

**2-1.**

An air-filled rubber ball has a diameter of 6 in. If the air pressure within the ball is increased until the diameter becomes 7 in., determine the average normal strain in the rubber.

**SOLUTION**

$$d_0 = 6 \text{ in.}$$

$$d = 7 \text{ in.}$$

$$\epsilon = \frac{\pi d - \pi d_0}{\pi d_0} = \frac{7 - 6}{6} = 0.167 \text{ in./in.}$$

**Ans.**

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**Ans:**  
 $\epsilon = 0.167 \text{ in./in.}$

**2-2.**

A thin strip of rubber has an unstretched length of 15 in. If it is stretched around a pipe having an outer diameter of 5 in., determine the average normal strain in the strip.

**SOLUTION**

$$L_0 = 15 \text{ in.}$$

$$L = \pi(5 \text{ in.})$$

$$\epsilon = \frac{L - L_0}{L_0} = \frac{5\pi - 15}{15} = 0.0472 \text{ in./in.}$$

**Ans.**

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**Ans:**  
 $\epsilon = 0.0472 \text{ in./in.}$

2-3.

If the load **P** on the beam causes the end **C** to be displaced 10 mm downward, determine the normal strain in wires **CE** and **BD**.

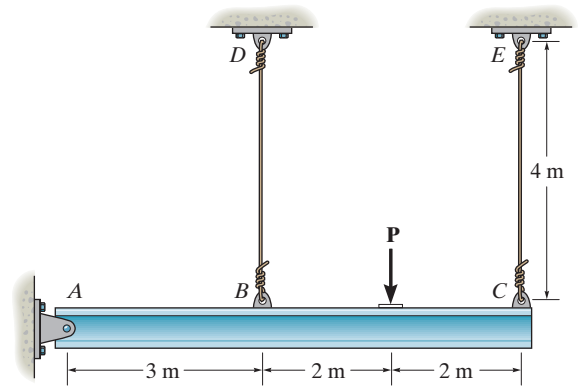
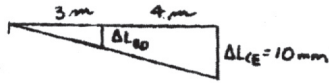
### SOLUTION

$$\frac{\Delta L_{BD}}{3} = \frac{\Delta L_{CE}}{7}$$

$$\Delta L_{BD} = \frac{3(10)}{7} = 4.286 \text{ mm}$$

$$\epsilon_{CE} = \frac{\Delta L_{CE}}{L} = \frac{10}{4000} = 0.00250 \text{ mm/mm}$$

$$\epsilon_{BD} = \frac{\Delta L_{BD}}{L} = \frac{4.286}{4000} = 0.00107 \text{ mm/mm}$$



**Ans.**

**Ans.**

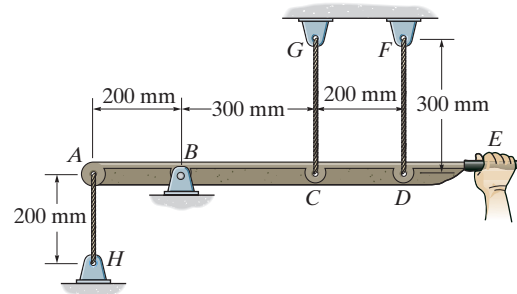
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**Ans:**

$$\epsilon_{CE} = 0.00250 \text{ mm/mm}, \epsilon_{BD} = 0.00107 \text{ mm/mm}$$

**\*2-4.**

The force applied at the handle of the rigid lever causes the lever to rotate clockwise about the pin  $B$  through an angle of  $2^\circ$ . Determine the average normal strain in each wire. The wires are unstretched when the lever is in the horizontal position.



**SOLUTION**

**Geometry:** The lever arm rotates through an angle of  $\theta = \left(\frac{2^\circ}{180}\right)\pi \text{ rad} = 0.03491 \text{ rad}$ .

Since  $\theta$  is small, the displacements of points  $A$ ,  $C$ , and  $D$  can be approximated by

$$\delta_A = 200(0.03491) = 6.9813 \text{ mm}$$

$$\delta_C = 300(0.03491) = 10.4720 \text{ mm}$$

$$\delta_D = 500(0.03491) = 17.4533 \text{ mm}$$

**Average Normal Strain:** The unstretched length of wires  $AH$ ,  $CG$ , and  $DF$  are

$L_{AH} = 200 \text{ mm}$ ,  $L_{CG} = 300 \text{ mm}$ , and  $L_{DF} = 300 \text{ mm}$ . We obtain

$$(\epsilon_{\text{avg}})_{AH} = \frac{\delta_A}{L_{AH}} = \frac{6.9813}{200} = 0.0349 \text{ mm/mm}$$

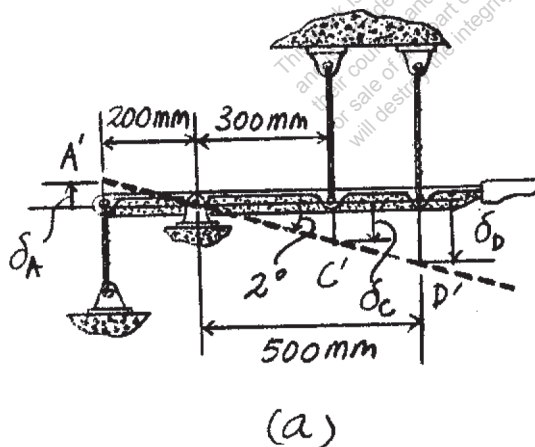
**Ans.**

$$(\epsilon_{\text{avg}})_{CG} = \frac{\delta_C}{L_{CG}} = \frac{10.4720}{300} = 0.0349 \text{ mm/mm}$$

**Ans.**

$$(\epsilon_{\text{avg}})_{DF} = \frac{\delta_D}{L_{DF}} = \frac{17.4533}{300} = 0.0582 \text{ mm/mm}$$

**Ans.**



**Ans:**

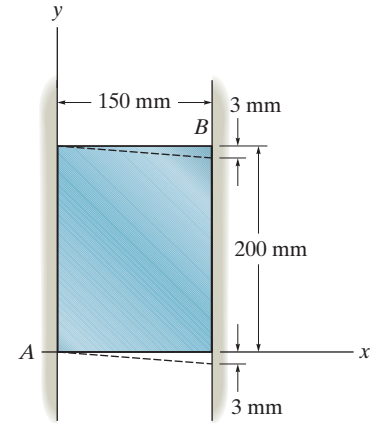
$$(\epsilon_{\text{avg}})_{AH} = 0.0349 \text{ mm/mm}$$

$$(\epsilon_{\text{avg}})_{CG} = 0.0349 \text{ mm/mm}$$

$$(\epsilon_{\text{avg}})_{DF} = 0.0582 \text{ mm/mm}$$

2-5.

The rectangular plate is subjected to the deformation shown by the dashed line. Determine the average shear strain  $\gamma_{xy}$  in the plate.



## SOLUTION

**Geometry:**

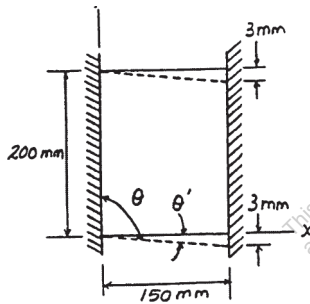
$$\theta' = \tan^{-1} \frac{3}{150} = 0.0200 \text{ rad}$$

$$\theta = \left( \frac{\pi}{2} + 0.0200 \right) \text{ rad}$$

**Shear Strain:**

$$\begin{aligned} \gamma_{xy} &= \frac{\pi}{2} - \theta = \frac{\pi}{2} - \left( \frac{\pi}{2} + 0.0200 \right) \\ &= -0.0200 \text{ rad} \end{aligned}$$

**Ans.**



**Ans:**

$$\gamma_{xy} = -0.0200 \text{ rad}$$

## 2-6.

The square deforms into the position shown by the dashed lines. Determine the shear strain at each of its corners,  $A$ ,  $B$ ,  $C$ , and  $D$ , relative to the  $x$ ,  $y$  axes. Side  $D'B'$  remains horizontal.

### SOLUTION

#### Geometry:

$$B'C' = \sqrt{(8 + 3)^2 + (53 \sin 88.5^\circ)^2} = 54.1117 \text{ mm}$$

$$\begin{aligned} C'D' &= \sqrt{53^2 + 58^2 - 2(53)(58) \cos 91.5^\circ} \\ &= 79.5860 \text{ mm} \end{aligned}$$

$$B'D' = 50 + 53 \sin 1.5^\circ - 3 = 48.3874 \text{ mm}$$

$$\begin{aligned} \cos \theta &= \frac{(B'D')^2 + (B'C')^2 - (C'D')^2}{2(B'D')(B'C')} \\ &= \frac{48.3874^2 + 54.1117^2 - 79.5860^2}{2(48.3874)(54.1117)} = -0.20328 \end{aligned}$$

$$\theta = 101.73^\circ$$

$$\beta = 180^\circ - \theta = 78.27^\circ$$

#### Shear Strain:

$$(\gamma_A)_{xy} = \frac{\pi}{2} - \pi \left( \frac{91.5^\circ}{180^\circ} \right) = -0.0262 \text{ rad}$$

Ans.

$$(\gamma_B)_{xy} = \frac{\pi}{2} - \theta = \frac{\pi}{2} - \pi \left( \frac{101.73^\circ}{180^\circ} \right) = -0.205 \text{ rad}$$

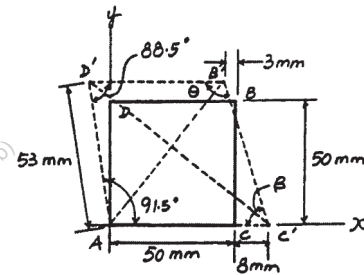
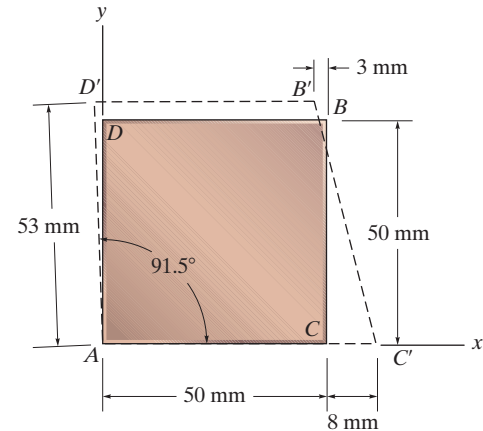
Ans.

$$(\gamma_C)_{xy} = \beta - \frac{\pi}{2} = \pi \left( \frac{78.27^\circ}{180^\circ} \right) - \frac{\pi}{2} = -0.205 \text{ rad}$$

Ans.

$$(\gamma_D)_{xy} = \pi \left( \frac{88.5^\circ}{180^\circ} \right) - \frac{\pi}{2} = -0.0262 \text{ rad}$$

Ans.

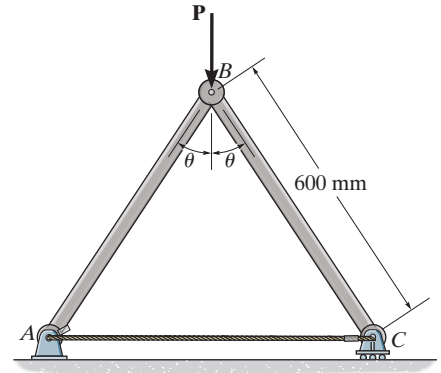


#### Ans:

$$\begin{aligned} (\gamma_A)_{xy} &= -0.0262 \text{ rad} \\ (\gamma_B)_{xy} &= -0.205 \text{ rad} \\ (\gamma_C)_{xy} &= -0.205 \text{ rad} \\ (\gamma_D)_{xy} &= -0.0262 \text{ rad} \end{aligned}$$

2-7.

The pin-connected rigid rods  $AB$  and  $BC$  are inclined at  $\theta = 30^\circ$  when they are unloaded. When the force  $\mathbf{P}$  is applied  $\theta$  becomes  $30.2^\circ$ . Determine the average normal strain in wire  $AC$ .



## SOLUTION

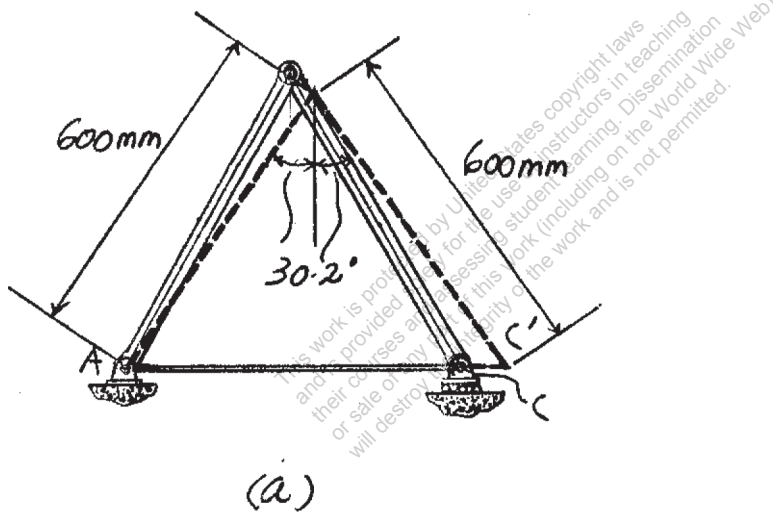
**Geometry:** Referring to Fig. *a*, the unstretched and stretched lengths of wire  $AD$  are

$$L_{AC} = 2(600 \sin 30^\circ) = 600 \text{ mm}$$

$$L_{AC'} = 2(600 \sin 30.2^\circ) = 603.6239 \text{ mm}$$

**Average Normal Strain:**

$$(\epsilon_{\text{avg}})_{AC} = \frac{L_{AC'} - L_{AC}}{L_{AC}} = \frac{603.6239 - 600}{600} = 6.04(10^{-3}) \text{ mm/mm} \quad \text{Ans.}$$



**Ans:**

$$(\epsilon_{\text{avg}})_{AC} = 6.04(10^{-3}) \text{ mm/mm}$$

**\*2-8.**

The wire  $AB$  is unstretched when  $\theta = 45^\circ$ . If a load is applied to the bar  $AC$ , which causes  $\theta$  to become  $47^\circ$ , determine the normal strain in the wire.

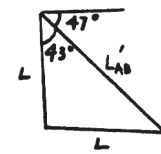
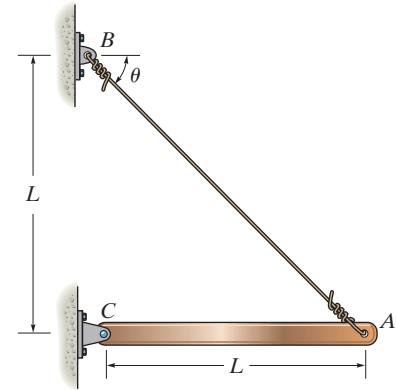
### SOLUTION

$$L^2 = L^2 + L_{AB}'^2 - 2LL_{AB}' \cos 43^\circ$$

$$L_{AB}' = 2L \cos 43^\circ$$

$$\begin{aligned} \epsilon_{AB} &= \frac{L_{AB}' - L_{AB}}{L_{AB}} \\ &= \frac{2L \cos 43^\circ - \sqrt{2}L}{\sqrt{2}L} \\ &= 0.0343 \end{aligned}$$

**Ans.**



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**Ans:**  
 $\epsilon_{AB} = 0.0343$

2-9.

If a horizontal load applied to the bar  $AC$  causes point  $A$  to be displaced to the right by an amount  $\Delta L$ , determine the normal strain in the wire  $AB$ . Originally,  $\theta = 45^\circ$ .

### SOLUTION

$$L'_{AB} = \sqrt{(\sqrt{2}L)^2 + \Delta L^2 - 2(\sqrt{2}L)(\Delta L) \cos 135^\circ}$$

$$= \sqrt{2L^2 + \Delta L^2 + 2L\Delta L}$$

$$\epsilon_{AB} = \frac{L'_{AB} - L_{AB}}{L_{AB}}$$

$$= \frac{\sqrt{2L^2 + \Delta L^2 + 2L\Delta L} - \sqrt{2}L}{\sqrt{2}L}$$

$$= \sqrt{1 + \frac{\Delta L^2}{2L^2} + \frac{\Delta L}{L}} - 1$$

Neglecting the higher-order terms,

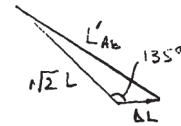
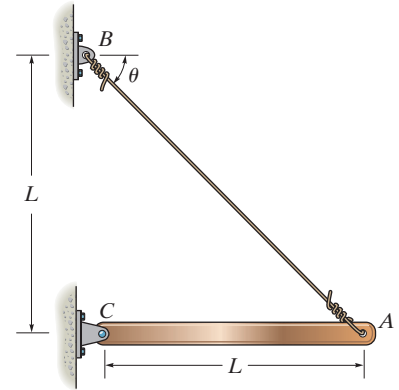
$$\epsilon_{AB} = \left(1 + \frac{\Delta L}{L}\right)^{\frac{1}{2}} - 1$$

$$= 1 + \frac{1}{2} \frac{\Delta L}{L} + \dots - 1$$

$$= \frac{0.5\Delta L}{L}$$

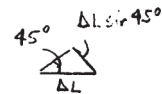
Also,

$$\epsilon_{AB} = \frac{\Delta L \sin 45^\circ}{\sqrt{2}L} = \frac{0.5 \Delta L}{L}$$



Ans.

Ans.

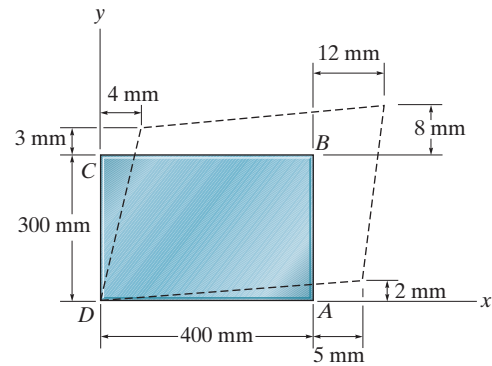


Ans:

$$\epsilon_{AB} = \frac{0.5\Delta L}{L}$$

## 2-10.

Determine the shear strain  $\gamma_{xy}$  at corners  $A$  and  $B$  if the plastic distorts as shown by the dashed lines.



## SOLUTION

**Geometry:** Referring to the geometry shown in Fig.  $a$ , the small-angle analysis gives

$$\alpha = \psi = \frac{7}{306} = 0.022876 \text{ rad}$$

$$\beta = \frac{5}{408} = 0.012255 \text{ rad}$$

$$\theta = \frac{2}{405} = 0.0049383 \text{ rad}$$

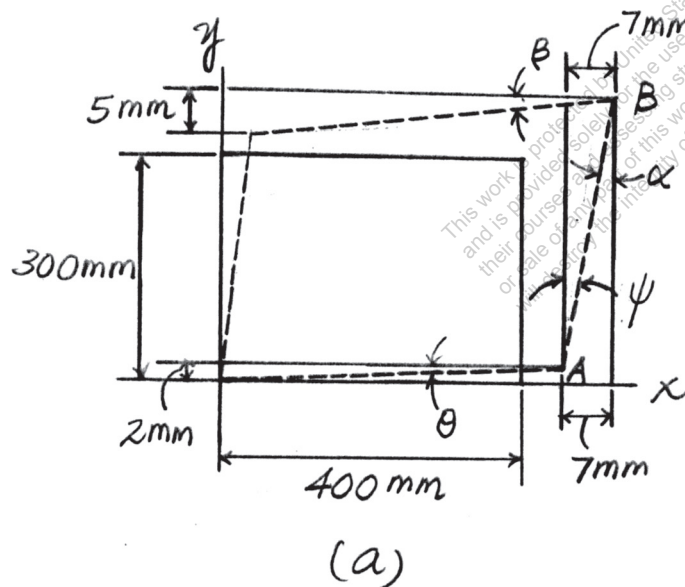
**Shear Strain:** By definition,

$$(\gamma_A)_{xy} = \theta + \psi = 0.02781 \text{ rad} = 27.8(10^{-3}) \text{ rad}$$

**Ans.**

$$(\gamma_B)_{xy} = \alpha + \beta = 0.03513 \text{ rad} = 35.1(10^{-3}) \text{ rad}$$

**Ans.**



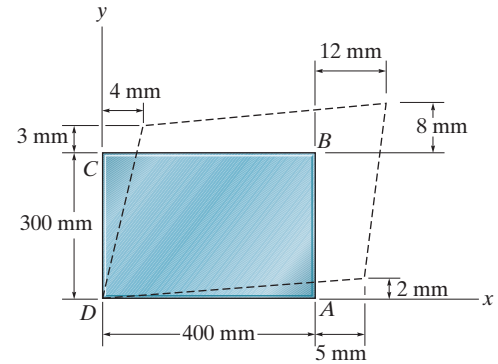
**Ans:**

$$(\gamma_A)_{xy} = 27.8(10^{-3}) \text{ rad}$$

$$(\gamma_B)_{xy} = 35.1(10^{-3}) \text{ rad}$$

## 2-11.

Determine the shear strain  $\gamma_{xy}$  at corners  $D$  and  $C$  if the plastic distorts as shown by the dashed lines.



### SOLUTION

**Geometry:** Referring to the geometry shown in Fig.  $a$ , the small-angle analysis gives

$$\alpha = \psi = \frac{4}{303} = 0.013201 \text{ rad}$$

$$\theta = \frac{2}{405} = 0.0049383 \text{ rad}$$

$$\beta = \frac{5}{408} = 0.012255 \text{ rad}$$

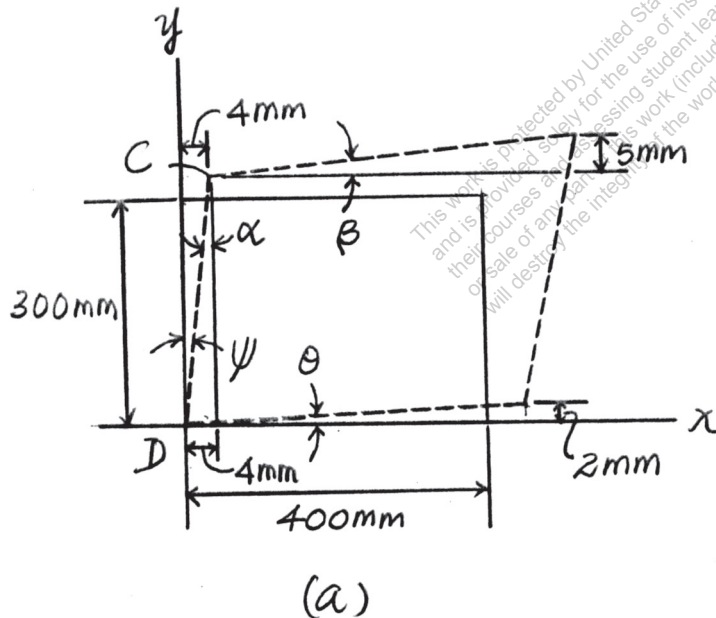
**Shear Strain:** By definition,

$$(\gamma_{xy})_C = \alpha + \beta = 0.02546 \text{ rad} = 25.5(10^{-3}) \text{ rad}$$

**Ans.**

$$(\gamma_{xy})_D = \theta + \psi = 0.01814 \text{ rad} = 18.1(10^{-3}) \text{ rad}$$

**Ans.**



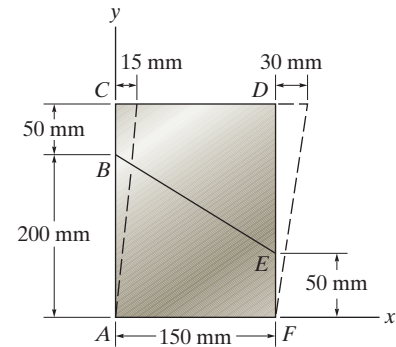
**Ans:**

$$(\gamma_{xy})_C = 25.5(10^{-3}) \text{ rad}$$

$$(\gamma_{xy})_D = 18.1(10^{-3}) \text{ rad}$$

**\*2-12.**

The material distorts into the dashed position shown. Determine the average normal strains  $\epsilon_x$ ,  $\epsilon_y$  and the shear strain  $\gamma_{xy}$  at A, and the average normal strain along line BE.



**SOLUTION**

**Geometry:** Referring to the geometry shown in Fig. a,

$$\tan \theta = \frac{15}{250}; \quad \theta = (3.4336^\circ) \left( \frac{\pi}{180^\circ} \text{ rad} \right) = 0.05993 \text{ rad}$$

$$L_{AC'} = \sqrt{15^2 + 150^2} = \sqrt{62725} \text{ mm}$$

$$\frac{BB'}{15} = \frac{200}{250}; \quad BB' = 12 \text{ mm} \quad \frac{EE'}{30} = \frac{50}{250}; \quad EE' = 6 \text{ mm}$$

$$x' = 150 + EE' - BB' = 150 + 6 - 12 = 144 \text{ mm}$$

$$L_{BE} = \sqrt{150^2 + 150^2} = 150\sqrt{2} \text{ mm} \quad L_{B'E'} = \sqrt{144^2 + 150^2} = \sqrt{43236} \text{ mm}$$

**Average Normal and Shear Strain:** Since no deformation occurs along x axis,

$$(\epsilon_x)_A = 0$$

**Ans.**

$$(\epsilon_y)_A = \frac{L_{AC'} - L_{AC}}{L_{AC}} = \frac{\sqrt{62725} - 250}{250} = 1.80(10^{-3}) \text{ mm/mm}$$

**Ans.**

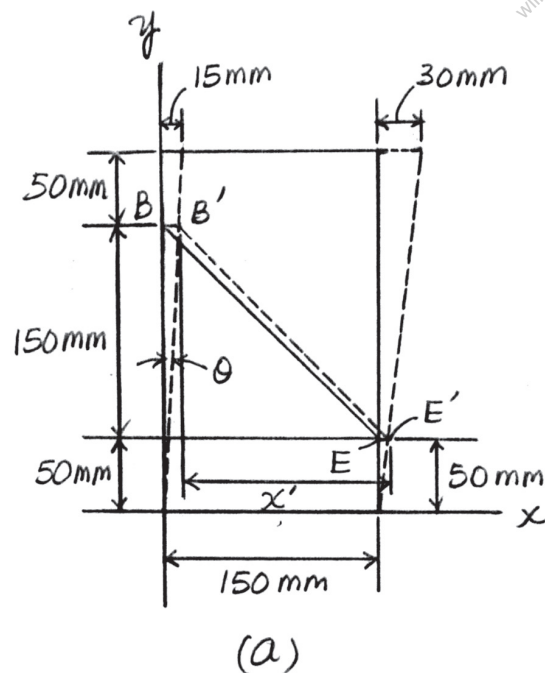
By definition,

$$(\gamma_{xy})_A = \theta = 0.0599 \text{ rad}$$

**Ans.**

$$\epsilon_{BE} = \frac{L_{B'E'} - L_{BE}}{L_{BE}} = \frac{\sqrt{43236} - 150\sqrt{2}}{150\sqrt{2}} = -0.0198 \text{ mm/mm}$$

**Ans.**



**Ans:**

$$(\epsilon_x)_A = 0$$

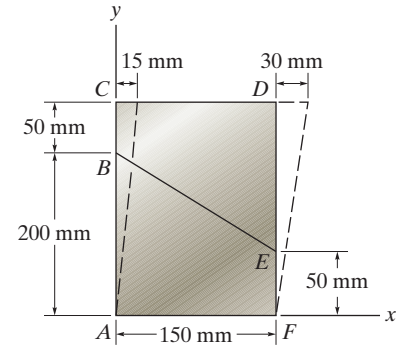
$$(\epsilon_y)_A = 1.80(10^{-3}) \text{ mm/mm}$$

$$(\gamma_{xy})_A = 0.0599 \text{ rad}$$

$$\epsilon_{BE} = -0.0198 \text{ mm/mm}$$

2-13.

The material distorts into the dashed position shown. Determine the average normal strains along the diagonals  $AD$  and  $CF$ .



SOLUTION

**Geometry:** Referring to the geometry shown in Fig.  $a$ ,

$$L_{AD} = L_{CF} = \sqrt{150^2 + 250^2} = \sqrt{85000} \text{ mm}$$

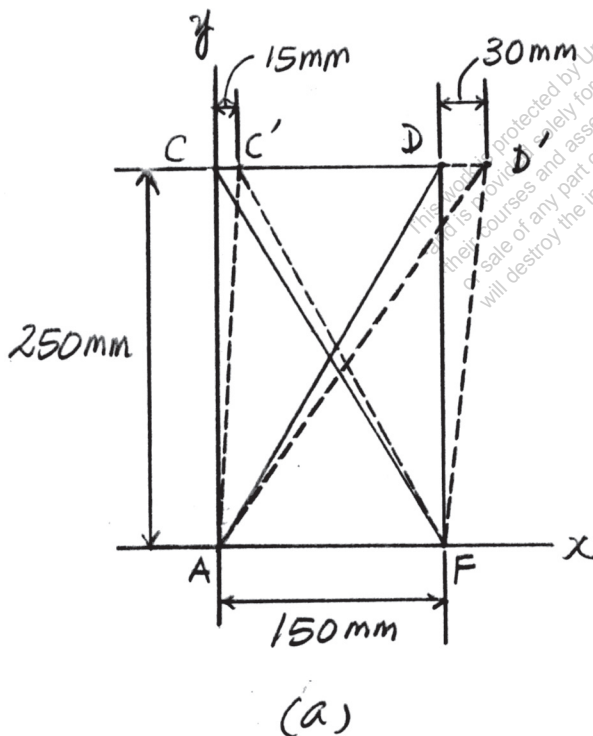
$$L_{AD'} = \sqrt{(150 + 30)^2 + 250^2} = \sqrt{94900} \text{ mm}$$

$$L_{C'F} = \sqrt{(150 - 15)^2 + 250^2} = \sqrt{80725} \text{ mm}$$

**Average Normal Strain:**

$$\epsilon_{AD} = \frac{L_{AD'} - L_{AD}}{L_{AD}} = \frac{\sqrt{94900} - \sqrt{85000}}{\sqrt{85000}} = 0.0566 \text{ mm/mm} \quad \text{Ans.}$$

$$\epsilon_{CF} = \frac{L_{C'F} - L_{CF}}{L_{CF}} = \frac{\sqrt{80725} - \sqrt{85000}}{\sqrt{85000}} = -0.0255 \text{ mm/mm} \quad \text{Ans.}$$



**Ans:**

$$\epsilon_{AD} = 0.0566 \text{ mm/mm}$$

$$\epsilon_{CF} = -0.0255 \text{ mm/mm}$$

## 2-14.

Part of a control linkage for an airplane consists of a rigid member  $CB$  and a flexible cable  $AB$ . If a force is applied to the end  $B$  of the member and causes it to rotate by  $\theta = 0.5^\circ$ , determine the normal strain in the cable. Originally the cable is unstretched.

### SOLUTION

**Geometry:** Referring to the geometry shown in Fig.  $a$ , the unstretched and stretched lengths of cable  $AB$  are

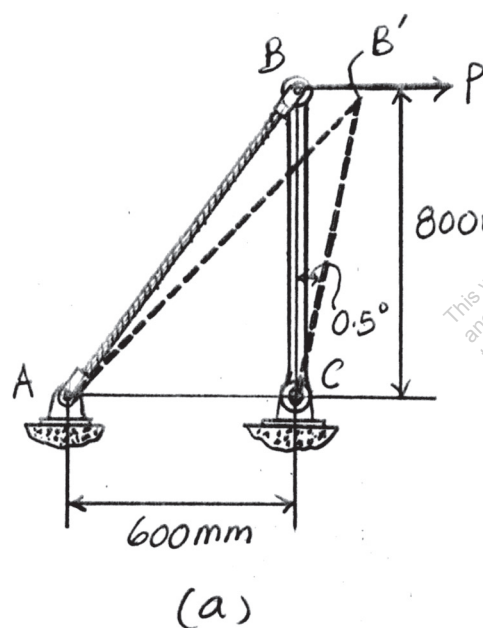
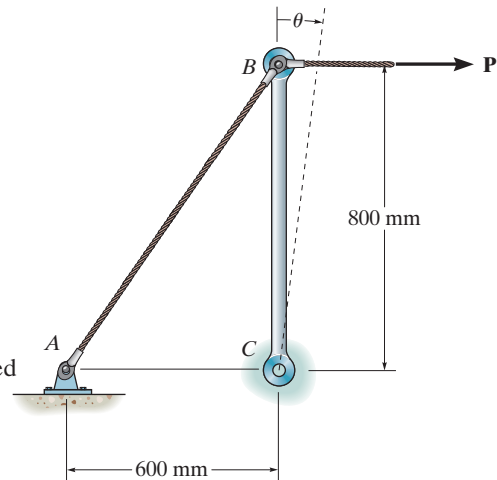
$$L_{AB} = \sqrt{600^2 + 800^2} = 1000 \text{ mm}$$

$$L_{AB'} = \sqrt{600^2 + 800^2 - 2(600)(800) \cos 90.5^\circ} = 1004.18 \text{ mm}$$

**Average Normal Strain:**

$$\epsilon_{AB} = \frac{L_{AB'} - L_{AB}}{L_{AB}} = \frac{1004.18 - 1000}{1000} = 0.00418 \text{ mm/mm}$$

**Ans.**



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**Ans:**

$$\epsilon_{AB} = 0.00418 \text{ mm/mm}$$

## 2-15.

Part of a control linkage for an airplane consists of a rigid member  $CB$  and a flexible cable  $AB$ . If a force is applied to the end  $B$  of the member and causes a normal strain in the cable of  $0.004 \text{ mm/mm}$ , determine the displacement of point  $B$ . Originally the cable is unstretched.

### SOLUTION

**Geometry:** Referring to the geometry shown in Fig.  $a$ , the unstretched and stretched lengths of cable  $AB$  are

$$L_{AB} = \sqrt{600^2 + 800^2} = 1000 \text{ mm}$$

$$L_{AB'} = \sqrt{600^2 + 800^2 - 2(600)(800) \cos(90^\circ + \theta)}$$

$$L_{AB'} = \sqrt{1(10^6) - 0.960(10^6) \cos(90^\circ + \theta)}$$

**Average Normal Strain:**

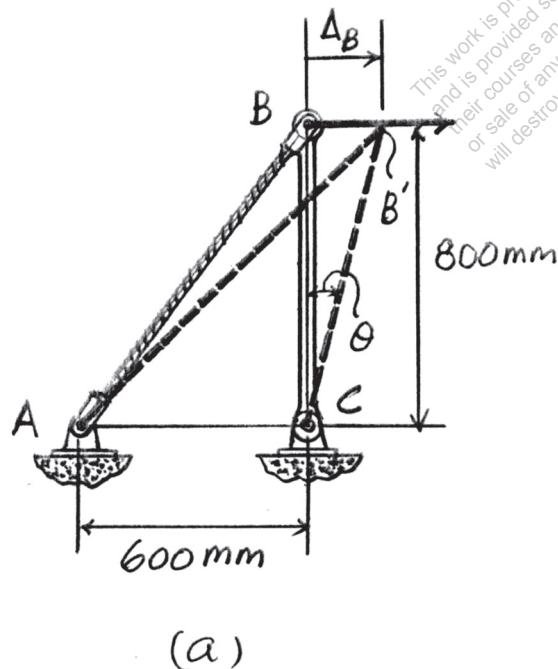
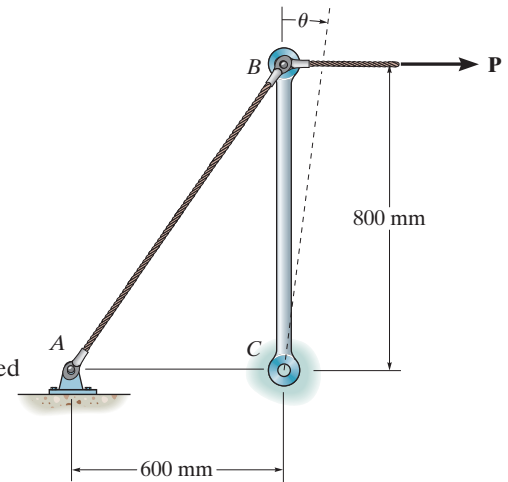
$$\epsilon_{AB} = \frac{L_{AB'} - L_{AB}}{L_{AB}}; \quad 0.004 = \frac{\sqrt{1(10^6) - 0.960(10^6) \cos(90^\circ + \theta)} - 1000}{1000}$$

$$\theta = 0.4784^\circ \left( \frac{\pi}{180^\circ} \right) = 0.008350 \text{ rad}$$

Thus,

$$\Delta_B = \theta L_{BC} = 0.008350(800) = 6.68 \text{ mm}$$

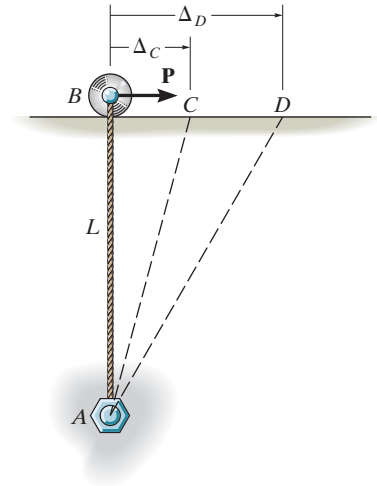
**Ans.**



**Ans:**  
 $\Delta_B = 6.68 \text{ mm}$

**\*2-16.**

The nylon cord has an original length  $L$  and is tied to a bolt at  $A$  and a roller at  $B$ . If a force  $\mathbf{P}$  is applied to the roller, determine the normal strain in the cord when the roller is at  $C$ , and at  $D$ . If the cord is originally unstrained when it is at  $C$ , determine the normal strain  $\epsilon'_D$  when the roller moves to  $D$ . Show that if the displacements  $\Delta_C$  and  $\Delta_D$  are small, then  $\epsilon'_D = \epsilon_D - \epsilon_C$ .



**SOLUTION**

$$L_C = \sqrt{L^2 + \Delta_C^2}$$

$$\begin{aligned}\epsilon_C &= \frac{\sqrt{L^2 + \Delta_C^2} - L}{L} \\ &= \frac{L\sqrt{1 + \left(\frac{\Delta_C^2}{L^2}\right)} - L}{L} = \sqrt{1 + \left(\frac{\Delta_C^2}{L^2}\right)} - 1\end{aligned}$$

For small  $\Delta_C$ ,

$$\epsilon_C = 1 + \frac{1}{2}\left(\frac{\Delta_C^2}{L^2}\right) - 1 = \frac{1}{2}\frac{\Delta_C^2}{L^2}$$

In the same manner,

$$\epsilon_D = \frac{1}{2}\frac{\Delta_D^2}{L^2}$$

$$\epsilon'_D = \frac{\sqrt{L^2 + \Delta_D^2} - \sqrt{L^2 + \Delta_C^2}}{\sqrt{L^2 + \Delta_C^2}} = \frac{\sqrt{1 + \frac{\Delta_D^2}{L^2}} - \sqrt{1 + \frac{\Delta_C^2}{L^2}}}{\sqrt{1 + \frac{\Delta_C^2}{L^2}}}$$

For small  $\Delta_C$  and  $\Delta_D$ ,

$$\epsilon'_D = \frac{\left(1 + \frac{1}{2}\frac{\Delta_D^2}{L^2}\right) - \left(1 + \frac{1}{2}\frac{\Delta_C^2}{L^2}\right)}{\left(1 + \frac{1}{2}\frac{\Delta_C^2}{L^2}\right)} = \frac{\frac{1}{2L^2}(\Delta_D^2 - \Delta_C^2)}{\frac{1}{2L^2}(2L^2 + \Delta_C^2)}$$

$$\epsilon'_D = \frac{\Delta_D^2 - \Delta_C^2}{2L^2 - \Delta_C^2} = \frac{1}{2L^2}(\Delta_D^2 - \Delta_C^2) = \epsilon_D - \epsilon_C$$

Also this problem can be solved as follows:

$$A_C = L \sec \theta_C; \quad A_D = L \sec \theta_D$$

$$\epsilon_C = \frac{L \sec \theta_C - L}{L} = \sec \theta_C - 1$$

$$\epsilon_D = \frac{L \sec \theta_D - L}{L} = \sec \theta_D - 1$$

Expanding  $\sec \theta$

$$\sec \theta = 1 + \frac{\theta^2}{2!} + \frac{5\theta^4}{4!} + \dots$$

**Ans.**

**Ans.**

**QED**

**\*2-16. Continued**

For small  $\theta$  neglect the higher order terms

$$\sec \theta = 1 + \frac{\theta^2}{2}$$

Hence,

$$\epsilon_C = 1 + \frac{\theta_C^2}{2} - 1 = \frac{\theta_C^2}{2}$$

$$\epsilon_D = 1 + \frac{\theta_D^2}{2} - 1 = \frac{\theta_D^2}{2}$$

$$\epsilon_D' = \frac{L \sec \theta_D - L \sec \theta_C}{L \sec \theta_C} = \frac{\sec \theta_D}{\sec \theta_C} - 1 = \sec \theta_D \cos \theta_C - 1$$

$$\text{Since } \cos \theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} \dots$$

$$\begin{aligned} \sec \theta_D \cos \theta_C &= \left(1 + \frac{\theta_D^2}{2} \dots\right) \left(1 - \frac{\theta_C^2}{2} \dots\right) \\ &= 1 - \frac{\theta_C^2}{2} + \frac{\theta_D^2}{2} - \frac{\theta_C^2 \theta_D^2}{4} \end{aligned}$$

Neglecting the higher order terms

$$\sec \theta_D \cos \theta_C = 1 + \frac{\theta_D^2}{2} - \frac{\theta_C^2}{2}$$

$$\epsilon_D' = \left[1 + \frac{\theta_D^2}{2} - \frac{\theta_C^2}{2}\right] - 1 = \frac{\theta_D^2}{2} - \frac{\theta_C^2}{2}$$

$$= \epsilon_D - \epsilon_C$$

**QED**

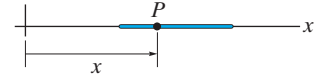
**Ans:**

$$\epsilon_C = \frac{1}{2} \frac{\Delta_C^2}{L^2}$$

$$\epsilon_D = \frac{1}{2} \frac{\Delta_D^2}{L^2}$$

**2-17.**

A thin wire, lying along the  $x$  axis, is strained such that each point on the wire is displaced  $\Delta x = kx^2$  along the  $x$  axis. If  $k$  is constant, what is the normal strain at any point  $P$  along the wire?



**SOLUTION**

$$\epsilon = \frac{d(\Delta x)}{dx} = 2 k x$$

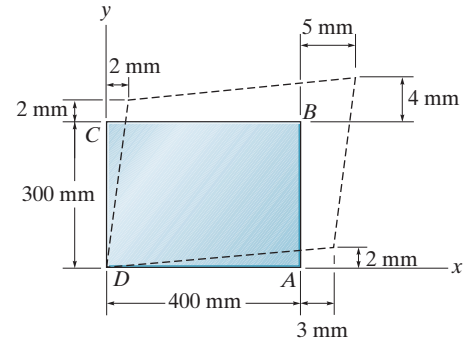
**Ans.**

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**Ans:**  
 $\epsilon = 2kx$

**2-18.**

Determine the shear strain  $\gamma_{xy}$  at corners  $A$  and  $B$  if the plate distorts as shown by the dashed lines.



**SOLUTION**

**Geometry:** For small angles,

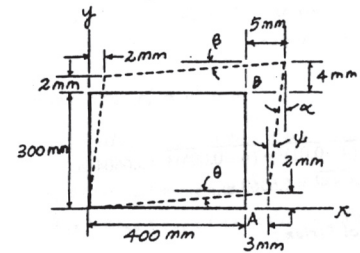
$$\alpha = \psi = \frac{2}{302} = 0.00662252 \text{ rad}$$

$$\beta = \theta = \frac{2}{403} = 0.00496278 \text{ rad}$$

**Shear Strain:**

$$\begin{aligned} (\gamma_B)_{xy} &= \alpha + \beta \\ &= 0.0116 \text{ rad} = 11.6(10^{-3}) \text{ rad} \end{aligned}$$

$$\begin{aligned} (\gamma_A)_{xy} &= \theta + \psi \\ &= 0.0116 \text{ rad} = 11.6(10^{-3}) \text{ rad} \end{aligned}$$



**Ans.**

**Ans.**

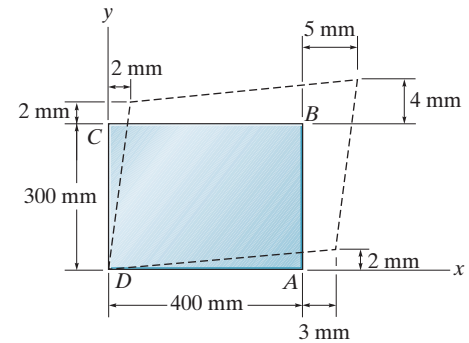
**Ans:**

$$(\gamma_B)_{xy} = 11.6(10^{-3}) \text{ rad},$$

$$(\gamma_A)_{xy} = 11.6(10^{-3}) \text{ rad}$$

## 2-19.

Determine the shear strain  $\gamma_{xy}$  at corners  $D$  and  $C$  if the plate distorts as shown by the dashed lines.



## SOLUTION

**Geometry:** For small angles,

$$\alpha = \psi = \frac{2}{403} = 0.00496278 \text{ rad}$$

$$\beta = \theta = \frac{2}{302} = 0.00662252 \text{ rad}$$

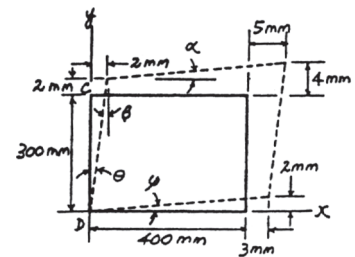
**Shear Strain:**

$$\begin{aligned} (\gamma_C)_{xy} &= \alpha + \beta \\ &= 0.0116 \text{ rad} = 11.6(10^{-3}) \text{ rad} \end{aligned}$$

$$\begin{aligned} (\gamma_D)_{xy} &= \theta + \psi \\ &= 0.0116 \text{ rad} = 11.6(10^{-3}) \text{ rad} \end{aligned}$$

**Ans.**

**Ans.**



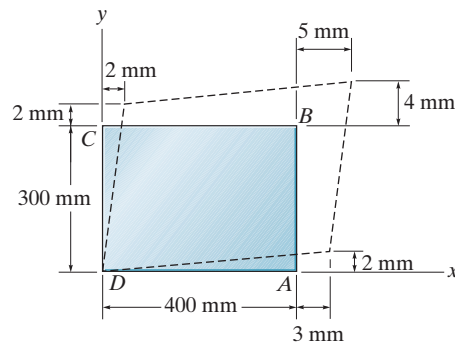
**Ans:**

$$(\gamma_C)_{xy} = 11.6(10^{-3}) \text{ rad},$$

$$(\gamma_D)_{xy} = 11.6(10^{-3}) \text{ rad}$$

**\*2-20.**

Determine the average normal strain that occurs along the diagonals  $AC$  and  $DB$ .



## SOLUTION

### Geometry:

$$AC = DB = \sqrt{400^2 + 300^2} = 500 \text{ mm}$$

$$DB' = \sqrt{405^2 + 304^2} = 506.4 \text{ mm}$$

$$A'C' = \sqrt{401^2 + 300^2} = 500.8 \text{ mm}$$

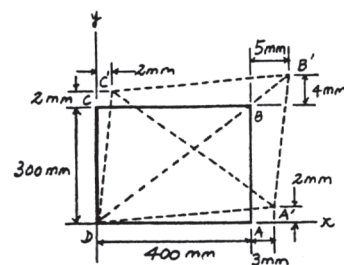
### Average Normal Strain:

$$\begin{aligned} \epsilon_{AC} &= \frac{A'C' - AC}{AC} = \frac{500.8 - 500}{500} \\ &= 0.00160 \text{ mm/mm} = 1.60(10^{-3}) \text{ mm/mm} \end{aligned}$$

$$\begin{aligned} \epsilon_{DB} &= \frac{DB' - DB}{DB} = \frac{506.4 - 500}{500} \\ &= 0.0128 \text{ mm/mm} = 12.8(10^{-3}) \text{ mm/mm} \end{aligned}$$

**Ans.**

**Ans.**



**Ans:**

$$\epsilon_{AC} = 1.60(10^{-3}) \text{ mm/mm}$$

$$\epsilon_{DB} = 12.8(10^{-3}) \text{ mm/mm}$$

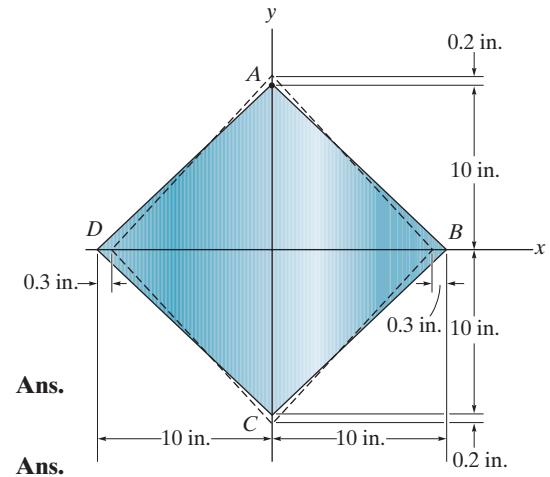
**2-21.**

The corners of the square plate are given the displacements indicated. Determine the average normal strains  $\epsilon_x$  and  $\epsilon_y$  along the  $x$  and  $y$  axes.

**SOLUTION**

$$\epsilon_x = \frac{-0.3}{10} = -0.03 \text{ in./in.}$$

$$\epsilon_y = \frac{0.2}{10} = 0.02 \text{ in./in.}$$



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**Ans:**

$$\epsilon_x = -0.03 \text{ in./in.}$$

$$\epsilon_y = 0.02 \text{ in./in.}$$

**2-22.**

The triangular plate is fixed at its base, and its apex  $A$  is given a horizontal displacement of 5 mm. Determine the shear strain,  $\gamma_{xy}$ , at  $A$ .

**SOLUTION**

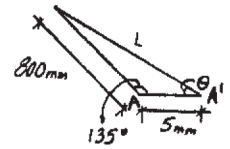
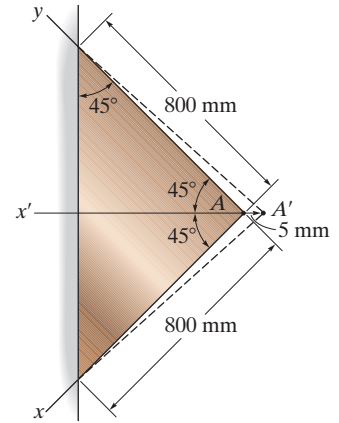
$$L = \sqrt{800^2 + 5^2 - 2(800)(5) \cos 135^\circ} = 803.54 \text{ mm}$$

$$\frac{\sin 135^\circ}{803.54} = \frac{\sin \theta}{800}; \quad \theta = 44.75^\circ = 0.7810 \text{ rad}$$

$$\gamma_{xy} = \frac{\pi}{2} - 2\theta = \frac{\pi}{2} - 2(0.7810)$$

$$= 0.00880 \text{ rad}$$

**Ans.**



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**Ans:**

$$\gamma_{xy} = 0.00880 \text{ rad}$$

**2-23.**

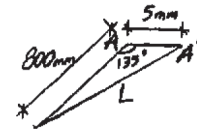
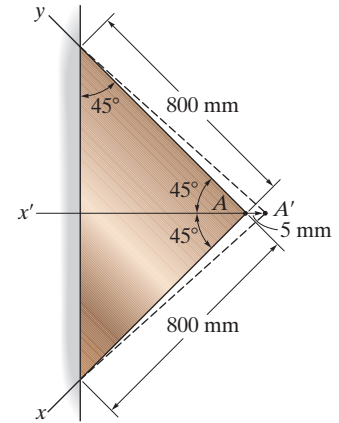
The triangular plate is fixed at its base, and its apex  $A$  is given a horizontal displacement of 5 mm. Determine the average normal strain  $\epsilon_x$  along the  $x$  axis.

**SOLUTION**

$$L = \sqrt{800^2 + 5^2 - 2(800)(5) \cos 135^\circ} = 803.54 \text{ mm}$$

$$\epsilon_x = \frac{803.54 - 800}{800} = 0.00443 \text{ mm/mm}$$

**Ans.**



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**Ans:**

$$\epsilon_x = 0.00443 \text{ mm/mm}$$

**\*2-24.**

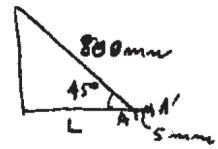
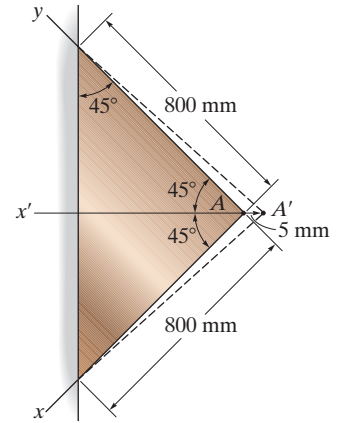
The triangular plate is fixed at its base, and its apex  $A$  is given a horizontal displacement of 5 mm. Determine the average normal strain  $\epsilon_{x'}$  along the  $x'$  axis.

**SOLUTION**

$$L = 800 \cos 45^\circ = 565.69 \text{ mm}$$

$$\epsilon_{x'} = \frac{5}{565.69} = 0.00884 \text{ mm/mm}$$

**Ans.**

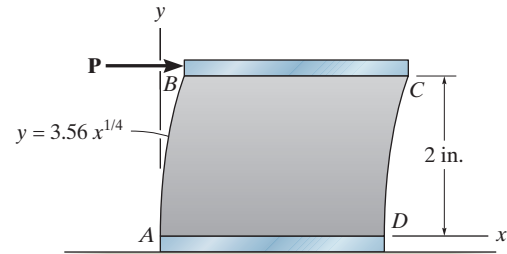


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**Ans:**  
 $\epsilon_{x'} = 0.00884 \text{ mm/mm}$

**2-25.**

The polysulfone block is glued at its top and bottom to the rigid plates. If a tangential force, applied to the top plate, causes the material to deform so that its sides are described by the equation  $y = 3.56 x^{1/4}$ , determine the shear strain at the corners  $A$  and  $B$ .



**SOLUTION**

$$y = 3.56 x^{1/4}$$

$$\frac{dy}{dx} = 0.890 x^{-3/4}$$

$$\frac{dx}{dy} = 1.123 x^{3/4}$$

At  $A$ ,  $x = 0$

$$\gamma_A = \frac{dx}{dy} = 0$$

At  $B$ ,

$$2 = 3.56 x^{1/4}$$

$$x = 0.0996 \text{ in.}$$

$$\gamma_B = \frac{dx}{dy} = 1.123(0.0996)^{3/4} = 0.199 \text{ rad}$$

**Ans.**

**Ans.**



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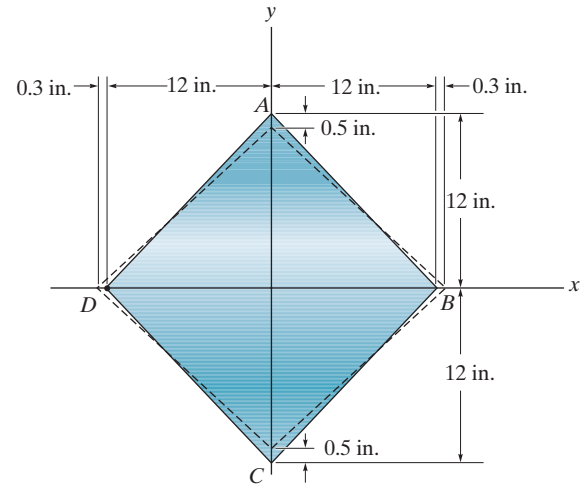
**Ans:**

$$\gamma_A = 0$$

$$\gamma_B = 0.199 \text{ rad}$$

2-26.

The corners of the square plate are given the displacements indicated. Determine the shear strain at  $A$  relative to axes that are directed along  $AB$  and  $AD$ , and the shear strain at  $B$  relative to axes that are directed along  $BC$  and  $BA$ .



**SOLUTION**

**Geometry:** Referring to the geometry shown in Fig.  $a$ ,

$$\tan \frac{\theta}{2} = \frac{12.3}{11.5} \quad \theta = (93.85^\circ) \left( \frac{\pi}{180^\circ} \text{ rad} \right) = 1.6380 \text{ rad}$$

$$\tan \frac{\phi}{2} = \frac{11.5}{12.3} \quad \phi = (86.15^\circ) \left( \frac{\pi}{180^\circ} \text{ rad} \right) = 1.5036 \text{ rad}$$

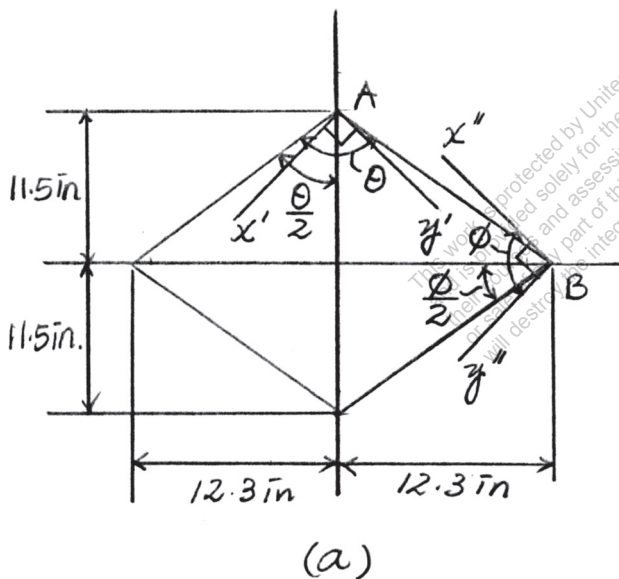
**Shear Strain:** By definition,

$$(\gamma_{x'y'})_A = \frac{\pi}{2} - \theta = \frac{\pi}{2} - 1.6380 = -0.0672 \text{ rad}$$

**Ans.**

$$(\gamma_{x''y''})_B = \frac{\pi}{2} - \phi = \frac{\pi}{2} - 1.5036 = 0.0672 \text{ rad}$$

**Ans.**



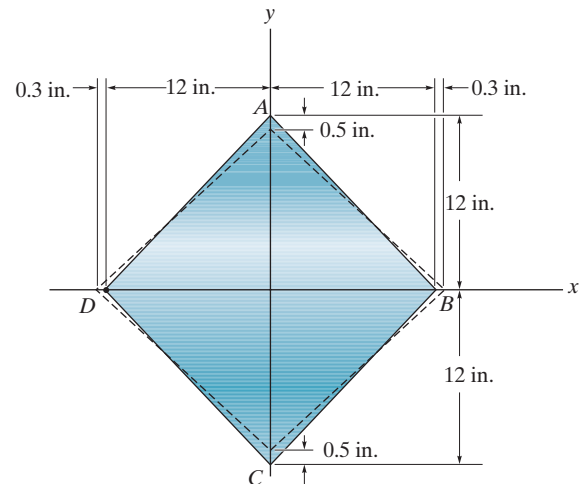
**Ans:**

$$(\gamma_{x'y'})_A = -0.0672 \text{ rad}$$

$$(\gamma_{x''y''})_B = 0.0672 \text{ rad}$$

## 2-27.

The corners of the square plate are given the displacements indicated. Determine the average normal strains along side  $AB$  and diagonals  $AC$  and  $BD$ .



## SOLUTION

**Geometry:** Referring to the geometry shown in Fig.  $a$ ,

$$L_{AB} = \sqrt{12^2 + 12^2} = 12\sqrt{2} \text{ in.}$$

$$L_{A'B'} = \sqrt{12.3^2 + 11.5^2} = \sqrt{283.54} \text{ in.}$$

$$L_{BD} = 2(12) = 24 \text{ in.}$$

$$L_{B'D'} = 2(12 + 0.3) = 24.6 \text{ in.}$$

$$L_{AC} = 2(12) = 24 \text{ in.}$$

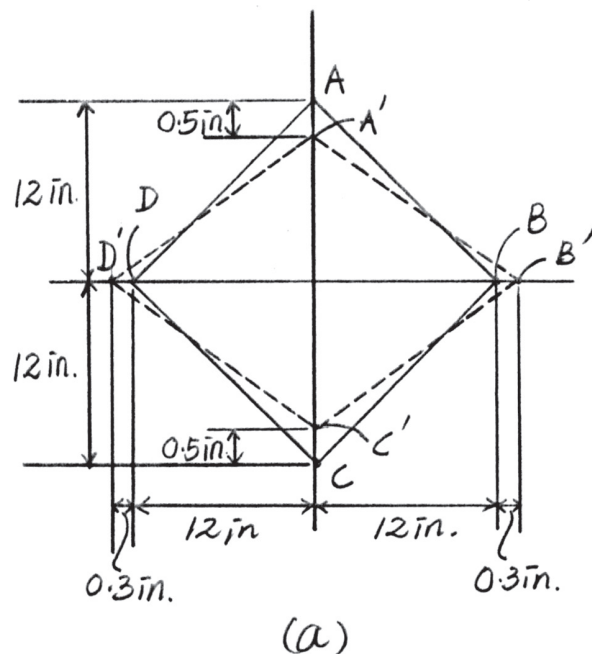
$$L_{A'C'} = 2(12 - 0.5) = 23 \text{ in.}$$

**Average Normal Strain:**

$$\epsilon_{AB} = \frac{L_{A'B'} - L_{AB}}{L_{AB}} = \frac{\sqrt{283.54} - 12\sqrt{2}}{12\sqrt{2}} = -7.77(10^{-3}) \text{ in./in.} \quad \text{Ans.}$$

$$\epsilon_{BD} = \frac{L_{B'D'} - L_{BD}}{L_{BD}} = \frac{24.6 - 24}{24} = 0.025 \text{ in./in.} \quad \text{Ans.}$$

$$\epsilon_{AC} = \frac{L_{A'C'} - L_{AC}}{L_{AC}} = \frac{23 - 24}{24} = -0.0417 \text{ in./in.} \quad \text{Ans.}$$



**Ans:**

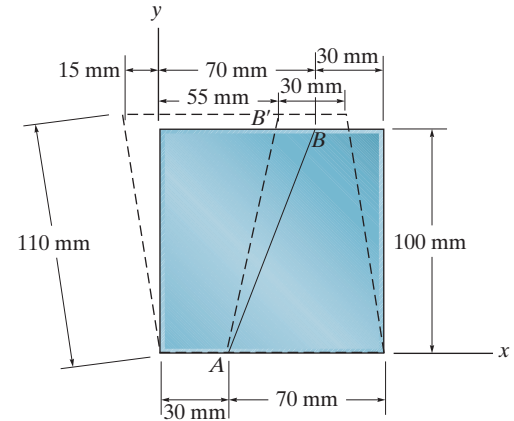
$$\epsilon_{AB} = -7.77(10^{-3}) \text{ in./in.}$$

$$\epsilon_{BD} = 0.025 \text{ in./in.}$$

$$\epsilon_{AC} = -0.0417 \text{ in./in.}$$

**\*2-28.**

The block is deformed into the position shown by the dashed lines. Determine the average normal strain along line  $AB$ .



## SOLUTION

**Geometry:**

$$AB = \sqrt{100^2 + (70 - 30)^2} = 107.7033 \text{ mm}$$

$$AB' = \sqrt{(70 - 30 - 15)^2 + (100^2 - 15^2)} = 111.8034 \text{ mm}$$

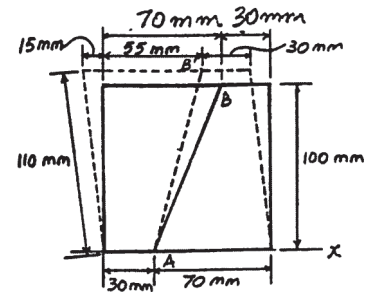
**Average Normal Strain:**

$$\epsilon_{AB} = \frac{AB' - AB}{AB}$$

$$= \frac{111.8034 - 107.7033}{107.7033}$$

$$= 0.0381 \text{ mm/mm} = 38.1 (10^{-3}) \text{ mm}$$

**Ans.**



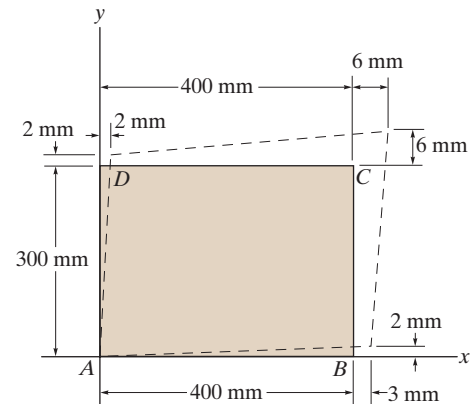
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**Ans:**

$$\epsilon_{AB} = 38.1 (10^{-3}) \text{ mm}$$

## 2-29.

The rectangular plate is deformed into the shape shown by the dashed lines. Determine the average normal strain along diagonal  $AC$ , and the average shear strain at corner  $A$  relative to the  $x, y$  axes.



## SOLUTION

**Geometry:** The unstretched length of diagonal  $AC$  is

$$L_{AC} = \sqrt{300^2 + 400^2} = 500 \text{ mm}$$

Referring to Fig. *a*, the stretched length of diagonal  $AC$  is

$$L_{AC'} = \sqrt{(400 + 6)^2 + (300 + 6)^2} = 508.4014 \text{ mm}$$

Referring to Fig. *a* and using small angle analysis,

$$\phi = \frac{2}{300 + 2} = 0.006623 \text{ rad}$$

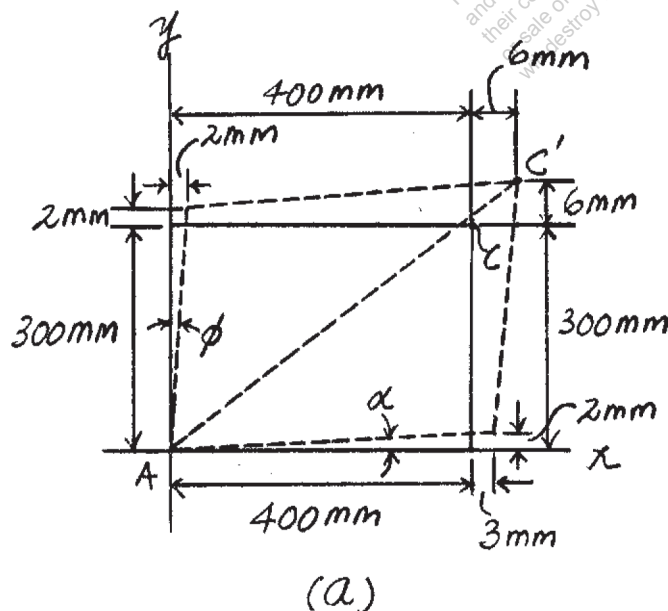
$$\alpha = \frac{2}{400 + 3} = 0.004963 \text{ rad}$$

**Average Normal Strain:** Applying Eq. 2,

$$(\epsilon_{\text{avg}})_{AC} = \frac{L_{AC'} - L_{AC}}{L_{AC}} = \frac{508.4014 - 500}{500} = 0.0168 \text{ mm/mm} \quad \text{Ans.}$$

**Shear Strain:** Referring to Fig. *a*,

$$(\gamma_A)_{xy} = \phi + \alpha = 0.006623 + 0.004963 = 0.0116 \text{ rad} \quad \text{Ans.}$$

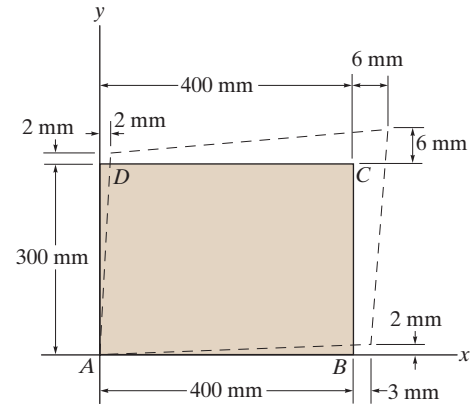


**Ans:**

$$(\epsilon_{\text{avg}})_{AC} = 0.0168 \text{ mm/mm}, (\gamma_A)_{xy} = 0.0116 \text{ rad}$$

2-30.

The rectangular plate is deformed into the shape shown by the dashed lines. Determine the average normal strain along diagonal  $BD$ , and the average shear strain at corner  $B$  relative to the  $x, y$  axes.



SOLUTION

**Geometry:** The unstretched length of diagonal  $BD$  is

$$L_{BD} = \sqrt{300^2 + 400^2} = 500 \text{ mm}$$

Referring to Fig. *a*, the stretched length of diagonal  $BD$  is

$$L_{B'D'} = \sqrt{(300 + 2 - 2)^2 + (400 + 3 - 2)^2} = 500.8004 \text{ mm}$$

Referring to Fig. *a* and using small angle analysis,

$$\phi = \frac{2}{403} = 0.004963 \text{ rad}$$

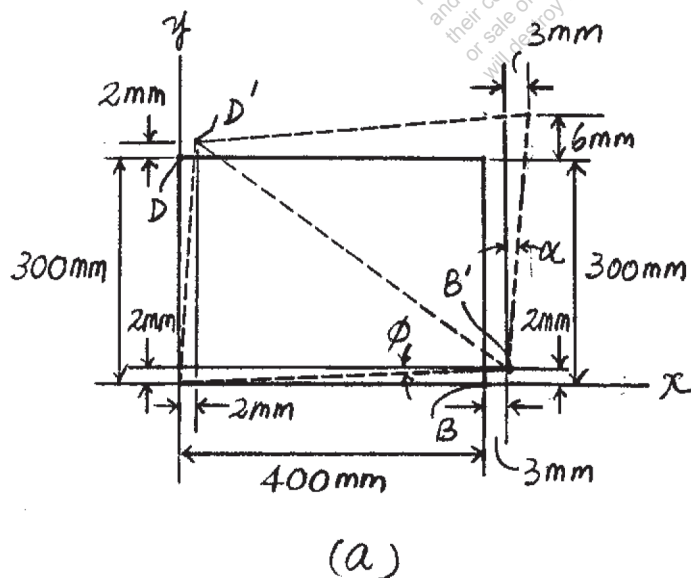
$$\alpha = \frac{3}{300 + 6 - 2} = 0.009868 \text{ rad}$$

**Average Normal Strain:** Applying Eq. 2,

$$(\epsilon_{\text{avg}})_{BD} = \frac{L_{B'D'} - L_{BD}}{L_{BD}} = \frac{500.8004 - 500}{500} = 1.60(10^{-3}) \text{ mm/mm} \quad \text{Ans.}$$

**Shear Strain:** Referring to Fig. *a*,

$$(\gamma_B)_{xy} = \phi + \alpha = 0.004963 + 0.009868 = 0.0148 \text{ rad} \quad \text{Ans.}$$



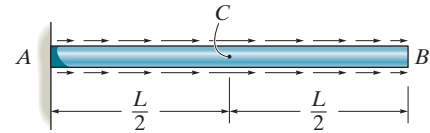
**Ans:**

$$(\epsilon_{\text{avg}})_{BD} = 1.60(10^{-3}) \text{ mm/mm},$$

$$(\gamma_B)_{xy} = 0.0148 \text{ rad}$$

## 2-31.

The nonuniform loading causes a normal strain in the shaft that can be expressed as  $\epsilon_x = k \sin\left(\frac{\pi}{L}x\right)$ , where  $k$  is a constant. Determine the displacement of the center  $C$  and the average normal strain in the entire rod.



## SOLUTION

$$\epsilon_x = k \sin\left(\frac{\pi}{L}x\right)$$

$$\begin{aligned} (\Delta x)_C &= \int_0^{L/2} \epsilon_x dx = \int_0^{L/2} k \sin\left(\frac{\pi}{L}x\right) dx \\ &= -k\left(\frac{L}{\pi}\right) \cos\left(\frac{\pi}{L}x\right) \Big|_0^{L/2} = -k\left(\frac{L}{\pi}\right) \left(\cos \frac{\pi}{2} - \cos 0\right) \\ &= \frac{kL}{\pi} \end{aligned}$$

**Ans.**

$$\begin{aligned} (\Delta x)_B &= \int_0^L k \sin\left(\frac{\pi}{L}x\right) dx \\ &= -k\left(\frac{L}{\pi}\right) \cos\left(\frac{\pi}{L}x\right) \Big|_0^L = -k\left(\frac{L}{\pi}\right) (\cos \pi - \cos 0) = \frac{2kL}{\pi} \end{aligned}$$

$$\epsilon_{\text{avg}} = \frac{(\Delta x)_B}{L} = \frac{2k}{\pi}$$

**Ans.**

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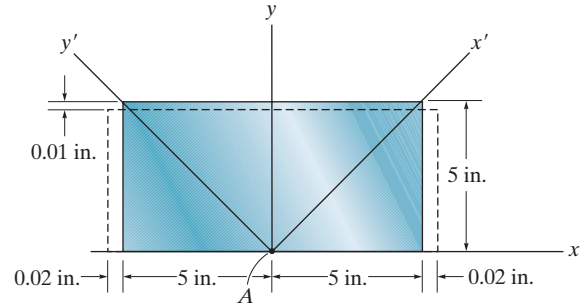
**Ans:**

$$(\Delta x)_C = \frac{kL}{\pi}$$

$$\epsilon_{\text{avg}} = \frac{2k}{\pi}$$

**\*2-32.**

The rectangular plate undergoes a deformation shown by the dashed lines. Determine the shear strain  $\gamma_{xy}$  and  $\gamma_{x'y'}$  at point A.



**SOLUTION**

Since the right angle of an element along the  $x, y$  axes does not distort, then

$$\gamma_{xy} = 0$$

**Ans.**

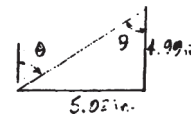
$$\tan \theta = \frac{5.02}{4.99}$$

$$\theta = 45.17^\circ = 0.7884 \text{ rad}$$

$$\gamma_{x'y'} = \frac{\pi}{2} - 2\theta$$

$$= \frac{\pi}{2} - 2(0.7884)$$

$$= -0.00599 \text{ rad}$$



**Ans.**

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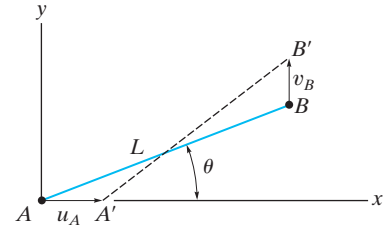
**Ans:**

$$\gamma_{xy} = 0$$

$$\gamma_{x'y'} = -0.00599 \text{ rad}$$

**2-33.**

The fiber  $AB$  has a length  $L$  and orientation  $\theta$ . If its ends  $A$  and  $B$  undergo very small displacements  $u_A$  and  $v_B$  respectively, determine the normal strain in the fiber when it is in position  $A'B'$ .



**SOLUTION**

**Geometry:**

$$\begin{aligned} L_{A'B'} &= \sqrt{(L \cos \theta - u_A)^2 + (L \sin \theta + v_B)^2} \\ &= \sqrt{L^2 + u_A^2 + v_B^2 + 2L(v_B \sin \theta - u_A \cos \theta)} \end{aligned}$$

**Average Normal Strain:**

$$\begin{aligned} \epsilon_{AB} &= \frac{L_{A'B'} - L}{L} \\ &= \sqrt{1 + \frac{u_A^2 + v_B^2}{L^2} + \frac{2(v_B \sin \theta - u_A \cos \theta)}{L}} - 1 \end{aligned}$$

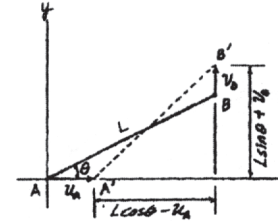
Neglecting higher terms  $u_A^2$  and  $v_B^2$

$$\epsilon_{AB} = \left[ 1 + \frac{2(v_B \sin \theta - u_A \cos \theta)}{L} \right]^{\frac{1}{2}} - 1$$

Using the binomial theorem:

$$\begin{aligned} \epsilon_{AB} &= 1 + \frac{1}{2} \left( \frac{2v_B \sin \theta}{L} - \frac{2u_A \cos \theta}{L} \right) + \dots - 1 \\ &= \frac{v_B \sin \theta}{L} - \frac{u_A \cos \theta}{L} \end{aligned}$$

**Ans.**



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**Ans:**

$$\epsilon_{AB} = \frac{v_B \sin \theta}{L} - \frac{u_A \cos \theta}{L}$$

**2-34.**

If the normal strain is defined in reference to the final length  $\Delta s'$ , that is,

$$\epsilon' = \lim_{\Delta s' \rightarrow 0} \left( \frac{\Delta s' - \Delta s}{\Delta s'} \right)$$

instead of in reference to the original length, Eq. 2-2, show that the difference in these strains is represented as a second-order term, namely,  $\epsilon - \epsilon' = \epsilon \epsilon'$ .

**SOLUTION**

$$\epsilon = \frac{\Delta s' - \Delta s}{\Delta s}$$

$$\epsilon - \epsilon' = \frac{\Delta s' - \Delta s}{\Delta s} - \frac{\Delta s' - \Delta s}{\Delta s'}$$

$$= \frac{\Delta s'^2 - \Delta s \Delta s' - \Delta s' \Delta s + \Delta s^2}{\Delta s \Delta s'}$$

$$= \frac{\Delta s'^2 + \Delta s^2 - 2\Delta s' \Delta s}{\Delta s \Delta s'}$$

$$= \frac{(\Delta s' - \Delta s)^2}{\Delta s \Delta s'} = \left( \frac{\Delta s' - \Delta s}{\Delta s} \right) \left( \frac{\Delta s' - \Delta s}{\Delta s'} \right)$$

$$= \epsilon \epsilon'$$

**(Q.E.D)**

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**Ans:**  
N/A