CHAPTER 1 Foundations for Learning Mathematics

SECTION 1.1 What Is Mathematics?

1. Answers will vary.

2. Answers will vary.

3. Answers will vary.

4. Answers will vary.

5. The habits and attitudes a mathematically proficient student will have are: They can make sense of problems and persevere in solving them, reason abstractly and quantitatively, construct viable arguments and critique the reasoning of others, model with mathematics, use appropriate tools strategically, attend to precision, look for and make use of structure, and look for and express regularity in repeated reasoning.

6. People who have a mathematical growth mindset hold the following beliefs: 1. They believe that if they try really hard, they can solve virtually every problem in this book. 2. There is usually more than one way to solve most problems. 3. The best way to learn is to make sure that I understand the concepts and each step I take to solve the question. 4. Everyone can learn math with the right opportunities to learn and hard work. 5. I am comfortable making mistakes because mistakes help me to learn. 6. Mathematics is about problem solving and critical thinking. 7. I need to develop a deeper understanding of all elementary school math topics.

7.

Student A:

$$\begin{array}{r} 86 \\ \times\ 47 \\ \hline 602 \\ 344 \\ \hline 4042 \end{array}$$

Student B:

$$\begin{array}{r} 86 \\ \times\ 47 \\ \hline 42 \\ 560 \\ 240 \\ 3200 \\ \hline 4042 \end{array}$$

8. Answers will vary.

9. Answers will vary.

10. Answers will vary.

11. In one week, Sally makes 40 hours × \$10.00 per hour = \$400. Assuming she works 5 days a week, she pays \$20 · 5 = \$100 per week for child care. After deducting child care expenses, she makes \$400 – \$100 = \$300. Therefore, she actually makes \$300 ÷ 40 = \$7.50 per hour.

12. **a.** If one penny has a diameter of 0.75 inches, then we have 288 × 0.75 = 216 billion inches; or 216 ÷ 12 = 18 billion feet; or 18 ÷ 5280 ≈ .003092784 billion miles = 3,092,784 miles.

b. 30,000,000 ÷ 24 = 1,250,000 per hour; 1,250,000 ÷ 60 = 20,833 per minute; 20,833 ÷ 60 = 347 per second. 30,000,000 × 365 = 10,950,000,000 pennies per year.

13. In one day, the arctic tern flies 25 miles per hour × 12 hours = 300 miles per day. A one-way flight is 10,000 miles, so it takes 10,000 miles ÷ 300 miles per day = $33\frac{1}{3}$ days, or about 33 days.

14. **a.** Need to make assumptions for the distance driven and the average cost of gasoline.

b. Answers will vary. Sample answer: If the distance is 2400 miles and the cost of gas averages $3.00 per gallon, then the cost of gas for each is as follows:

Van: (2400 ÷ 18) × 3 = $400 Sedan: (2400 ÷ 32) × 3 = $225

The difference is $400 – $225 = $175. So, it will take $175 extra to drive the van.

c. Using the assumptions in part b, if the price of gas rises 40¢, then it is now averaging $3.40 per gallon. Using the same procedure as in part b, it costs $53.33 more for the van and $30 more for the sedan.

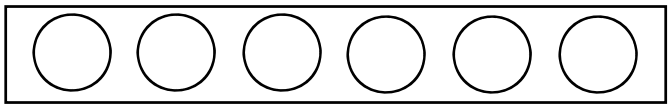
15. **a.** The numbers decrease clockwise instead of increasing.

b. It is 3:00.

c. The minute hand will point to the 9 and the hour hand will be between 9 and 10 (closer to 10).

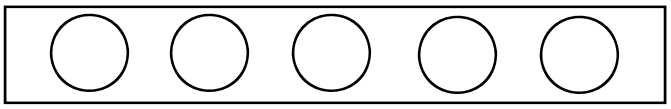
d. Answers will vary.

16. **a.**



$\frac{104-(8\times6)}{7}=\frac{104-48}{7}=\frac{56}{7}=8$; She should hang each plate with an 8-inch space between each plate and between the end plates and the wall.

b. If one plate breaks, she would have 5 plates to hang.



$\frac{104-(8\times5)}{6}=\frac{104-40}{6}=\frac{64}{6}=10\frac{4}{6}=10\frac{2}{3}$; She should hang each plate with a $10\frac{2}{3}$-inch space between each plate and between each end plate.

c. $\frac{104-(12\times2)-(8\times6)}{5}=\frac{104-24-48}{5}=\frac{32}{5}=6\frac{2}{5}$; She should hang each plate with a $6\frac{2}{5}$-inch space between them.

17. There are 10 pigs and 30 chickens.

18. There are 3 possibilities: There could be 6 bicycles, there could be 3 bicycles and 2 tricycles, or there could be 4 tricycles.

19. There are 30 pigs and 139 chickens.

20. **a.** The possible scores are: 4, 8, 12, 13, 16, 17, 20, 21, 22, 25, 26, 28, 30, 31, 32, 35, 36, 37, 40, 41, 45, 46, 50, 52, 55, 56, 60, 61, 65, 70, 76, 80, 85, and 100.

b. Answers will vary.

21. For all parts, let the bottom of the well be 0 feet.

a. 9 hours

Hour	1	2	3	4	5	6	7	8	9
Height	3	2	5	4	7	6	9	8	11

b. 23 hours

Hour	1	2	3	4	5	6
Height	7	4	11	8	15	12

It appears that the height is a multiple of 4 when the hours are even; or, more simply, the height is 2 times the number representing the hour. After 22 hours, the frog has climbed 44 feet. Therefore, it will reach the top of the well in 23 hours.

c. Using a similar strategy as in parts (a) and (b), ot will take the caterpillar 19 hours to climb to the top of the jar.

22. Bring over the goose (the fox will not eat the corn). Return and bring over the fox, but return with the goose. Bring over the corn. Return and bring over the goose again.

23. **a.** There are 48 ways

Row	Quarters	Dimes	Nickels	Pennies
1	2	0	0	0
2	1	2	1	0
3	1	2	0	5
4	1	1	3	0
5	1	1	2	5
6	1	1	1	10
7	1	0	5	0
8	1	0	4	5
9	1	0	3	10
10	1	0	2	15
11	1	0	1	20
12	1	0	0	25
13	0	5	0	0
14	0	4	2	0
15	0	4	1	5
16	0	4	0	10
17	0	3	4	0
18	0	3	3	5
19	0	3	2	10
20	0	3	1	15
21	0	3	0	20
22	0	2	6	0
23	0	2	5	5
24	0	2	4	10
25	0	2	3	15
26	0	2	2	20

Row	Quarters	Dimes	Nickels	Pennies
27	0	2	1	25
28	0	2	0	30
29	0	1	8	0
30	0	1	7	5
31	0	1	6	10
32	0	1	5	15
33	0	1	4	20
34	0	1	3	25
35	0	1	2	30
36	0	1	1	35
37	0	1	0	40
38	0	0	10	0
39	0	0	9	5
40	0	0	8	10
41	0	0	7	15
42	0	0	6	20
43	0	0	5	25
44	0	0	4	30
45	0	0	3	35
46	0	0	2	40
47	0	0	1	45
48	0	0	0	50

b. There are 29 ways

Row	Quarters	Dimes	Nickels
1	4	0	0
2	3	2	1
3	3	1	3
4	3	0	5
5	2	5	0
6	2	4	2
7	2	3	4
8	2	2	6
9	2	1	8
10	2	0	10
11	1	7	1
12	1	6	3
13	1	5	5
14	1	4	7
15	1	3	9
16	1	2	11
17	1	1	13
18	1	0	5
19	0	10	0
20	0	9	2
21	0	8	4
22	0	7	6
23	0	6	8
24	0	5	10
25	0	4	12
26	0	3	14
27	0	2	16
28	0	1	18
29	0	0	20

24. Answers will vary.

25. **a.** (1) There are many patterns; (2) Next row: 6 12 18 24 30 36

b. (1) There are many patterns; (2) Next row: 1 13 61 129 129 61 13 1

c. (1) There are many patterns; (2) Next row: 49 54 49 54 65 78 84 95 96

d. (1) There are many patterns; (2) Next row: 3 21 63 105 63 21 3

26. Since Sheila is using one 44 cent stamp, we only need to calculate how many ways we can make 61 cents - 44 cents = 17 cents using 5, 3, and 2 cent stamps. There are 8 ways to do this:

5 5 5 2

5 3 3 3 3

5 3 3 2 2 2

5 2 2 2 2 2 2

3 3 3 3 3 2

3 3 3 2 2 2 2

3 2 2 2 2 2 2 2

5 5 3 2 2

27. Since Karen used a total of 24 toothpicks to make the square and each side of the square has the same length, each side used 6 toothpicks. Since a hexagon has 6 sides with each side having 6 toothpicks, there are a total of 36 toothpicks in the hexagon.

28. The least amount of money you could have is one quarter, one dime, and six pennies, which is $25¢ + 10¢ + 6¢ = 41¢$.

29. Since you want to minimize the number of tables you would need to use as many round tables as possible because they each hold more. You cannot use 6 round tables because that would be a total of 36 chairs, too may. You cannot use 5 round tables because you would then need 1 square table giving you 34 chairs, too many. So you must need 4 round tables and 2 square tables for a total of exactly 32 chairs.

30. The fewest number of trips she can take to walk all 16 dogs is 6; 5 trips with 3 dogs each time and 1 trip with 1 dog or 4 trips with 3 dogs each time and 2 trips with 2 dogs each time. The correct answer is c.

SECTION 1.2 Sets

1. **a.** $0 \notin \varnothing$ or $0 \notin \{\}$ **b.** $3 \notin B$

2. **a.** $D \nsubseteq E$ **b.** $A \subseteq U$

3. **a.** {e, l, m, n, t, a, r, y} and $\{x \mid x$ is a letter in the word "elementary"$\}$
or $\{x \mid x$ is one of these letters: e, l, m, n, t, a, r, y$\}$.

b. {Spain, Portugal, France, Ireland, United Kingdom (England/Scotland), Western Russia, Germany, Italy, Austria, Switzerland, Belgium, Netherlands, Estonia, Latvia, Denmark, Sweden, Norway, Finland, Poland, Bulgaria, Yugoslavia, The Czech Republic, Slovakia, Romania, Greece, Macedonia, Albania, Croatia, Hungary, Bosnia and Herzegovina, Ukraine, Belarus, Lithuania}.
Also $\{x \mid x$ is a country in Europe$\}$.

c. {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97}.
Also $\{x \mid x$ is a prime less than 100$\}$.

d. The set of fractions between 0 and 1 is infinite.
$\{x \mid x$ is a fraction between zero and one$\}$.

e. {name1, name2, name3, etc.}.
$\{x \mid x$ is a student in this class$\}$.

4. **a.** $\subset$ **b.** $\in$ **c.** $\subset$ **d.** $\subset$

e. True **f.** False; red is an element, not a set.

g. False; gray is not in set *S*. **h.** True

5. **a.** $\in$; 3 is an element of the set. **b.** $\subset$; {3} is a subset of the set.

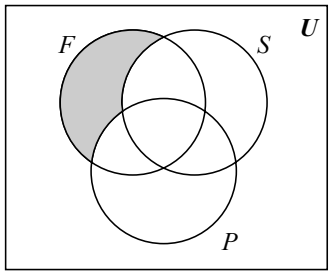
c. $\in$; {1} is an element of this set of sets. **d.** $\subset$; $\{a\}$ is a subset of the set.

e. $\not\subset$ or $\notin$; $\{ab\}$ $\not\subset$ is neither a subset nor an element.

f. $\subset$; the null set is a subset of every set.

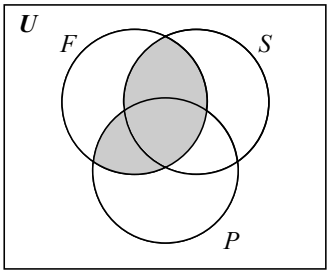
6. **a.** 64 **b.** A set with n elements has 2^n subsets.

7. **a.**



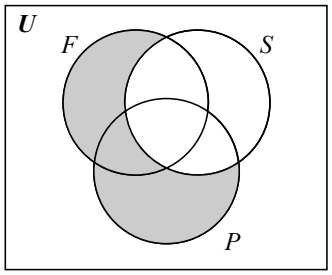
$F \cap \left(\overline{S \cup P}\right)$ or $F \cap \overline{S} \cap \overline{P}$

b.



American females who smoke and/or have a health problem.

c.



Nonsmokers who either are female or have health problems.

d. $F \cap S$

Females who smoke.

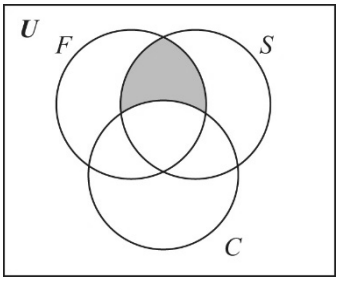
e. $F \cap (S \cap P)$

Males who smoke and have a health problem.

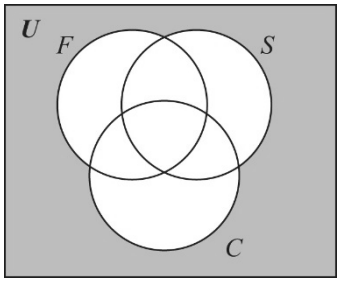
8. **a.** Students who are members of at least two of the film, science, and computer clubs.
$(F \cap S) \cup (S \cap C) \cup (C \cap F)$

b. Students who are members of both the science and computer clubs, but not the film club.
$\overline{F} \cap (S \cap C)$

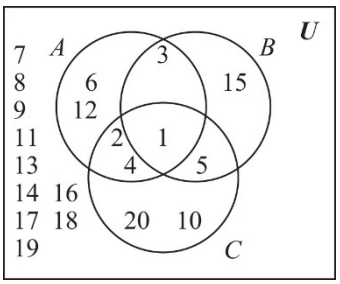
c. $\overline{C} \cap (S \cap F)$



d. $\overline{F \cup S \cup C}$



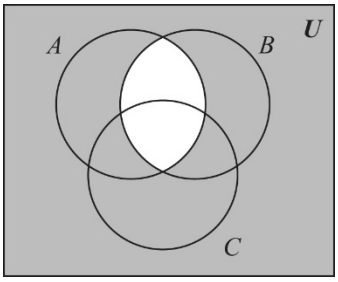
9. **a.**



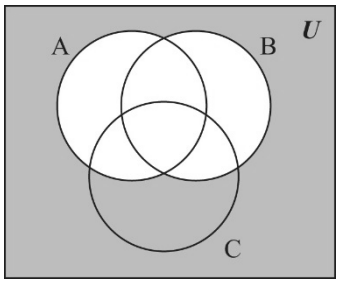
b. All numbers that don't evenly divide 12, 15, or 20; $\overline{A \cup B \cup C}$ or $\overline{A} \cap \overline{B} \cap \overline{C}$

c. All numbers that evenly divide 12 and 20, but not 15; $\overline{B} \cap (A \cap C)$

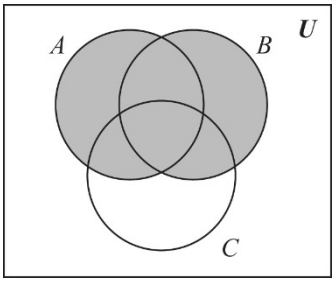
d. All numbers except 1 and 3.



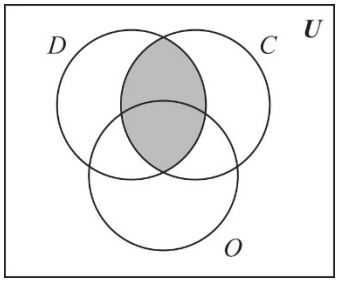
e. All numbers from 1 to 20, except those that divide 12 or 15 evenly.



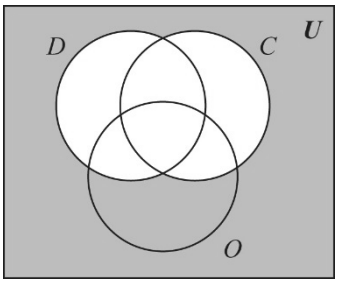
f. Note: This description is ambiguous; it depends on how one interprets "or." $A \cup B$



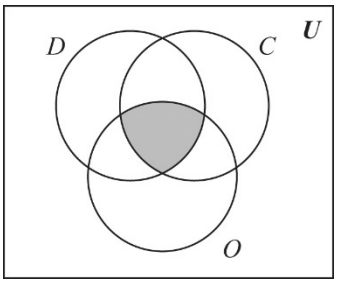
10. a. Students who have at least one cat and at least one dog.



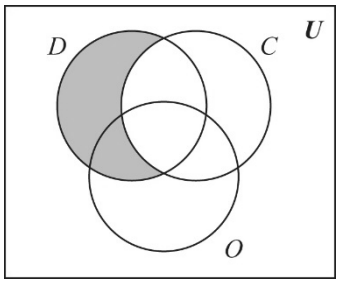
b. Students who have neither cats nor dogs.



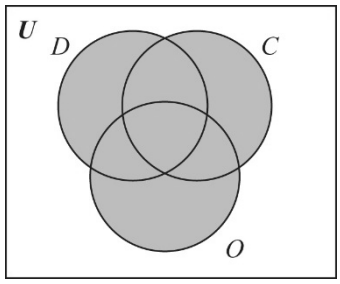
c. Students who have at least one cat, at least one dog, and at least one other pet.



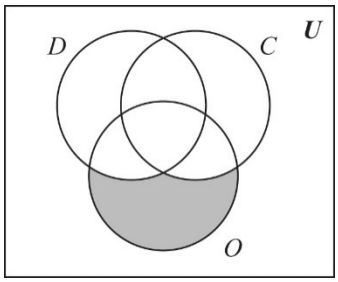
d. $D \cap \overline{C}$



e. $D \cup C \cup O$



f. $O \cap (\overline{D \cup C})$



g. $\overline{D \cup C \cup O}$ or $\overline{D} \cap \overline{C} \cap \overline{O}$

Students who have no pets.

h. $C \cap (\overline{D \cup O})$

Students who have at least one cat and no other pets.

11. 15 possible committees. Label the members with A, B, C, D, E, and F.

The committees could be: *AB, AC, AD, AE, AF, BC, BD, BE, BF, CD, CE, CF, DE, DF, EF*.

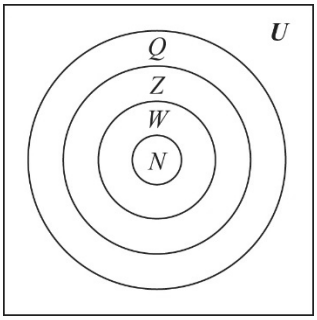
12. a. and b. Answers will vary.

13. Answers will vary.

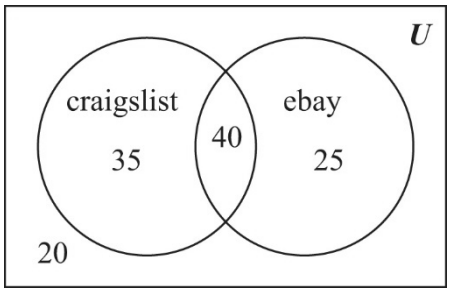
14. Answers will vary.

15. The circles enable us to easily represent visually all the possible subsets.
The diagram is not equivalent because there is no region corresponding to elements that are in all three sets.

16.

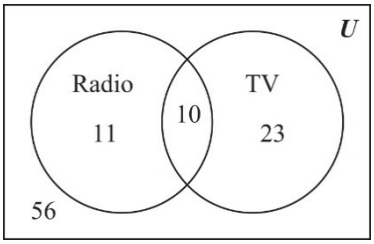


17. a.



b. Yes, they are well defined.

18. **a.** Construct a Venn diagram. 100 – 11 – 10 – 23 = 56%



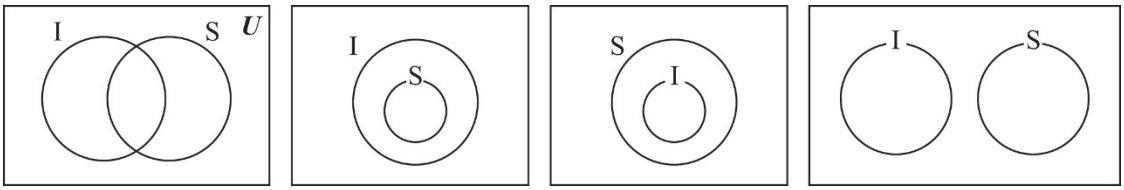
b. 11 + 23 = 34%

19. **a.** A lesson in which the teacher would be using a lab approach with small groups.

b. Lessons that use a lab approach and concrete materials and/or small groups.

20. Answers will vary.

21. **a.** Theoretically, there are four possibilities. I would pick the one at the left, because I think there can be successful people who are not very intelligent, intelligent people who are not successful, people who are successful and intelligent, and people who are neither.



b. Answers will vary.

CHAPTER 1 REVIEW EXERCISES

1. 15 coins equaling 92 cents

50	25	10	5	1
1	1	0	1	12
1	0	0	7	7
0	1	1	11	2
0	1	5	2	7
0	2	1	5	7
0	0	5	8	2

2. The dimensions of the patio in inches is 192 inches by 144 inches; so the Perez family needs $(192 \times 144) \div (12 \times 8) = 288$ tiles. At 75 cents for each tile, the patio will cost $216.

3. Nine ways.

Quarter	Dimes	Nickels
1	2	1
1	1	3
1	0	5
0	5	0
0	4	2
0	3	4
0	2	6
0	1	8
0	0	10

4. 40 posts are needed for a fence that measures 100 feet by 100 feet.

5. There are 3600 seconds in 1 hour. Therefore, the monarch butterfly flaps its wings $12 \times 3600 = 43{,}200$ times in 1 hour.

6. Answers will vary. Some possibilities are:

The magic sum is 34.

The sum of the four numbers in the center is also 34.

If you partition the 4×4 square into 2×2 squares, the sum of the numbers in each of the 2×2 squares is also 34.

7. **a.** The sum of the numbers along the length of the stick equals the number at the end of the stick.

b. Since the largest number on the chart is 924, working backwards: 1, 6, 21, 56, 126, 252, 462.

c. The handle would be the diagonal row of 13 ones along the left edge of the chart, and at the end of the stick would be 12.

d. Answers will vary.

8. **a.** Three ways: 1, 1, 4, 8; 2, 4, 4, 4; and 2, 2, 2, 8.

b. Scores of 23, 27, 29, 30, and 31 are impossible.

9. **a.** Fill the 9-gallon pail and use it to fill the 4-gallon pail. Empty the 4-gallon pail and fill it again from the 9-gallon pail. You now have 1 gallon left in the 9-gallon pail.

b. Fill the 4-gallon pail and empty it into the 9-gallon pail. Do it again so the 9-gallon pail has 8 gallons. Fill the 4-gallon pail a third time and finish filling the 9-gallon pail—it takes one more gallon to do so. You now have exactly 3 gallons left in the 4-gallon pail.

10. 11 hours

Hour	1	2	3	4	5	6	7	8	9	10	11
Height	3	2	5	4	7	6	9	8	11	10	13

11. There are 8 tricycles and 24 bicycles.

12. There are 81 tribbles and 16 chalkas.

13. **a.** The possible scores are 4 through 22, 24, 25, 26, 28, and 32.

b. Two ways: $1 + 1 + 2 + 8$ and $2 + 2 + 4 + 4$

c. There must be an even number of ones for the result to be even. There are three ways to get an even number: no 1s, two 1s, or four 1s. Of course, the last possibility will not give us 12. If we use two 1s, then the remaining two throws must result in a total of 10. The only way to do this is $2 + 8$. Finally, using no 1s, suppose we have one 8-point throw, then we must score the remaining 4 points in three throws, which is impossible. Thus, an 8-point dart cannot be used, If we have a 4-point throw, then the remaining three throws must total 8 points. The only way to do this is $2 + 2 + 4$.

14. The ball bounces 4 times. After the first bounce, the ball rises $\frac{1}{2} \times 16 = 8$ feet. After the second bounce, the ball rises 4 feet; after the third bounce, the ball rises 2 feet; and after the fourth bounce the ball rises 1 foot.

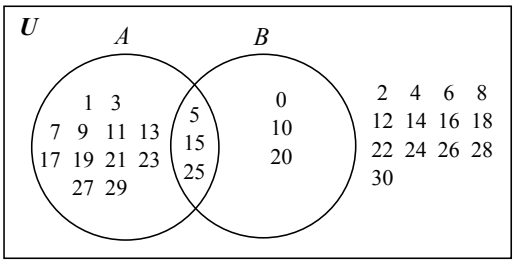
15. **a.** $\{x \mid x = 10^n, n = 0, 1, 2, 3, \ldots\}$

b. $\{10, 100, 1000, 10{,}000, \ldots\}$ or $\{10^1, 10^2, 10^3, \ldots\}$

16. **a.** $\in$ **b.** $\subset$ **c.** $\subset$ **d.** $\not\subset$

17. **a.** $D \not\subset E$ **b.** $0 \notin \{\ \}$

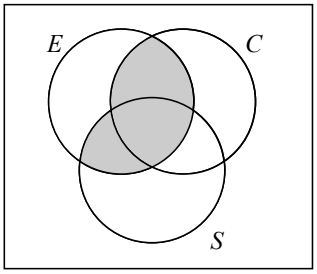
18. **a.**



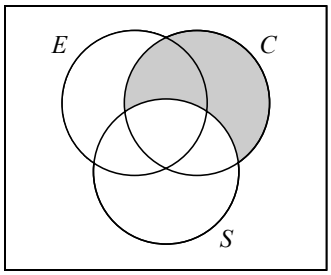
b. 5, 15, 25

c. The set of even numbers between 0 and 30.

19. **a.**



b.



20. 70 + 60 +20 = 150, so there are 150 – 100 = 50 counted twice, thus there are 50 who have both.

21. Two sets are equal if they have exactly the same elements. Two sets are equivalent if they have the same number of elements.

Chapter 2 The Number System

SECTION 2.1 Whole Numbers

1. a.

	Maya	Luli	South American
7		lokep moile tamlip	
8			teyente toazumba
12			caya-ente-cayupa
13		is yaoum moile tamlip	caya-ente-toazumba
15		is yaoum is alapea	
16	uac-lahun	is yaoum moile lokep moile tamop	toazumba-ente-tey
21	hun hunkal	is eln yaoum moile alapea	cajesa-ente-tey
22	ca huncal	is eln yaoum moile tamop	cajesa-ente-cayupa

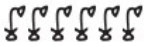
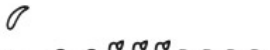
b. Answers will vary.

c. Answers will vary.

2. **a.** 3031 **b.** 230,012 **c.** 1666 **d.** 1519

e. 109 **f.** 75,602 **g.** 133 **h.** 23

3.

	Egyptian	Roman	Babylonian
a. 312		CCCXII	
b. 1206		MCCIIIIII or MCCVI	
c. 6000		MMMMMM	
d. 10,000		MMMMMMMMM	
e. 123,456		Can't do	

4 **a.** 87 **b.** 360 **c.** 5407

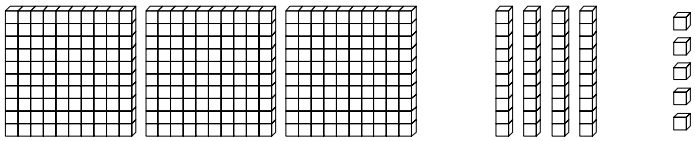
d.  **e.**  **f.** 

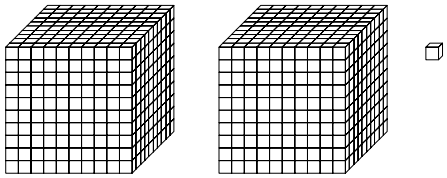
5. **a.** 26 **b.** 240 **c.** 25 **d.** 450

e. three thousand four hundred **f.** 3450

6. **a.** –400 = **b.** –7770 = **c.** +80,000 = **d.** +4040 =

e. $-346,733 =$ **f.** –111,111 = **g.** Answers will vary.

7. **a.** 

b. 

8. **a.** $3 \cdot 10^2 + 4 \cdot 10 + 5$ **b.** $2 \cdot 10^3 + 1$ **c.** $1 \cdot 10^4 + 1 \cdot 10^2 + 1$

9. **a.** 4859 **b.** 30,240 **c.** 750,003

10. **a.** 40_{five} **b.** 1100_{two} **c.** $9a_{\text{sixteen}}$ **d.** $709a_{\text{sixteen}}$

e. 110_{two} **f.** 112_{twelve} **g.** 130_{five} **h.** 410_{six}

11. **a.** 1004_{five} **b.** 334_{five} **c.** $0ff_{\text{sixteen}}$ **d.** 1101_{two}

e. 1001_{two} **f.** $10f_{\text{sixteen}}$ **g.** 113_{four} **h.** 56_{seven}

12. **a.** 1009

b. $\text{MIMIC} = 1000 + 1 + 1000 + 1 + 100 = 2102$

c. Answers will vary.

13. **a.** $50 + 10 + 10 + 5 + 1 + 1 = 77$

b. $100 + 50 + 10 + 10 + 5 + 1 = 176$

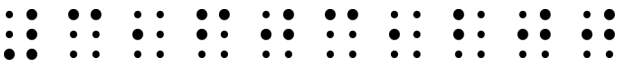
c. 

d. 

14. **a.** 32,570 **b.** 646

c. **d.**

15. **a.** 

b. 

c. 460,859

d. 135,246

16. **a.** (1) No: Needs a new symbol for each new power of ten.
(2) Sort of: The value of each numeral is 10 times the value of the previous numeral.
(3) Sort of: By decorating each basic symbol, you now have one basic symbol for each place, the number of dots on the symbol varies.
(4) No.
(5) Sort of: though, given the origin of this system, it would be more likely to be counted. For example, 2 thousands, 8 hundreds, etc. However, technically, you would multiply the value of each basic symbol by the number of dots on the symbol.
(6) No zero.

b. It has characteristic 2: The value of each place is 10 times the value of the previous place. It "sort of" has characteristic 3, with the modification that each "place" contains two symbols. Some might say that it has characteristics 4 and 5, but the order of the numerals is still a matter of convention – unlike base 10, where changing the order changes the value.

c. This system has all characteristics.

17. **a.** 585 cartons of milk

b. It has all 6 characteristics because this system is essentially base 6. The places are called cartons, boxes, crates, flats, and pallets. The value of each place is 6 times that of the previous place.

18. 1:0:58:4 or 1 hour, 58.04 seconds

19. The child is not making the connection between the physical representation of the number (singles and longs) and the verbal representation. They don't understand that once a place is filled (no place can hold more than 9) the 10 units are regrouped into a long, increasing the number of longs and making the number after twenty-nine, thirty.

20. Thirty-one means 3 longs, or tens, and 1 single, or ones. The child is not making the connection between the physical representation of the number (singles and longs) and the verbal representation. They don't understand that once the 10 units are regrouped into a long, there are no singles left, so the number after twenty-nine is thirty, not thirty-one.

21. Yes, 5 is the middle number between 0 and 10

22. Because the Hindu-Arabic system has place value and a place holder (zero, 0), it allows extremely large numbers to be represented with only 10 symbols. It is also much less cumbersome, since it only takes six digits to represent one hundred thousand.

23. We mark our years, in retrospect, with respect to the approximate birth year of Jesus Christ—this is why they are denoted 1996 A.D.; A.D. stands for Anno Domini, Latin for "in the year of our Lord." Because we are marking in retrospect from a fixed point, we call the first hundred years after that point the first century, the second hundred years the second century, and so on. The first hundred years are numbered zero (for the period less than a year after Jesus' birth) through ninety-nine. This continues until we find that the twentieth century is numbered 1900 A.D. through 1999 A.D.

24. In our numeration system every three digits have a different name, such as thousands, millions, and billions.

25. Place value is the idea of assigning different *number values* to *digits* depending on their position in a number. This means that the numeral 4 (four) would have a different value in the "ones" place than in the "hundreds" place, because 4 ones are very different from 4 hundreds. (That's why 4 isn't equal to 400.)

26. If we use 2 feet as our average shoulder width, and we use 25,000 miles as the circumference of Earth, we have 25,000 miles × 5280 feet per mile divided by 2 feet per person = 66,000,000.

27. **a.** 11.57 days

b. 11,570 days, or 31.7 years.

28. **a.** 94.7 miles. Depending on the value you use for the length of a dollar bill, you might get a slightly different amount.

b. 94,700 miles long, or almost 4 times around Earth!

29. **a.** 21 **b.** 35 **c.** 55 **d.** 279

e. 26 **f.** 259 **g.** 51 **h.** 300

i. 13 **j.** 17 **k.** 153 **l.** 2313

30. **a.** 134_{five} **b.** 1102_{five} **c.** 1011100_{two} **d.** 11001110_{two}

e. 60_{twelve} **f.** 192_{sixteen} **g.** 112_{six} **h.** 5444_{six}

i. 90_{sixteen} **j.** 400_{five} **k.** 63_{sixteen} **l.** 13202_{five}

m. 1120000000_{five}

n. 100 110 001 001 011 010 000 0 (The spaces are only for readability.)

31. base 9; We know that 5 + 5 is equal to 10, but in the sum we get 1, so that means 10 is 1 more than the base. Thus the base is 9.

32. base 9; We know that 23 base x means $2x + 3 = 25$ in base ten. Solving for x we get $2x = 18$, or $x = 9$. Thus the base is 9.

33. $x = 9$; Since one of the digits is a 7, we know that the base must be at least 8. Trial and error shows the base is 9.

34. 50_x candy bars, $x = 6$; 44 base x means $4x + 4 = 28$. Solving for x we get $x = 6$. Since 110 base five equals 30 in base ten, and 30 = 5(6), we know 50 base 6 candy bars will fit in the box.

35. This has to do with dimensions. The base 10 long is 2 times the length of the base 5 long. When we go to the next place, we now have a new dimension, so the value will be 2×2 as much. This links to measurement. If we compare two cubes, one of whose sides is double the length of the other, the ratio of lengths of sides is 2:1, the ratio of the surface area is 4:1, the ratio of the volumes is 8:1.

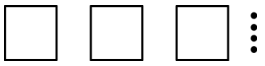
36. **a.** Just as each of the places in a base 10 numeral has a specific value that is a power of 10, each of the places in a base 5 numeral also has a value, but in a base 5 numeral these values are powers of 5. Let's look at this diagram:

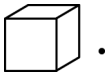
___	__	_	___ 5
125	25	5	1
5^3	5^2	5^1	5^0

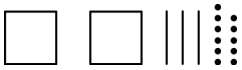
We can see that the places of a base 5 number, starting from the right, have the values 1, 5, 25, and 125. Now we ask ourselves how many times these go into 234_{ten} · 125 goes into 234 once, leaving 109; there are four 25s in 109, leaving 9; and, finally, the 9 can be written as one 5 and four 1s, so our number is 1214_5.

1	4	1	4 five
125	25	5	1
5^3	5^2	5^1	5^0

b. A similar chart can be created to show that $405_{\text{eight}} = (4 \times 64) + (0 \times 8) + (5 \times 1) = 261_{\text{ten}}$

37. **a.** 

b. 

c. 

38. **a.** 7777

b. f f f

39. Answers will vary.

40. 1_{four}, 2_{four}, 3_{four}, 10_{four}, 11_{four}, 12_{four}, 13_{four}, 20_{four}, 21_{four}, 22_{four}, 23_{four}, 30_{four}, 31_{four}, 32_{four}, 33_{four}, 100_{four}, …

41. There are 10 palindromes between 100 and 199. They are 101, 111, 121, 131, 141, 151, 161, 171, 181, and 191.

42. 6 hundreds + 23 tens + 5 ones = 6(100) + 23(10) + 5 = 600 +230 + 5 = 835

43. (9 × 10) + (5 × 1) + (8 × 100) = 90 + 5 + 800 = 895; the correct answer is b.

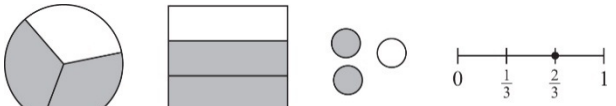
44. (12 × 10) + 30,605 = 120 + 30,605 = 30,725; the correct answer is b.

45. (6 × 100,000) + (23 × 100) = 600,000 + 2300 = 602,300; the correct answer is c.

46. The digit being replaced is in the tens place. So, if the digit 1 is replaced by the digit 5, the number is increased by (5 × 10) – (1 × 10) = 50 – 10 = 40. The correct answers is b.

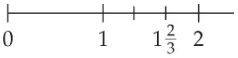
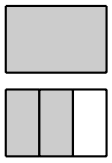
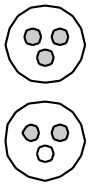
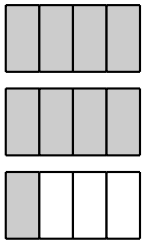
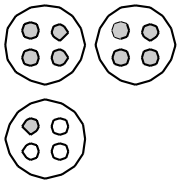
SECTION 2.2 Fractions

1. Answers will vary. Some possibilities include:



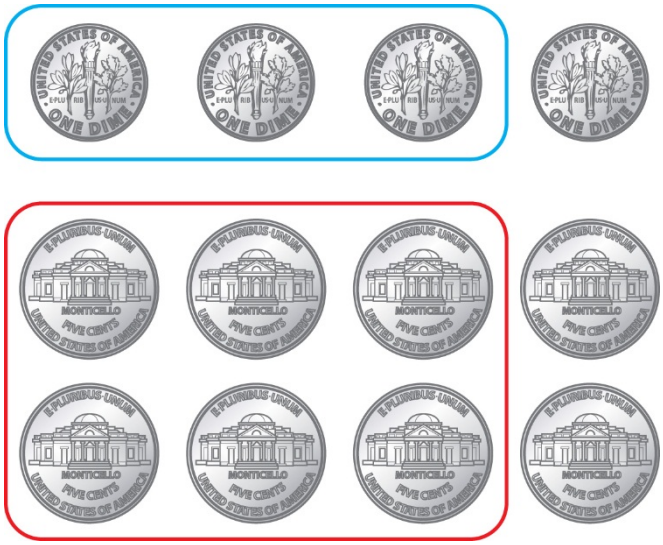
$\frac{4}{6}$ $.\overline{6}$

2. Note: Answers will vary. Below is one valid representation for each part.

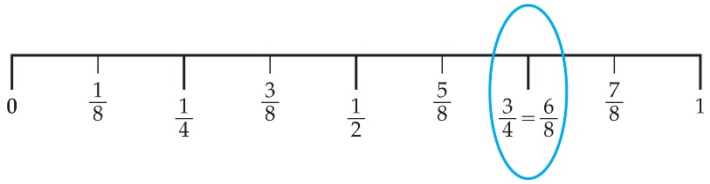
$\frac{3}{5}$			
$1\frac{2}{3}$			
$\frac{9}{4}$			

3. Since 3 of the larger pieces make up 1 brownie, the larger piece = $\frac{1}{3}$, since 5 of the smaller pieces makes $\frac{1}{3}$, 15 smaller pieces make 1 brownie, so the smaller piece = $\frac{1}{15}$

4. a.

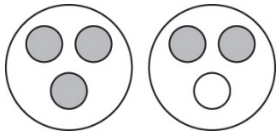


b.



c.

5.



6. $\frac{2}{3}$

7. a. $\frac{9}{12} = \frac{3}{4}$ (1 foot = 12 inches) **b.** $\frac{9}{36} = \frac{1}{4}$ (1 yard = 36 inches)