1.7 Compare the load-carrying capabilities of two beams having the respective cross-sections shown in Fig. 1.19. Use bending rigidity as the criterion for comparison. It is given that a = 4 cm, t = 0.2 cm, and the two cross-sections have the same area.

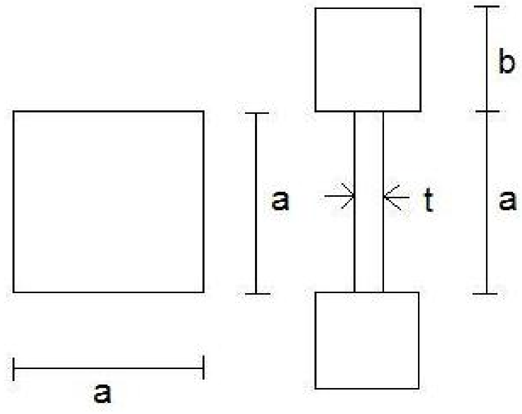

Figure 1.19 Cross-sections of two beams

<u>Solution:</u>

When using the bending rigidity (EI) as a criterion for comparison, Young's modulus E and the area moment of inertia I should be estimated.

(1) Young's modulus E :

Assume the Young's modulus of the beam having the left-hand-side cross-section and the right-hand-side cross-section are E_l and E_r , respectively.

(2) Moment of inertia I :

(i) Left cross-section:

$$I_l = \frac{1}{12}a^4 = \frac{1}{12}\times 4^4 = 21.33cm^4$$

(ii) Right cross-section:

$$I_r = \frac{b}{12}(a+2b)^3 - \frac{b-t}{12}a^3 \qquad \text{--- (a)}$$

or { $I_r = \frac{t}{12}a^3 + [\frac{1}{12}b^4 + b^2 \cdot (\frac{a}{2}+\frac{b}{2})^2]\times 2$ }

where b remains unknown. There is another condition, two cross-section have the same area, which will help to solve b.

$$A_l = a^2 = 4^2 = 16cm^2 \quad , \quad A_r = 2\cdot b^2 + a\cdot t = 2\cdot b^2 + 4\cdot 0.2$$

let A_l=A_r => $b = 2.7568cm$, then we have

$$I_r = \frac{2.7568}{12}(4 + 2 \times 2.7568)^3 - \frac{(2.7568 - 0.2)}{12} \cdot 4^3 = 184cm^4$$

or $\{ I_r = \frac{0.2}{12}4^3 + [\frac{1}{12}2.7568^4 + 2.7568^2 \cdot (\frac{4}{2} + \frac{2.7568}{2})^2] \times 2 = 184cm^4 \}$

(3) Performance:

The ratio of the moments of inertia of the two cross-sections can be expressed as

$$\frac{(EI)_l}{(EI)_r} = \frac{E_l I_l}{E_r I_r} = \frac{21.33E_l}{184.18E_r} = \frac{E_l}{8.635E_r} = 0.12\frac{E_l}{E_r}$$

The cross-section to the right is much better if the same material is used for both beams.

(i) If $E_r < 0.12E_l$

The left cross-section outperforms the right one.

(ii) If $E_r = 0.12E_l$

They are equivalent.

(iii) If $E_r > 0.12E_l$

The right cross-section outperforms the left one.

--- ANS