounded.

- 18.
- x
- is greater than, or equal to,
- 4
- .

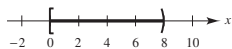


- 19.
- x
- is less than, or equal to,
- 5
- .



The interval is unbounded.

- 20.
- x
- is greater than or equal to
- 0
- , and less than
- 8
- .



- 21.
- $y \geq 4, [4, \infty)$

- 22.
- $q \geq 0, [0, \infty)$

- 23.
- $0.03 < r \leq 0.07,$
-
- $(0.03, 0.07]$

- 24.
- $T > 90^\circ, (90^\circ, \infty)$

- 25.
- $2x - 1 \geq 0$

$$2x \geq 1$$

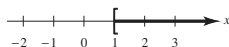
$$x \geq \frac{1}{2}$$



- 26.
- $3x + 1 \geq 2x + 2$

$$3x \geq 2x + 1$$

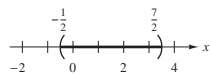
$$x \geq 1$$



- 27.
- $-4 < 2x - 3 < 4$

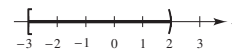
$$-1 < 2x < 7$$

$$-\frac{1}{2} < x < \frac{7}{2}$$



- 28.
- $0 \leq x + 3 < 5$

$$-3 \leq x < 2$$

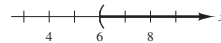


- 29.
- $\frac{x}{2} + \frac{x}{3} > 5$

$$3x + 2x > 30$$

$$5x > 30$$

$$x > 6$$

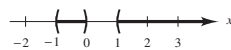


- 30.
- $x > \frac{1}{x}$

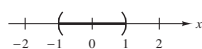
$$x - \frac{1}{x} > 0$$

$$\frac{x^2 - 1}{x} > 0$$

$$\frac{(1+x)(x-1)}{x} > 0$$

Test intervals: $(-\infty, -1)$, $(-1, 0)$, $(0, 1)$, $(1, \infty)$ Solution: $-1 < x < 0$ or $x > 1$

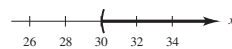
- 31.
- $|x| < 1 \Rightarrow -1 < x < 1$



- 32.
- $\frac{x}{2} - \frac{x}{3} > 5$

$$3x - 2x > 30$$

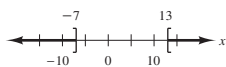
$$x > 30$$



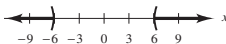
$$33. \left| \frac{x-3}{2} \right| \geq 5$$

$$x-3 \geq 10 \quad \text{or} \quad x-3 \leq -10$$

$$x \geq 13 \quad x \leq -7$$



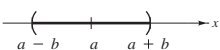
$$34. \left| \frac{x}{2} \right| > 3 \Rightarrow x > 6 \text{ or } x < -6$$



$$35. |x-a| < b$$

$$-b < x-a < b$$

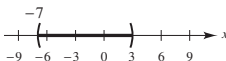
$$a-b < x < a+b$$



$$36. |x+2| < 5$$

$$-5 < x+2 < 5$$

$$-7 < x < 3$$

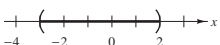


$$37. |2x+1| < 5$$

$$-5 < 2x+1 < 5$$

$$-6 < 2x < 4$$

$$-3 < x < 2$$

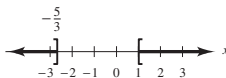


$$38. |3x+1| \geq 4$$

$$3x+1 \geq 4 \quad \text{or} \quad 3x+1 \leq -4$$

$$3x \geq 3 \quad 3x \leq -5$$

$$x \geq 1 \quad x \leq -\frac{5}{3}$$

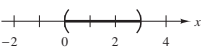


$$39. \left| 1 - \frac{2x}{3} \right| < 1$$

$$-1 < 1 - \frac{2x}{3} < 1$$

$$-2 < -\frac{2x}{3} < 0$$

$$3 > x > 0$$

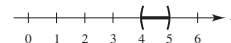


$$40. |9-2x| < 1$$

$$-1 < 9-2x < 1$$

$$-10 < -2x < -8$$

$$5 > x > 4$$



$$41. x^2 \leq 3-2x$$

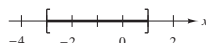
$$x^2 + 2x - 3 \leq 0$$

$$(x+3)(x-1) \leq 0$$

Test intervals:

$$(-\infty, -3), (-3, 1), (1, \infty)$$

Solution: $-3 \leq x \leq 1$



$$42. x^4 - x \leq 0$$

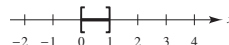
$$x(x^3 - 1) \leq 0$$

$$x = 0$$

$$x = 1$$

Test intervals: $(-\infty, 0), (0, 1), (1, \infty)$

Solution: $0 \leq x \leq 1$



$$43. x^2 + x - 1 \leq 5$$

$$x^2 + x - 6 \leq 0$$

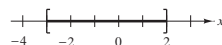
$$(x+3)(x-2) \leq 0$$

$$x = -3$$

$$x = 2$$

Test intervals: $(-\infty, -3), (-3, 2), (2, \infty)$

Solution: $-3 \leq x \leq 2$



$$44. 2x^2 + 1 < 9x - 3$$

$$2x^2 - 9x + 4 < 0$$

$$(2x-1)(x-4) < 0$$

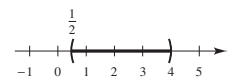
$$x = \frac{1}{2}$$

$$x = 4$$

Test intervals:

$$\left(-\infty, \frac{1}{2}\right), \left(\frac{1}{2}, 4\right), (4, \infty)$$

Solution: $\frac{1}{2} < x < 4$



45. $a = -1, b = 3$

Directed distance from a to b : 4Directed distance from b to a : -4 Distance between a and b : 4

46. $a = -\frac{5}{2}, b = \frac{13}{4}$

Directed distance from a to b : $\frac{23}{4}$ Directed distance from b to a : $-\frac{23}{4}$ Distance between a and b : $\frac{23}{4}$

47. (a) $a = 126, b = 75$

Directed distance from a to b : -51 Directed distance from b to a : 51Distance between a and b : 51

(b) $a = -126, b = -75$

Directed distance from a to b : 51Directed distance from b to a : -51 Distance between a and b : 51

48. (a) $a = 9.34, b = -5.65$

Directed distance from a to b : -14.99 Directed distance from b to a : 14.99Distance between a and b : 14.99

(b) $a = \frac{16}{5}, b = \frac{112}{75}$

Directed distance from a to b : $-\frac{128}{75}$ Directed distance from b to a : $\frac{128}{75}$ Distance between a and b : $\frac{128}{75}$

49. $a = -2, b = 2$

Midpoint: 0

Distance between midpoint and each endpoint: 2

$|x - 0| \leq 2$

$|x| \leq 2$

50. $a = -3, b = 3$

Midpoint: 0

Distance between midpoint and each endpoint: 3

$|x - 0| \geq 3$

$|x| \geq 3$

51. $a = 0, b = 4$

Midpoint: 2

Distance between midpoint and each endpoint: 2

$|x - 2| > 2$

52. $a = 20, b = 24$

Midpoint: 2

Distance between midpoint and each endpoint: 2

$|x - 22| \geq 2$

53. (a) All numbers that are at most 10 units from 12

$|x - 12| \leq 10$

(b) All numbers that are at least 10 units from 12

$|x - 12| \geq 10$

54. (a) y is at most 2 units from a : $|y - a| \leq 2$

(b) y is less than δ units from c : $|y - c| < \delta$

55. $a = -1, b = 3$

Midpoint: $\frac{-1 + 3}{2} = 1$

56. $a = -5, b = -\frac{3}{2}$

Midpoint: $\frac{-5 + (-3/2)}{2} = -\frac{13}{4}$

57. (a) $[7, 21]$

Midpoint: 14

(b) $[8.6, 11.4]$

Midpoint: 10

58. (a) $[-6.85, 9.35]$

Midpoint: 1.25

(b) $[-4.6, -1.3]$

Midpoint: -2.95