

CHAPTER 2

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2-1.*

a) $\overline{XYZ} = \bar{X} + \bar{Y} + \bar{Z}$

Verification of DeMorgan's Theorem

X	Y	Z	XYZ	$\overline{XYZ}$	$\bar{X} + \bar{Y} + \bar{Z}$
0	0	0	0	1	1
0	0	1	0	1	1
0	1	0	0	1	1
0	1	1	0	1	1
1	0	0	0	1	1
1	0	1	0	1	1
1	1	0	0	1	1
1	1	1	1	0	0

b) $X + YZ = (X + Y) \cdot (X + Z)$

The Second Distributive Law

X	Y	Z	YZ	X+YZ	X+Y	X+Z	(X+Y)(X+Z)
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

c) $\bar{X}Y + \bar{Y}Z + X\bar{Z} = X\bar{Y} + Y\bar{Z} + \bar{X}Z$

X	Y	Z	$\bar{X}Y$	$\bar{Y}Z$	$X\bar{Z}$	$\bar{X}Y + \bar{Y}Z + X\bar{Z}$	$X\bar{Y}$	$Y\bar{Z}$	$\bar{X}Z$	$X\bar{Y} + Y\bar{Z} + \bar{X}Z$
0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	1	0	1	0	0	1	1
0	1	0	1	0	0	1	0	1	0	1
0	1	1	1	0	0	1	0	0	1	1
1	0	0	0	0	1	1	1	0	0	1
1	0	1	0	1	0	1	1	0	0	1
1	1	0	0	0	1	1	0	1	0	1
1	1	1	0	0	0	0	0	0	0	0

2-2.*

a) $\bar{X}\bar{Y} + \bar{X}Y + XY = \bar{X} + Y$
 $= (\bar{X}Y + \bar{X}\bar{Y}) + (\bar{X}Y + XY)$
 $= \bar{X}(Y + \bar{Y}) + Y(X + \bar{X})$
 $= \bar{X} + Y$

b) $\bar{A}B + \bar{B}\bar{C} + AB + \bar{B}C = 1$
 $= (\bar{A}B + AB) + (\bar{B}\bar{C} + \bar{B}C)$
 $= B(A + \bar{A}) + \bar{B}(C + \bar{C})$

$$B + \bar{B} = 1$$

$$\begin{aligned} \text{c) } Y + \bar{X}Z + X\bar{Y} &= X + Y + Z \\ &= Y + X\bar{Y} + \bar{X}Z \\ &= (Y + X)(Y + \bar{Y}) + \bar{X}Z \\ &= Y + X + \bar{X}Z \\ &= Y + (X + \bar{X})(X + Z) \\ &= X + Y + Z \end{aligned}$$

$$\begin{aligned} \text{d) } \bar{X}\bar{Y} + \bar{Y}Z + XZ + XY + Y\bar{Z} &= \bar{X}\bar{Y} + XZ + Y\bar{Z} \\ &= \bar{X}\bar{Y} + \bar{Y}Z(X + \bar{X}) + XZ + XY + Y\bar{Z} \\ &= \bar{X}\bar{Y} + X\bar{Y}Z + \bar{X}\bar{Y}Z + XZ + XY + Y\bar{Z} \\ &= \bar{X}\bar{Y}(1 + Z) + X\bar{Y}Z + XZ + XY + Y\bar{Z} \\ &= \bar{X}\bar{Y} + XZ(1 + \bar{Y}) + XY + Y\bar{Z} \\ &= \bar{X}\bar{Y} + XZ + XY(Z + \bar{Z}) + Y\bar{Z} \\ &= \bar{X}\bar{Y} + XZ + XYZ + Y\bar{Z}(1 + X) \\ &= \bar{X}\bar{Y} + XZ(1 + Y) + Y\bar{Z} \\ &= \bar{X}\bar{Y} + XZ + Y\bar{Z} \end{aligned}$$

2-3.†

$$\begin{aligned} \text{a) } ABC\bar{C} + B\bar{C}\bar{D} + BC + \bar{C}D &= B + \bar{C}D \\ &= ABC\bar{C} + ABC + BC + B\bar{C}\bar{D} + B\bar{C}D + \bar{C}D \\ &= AB(\bar{C} + C) + B\bar{C}(\bar{D} + D) + BC + \bar{C}D \\ &= AB + B\bar{C} + BC + \bar{C}D \\ &= B + AB + \bar{C}D \\ &= B + \bar{C}D \end{aligned}$$

$$\begin{aligned} \text{b) } WY + \bar{W}Y\bar{Z} + WXZ + \bar{W}X\bar{Y} &= WY + \bar{W}X\bar{Z} + \bar{X}Y\bar{Z} + X\bar{Y}Z \\ &= (WY + W\bar{X}Y\bar{Z}) + (\bar{W}XY\bar{Z} + \bar{W}\bar{X}Y\bar{Z}) + (WXYZ + WX\bar{Y}Z) + (\bar{W}X\bar{Y}Z + \bar{W}X\bar{Y}\bar{Z}) \\ &= (WY + WXYZ) + (\bar{W}XY\bar{Z} + \bar{W}X\bar{Y}\bar{Z}) + (\bar{W}\bar{X}Y\bar{Z} + W\bar{X}Y\bar{Z}) + (WX\bar{Y}Z + \bar{W}X\bar{Y}\bar{Z}) \\ &= WY + \bar{W}X\bar{Z}(Y + \bar{Y}) + \bar{X}Y\bar{Z}(\bar{W} + W) + X\bar{Y}Z(W + \bar{W}) \\ &= WY + \bar{W}X\bar{Z} + \bar{X}Y\bar{Z} + X\bar{Y}Z \end{aligned}$$

$$\begin{aligned} \text{c) } \overline{AD + \bar{A}B + \bar{C}D + \bar{B}C} &= (\bar{A} + \bar{B} + \bar{C} + \bar{D})(A + B + C + D) \\ &= \overline{AD + \bar{A}B + \bar{C}D + \bar{B}C} \\ &= (\bar{A} + D)(\bar{A} + \bar{B})(C + \bar{D})(B + \bar{C}) \\ &= (\bar{A}\bar{B} + AD + \bar{B}D)(BC + B\bar{D} + \bar{C}\bar{D}) \\ &= \overline{\bar{A}\bar{B}\bar{C}\bar{D}} + ABCD \\ &= (A + B + C + D)(\bar{A} + \bar{B} + \bar{C} + \bar{D}) = (\bar{A} + \bar{B} + \bar{C} + \bar{D})(A + B + C + D) \end{aligned}$$

2-4.†

$$\begin{aligned} \text{Given: } A \cdot B &= 0, A + B = 1 \\ \text{Prove: } (A + C)(\bar{A} + B)(B + C) &= BC \\ &= (AB + \bar{A}C + BC)(B + C) \\ &= AB + \bar{A}C + BC \end{aligned}$$

$$\begin{aligned}
 &= 0 + C(\bar{A} + B) \\
 &= C(\bar{A} + B)(0) \\
 &= C(\bar{A} + B)(A + B) \\
 &= C(AB + \bar{A}B + B) \\
 &= BC
 \end{aligned}$$

2-5.†

- Step 1: Define all elements of the algebra as four bit vectors such as A , B and C :
- $$\begin{aligned}
 A &= (A_3, A_2, A_1, A_0) \\
 B &= (B_3, B_2, B_1, B_0) \\
 C &= (C_3, C_2, C_1, C_0)
 \end{aligned}$$
- Step 2: Define OR_1 , AND_1 and NOT_1 so that they conform to the definitions of AND, OR and NOT presented in Table 2-1.
- a) $A + B = C$ is defined such that for all $i, i = 0, \dots, 3$, C_i equals the OR_1 of A_i and B_i .
 - b) $A B = C$ is defined such that for all $i, i = 0, \dots, 3$, C_i equals the AND_1 of A_i and B_i .
 - c) The element 0 is defined such that for $A = "0"$, for all $i, i = 0, \dots, 3$, A_i equals logical 0.
 - d) The element 1 is defined such that for $A = "1"$, for all $i, i = 0, \dots, 3$, A_i equals logical 1.
 - e) For any element A , $\bar{A}$ is defined such that for all $i, i = 0, \dots, 3$, $\bar{A}_i$ equals the NOT_1 of A_i .

2-6.

- a) $\bar{A}\bar{C} + \bar{A}BC + \bar{B}C = \bar{A}\bar{C} + \bar{A}BC + (\bar{A}\bar{B}C + \bar{B}C)$
 $= \bar{A}\bar{C} + (\bar{A}BC + \bar{A}\bar{B}C) + \bar{B}C$
 $= (\bar{A}\bar{C} + \bar{A}C) + \bar{B}C = \bar{A} + \bar{B}C$
- b) $\overline{(A + B + C)(\bar{A}\bar{B}\bar{C})}$
 $= \bar{A}\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}\bar{C}$
 $= (\bar{A}\bar{A})\bar{B}\bar{C} + \bar{A}(\bar{B}\bar{B})\bar{C} + \bar{A}\bar{B}(\bar{C}\bar{C})$
 $= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} = \bar{A}\bar{B}\bar{C}$
- c) $AB\bar{C} + AC = A(\bar{B}\bar{C} + C) = A(B + C)$
- d) $\bar{A}\bar{B}D + \bar{A}\bar{C}D + BD$
 $= (\bar{A}\bar{B} + B + \bar{A}\bar{C})D$
 $= (\bar{A} + \bar{A}\bar{C} + B)D$
 $= (\bar{A} + B)D$
- e) $\overline{(\bar{A} + B)(\bar{A} + \bar{C})(\bar{A}\bar{B}\bar{C})}$
 $= (A\bar{B})(AC)(\bar{A} + B + \bar{C}) = A\bar{B}C(\bar{A} + B + \bar{C})$
 $= 0$

2-7.*

- a) $\bar{X}\bar{Y} + XYZ + \bar{X}Y = \bar{X} + XYZ = (\bar{X} + XY)(\bar{X} + Z) = (\bar{X} + X)(\bar{X} + Y)(\bar{X} + Z)$
 $= (\bar{X} + Y)(\bar{X} + Z) = \bar{X} + YZ$
- b) $X + Y(Z + \bar{X} + \bar{Z}) = X + Y(Z + \bar{X}\bar{Z}) = X + Y(Z + \bar{X})(Z + \bar{Z}) = X + YZ + \bar{X}Y$
 $= (X + \bar{X})(X + Y) + YZ = X + Y + YZ = X + Y$
- c) $\bar{W}X(\bar{Z} + \bar{Y}Z) + X(W + \bar{W}YZ) = \bar{W}X\bar{Z} + \bar{W}X\bar{Y}Z + WX + \bar{W}XYZ$

Problem Solutions – Chapter 2

$$= \overline{W}X\overline{Z} + \overline{W}XZ + WX = \overline{W}X + WX = X$$

$$\begin{aligned} \text{d)} \quad & (AB + \overline{A}\overline{B})(\overline{C}\overline{D} + CD) + \overline{A}\overline{C} = \overline{A}B\overline{C}\overline{D} + \overline{A}BCD + \overline{A}\overline{B}CD + \overline{A}\overline{B}\overline{C}\overline{D} + \overline{A} + \overline{C} \\ & = \overline{A}BCD + \overline{A} + \overline{C} = \overline{A} + \overline{C} + A(BCD) = \overline{A} + \overline{C} + C(BD) = \overline{A} + \overline{C} + BD \end{aligned}$$

2-8.

$$\begin{aligned} \text{a)} \quad F &= \overline{A}\overline{B}C + \overline{A}\overline{C} + AB \\ &= (\overline{A} + B + \overline{C}) + (\overline{A} + C) + (\overline{A} + \overline{B}) \end{aligned}$$

$$\begin{aligned} \text{b)} \quad \overline{\overline{F}} &= \overline{\overline{A}\overline{B}C + \overline{A}\overline{C} + AB} \\ &= (\overline{A}\overline{B}C)(\overline{A}\overline{C})(AB) \end{aligned}$$

2-9.*

$$\begin{aligned} \text{a)} \quad \overline{F} &= (\overline{A} + B)(A + \overline{B}) \\ \text{b)} \quad \overline{F} &= ((V + \overline{W})\overline{X} + \overline{Y})Z \\ \text{c)} \quad \overline{F} &= [\overline{W} + \overline{X} + (Y + \overline{Z})(\overline{Y} + Z)][W + X + Y\overline{Z} + \overline{Y}Z] \\ \text{d)} \quad \overline{F} &= \overline{A}\overline{B}\overline{C} + (A + B)\overline{C} + \overline{A}(B + C) \end{aligned}$$

2-10.*

Truth Tables a, b, c

X	Y	Z	a	A	B	C	b	W	X	Y	Z	c
0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	1	0	0	0	1	1	0	0	0	1	0
0	1	0	0	0	1	0	0	0	0	1	0	1
0	1	1	1	0	1	1	1	0	0	1	1	0
1	0	0	0	1	0	0	0	0	1	0	0	0
1	0	1	1	1	0	1	0	0	1	0	1	0
1	1	0	1	1	1	0	0	0	1	1	0	1
1	1	1	1	1	1	1	1	0	1	1	1	0
								1	0	0	0	0
								1	0	0	1	0
								1	0	1	0	1
								1	0	1	1	0
								1	1	0	0	1
								1	1	0	1	1
								1	1	1	0	1
								1	1	1	1	1

$$\begin{aligned} \text{a)} \quad \text{Sum of Minterms:} \quad & \overline{X}YZ + X\overline{Y}Z + XY\overline{Z} + XYZ \\ \text{Product of Maxterms:} \quad & (X + Y + Z)(X + Y + \overline{Z})(X + \overline{Y} + Z)(\overline{X} + Y + Z) \\ \text{b)} \quad \text{Sum of Minterms:} \quad & \overline{A}\overline{B}\overline{C} + \overline{A}\overline{B}C + \overline{A}BC + ABC \\ \text{Product of Maxterms:} \quad & (A + \overline{B} + C)(\overline{A} + B + C)(\overline{A} + B + \overline{C})(\overline{A} + \overline{B} + C) \\ \text{c)} \quad \text{Sum of Minterms:} \quad & \overline{W}\overline{X}Y\overline{Z} + \overline{W}XY\overline{Z} + W\overline{X}Y\overline{Z} + WX\overline{Y}\overline{Z} + WX\overline{Y}Z + WXY\overline{Z} \\ & + WXYZ \\ \text{Product of Maxterms:} \quad & (W + X + Y + Z)(W + X + Y + \overline{Z})(W + X + \overline{Y} + \overline{Z}) \\ & (W + \overline{X} + Y + Z)(W + \overline{X} + Y + \overline{Z})(W + \overline{X} + \overline{Y} + \overline{Z}) \\ & (\overline{W} + X + Y + Z)(\overline{W} + X + Y + \overline{Z})(\overline{W} + X + \overline{Y} + \overline{Z}) \end{aligned}$$

2-11.

$$\begin{aligned} \text{a)} \quad E &= \Sigma m(1, 2, 4, 6) = \Pi M(0, 3, 5, 7), & F &= \Sigma m(0, 2, 4, 7) = \Pi M(1, 3, 5, 6) \\ \text{b)} \quad \overline{E} &= \Sigma m(0, 3, 5, 7), & \overline{F} &= \Sigma m(1, 3, 5, 6) \end{aligned}$$

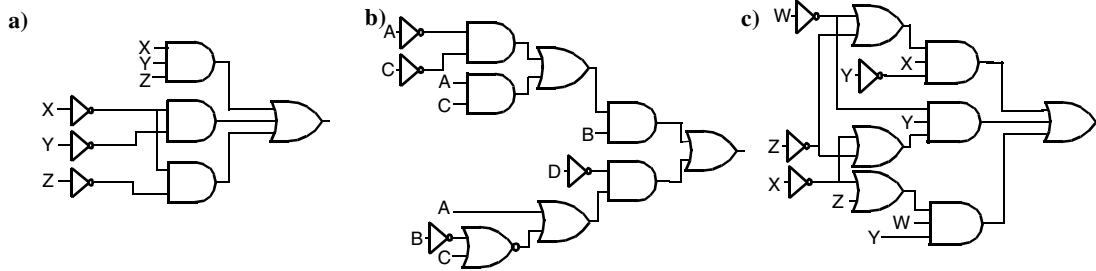
Problem Solutions – Chapter 2

- c) $E + F = \Sigma m(0, 1, 2, 4, 6, 7), \quad E \cdot F = \Sigma m(2, 4)$
d) $E = \bar{X}\bar{Y}Z + \bar{X}Y\bar{Z} + X\bar{Y}\bar{Z} + XYZ, \quad F = \bar{X}\bar{Y}\bar{Z} + \bar{X}Y\bar{Z} + X\bar{Y}\bar{Z} + XYZ$
e) $E = \bar{Z}(X + Y) + \bar{X}\bar{Y}Z, \quad F = \bar{Z}(\bar{X} + \bar{Y}) + XYZ$

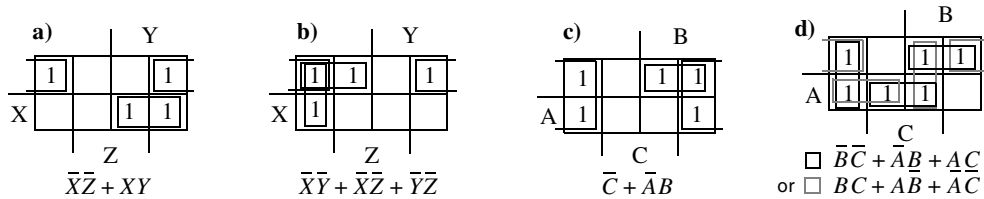
2-12.*

- a) $(AB + C)(B + \bar{C}D) = AB + AB\bar{C}D + BC = AB + BC \text{ s.o.p.}$
 $= B(A + C) \text{ p.o.s.}$
b) $\bar{X} + X(X + \bar{Y})(Y + \bar{Z}) = (\bar{X} + X)(\bar{X} + (X + \bar{Y})(Y + \bar{Z}))$
 $= (\bar{X} + X + \bar{Y})(\bar{X} + Y + \bar{Z}) \text{ p.o.s.}$
 $= (1 + \bar{Y})(\bar{X} + Y + \bar{Z}) = \bar{X} + Y + \bar{Z} \text{ s.o.p.}$
c) $(A + B\bar{C} + CD)(\bar{B} + EF) = (A + B + C)(A + B + D)(A + \bar{C} + D)(\bar{B} + EF)$
 $= (A + B + C)(A + B + D)(A + \bar{C} + D)(\bar{B} + E)(\bar{B} + F) \text{ p.o.s.}$
 $(A + B\bar{C} + CD)(\bar{B} + EF) = A(\bar{B} + EF) + B\bar{C}(\bar{B} + EF) + CD(\bar{B} + EF)$
 $= A\bar{B} + AEF + B\bar{C}EF + \bar{B}CD + CDEF \text{ s.o.p.}$

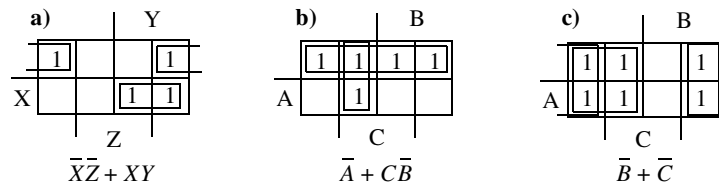
2-13.



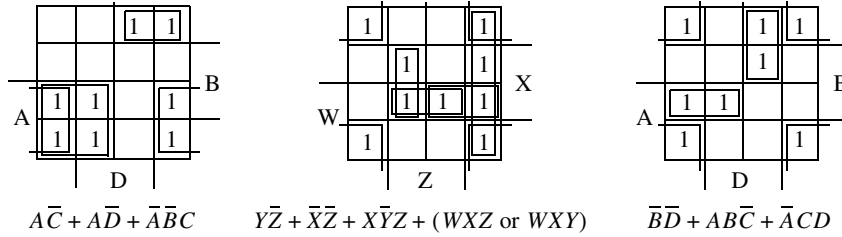
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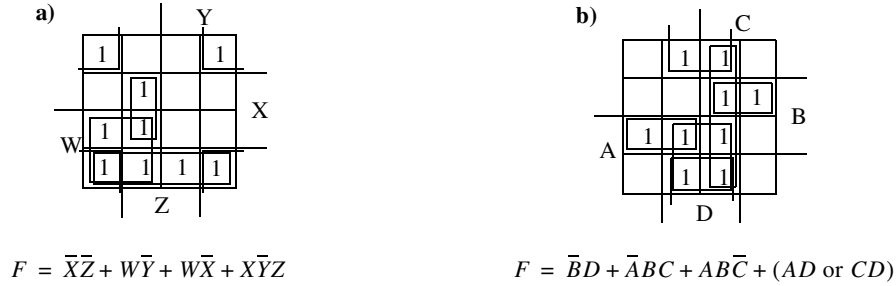
2-15.*



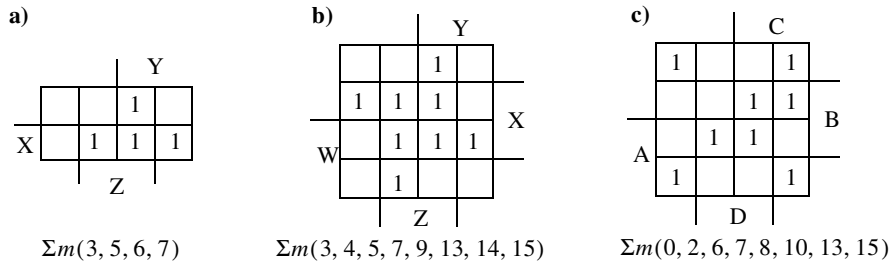
2-16.



2-17.



2-18. *

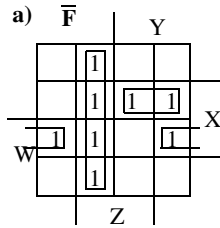


2-19. *

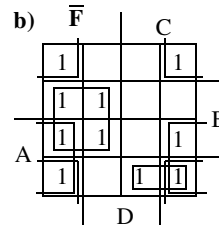
- a) Prime = $XZ, WX, \bar{X}\bar{Z}, W\bar{Z}$ b) Prime = $CD, AC, \bar{B}\bar{D}, \bar{A}BD, \bar{B}C$ c) Prime = $AB, AC, AD, \bar{B}\bar{C}, \bar{B}D, \bar{C}D$
 Essential = $XZ, \bar{X}\bar{Z}$ Essential = $AC, \bar{B}\bar{D}, \bar{A}BD$ Essential = $AC, \bar{B}\bar{C}, \bar{B}D$

- 2-20. a) Prime = $\bar{X}Y, \bar{X}\bar{Z}, W\bar{Y}\bar{Z}, WX\bar{Y}, X\bar{Y}Z, \bar{W}XZ, \bar{W}YZ$ b) Prime = $\bar{A}\bar{B}\bar{C}, \bar{A}CD, ABC, A\bar{C}D, BD$
 Essential = $\bar{X}Y, \bar{X}\bar{Z}$ Essential = $\bar{A}\bar{B}\bar{C}, \bar{A}CD, ABC, A\bar{C}D$
 $F = \bar{X}Y + XZ + WX\bar{Y} + \bar{W}XZ$ Redundant = BD
 $F = \bar{A}\bar{B}\bar{C} + \bar{A}CD + ABC + A\bar{C}D$
 c) Prime = $\bar{Y}\bar{Z}, W\bar{Y}, \bar{W}\bar{Z}, WXZ, XYZ, \bar{W}XY$
 Essential = $W\bar{Y}, \bar{W}\bar{Z}$
 Redundant = $\bar{Y}\bar{Z}$
 $F = W\bar{Y} + \bar{W}\bar{Z} + XYZ$

2-21.



$$\begin{aligned}\bar{F} &= \Sigma m(1, 5, 6, 7, 9, 12, 13, 14) \\ F &= \overline{YZ} + \overline{WXZ} + \overline{WXY} \\ F &= (Y + \bar{Z})(\bar{W} + \bar{X} + Z)(W + \bar{X} + \bar{Y})\end{aligned}$$



$$\begin{aligned}\bar{F} &= \Sigma m(0, 2, 4, 5, 8, 10, 11, 12, 13, 14) \\ F &= \overline{BC} + \overline{BD} + \overline{AD} + \overline{ABC} \\ F &= (\bar{B} + C)(B + D)(\bar{A} + D)(\bar{A} + B + \bar{C})\end{aligned}$$

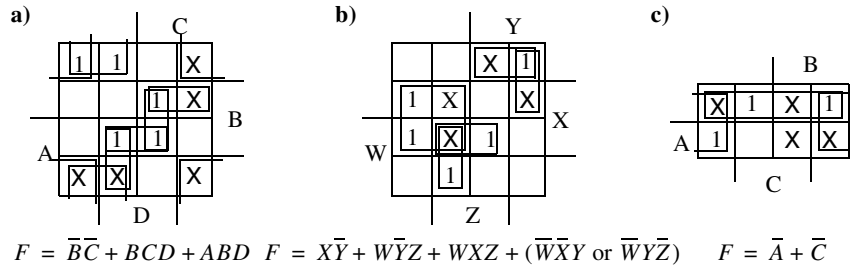
2-22.*

- a) s.o.p. $CD + A\bar{C} + \bar{B}D$ p.o.s. $(\bar{C} + D)(A + D)(A + \bar{B} + C)$
 b) s.o.p. $\bar{A}\bar{C} + \bar{B}\bar{D} + A\bar{D}$ p.o.s. $(\bar{C} + \bar{D})(\bar{A} + \bar{D})(A + \bar{B} + \bar{C})$
 c) s.o.p. $\bar{B}\bar{D} + \bar{A}BD + (\bar{A}BC \text{ or } \bar{A}C\bar{D})$ p.o.s. $(\bar{A} + \bar{B})(B + \bar{D})(\bar{B} + C + D)$

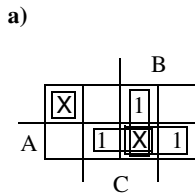
2-23.

- a) s.o.p. $\bar{A}\bar{B}\bar{C} + \bar{A}BD + ABC + A\bar{B}\bar{D}$
 or $\bar{A}\bar{C}D + BCD + AC\bar{D} + \bar{B}\bar{C}\bar{D}$
 p.o.s. $(A + B + \bar{C})(A + \bar{B} + D)(\bar{A} + \bar{B} + C)(\bar{A} + B + \bar{D})$
 or $(A + \bar{C} + D)(\bar{B} + C + D)(\bar{A} + C + \bar{D})(B + \bar{C} + \bar{D})$
 b) s.o.p. $\bar{Z} + \bar{W}X + \bar{X}\bar{Y}$
 p.o.s. $(\bar{W} + \bar{X} + \bar{Z})(X + \bar{Y} + \bar{Z})$

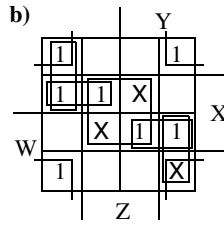
2-24.



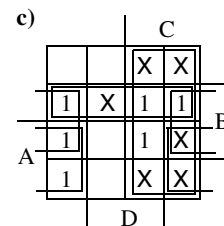
2-25.*



Primes = $AB, AC, BC, \bar{A}\bar{B}\bar{C}$
 Essential = AB, AC, BC
 $F = AB + AC + BC$



Primes = $\bar{X}\bar{Z}, XZ, \bar{W}X\bar{Y}, WXY, \bar{W}\bar{Y}\bar{Z}, WY\bar{Z}$
 Essential = $\bar{X}\bar{Z}$
 $F = \bar{X}\bar{Z} + \bar{W}X\bar{Y} + WXY$



Primes = $\bar{A}B, C, A\bar{D}, B\bar{D}$
 Essential = $C, A\bar{D}$
 $F = C + A\bar{D} + (B\bar{D} \text{ or } \bar{A}B)$

2-26.

a)(1)

		C	
X	0	0	X
0	X	1	0
X	1	1	X
X	0	0	X
		D	

$F = BD$

a)(2)

		C	
X	0	0	X
0	X		0
X			X
X	0	0	X
		D	

$\bar{F} = \bar{B} + \bar{D}$
 $F = BD$

b)(1)

		Y	
X	X	1	X
1	X		
X		1	X
	1		X
		Z	

$F = \bar{W}\bar{X} + \bar{W}\bar{Y} + \bar{X}\bar{Y}Z + WXY$

b)(2)

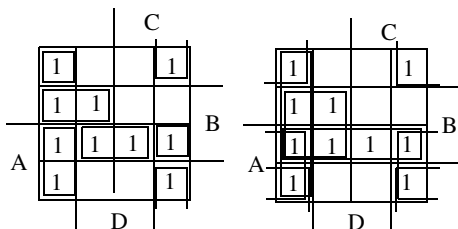
		Y	
X	X	1	X
1	X		
X		1	X
	1		X
		Z	

$\bar{F} = (W\bar{Z} \text{ or } \bar{X}\bar{Z}) + \bar{W}XY + W\bar{X}Y + (W\bar{X}\bar{Y} \text{ or } X\bar{Y}Z)$

$F = ((\bar{W} + Z) \text{ or } (X + Z)) (W + \bar{X} + \bar{Y})(\bar{W} + X + \bar{Y}) + (\bar{W} + \bar{X} + Y) \text{ or } (\bar{X} + Y + \bar{Z})$

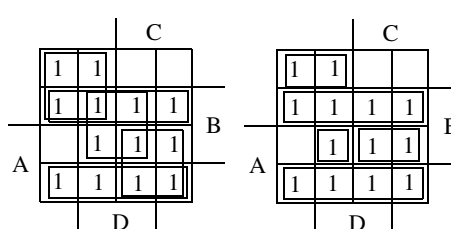
2-27.

a) $F = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}B\bar{C} + A\bar{C}\bar{D} + ABD + ABC\bar{D} + \bar{B}C\bar{D}$



There are other solutions depending on how ties are resolved.

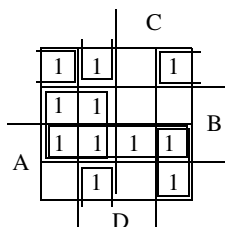
b) $F = \bar{A}\bar{C} + \bar{A}B + BD + AC + A\bar{B}$



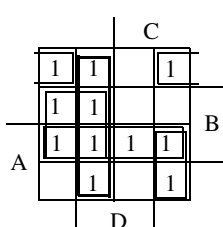
There are other solutions depending on how ties are resolved.

2-28.+

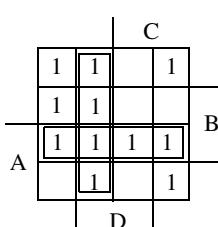
$F = \bar{A}\bar{B}\bar{D} + \bar{B}\bar{C}D + B\bar{C} + AB + AC\bar{D}$



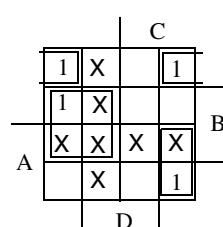
F and $\bar{F}$; Cost = 18



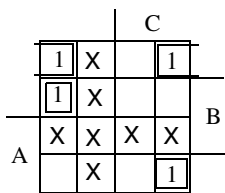
EXPAND



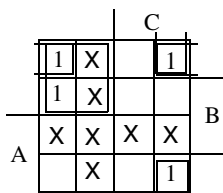
ESSENTIAL PRIMES



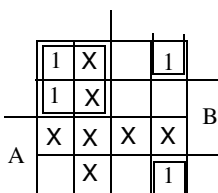
IRREDUNDANT COVER;
Cost = 17



REDUCE



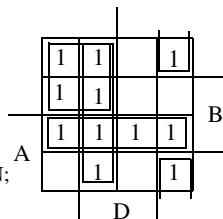
EXPAND



IRREDUNDANT COVER;
Cost = 13

FINAL SOLUTION;
Cost = 13

REDUCE, EXPAND,
IRREDUNDANT COVER,
and LAST GASP produce
no lasting changes.



2-29.

a) $F = A\bar{B}C + \bar{A}BC + A\bar{B}D + \bar{A}BD$

$X_1 = A\bar{B}$

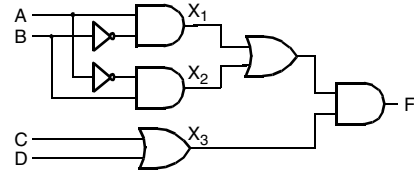
$X_2 = \bar{A}B$

$F = X_1C + X_1D + X_2C + X_2D$

$= (X_1 + X_2)(C + D)$

$X_3 = C + D$

$F = (X_1 + X_2)X_3$



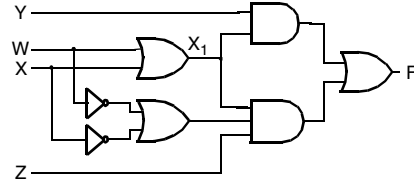
b) $F = WY + XY + \bar{W}XZ + W\bar{X}Z$

$= (W + X)Y + (\bar{W}X + W\bar{X})Z$

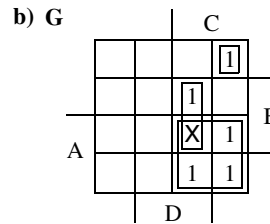
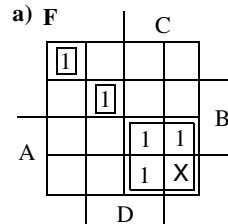
$= (W + X)Y + (W + X)(\bar{W} + \bar{X})Z$

$X_1 = W + X$

$F = X_1Y + X_1(\bar{W} + \bar{X})Z$



2-30.



$F = AC + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}C\bar{D}$

$= AC + \bar{A}\bar{C}(BD + \bar{B}\bar{D})$

$G = AC + BCD + \bar{A}\bar{B}\bar{C}\bar{D}$

$= AC + (ABCD + \bar{A}\bar{B}CD) + \bar{A}\bar{B}\bar{C}\bar{D}$

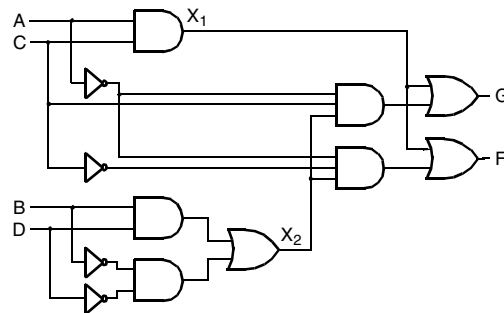
$= AC + \bar{A}\bar{C}(BD + \bar{B}\bar{D})$

$X_1 = AC$

$X_2 = BD + \bar{B}\bar{D}$

$F = X_1 + \bar{A}\bar{C}X_2$

$G = X_1 + \bar{A}\bar{C}X_2$



2-31.

a) $F = AB(\bar{C}\bar{D} + \bar{C}D) + \bar{B}(C\bar{D} + \bar{C}D) + \bar{A}(\bar{B} + CD)$
 $= AB(\bar{C} + D)(C + \bar{D}) + \bar{B}(C\bar{D} + \bar{C}D) + \bar{A}(\bar{B}(\bar{C} + D))$
 $= AB\bar{C}\bar{D} + ABCD + \bar{B}\bar{C}\bar{D} + \bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}D$

b) $T = YZ(W + \bar{X}) + \bar{Y}\bar{Z}(\bar{W}Y + X)$
 $= WYZ + \bar{X}YZ + X\bar{Y}\bar{Z}$

2-32.*

$$\begin{aligned}
 X \oplus Y &= X\bar{Y} + \bar{X}Y \\
 \text{Dual}(X \oplus Y) &= \text{Dual}(X\bar{Y} + \bar{X}Y) \\
 &= (X + \bar{Y})(\bar{X} + Y) \\
 &= \overline{\bar{X}Y + X\bar{Y}} \\
 &= \overline{X\bar{Y} + \bar{X}Y} \\
 &= \bar{X} \oplus \bar{Y}
 \end{aligned}$$

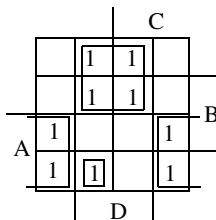
2-33.

$$AB\bar{C}D + A\bar{D} + \bar{A}D = AB\bar{C}D + (A \oplus D)$$

$$\text{Note that } X + Y = (X \oplus Y) + XY$$

$$\text{Letting } X = AB\bar{C}D \text{ and } Y = A \oplus D,$$

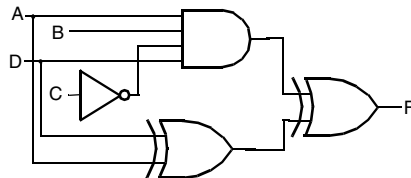
We can observe from the map below or determine algebraically that XY is equal to 0.



For this situation,

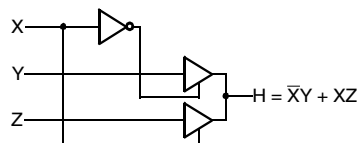
$$\begin{aligned}
 X + Y &= (X \oplus Y) + XY \\
 &= (X \oplus Y) + 0 \\
 &= X \oplus Y
 \end{aligned}$$

$$\text{So, we can write } F(A, B, C, D) = X \oplus Y = AB\bar{C}D \oplus (A \oplus D)$$

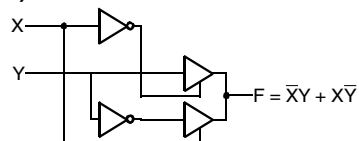


2-34.

a)

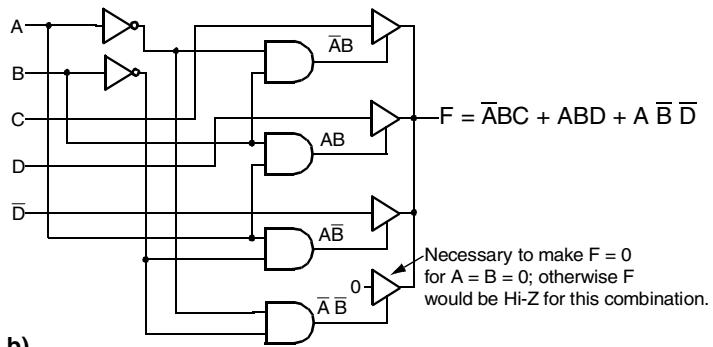


b)



2-35.

a)



b)

There are no three-state output conflicts.