CHAPTER 7

Internal Flows

7.1 $\quad \mathrm{Re} = \dfrac{VD}{n} = \dfrac{VD}{1\times 10^{-6}}.$ $\quad$ a) $2000 = \dfrac{V\times 2}{1\times 10^{-6}}.$ $\quad \therefore V = \underline{0.001\text{ m/ s}}.$

b) $2000 = \dfrac{V\times .02}{1\times 10^{-6}}.$ $\quad \therefore V = \underline{0.1\text{ m/ s}}.$

c) $2000 = \dfrac{V\times .002}{1\times 10^{-6}}.$ $\quad \therefore V = \underline{1.0\text{ m/ s}}.$

7.2 $\quad \mathrm{Re} = \dfrac{Vh}{n} = \dfrac{Vh}{1\times 10^{-6}}.$ $\quad$ a) $u(r) = \dfrac{1}{4m}\dfrac{dp_k}{dx}(r^2 - r_0^2) = \dfrac{1}{4m}\dfrac{\Delta p_k}{L}(r^2 - r_0^2).$

b) $1500 = \dfrac{V\times 1}{1\times 10^{-6}}.$ $\quad \therefore V = \underline{0.0015\text{ m/s}}.$ $\quad$ c) $1500 = \dfrac{V\times 0.3}{1\times 10^{-6}}.$ $\quad \therefore V = \underline{0.005\text{ m/s}}.$

7.3 $\quad \mathrm{Re} = \dfrac{Vh}{n} = \dfrac{1.5\times .2/12}{1.4\times 10^{-5}} = \underline{1790}.$ $\quad$ Using $R_{\text{crit}} = 1500$, the flow is $\underline{\text{turbulent}}$.

7.4 $\quad \mathrm{Re} = \dfrac{Vh}{n} = \dfrac{(1/2)\times 1.4}{10^{-6}} = 700\ 000.$ $\quad \therefore \underline{\text{Very turbulent}}$

7.5 $\quad \mathrm{Re} = \dfrac{VD}{n}.$ $\quad$ a) $V = \dfrac{\mathrm{Re}\times n}{D} = \dfrac{2000\times 10^{-6}}{0.02} = \underline{0.1\text{ m/ s}}.$

b) $V = \dfrac{\mathrm{Re}\times n}{D} = \dfrac{40\ 000\times 10^{-6}}{0.02} = \underline{2\text{ m/ s}}.$

7.6 $\quad \dfrac{L_E}{D} = 0.065\,\mathrm{Re}$ $\quad \mathrm{Re} = \dfrac{VD}{n}.$ $\quad V = \dfrac{.0002}{p\times .02^2} = .1592\text{ m/ s}.$

a) $L_E = .065\dfrac{.1592\times .04}{1.31\times 10^{-6}}\times .04 = \underline{12.6\text{ m}}.$

b) $L_E = .065\dfrac{.1592\times .04}{1.007\times 10^{-6}}\times .04 = \underline{16.4\text{ m}}.$

c) $L_E = .065\dfrac{.1592\times .04}{.661\times 10^{-6}}\times .04 = \underline{25.0\text{ m}}.$

d) $L_E = .065\dfrac{.1592\times .04}{.367\times 10^{-6}}\times .04 = \underline{45.1\text{ m}}.$

7.7 a) $V = \dfrac{\text{Re} \times n}{D} = \dfrac{1000 \times 1.51 \times 10^{-5}}{0.04} = \underline{0.378 \text{ m / s}}.$

$L_E = 0.065\,\text{Re} \times D = 0.065 \times 1000 \times 0.04 = \underline{2.6 \text{ m}}.$

$L_i \cong L_E / 4 = 2.6 / 4 = \underline{0.65 \text{ m}}.$

b) $V = \dfrac{\text{Re} \times n}{D} = \dfrac{80\,000 \times 1.51 \times 10^{-5}}{0.04} = \underline{30.2 \text{ m / s}}.$

$L_E \cong 120D = 120 \times 0.04 = \underline{4.8 \text{ m}}.$

$L_i \cong 10D = 10 \times 0.04 = \underline{0.4 \text{ m}}.$

7.8 $V = \dfrac{0.025}{p \times 0.03^2} = 8.84. \quad \text{Re} = \dfrac{8.84 \times 0.06}{1.007 \times 10^{-6}} = 5.3 \times 10^5. \quad \therefore \text{Turbulent.}$

$\therefore L_E = 120 \times .06 = 7.2 \text{ m}. \qquad \therefore \underline{\text{Developed}}.$

7.9 $V = \dfrac{Q}{A} = \dfrac{(18/1000)/2 \times 3600}{p \times 0.001^2} = 0.796 \text{ m / s}$

$\text{Re} = \dfrac{VD}{n} = \dfrac{0.796 \times 0.002}{1.14 \times 10^{-5}} = 139.6. \qquad \therefore \text{laminar.}$

$L_E = 0.065\,\text{Re} \times D = 0.065 \times 139.6 \times 0.002 = 0.0181 \text{ m}. \qquad \therefore \underline{\text{negligible}}$

7.10 $L_E = 0.04\text{Re} \times h = 0.04 \times 7700 \times .012 = \underline{3.7 \text{ m}}.$

$\left(L_E\right)_{\min} = .04 \times 1500 \times .012 = \underline{0.72 \text{ m}}.$

7.11 $\left(L_E\right)_{\text{lam}} = 0.065\,\text{Re}\,D = 0.065 \dfrac{5 \times .06}{1.55 \times 10^{-5}} \times .06 = \underline{75.5 \text{ m}}.$

$\left(L_E\right)_{\text{turb}} = 120 \times .06 = \underline{7.2 \text{ m}}. \qquad (\text{Re} = 32\,300)$

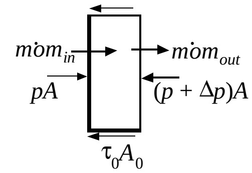
7.12 $\text{Re} = \dfrac{VD}{n} = \dfrac{0.2 \times 0.04}{10^{-6}} = 8000.$

a) If laminar, $L_E = 0.065 \times \text{Re} \times D = 0.065 \times 8000 \times 0.04 = \underline{20.8 \text{ m}}$

$L_i = L_E / 4 = 20.8 / 4 = \underline{5.2 \text{ m}}$

b) This is a low Reynolds number turbulent flow. A minimum entrance length would be $L_E = 120D = 120 \times 0.04 = \underline{4.8 \text{ m}}$ with a minimum inviscid core length of $L_i = 10D = 10 \times 0.04 = \underline{0.4 \text{ m}}$.

7.13 $\Sigma F_x = \Delta p \pi r_0^2 - \tau_0 2\pi r_0 \Delta x = \dot{m}\text{om}_{\text{out}} - \dot{m}\text{om}_{\text{in}}$.



$$\therefore \frac{\Delta p}{\Delta x} = \frac{2\tau_0}{r_0} + \frac{\Delta \dot{m}\text{om}}{\Delta x}$$

For developed flow $\dfrac{\Delta p}{\Delta x} = \dfrac{2\tau_0}{r_0}$ since $\dot{m}\text{om}$ = Const. From the velocity distribution in an entrance (see Fig. 7.1) it is obvious that $(\tau_0)_{\text{entrance}} > (\tau_0)_{\text{developed}}$ since $\left.\dfrac{\partial u}{\partial y}\right|_{\text{wall}}$ is greater in the entrance. Also, $\dfrac{\Delta \dot{m}\text{om}}{\Delta x} > 0$ since the momentum flux increases from the inlet to the developed flow. Hence, $\left(\dfrac{\Delta p}{\Delta x}\right)_{\text{entrance}} > \left(\dfrac{\Delta p}{\Delta x}\right)_{\text{developed}}$.

7.14 a) For a high Re flow transition to turbulence occurs near the origin. In the entrance region the velocity gradient $\partial u/\partial y$ at the wall is very large resulting in a large wall shear. This large wall shear requires a large pressure gradient. In addition, the momentum flux is increasing in the *x*-direction, also requiring an increased pressure gradient (see the solution to 7.13 for more detail).

b) For a low Re turbulent flow, the flow is laminar through much of the entrance region, up to about L_d (see Fig. 7.2). The laminar flow results in a much smaller velocity gradient at the wall compared with that of the turbulent flow of part (a) requiring a much smaller pressure gradient. This results in the lower distribution of Fig. 7.3.

c) The pressure distribution must move from the lower distribution to the higher distribution of Fig. 7.3 as the Re increases. This occurs at an intermediate Re when transition occurs near L_i. Research that gives accurate data for such low turbulent Re transition does not exist.

7.15 $u(r) = \dfrac{1}{4\mu}\dfrac{dp_k}{dx}(r^2 - r_0^2) = \dfrac{1}{4\mu}\dfrac{\Delta p_k}{L}(r^2 - r_0^2)$.

7.16 In a developed flow, *dh/dx* = slope of the pipe and *p* is a linear function of *x* so that dp/dx = const. Therefore, $d(p + \gamma h)/dx$ = const and it can be moved outside the integral. Then,

$$\frac{2}{4\mu r_0^2}\frac{d(p+\gamma h)}{dx}\int_0^{r_0}(r^2 - r_0^2)r\,dr = \frac{1}{2\mu r_0^2}\frac{d(p+\gamma h)}{dx}\left(\frac{r_0^4}{4} - \frac{r_0^2}{2}\times r_0^2\right) = \frac{r_0^2}{8\mu}\frac{d(p+\gamma h)}{dx}$$

7.17 (**D**)

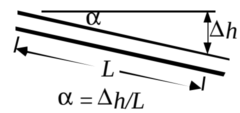
7.18 $1600 = \dfrac{VD}{n} = \dfrac{V \times .8/\ 12}{1.06 \times 10^{-5}}$. $\quad \therefore V = 0.254$ fps.

Using Eq. 7.3.14: $\quad L = \dfrac{\Delta p\, r_0^2}{8mV} = \dfrac{0.07 \times 144 \times .4^2 /\ 144}{8 \times 2.06 \times 10^{-5} \times .254} = \underline{268\text{ ft}}.$

Using Eq. 7.3.18: $\quad t_0 = \dfrac{r_0 \Delta p}{2L} = \dfrac{.4/\ 12 \times .07 \times 144}{2 \times 268} = \underline{6.27 \times 10^{-4}\ \text{psf}}.$

$$\therefore f = \frac{t_0}{\frac{1}{8} rV^2} = \frac{6.27 \times 10^{-4}}{\frac{1}{8} \times 1.94 \times .254^2} = \underline{0.04}.$$

7.19 $1500 = \dfrac{VD}{n} = \dfrac{V \times .01}{6.61 \times 10^{-7}}$. $\quad \therefore V = 0.0992$ m / s.



Eq. 7.3.14: $\quad \dfrac{\cancel{\Delta p}}{\cancel{L}} + g\dfrac{\Delta h}{L} = \dfrac{8mV}{r_0^2}. \qquad \therefore \dfrac{\Delta h}{L} = \dfrac{8mV}{r_0^2\, r\, g}.$

$\therefore a = \dfrac{8nV}{r_0^2 g} = \dfrac{8 \times 6.61 \times 10^{-7} \times .0992}{.005^2 \times 9.81} = 0.00214$ rad or $\underline{0.123^\circ}$

$$Q = AV = p \times .005^2 \times 0.0992 = \underline{7.79 \times 10^{-6}\ \text{m}^3/\ \text{s}}.$$

7.20 $V = \dfrac{Q}{A} = \dfrac{0.0002}{p \times .01^2} = 0.637$ m / s. $\qquad$ Use Eq. 7.3.14.

a) $\Delta p = \dfrac{8mVL}{r_0^2} = \dfrac{8 \times 0.1 \times .637 \times 10}{.01^2} = \underline{51\ 000\text{ Pa}}.$

b) $\Delta p = \dfrac{8mVL}{r_0^2} = \dfrac{8 \times 1 \times 10^{-3} \times .637 \times 10}{.01^2} = \underline{510\text{ Pa}}.$

c) $\Delta p = \dfrac{8mVL}{r_0^2} = \dfrac{8 \times 1.5 \times .637 \times 10}{.01^2} = \underline{764\ 000\text{ Pa}}.$

7.21 Eq. 7.3.14: $\quad Q = \displaystyle\int_0^{.01} \frac{9810(.00015)}{2\times10^{-3}}(.02y - y^2)100\,dy$ $\qquad$ For a vertical pipe $\Delta h = L$.

$$= 73\ 600(.01 \times .01^2 - \frac{.01^3}{3}) = \underline{0.049\ \text{m}^3/\text{s}}$$

Thus,

$$V = \frac{rgr_0^2}{8m} = \frac{gr_0^2}{8n} = \frac{9.81 \times .01^2}{8n} = \frac{1.226 \times 10^{-4}}{n}.$$

a) $V = \dfrac{1.226 \times 10^{-4}}{1.52 \times 10^{-6}} = 80.7\ \dfrac{\text{m}}{\text{s}}. \quad \therefore Q = p \times .01^2 \times 80.7 = \underline{0.0254\ \dfrac{\text{m}^3}{\text{s}}}.$

$$\text{Re} = \frac{80.7 \times .02}{1.52 \times 10^{-6}} = 1.06 \times 10^{6}. \qquad \therefore \underline{\text{not laminar}}$$

b) $V = \dfrac{1.226 \times 10^{-4}}{0.34 /\ 917} = 0.33\ \dfrac{\text{m}}{\text{s}}. \quad \therefore Q = \underline{1.04 \times 10^{-4}\ \dfrac{\text{m}^3}{\text{s}}}. \quad \text{Re} = 17.8. \quad \therefore \underline{\text{laminar.}}$

c) $V = \dfrac{1.226 \times 10^{-4}}{1.5 /\ 1258} = 0.103\ \dfrac{\text{m}}{\text{s}}. \quad \therefore Q = \underline{3.24 \times 10^{-5}\ \dfrac{\text{m}^3}{\text{s}}}. \quad \text{Re} = 1.73. \quad \therefore \underline{\text{laminar.}}$

7.22 $\quad 2000 = \dfrac{VD}{n} = \dfrac{4QDr}{p D^2 m} = \dfrac{4 \times 0.12 \times 1.78}{p \times 2 \times 10^{-4} \times D} = \dfrac{1360}{D}. \qquad \therefore D = \underline{0.680 \text{ ft.}}$

$$\Delta p = \frac{8mVL}{r_0^2} = \frac{8 \times 2 \times 10^{-4} \times 30}{0.34^2} = \underline{0.415 \text{ psf}}.$$

7.23 Neglect the effects of the entrance region and assume developed flow for the whole length. Also, assume $p_{\text{inlet}} = gh$ (neglect $V^2 /\ 2g$ compared to 4 m).

$\therefore \Delta p = gh - 0. \qquad \therefore\ rgh = \dfrac{8mVL}{r_0^2}. \qquad \therefore V = \dfrac{ghr_0^2}{8nL} = \dfrac{9.81 \times 4 \times .0025^2}{8 \times 1 \times 10^{-6} \times 40} = 0.766 \text{ m} /\ \text{s}.$

$\therefore Q = AV = p \times .0025^2 \times .766 = \underline{1.5 \times 10^{-5}\ \dfrac{\text{m}^3}{\text{s}}}. \quad L_E = .065 \times \dfrac{.766 \times .005}{1 \times 10^{-6}} \times .005 = \underline{1.2 \text{ m}}.$

7.24 Neglect the effects of the entrance region and assume developed flow for the whole length. Also, assume $p_{\text{inlet}} = gh$ (neglect $V^2 /\ 2g$ compared to 4 m).

$\therefore \Delta p = gh = \dfrac{8mVL}{r_0^2}$ and $r_0^2 = \dfrac{8VmL}{gh} = \dfrac{8(0.0034 /\ 60 \times 60) \times 10^{-3} \times 4}{9800 \times 4 \times pr_0^2}$

where $V = Q /\ A = Q /\ pr_0^2$. This gives $r_0 = 7.04 \times 10^{-4}$ m or $\underline{0.704 \text{ mm}}$.

The velocity is then $V = \dfrac{0.0034 / 60 \times 60}{p \times (0.000704)^2} = 0.6066 \text{ m/s}.$

$\dfrac{V^2}{2g} = \dfrac{0.6066^2}{2 \times 9.81} = 0.0188 \text{ m}.$ This is negligible compared to 4 m.

$$L_E = 0.065\,\text{Re} \times D = 0.065 \frac{0.6066 \times 14.08 \times 10^{-4}}{10^{-6}} \times 14.08 \times 10^{-4} = 1.2 \text{ m}$$

This entrance region may be significant; to include its effects would be quite difficult, certainly beyond the scope of an introductory course.

7.25 $\quad \text{Re} = 2000 = \dfrac{VD}{n} = \dfrac{V \times .8 /\ 12}{1.6 \times 10^{-4}}. \qquad \therefore V = 4.8 \text{ fps}.$

$$\Delta p = \frac{8mVL}{r_0^2} = \frac{8 \times 3.82 \times 10^{-7} \times 4.8 \times 30}{(.4 /\ 12)^2} = \underline{0.396 \text{ psf}}.$$

7.26 $\Delta p + g\Delta h = \dfrac{8mVL}{r_0^2}$. $\quad -6000 + 9810 \times 10 \sin 10^\circ = \dfrac{8 \times 1 \times 10^{-3} V \times 10}{.002^2}$.

$\therefore V = 0.552 \text{ m / s}$. $\quad \text{Re} = \dfrac{VD}{n} = \dfrac{.552 \times .004}{1 \times 10^{-6}} = \underline{2210}$.

$\Delta p + g\Delta h = \dfrac{2t_0 L}{r_0}$. $\quad -6000 + 9810 \times 10 \sin 10^\circ = \dfrac{2 \times t_0 \times 10}{.002}$. $\quad \therefore t_0 = \underline{1.1 \text{ Pa}}$.

7.27 $\text{Re} = 40\ 000 = \dfrac{VD}{n} = \dfrac{V \times 0.1}{1.51 \times 10^{-5}}$. $\quad \therefore V = 6.04 \text{ m / s}$. $\quad \therefore V_{\max} = 2V = \underline{12.1 \text{ m / s}}$

$\Delta p = \dfrac{8mVL}{r_0^2} = \dfrac{8 \times 1.81 \times 10^{-5} \times 6.04 \times 10}{.05^2} = \underline{3.5 \text{ Pa}}$. $\quad L_E = .065 \times 40\ 000 \times .1 = \underline{260 \text{ m}}$.

7.28 If $Q = pR^2\sqrt{2gH}$, then $V = \sqrt{2gH}$. The parabolic velocity profile is

$$u(r) = u_{\max}\left(1 - \frac{r^2}{R^2}\right) = 2V\left(1 - \frac{r^2}{R^2}\right) = 2\sqrt{2gH}\left(1 - \frac{r^2}{R^2}\right).$$

But, the manometer requires

$$p + \frac{u^2}{2} r - p = gH. \qquad \therefore H = \frac{u^2}{2g}.$$

Substituting yields

$$u = 2u\left(1 - \frac{r^2}{R^2}\right). \qquad \therefore r^2 = \frac{R^2}{2} \text{ or } \underline{r = R / \sqrt{2}}.$$

7.29 $V = -\dfrac{R^2}{8m}\dfrac{\Delta p + g\Delta h}{L} = -\dfrac{R^2}{8m}\dfrac{g(-L)}{L} = \dfrac{gR^2}{8m} = \dfrac{9800 \times 0.001^2}{8 \times 10^{-3}} = 1.225 \text{ m/s}$

$$Q = AV = p \times 0.001^2 \times 1.225 = \underline{3.85 \times 10^{-6} \text{ m}^3 \text{ / s}}$$

$$\text{Re} = \frac{VD}{n} = \frac{122.5 \times 00.02}{10^{-6}} = 2450$$

It probably is not a laminar flow. Since $\text{Re} > 2000$ it would most likely be turbulent. If the pipe were smooth, rigidly held, vibration free, with a well-rounded entrance and disturbance-free water entering the pipe it could be laminar.

7.30 a) Combine Eq. 7.3.12 and 7.3.15: $\quad u = u_{\max}\left(1 - \dfrac{r^2}{r_0^2}\right)$.

$$V = 2V\left(1 - \frac{r^2}{r_0^2}\right). \qquad \therefore r^2 = \frac{r_0^2}{2} \text{ and } \underline{r = 0.707\ r_0}$$

b) From Eq. 7.3.17: $t = Cr$ where C is a constant. Then $t_w = Cr_0$.

If $t = t_w / 2$, then $t_w / 2 = Cr$ and $\quad r = \dfrac{1}{2}\left(\dfrac{t_w}{C}\right) = \dfrac{r_0}{2}$

7.31 $$\frac{Q_{\text{pipe}}}{Q_{\text{annulus}}}=\frac{\pi r_0^4}{8\mu}\frac{\Delta p}{L}\Big/\frac{\pi}{8\mu}\frac{\Delta p}{L}\left\{r_0^4-\left(\frac{r_0}{2}\right)^4-\frac{\left[r_0^2-(r_0/2)^2\right]^2}{\ln(2r_0/r_0)}\right\}$$

$$=\frac{r_0^4}{r_0^4-\frac{r_0^4}{16}-\frac{9r_0^4/16}{\ln 2}}=\underline{7.938}$$

7.32 $$\text{Re}=20\ 000=\frac{VD}{\nu}=\frac{V\times 2/\ 12}{1.2\times 10^{-5}}.\qquad \therefore V=1.44\text{ fps.}$$

$$h_L=\frac{\Delta p}{\gamma}=\frac{8\mu VL}{\gamma\, r_0^2}=\frac{8\times 2.36\times 10^{-5}\times 1.44\times 30}{62.4\times(1\ /\ 12)^2}=\underline{0.0188\text{ ft}}.$$

$$\tau_0=\frac{r_0\Delta p}{2L}=\frac{(1\ /\ 12).0188\times 62.4}{2\times 30}=\underline{0.00163\text{ psf}}.\qquad L_E=.065\times 20\ 000\times\frac{2}{12}=\underline{217\text{ ft.}}$$

7.33 See Example 7.2: $$Q=-\frac{\pi}{8\mu}\frac{(-\Delta p)}{L}\left[r_2^4-r_1^4-\frac{(r_2^2-r_1^2)^2}{\ell n(r_2/\ r_1)}\right]$$

$$\therefore Q=-\frac{\pi}{8\times 1\times 10^{-3}}\frac{(-100)}{10}\left[.03^4-.02^4-\frac{(.03^2-.02^2)^2}{\ell n(.03/.02)}\right]=\underline{1.31\times 10^{-4}\ \text{m}^3\ /\text{ s}}.$$

$$\tau_{r_1}=\mu\frac{\partial u}{\partial r}\bigg|_{r=r_1}=\frac{1}{4}\frac{(-\Delta p)}{L}\left[2r_1-\frac{r_2^2-r_1^2}{\ell n(r_2/\ r_1)}\frac{1}{r_1}\right]$$

$$=-\frac{100}{4\times 10}\left[2\times.02-\frac{.03^2-.02^2}{\ell n(.03/\ .02)}\frac{1}{.02}\right]=\underline{0.054\text{ Pa}}.$$

7.34 See Example 7.2: $$Q=-\frac{\pi}{8\mu}\frac{(-\Delta p)}{L}\left[r_2^4-r_1^4-\frac{(r_2^2-r_1^2)^2}{\ell n(r_2/\ r_1)}\right]$$

$$\therefore Q=\frac{\pi\times 10}{8\times 1.81\times 10^{-5}\times 10}\left[.03^4-.02^4-\frac{(.03^2-.02^2)^2}{\ell n(.03/\ .02)}\right]=7.25\times 10^{-4}\ \text{m}^3/\text{ s}.$$

$$\therefore V=\frac{Q}{A}=\frac{7.25\times 10^{-4}}{\pi(.03^2-.02^2)}=\underline{0.462\text{ m / s}}.$$

$$\tau_{r_1}=\mu\frac{\partial u}{\partial r}\bigg|_{r=r_1}=-\frac{\Delta p}{4L}\left[2r_1-\frac{r_2^2-r_1^2}{\ell n(r_2\ /r_1)}\frac{1}{r_1}\right]$$

$$=-\frac{10}{4\times 10}\left[2\times.02-\frac{.03^2-.02^2}{\ell n(.03/\ .02)}\frac{1}{.02}\right]=\underline{0.0054\text{ Pa}}.$$

7.35 See Example 7.2: $\frac{dp}{dx}=\frac{m(T)}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right)$. The function $m(T)$ requires that $T(r)$ be known. The energy equation is needed to find $T(r)$. But, as Eq. 5.5.1 shows, we must know $u(r)$ to find $T(r)$. Hence, the above momentum equation and the energy equation are coupled. A simultaneous solution (a numerical approach is needed) would provide $u(r)$ and $T(r)$.

7.36 From Example 7.2 $u(r)=\frac{1}{4m}\frac{dp}{dx}\left[r^2-r_2^2+\frac{r_2^2-r_1^2}{\ell n(r_1/r_2)}\ell n\frac{r}{r_2}\right]$

As $r_1 \to 0$, $\ell n(r_1/r_2)\to -\infty$ so that $\frac{r_2^2-r_1^2}{\ell n(r_1/r_2)}\ell n\frac{r}{r_2}\to 0.$

Thus, $u(r)=\frac{1}{4m}\frac{dp}{dx}\left(r^2-r_0^2\right)$ where $r_2=r_0$. See Eq. 7.3.11.

As $r_1\to r_2$, $\frac{r_2^2-r_1^2}{\ell n(r_1/r_2)}=\frac{0}{0}$. $\therefore$ differentiate $w.r.t.\ r_1$: $\frac{-2r_1}{1/r_1}=-2r_1^2$.

Also, $\ell n\frac{r}{r_2}=\ell n\left(1-\frac{y}{r_2}\right)\cong -\frac{y}{r_2}$, where $y=r_2-r$.

$$\therefore u(r)=\frac{1}{4m}\frac{dp}{dx}\left(r^2-r_2^2+2r_1^2y/r_2\right)=\frac{1}{4m}\frac{dp}{dx}\left[y^2-2r_2y+\frac{2r_1^2}{r_2}y\right]=\frac{1}{4m}\frac{dp}{dx}(y^2-ay).$$

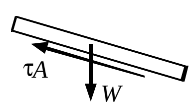
7.37 (**A**)

7.38 a) $2000=\frac{Vh}{n}$. $\therefore V=\frac{2000\times 1.2\times 10^{-5}}{1/24}=0.576$ fps. $\therefore Q=AV=\underline{0.04\text{ ft}^3/\text{sec}}$.

b) $V=\frac{2000\times 1.6\times 10^{-5}}{1/24}=0.768$ fps. $\therefore Q=\frac{1}{24}\times\frac{20}{12}\times .768=\underline{0.053\text{ cfs}}$.

7.39 There is no pressure gradient. $\therefore$ Eq. 7.4.13 gives $u=\frac{V}{a}y$.

The friction balances the weight component.



$$tA=W\sin q. \qquad t=m\frac{\P u}{\P y}=m\frac{V}{a}=m\frac{.2}{.0004}.$$

a) $m\frac{.2}{.0004}\times 1\times 1=40\sin 20^\circ$. $\therefore m=\underline{0.0274\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}}$.

b) $m\frac{.2}{.0004}\times 1\times 1=40\sin 30^\circ$. $\therefore m=\underline{0.04\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}}$.

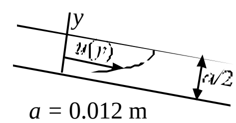
7.40 With the pressure gradient zero, Eq. 7.4.13 gives $u = \frac{U}{a} y$. Thus,

$$\tau A = W \sin\theta. \qquad \tau = \mu \frac{\partial u}{\partial y} = \mu \frac{V}{a} = \frac{1\times 10^{-3}}{.0004} V.$$

a) $\frac{1\times 10^{-3} V}{.0004} \times 1 \times 1 = 40 \sin 20^\circ. \qquad \therefore V = \underline{5.47 \text{ m/s}}.$

b) $\frac{1\times 10^{-3} V}{.0004} \times 1 \times 1 = 40 \sin 30^\circ. \qquad \therefore V = \underline{8 \text{ m/s}}.$

7.41 The depth of water is $a/2$, with the maximum velocity at the surface.



a = 0.012 m

$$-\frac{dh}{dx} = \sin\theta. \qquad \text{Hence, } u(y) = -\frac{\gamma \sin\theta}{2\mu}(y^2 - ay).$$

$$Q = -\int_0^{a/2} \frac{\gamma \sin\theta}{2\mu}(y^2 - ay) 50 dy = -\frac{50\gamma \sin\theta}{2\mu}\left(\frac{a^3}{24} - \frac{a^3}{8}\right) = \frac{25}{12\mu}\gamma \sin\theta\, a^3$$

$$= \frac{25}{12\times 10^{-3}} \times 9810 \times \sin 20^\circ \times .012^3 = \underline{12.1 \text{ m}^3/\text{s}}. \qquad \therefore V = \frac{12.1}{.006\times 50} = 40.3 \text{ m/s}.$$

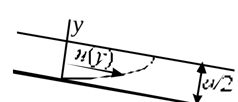
$\therefore \text{Re} = \frac{Va/2}{\nu} = \frac{40.3\times .006}{10^{-6}} = 241\,000. \qquad \therefore$ The assumption of laminar flow was not a good one, but we shall stay with it to answer the remaining parts!

$$u_{max} = \frac{9810 \times \sin 20^\circ}{2\times 10^{-3}}\left(\frac{.012^2}{4}\right) = \underline{60.4 \text{ m/s}}.$$

$$\tau_0 = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = -\frac{\gamma \sin\theta}{2}(-a) = \frac{9810 \sin 20^\circ}{2} \times .012 = \underline{20.1 \text{ Pa}}.$$

Obviously the flow would be turbulent and the above analysis would have to be modified substantially for an actual flow.

7.42 $u(y) = \frac{\gamma}{2\mu}\frac{dh}{dx}(y^2 - ay) = \frac{9810}{2\times 10^{-3}} \times (-.00015)(y^2 - .02y)$



a = 0.02 m

$$Q = \int_0^{.01} \frac{9810(.00015)}{2\times 10^{-3}}(.02y - y^2) 100 dy$$

$$= 73\,600(.01\times .01^2 - \frac{.01^3}{3}) = \underline{0.049 \text{ m}^3\text{/s}}$$

$$V = \frac{.049}{100\times .01} = 0.049 \text{ m/s}. \qquad \tau_0 = \mu \left.\frac{du}{dy}\right|_0 = \frac{9810\times .00015\times .02}{2} = \underline{0.015 \text{ Pa}}.$$

$$f = \frac{8\tau_0}{\rho V^2} = \underline{0.050} \qquad \text{Re} = \underline{490}.$$

7.43 Eq. 7.4.17: $\Delta p = \dfrac{12mVL}{a^2}$. $\therefore V = \dfrac{50\times.02^2}{12\times1.81\times10^{-5}\times60} = 1.53$ m / s.

$\therefore Q = AV = .02\times.9\times1.53 = \underline{0.028 \text{ m}^3/\text{ s}}$. This is maximum since laminar flow is assumed. Check the Reynolds number: $\text{Re} = \dfrac{Va}{n} = \dfrac{1.53\times.02}{1.51\times10^{-5}} = \underline{2030}$. This is quite high unless care is taken to eliminate vibrations, disturbances, rough walls.

7.44 $(p_A - p_B)_{\text{static}} = gh = 9810\times20\sin30^\circ = 98\,100$ or 98.1 kPa. $\therefore$ flow is down.

Eq. 7.4.17: $\Delta p + g\Delta h = \dfrac{12mLV}{a^2}$. $\therefore -96\,000 + 98\,100 = \dfrac{12\times10^{-3}\times20V}{.008^2}$.

$\therefore V = 0.56$ m / s. $t_0 = \dfrac{a\Delta p}{2L} + \dfrac{ag\Delta h}{2L} = \dfrac{.008}{2\times20}[-96\,000 + 98\,100] = 0.42$ Pa.

$$\therefore f = \frac{8t_0}{rV^2} = \frac{8\times.42}{1000\times.56^2} = \underline{0.0107}.$$

7.45 Assume laminar flow:

$$\Delta p = \frac{12mVL}{a^2}. \quad \therefore 600\times144 = \frac{12\times1.2\times10^{-3}V\times2/12}{(.02/12)^2}. \quad \therefore V = 100 \text{ fps}.$$

$\therefore Q = AV = (.02\times4/144)\times100 = \underline{0.0556 \text{ ft}^3/\text{ sec}}$

7.46 $u(y) = \dfrac{1}{2m}\dfrac{dp}{dx}(y^2 - ay) + \dfrac{U}{a}y$. $t = m\dfrac{du}{dy} = \dfrac{1}{2}\dfrac{dp}{dx}(2y-a) + m\dfrac{U}{a}$.

a) $t_{y=a} = 0 = \dfrac{1}{2}\dfrac{dp}{dx}(2a-a) + m\dfrac{U}{a}$. $\therefore \dfrac{dp}{dx} = -\dfrac{2mU}{a^2} = -\dfrac{2\times1.81\times10^{-5}\times6}{.004^2} = \underline{-13.6 \text{ Pa/m.}}$

b) $t_{y=o} = 0 = -\dfrac{a}{2}\dfrac{dp}{dx} + m\dfrac{U}{a}$. $\therefore \dfrac{dp}{dx} = \dfrac{2mU}{a^2} = \dfrac{2\times1.81\times10^{-5}\times6}{.004^2} = \underline{+13.6 \text{ Pa/m}}.$

c) $Q = \displaystyle\int_0^{.004}\left[\frac{1}{2m}\frac{dp}{dx}(y^2 - ay) + \frac{U}{a}y\right]dy = \frac{1}{2\times1.81\times10^{-5}}\frac{dp}{dx}\left(\frac{.004^3}{3} - \frac{.004^3}{2}\right) + \frac{6}{.004}\times\frac{.004^2}{2} = 0.$

$\therefore \dfrac{dp}{dx} = \underline{40.7 \text{ Pa/ m}}.$

d) $u(.002) = 4 = \dfrac{1}{2\times1.81\times10^{-5}}\dfrac{dp}{dx}(.002^2 - .004\times.002) + \dfrac{6}{.004}\times.002$. $\therefore \dfrac{dp}{dx} = \underline{9.05 \text{ Pa/ m}}.$

7.47 $u = \dfrac{1}{2m}\dfrac{dp}{dx}(y^2 - ay) + \dfrac{U}{a}y$. $t = m\dfrac{du}{dy} = \dfrac{1}{2}\dfrac{dp}{dx}(2y-a) + m\dfrac{U}{a}$.

a) $t_{y=.006} = \dfrac{1}{2}(-20)(.006) + 1.95\times10^{-5}\dfrac{U}{.006} = 0$. $\therefore U = \underline{18.5 \text{ m/s.}}$

b) $t_{y=0} = \frac{1}{2}(-20)(.006) + 1.95\times10^{-5}\frac{U}{.006} = 0. \qquad \therefore U = \underline{-18.5\text{ m/s.}}$

c) $Q = \int_0^{.006}\left[\frac{1}{2\times1.95\times10^{-5}}(-20)(y^2 - .006y) + \frac{U}{.006}y\right]dy \qquad \therefore U = \underline{-6.15\text{ m/s}}.$

d) $u(.002) = \frac{1}{2\times1.95\times10^{-5}}(-20)(.002^2 - .006\times.002) + \frac{U}{.006}\times.002. \quad \therefore U = \underline{-12.3\text{ m/s}}.$

7.48 $u = \frac{U}{a}y = \frac{45}{.008/12}y = 67{,}500y. \qquad t = m\frac{du}{dy} = 10^{-4}\times67{,}500 = 6.75\text{ psf.}$

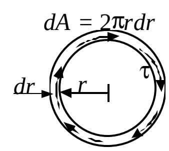
$\therefore F = tA = 6.75\times(2p\times10/\ 144) = \underline{2.95\text{ lb}}.$

7.49 $v_q = \frac{U}{a}y. \qquad \therefore t = m\frac{dv_q}{dy} = 0.1\times\frac{.2\times30}{.0008} = 750\text{ Pa.}$

$T = F\times R = tpDL\times R = 750p\times.4\times.8\times.2 = \underline{151\text{ N}\cdot\text{m}}.$

7.50 Assume a linear velocity profile between the rotating disc and the wall.

$\therefore v_q = \frac{U}{a}y = \frac{rw}{a}y. \qquad \therefore t = m\frac{dv_q}{dy} = \frac{mw}{a}r = \frac{.01\times60}{.0012}r = 500\text{ r.}$



$T = \int tdA\times r = \int_0^{.2}500r\times r2prdr = 1000p\int_0^{.2}r^3dr = \underline{1.26\text{ N}\cdot\text{m}}.$

The largest Re occurs at $r = 0.2$ m: $(\text{Re})_{\max} = \frac{Rw\times a}{n} = \frac{.2\times60\times.0012}{.01/(.86\times1000)} = \underline{1240}.$

The laminar flow assumption is valid.

7.51 Neglect the shear on the cylinder bottom; assume a linear velocity profile:

$v_q = \frac{U}{a}y = \frac{.1\times30}{.001}y = 3000y. \qquad \therefore t = m\frac{dv_q}{dy} = 0.42\times3000 = 1260\text{ Pa.}$

$\therefore T = F\times R = pDL\times t\times R = p\times.2\times.1\times1260\times.1 = \underline{7.9\text{ N}\cdot\text{m}}.$

7.52 Assume a linear velocity profile: $v_q = \frac{U}{a}y = \frac{rw}{a}y = \frac{50r}{.002}y.$

$t = m\frac{dv_q}{dy} = 25\,000\,r. \quad T = \int_0^{.0707}t\times dA\times r = \int_0^{.0707}(25\,000\,r)(2pr\frac{dr}{.707})\times r.$

$\therefore T = 25\,000\times\frac{2p}{.707}\int_0^{.0707}r^3dr = \underline{0.694\text{ N}\cdot\text{m}}.$

7.53 Assume that all losses occur in the 8-m-long channel. The velocity through the straws and screens is so low that the associated losses will be neglected. Assume a developed flow in the channel:

$$V = \frac{\mathrm{Re}\times n}{h} = \frac{7000\times 1.5\times 10^{-5}}{0.012} = 8.75\ \mathrm{m/\ s}$$

$$\Delta p = \frac{12\times 1.8\times 10^{-5}\times 8.75\times 8}{0.012^2} = 105\ \mathrm{Pa}$$

Energy: $\dot{W}_{\mathrm{fan}} = \frac{\Delta p}{rh}\dot{m} = \frac{\Delta p}{rh}(rAV) = \Delta pAV/\ h$

$$= 105\times 1.2\times 0.012\times 8.75/\ 0.7 = \underline{18.9\ \mathrm{W}}$$

7.54 The solution for $v_q(r)$ is (see Eq. 7.5.15) $v_q = \frac{A}{2}r + \frac{B}{r}$.

If $v_q = 0$ as $r \to \infty$, $A = 0$. $\therefore v_q = \frac{B}{r}$. Also, $v_q = Rw$ at $r = R$.

$\therefore Rw = \frac{B}{R}$. $\therefore B = wR^2$. $\therefore v_q = \underline{wR^2/r}$. The shear stress at

$r = R$ is $t_1 = -\left[mr\frac{dv_q/\ r}{dr}\right] = 2mw$. $\therefore T = t_1 2pRL\times R = 4pmwR^2L$.

$$\therefore T = 4p\times 2.36\times 10^{-5}\times\left(\frac{1000\times 2p}{60}\right)\times\left(\frac{1}{12}\right)^2\times\frac{40}{12} = \underline{7.19\times 10^{-4}\ \mathrm{ft\text{-}lb}}.$$

7.55 Use Eq. 7.5.19: $T = \frac{4pmr_1^2r_2^2Lw_1}{r_2^2 - r_1^2} = \frac{4p\times .035\times .02^2\times .03^2\times .4\times(3000\times 2p/\ 60)}{.03^2 - .02^2}$

$\therefore T = \underline{0.040\ \mathrm{N\cdot m}}$ $\therefore \dot{W} = Tw = .04\times(3000\times 2p/\ 60) = \underline{12.6\ \mathrm{W}}$.

$\mathrm{Re} = \frac{wr_1(r_2 - r_1)r}{m} = \frac{3000\times 2p\times .02\times(.01)\times 917}{60\times .035} = \underline{1650}$. $\therefore$Eq. 7.5.15 is OK.

7.56 Use Eq. 7.5.19: $T = \frac{4mpr_1^2r_2^2Lw_1}{r_2^2 - r_1^2} = .015 = \frac{4pm.04^2\times .05^2\times .5\times 40}{.05^2 - .04^2}$

$\therefore m = \underline{0.0134\ \frac{\mathrm{N\cdot s}}{\mathrm{m}^2}}$. $\mathrm{Re} = \frac{40\times .04(.05 - .04)\times 1000\times .9}{.0134} = \underline{1070}$. Eq. 7.5.17 is OK.

7.57 With $w_1 = 0$, Eq. 7.5.15 is $v_q = \frac{r_2^2w_2}{r_2^2 - r_1^2}\left(r - \frac{r_1^2}{r}\right)$. $t_2 = mr\frac{d}{dr}\left(\frac{v_q}{r}\right) = \frac{2mr_1^2w_2}{r_2^2 - r_1^2}$.

$$\therefore T_2 = t_2A_2r_2 = \frac{2mr_1^2w_2}{r_2^2 - r_1^2}2pr_2Lr_2 = \frac{4pmr_1^2r_2^2Lw_2}{r_2^2 - r_1^2}.$$

7.58 $T = \frac{4pmr_1^2r_2^2Lw_1}{r_2^2 - r_1^2} = \frac{4p\times 0.1\times 0.2^2\times 0.2008^2\times 0.8\times 30}{0.2008^2 - 0.2^2} = 152\ \mathrm{N\cdot m}$

$$\%\text{ error} = \frac{152-151}{151}\times 100 = \underline{0.66\%}$$

7.59 (**D**) The flow in a pipe may be laminar at any Reynolds number between 2000 and perhaps 40 000 depending on the character of the flow.

7.60 Let $u = \bar{u} + u'$, $v = \bar{v} + v'$, $w = \bar{w} + w'$. The continuity equation becomes

$$\frac{\partial}{\partial x}(\bar{u}+u') + \frac{\partial}{\partial y}(\bar{v}+v') + \frac{\partial}{\partial z}(\bar{w}+w') = \frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} + \frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z}.$$

Now, time-average the above equation recognizing that $\overline{\dfrac{\partial \bar{u}}{\partial x}} = \dfrac{\partial \bar{u}}{\partial x}$ and $\overline{\dfrac{\partial u'}{\partial x}} = \dfrac{\partial}{\partial x}\overline{u}' = 0.$ Then, $\dfrac{\partial \bar{u}}{\partial x} + \dfrac{\partial \bar{v}}{\partial y} + \dfrac{\partial \bar{w}}{\partial z} = 0.$ Substitute this back into the continuity equation, so that $\dfrac{\partial u'}{\partial x} + \dfrac{\partial v'}{\partial y} + \dfrac{\partial w'}{\partial z} = 0.$

7.61
$$\overline{\frac{Du}{Dt}} = \overline{(\bar{u}+u')\frac{\partial}{\partial x}(\bar{u}+u')} + \overline{(\bar{v}+v')\frac{\partial}{\partial y}(\bar{u}+u')} + \overline{(\bar{w}+w')\frac{\partial}{\partial z}(\bar{u}+u')} + \overline{\frac{\partial}{\partial t}(\bar{u}+u')}$$

$$= \bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{u}\cancel{\overline{\frac{\partial u'}{\partial x}}} + \cancel{\overline{u'}}\,\overline{\frac{\partial \bar{u}}{\partial x}} + \overline{u'\frac{\partial u'}{\partial x}} + \bar{v}\frac{\partial \bar{u}}{\partial y} + \bar{v}\cancel{\overline{\frac{\partial u'}{\partial y}}} + \cancel{\overline{v'}}\,\overline{\frac{\partial \bar{u}}{\partial y}} + \overline{v'\frac{\partial u'}{\partial y}} + \bar{w}\frac{\partial \bar{u}}{\partial z} + \bar{w}\cancel{\overline{\frac{\partial u'}{\partial z}}}$$

$$+ \cancel{\overline{w'}}\,\overline{\frac{\partial \bar{u}}{\partial z}} + \overline{w'\frac{\partial u'}{\partial z}} + \frac{\partial \bar{u}}{\partial t}.$$

$$= \bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} + \bar{w}\frac{\partial \bar{u}}{\partial t} + \frac{\partial \bar{u}}{\partial t} + \overline{u'\frac{\partial u'}{\partial x}} + \overline{v'\frac{\partial u'}{\partial y}} + \overline{w'\frac{\partial u'}{\partial z}}.$$

$$\frac{\overline{Du}}{Dt} = \bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial z} + \bar{w}\frac{\partial \bar{u}}{\partial z} + \frac{\partial \bar{u}}{\partial t}.$$

$$\therefore \overline{\frac{Du}{Dt}} - \frac{D\bar{u}}{Dt} = \underline{\frac{\partial}{\partial x}\overline{u'^2} + \frac{\partial}{\partial y}\overline{u'v'} + \frac{\partial}{\partial z}\overline{u'w'}}. \quad \text{(We used continuity.)}$$

7.62 Use the fact that $\dfrac{\partial}{\partial y}\overline{u'v'} = \overline{\dfrac{\partial}{\partial y}u'v'}$. See Eq. 7.6.2. This is equivalent to $\dfrac{\partial}{\partial y}\left(\dfrac{1}{T}\displaystyle\int_0^T u'v'dt\right) = \dfrac{1}{T}\displaystyle\int_0^T \dfrac{\partial}{\partial y}(u'v')dt$, which is obviously correct. Also,

$$\frac{\partial}{\partial x}u'u' + \frac{\partial}{\partial y}u'v' + \frac{\partial}{\partial z}u'w' = u'\left(\cancel{\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z}}\right) + u'\frac{\partial u'}{\partial x} + v'\frac{\partial u'}{\partial y} + w'\frac{\partial u'}{\partial z}.$$

Time average both sides and obtain the result.

7.63 The x-component Navier-Stokes equation for a horizontal channel is

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

Substitute $u=\bar{u}+u'$, $v=v'$, $w=w'$ into the N-S eq and time-average:

$$\rho\left[\frac{\partial \bar{u}}{\partial t}+\frac{\partial \overline{u'}}{\partial t}+\frac{\partial}{\partial x}\overline{u'^2}+\frac{\partial}{\partial y}\overline{u'v'}+\frac{\partial}{\partial z}\overline{u'w'}\right]=-\frac{\partial \bar{p}}{\partial x}-\frac{\partial \overline{p'}}{\partial x}+\mu\left(\frac{\partial^2 \bar{u}}{\partial x^2}+\frac{\partial^2 \bar{u}}{\partial y^2}+\frac{\partial^2 \bar{u}}{\partial z^2}\right)$$

where we used the result written in Problem 7.61. Then

$$\rho\frac{\partial}{\partial y}\overline{u'v'}=-\frac{\partial \bar{p}}{\partial x}+\mu\frac{\partial^2 \bar{u}}{\partial y^2}$$

In terms of stresses

$$-\frac{\partial \bar{\tau}_{\text{turb}}}{\partial y}=-\frac{\partial \bar{p}}{\partial x}+\frac{\partial \bar{\tau}_{\text{lam}}}{\partial y}\qquad\text{or}\qquad\frac{\partial \bar{p}}{\partial x}=\frac{\partial}{\partial y}\left(\bar{\tau}_{\text{turb}}+\bar{\tau}_{\text{lam}}\right)$$

7.64 $\bar{u}=\Sigma u_i/\ 11=\underline{16.2\text{ m/ s}}$. $\quad\bar{v}=\Sigma v_i/\ 11=\underline{-1.6\text{ m/ s}}$. $\quad u'=u-\bar{u},\quad v'=v-\bar{v}$.

t	0	.01	.02	.03	.04	.05	.06	.07	.08	.09	.1	
u'	-.1	9.5	-5.6	1.1	-11	-6	0.9	12.4	-9.5	3	5.4	
u'^2	.01	90.2	31.4	1.2	121	36	.81	153.8	90.2	9	29.2	$\overline{u'^2}=\underline{51.2\text{ m}^2/\text{s}^2}$
$0=\cancel{\frac{V_2^2-V_1^2}{2g}}+\frac{p_2-p_1}{g}+K\frac{V^2}{2g}$.					3.2	-3.6	-7.0	5.1	5.7	-4.4	0.2	8.3
	-3.6	-6.6	3.1									
v'^2	10.2	14.4	49	26	32.5	19.4	.04	68.9	13.0	43.6	9.6	$\overline{v'^2}=\underline{26.1\text{ m}^2/\text{s}^2}$
$u'v'$	-.3	-36.1	39.2	5.6	-62.7	26.4	.2	102.9	34.2	-19.8	16.7	$\overline{u'v'}=\underline{9.7\text{ m}^2/\text{s}^2}$

7.65 $$\overline{u'v'}=\nu\frac{\partial\bar{u}}{\partial y}-\frac{\bar{\tau}}{\rho}=\nu\frac{\partial\bar{u}}{\partial y}-\frac{r}{2}\frac{\Delta p}{L\rho}=1.6\times10^{-4}\,\frac{76.9-60.7}{0.09}-\frac{.69}{2}\,\frac{8}{30\times.0035}=\underline{-26.3\text{ ft}^2/\sec^2}$$

7.66 $$\eta=-\frac{\overline{u'v'}}{d\bar{u}/dy}=-\frac{-26.3}{(76.9-60.7)/.09}=\underline{0.146\text{ ft}^2/\text{s}}.$$

$$K_{uv}=\frac{\overline{u'v'}}{\sqrt{\overline{u'^2}}\sqrt{\overline{v'^2}}}=\frac{-26.3}{\sqrt{316}\sqrt{156}}=\underline{-0.118}$$

$$\ell_m=\sqrt{\eta/\partial\bar{u}/\partial y}=\sqrt{0.146/(76.9-60.7)/0.09}=0.0285\text{ ft or }\underline{0.342\text{ in}}.$$

7.67 $$v'=\frac{1}{2}\sin\frac{2\pi}{.2}t.\quad \overline{u'v'}=\frac{1}{.2}\int_0^{.2}\left(\frac{1}{2}\sin10\pi t\right)\left(\frac{1}{2}\sin10\pi t\right)dt=\frac{1}{.8}\int_0^{.2}\sin^2 10\pi t\,dt$$

$$= \frac{1}{.8}\int_0^{.2}\left(\frac{1}{2}+\frac{1}{2}\cos 20\pi t\right)dt = \frac{1}{.8}\left(\frac{.2}{2}\right) = \underline{0.125\ \mathrm{m}^2/\mathrm{s}^2}.$$

$$\eta = -\frac{\overline{u'v'}}{d\bar{u}/dy} = -\frac{.125}{-10} = \underline{0.0125\ \mathrm{m}^2/\mathrm{s}}.$$

$$\ell_m = \sqrt{\eta/\left|\partial\bar{u}/\partial y\right|} = \sqrt{0.0125/10} = 0.0354\ \mathrm{m}\ \text{or}\ \underline{3.54\ \mathrm{cm}}$$

$$u'^2 = \frac{1}{4}\sin^2 10\pi t = v'^2 \qquad \therefore \overline{u'^2} = \overline{v'^2} = .125.$$

$$K_{uv} = \frac{\overline{u'v'}}{\sqrt{\overline{u'^2}}\sqrt{\overline{v'^2}}} = \frac{.125}{\sqrt{.125}\sqrt{.125}} = \underline{1.0}$$

7.68 a) $\mathrm{Re} = \dfrac{VD}{\nu} = \dfrac{0.02\times 0.2}{10^{-6}} = 4000,\quad \dfrac{e}{D} = \dfrac{0.26}{200} = 0.0013.$

From the Moody diagram, this is effectively a <u>"smooth" pipe</u> so we conclude that $\delta_\nu > e$.

b) $\mathrm{Re} = \dfrac{0.2\times 0.2}{10^{-6}} = 40\ 000,\quad \dfrac{e}{D} = 0.0013.$

From the Moody diagram, this is in the transition zone where δ_ν may be near, in magnitude, to e. The pipe is <u>rough</u>.

c) $\mathrm{Re} = 400\ 000,\ \dfrac{e}{D} = 0.0013$ and the Moody diagram indicates a <u>rough pipe</u>.

7.69 $\mathrm{Re} = \dfrac{VD}{\nu} = \dfrac{6\times 0.1}{1.1\times 10^{-4}} = 5455.$ From the Moody diagram, if $\dfrac{e}{D} = 0.001$, then the pipe is "smooth". Thus, $e = 0.001\,D = 0.001\times 100 = \underline{0.1\ \mathrm{mm}}$.

7.70 a) Using $\mathrm{Re} = 4000$ and $e/D = 0.0013$, the Moody diagram provides $f = 0.04$.

Then
$$\tau_0 = \frac{f}{8}\rho V^2 = \frac{0.04}{8}1000\times 0.02^2 = 0.002\ \mathrm{Pa}$$
$$\therefore u_\tau = \sqrt{\tau_0/\rho} = \sqrt{0.002/1000} = 0.001414\ \mathrm{m/s}$$

Eq. 7.6.16 gives
$$u_{max} = u_\tau\left(2.44\,\ell n\frac{u_\tau r_o}{\nu} + 5.7\right)$$
$$= 0.001414\left(2.44\,\ell n\frac{0.001414\times 0.1}{10} + 5.7\right) = \underline{0.0251\ \mathrm{m/s}}$$

b) With $\mathrm{Re} = 40\ 000$, $e/D = 0.0013$ the Moody diagram gives $f = 0.026$. Then
$$\tau_0 = \frac{0.026}{8}1000\times 0.2^2 = 0.130\ \mathrm{Pa}$$
$$u_\tau = \sqrt{0.130/1000} = 0.01140\ \mathrm{m/s}$$

Eq. 7.6.17 gives

$$u_{\max} = u_t\left(2.44\ell n\frac{r_o}{e} + 8.5\right)$$

$$= 0.0114\left(2.44\ell n\frac{0.1}{0.00026} + 8.5\right) = \underline{0.262\text{ m/ s}}$$

c) With $\text{Re} = 400\ 000,\ e/\ D = 0.0013$ we find $f = 0.022$:

$$t_0 = \frac{0.022}{8}1000\times 2^2 = 11\text{ Pa}$$

$$u_t = \sqrt{11/\ 1000} = 0.105\text{ m/ s}$$

$$u_{\max} = u_t\left(2.44\ell n\frac{r_o}{e} + 8.5\right)$$

$$= 0.105\left(2.44\ell n\frac{0.1}{0.00026} + 8.5\right) = \underline{2.41\text{ m/ s}}$$

7.71 Here $n = 7$ so that, from Eq. 7.6.21, $f = \frac{1}{n^2} = \frac{1}{49} = 0.0204$. And from Eq. 7.6.20, $V = 4.9$ m/s.

a) $\therefore t_0 = \frac{1}{8}rV^2 f = \frac{1}{8}\times 1000\times 4.9^2\times .0204 = \underline{61.2\text{ Pa}}$.

b) $\frac{d\bar{u}}{dy} = 9.2\times\frac{1}{7}y^{-6/7}\Big|_{y=0} = \underline{\infty}$. (The profile is not too good near the wall!)

c) $\frac{\P p}{\P x} = -\frac{\Delta p}{L} = -\frac{2t_0}{r_o} = -\frac{2\times 61.2}{.05} = \underline{-2450\text{ Pa/ m}}$.

d) t varies linearly with r. $\therefore t_{r=2.5\text{ cm}} = \frac{1}{2}t_0 = 30.6$ Pa. $(r_o = 5\text{ cm})$

$$t = rh\frac{d\bar{u}}{dy}.\quad \therefore 30.6 = 1000h\left(9.2\times\frac{1}{7}\times .025^{-6/7}\right).\quad \therefore h = \underline{9.85\times 10^{-4}\text{m}^2/\text{s}}.$$

7.72 $V = \frac{Q}{A} = \frac{2.5}{p\times(2.5/\ 12)^2} = 18.33$ fps. $\text{Re} = \frac{VD}{n} = \frac{18.33\times 5/\ 12}{1.08\times 10^{-5}} = 7.09\times 10^5$.

From Table 7.1, $\underline{n \cong 8.5}$. From Eq. 7.6.20

$$u_{\max} = \frac{(n+1)(2n+1)}{2n^2}\times V = \frac{9.5\times 18}{2\times 8.5^2}\times 18.33 = \underline{21.7\text{ fps}}.$$

7.73 $n = 5$: $V = \frac{2n^2}{(n+1)(2n+1)}u_{\max} = \frac{2\times 5^2}{6\times 11}u_{\max} = 0.758u_{\max}$.

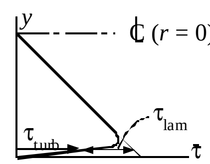
$$\alpha = \frac{\int u^3 dA}{V^3 A} = \frac{\int_0^{r_o} u_{\max}^3 y^{3/n} 2\pi (r_o - y)(dy)}{.758^3 u_{\max}^3 \pi r_o^2 r_o^{3/n}} = \frac{2}{.758^3 r_o^{2+3/n}} \int_0^{r_o} \left(r_o y^{3/n} - y^{1+3/n} \right) dy$$

$$= \frac{4.59}{r_o^{2+3/n}} \left[\frac{r_o^{2+3/n}}{1+3/n} - \frac{r_o^{2+3/n}}{2+3/n} \right] = 4.59 \left(\frac{1}{1.6} - \frac{1}{2.6} \right) = \underline{1.10}$$

With $n = 10$, $V = \dfrac{2 \times 10^2}{11 \times 21} u_{\max} = .866 u_{\max}$.

$$\alpha = \frac{2}{.866^3} \left(\frac{1}{1.3} - \frac{1}{2.3} \right) = \underline{1.03}.$$

7.74 With $n = 7$, Eq. 7.6.21 gives $f = \dfrac{1}{n^2} = \dfrac{1}{49} = 0.0204$.



$$\therefore \tau_0 = \frac{1}{8} \rho V^2 \times f = \frac{1}{2} \times 1000 \times 10^2 \times .0204 = 1020 \text{ Pa}.$$

Since τ varies linearly with r and is zero at $r = 0$,

$$\bar{\tau} = \tau_0 \frac{r}{r_o} = \frac{1020}{.05} r = 20\,400\ r.$$

$$\tau_{\text{lam}} = \mu \frac{\partial \bar{u}}{\partial y} = 10^{-3} \left[u_{\max} \frac{1}{7} \frac{1}{r_0^{1/7}} y^{-6/7} \right] = \frac{10^{-3} \times 12.24}{7 \times .05^{1/7}} y^{-6/7} = 0.00268 y^{-6/7}.$$

$$\tau_{\text{turb}} = \bar{\tau} - \tau_{\text{lam}} = 20\,400\ r - 0.00268 y^{-6/7} \quad \text{where } y + r = .05.$$

$\tau_{\text{lam}}(y)$ is good away from $y = 0$ (the wall).

$$\tau_{\text{lam}}(.00625) = .21, \quad \tau_{\text{lam}}(.003125) = .38, \quad \tau_{\text{lam}}(.00156) = .68, \quad \tau_{\text{lam}}(.00078) = 1.24$$

$$\frac{d\bar{p}}{dx} = -\frac{2\tau_0}{r_0} = -\frac{2 \times 1020}{.05} = \underline{-40\,800 \text{ Pa / m}}.$$

7.75 a) $V = \dfrac{Q}{A} = \dfrac{1.2}{\pi \times .4^2} = 19.63$ m / s. $\quad \text{Re} = \dfrac{VD}{\nu} = \dfrac{19.63 \times 0.8}{2.2 \times 10^{-4}} = \underline{71\,400}$.

b) $\dfrac{e}{D} = 0$. $\quad \therefore$ From Fig. 7.13 $f = \underline{0.019}$. $\quad \tau_0 = \dfrac{1}{8} \times 917 \times 19.63^2 \times .019 = 840$ Pa.

c) $V = \dfrac{2n^2}{(n+1)(2n+1)} u_{\max}$. $\quad \therefore u_{\max} = \dfrac{19.63 \times 8 \times 15}{2 \times 49} = \underline{24.0 \text{ m / s}}$

d) $\delta_\nu = \dfrac{5\nu}{u_\tau} = \dfrac{5 \times 2.2 \times 10^{-4}}{0.957} = \underline{0.0015 \text{ m}}$ where $u_\tau = \sqrt{840/917} = 0.957$ m/s.

e) $u_{\max} = 0.957 \left[2.44 \ln \dfrac{0.957 \times .4}{2.2 \times 10^{-4}} + 5.7 \right] = \underline{22.9 \text{ m / s}}$.

Note that $u_{\max}$ is nearly the same using either form of the velocity profile.

7.76 We must find u_t:

$$V=\frac{Q}{A}=\frac{1.2}{p\times 0.4^2}=19.63\text{ m/ s.}\qquad \text{Re}=\frac{19.63\times 0.8}{2.2\times 10^{-4}}=71\,400$$

$$\frac{e}{D}=\frac{0.26}{800}=0.000325.\quad \therefore \text{ Moody diagram }\Rightarrow f=0.021$$

$$t_0=\frac{1}{8}frV^2=\frac{1}{8}\times 0.021\times 917\times 19.63^2=928\text{ Pa}$$

$$u_t=\sqrt{t_0/r}=\sqrt{928/917}=1.006\text{ m/ s.}$$

$$\therefore u_{\max}=1.006\left[2.44\ell n\frac{1.006\times 0.4}{2.2\times 10^{-4}}+5.7\right]=\underline{24.2\text{ m/ s}}$$

7.77 a) $\dfrac{\Delta p}{L}=\dfrac{1.5\times 144}{15}=14.4=\dfrac{2t_0}{r_0}=\dfrac{2t_0}{2/12}.\quad \therefore t_0=\underline{1.2\text{ psf.}}\quad \therefore u_t=\sqrt{\dfrac{1.2}{1.94}}=0.786\text{ fps.}$

b) $u_{\max}=u_t\left[2.44\ell n\dfrac{u_t r_0}{n}+5.7\right]=.786\left[2.44\ell n\dfrac{.786\times 2/12}{.736\times 10^{-5}}+5.7\right]=\underline{23.2\text{ fps.}}$

c) Assume $n=8$: $V=\dfrac{2n^2}{(n+1)(2n+1)}u_{\max}=\dfrac{2\times 64}{9\times 17}\times 23.2=\underline{19.41\text{ fps.}}$

d) Check Re: $\text{Re}=\dfrac{VD}{n}=\dfrac{19.41\times 4/12}{.736\times 10^{-5}}=\underline{8.79\times 10^5}.\quad \therefore n\cong 8$ is OK.

e) $Q=AV=p\times(2/12)^2\times 19.41=\underline{1.69\text{ cfs.}}$

7.78 a) From a control volume of the 10-m section of pipe

$$\Delta p\frac{pD^2}{4}=t_0 pDL.\qquad \therefore t_0=\frac{0.12\times 5000}{4\times 10}=\underline{15\text{ Pa}}$$

Assume n = 7. Then Eq. 7.6.21 gives $f=\dfrac{1}{n^2}=\dfrac{1}{49}=0.0204$.

From Eq. 7.3.19,

$$V^2=\frac{8t_0}{rf}=\frac{8\times 15}{917\times 0.0204}=6.41\quad\text{or}\quad V=2.53\text{ m/ s}$$

Check: $\text{Re}=\dfrac{2.53\times 0.12}{2.2\times 10^{-4}}=1381.$ This suggests laminar flow. Use Eq. 7.3.14:

$$V=\frac{r_0^2\Delta p}{8mL}=\frac{0.06^2\times 5000}{8\times(917\times 2.2\times 10^{-4})\times 10}=\underline{1.12\text{ m/ s}}.$$

Check: $\text{Re}=\dfrac{1.12\times 0.12}{2.2\times 10^{-4}}=611.$ OK.

Finally, $Q=AV=p\times 0.06^2\times 1.12=\underline{0.0127\text{ m}^3/\text{ s}}$

b) From a control volume or Eq. 7.6.18: $t_0 = -\dfrac{0.06(-20\ 000)}{2 \times 10} = 60$ Pa

Assume n = 6. Then Eq. 7.6.21 gives $f = \dfrac{1}{36} = 0.0278$

and

$$V^2 = \frac{8t_0}{rf} = \frac{8 \times 60}{917 \times 0.0278} = 18.83 \text{ or } V = 4.34 \text{ m/s}.$$

Check: $\text{Re} = \dfrac{4.34 \times 0.12}{2.2 \times 10^{-4}} = 2370.$ OK.

$$Q = p \times 0.06^2 \times 4.34 = \underline{0.0491 \text{ m}^3/\text{s}}$$

c) $t_0 = -\dfrac{0.06(-200\ 000)}{2 \times 10} = 600$ Pa

Assume n = 7. Then $f = \dfrac{1}{49} = 0.0204$

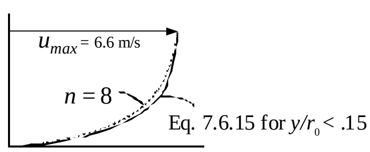
and

$$V = \sqrt{\frac{8t_0}{rf}} = \sqrt{\frac{8 \times 600}{917 \times 0.0204}} = \underline{16.0 \text{ m/s}}$$

Check: $\text{Re} = \dfrac{16.0 \times 0.12}{2.2 \times 10^{-4}} = 8740.$ OK.

$$Q = p \times 0.06^2 \times 16 = \underline{0.181 \text{ m}^3/\text{s}}$$

7.79 $\dfrac{u}{u_{\max}} = \left(\dfrac{y}{r_0}\right)^{1/8}$



7.80 (**D**) The friction factor f depends on the velocity (the Reynolds number).

7.81 (**A**)

7.82 a) $\text{Re} = \dfrac{VD}{n} = \dfrac{(0.020/p \times 0.04^2) \times 0.08}{10^{-6}} = 3.18 \times 10^5.$ $\dfrac{e}{D} = 0.$ $\therefore\ f = 0.0143.$

b) Eq. 7.6.26 provides f by trial-and-error. Try f = 0.0143 from Moody's diagram:

$$\frac{1}{\sqrt{f}} = 0.86 \ln\left(3.18 \times 10^5 \sqrt{0.0143}\right) - 0.8. \quad \therefore\ f = 0.0146.$$

Another iteration may be recommended but this is quite close. The value for f is essentially the same using either method. The equations could, however, be programmed on a computer.

7.83 a) $\text{Re} = \frac{VD}{\nu} = \frac{(0.03/\pi \times 0.05^2) \times 0.1}{1.14 \times 10^{-6}} = 3.35 \times 10^5. \quad \frac{e}{D} = \frac{0.26}{100} = 0.0026. \quad \therefore f = 0.026.$

b) Eq. 7.6.28 provides f by trial-and-error. Try $f = 0.026$ from the Moody diagram:

$$\frac{1}{\sqrt{f}} = -0.86 \ln\left(\frac{0.26}{3.7 \times 100} + \frac{2.51}{3.35 \times 10^5 \times \sqrt{0.026}}\right) = 6.189. \quad \therefore f = 0.0261$$

The value for f is essentially the same using either method. The equations could, however, be programmed on a computer.

7.84 a) $\text{Re} = \frac{0.025 \times 0.04}{10^{-6}} = 1000. \quad \therefore \text{laminar.} \quad \therefore f = \frac{64\nu}{VD} = \frac{64 \times 10^{-6}}{0.025 \times 0.04} = \underline{0.064}$

b) $\text{Re} = \frac{0.25 \times 0.04}{10^{-6}} = 10\,000, \quad \frac{e}{D} = \frac{0.26}{40} = 0.0065. \quad \therefore f = \underline{0.04}$

c) $\text{Re} = \frac{2.5 \times 0.04}{10^{-6}} = 100\,000, \quad \frac{e}{D} = \frac{0.26}{40} = 0.0065. \quad \therefore f = \underline{0.034}$

d) $\text{Re} = 10^6, \quad \frac{e}{D} = 0.0065. \quad \therefore f = \underline{0.033}$

7.85 (**B**) $\Delta p = f \frac{L}{D} \frac{V^2}{2g}. \quad \frac{e}{D} = \frac{0.15}{20} = 0.0075. \quad \therefore$ Moody's diagram gives, assuming $\text{Re} > 10^5$,

$f = 0.034. \quad$ Then $60000 = 0.034 \times \frac{15}{0.02} \frac{V^2}{2 \times 9.81}. \quad \therefore V = 6.79$ m/s and

$$Q = AV = \pi \times 0.01^2 \times 6.79 = 0.00214 \text{ m}^3\text{/s}.$$

Check the Reynolds number: $\text{Re} = \frac{6.79 \times 0.02}{10^{-6}} = 1.36 \times 10^5. \quad \therefore OK.$

7.86 (**D**) $\frac{e}{D} = \frac{0.26}{80} = 0.00325. \qquad$ Assume $\text{Re} > 3 \times 10^5$. Then $f = 0.026$.

$$\frac{h_L}{L} = \sin\theta = f \frac{1}{D} \frac{V^2}{2g}. \qquad \sin 30^\circ = 0.026 \frac{1}{0.08} \frac{V^2}{2 \times 9.81}. \qquad \therefore V = 5.49 \text{ m/s}.$$

Check Re: $\text{Re} = \frac{VD}{\nu} = \frac{5.49 \times 0.08}{10^{-6}} = 4.39 \times 10^5. \qquad \therefore \text{OK}.$

7.87 $V = \frac{Q}{A} = \frac{.02}{\pi \times .05^2} = 2.55 \text{ m/s}. \quad \Delta p = \gamma f \frac{L}{D} \frac{V^2}{2g} = \rho f \frac{100}{.10} \frac{2.55^2}{2} = 3251 \rho f.$

a) $\text{Re} = \frac{2.55 \times .1}{10^{-6}} = 2.55 \times 10^5. \qquad \frac{e}{D} = \frac{.046}{100} = .00046. \quad \therefore f = .0185$

$$\therefore \Delta p = 1000 \times .0185 \times 3251 = \underline{60\ 100 \text{ Pa}}.$$