

Chapter 1

Problems 1-1 through 1-6 are for student research. No standard solutions are provided.

- 1-7** From Fig. 1-2, cost of grinding to ± 0.0005 in is 270%. Cost of turning to ± 0.003 in is 60%.

$$\text{Relative cost of grinding vs. turning} = 270/60 = 4.5 \text{ times} \quad \text{Ans.}$$

- 1-8** $C_A = C_B$,

$$10 + 0.8 P = 60 + 0.8 P - 0.005 P^2$$

$$P^2 = 50/0.005 \quad \Rightarrow \quad P = 100 \text{ parts} \quad \text{Ans.}$$

- 1-9** Max. load = $1.10 P$

$$\text{Min. area} = (0.95)^2 A$$

$$\text{Min. strength} = 0.85 S$$

To offset the absolute uncertainties, the design factor, from Eq. (1-1) should be

$$n_d = \frac{1.10}{0.85(0.95)^2} = 1.43 \quad \text{Ans.}$$

- 1-10 (a)** $X_1 + X_2$:

$$x_1 + x_2 = X_1 + e_1 + X_2 + e_2$$

$$\text{error} = e = (x_1 + x_2) - (X_1 + X_2)$$

$$= e_1 + e_2 \quad \text{Ans.}$$

- (b)** $X_1 - X_2$:

$$x_1 - x_2 = X_1 + e_1 - (X_2 + e_2)$$

$$e = (x_1 - x_2) - (X_1 - X_2) = e_1 - e_2 \quad \text{Ans.}$$

- (c)** $X_1 X_2$:

$$x_1 x_2 = (X_1 + e_1)(X_2 + e_2)$$

$$e = x_1 x_2 - X_1 X_2 = X_1 e_2 + X_2 e_1 + e_1 e_2$$

$$\approx X_1 e_2 + X_2 e_1 = X_1 X_2 \left(\frac{e_1}{X_1} + \frac{e_2}{X_2} \right) \quad \text{Ans.}$$

(d) X_1/X_2 :

$$\frac{x_1}{x_2} = \frac{X_1 + e_1}{X_2 + e_2} = \frac{X_1}{X_2} \left(\frac{1 + e_1/X_1}{1 + e_2/X_2} \right)$$

$$\left(1 + \frac{e_2}{X_2} \right)^{-1} \approx 1 - \frac{e_2}{X_2} \quad \text{then} \quad \left(\frac{1 + e_1/X_1}{1 + e_2/X_2} \right) \approx \left(1 + \frac{e_1}{X_1} \right) \left(1 - \frac{e_2}{X_2} \right) \approx 1 + \frac{e_1}{X_1} - \frac{e_2}{X_2}$$

Thus, $e = \frac{x_1}{x_2} - \frac{X_1}{X_2} \approx \frac{X_1}{X_2} \left(\frac{e_1}{X_1} - \frac{e_2}{X_2} \right)$ *Ans.*

1-11 (a) $x_1 = \sqrt{7} = 2.645\,751\,311\,1$
 $X_1 = 2.64$ (3 correct digits)
 $x_2 = \sqrt{8} = 2.828\,427\,124\,7$
 $X_2 = 2.82$ (3 correct digits)
 $x_1 + x_2 = 5.474\,178\,435\,8$
 $e_1 = x_1 - X_1 = 0.005\,751\,311\,1$
 $e_2 = x_2 - X_2 = 0.008\,427\,124\,7$
 $e = e_1 + e_2 = 0.014\,178\,435\,8$
Sum = $x_1 + x_2 = X_1 + X_2 + e$
 $= 2.64 + 2.82 + 0.014\,178\,435\,8 = 5.474\,178\,435\,8$ Checks

(b) $X_1 = 2.65$, $X_2 = 2.83$ (3 digit significant numbers)
 $e_1 = x_1 - X_1 = -0.004\,248\,688\,9$
 $e_2 = x_2 - X_2 = -0.001\,572\,875\,3$
 $e = e_1 + e_2 = -0.005\,821\,564\,2$
Sum = $x_1 + x_2 = X_1 + X_2 + e$
 $= 2.65 + 2.83 - 0.001\,572\,875\,3 = 5.474\,178\,435\,8$ Checks

1-12 $\sigma = \frac{S}{n_d} \Rightarrow \frac{32(1000)}{\pi d^3} = \frac{25(10^3)}{2.5} \Rightarrow d = 1.006 \text{ in}$ *Ans.*

Table A-17: $d = 1\frac{1}{4} \text{ in}$ *Ans.*

$$n = \frac{S}{\sigma} = \frac{25(10^3)}{\frac{32(1000)}{\pi(1.25)^3}} = 4.79$$
 Ans.

Factor of safety:

1-13 (a)

x	f	fx	fx^2
60	2	120	7200
70	1	70	4900
80	3	240	19200
90	5	450	40500
100	8	800	80000
110	12	1320	145200
120	6	720	86400
130	10	1300	169000
140	8	1120	156800
150	5	750	112500
160	2	320	51200
170	3	510	86700
180	2	360	64800
190	1	190	36100
200	0	0	0
210	1	210	44100
Σ	69	8480	1 104 600

Eq. (1-6)
$$\bar{x} = \frac{1}{N} \sum_{i=1}^k f_i x_i = \frac{8480}{69} = 122.9 \text{ kcycles}$$

Eq. (1-7)

$$s_x = \sqrt{\frac{\sum_{i=1}^k f_i x_i^2 - N \bar{x}^2}{N - 1}} = \left[\frac{1\,104\,600 - 69(122.9)^2}{69 - 1} \right]^{1/2} = 30.3 \text{ kcycles } \textit{Ans.}$$

(b) Eq. (1-5)
$$z_{115} = \frac{x - \mu_x}{\hat{\sigma}_x} = \frac{x_{115} - \bar{x}}{s_x} = \frac{115 - 122.9}{30.3} = -0.2607$$

Interpolating from Table (A-10)

$$\begin{array}{cc} 0.2600 & 0.3974 \\ 0.2607 & x \\ 0.2700 & 0.3936 \end{array} \Rightarrow x = 0.3971$$

$$N\Phi(-0.2607) = 69 (0.3971) = 27.4 \approx 27 \text{ } \textit{Ans.}$$

From the data, the number of instances less than 115 kcycles is
 $2 + 1 + 3 + 5 + 8 + 12 = 31$ (the data is not perfectly normal)

1-14

x	f	fx	fx^2
174	6	1044	181656
182	9	1638	298116
190	44	8360	1588400
198	67	13266	2626668
206	53	10918	2249108
214	12	2568	549552
222	6	1332	295704
Σ	197	39126	7789204

$$\text{Eq. (1-6)} \quad \bar{x} = \frac{1}{N} \sum_{i=1}^k f_i x_i = \frac{39\,126}{197} = 198.61 \text{ kpsi}$$

$$\text{Eq. (1-7)} \quad s_x = \sqrt{\frac{\sum_{i=1}^k f_i x_i^2 - N \bar{x}^2}{N-1}} = \left[\frac{7\,789\,204 - 197(198.61)^2}{197-1} \right]^{1/2} = 9.68 \text{ kpsi } \textit{Ans.}$$

1-15 $\bar{L} = 122.9$ kcycles and $s_L = 30.3$ kcycles

$$\text{Eq. (1-5)} \quad z_{10} = \frac{x - \mu_x}{\hat{\sigma}} = \frac{x_{10} - \bar{L}}{s_L} = \frac{x_{10} - 122.9}{30.3}$$

$$\text{Thus,} \quad x_{10} = 122.9 + 30.3 z_{10} = L_{10}$$

From Table A-10, for 10 percent failure, $z_{10} = -1.282$. Thus,

$$L_{10} = 122.9 + 30.3(-1.282) = 84.1 \text{ kcycles } \textit{Ans.}$$

1-16

x	f	fx	fx^2
93	19	1767	164331
95	25	2375	225625
97	38	3686	357542
99	17	1683	166617
101	12	1212	122412
103	10	1030	106090
105	5	525	55125
107	4	428	45796
109	4	436	47524
111	2	222	24642
Σ	136	13364	1315704

Eq. (1-6)
$$\bar{x} = \frac{1}{N} \sum_{i=1}^k f_i x_i = 13\,364 / 136 = 98.26471 = 98.26 \text{ kpsi}$$

Eq. (1-7)
$$s_x = \sqrt{\frac{\sum_{i=1}^k f_i x_i^2 - N \bar{x}^2}{N-1}} = \left(\frac{1\,315\,704 - 136(98.26471)^2}{136-1} \right)^{1/2} = 4.30 \text{ kpsi}$$

Note, for accuracy in the calculation given above, $\bar{x}$ needs to be of more significant figures than the rounded value.

For a normal distribution, from Eq. (1-5), and a yield strength exceeded by 99 percent ($R = 0.99, p_f = 0.01$),

$$z_{0.01} = \frac{x - \mu_x}{\hat{\sigma}_x} = \frac{x_{0.01} - \bar{x}}{s_x} = \frac{x_{0.01} - 98.26}{4.30}$$

Solving for the yield strength gives

$$x_{0.01} = 98.26 + 4.30 z_{0.01}$$

From Table A-10, $z_{0.01} = -2.326$. Thus

$$x_{0.01} = 98.26 + 4.30(-2.326) = 88.3 \text{ kpsi} \quad \text{Ans.}$$

1-17 Eq. (1-9):
$$R = \prod_{i=1}^n R_i = 0.98(0.96)0.94 = 0.88$$

Overall reliability = 88 percent *Ans.*

1-18 Obtain the coefficients of variance for strength and stress

$$C_S = \frac{\hat{\sigma}_{S_{sy}}}{\bar{S}_{sy}} = \frac{23.5}{312} = 0.07532$$

$$C_\sigma = \frac{\hat{\sigma}_\tau}{\bar{\tau}} = \frac{\hat{\sigma}_T}{\bar{T}} = \frac{145}{1500} = 0.09667$$

For $R = 0.99$, from Table A-10, $z = -2.326$.

Eq. (1-12):

$$\begin{aligned}\bar{n} &= \frac{1 + \sqrt{1 - (1 - z^2 C_S^2)(1 - z^2 C_\sigma^2)}}{1 - z^2 C_S^2} \\ &= \frac{1 + \sqrt{1 - [1 - (-2.326)^2 (0.07532)^2][1 - (-2.326)^2 (0.09667)^2]}}{1 - (-2.326)^2 (0.07532)^2} = 1.3229 = 1.32 \quad \text{Ans.}\end{aligned}$$

From the given equation for stress,

$$\tau_{\max} = \frac{S_{sy}}{\bar{n}} = \frac{16T}{\pi d^3}$$

Solving for d gives

$$d = \left(\frac{16T\bar{n}}{\pi S_{sy}} \right)^{1/3} = \left[\frac{16(1500)(1.3229)}{\pi(312)10^6} \right]^{1/3} = 0.0319 \text{ m} = 31.9 \text{ mm} \quad \text{Ans.}$$

1-19 Obtain the coefficients of variance for stress and strength

$$C_\sigma = \frac{\hat{\sigma}_\sigma}{\mu_\sigma} = \frac{\hat{\sigma}_P}{\bar{P}} = \frac{5}{65} = 0.09231$$

$$C_S = \frac{\hat{\sigma}_S}{\mu_S} = \frac{\hat{\sigma}_{S_y}}{\bar{S}_y} = \frac{6.59}{95.5} = 0.06901$$

(a) $\bar{n} = 1.2$

$$\text{Eq. (1-11): } z = -\frac{\bar{n}_d - 1}{\sqrt{\bar{n}_d^2 C_s^2 + C_\sigma^2}} = -\frac{1.2 - 1}{\sqrt{1.2^2 (0.06901^2) + 0.09231^2}} = -1.6127$$

Interpolating Table A-10,

1.61	0.0537	
1.6127	Φ	$\Rightarrow \Phi = 0.0534$
1.62	0.0526	

$$R = 1 - 0.0534 = 0.9466 \quad \text{Ans.}$$

$$\bar{n} = \frac{\bar{S}_y}{\bar{\sigma}} = \frac{\bar{S}_y}{\bar{P} / (\pi d^2 / 4)} = \frac{\pi d^2 \bar{S}_y}{4 \bar{P}} \Rightarrow d = \sqrt{\frac{4 \bar{P} \bar{n}}{\pi \bar{S}_y}} = \sqrt{\frac{4(65)1.2}{\pi(95.5)}} = 1.020 \text{ in} \quad \text{Ans.}$$

(b) $\bar{n} = 1.5$

$$z = -\frac{1.5 - 1}{\sqrt{1.5^2 (0.06901^2) + 0.09231^2}} = -3.605$$

3.6	0.000159	
3.605	Φ	$\Rightarrow \Phi = 0.00015645$
3.7	0.000108	

$$R = 1 - 0.00015645 = 0.9998 \quad \text{Ans.}$$

$$d = \sqrt{\frac{4 \bar{P} \bar{n}}{\pi \bar{S}_y}} = \sqrt{\frac{4(65)1.5}{\pi(95.5)}} = 1.140 \text{ in} \quad \text{Ans.}$$

1-20

$$\mu_{\sigma_{\max}} = \bar{\sigma}_{\max} = \bar{\sigma}_a + \bar{\sigma}_b = 90 + 383 = 473 \text{ MPa}$$

From footnote 9, p. 25 of text,

$$\hat{\sigma}_{\sigma_{\max}} = \left(\hat{\sigma}_{\sigma_a}^2 + \hat{\sigma}_{\sigma_b}^2 \right)^{1/2} = (8.4^2 + 22.3^2)^{1/2} = 23.83 \text{ MPa}$$

$$C_{\sigma_{\max}} = \frac{\hat{\sigma}_{\sigma_{\max}}}{\mu_{\sigma_{\max}}} = \frac{\hat{\sigma}_{\sigma_{\max}}}{\bar{\sigma}_{\max}} = \frac{23.83}{473} = 0.0504$$

$$C_{S_y} = \frac{\hat{\sigma}_{S_y}}{\mu_{S_y}} = \frac{\hat{\sigma}_{S_y}}{\bar{S}_y} = \frac{42.7}{553} = 0.0772$$

$$\bar{n} = \frac{\bar{S}_y}{\bar{\sigma}_{\max}} = \frac{553}{473} = 1.169 = 1.17 \text{ Ans.}$$

$$\text{Eq. (1-11): } z = -\frac{\bar{n}_d - 1}{\sqrt{\bar{n}_d^2 C_s^2 + C_\sigma^2}} = -\frac{1.169 - 1}{\sqrt{1.169^2 (0.0772^2) + 0.0504^2}} = -1.635$$

From Table A-10, $\Phi(-1.635) = 0.05105$

$$R = 1 - 0.05105 = 0.94895 = 94.9 \text{ percent Ans.}$$

1-21

$$a = 1.500 \pm 0.001 \text{ in}$$

$$b = 2.000 \pm 0.003 \text{ in}$$

$$c = 3.000 \pm 0.004 \text{ in}$$

$$d = 6.520 \pm 0.010 \text{ in}$$

$$(a) \quad \bar{w} = \bar{d} - \bar{a} - \bar{b} - \bar{c} = 6.520 - 1.5 - 2 - 3 = 0.020 \text{ in}$$

$$t_w = \sum t_{\text{all}} = 0.001 + 0.003 + 0.004 + 0.010 = 0.018$$

$$w = 0.020 \pm 0.018 \text{ in Ans.}$$

(b) From part (a), $w_{\min} = 0.002 \text{ in}$. Thus, must add 0.008 in to $\bar{d}$. Therefore,

$$\bar{d} = 6.520 + 0.008 = 6.528 \text{ in Ans.}$$

1-22 $V = xyz$, and $x = a \pm \Delta a$, $y = b \pm \Delta b$, $z = c \pm \Delta c$,

$$\bar{V} = abc$$

$$V = (a \pm \Delta a)(b \pm \Delta b)(c \pm \Delta c)$$

$$= abc \pm bc\Delta a \pm ac\Delta b \pm ab\Delta c \pm a\Delta b\Delta c \pm b\Delta c\Delta a \pm c\Delta a\Delta b \pm \Delta a\Delta b\Delta c$$

The higher order terms in Δ are negligible. Thus,

$$\Delta V \approx bc\Delta a + ac\Delta b + ab\Delta c$$

$$\text{and, } \frac{\Delta V}{\bar{V}} \approx \frac{bc\Delta a + ac\Delta b + ab\Delta c}{abc} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c} = \frac{\Delta a}{\bar{a}} + \frac{\Delta b}{\bar{b}} + \frac{\Delta c}{\bar{c}} \text{ Ans.}$$

For the numerical values given, $\bar{V} = 1.500(1.875)3.000 = 8.4375 \text{ in}^3$

$$\frac{\Delta V}{\bar{V}} \approx \frac{0.002}{1.500} + \frac{0.003}{1.875} + \frac{0.004}{3.000} = 0.004267 \Rightarrow \Delta V \approx 0.004267(8.4375) = 0.0360 \text{ in}^3$$

$$V = 8.4375 \pm 0.0360 \text{ in}^3 \quad \text{Ans.}$$

This answer yields $V \approx \frac{8.4735}{8.4015} \text{ in}$, whereas, exact is $V = \frac{8.473551..}{8.401551..} \text{ in}$

1-23

$$w_{\max} = 0.05 \text{ in}, \quad w_{\min} = 0.004 \text{ in}$$

$$\bar{w} = \frac{0.05 + 0.004}{2} = 0.027 \text{ in}$$

Thus, $\Delta w = 0.05 - 0.027 = 0.023 \text{ in}$, and then, $w = 0.027 \pm 0.023 \text{ in}$.

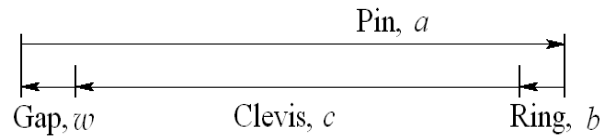
$$\bar{w} = \bar{a} - \bar{b} - \bar{c}$$

$$0.027 = \bar{a} - 0.042 - 1.5$$

$$\bar{a} = 1.569 \text{ in}$$

$$t_w = \sum t_{\text{all}} \Rightarrow 0.023 = t_a + 0.002 + 0.005 \Rightarrow t_a = 0.016 \text{ in}$$

Thus, $a = 1.569 \pm 0.016 \text{ in} \quad \text{Ans.}$



1-24

$$\bar{D}_o = \bar{D}_i + 2\bar{d} = 3.734 + 2(0.139) = 4.012 \text{ in}$$

$$t_{D_o} = \sum t_{\text{all}} = 0.028 + 2(0.004) = 0.036 \text{ in}$$

$$D_o = 4.012 \pm 0.036 \text{ in} \quad \text{Ans.}$$

1-25 From O-Rings, Inc. (oringsusa.com), $D_i = 9.19 \pm 0.13 \text{ mm}$, $d = 2.62 \pm 0.08 \text{ mm}$

$$\bar{D}_o = \bar{D}_i + 2\bar{d} = 9.19 + 2(2.62) = 14.43 \text{ mm}$$

$$t_{D_o} = \sum t_{\text{all}} = 0.13 + 2(0.08) = 0.29 \text{ mm}$$

$$D_o = 14.43 \pm 0.29 \text{ mm} \quad \text{Ans.}$$

1-26 From O-Rings, Inc. (oringsusa.com), $D_i = 34.52 \pm 0.30$ mm, $d = 3.53 \pm 0.10$ mm

$$\bar{D}_o = \bar{D}_i + 2\bar{d} = 34.52 + 2(3.53) = 41.58 \text{ mm}$$

$$t_{D_o} = \sum t_{\text{all}} = 0.30 + 2(0.10) = 0.50 \text{ mm}$$

$$D_o = 41.58 \pm 0.50 \text{ mm} \quad \text{Ans.}$$

1-27 From O-Rings, Inc. (oringsusa.com), $D_i = 5.237 \pm 0.035$ in, $d = 0.103 \pm 0.003$ in

$$\bar{D}_o = \bar{D}_i + 2\bar{d} = 5.237 + 2(0.103) = 5.443 \text{ in}$$

$$t_{D_o} = \sum t_{\text{all}} = 0.035 + 2(0.003) = 0.041 \text{ in}$$

$$D_o = 5.443 \pm 0.041 \text{ in} \quad \text{Ans.}$$

1-28 From O-Rings, Inc. (oringsusa.com), $D_i = 1.100 \pm 0.012$ in, $d = 0.210 \pm 0.005$ in

$$\bar{D}_o = \bar{D}_i + 2\bar{d} = 1.100 + 2(0.210) = 1.520 \text{ in}$$

$$t_{D_o} = \sum t_{\text{all}} = 0.012 + 2(0.005) = 0.022 \text{ in}$$

$$D_o = 1.520 \pm 0.022 \text{ in} \quad \text{Ans.}$$

1-29 From Table A-2,

(a) $\sigma = 150/6.89 = 21.8$ kpsi *Ans.*

(b) $F = 2/4.45 = 0.449$ kip = 449 lbf *Ans.*

(c) $M = 150/0.113 = 1330$ lbf · in = 1.33 kip · in *Ans.*

(d) $A = 1500/25.4^2 = 2.33$ in² *Ans.*

(e) $I = 750/2.54^4 = 18.0$ in⁴ *Ans.*

(f) $E = 145/6.89 = 21.0$ Mpsi *Ans.*

(g) $v = 75/1.61 = 46.6$ mi/h *Ans.*

(h) $V = 1000/946 = 1.06 \text{ qt}$ *Ans.*

1-30 From Table A-2,

(a) $l = 5(0.305) = 1.53 \text{ m}$ *Ans.*

(b) $\sigma = 90(6.89) = 620 \text{ MPa}$ *Ans.*

(c) $p = 25(6.89) = 172 \text{ kPa}$ *Ans.*

(d) $Z = 12(16.4) = 197 \text{ cm}^3$ *Ans.*

(e) $w = 0.208(175) = 36.4 \text{ N/m}$ *Ans.*

(f) $\delta = 0.00189(25.4) = 0.0480 \text{ mm}$ *Ans.*

(g) $v = 1200(0.0051) = 6.12 \text{ m/s}$ *Ans.*

(h) $\epsilon = 0.00215(1) = 0.00215 \text{ mm/mm}$ *Ans.*

(i) $V = 1830(25.4^3) = 30.0(10^6) \text{ mm}^3$ *Ans.*

1-31

(a) $\sigma = M/Z = 1770/0.934 = 1895 \text{ psi} = 1.90 \text{ kpsi}$ *Ans.*

(b) $\sigma = F/A = 9440/23.8 = 397 \text{ psi}$ *Ans.*

(c) $y = Fl^3/3EI = 270(31.5)^3/[3(30)10^6(0.154)] = 0.609 \text{ in}$ *Ans.*

(d) $\theta = Tl/GJ = 9740(9.85)/[11.3(10^6)(\pi/32)1.00^4] = 8.648(10^{-2}) \text{ rad} = 4.95^\circ$ *Ans.*

1-32

(a) $\sigma = F/wt = 1000/[25(5)] = 8 \text{ MPa}$ *Ans.*

(b) $I = bh^3/12 = 10(25)^3/12 = 13.0(10^3) \text{ mm}^4$ *Ans.*

(c) $I = \pi d^4/64 = \pi(25.4)^4/64 = 20.4(10^3) \text{ mm}^4$ *Ans.*

(d) $\tau = 16T/\pi d^3 = 16(25)10^3/[\pi(12.7)^3] = 62.2 \text{ MPa}$ *Ans.*

1-33

(a) $\tau = F/A = 2\,700/[\pi(0.750)^2/4] = 6110 \text{ psi} = 6.11 \text{ kpsi}$ *Ans.*

(b) $\sigma = 32Fa/\pi d^3 = 32(180)31.5/[\pi(1.25)^3] = 29\,570 \text{ psi} = 29.6 \text{ kpsi}$ *Ans.*

(c) $Z = \pi(d_o^4 - d_i^4)/(32 d_o) = \pi(1.50^4 - 1.00^4)/[32(1.50)] = 0.266 \text{ in}^3$ *Ans.*

(d) $k = (d^4G)/(8D^3 N) = 0.062\,5^4(11.3)10^6/[8(0.760)^3 32] = 1.53 \text{ lbf/in}$ *Ans.*
