1.2 Do problem 1.1 with $M = \alpha T$ where $\alpha = 0 \ to \ \infty$.

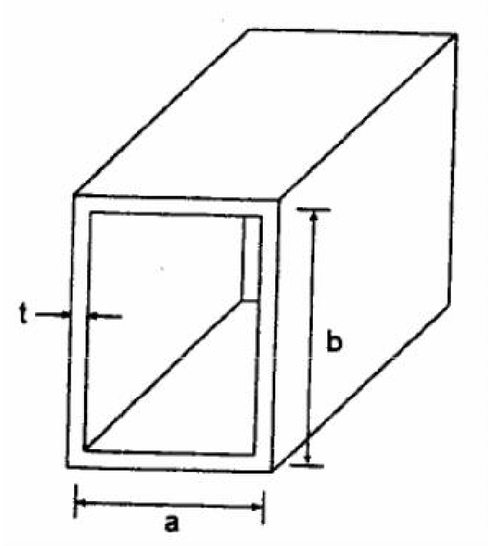

Figure 1.16 Closed thin-walled section

Solution:

(1) The bending stress of beams is $\sigma = \dfrac{My}{I}$, where y is the distance from the neutral axis. The moment of inertia I of the cross-section can be calculated by considering the four segments of thin walls and using the formula for a rectangular section with height h and width w. $I = \sum(\frac{1}{12}wh^3 + Ad^2)$ in which A is the cross-sectional area of the segment and d is the distance of the centroid of the segment to the neutral axis. Note that the Parallel Axis Theorem is applied. The result is $I = 2\cdot\frac{1}{12}tb^3 + 2\cdot[\frac{1}{12}\cdot at^3 + (at)\cdot(\frac{b}{2})^2] \approx \frac{tb^2}{6}(3a+b)$, assuming that t is very small.

(2) The shear stress due to torsion for a closed thin-walled section shown above is
$\tau = \dfrac{T}{2abt}$.

(3) Two approaches are employed to find the solution.
(i) Assume that the bending stress reaches the allowable $\sigma_{allowable}$ first and find the corresponding bending maximum bending moment. Then apply the stated loading condition of $M = \alpha T$ to check whether the corresponding $\tau_{\max}$ has exceeded the allowable shear stress $\tau_{allowable}$. If this condition is violated, then the optimized b/a ratio is not valid.

(a) $$\sigma|_{y=\frac{b}{2}} = \frac{My}{I} = \frac{M\cdot\frac{b}{2}}{\frac{tb^2}{6}(3a+b)} = \frac{3M}{tb(3a+b)}$$

When given $L = 2(a+b)$ as a constant, a can be expressed in terms of b

and L as $a = \frac{L}{2} - b$. Then we can minimize

$$S = \frac{tb(3a+b)}{3} = \frac{tb(3L-4b)}{6}$$ in order to maximize σ, i.e.,

$$\frac{\partial S}{\partial b} = 0 \Rightarrow \frac{t}{6}(3L-8b) = 0 \Rightarrow b = \frac{3L}{8}, \text{ so } a = \frac{L}{2} - b = \frac{L}{8}$$

where the optimum ratio is $\frac{b}{a} = 3$

Thus, $$\sigma_{\max} = \frac{3M}{tb(3a+b)} = \frac{3M}{t\cdot(3L/8)\cdot(3\cdot L/8+3L/8)} = \frac{32M}{3tL^2}$$

(b) Check $\tau_{\max}$ with $M = \alpha T$ and b/a = 3 and check whether $\tau_{\max}$ is within the allowable shear stress $\tau_{allowable}$.

$$\tau_{\max} = \frac{T}{2abt} = \frac{M/\alpha}{2\cdot(L/8)\cdot(3L/8)\cdot t} = \frac{32M}{3\alpha tL^2} = \frac{1}{\alpha}\sigma_{\max}$$
$$= \frac{1}{\alpha}\sigma_{allowable} = \frac{2}{\alpha}\tau_{allowable}$$

We have $\tau_{\max} \le \tau_{allowable} \Rightarrow \frac{2}{\alpha}\tau_{allowable} \le \tau_{allowable}$

➔ $\alpha \ge 2$ (since $\tau_{allowable} > 0$ is always satisfied)

(ii) Assume now that failure is controlled by shear stress. We assume that $\tau_{\max} = \tau_{allowable}$ is reached first and then find the corresponding bending stress according to the loading condition $M = \alpha T$.

(a) $\tau_{\max} = \frac{T}{2abt}$

Again we minimize $S = 2abt = (L-2b)bt$ in order to maximize τ, i.e.,

$$\frac{\partial S}{\partial b} = 0 \Rightarrow (L-4b) = 0 \Rightarrow b = \frac{L}{4}, \text{ so } a = \frac{L}{2} - b = \frac{L}{4}$$

and the optimum ratio is $\frac{b}{a} = 1$

and $$\tau_{\max} = \frac{T}{2abt} = \frac{T}{2\cdot(L/4)\cdot(L/4)\cdot t} = \frac{8T}{tL^2}$$

(b) Then corresponding $\sigma_{\max}$ under the optimum condition stated above can be obtained using $M = \alpha T$. We have

$$\sigma_{\max} = \frac{3M}{tb(3a+b)} = \frac{3\alpha T}{t\cdot(L/4)\cdot(3\cdot L/4+L/4)} = \frac{12\alpha T}{tL^2} = \frac{3}{2}\alpha\tau_{\max}$$
$$= \frac{3}{2}\alpha\tau_{allowable} = \frac{3}{2}\alpha\cdot\left(\frac{\sigma_{allowable}}{2}\right) = \frac{3}{4}\alpha\sigma_{allowable}$$

Since $\sigma_{\max} \le \sigma_{allowable} \Rightarrow \frac{3}{4}\alpha\sigma_{allowable} \le \sigma_{allowable}$

➔ $\alpha \le \frac{4}{3}$ (since $\sigma_{allowable} > 0$ is always satisfied)

(4) From the above two approaches, we have the conclusions.

(i) For $0 < \alpha \le \frac{4}{3}$, the failure is controlled by shear and the optimum ratio of $\frac{b}{a} = 1$ achieves the most efficient section..

(ii) For $\alpha \ge 2$, the failure is controlled by bending and the optimum ratio of $\frac{b}{a} = 3$ achieves the most efficient section.

(iii) For $\frac{4}{3} < \alpha < 2$, the optimal ratio lies between 1 and 3. The most straightforward way in finding the best ratio for a given α in this range is to calculate the maximum bending moments and torques for different values of b/a ratios between 1 and 3 and pick the ratio that produces the greatest minimum failure load, either T or M.

--- ANS