SOLUTIONS MANUAL

For

MECHANICS OF FLUIDS

FOURTH EDITION

by

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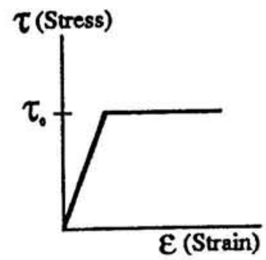
CHAPTER 1

1.1 Resnick and Halliday in "Physics for Students in Science and Engineering" define a fluid as "a substance that can **flow**". This is very close to our definition but would include materials undergoing creep and other viscoelastic behavior. However, as a first introduction to the subject of mechanics this definition is adequate.

Pauling in "General Chemistry" does not define fluid but is concerned about gases and liquids. These definitions are concerned with molecular behavior of these two phases.

1.2 The diagram referred to in this problem appears as

In strength of materials, we used the concept of an elastic, perfectly plastic stress-strain diagram, as shown. Does such a material satisfy the definition of a fluid? Explain.



This material does not satisfy our definition because a certain stress τ_o must be reached before there is the possibility of plastic flow. A fluid flows for any shear stress, however small.

1.3

What is the dimensional representation of:
(*a*) Power
(*b*) Modulus of elasticity
(*c*) Specific weight
(*d*) Angular velocity
(*e*) Energy
(*f*) Moment of a force
(*g*) Poisson's ratio
(*h*) Strain

$\left(\frac{FL}{t}\right)$ (a) $\qquad (FL)$ (e)

$\left(\frac{F}{L^2}\right)$ (b) $\qquad (FL)$ (f)

$\left(\frac{F}{L^3}\right)$ (c) $\qquad (1)$ (g)

$\left(\frac{1}{t}\right)$ (d) $\qquad (1)$ (h)

1.4

What is the relation between a scale unit of acceleration in USCS (pound-mass-foot-second) and SI (kilogram-meter-second)?

$$(a) = \left(\frac{L}{t^2}\right) = \frac{ft}{sec^2} \equiv \frac{ft\left(\dfrac{.305\,meter}{1\,ft}\right)}{sec^2}$$

$$= .305\ \frac{meter}{sec^2} \qquad \therefore\ 1\ \frac{ft}{sec^2} = .305\ \frac{meter}{sec^2}$$

1.5

How many scale units of power in SI using newtons, meters, and seconds are there to a scale unit in USCS using pounds of force, feet, and seconds?

$$Power = \frac{FL}{t} = \frac{lbf \cdot ft}{s}$$

$$= \frac{lbf\left(\frac{4.45N}{lbf}\right) ft \left(\frac{.305m}{1 ft}\right)}{s}$$

$$\therefore \; 1 \; \frac{ft \cdot lbf}{s} = 1.373 \; \frac{N \cdot m}{s}$$

1.6

Is the following equation a dimensionally homogeneous equation:

$$a = \frac{2d}{t^2} - \frac{2V_0}{t}$$

where a = acceleration
d = distance
V_0 = velocity
t = time

$$a = \frac{2d}{t^2} - 2\,\frac{V_o}{t}$$

$$\left(\frac{L}{t^2}\right) = \left(\frac{L}{t^2}\right) - \left(\frac{\frac{L}{t}}{t}\right)$$

$\therefore$ Dimensionally homogeneous

1.7

The following equation is dimensionally homogeneous:

$$F = \frac{4Ey}{(1-\nu^2)(Rd^2)}\left[(h-y)\left(h-\frac{y}{2}\right)A - A^3\right]$$

where E = Young's modulus
ν = Poisson's ratio
d, y, h = distances
R = ratio of distances
F = force
What are the dimensions of A?

$$(F) = \left(\frac{\frac{F}{L^2}\,L}{L^2}\right)[(L)(L)(A)-(A)^3]$$

$$(A) = (L)$$

1.8

The shape of a hanging drop of liquid is expressible by the following formulation developed from photographic studies of the drop:

$$T = \frac{(\gamma - \gamma_0)(d_e)^2}{H}$$

where γ = specific weight of liquid drop
γ_0 = specific weight of vapor around it
d_e = diameter of drop at its equator
T = surface tension, i.e., force per unit length
H = a function determined by experiment
For the equation above to be dimensionally homogeneous, what dimensions must H possess?

$$\left(\frac{F}{L}\right) = \frac{\left(\frac{F}{L^3}\right)(L^2)}{(H)}$$

$(H) = (1)$ *H is dimensionless*

1.9

In the study of elastic solids we must solve the following partial differential equation for the case of a plane where body forces are conservative:

$$\frac{\partial^4\phi}{\partial x^4} + 2\frac{\partial^4\phi}{\partial x^2\,\partial y^2} + \frac{\partial^4\phi}{\partial y^4} = -(1-\nu)\left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}\right)$$

where ϕ = stress function
ν = Poisson's ratio
V = scalar function whose gradient $[(\partial V/\partial x)\mathbf{i} + (\partial V/\partial y)\mathbf{j}]$ is the body-force distribution where the body force is given per unit volume
What would the dimensions have to be of the stress function?

$$(\nabla V) = \frac{(F)}{L^3}$$

$$\frac{(V)}{(L)} = \frac{(F)}{(L^3)}$$

$$(V) = \frac{(F)}{(L^2)}$$

$$\therefore \; \frac{(\phi)}{(L^4)} = \frac{(F)/(L^2)}{L^2}$$

$$\therefore \; (\phi) = (F)$$

1.10

Convert the coefficient of viscosity μ from units of dynes, seconds, and centimeters (i.e., poises) to units of pound-force, seconds, and feet.

From **Newton's viscosity** law we have:

$$\mu = \frac{\tau}{\left(\frac{\partial v}{\partial n}\right)}$$

$$\therefore \; (\mu) = \frac{\frac{F}{L^2}}{\left(\frac{1}{L}\right)\left(\frac{L}{t}\right)} = \left(\frac{Ft}{L^2}\right)$$

$$\therefore \; 1 \; poise = \frac{(dyne)(\text{sec})}{(cm)^2} =$$

$$\frac{(dyne)\,\dfrac{1\; lbf}{4.45\times 10^5 \; dynes}\,(\text{sec})}{(cm)^2\left(\dfrac{1\; ft}{30.5\; cm}\right)^2}$$

$$= 2.09 \times 10^{-3} \; \frac{(lbf)(\text{sec})}{(ft^2)}$$

What are the dimensions of kinematic viscosity? If the viscosity of water at 68°F is 2.11×10^{-5} lb · s/ft², what is the kinematic viscosity at these conditions? How many stokes of kinematic viscosity does the water have?

1.11

a) $$(\nu) = \left(\frac{\mu}{\rho}\right) = \frac{(M)}{(L)(t)}\,\frac{(L^3)}{(M)} = \frac{(L^2)}{(t)}$$

b)

$$\mu = 2.11\times10^{-5}\ \frac{lb\cdot sec}{ft^2}$$

$$\nu = \frac{2.11\times10^{-5}\dfrac{lb\cdot sec}{ft^2}}{1.94\ \dfrac{slugs}{ft^3}}$$

$$= \frac{(2.11\times10^{-5})\ \dfrac{lb\cdot sec}{ft^2}}{1.94\ \dfrac{slugs}{ft^3}\left(\dfrac{lb/ft/sec^2}{1\ slug}\right)}$$

$$= 1.088\times10^{-5}\ \frac{ft^2}{sec}$$

c)

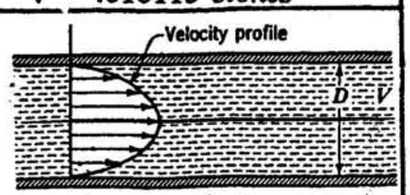
$$1\ stoke = \frac{1\ cm^2}{sec} = \frac{1(.01m)^2}{sec}$$

$$= \frac{\left[.01m\left(\dfrac{3.28\ ft}{1\ m}\right)\right]^2}{sec}$$

$$= .001076\ \frac{ft^2}{sec}$$

$\therefore$ For water

$$\nu = .010113\ stokes$$



1.12

Water is moving through a pipe. The velocity profile at some section is shown and is given mathematically as

$$V = \frac{\beta}{4\mu}\left(\frac{D^2}{4} - r^2\right)$$

where β = a constant
r = radial distance from centerline
V = velocity at any position r

What is the shear stress at the wall of the pipe from the water? What is the shear stress at a position $r = D/4$? If the profile above persists a distance L along the pipe, what drag is induced on the pipe by the water in the direction of flow over this distance?

$$V = \frac{\beta}{4\mu}\left(\frac{D^2}{4} - r^2\right)$$

$$\tau_w = \mu\left(\frac{\partial V}{\partial r}\right)_{r=\frac{D}{2}} = \mu\ \frac{\beta}{4\mu}\ (-2r)\Big|_{r=\frac{D}{2}}$$

a) $$\tau_w = -\frac{\beta D}{4}$$

b) at $$r = \frac{D}{4},$$

$$\tau = \frac{\beta}{4}\left(-2\ \frac{D}{4}\right) = -\frac{\beta D}{8}$$

c)

$$Drag = \tau_w(\pi)(D)(L)$$

$$= \frac{\beta D}{4}\ (\pi)(D)(L)$$

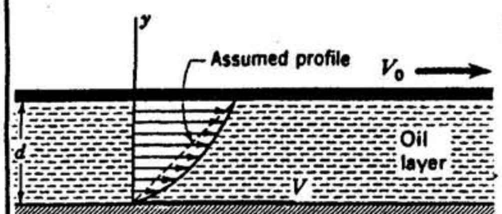
$$= \frac{\beta D^2 \pi L}{4}$$

A large plate moves with speed V_0 over a stationary plate on a layer of oil. If the velocity profile is that of a parabola, with the oil at the plates having the same velocity as the plates, what is the shear stress on the moving plate from the oil? If a linear profile is assumed, what is then the shear stress on the upper plate?

1.13 a) For a **parabolic profile**

$$V^2 = ay$$

When $y = d$, $V = V_o$. Hence:



$$V_o^2 = ad$$

$$a = \frac{V_0^2}{d}$$

Hence:

$$V^2 = V_o^2\left(\frac{y}{d}\right) \qquad \therefore\ V = V_o\sqrt{\frac{y}{d}}$$

$$\tau_w = \mu\left(\frac{\partial V}{\partial y}\right)_{y=d} = \mu\ V_o\ \frac{1}{\sqrt{d}}\ \frac{1}{2}\left(y^{-1/2}\right)_{y=d}$$

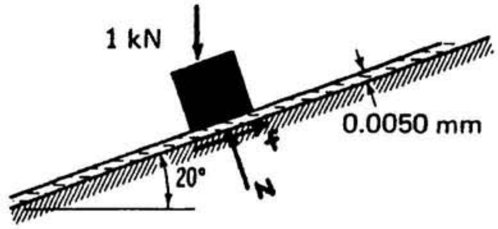
$$(\tau_w)_a = \frac{\mu V_o}{2d}$$

b) For a **linear profile**:

$$(\tau_w)_b = \mu\left(\frac{V_o}{d}\right)$$

A block weighing 1 kN and having dimensions 200 mm on an edge is allowed to slide down an incline on a film of oil having a thickness of 0.0050 mm. If we use a linear velocity profile in the oil, what is the terminal speed of the block? The viscosity of the oil is 7×10^{-2} P.

1.14



1.14 Note that **1** ***poise*** $= \dfrac{1}{10}$ of a viscosity unit in S.I. units. Hence

$$\mu = 7\times10^{-3} \quad N\cdot s/m^2$$

Using **Newton's** viscosity law:

$$\tau = \mu \frac{\partial V}{\partial n} = 7\times10^{-3} \frac{V_T}{(.005\times10^{-3})} = 1400\,V_T \quad Pa$$

The force f is then

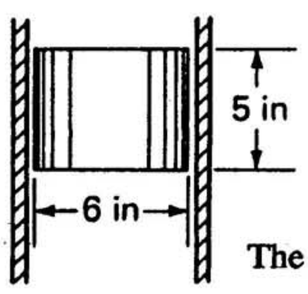
$$f = \tau A = (1400\,V_T)(.200)^2 = 56\,V_T$$

At the **terminal** condition we have **Equilibrium.**

$$\therefore \quad (1{,}000)\sin 20° - 56\,V_T = 0$$

$$V_T = \underline{6.11 \quad m/sec}$$

1.15 The shear stress on the cylinder assuming a linear profile is:



$$\tau = \mu\left(\frac{lb}{ft^2}\sec\right)\frac{V\left(\frac{ft}{\sec}\right)}{\left(\frac{.001}{12}\right)(ft)} = (12{,}000)\mu V \quad \frac{lb}{ft^2}$$

The force of resistance is then

$$f = [(12{,}000)(\mu)(V)](\pi)\left(\frac{6}{12}\right)\left(\frac{5}{12}\right)$$

$$= 7850\mu V \; lbf$$

A cylinder of weight 20 lb slides in a lubricated pipe. The clearance between cylinder and pipe is 0.001 in. If the cylinder is observed to decelerate at a rate of 2 ft/s² when the speed is 20 ft/s, what is the viscosity of the oil? The diameter of the cylinder D is 6.00 in and the length L is 5.00 in.

1.15 (continued)

Newton's law then gives us:

$$20 - 7850\mu V = \frac{20}{g}\,(a)$$

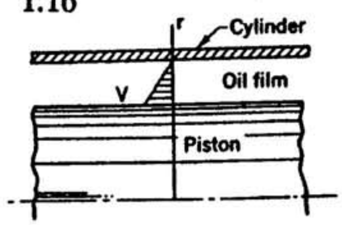
At the instant of interest:

$$20 - 7850\mu(20) = -\frac{20}{g}\,(2)$$

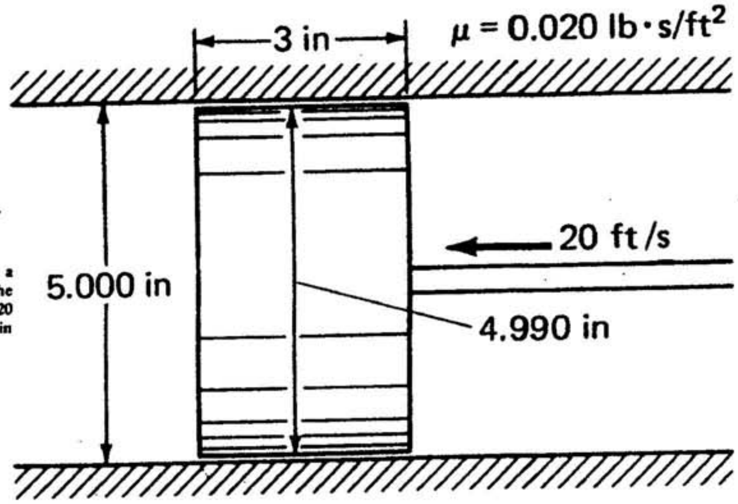
Solving for μ we get:

$$\mu = \frac{\left(20 + \dfrac{40}{g}\right)}{(7850)(20)} = 1.353\times10^{-4}\ \frac{lb\cdot sec}{ft^2}$$

1.16



A plunger is moving through a cylinder at a speed of 20 ft/s. The film of oil separating the plunger from the cylinder has a viscosity of 0.020 lb · s/ft². What is the force required to maintain this motion?



We shall assume that the thickness of the film is uniform over the entire peripheral surface of the plunger. Furthermore, because the film is thin, we shall assume a **linear** velocity profile for the flow of oil in the film. To find the frictional resistance, we must compute the shear stress at the plunger surface. Thus we have:

$$\tau = -\mu\,\frac{\partial V}{\partial r} = 0.020\,\frac{20}{\dfrac{5.000-4.990}{(2)(12)}}$$

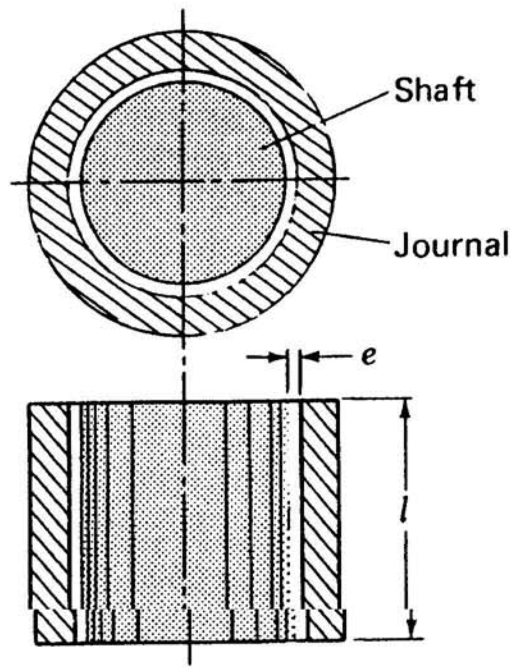
$$= 960\ \frac{lb}{ft^2}$$

The frictional force then becomes

$$F_f = \tau A = 960\pi\,\frac{4.990}{12}\,\frac{3}{12} = 314\ \ lbf$$

1.17

A vertical shaft rotates in a bearing. It is assumed that the shaft is concentric with the bearing journal. A film of oil of thickness e and viscosity μ separates the shaft from the bearing journal. If the shaft rotates at a speed of ω radians per second and has a diameter D, what is the frictional torque to be overcome at this speed? Neglect centrifugal effects at the bearing ends and assume a linear velocity profile. What is the power dissipated?



Even though the fluid particles move along lines which are not straight, we can with reasonably good accuracy, still employ **Newton's viscosity law.** Thus, the shear stress τ on the shaft is:

$$\tau = -\frac{0-\frac{\omega D}{2}}{e}\,\mu$$

The torque is:

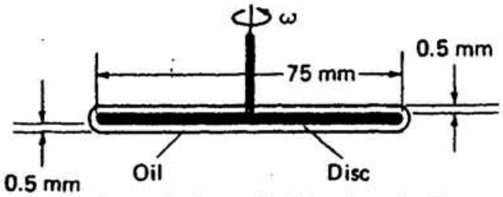
$$torque = \frac{\omega D}{2e}\,\mu\,\frac{D}{2}\,\pi D\ell = \frac{\mu\pi D^3\ell\omega}{4e}$$

The power is then

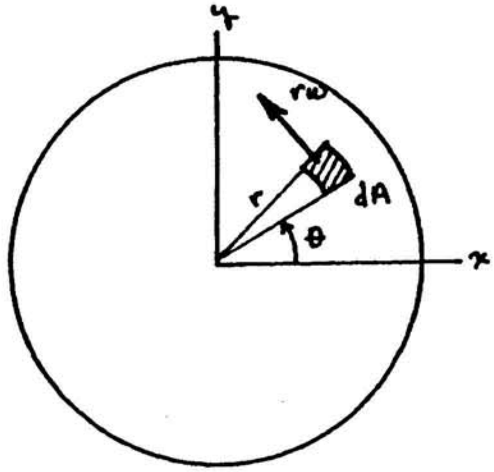
$$P = T\omega = \frac{\mu\pi D^3\ell\dot{\omega}^2}{4e}$$

1.18

We assume at any point that the velocity profile of the oil is linear.



In some electric measuring devices, the motion of the pointer mechanism is dampened by having a circular disc turn (with the pointer) in a container of oil. In this way, extraneous rotations are damped out. What is the damping torque for $\omega = 0.2$ rad/s if the oil has a viscosity of 8×10^{-3} $N \cdot s/m^2$? Neglect effects on the outer edge of the rotating plate.



The slope of the profile is then:

$$\frac{\partial V}{\partial n} = \frac{r\omega}{.5\,mm} = \frac{(.2)r}{.0005\,m} = 400r$$

$$\therefore \tau = (400r)(\mu) = 8\times 10^{-3}(400r)$$

$$= 3.2r$$

The force df on dA on the upper face of the disc is then:

$$df = \tau dA = (3.2r)(r d\theta dr)$$

$$= 3.2r^2 d\theta dr$$

The torque for dA on upper face is then:

$$dT = 3.2r^3 d\theta dr$$

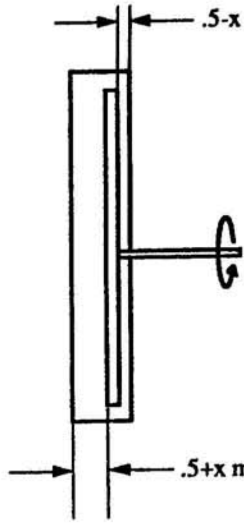
The total resisting torque on both faces is:

$$T = 2\left[\int_0^{.075/2}\int_0^{2\pi} 3.2r^3 d\theta dr\right]$$

$$= 6.4\,\frac{r^4}{4}\,(2\pi)\Bigg|_0^{0.75/2} = 1.988\times 10^{-5}\ N\text{-}m$$

1.19

For the apparatus in Prob. 1.18, develop an expression giving the damping torque as a function of x (the distance that the midplane of the rotating plate is from its center position). Do this for an angular rotation $\omega = 0.2$ rad/s.



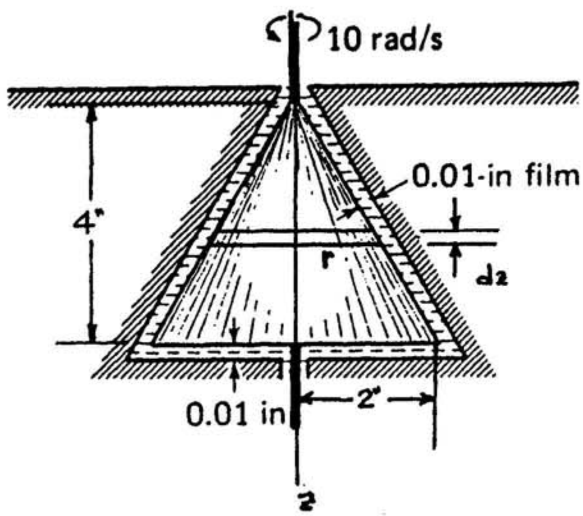
$$Torque = \int_0^{.075/2}\int_0^{2\pi}\left[\frac{r\omega}{(.5-x)(10^{-3})}\right]\mu r(rd\theta dr) + \int_0^{.075/2}\int_0^{2\pi}\left[\frac{r\omega}{(.5+x)(10^{-3})}\right]\mu r(rd\theta dr)$$

$$Torque = \frac{\mu\omega}{(.5-x)(10^{-3})}(2\pi)\frac{\left(\frac{.075}{2}\right)^4}{4} + \frac{\mu\omega}{(.5+x)(10^{-3})}(2\pi)\frac{\left(\frac{.075}{2}\right)^4}{4}$$

$$= \left[\left(\frac{1}{.5-x}\right)+\left(\frac{1}{.5+x}\right)\right]\frac{(8\times10^{-3})}{10^{-3}}(.2)(2\pi)\frac{\left(\frac{.075}{2}\right)^4}{4} = \left[\frac{.5+x\ +\ .5-x}{.25-x^2}\right]4.97\times10^{-6}$$

$$= \frac{4.97\times10^{-6}}{.25-x^2}\quad N\cdot m\quad (x\ in\ mm)$$

1.20



A conical body is made to rotate at a constant speed of 10 rad/s. A film of oil having a viscosity of 4.5×10^{-5} lb · s/ft² separates the cone from the container. The film thickness is 0.01 in. What torque is required to maintain this motion? The cone has a 2-in radius at the base and is 4 in tall. Use the straight-line-profile assumption and Newton's viscosity law.

Consider conical surface first. Area of the strip shown is:

$$dA = (2\pi r)(ds) = (2\pi r)\,\frac{dz}{\dfrac{4}{\sqrt{20}}} \tag{a}$$

But

$$\frac{r}{2} = \frac{z}{4} \qquad \therefore\; r = \frac{1}{2}z$$

Hence,

$$dA = 2\pi\,\frac{z}{2}\left(\frac{dz}{\dfrac{4}{\sqrt{20}}}\right) = \frac{\sqrt{20}\,\pi}{4}\,z\,dz$$

The stress on this element is:

$$\tau = \mu\frac{V}{\delta} = \mu\left(\frac{\omega r}{.01}\right) = \frac{\mu(10)\left(\dfrac{z}{2}\right)}{.01} = 500\,\mu z$$

with z in inches. The torque on the strip is

$$dT = \tau(dA)r = (500\,\mu z)\left(\frac{\sqrt{20}\,\pi}{4}\,z\,dz\right)\left(\frac{z}{2}\right) = 878\;\mu z^3 dz$$

The total torque is:

1.20 (cont.)

$$T_1 = \int_0^4 878\, uz^3 dz = 878\,\frac{(4)^4}{4}\,\mu = 56{,}198\mu$$

Subst. for μ to get in-lb.

$$T_1 = \left(\frac{4.5\times10^{-5}}{144}\right)(56{,}198) = .01756 \quad in-lb$$

Next consider the base. The friction force is:

$$df = \left[\mu\,\frac{(r\omega)}{.01}\right] r\, d\theta\, dr \;=\; 1{,}000\,\mu r^2\, d\theta\, dr$$

Torque for df :

$$dT = 1{,}000\mu r^3 d\theta\, dr$$

Total torque T_2

$$T_2 = \int_0^2\int_0^{2\pi} (1{,}000)(\mu) r^3 d\theta\, dr$$

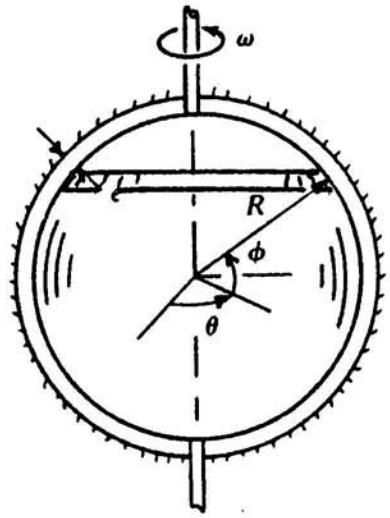
$$T_2 = (1{,}000)\left(\frac{4.5\times10^{-5}}{144}\right)\left(\frac{2^4}{4}\right)(2\pi) = .00785 \quad in\cdot lb$$

The total torque is then:

$$T_{total} = .01756 + .00785 = .0254 \quad in\cdot lb$$

1.21

A sphere of radius R rotates at constant speed of ω rad/s. A thin film of oil separates the rotating sphere from a stationary spherical container. Develop an expression for the resisting torque in terms of R, ω, μ, and e. Spherical coordinates are shown.



Velocity V of spherical surface is

$$V = R\cos\phi\,\omega$$

Stress from **Newton's viscosity** law:

$$\tau = \mu\frac{V}{e} = \frac{\mu}{e}(R\cos\phi\,\omega)$$

Torque increment on strip $R\,d\phi$

$$dT = (\tau)(2\pi\,R\cos\phi)(R\,d\phi)(R\cos\phi)$$

circumference of strip   width of strip   moment of arm

$$\therefore\quad dT = \frac{\mu}{e}(R\cos\phi\,\omega)(2\pi\,R\cos\phi)(R\,d\phi)(R\cos\phi) = \frac{2\pi\mu\omega R^4}{e}\cos^3\phi\,d\phi$$

$$T = \frac{2\pi\mu\omega R^4}{e}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos^3\phi\,d\phi$$

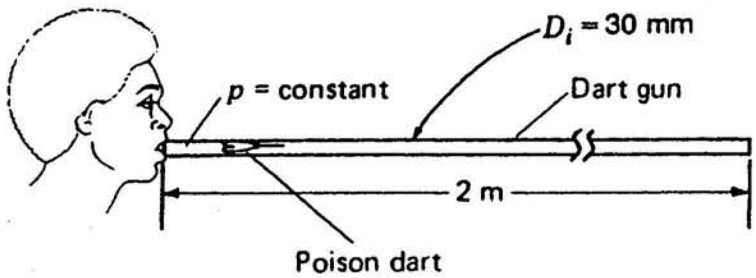
$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos^3\phi\,d\phi = \frac{1}{3}\sin\phi(\cos^2\phi + 2)\Bigg|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{4}{3}$$

$$\therefore\quad T = \left(\frac{2\pi\mu\omega R^4}{e}\right)\left(\frac{4}{3}\right)$$

$$\boxed{T = \frac{8\pi\mu\omega R^4}{3e}}$$

1.22

An African hunter is operating a blow gun with a poison dart. He maintains a constant pressure of 5 kPa gage behind the poison dart, which has a weight of $\frac{1}{2}$ N and a peripheral area directly adjacent to the inside surface of the blow gun of 1500 mm^2. The average clearance of this 1500-mm^2 peripheral area of the dart with the inside surface of the gun is 0.01 mm when shooting directly upward (at a bird in a tree). What is the speed of the dart on leaving the blow gun when fired directly upward? The inside surface of the gun is dry with air and vapor from the hunter's breath as the lubricating fluid between dart and gun. This mixture has a viscosity of 3×10^{-5} $N \cdot s/m^2$. *Hint:* Express dV/dt as $V(dV/dx)$ in Newton's law.



The shear force is: $$f_s = \mu \frac{\partial V}{\partial r}(A) = (3\times10^{-5})\left(\frac{V}{(.01)\times10^{-3}}\right)(1{,}500)(10^{-6}) = .0045\ V\ N$$

<u>Newton's Law</u>: $$p\,\frac{\pi D_i^2}{4} - W - .0045V = \frac{W}{g}V\frac{dV}{dx}$$

$$(5{,}000)\,\frac{(\pi)(.030)^2}{4} - .5 - .0045V = \frac{.5}{9.81}\,\frac{VdV}{dx}$$

$$3.534 - .5 - .0045V = \frac{.5}{9.81}\,\frac{VdV}{dx}$$

$$674 - V = 11.33\,\frac{VdV}{dx}$$

Separate variables: $$dx = 11.33\left(\frac{VdV}{674-V}\right)$$

Integrating:

$$x = 11.33[(674-V) - 674(\ln 674-V)] + C$$

When $x = 0$, $V = 0$. Hence $$0 = 11.33[674 - 674 \ln 674] + C$$

$$C = 42{,}101$$

$$\therefore\ x = 11.33[(674-V) - 674\ln(674-V)] + 42.101\times10^3$$

Set $x = 2m$. Solve by trial and error. $V = 16.55$ m/sec

$$\therefore\ \boxed{V = 16.55\ m/\text{sec}}$$

1.23

If specific volume υ is given in units of volume per unit mass, and density ρ is given in terms of mass per unit volume, how are they related? Also, if specific weight γ is given in units of weight per unit volume, how is it related to the other quantities?

$$(\upsilon) = \left(\frac{L^3}{M}\right)$$

$$(\rho) = \left(\frac{M}{L^3}\right) \qquad \therefore\ \upsilon = \frac{1}{\rho}$$

$$(\gamma) = \left(\frac{F}{L^3}\right)$$

$$\therefore \begin{cases} \gamma = \rho g \\ \gamma = \dfrac{g}{\upsilon} \end{cases}$$

1.24

What are the dimensions of R, the gas constant, in Eq.(1.89) Using for air the value 53.3 for R for units degrees Rankine, pound-mass, pound-force, and feet, determine the specific volume of air at a pressure of 50 lb/in^2 absolute and a temperature of 100°F.

$$p\upsilon = RT \qquad \therefore\ R = \frac{p\upsilon}{T}$$

$$(R) = \frac{\left(\frac{F}{L^2}\right)\left(\frac{L^3}{M}\right)}{(^\circ R)} = \left(\frac{FL}{M^\circ R}\right)$$

$\therefore$ for air

$$R = 53.3\ \frac{ft\text{-}lbf}{lbm^\circ R}$$

$$\upsilon = \frac{RT}{p} = \frac{(53.3)(460+100)}{(50)(144)}$$

$$\boxed{\upsilon = 4.15\ ft^3/lbm}$$

1.25

A perfect gas undergoes a process whereby its pressure is doubled and its specific volume is decreased by two-thirds. If the initial temperature is 100°F, what is the final temperature in degrees Fahrenheit?

$$T_1 = 100°F \qquad \frac{p_2}{p_1} = 2 \qquad \upsilon_2 = \frac{1}{3}\upsilon_1$$

$$p_1\upsilon_1 = RT_1$$

$$p_2\upsilon_2 = RT_2$$

Divide

$$\frac{p_1}{p_2}\frac{\upsilon_1}{\upsilon_2} = \frac{T_1}{T_2} \qquad \left(\frac{1}{2}\right)\left(\frac{3}{1}\right) = \frac{560}{T_2}$$

$$T_2 = \left(\frac{2}{3}\right)(560) = 373°R$$

$$\therefore \quad \boxed{\therefore \ T_2 = 373 - 460 = -87}$$

1.26 Initially we can say from the equation of state: $p_1\upsilon_1 = RT_1$

$$\therefore \ (200)(1{,}000)(\upsilon_1) = (287)(303) \qquad \upsilon_1 = .435 \ \frac{m^3}{kg}$$

If the volume V_1 is $80L$, then the mass of the gas is: $V_1 = (80)(.001)m^3$

$$\therefore \ M = \frac{(80)(.001)}{.435} = .1839 \ kg$$

At the final stage, $p_2\upsilon_2 = RT_2$

$$(500\times10^3)\left[\frac{40\times10^{-3}}{M-.003}\right] = (287)(T_2)$$

$$\boxed{T_2 = 385°K = 112.2°C}$$

In order to reduce gasoline consumption in city driving, the Department of Energy of the federal government is studying the so-called "inertial transmission" system. In this system, when drivers want to slow up, the wheels are made to drive pumps which pump oil into the compressor tank so as to increase the pressure of the trapped air in the tank. The pumps thus act as brakes. As long as the pressure in the tank stays above a certain minimum value, the tank can supply energy to the aforestated pumps, which then act as motors to drive the wheels when a driver wishes to accelerate. If sufficient braking does not take place to keep the air pressure up, a conventional gas engine cuts in to build up the pressure in the tank. It is expected that a doubling of mileage per gallon can take place in city driving by this system.

Suppose that the volume of air initially in the tank is 80 L and the temperature is 30°C with a pressure of 200 kPa gage. As a result of braking on going down a long hill, the volume decreases to 40 L and the air reaches a pressure of 500 kPa gage. What is the final temperature of the air if there is a loss of air due to a leak of 0.003 kg?

1.27 Use equation of state initially. $$p_1 \frac{V_1}{M} = RT_1$$

For Prob. 1.26 suppose that the initial volume of air in the tank is 80 L at a pressure of 120 kPa at $T = 20°C$. The gasoline engine cuts in to double the pressure in the tank while the volume is decreased to 50 L. What is the final temperature and density of the air?

$$(120\times10^3)\left[\frac{(80\times10^{-3})}{M}\right] = (287)(293) \qquad M = .1142\ kg$$

At final state: $$p_2\left(\frac{V_2}{M}\right) = RT_2$$

$$(2)(120)(10^3)\left(\frac{50\times10^{-3}}{.1142}\right) = (287)(T_2)$$

$$\boxed{\begin{aligned} T_2 &= 366\ K = 93.1°C \\ \rho &= \frac{.1142}{50\times10^{-3}} = 2.28\ \frac{kg}{m^3} \end{aligned}}$$

1.28 $$p\left(\frac{V}{nM}\right) = RT$$

As you may recall from chemistry, a *pound-mole* of a gas is the number of pounds-mass of the gas equal to its molecular weight M. For 2 lb-mol of air with a molecular weight of 29, a temperature of 100°F, and a pressure of 2 atm, what is the volume V? Show that $pv = RT$ can be expressed as $pV = nMRT$, where n is the number of moles.

$$(2)(14.7)(144)\left[\frac{V}{(2)(29)}\right] = (53.3)(560)$$

$$V = 408.9\ ft^3$$

From Eq. (1)

$$pV = nMRT$$

1.29

You may recall from chemistry that the gas constant R for a particular gas can be determined from a universal gas constant R_u, having a constant value for all perfect gases, and the molecular weight M of the particular gas. That is, $R = R_u/M$.

The value of R_u in USCS is $R_u = 49{,}700\ ft^2/(s^2)(°R)$.

Show that for SI units, we get, $R_u = 8310\ m^2/(s^2)(K)$. What is the gas constant R for helium in SI units?

$$R_u = 49{,}700 ft^2 \frac{1m^2}{(3.28\ ft)^2 \cdot s^2 \cdot (R)\ \dfrac{(5/9)K}{(R)}}$$

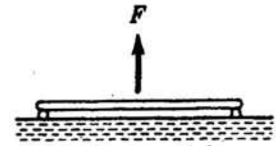
$$= 8{,}310\ \frac{m^2}{s^2 \cdot K}$$

$$R = \frac{8{,}310}{M} = \frac{8{,}310}{4} = 2{,}077\ \frac{m^2}{s^2 \cdot K}$$

1.30 There are two surfaces exerting surface forces on the wire. Hence for force F is:

A thin, circular wire is being lifted from contact with water. What force F is required for this action over and above the weight of the wire? The water forms a zero degree contact angle with the outermost and innermost peripheries of the wire for certain metals such as platinum. Compute F for a platinum wire. Explain how you could use this system to measure σ.

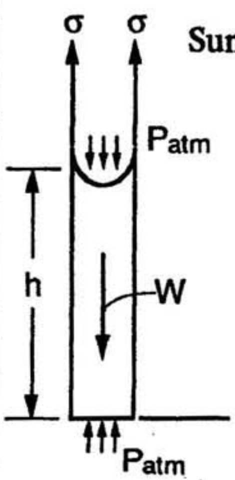
$$F = 2[(2\pi r)(\sigma)] = (2\pi)(.200)(.0730)(2) = 0.1835\ N$$



You could measure F experimentally and then compute σ from this formula.

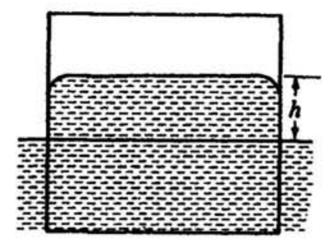
$$F = 2[2\pi r]\sigma$$

1.31 Because the plates are clean, the angle of contact between the water and the glass is taken as zero. Consider the free body diagram of a unit width of the raised water away from the ends.



Summing forces in the vertical direction we have

$$2[(\sigma)(L)] - (Ld)(h)(\gamma) = 0$$

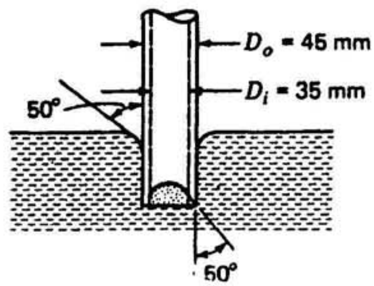


$$2(.0730) - (.001)(h)(9806) = 0$$

$$\boxed{h = 14.89\ mm}$$

Two parallel, wide, clean, glass plates separated by a distance d of 1 mm are placed in water. How far does the water rise due to capillary action away from the ends of the plates? *Hint:* See footnote 10.

1.32



$$F = \sigma(\pi D_0)\cos 50^\circ + \sigma(\pi D_i)\cos 50^\circ$$

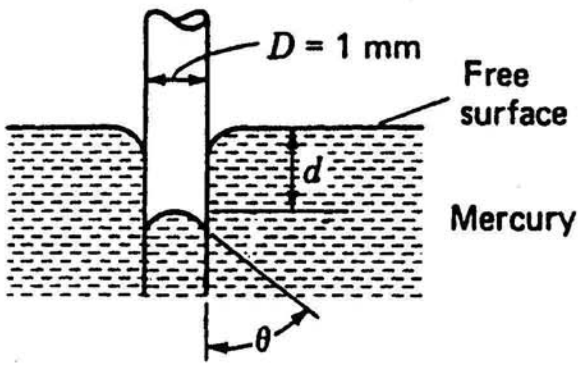
$$= (0.514)(\pi)(.045)(.643) + (0.514)(\pi)(.035)(.643)$$

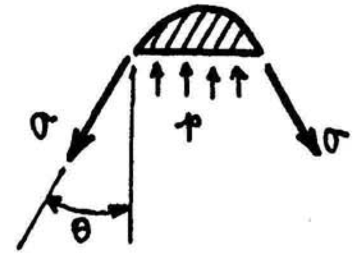
$$F = .0831\ N$$

A glass tube is inserted in mercury. What is the upward force on the glass as a result of surface effects? Note that the contact angle is 50° inside and outside. Temperature is 20°C.

1.33

Compute an approximate distance d for mercury in a glass capillary tube. The surface tension σ for mercury and air here is 0.514 N/m, and the angle θ is 40°. The specific gravity of mercury is 13.6. *Hint:* The pressure p_{gage} below the main free surface is the specific weight times the depth below the free surface. Do your assumptions render the actual d larger or smaller than the computed d?





Consider as a free body the meniscus of the mercury. Neglect the **weight** of this free body. We have for equilibrium:

$$-(\sigma)(\pi D)(\cos\theta) + p\left(\frac{\pi D^2}{4}\right) = 0$$

$$-(.514)(\pi)(.001)\cos 40^\circ + (13.6)(9806)(d)\ \frac{(\pi)(.001)^2}{4} = 0$$

$$\boxed{d = 11.81\ mm}$$

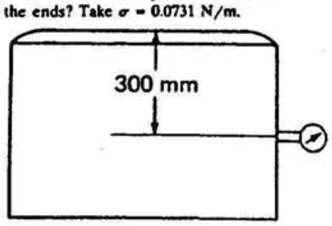
Actual d must be larger because we neglected weight at meniscus.

1.34

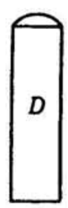
At the top of the water surface

$$\Delta p = 2{,}943.7 - (9{,}806)(.300) = 1.900\ \ Pa\ gauge$$

A narrow tank with one end open is filled with water at 45°C carefully and slowly to get the maximum amount of water in without spilling any water. If the pressure gage measures a gage pressure of 2943.7 Pa, what is the radius of curvature of the water surface at the top of the surface away from the ends? Take σ = 0.0731 N/m.



Going to Eq. (1.11), we have

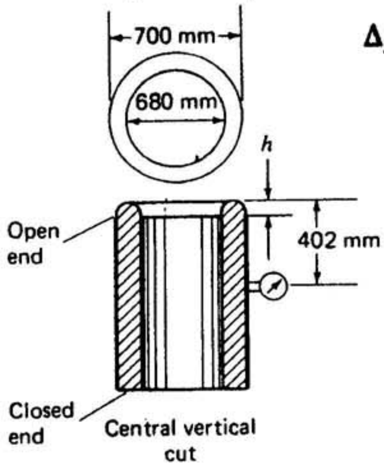


$$\Delta p = 1.900 = \sigma\left(\frac{1}{R_1} + \frac{1}{\infty}\right) = \frac{\sigma}{R_1}$$

$$\therefore\ R_1 = \frac{(.0730)}{1.900} = .0384\ m = \boxed{38.42\ \ mm}$$

Water at 10°C is poured into a region between concentric cylinders until water appears above the top of the open end. If the pressure measured by the gage is 3970.80 Pa gage, what is the curvature of the water at the top? Using the Taylor series, estimate the height *h* of the water above the edge of the cylinders. Assume that the highest point of the water is at the midradius of the cylinders.

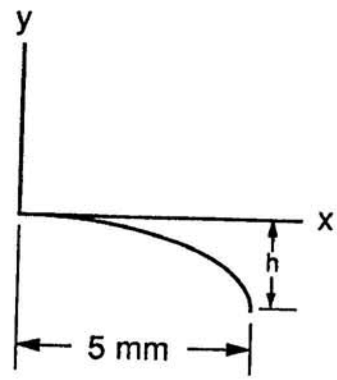
1.35 To find Δp to be used in Eq. (1.10) we have:



$$\Delta p = 3970.80 - (9803)(.402) = 30\ Pa$$

$$\therefore\ \Delta p = 30 = (.0730)\left[\left(\frac{d^2y}{dr^2}\right) + 0\right]$$

$$\therefore\ \frac{d^2y}{dr^2} = 411\ m^{-1}$$



$$h = 0 + \frac{dy}{dx}(.005) + \frac{d^2y}{dx^2}\left(\frac{.005^2}{2}\right) + \ldots$$

$$= 0 + 0 + (411)\left(\frac{(.005)^2}{2}\right) + \ldots$$

$$\boxed{h \sim 5.14\ \ mm}$$

1.36

Given the velocity field

$$\mathbf{V}(x, y, z, t) = (6xy^2 + t)\mathbf{i} + (3z + 10)\mathbf{j} + 20\mathbf{k} \quad \text{m/s}$$

with x, y, z in meters and t in seconds, what is the velocity vector at position $x = 10$ m, $y = -1$ m, and $z = 2$ m when $t = 5$ s? What is the magnitude of this velocity?

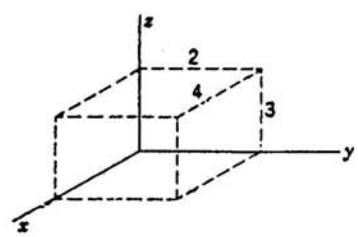
$$\vec{V} = (6xy^2+t)\hat{i} + (3z+10)\hat{j} + 20\hat{k}$$

$$= [(6)(10)(1)+5]\hat{i} + [(3)(2)+10]\hat{j} + 20\hat{k}$$

$$= 65\hat{i} + 16\hat{j} + 20\hat{k} \ m/sec$$

$$|\vec{V}| = \sqrt{65^2+16^2+20^2} = 69.86 \ m/sec$$

1.37



A body-force distribution is given as

$$\mathbf{B} = 16x\mathbf{i} + 10\mathbf{j} \quad \text{N/kg}$$

per unit mass of the material acted on. If the density of the material is given as

$$\rho = x^2 + 2z \quad \text{kg/m}^3$$

what is the resultant body force on material in the region shown in the diagram?

$$\vec{F} = \iiint_M \vec{B}\, dm = \iiint_V \vec{B}\rho\, dv$$

$$= \int_0^3\int_0^2\int_0^4 (16x\hat{i}+10\hat{j})(x^2+2z)\,dxdydz$$

$$= \int_0^3\int_0^2\int_0^4 (16x^3\hat{i}+32xz\hat{i}+10x^2\hat{j}+20z\hat{j})\,dxdydz$$

$$= \int_0^3\int_0^2 \left[\left(\frac{16x^4}{4} + \frac{32x^2z}{2}\right)\hat{i} + \left(\frac{10x^3}{3} + 20zx\right)\hat{j}\right]_0^4 dydz$$

$$= \int_0^3\int_0^2 (1024+256z)\hat{i}+(213.3+80z)\hat{j}\,dydz$$

$$= \int_0^3 [(1024)(2)+(256)(z)(2)]\hat{i}+[(213.3)(2)+(80z)(2)]\hat{j}dz$$

$$= \int_0^3 [(2048+512z)\hat{i}+(426.6+160z)\hat{j}]dz$$

$$= [(2048)(3) + \frac{512}{2}(3^2)]\hat{i}+[426.6(3) + \frac{160}{2}3^2]\hat{j}$$

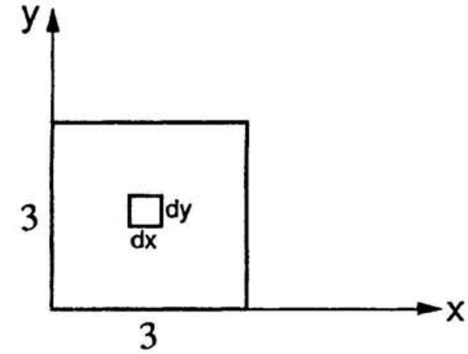
$$\boxed{\vec{F} = 8{,}448\hat{i} + 1{,}999.8\hat{j} \ \ N}$$

1.38

Oil is moving over a flat surface. We are observing this flow from above in the diagram. A traction force field T is developed on the flat surface given as

$$\mathbf{T} = (6y + 3)\mathbf{i} + (3x^2 + y)\mathbf{j} + (5 + x^2)\mathbf{k} \quad \text{lb/ft}^2$$

What is the total force on the 3×3 square of area shown in the diagram.



On flat surface we have:

$$\vec{F} = \int_0^3\int_0^3 [(6y+3)\hat{i}+(3x^2+y)\hat{j}+(5+x^2)\hat{k}]dxdy$$

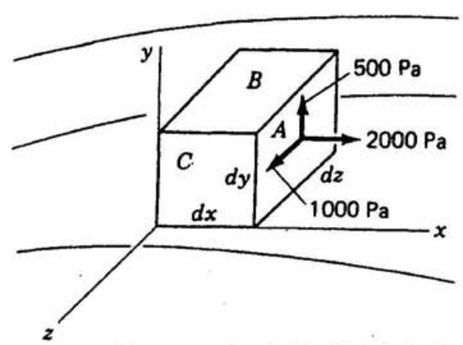
$$= \int_0^2 \{[(6y)(3)+(3)(3)]\hat{i} + \left[\frac{(3)(27)}{3} + y(3)\right]\hat{j} + \left[(5)(3) + \frac{(3^3)}{3}\right]\hat{k}\}dy$$

$$= \int_0^3 \{[18y+9]\hat{i}+[27+3y]\hat{j} + [24]\hat{k}\}dy$$

$$= \left\{\left(18\,\frac{(3)^2}{2} + (9)(3)\right)\hat{i} + \left[(27)(3) + \frac{(3)(3^2)}{2}\right]\hat{j}+[(24)(3)]\hat{k}\right\}$$

$$\boxed{\vec{F} = 108\hat{i} + 94.5\hat{j} + 72\hat{k} \quad lb}$$

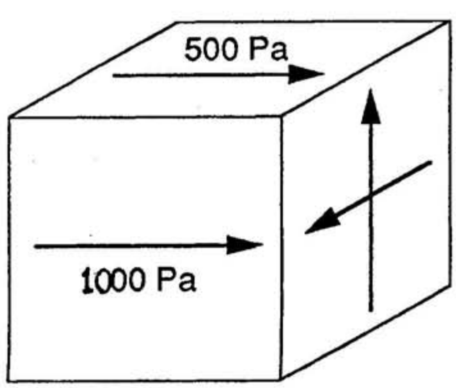
1.39



The stresses on face A of an infinitesimal rectangular parallelepiped of fluid in a flow are shown . . . . at time t. What is the traction vector for this face at the instant shown? What can you say about shear stresses on faces B and C at this instant?

a) $\vec{T} = 2{,}000\hat{i} + 500\hat{j} + 1{,}000\hat{k}$

b)

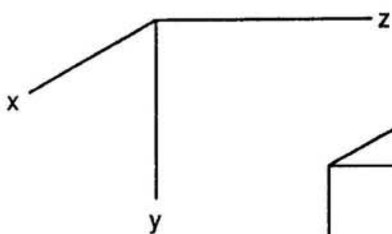


1.40

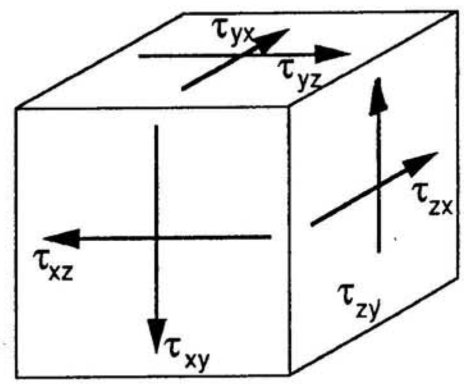
Explain why in hydrostatics the traction force exerted on an area element by the fluid is always normal to the area element of the boundary.

In a static fluid there can be no shear stress and so there is only normal stress on the area element. This can only result in a force normal to the area element.

1.41



Label · ` Fig. using the convention set forth i



1.42

We are given the following stress field in megapascals:

$$\tau_{xx} = 16x + 10 \qquad \tau_{zz} = \tau_{xz} = \tau_{yz} = 0$$
$$\tau_{yy} = 10y^2 + 6xy$$
$$\tau_{xy} = -5x^2$$

Express the bulk stress distribution as a scalar field. What is the bulk stress at (0, 10, 2) m?

$$\bar{\sigma} = \frac{1}{3}(\tau_{xx} + \tau_{yy} + \tau_{zz})$$

$$= \frac{1}{3}(16x + 10 + 10y^2 + 6xy + 0)$$

$$= (5.33x + 3.33y^2 + 2xy + 3.33)\ MPa$$

At (0,10,2) we have:

$$\bar{\sigma} = 333 + 3.33 = \boxed{336\ MPa}$$

1.43

In a viscous flow, the stress tensor at a point is

$$\tau_{ij} = \begin{bmatrix} -4000 & 3000 & 1000 \\ 3000 & 2000 & -1000 \\ 1000 & -1000 & -5000 \end{bmatrix} \text{ lb/in}^2$$

What is the thermodynamic pressure at this point?

$$\overline{\sigma} = \frac{1}{3}(-4,000 + 2,000 = 5,000)$$

$$= -2,333 \; psi$$

$$\boxed{p = 2,333 \quad psi}$$

1.44

A vector field may be formed by taking the gradient of a scalar field. If $\phi = xy + 16t^2 + yz^3$, what is the field **grad** ϕ? What is the magnitude of the vector **grad** ϕ at position (0, 3, 2) when $t = 0$?

$$\phi = xy + 16t^2 + yz^3$$

$$\nabla\phi = y\hat{i} + (x+z^3)\hat{j} + 3yz^2\hat{k}$$

At (0,3,2) and t=0

$$\nabla\phi = 3\hat{i} + 8\hat{j} + 36\hat{k}$$

$$|\nabla\phi| = \sqrt{3^2+8^2+36^2} = 37.0$$

1.45

$$p = xy + (x+z^2) + 10$$

$$\nabla p = (y+1)\hat{i} + x\hat{j} + 2z\hat{k} \quad kN/m^3$$

$$\vec{f} = -\nabla p = -(y+1)\hat{i} - x\hat{j} - 2z\hat{k} \quad kN/m^3$$

At (10,3,4)

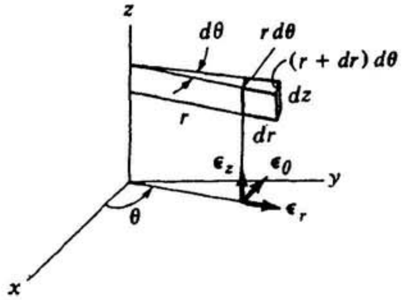
$$\vec{f} = -4\hat{i} - 10\hat{j} - 8\hat{k} \quad kN/m^3$$

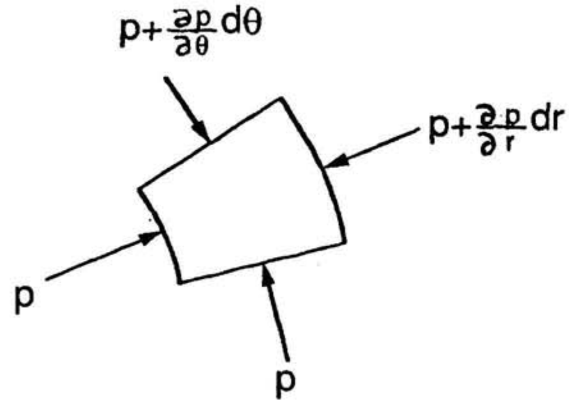
$$\hat{e} = .95\hat{i} + .32\hat{j} \; m$$

$$\therefore \; \vec{f}\cdot\hat{e} + (-4)(.95) - (10)(.32) + 0$$

$$\boxed{= -7 \quad kN/m^3}$$

1.46





$$dF_r = \left[prd\theta - \left(p + \frac{\partial p}{\partial r}dr\right)(r+dr)d\theta\right]dz$$

$$+ \,p\,drdz\,\sin\frac{d\theta}{2} + \left(p + \frac{\partial p}{\partial \theta}d\theta\right)drdz\,\sin\frac{d\theta}{2}$$

$$= -p\,drd\theta dz - \frac{\partial p}{\partial r}dr\,r\,d\theta dz - \frac{\partial p}{\partial r}drdrd\theta dz$$

$$+ \,pdrdzd\theta + \frac{\partial p}{\partial \theta}d\theta drdz\frac{d\theta}{2}$$

Drop second-order terms and cancel where possible.

$$dF_R = -\frac{\partial p}{\partial r}rd\theta drdz = -\frac{\partial p}{\partial r}d\upsilon$$

$$dF_\theta = p\,drdz\,\cos\frac{d\theta}{2} - (p + \frac{\partial p}{\partial \theta}d\theta)\left(drdz\,\cos\frac{d\theta}{2}\right)$$

$$= -\frac{\partial p}{\partial \theta}d\theta drdz = -\frac{\partial p}{r\partial \theta}d\upsilon$$

$$dF_z = prd\theta dr - \left(p + \frac{\partial p}{\partial z}dz\right)rd\theta dr$$

$$= -\frac{\partial p}{\partial z}rd\theta drdz$$

$$\therefore\ d\vec{F} = -\frac{\partial p}{\partial r}d\upsilon\ \hat{e}_r - \frac{\partial p}{r\partial \theta}d\upsilon\ \hat{e}_\theta - \frac{\partial p}{\partial z}d\upsilon\ \hat{e}_z$$

$$\frac{d\vec{F}}{d\upsilon} = -\nabla p = -\left(\frac{\partial p}{\partial r}\hat{e}_r + \frac{\partial p}{r\partial \theta}\hat{e}_\theta + \frac{\partial p}{\partial z}\hat{e}_z\right) \qquad \therefore\ \nabla = \frac{\partial}{\partial r}\hat{e}_r + \frac{\partial}{r\partial \theta}\hat{e}_\theta + \frac{\partial}{\partial z}\hat{e}_z$$