1.3 The dimensions of a steel (300M) I-beam are b = 50 mm, t = 5 mm, and h = 200 mm (Fig. 1.17). Assume that t and h are to be fixed for an aluminum(7075-T6) I-beam. Find the width b for the aluminum beam so that its bending stiffness EI is equal to that of the steel beam. Compare the weights-per-unit length of these two beams. Which is more efficient weightwise?

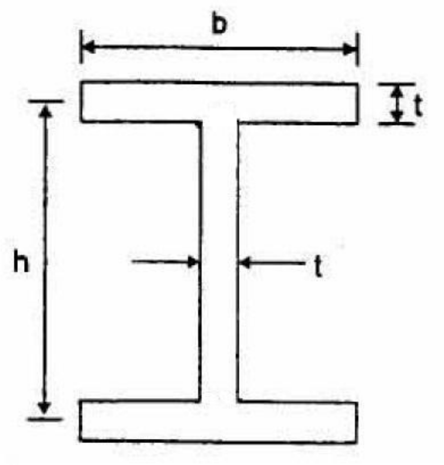

Figure 1.17 Dimensions of the cross-section of an I-beam

Solution:

(1) The expression of area moment of inertia I for an I-beam is:

$$I = \frac{t}{12}(h-t)^3 + [\frac{b}{12}t^3 + (bt)(\frac{h}{2})^2] \times 2 \quad , \quad \text{by applying Parallel Axis Theorem.}$$

(2) First obtaining the area moment of inertia of the steel (300M) I-beam with given b, t, and h.

$$I_{Steel} = \frac{5}{12}(200-5)^3 + [\frac{50}{12} \cdot 5^3 + (50 \cdot 5)(\frac{200}{2})^2] \times 2 = 8090573 mm^4$$

(3) For the given condition $(EI)_{Aluminum} = (EI)_{Steel}$

we have $I_{Al} = \frac{E_{St}}{E_{Al}} I_{St} = \frac{200}{71} \times 8090573 = 22790000 mm^4$

which allows to calculate the width b for the aluminum beam with the following result:

$$I_{Al} = \frac{5}{12}(200-5)^3 + [\frac{b}{12} \cdot 5^3 + (b \cdot 5)(\frac{200}{2})^2] \times 2$$
$$= 3089531.3 + 100020.8b = 22790000$$

and $b = 197mm$

---- ANS

(4) Then we compare the weights-per-unit length of these two beams.

The weights-per-unit length is defined as

$w = \rho \cdot A$, where ρ = density , and A = cross-sectional area

(i) For the Steel beam

$$\rho_{St} = 7.8(g/cm^3) = 7.8 \times 10^{-3}(g/mm^3)$$

$$A_{St} = (200-5)\times 5 + 2\times 50\times 5 = 1475(mm^2)$$

$$w_{St} = \rho_{St} \cdot A_{St} = 7.8\times 10^{-3} \times 1475 = 11.5(g/mm)$$

(ii) For the Aluminum beam

$$\rho_{Al} = 2.78(g/cm^3) = 2.78\times 10^{-3}(g/mm^3)$$

$$A_{Al} = (200-5)\times 5 + 2\times 196.97\times 5 = 2945(mm^2)$$

$$w_{Al} = \rho_{Al} \cdot A_{Al} = 2.78\times 10^{-3} \times 2944.7 = 8.2(g/mm)$$

For a unit length of both materials, the aluminum beam is much lighter than the steel beam. It means that the *ALUMINUM BEAM IS MORE EFFICIENT!*

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