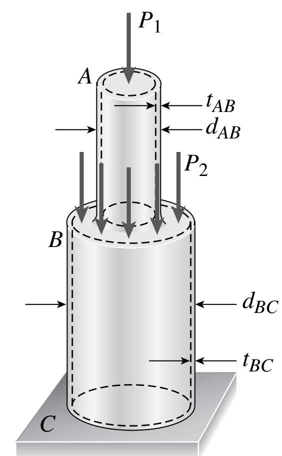
1

Tension, Compression, and Shear

Normal Stress and Strain

Problem 1.2-1 A hollow circular post *ABC* (see figure) supports a load $P_1 = 1700$ lb acting at the top. A second load P_2 is uniformly distributed around the cap plate at *B*. The diameters and thicknesses of the upper and lower parts of the post are $d_{AB} = 1.25$ in., $t_{AB} = 0.5$ in., $d_{BC} = 2.25$ in., and $t_{BC} = 0.375$ in., respectively.

(a) Calculate the normal stress σ_{AB} in the upper part of the post.
(b) If it is desired that the lower part of the post have the same compressive stress as the upper part, what should be the magnitude of the load P_2?
(c) If P_1 remains at 1700 lb and P_2 is now set at 2260 lb, what new thickness of *BC* will result in the same compressive stress in both parts?



Solution 1.2-1

PART (a)

$P_1 = 1700 \qquad d_{AB} = 1.25 \qquad t_{AB} = 0.5$

$d_{BC} = 2.25 \qquad t_{BC} = 0.375$

$$A_{AB} = \frac{\pi[\,d_{AB}^2 - (d_{AB} - 2t_{AB})^2]}{4}$$

$$A_{AB} = 1.178 \qquad \sigma_{AB} = \frac{P_1}{A_{AB}}$$

$$\sigma_{AB} = 1443 \text{ psi} \quad \longleftarrow$$

PART (b)

$$A_{BC} = \frac{\pi[\,d_{BC}^2 - (d_{BC} - 2t_{BC})^2]}{4}$$

$$A_{BC} = 2.209 \qquad P_2 = \sigma_{AB}A_{BC} - P_1$$

$$P_2 = 1488 \text{ lbs} \quad \longleftarrow$$

CHECK: $\dfrac{P_1 + P_2}{A_{BC}} = 1443$ psi

Part (c)

$$P_2 = 2260 \qquad \frac{P_1 + P_2}{\sigma_{AB}} = A_{BC}$$

$$\frac{P_1 + P_2}{\sigma_{AB}} = 2.744$$

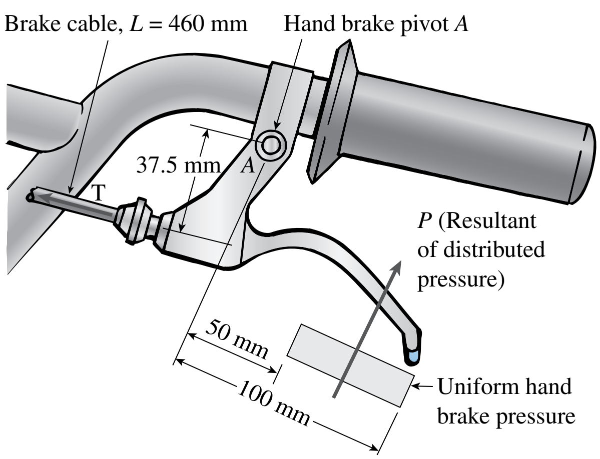
$$(d_{BC} - 2t_{BC})^2 = d_{BC}^2 - \frac{4}{\pi}\left(\frac{P_1 + P_2}{\sigma_{AB}}\right)$$

$$d_{BC} - 2t_{BC} = \sqrt{d_{BC}^2 - \frac{4}{\pi}\left(\frac{P_1 + P_2}{\sigma_{AB}}\right)}$$

$$t_{BC} = \frac{d_{BC} - \sqrt{d_{BC}^2 - \frac{4}{\pi}\left(\frac{P_1 + P_2}{\sigma_{AB}}\right)}}{2}$$

$$t_{BC} = 0.499 \text{ inches} \quad \leftarrow$$

Problem 1.2-2 A force P of 70 N is applied by a rider to the front hand brake of a bicycle (P is the resultant of an evenly distributed pressure). As the hand brake pivots at A, a tension T develops in the 460-mm long brake cable ($A_e = 1.075 \text{ mm}^2$) which elongates by $\delta = 0.214$ mm. Find normal stress σ and strain ε in the brake cable.



Solution 1.2-2

$P = 70$ N $\qquad A_e = 1.075 \text{ mm}^2$

$L = 460$ mm $\qquad \delta = 0.214$ mm

Statics: sum moments about A to get $T = 2P$

$$\sigma = \frac{T}{A_e} \qquad \sigma = 103.2 \text{ MPa} \quad \leftarrow$$

$$\varepsilon = \frac{\delta}{L} \qquad \varepsilon = 4.65 \times 10^{-4} \quad \leftarrow$$

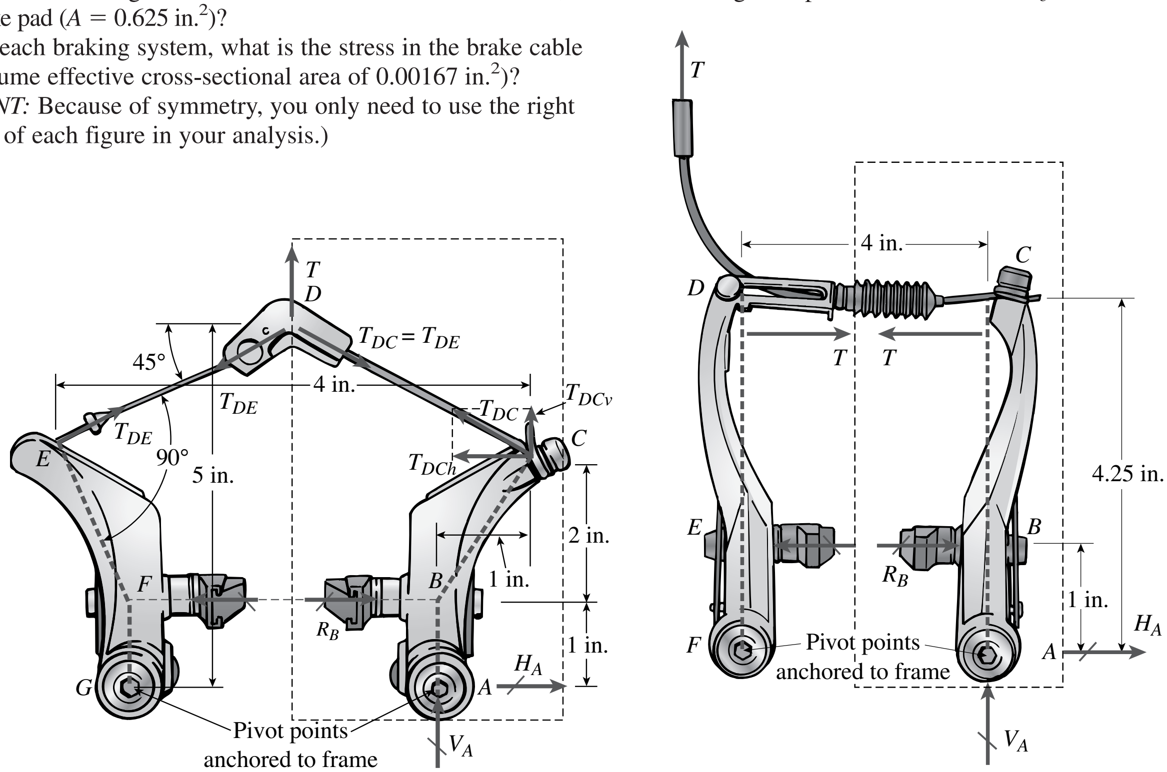
$$E = \frac{\sigma}{\varepsilon} = 1.4 \times 10^5 \text{ MPa}$$

NOTE: (E for cables is approx. 140 GPa)

Problem 1.2-3 A bicycle rider would like to compare the effectiveness of cantilever hand brakes [see figure part (a)] versus V brakes [figure part (b)].

(a) Calculate the braking force R_B at the wheel rims for each of the bicycle brake systems shown. Assume that all forces act in the plane of the figure and that cable tension $T = 45$ lbs. Also, what is the average compressive normal stress σ_c on the brake pad ($A = 0.625$ in.^{2})?

(b) For each braking system, what is the stress in the brake cable (assume effective cross-sectional area of 0.00167 in.^{2})? (*HINT:* Because of symmetry, you only need to use the right half of each figure in your analysis.)



(a) Cantilever brakes (b) V brakes

Solution 1.2-3

T = 45 lbs $\quad A_{pad} = 0.625$ in.^{2}

$A_{cable} = 0.00167$ in.^{2}

(a) CANTILEVER BRAKES-BRAKING FORCE
R_B & PAD PRESSURE

Statics: sum forces at D to get $T_{DC} = T/2$

$\sum M_A = 0$

$R_B(1) = T_{DCh}(3) + T_{DCv}(1)$

$T_{DCh} = T_{DCv} \qquad T_{DCh} = T/2$

$R_B = 2T \qquad R_B = 90 \text{ lbs} \quad \leftarrow$

so $R_B = 2T$ vs 4.25T for V brakes (below)

$$\sigma_{\text{pad}} = \frac{R_B}{A_{\text{pad}}} \qquad \sigma_{\text{pad}} = 144 \text{ psi} \quad \longleftarrow \qquad \frac{4.25}{2} = 2.125$$

$$\sigma_{\text{cable}} = \frac{T}{A_{\text{cable}}} \qquad \sigma_{\text{cable}} = 26{,}946 \text{ psi} \quad \longleftarrow$$

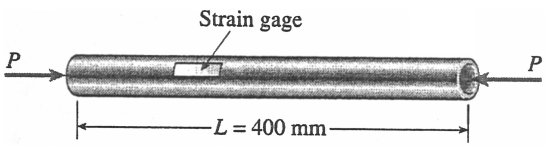
(same for V-brakes (below))

(b) V BRAKES - BRAKING FORCE R_B & PAD PRESSURE

$$\sum M_A = 0 \qquad R_B = 4.25T \qquad R_B = 191.3 \text{ lbs} \quad \longleftarrow$$

$$\sigma_{\text{pad}} = \frac{R_B}{A_{\text{pad}}} \qquad \sigma_{\text{pad}} = 306 \text{ psi} \quad \longleftarrow$$

Problem 1.2-4 A circular aluminum tube of length $L = 400$ mm is loaded in compression by forces P (see figure). The outside and inside diameters are 60 mm and 50 mm, respectively. A strain gage is placed on the outside of the bar to measure normal strains in the longitudinal direction.



(a) If the measured strain in $\epsilon = 550 \times 10^{-6}$, what is the shortening δ of the bar?
(b) If the compressive stress in the bar is intended to be 40 MPa, what should be the load P?

Solution 1.2-4 Aluminum tube in compression

$\varepsilon = 550 \times 10^{-6}$

$L = 400$ mm

$d_2 = 60$ mm

$d_1 = 50$ mm

(a) SHORTENING δ OF THE BAR

$$\delta = \varepsilon L = (550 \times 10^{-6})(400 \text{ mm})$$
$$= 0.220 \text{ mm} \quad \longleftarrow$$

(b) COMPRESSIVE LOAD P

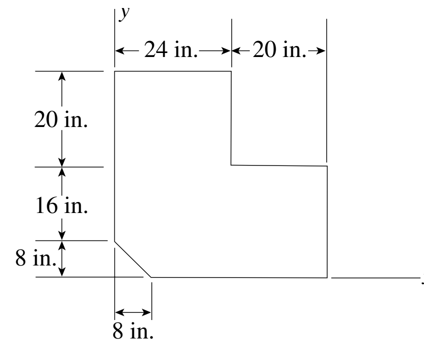
$\sigma = 40$ MPa

$$A = \frac{\pi}{4}[d_2^2 - d_1^2] = \frac{\pi}{4}[(60 \text{ mm})^2 - (50 \text{ mm})^2]$$

$$P = \sigma A = (40 \text{ MPa})(863.9 \text{ mm}^2)$$
$$= 34.6 \text{ kN} \quad \longleftarrow$$

Problem 1.2-5 The cross section of a concrete corner column that is loaded uniformly in compression is shown in the figure.

(a) Determine the average compressive stress σ_c in the concrete if the load is equal to 3200 k.
(b) Determine the coordinates x_c and y_c of the point where the resultant load must act in order to produce uniform normal stress in the column.



Solution 1.2-5

$$P = 3200 \text{ kips}$$

$$A = (24 + 20)(20 + 16 + 8) - \left(\frac{1}{2}8^2\right) - 20^2$$

$$A = 1.504 \times 10^3 \ \text{in}^2$$

(a) $\sigma_c = \dfrac{P}{A}$ $\quad \sigma_c = 2.13 \text{ ksi} \quad \longleftarrow$

(b)
$$x_c = \frac{\left[(24)(20+16)(12) + (24-8)(8)\left(8 + \dfrac{24-8}{2}\right) + (20)(16+8)(24+10) + \dfrac{1}{2}(8^2)\left(\dfrac{2}{3}8\right)\right]}{A}$$

$x_c = 19.22 \text{ inches} \quad \longleftarrow$

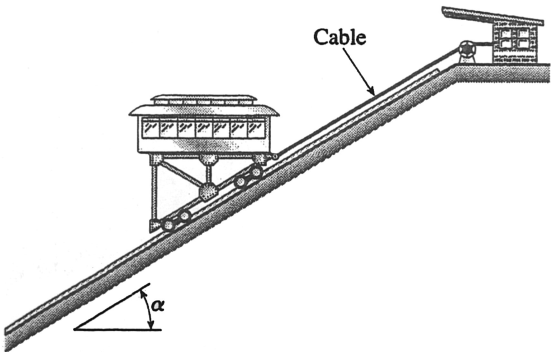
$$y_c = \frac{\left[(24)(20+16)\left(8 + \dfrac{20+16}{2}\right) + (20)(16+8)\left(\dfrac{16+8}{2}\right) + (24-8)(8)(4) + \dfrac{1}{2}(8^2)\left(\dfrac{2}{3}8\right)\right]}{A}$$

$y_c = 19.22 \text{ inches} \quad \longleftarrow$

NOTE: x_c & y_c are the same as expected due to symmetry about a diagonal

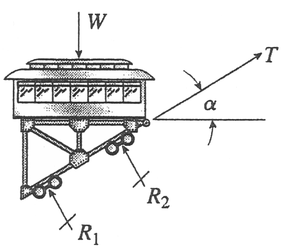
Problem 1.2-6 A car weighing 130 kN when fully loaded is pulled slowly up a steep inclined track by a steel cable (see figure). The cable has an effective cross-sectional area of 490 mm^2, and the angle α of the incline is 30°.

Calculate the tensile stress σ_t in the cable.



Solution 1.2-6 Car on inclined track

FREE-BODY DIAGRAM OF CAR



W = Weight of car

T = Tensile force in cable

α = Angle of incline

A = Effective area of cable

R_1, R_2 = Wheel reactions (no friction force between wheels and rails)

EQUILIBRIUM IN THE INCLINED DIRECTION

$\Sigma F_T = 0 \quad \nearrow_{+} \swarrow^{-} \quad T - W\, sin\, \alpha = 0$

$T = W \sin \alpha$

TENSILE STRESS IN THE CABLE

$$\sigma_t = \frac{T}{A} = \frac{W \sin \alpha}{A}$$

SUBSTITUTE NUMERICAL VALUES:

$W = 130 \text{ kN} \quad \alpha = 30°$

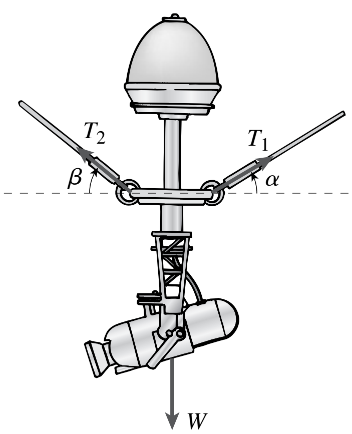
$A = 490 \text{ mm}^2$

$$\sigma_t = \frac{(130 \text{ kN})(\sin 30°)}{490 \text{ mm}^2}$$

$$= 133 \text{ MPa} \quad \leftarrow$$

Problem 1.2-7 Two steel wires support a moveable overhead camera weighing W = 25 lb (see figure) used for close-up viewing of field action at sporting events. At some instant, wire 1 is at on angle $\alpha = 20°$ to the horizontal and wire 2 is at an angle $\beta = 48°$. Both wires have a diameter of 30 mils. (Wire diameters are often expressed in mils; one mil equals 0.001 in.)

Determine the tensile stresses σ_1 and σ_2 in the two wires.



Solution 1.2-7

NUMERICAL DATA

$W = 25 \text{ lb} \qquad d = 30 \times 10^{-3} \text{ in.}$

$\alpha = 20\dfrac{\pi}{180} \qquad \beta = 48\dfrac{\pi}{180} = \text{radians}$

EQUILIBRIUM EQUATIONS

$\sum F_h = 0 \qquad T_1\cos(\alpha) = T_2\cos(\beta)$

$$T_1 = T_2\frac{\cos(\beta)}{\cos(\alpha)}$$

$\sum F_v = 0 \qquad T_1\sin(\alpha) + T_2\sin(\beta) = W$

$$T_2\left(\frac{\cos(\beta)}{\cos(\alpha)}\sin(\alpha) + \sin(\beta)\right) = W$$

TENSION IN WIRES

$$T_2 = \frac{W}{\left(\dfrac{\cos(\beta)}{\cos(\alpha)}\sin(\alpha) + \sin(\beta)\right)}$$

$T_2 = 25.337 \text{ lb}$

$T_1 = T_2\dfrac{\cos(\beta)}{\cos(\alpha)} \qquad T_1 = 18.042 \text{ lb}$

TENSILE STRESSES IN WIRES

$$A_{wire} = \frac{\pi}{4}d^2$$

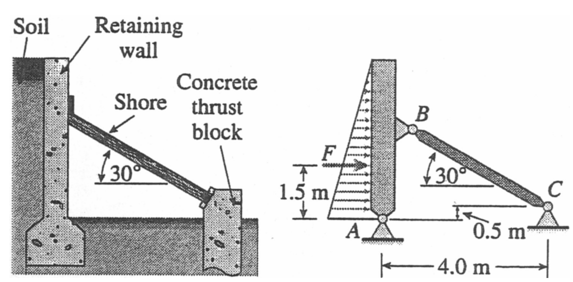
$\sigma_1 = \dfrac{T_1}{A_{wire}} \qquad \sigma_1 = 25.5 \text{ ksi} \quad \leftarrow$

$\sigma_2 = \dfrac{T_2}{A_{wire}} \qquad \sigma_2 = 35.8 \text{ ksi} \quad \leftarrow$

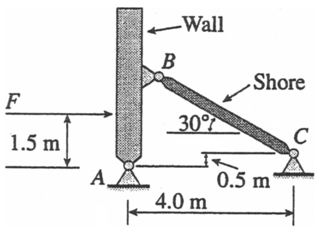
Problem 1.2-8 A long retaining wall is braced by wood shores set at an angle of 30° and supported by concrete thrust blocks, as shown in the first part of the figure. The shores are evenly spaced, 3 m apart.

For analysis purposes, the wall and shores are idealized as shown in the second part of the figure. Note that the base of the wall and both ends of the shores are assumed to be pinned. The pressure of the soil against the wall is assumed to be triangularly distributed, and the resultant force acting on a 3-meter length of the wall is $F = 190$ kN.

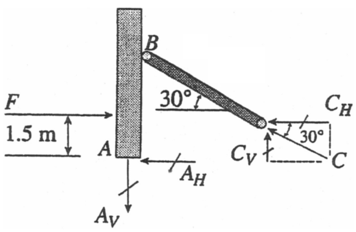
If each shore has a 150 mm × 150 mm square cross section, what is the compressive stress σ_c in the shores?



Solution 1.2-8 Retaining wall braced by wood shores



FREE-BODY DIAGRAM OF WALL AND SHORE



C = compressive force in wood shore

C_H = horizontal component of C

C_V = vertical component of C

$C_H = C \cos 30°$

$C_V = C \sin 30°$

$F = 190$ kN

A = area of one shore

$A = (150 \text{ mm})(150 \text{ mm})$

$= 22{,}500 \text{ mm}^2$

$= 0.0225 \text{ m}^2$

SUMMATION OF MOMENTS ABOUT POINT A

$\Sigma M_A = 0$

$-F(1.5 \text{ m}) + C_V(4.0 \text{ m}) + C_H(0.5 \text{ m}) = 0$

or

$$-(190 \text{ kN})(1.5 \text{ m}) + C(\sin 30°)(4.0 \text{ m}) + C(\cos 30°)(0.5 \text{ m}) = 0$$

$\therefore C = 117.14$ kN

COMPRESSIVE STRESS IN THE SHORES

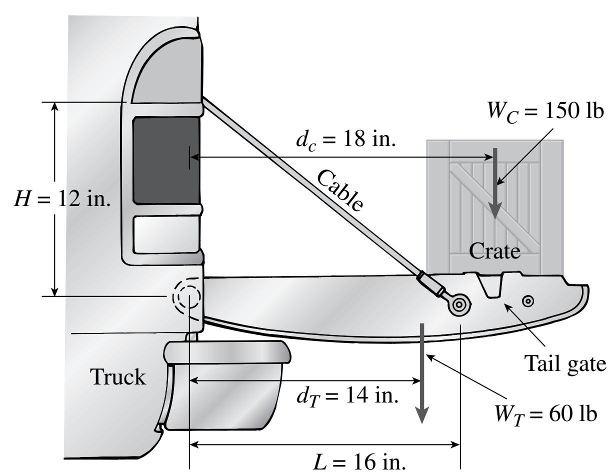
$$\sigma_c = \frac{C}{A} = \frac{117.14 \text{ kN}}{0.0225 \text{ m}^2}$$

$$= 5.21 \text{ MPa} \quad \leftarrow$$

Problem 1.2-9 A pickup truck tailgate supports a crate ($W_C = 150$ lb), as shown in the figure. The tailgate weighs $W_T = 60$ lb and is supported by two cables (only one is shown in the figure). Each cable has an effective cross-sectional area $A_e = 0.017 \text{ in}^2$.

(a) Find the tensile force T and normal stress σ in each cable.

(b) If each cable elongates $\delta = 0.01$ in. due to the weight of both the crate and the tailgate, what is the average strain in the cable?



Solution 1.2-9

$W_c = 150 \text{ lb}$

$A_e = 0.017 \text{ in}^2$

$W_T = 60$

$\delta = 0.01$

$d_c = 18$

$d_T = 14$

$H = 12$

$L = 16$

$L_c = \sqrt{L^2 + H^2} \qquad L_c = 20$

$\sum M_{hinge} = 0 \qquad 2T_vL = W_cd_c + W_Td_T$

$$T_v = \frac{W_cd_c + W_Td_T}{2L} \qquad T_v = 110.625 \text{ lb}$$

$$T_h = \frac{L}{H}T_v \qquad T_h = 147.5$$

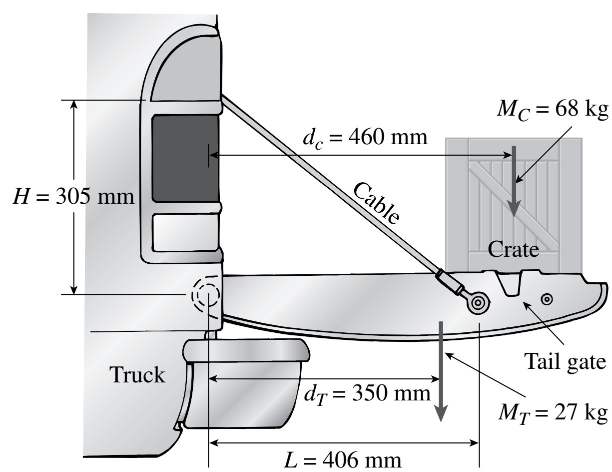
(a) $T = \sqrt{T_v^2 + T_h^2} \qquad T = 184.4 \text{ lb} \quad \leftarrow$

$$\sigma_{cable} = \frac{T}{A_e} \qquad \sigma_{cable} = 10.8 \text{ ksi} \quad \leftarrow$$

(b) $\varepsilon_{cable} = \dfrac{\delta}{L_c} \qquad \varepsilon_{cable} = 5 \times 10^{-4} \quad \leftarrow$

Problem 1.2-10 Solve the preceding problem if the mass of the tail gate is $M_T = 27$ kg and that of the crate is $M_C = 68$ kg. Use dimensions $H = 305$ mm, $L = 406$ mm, $d_C = 460$ mm, and $d_T = 350$ mm. The cable cross-sectional area is $A_e = 11.0 \text{ mm}^2$.

(a) Find the tensile force T and normal stress σ in each cable.
(b) If each cable elongates $\delta = 0.25$ mm due to the weight of both the crate and the tailgate, what is the average strain in the cable?



Solution 1.2-10

$M_c = 68$

$M_T = 27\ \text{kg} \qquad g = 9.81\dfrac{m}{s^2}$

$W_c = M_c g \qquad W_T = M_T g$

$W_c = 667.08 \qquad W_T = 264.87$

$N = kg\dfrac{m}{s^2}$

$A_e = 11.0\ \text{mm}^2 \qquad \delta = 0.25$

$d_c = 460 \qquad d_T = 350$

$H = 305 \qquad L = 406$

$L_c = \sqrt{L^2 + H^2} \qquad L_c = 507.8\ \text{mm}$

$\sum M_{hinge} = 0 \qquad 2T_vL = W_cd_c + W_Td_T$

$$T_v = \frac{W_cd_c + W_Td_T}{2L} \qquad T_v = 492.071\ \text{N}$$

$$T_h = \frac{L}{H}T_v \qquad T_h = 655.019\ \text{N}$$

(a) $T = \sqrt{T_v^2 + T_h^2} \qquad T = 819\ \text{N} \quad \leftarrow$

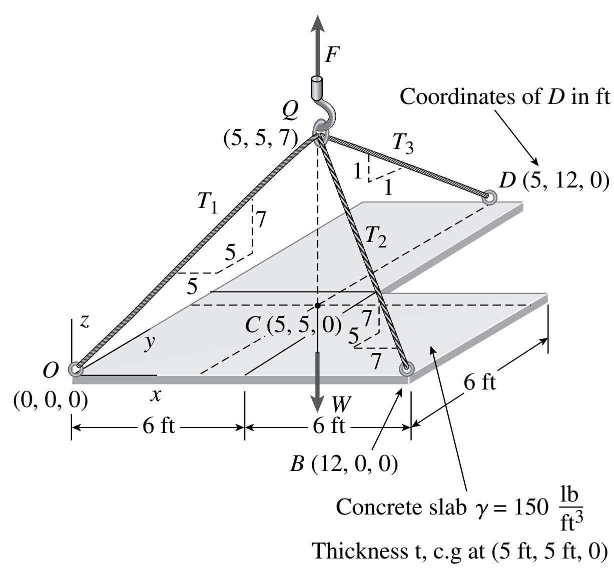
$$\sigma_{cable} = \frac{T}{A_e} \qquad \sigma_{cable} = 74.5\ \text{MPa} \quad \leftarrow$$

(b) $\varepsilon_{cable} = \dfrac{\delta}{L_c} \qquad \varepsilon_{cable} = 4.92 \times 10^{-4} \quad \leftarrow$

Problem ★1.2-11 An L-shaped reinforced concrete slab 12 ft × 12 ft (but with a 6 ft × 6 ft cutout) and thickness $t = 9.0$ in. is lifted by three cables attached at O, B and D, as shown in the figure. The cables are combined at point Q, which is 7.0 ft above the top of the slab and directly above the center of mass at C. Each cable has an effective cross-sectional area of $A_e = 0.12\ \text{in}^2$.

(a) Find the tensile force T_i ($i = 1, 2, 3$) in each cable due to the weight W of the concrete slab (ignore weight of cables).

(b) Find the average stress σ_i in each cable. (See Table H-1 in Appendix H for the weight density of reinforced concrete.)



Solution 1.2-11

CABLE LENGTHS

$$L_1 = \sqrt{5^2 + 5^2 + 7^2} \qquad L_1 = 9.95$$

$$5^2 + 5^2 + 7^2 = 99 \qquad L_1 = \sqrt{99}$$

$$L_2 = \sqrt{5^2 + 7^2 + 7^2} \qquad L_2 = 11.091$$

$$5^2 + 7^2 + 7^2 = 123 \qquad L_2 = \sqrt{123}$$

$$L_3 = \sqrt{7^2 + 7^2} \qquad L_3 = 9.899$$

$$7^2 + 7^2 = 98 \qquad L_3 = 7\sqrt{2}$$

(a) SOLUTION FOR CABLE FORCES USING STATICS (3 EQU, 3 UNKNOWNS)

$$T_1 = \frac{7\sqrt{99}}{144} \qquad T_1 = 0.484 \qquad \delta_1 = \frac{T_1L_1}{EA}$$

$$T_2 = \frac{5\sqrt{123}}{144} \qquad T_2 = 0.385 \qquad \delta_2 = \frac{T_2L_2}{EA}$$

$$T_3 = \frac{5\sqrt{2}}{12} \qquad T_3 = 0.589 \qquad \delta_3 = \frac{T_3L_3}{EA}$$

$$\sum T_{verti} = 0$$

$$T_1\frac{7}{\sqrt{99}} + T_2\frac{7}{\sqrt{123}} + T_3\frac{1}{\sqrt{2}} = 1 \quad \text{CHECK}$$

For unit force in Z-direction

$$\begin{pmatrix} T_1 \\ T_2 \\ T_3 \end{pmatrix} = \begin{pmatrix} \frac{-5}{\sqrt{99}} & \frac{7}{\sqrt{123}} & 0 \\ \frac{-5}{\sqrt{99}} & \frac{-5}{\sqrt{123}} & \frac{1}{\sqrt{2}} \\ \frac{7}{\sqrt{99}} & \frac{7}{\sqrt{123}} & \frac{1}{\sqrt{2}} \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} T_1 \\ T_2 \\ T_3 \end{pmatrix} = \begin{pmatrix} 0.484 \\ 0.385 \\ 0.589 \end{pmatrix} \qquad T_u = \begin{pmatrix} T_1 \\ T_2 \\ T_3 \end{pmatrix}$$

$$\text{check:} \quad T_1\frac{7}{\sqrt{99}} + T_2\frac{7}{\sqrt{123}} + T_3\frac{1}{\sqrt{2}} = 1$$

NOTE: preferred solution uses sum of moments about a line as follows –

1. sum about x-axis to get T3v, then T3
2. sum about y-axis to get T2v, then T2
3. sum vertical forces to get T1v, then T1 OR sum forces in x-dir to get T1x in terms of T2x

SLAB WEIGHT & C.G.

$$W = 150\left(12^2 - 6^2\right)\frac{9}{12} \qquad W = 1.215 \times 10^4$$

$$x_{cg} = \frac{2A3 + A(6 + 3)}{3A}$$

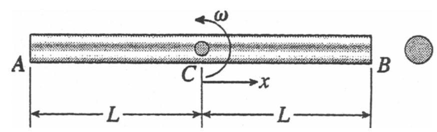
$$x_{cg} = 5 \qquad \text{same for ycg} \qquad y_{cg} = x_{cg}$$

Multiply unit forces by W

$$T = T_uW \qquad T = \begin{pmatrix} 5877 \\ 4679 \\ 7159 \end{pmatrix} \text{lb} \quad \leftarrow$$

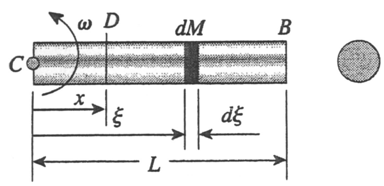
$$\text{(b)} \quad \sigma = \frac{T}{0.12} \qquad \sigma = \begin{pmatrix} 49.0 \text{ ksi} \\ 39.0 \text{ ksi} \\ 60.0 \text{ ksi} \end{pmatrix} \text{psi} \quad \leftarrow$$

Problem *1.2-12 A round bar *ACB* of length $2L$ (see figure) rotates about an axis through the midpoint *C* with constant angular speed ω (radians per second). The material of the bar has weight density γ.



(a) Derive a formula for the tensile stress σ_x in the bar as a function of the distance x from the midpoint *C*.
(b) What is the maximum tensile stress σ_{max}?

Solution 1.2-12 Rotating Bar



ω = angular speed (rad/s)

A = cross-sectional area

γ = weight density

$\frac{\gamma}{g}$ = mass density

We wish to find the axial force F_x in the bar at Section D, distance x from the midpoint *C*.

The force F_x equals the inertia force of the part of the rotating bar from *D* to *B*.

Consider an element of mass dM at distance ξ from the midpoint *C*. The variable ξ ranges from x to L.

$$dM = \frac{\gamma}{g} A\, dj$$

dF = Inertia force (centrifugal force) of element of mass dM

$$dF = (dM)(j\omega^2) = \frac{\gamma}{g} \quad A\omega^2 j dj$$

$$F_x = \int_D^B dF = \int_x^L \frac{\gamma}{g} A\omega^2 j dj = \frac{\gamma A\omega^2}{2g}(L^2 - x^2)$$

(a) TENSILE STRESS IN BAR AT DISTANCE x

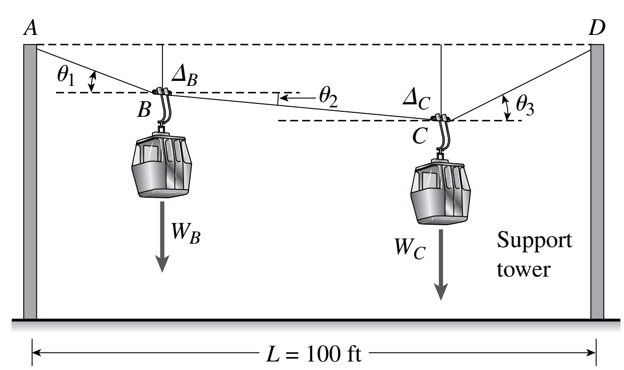
$$\sigma_x = \frac{F_x}{A} = \frac{\gamma\omega^2}{2g}(L^2 - x^2) \quad \leftarrow$$

(b) MAXIMUM TENSILE STRESS

$$x = 0 \quad \sigma_{max} = \frac{\gamma\omega^2 L^2}{2g} \quad \leftarrow$$

Problem 1.2-13 Two gondolas on a ski lift are locked in the position shown in the figure while repairs are being made elsewhere. The distance between support towers is L = 100 ft. The length of each cable segment under gondola weights W_B = 450 lb and W_C = 650 lb are D_{AB} = 12 ft, D_{BC} = 70 ft, and D_{CD} = 20 ft. The cable sag at *B* is Δ_B = 3.9 ft and that at C(ΔC) is 7.1 ft. The effective cross-sectional area of the cable is $A_e = 0.12\ in^2$.

(a) Find the tension force in each cable segment; neglect the mass of the cable.
(b) Find the average stress (σ) in each cable segment.



Solution 1.2-13

$W_B = 450$

$Wc = 650$ lb

$\Delta_B = 3.9$ ft

$\Delta_C = 7.1$ ft

$L = 100$ ft

$D_{AB} = 12$ ft

$D_{BC} = 70$ ft

$D_{CD} = 20$ ft

$D_{AB} + D_{BC} + D_{CD} = 102$ ft

$A_e = 0.12\ \text{in}^2$

Compute initial values of theta angles (radians)

$$\theta_1 = \operatorname{asin}\left(\frac{\Delta_B}{D_{AB}}\right) \qquad \theta_1 = 0.331$$

$$\theta_2 = \operatorname{asin}\left(\frac{\Delta_C - \Delta_B}{D_{BC}}\right) \qquad \theta_2 = 0.046$$

$$\theta_3 = \operatorname{asin}\left(\frac{\Delta_C}{D_{CD}}\right) \qquad \theta_3 = 0.363$$

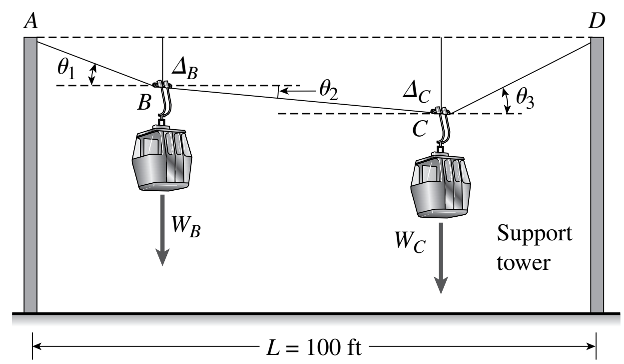
(a) Statics at B & C

$$-T_{AB}\cos(\theta_1) + T_{BC}\cos(\theta_2) = 0$$

$$T_{AB}\sin(\theta_1) - T_{BC}\sin(\theta_2) = W_B$$

$$-T_{BC}\cos(\theta_2) + T_{CD}\cos(\theta_3) = 0$$

$$T_{BC}\sin(\theta_2) - T_{CD}\sin(\theta_3) = W_C$$



Contraint equations

$$D_{AB}\cos(\theta_1) + D_{BC}\cos(\theta_2) + D_{CD}\cos(\theta_3) = L$$

$$D_{AB}\sin(\theta_1) + D_{BC}\sin(\theta_2) = D_{CD}\sin(\theta_3)$$

Solve simultaneous equations numerically for tension force in each cable segment

$T_{AB} = 1620$ lb $\quad T_{CB} = 1536$ lb $\quad T_{CD} = 1640$ lb $\quad \leftarrow$

Check equilibrium at B & C

$$T_{AB}\sin(\theta_1) - T_{BC}\sin(\theta_2) = 450$$

$$T_{BC}\sin(\theta_2) - T_{CD}\sin(\theta_3) = 650$$

(b) Compute stresses in cable segments

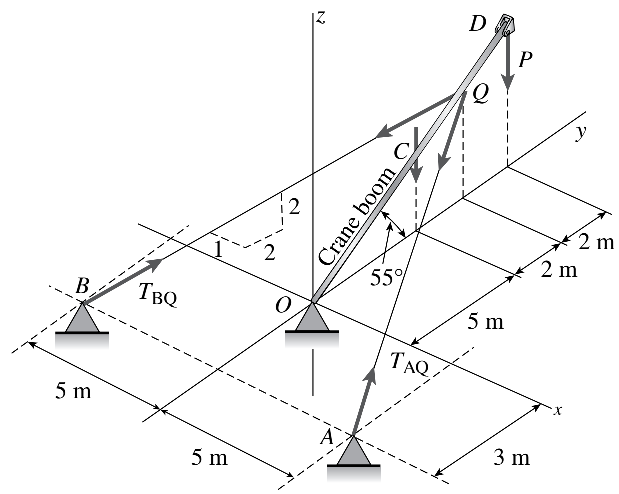
$$\sigma_{AB} = \frac{T_{AB}}{A_e} \qquad \sigma_{BC} = \frac{T_{BC}}{A_e} \qquad \sigma_{CD} = \frac{T_{CD}}{A_e}$$

$\sigma_{AB} = 13.5$ ksi $\quad \sigma_{BC} = 12.8$ ksi

$\sigma_{CD} = 13.67$ ksi $\quad \leftarrow$

Problem 1.2-14 A crane boom of mass 450 kg with its center of mass at *C* is stabilized by two cables *AQ* and *BQ* (A_e = 304 mm^2 for each cable) as shown in the figure. A load *P* = 20 kN is supported at point *D*. The crane boom lies in the *y*–*z* plane.

(a) Find the tension forces in each cable: T_{AQ} and T_{BQ} (kN); neglect the mass of the cables, but include the mass of the boom in addition to load *P*.
(b) Find the average stress (σ) in each cable.



Solution 1.2-14

Data $\quad M_{boom} = 450$ kg

$g = 9.81\,\dfrac{m}{s^2} \qquad W_{boom} = M_{boom}\,g$

$W_{boom} = 4415$ N

$P = 20$ kN

$A_e = 304$ mm^2

(a) symmetry: $T_{AQ} = T_{BQ}$

$$\sum M_x = 0$$

$$2T_{AQZ}(3000) = W_{boom}(5000) + P(9000)$$

$$T_{AQZ} = \frac{W_{boom}(5000) + P(9000)}{2(3000)}$$

$$T_{AQ} = \sqrt{\frac{2^2 + 2^2 + 1^2}{2}}\,T_{AQz}$$

$$T_{AQ} = 50.5 \text{ kN} = T_{BQ} \quad \leftarrow$$

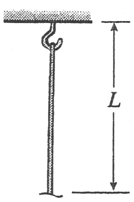
(b) $\sigma = \dfrac{T_{AQ}}{A_e} \qquad \sigma = 166.2$ MPa $\quad \leftarrow$

Mechanical Properties of Materials

Problem 1.3-1 Imagine that a long steel wire hangs vertically from a high-altitude balloon.

(a) What is the greatest length (feet) it can have without yielding if the steel yields at 40 ksi?
(b) If the same wire hangs from a ship at sea, what is the greatest length? (Obtain the weight densities of steel and sea water from Table H-1, Appendix H.)

Solution 1.3-1 Hanging wire of length *L*



W = total weight of steel wire

γ_S = weight density of steel

$= 490 \text{ lb/ft}^3$

γ_w = weight density of sea water

$= 63.8 \text{ lb/ft}^3$

A = cross-sectional area of wire

$\sigma_{max} = 40$ ksi (yield strength)

(a) Wire hanging in air

$$W = \gamma_S A L$$

$$\sigma_{max} = \frac{W}{A} = \gamma_S L$$

$$L_{max} = \frac{\sigma_{max}}{\gamma_S} = \frac{40{,}000 \text{ psi}}{490 \text{ lb/ft}^3}(144 \text{ in.}^2/\text{ft}^2)$$

$$= 11{,}800 \text{ ft} \quad \longleftarrow$$

(b) Wire hanging in sea water

F = tensile force at top of wire

$$F = (\gamma_S - \gamma_W)AL \quad \sigma_{max} = \frac{F}{A} = (\gamma_S - \gamma_W)L$$

$$L_{max} = \frac{\sigma_{max}}{\gamma_S - \gamma_W}$$

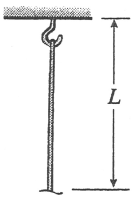
$$= \frac{40{,}000 \text{ psi}}{(490 - 63.8) \text{ lb/ft}^3}(144 \text{ in.}^2/\text{ft}^2)$$

$$= 13{,}500 \text{ ft} \quad \longleftarrow$$

Problem 1.3-2 Imagine that a long wire of tungsten hangs vertically from a high-altitude balloon.

(a) What is the greatest length (meters) it can have without breaking if the ultimate strength (or breaking strength) is 1500 MPa?
(b) If the same wire hangs from a ship at sea, what is
the greatest length? (Obtain the weight densities of tungsten and sea water from Table H-1, Appendix H.)

Solution 1.3-2 Hanging wire of length L



W = total weight of tungsten wire

γ_T = weight density of tungsten $= 190 \text{ kN/m}^3$

γ_W = weight density of sea water $= 10.0 \text{ kN/m}^3$

A = cross-sectional area of wire

$\sigma_{\max} = 1500$ MPa (breaking strength)

(a) Wire hanging in air

$$W = \gamma_T A L$$

$$\sigma_{\max} = \frac{W}{A} = \gamma_T L$$

$$L_{\max} = \frac{\sigma_{\max}}{\gamma_T} = \frac{1500\text{MPa}}{190 \text{ kN/m}^3}$$

$$= 7900 \text{ m} \quad \leftarrow$$

(b) Wire hanging in sea water

F = tensile force at top of wire

$$F = (\gamma_T - \gamma_W)AL$$

$$\sigma_{\max} = \frac{F}{A} = (\gamma_T - \gamma_W)L$$

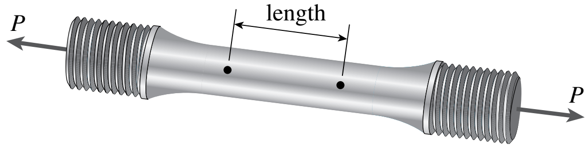
$$L_{\max} = \frac{\sigma_{\max}}{\gamma_T - \gamma_W}$$

$$= \frac{1500\text{MPa}}{(190 - 10.0) \text{ kN/m}^3}$$

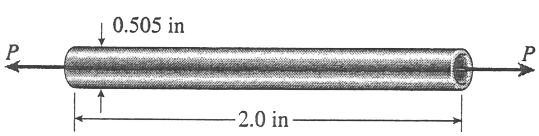
$$= 8300 \text{ m} \quad \leftarrow$$

Problem 1.3-3 Three different materials, designated A, B, and C, are tested in tension using test specimens having diameters of 0.505 in. and gage lengths of 2.0 in. (see figure). At failure, the distances between the gage marks are found to be 2.13, 2.48, and 2.78 in., respectively. Also, at the failure cross sections the diameters are found to be 0.484, 0.398, and 0.253 in., respectively.

Determine the percent elongation and percent reduction in area of each specimen, and then, using your own judgment, classify each material as brittle or ductile.



Solution 1.3-3 Tensile tests of three materials



$$\text{Percent elongation} = \frac{L_1 - L_0}{L_0}(100) = \left(\frac{L_1}{L_0} - 1\right)100$$

$L_0 = 2.0$ in.

$$\text{Percent elongation} = \left(\frac{L_1}{2.0} - 1\right)(100) \qquad \text{(Eq. 1)}$$

where L_1 is in inches.

$$\text{Percent reduction in area} = \frac{A_0 - A_1}{A_0}(100) = \left(1 - \frac{A_1}{A_0}\right)(100)$$

d_0 = initial diameter $\qquad d_1$ = final diameter

$$\frac{A_1}{A_0} = \left(\frac{d_1}{d_0}\right)^2 \qquad d_0 = 0.505 \text{ in.}$$

Percent reduction in area

$$= \left[1 - \left(\frac{d_1}{0.505}\right)^2\right](100) \qquad \text{(Eq. 2)}$$

where d_1 is in inches.

Material	L_1 (in.)	d_1 (in.)	% Elongation (Eq. 1)	% Reduction (Eq. 2)	Brittle or Ductile?
A	2.13	0.484	6.5%	8.1%	Brittle
B	2.48	0.398	24.0%	37.9%	Ductile
C	2.78	0.253	39.0%	74.9%	Ductile

Problem 1.3-4 The *strength-to-weight ratio* of a structural material is defined as its load-carrying capacity divided by its weight. For materials in tension, we may use a characteristic tensile stress (as obtained from a stress-strain curve) as a measure of strength. For instance, either the yield stress or the ultimate stress could be used, depending upon the particular application. Thus, the strength-to-weight ratio $R_{S/W}$ for a material in tension is defined as

$$R_{S/W} = \frac{\sigma}{\gamma}$$

in which σ is the characteristic stress and γ is the weight density. Note that the ratio has units of length.

Using the ultimate stress σ_U as the strength parameter, calculate the strength-to-weight ratio (in units of meters) for each of the following materials: aluminum alloy 6061-T6, Douglas fir (in bending), nylon, structural steel ASTM-A572, and a titanium alloy. (Obtain the material properties from Tables H-1 and H-3 of Appendix H. When a range of values is given in a table, use the average value.)

Solution 1.3-4 Strength-to-weight ratio

The ultimate stress σ_U for each material is obtained from Table H-3, Appendix H, and the weight density γ is obtained from Table H-1.

The strength-to-weight ratio (meters) is

$$R_{S/W} = \frac{\sigma_U\,(\text{MPa})}{\gamma(\text{kN/m}^3)}\,(10^3)$$

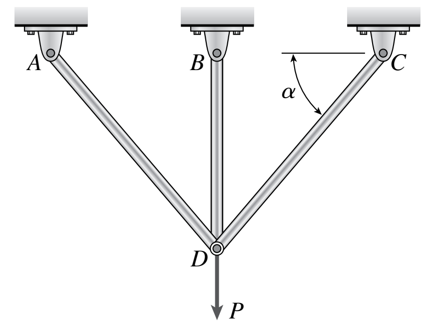
Values of σ_U, γ, and $R_{S/W}$ are listed in the table.

	σ_U (MPa)	γ (kN/m^3)	$R_{S/W}$ (m)
Aluminum alloy 6061-T6	310	26.0	11.9×10^3
Douglas fir	65	5.1	12.7×10^3
Nylon	60	9.8	6.1×10^3
Structural steel ASTM-A572	500	77.0	6.5×10^3
Titanium alloy	1050	44.0	23.9×10^3

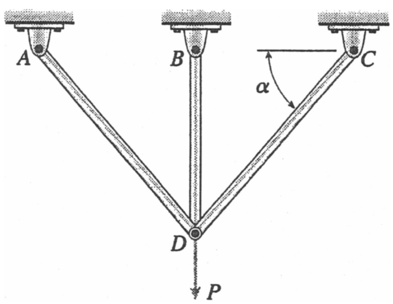
Titanium has a high strength-to-weight ratio, which is why it is used in space vehicles and high-performance airplanes. Aluminum is higher than steel, which makes it desirable for commercial aircraft. Some woods are also higher than steel, and nylon is about the same as steel.

Problem 1.3-5 A symmetrical framework consisting of three pin-connected bars is loaded by a force P (see figure). The angle between the inclined bars and the horizontal is $\alpha = 48°$. The axial strain in the middle bar is measured as 0.0713.

Determine the tensile stress in the outer bars if they are constructed of aluminum alloy having the stress-strain diagram shown in Fig. 1-13. (Express the stress in USCS units.)



Solution 1.3-5 Symmetrical framework

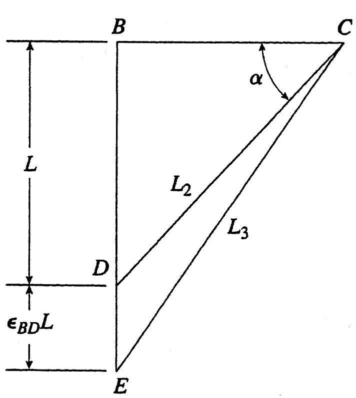


Aluminum alloy

$\alpha = 48°$

$\varepsilon_{BD} = 0.0713$

Use stress-strain diagram of Figure 1-13



L = length of bar BD

L_1 = distance BC

$= L \cot \alpha = L(\cot 48°) = 0.9004\,L$

L_2 = length of bar CD

$= L \csc \alpha = L(\csc 48°) = 1.3456\,L$

Elongation of bar BD = distance $DE = \varepsilon_{BD}L$

$\varepsilon_{BD}L = 0.0713\,L$

L_3 = distance CE

$$L_3 = \sqrt{L_1^2 + (L + \varepsilon_{BD}L)^2}$$

$$= \sqrt{(0.9004L)^2 + L^2(1 + 0.0713)^2}$$

$$= 1.3994\,L$$

δ = elongation of bar CD

$\delta = L_3 - L_2 = 0.0538L$

Strain in bar CD

$$= \frac{\delta}{L_2} = \frac{0.0538L}{1.3456L} = 0.0400$$

From the stress-strain diagram of Figure 1-13:

$\sigma \approx 31$ ksi $\quad\leftarrow$

Problem 1.3-6 A specimen of a methacrylate plastic is tested in tension at room temperature (see figure), producing the stress-strain data listed in the accompanying table.

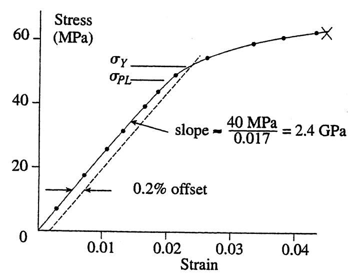
Plot the stress-strain curve and determine the proportional limit, modulus of elasticity (i.e., the slope of the initial part of the stress-strain curve), and yield stress at 0.2% offset. Is the material ductile or brittle?

STRESS-STRAIN DATA FOR PROBLEM 1.3-6

Stress (MPa)	Strain
8.0	0.0032
17.5	0.0073
25.6	0.0111
31.1	0.0129
39.8	0.0163
44.0	0.0184
48.2	0.0209
53.9	0.0260
58.1	0.0331
62.0	0.0429
62.1	Fracture

Solution 1.3-6 Tensile test of a plastic

Using the stress-strain data given in the problem statement, plot the stress-strain curve:



σ_{PL} = proportional limit $\quad \sigma_{PL} \approx 47$ MPa $\quad \leftarrow$

Modulus of elasticity (slope) $\quad \approx 2.4$ GPa $\quad \leftarrow$

σ_Y = yield stress at 0.2% offset

$\sigma_Y \approx 53$ MPa $\quad \leftarrow$

Material is *brittle*, because the strain after the proportional limit is exceeded is relatively small. $\leftarrow$

Problem 1.3-7 The data shown in the accompanying table were obtained from a tensile test of high-strength steel. The test specimen had a diameter of 0.505 in. and a gage length of 2.00 in. (see figure for Prob. 1.3-3). At fracture, the elongation between the gage marks was 0.12 in. and the minimum diameter was 0.42 in.

Plot the conventional stress-strain curve for the steel and determine the proportional limit, modulus of elasticity (i.e., the slope of the initial part of the stress-strain curve), yield stress at 0.1% offset, ultimate stress, percent elongation in 2.00 in., and percent reduction in area.

TENSILE-TEST DATA FOR PROBLEM 1.3-7

Load (lb)	Elongation (in.)
1,000	0.0002
2,000	0.0006
6,000	0.0019
10,000	0.0033
12,000	0.0039
12,900	0.0043
13,400	0.0047
13,600	0.0054
13,800	0.0063
14,000	0.0090
14,400	0.0102
15,200	0.0130
16,800	0.0230
18,400	0.0336
20,000	0.0507
22,400	0.1108
22,600	Fracture

Solution 1.3-7 Tensile test of high-strength steel

$d_0 = 0.505$ in. $\quad L_0 = 2.00$ in.

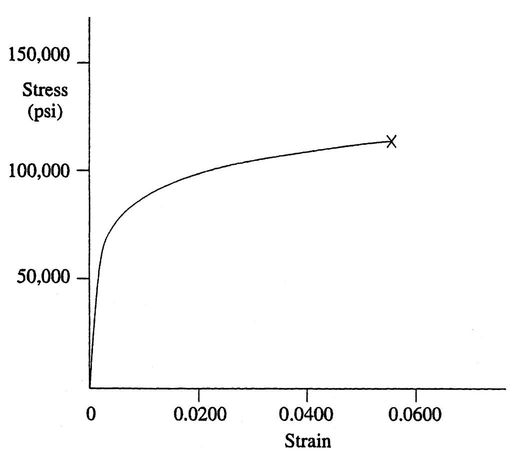
$$A_0 = \frac{\pi d_0^2}{4} = 0.200 \text{ in.}^2$$

CONVENTIONAL STRESS AND STRAIN

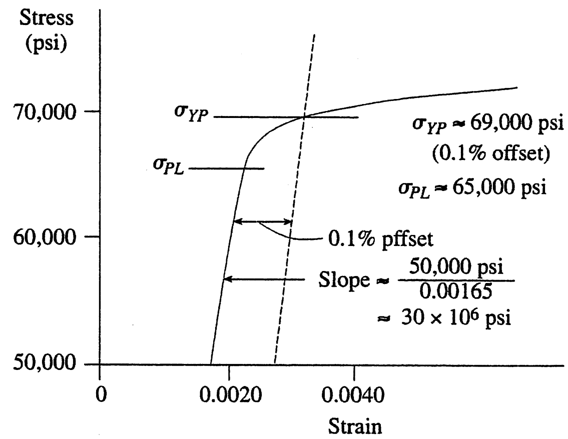
$$\sigma = \frac{P}{A_0} \quad \varepsilon = \frac{\delta}{L_0}$$

Load P (lb)	Elongation δ (in.)	Stress σ (psi)	Strain ε
1,000	0.0002	5,000	0.00010
2,000	0.0006	10,000	0.00030
6,000	0.0019	30,000	0.00100
10,000	0.0033	50,000	0.00165
12,000	0.0039	60,000	0.00195
12,900	0.0043	64,500	0.00215
13,400	0.0047	67,000	0.00235
13,600	0.0054	68,000	0.00270
13,800	0.0063	69,000	0.00315
14,000	0.0090	70,000	0.00450
14,400	0.0102	72,000	0.00510
15,200	0.0130	76,000	0.00650
16,800	0.0230	84,000	0.01150
18,400	0.0336	92,000	0.01680
20,000	0.0507	100,000	0.02535
22,400	0.1108	112,000	0.05540
22,600	Fracture	113,000	

STRESS-STRAIN DIAGRAM



ENLARGEMENT OF PART OF THE STRESS-STRAIN CURVE



RESULTS

Proportional limit ≈ 65,000 psi ←

Modulus of elasticity (slope) ≈ 30×10^6 psi ←

Yield stress at 0.1% offset ≈ 69,000 psi ←

Ultimate stress (maximum stress)

≈ 113,000 psi ←

Percent elongation in 2.00 in.

$$= \frac{L_1 - L_0}{L_0}(100)$$

$$= \frac{0.12 \text{ in.}}{2.00 \text{ in.}}(100) = 6\% \quad \leftarrow$$

Percent reduction in area

$$= \frac{A_0 - A_1}{A_0}(100)$$

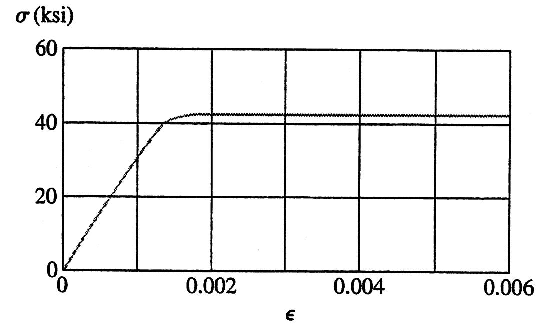
$$= \frac{0.200 \text{ in.}^2 - \frac{\pi}{4}(0.42 \text{ in.})^2}{0.200 \text{ in.}^2}(100)$$

$$= 31\% \quad \leftarrow$$

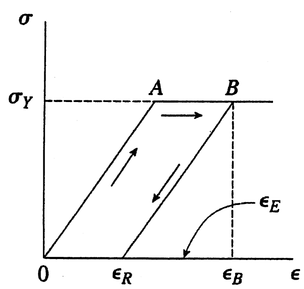
Elasticity, Plasticity, and Creep

Problem 1.4-1 A bar made of structural steel having the stress-strain diagram shown in the figure has a length of 48 in. The yield stress of the steel is 42 ksi and the slope of the initial linear part of the stress-strain curve (modulus of elasticity) is 30×10^3 ksi. The bar is loaded axially until it elongates 0.20 in., and then the load is removed.

How does the final length of the bar compare with its original length? (*Hint*: Use the concepts illustrated in Fig. 1-18b.)



Solution 1.4-1 **Steel bar in tension**



$L = 48$ in.

Yield stress $\sigma_Y = 42$ ksi

Slope $= 30 \times 10^3$ ksi

$\delta = 0.20$ in.

Stress and strain at point B

$\sigma_B = \sigma_Y = 42$ ksi

$$\varepsilon_B = \frac{\delta}{L} = \frac{0.20 \text{ in.}}{48 \text{ in.}} = 0.00417$$

Elastic recovery ε_E

$$\varepsilon_E = \frac{\sigma_B}{\text{Slope}} = \frac{42 \text{ ksi}}{30 \times 10^3 \text{ ksi}} = 0.00140$$

Residual strain ε_R

$$\varepsilon_R = \varepsilon_B - \varepsilon_E = 0.00417 - 0.00140$$
$$= 0.00277$$

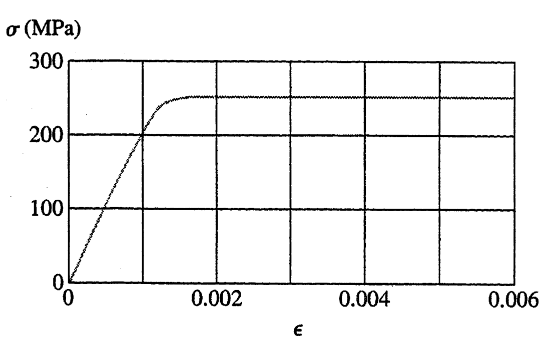
Permanent set

$$\varepsilon_R L = (0.00277)(48 \text{ in.})$$
$$= 0.13 \text{ in.}$$

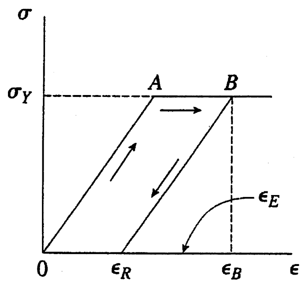
Final length of bar is 0.13 in. greater than its original length. ←

Problem 1.4-2 A bar of length 2.0 m is made of a structural steel having the stress-strain diagram shown in the figure. The yield stress of the steel is 250 MPa and the slope of the initial linear part of the stress-strain curve (modulus of elasticity) is 200 GPa. The bar is loaded axially until it elongates 6.5 mm, and then the load is removed.

How does the final length of the bar compare with its original length? (*Hint*: Use the concepts illustrated in Fig. 1-18b.)



Solution 1.4-2 Steel bar in tension



$L = 2.0$ m $= 2000$ mm

Yield stress $\sigma_Y = 250$ MPa

Slope $= 200$ GPa

$\delta = 6.5$ mm

STRESS AND STRAIN AT POINT B

$\sigma_B = \sigma_Y = 250$ MPa

$$\varepsilon_B = \frac{\delta}{L} = \frac{6.5\text{ mm}}{2000\text{ mm}} = 0.00325$$

ELASTIC RECOVERY ε_E

$$\varepsilon_E = \frac{\sigma_B}{\text{Slope}} = \frac{250\text{ MPa}}{200\text{ GPa}} = 0.00125$$

RESIDUAL STRAIN ε_R

$$\varepsilon_R = \varepsilon_B - \varepsilon_E = 0.00325 - 0.00125$$
$$= 0.00200$$

Permanent set $= \varepsilon_R L = (0.00200)(2000\text{ mm})$
$= 4.0$ mm

Final length of bar is 4.0 mm greater than its original length. ←

Problem 1.4-3 An aluminum bar has length $L = 5$ ft and diameter $d = 1.25$ in. The stress-strain curve for the aluminum is shown in Fig. 1-13 of Section 1.3. The initial straight-line part of the curve has a slope (modulus of elasticity) of 10×10^6 psi. The bar is loaded by tensile forces $P = 39$ k and then unloaded.

(a) What is the permanent set of the bar?
(b) If the bar is reloaded, what is the proportional limit? (*Hint*: Use the concepts illustrated in Figs. 1-18b and 1-19.)

Solution 1.4-3

(a) PERMAMENT SET

Numerical data $\mathrm{L} = 60$ in

$\mathrm{d} = 1.25$ in $\quad \mathrm{P} = 39$ kips

STRESS AND STRAIN AT PT B

$$\sigma_{\mathrm{B}} = \frac{\mathrm{P}}{\frac{\pi}{4}\mathrm{d}^2}$$

$\sigma_{\mathrm{B}} = 31.78$ ksi

From Figure 1-13 $\varepsilon_{\mathrm{B}} = 0.05$

ELASTIC RECOVERY

$$\varepsilon_{\mathrm{E}} = \frac{\sigma_{\mathrm{B}}}{10(10)^3} \qquad \varepsilon_{\mathrm{E}} = 3.178 \times 10^{-3}$$

RESIDUAL STRAIN

$\varepsilon_{\mathrm{E}} = \varepsilon_{\mathrm{B}} - \varepsilon_{\mathrm{E}} \qquad \varepsilon_{\mathrm{R}} = 0.047$

PERMANENT SET

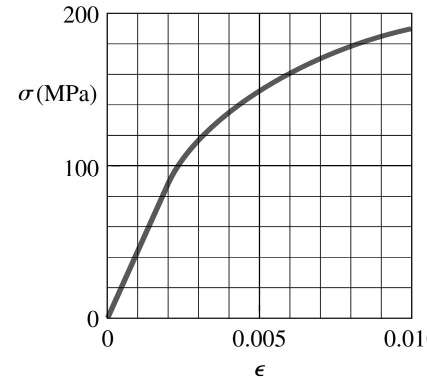
$\varepsilon_{\mathrm{R}}\mathrm{L} = 2.81$ in. ←

(b) PROPORTIONAL LIMIT WHEN RELOADED

$\sigma_{\mathrm{B}} = 31.8$ ksi ←

Problem 1.4-4 A circular bar of magnesium alloy is 750 mm long. The stress-strain diagram for the material is shown in the figure. The bar is loaded in tension to an elongation of 6.0 mm, and then the load is removed.

(a) What is the permanent set of the bar?
(b) If the bar is reloaded, what is the proportional limit? (*Hint*: Use the concepts illustrated in Figs. 1-18b and 1-19.)



Solution 1.4-4

NUMERICAL DATA $L = 750$ mm $\delta = 6$ mm

STRESS AND STRAIN AT PT B

$\varepsilon_B = \dfrac{\delta}{L}$ $\varepsilon_B = 8 \times 10^{-3}$ $\sigma_B = 180$ MPa

ELASTIC RECOVERY

$\text{slope} = \dfrac{178}{0.004}$ $\text{slope} = 4.45 \times 10^4$

$\varepsilon_E = \dfrac{\sigma_B}{\text{slope}}$

$\varepsilon_E = 4.045 \times 10^{-3}$

RESIDUAL STRAIN

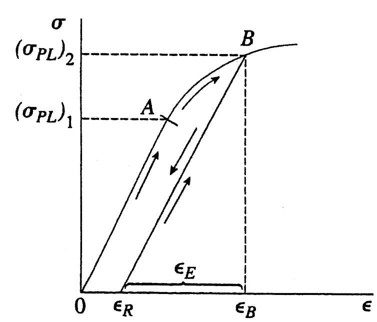
$\varepsilon_R = \varepsilon_B - \varepsilon_E$ $\varepsilon_R = 3.955 \times 10^{-3}$

(a) PERMANENT SET

$\varepsilon_R L = 2.97$ mm ←

(b) PROPORTIONAL LIMIT WHEN RELOADED

$\sigma_B = 180$ MPa ←



Problem 1.4-5 A wire of length $L = 4$ ft and diameter $d = 0.125$ in. is stretched by tensile forces $P = 600$ lb. The wire is made of a copper alloy having a stress-strain relationship that may be described mathematically by the following equation:

$$\sigma = \frac{18{,}000\epsilon}{1 + 300\epsilon} \qquad 0 \le \epsilon \le 0.03 \quad (\sigma = \text{ksi})$$

in which ϵ is nondimensional and σ has units of kips per square inch (ksi).

(a) Construct a stress-strain diagram for the material.
(b) Determine the elongation of the wire due to the forces P.
(c) If the forces are removed, what is the permanent set of the bar?
(d) If the forces are applied again, what is the proportional limit?

Solution 1.4-5 Wire stretched by forces P

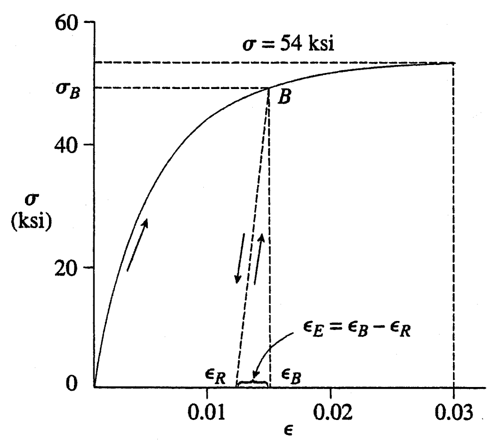
$L = 4 \text{ ft} = 48 \text{ in.} \qquad d = 0.125 \text{ in.}$

$P = 600 \text{ lb}$

COPPER ALLOY

$$\sigma = \frac{18{,}000\varepsilon}{1 + 300\varepsilon} \quad 0 \le \varepsilon \le 0.03 \ (\sigma = \text{ksi}) \quad \text{(Eq. 1)}$$

(a) STRESS-STRAIN DIAGRAM (From Eq. 1)



INITIAL SLOPE OF STRESS-STRAIN CURVE

Take the derivative of σ with respect to ε:

$$\frac{d\sigma}{d\varepsilon} = \frac{(1 + 300\varepsilon)(18{,}000) - (18{,}000)(300)\sigma}{(1 + 300\varepsilon)^2}$$

$$= \frac{18{,}000}{(1 + 300\varepsilon)^2}$$

At $\varepsilon = 0, \quad \dfrac{d\sigma}{d\varepsilon} = 18{,}00 \text{ ksi}$

$\therefore$ Initial slope $= 18{,}000$ ksi

ALTERNATIVE FORM OF THE STRESS-STRAIN RELATIONSHIP

Solve Eq. (1) for ε in terms of σ:

$$\varepsilon = \frac{\sigma}{18{,}000 - 300\sigma} \quad 0 \le \sigma \le 54 \text{ ksi} \ (\sigma = \text{ksi}) \quad \text{(Eq. 2)}$$

This equation may also be used when plotting the stress-strain diagram.

(b) ELONGATION δ OF THE WIRE

$$\sigma = \frac{P}{A} = \frac{600 \text{ lb}}{\frac{\pi}{4}(0.125 \text{ in.})^2} 48{,}900 \text{ psi} = 48.9 \text{ ksi}$$

From Eq. (2) or from the stress-strain diagram:

$\varepsilon = 0.0147$

$\delta = \varepsilon L = (0.0147)(48 \text{ in.}) = 0.71 \text{ in.} \quad \leftarrow$

STRESS AND STRAIN AT POINT B (see diagram)

$\sigma_B = 48.9 \text{ ksi} \quad \varepsilon_B = 0.0147$

ELASTIC RECOVERY ε_E

$$\varepsilon_E = \frac{\sigma_B}{\text{Slope}} = \frac{48.9 \text{ ksi}}{18{,}000 \text{ ksi}} = 0.00272$$

RESIDUAL STRAIN ε_R

$\varepsilon_R = \varepsilon_B - \varepsilon_E = 0.0147 - 0.0027 = 0.0120$

(c) Permanent set $= \varepsilon_R L = (0.0120)(48 \text{ in.})$

$= 0.58 \text{ in.} \quad \leftarrow$

(d) Proportional limit when reloaded $= \sigma_B$

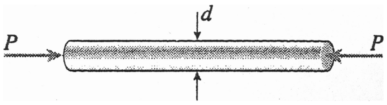
$\sigma_B = 49 \text{ ksi} \quad \leftarrow$

Linear Elasticity, Hooke's Law, and Poisson's Ratio

When solving the problems for Section 1.5, assume that the material behaves linearly elastically.

Problem 1.5-1 A high-strength steel bar used in a large crane has diameter $d = 2.00$ in. (see figure). The steel has modulus of elasticity $E = 29 \times 10^6$ psi and Poisson's ratio $\nu = 0.29$. Because of clearance requirements, the diameter of the bar is limited to 2.001 in. when it is compressed by axial forces.

What is the largest compressive load P_{max} that is permitted?



Solution 1.5-1 Steel bar in compression

STEEL BAR

$d = 2.00$ in. Max. $\Delta d = 0.001$ in.

$E = 29 \times 10^6$ psi $\nu = 0.29$

LATERAL STRAIN

$$\varepsilon' = \frac{\Delta d}{d} = \frac{0.001 \text{ in.}}{2.00 \text{ in.}} = 0.0005$$

AXIAL STRAIN

$$\varepsilon = -\frac{\varepsilon'}{\nu} = -\frac{0.0005}{0.29} = -0.001724$$

(shortening)

AXIAL STRESS

$$\sigma = E\varepsilon = (29 \times 10^6 \text{ psi})(-0.001724)$$
$$= -50.00 \text{ ksi (compression)}$$

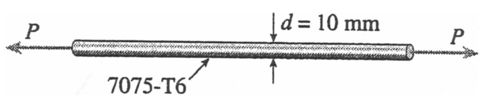
Assume that the yield stress for the high-strength steel is greater than 50 ksi. Therefore, Hooke's law is valid.

MAXIMUM COMPRESSIVE LOAD

$$P_{max} = \sigma A = (50.00 \text{ ksi})\left(\frac{\pi}{4}\right)(2.00 \text{ in.})^2$$
$$= 157 \text{ k} \quad \leftarrow$$

Problem 1.5-2 A round bar of 10 mm diameter is made of aluminum alloy 7075-T6 (see figure). When the bar is stretched by axial forces P, its diameter decreases by 0.016 mm.

Find the magnitude of the load P. (Obtain the material properties from Appendix H.)



Solution 1.5-2 Aluminum bar in tension

$d = 10$ mm $\Delta d = 0.016$ mm

(Decrease in diameter)

7075-T6

From Table H-2: $E = 72$ GPa $\nu = 0.33$

From Table H-3: Yield stress $\sigma_Y = 480$ MPa

LATERAL STRAIN

$$\varepsilon' = \frac{\Delta d}{d} = \frac{-0.016\text{mm}}{10\text{mm}} = -0.0016$$

AXIAL STRAIN

$$\varepsilon = -\frac{\varepsilon'}{\nu} = \frac{0.0016}{0.33}$$
$$= 0.004848 \text{ (Elongation)}$$

AXIAL STRESS

$$\sigma = E\varepsilon = (72 \text{ GPa})(0.004848)$$
$$= 349.1 \text{ MPa (Tension)}$$

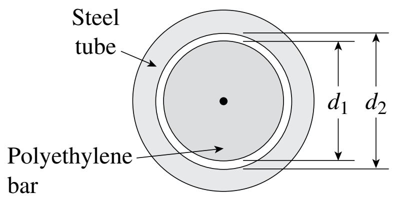
Because $\sigma < \sigma_Y$, Hooke's law is valid.

LOAD P (TENSILE FORCE)

$$P = \sigma A = (349.1 \text{ MPa})\left(\frac{\pi}{4}\right)(10 \text{ mm})^2$$
$$= 27.4 \text{ kN} \quad \leftarrow$$

Problem 1.5-3 A polyethylene bar having diameter $d_1 = 4.0$ in. is placed inside a steel tube having inner diameter $d_2 = 4.01$ in. (see figure). The polyethylene bar is then compressed by an axial force P.

At what value of the force P will the space between the nylon bar and the steel tube be closed? (For nylon, assume $E = 400$ ksi and $\nu = 0.4$.)



Solution 1.5-3

NUMERICAL DATA

$d_1 = 4$ in $\qquad d_2 = 4.01$ in. $\qquad E = 200$ ksi

$\nu = 0.4 \qquad \Delta d_1 = 0.01$ in

$$A_1 = \frac{\pi}{4}d_1^2 \qquad A_2 = \frac{\pi}{4}d_2^2 \qquad A_1 = 12.566\ \text{in}^2$$

$A_2 = 12.629\ \text{in}^2$

LATERAL STRAIN

$$\varepsilon_p = \frac{\Delta d_1}{d_1} \qquad \varepsilon_p = \frac{0.01}{4} \qquad \varepsilon_p = 2.5 \times 10^{-3}$$

NORMAL STRAIN

$$\varepsilon_1 = \frac{-\varepsilon_p}{\nu} \qquad \varepsilon_1 = -6.25 \times 10^{-3}$$

AXIAL STRESS

$\sigma_1 = E\,\varepsilon_1 \qquad \sigma_1 = -1.25$ ksi

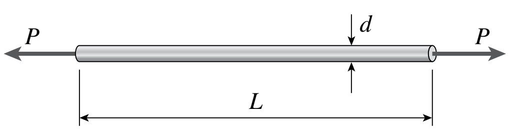
COMPRESSION FORCE

$P = EA_1\varepsilon_1$

$P = -15.71$ kips $\qquad \leftarrow$

Problem 1.5-4 A prismatic bar with a circular cross section is loaded by tensile forces $P = 65$ kN (see figure). The bar has length $L = 1.75$ m and diameter $d = 32$ mm. It is made of aluminum alloy with modulus of elasticity $E = 75$ GPa and Poisson's ratio $\nu = 1/3$.

Find the increase in length of the bar and the percent decrease in its cross-sectional area.



Solution 1.5-4

NUMERICAL DATA

$P = 65\ \text{kN} \qquad \nu = \frac{1}{3}$

$d = 32\ \text{mm} \quad L = 1.75(1000)\ \text{mm}$

$E = 75\ \text{GPa}$

INITIAL AREA OF CROSS SECTION

$A_i = \frac{\pi}{4}d^2 \qquad A_i = 804.248\ \text{mm}^2$

AXIAL STRAIN

$\varepsilon = \frac{P}{EA_i} \qquad \varepsilon = 1.078 \times 10^{-3}$

INCREASE IN LENGTH

$\Delta L = \varepsilon L \qquad \Delta L = 1.886\ \text{mm} \quad \leftarrow$

LATERAL STRAIN

$\varepsilon_p = -\nu\varepsilon \qquad \varepsilon_p = -3.592 \times 10^{-4}$

DECREASE IN DIAMETER

$\Delta d = |\varepsilon_p d| \qquad \Delta d = 0.011\ \text{mm}$

FINAL AREA OF CROSS SECTION

$A_f = \frac{\pi}{4}(d - \Delta d)^2$

$A_f = 803.67\ \text{mm}^2$

$\%\ \text{decrease in x-sec area} = \frac{A_f - A_i}{A_i}(100) \quad \leftarrow$

$= -0.072 \quad \leftarrow$

Problem 1.5-5 A bar of monel metal as in the figure (length $L = 9$ in., diameter $d = 0.225$ in.) is loaded axially by a tensile force P. If the bar elongates by 0.0195 in., what is the decrease in diameter d? What is the magnitude of the load P? Use the data in Table H-2, Appendix H.

Solution 1.5-5

NUMERICAL DATA

$E = 25000\ \text{ksi}$

$\nu = 0.32$

$L = 9\ \text{in.}$

$\delta = 0.0195\ \text{in.}$

$d = 0.225\ \text{in.}$

NORMAL STRAIN

$\varepsilon = \frac{\delta}{L} \quad \varepsilon = 2.167 \times 10^{-3}$

LATERAL STRAIN

$\varepsilon_p = -\nu\varepsilon \quad \varepsilon_p = -6.933 \times 10^{-4}$

DECREASE IN DIAMETER

$\Delta d = \varepsilon_p d$

$\Delta d = -1.56 \times 10^{-4}\ \text{in.} \quad \leftarrow$

INITIAL CROSS SECTIONAL AREA

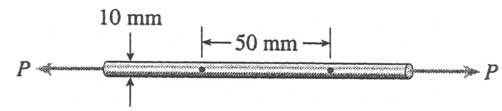
$A_i = \frac{\pi}{4}d^2 \quad A_i = 0.04\ \text{in.}^2$

MAGNITUDE OF LOAD P

$P = EA_i\varepsilon$

$P = 2.15\ \text{kips} \quad \leftarrow$

Problem 1.5-6 A tensile test is peformed on a brass specimen 10 mm in diameter using a gage length of 50 mm (see figure). When the tensile load P reaches a value of 20 kN, the distance between the gage marks has increased by 0.122 mm.



(a) What is the modulus of elasticity E of the brass?
(b) If the diameter decreases by 0.00830 mm, what is Poisson's ratio?

Solution 1.5-6 Brass specimen in tension

$d = 10$ mm Gage length $L = 50$ mm

$P = 20$ kN $\delta = 0.122$ mm $\Delta d = 0.00830$ mm

AXIAL STRESS

$$\sigma = \frac{P}{A} = \frac{20\text{ k}}{\frac{\pi}{4}(10\text{ mm})^2} = 254.6\text{ MPa}$$

Assume σ is below the proportional limit so that Hooke's law is valid.

AXIAL STRAIN

$$\varepsilon = \frac{\delta}{L} = \frac{0.122\text{ mm}}{50\text{ mm}} = 0.002440$$

(a) MODULUS OF ELASTICITY

$$E = \frac{\sigma}{\varepsilon} = \frac{254.6\text{ MPa}}{0.002440} = 104\text{ Gpa} \quad \leftarrow$$

(b) POISSON'S RATIO

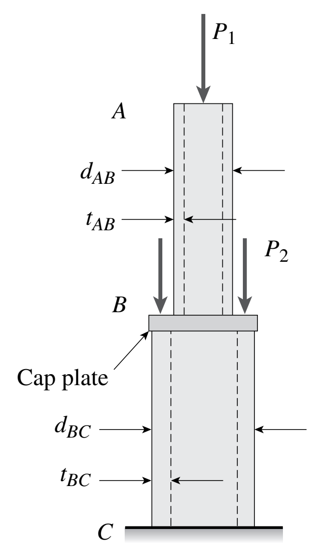
$$\varepsilon' = \nu\varepsilon$$

$$\Delta d = \varepsilon' d = \nu\varepsilon d$$

$$\nu = \frac{\Delta d}{\varepsilon d} = \frac{0.00830\text{ mm}}{(0.002440)(10\text{ mm})} = 0.34 \quad \leftarrow$$

Problem 1.5-7 A hollow, brass circular pipe ABC (see figure) supports a load $P_1 = 26.5$ kips acting at the top. A second load $P_2 = 22.0$ kips is uniformly distributed around the cap plate at B. The diameters and thicknesses of the upper and lower parts of the pipe are $d_{AB} = 1.25$ in., $t_{AB} = 0.5$ in., $d_{BC} = 2.25$ in., and $t_{AB} = 0.375$ in., respectively. The modulus of elasticity is 14,000 ksi. When both loads are fully applied, the wall thickness of pipe BC increases by $200\ 3\ 10^{-6}$ in.

(a) Find the increase in the inner diameter of pipe segment BC.
(b) Find Poisson's ratio for the brass.
(c) Find the increase in the wall thickness of pipe segment AB and the increase in the inner diameter of AB.



Solution 1.5-7

NUMERICAL DATA

$P_1 = 26.5$ kips

$P_2 = 22$ kips

$d_{AB} = 1.25$ in.

$t_{AB} = 0.5$ in.

$d_{BC} = 2.25$ in.

$t_{BC} = 0.375$ in.

$E = 14000$ ksi

$\Delta t_{BC} = 200 \times 10^{-6}$

(a) INCREASE IN THE INNER DIAMETER OF PIPE SEGMENT BC

$$\varepsilon_{pBC} = \frac{\Delta t_{BC}}{t_{BC}} \qquad \varepsilon_{pBC} = 5.333 \times 10^{-4}$$

$$\Delta d_{BCinner} = \varepsilon_{pBC}(d_{BC} - 2t_{BC})$$

$$\Delta\, d_{BCinner} = 8 \times 10^{-4} \text{ inches} \quad \leftarrow$$

(b) POISSON'S RATIO FOR THE BRASS

$$A_{BC} = \frac{\pi}{4}\left[d_{BC}^{\,2} - (d_{BC} - 2t_{BC})^2\right]$$

$$A_{BC} = 2.209 \text{ in.}^2$$

$$\varepsilon_{BC} = \frac{-(P_1 + P_2)}{(EA_{BC})} \qquad \varepsilon_{BC} = -1.568 \times 10^{-3}$$

$$\nu_{brass} = \frac{-\varepsilon_{pBC}}{\varepsilon_{BC}} \qquad \nu_{brass} = 0.34$$

(agrees with App. H (Table H-2))

(c) INCREASE IN THE WALL THICKNESS OF PIPE SEGMENT AB AND THE INCREASE IN THE INNER DIAMETER OF AB

$$A_{AB} = \frac{\pi}{4}\left[d_{AB}^{\,2} - (d_{AB} - 2t_{AB})^2\right]$$

$$\varepsilon_{AB} = \frac{-P_1}{EA_{AB}} \qquad \varepsilon_{AB} = -1.607 \times 10^{-3}$$

$$\varepsilon_{pAB} = -\nu_{brass}\varepsilon_{AB} \qquad \varepsilon_{pAB} = 5.464 \times 10^{-4}$$

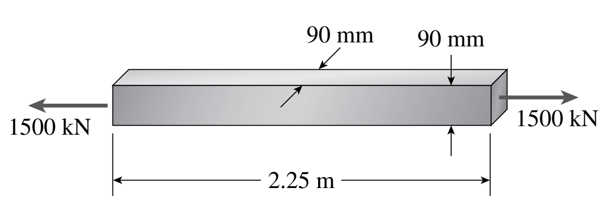
$$\Delta t_{AB} = \varepsilon_{pAB} t_{AB} \qquad \Delta t_{AB} = 2.73 \times 10^{-4} \text{ in.} \quad \leftarrow$$

$$\Delta d_{ABinner} = \varepsilon_{pAB}(d_{AB} - 2t_{AB})$$

$$\Delta d_{ABinner} = 1.366 \times 10^{-4} \text{ inches}$$

Problem 1.5-8 A brass bar of length 2.25 m with a square cross section of 90 mm on each side is subjected to an axial tensile force of 1500 kN (see figure). Assume that $E = 110$ GPa and $\nu = 0.34$.

Determine the increase in volume of the bar.



Solution 1.5-8

NUMERICAL DATA

$E = 110$ GPa $\quad \nu = 0.34 \quad P = 1500$ kN

$b = 90$ mm $\quad L = 2250$ mm

INITIAL VOLUME

$\text{Vol}_i = Lb^2$

$\text{Vol}_i = 1.822 \times 10^7 \text{ mm}^3$

NORMAL STRESS AND STRAIN

$\sigma = \dfrac{P}{b^2} \qquad \sigma = 185$ MPa (less than yield so Hooke's Law applies)

$\varepsilon = \dfrac{\sigma}{E} \qquad \varepsilon = 1.684 \times 10^{-3}$

LATERAL STRAIN

$\varepsilon_p = \nu\varepsilon \quad \varepsilon_p = 5.724 \times 10^{-4}$

CHANGE IN DIMENSIONS

$\Delta b = \varepsilon_p b \qquad \Delta b = 0.052$ mm

$\Delta L = \varepsilon L \qquad \Delta L = 3.788$ mm

FINAL LENGTH AND WIDTH

$L_f = L + \Delta L \quad L_f = 2.254 \times 10^3$ mm

$b_f = b - \Delta b \qquad b_f = 89.948$ mm

FINAL VOLUME

$\text{Vol}_f = L_f b_f^2 \quad \text{Vol}_f = 1.823 \times 10^7 \text{ mm}^3$

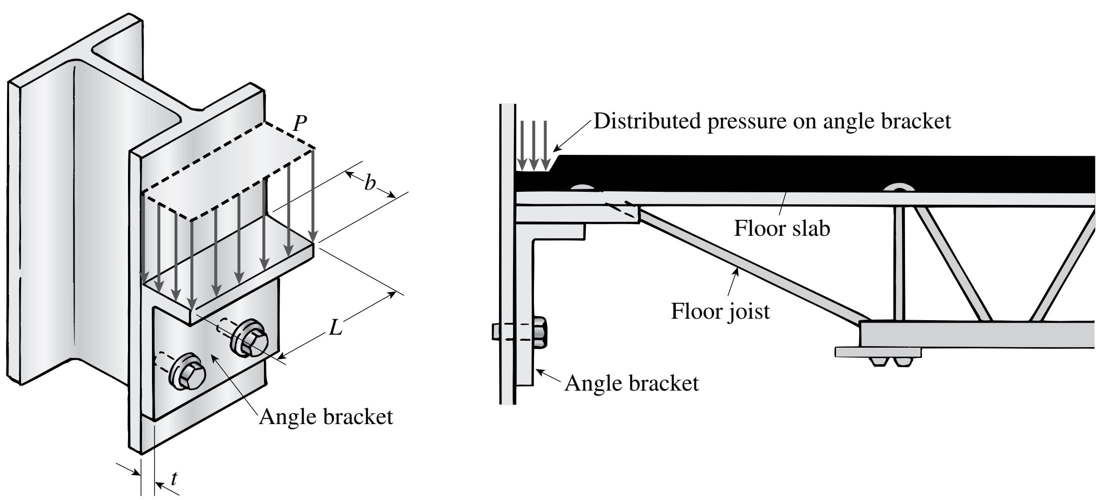
INCREASE IN VOLUME

$\Delta V = \text{Vol}_f - \text{Vol} \quad \Delta V = 9789 \text{ mm}^3$

Shear Stress and Strain

Problem 1.6-1 An angle bracket having thickness $t = 0.75$ in. is attached to the flange of a column by two 5/8-inch diameter bolts (see figure). A uniformly distributed load from a floor joist acts on the top face of the bracket with a pressure $p = 275$ psi. The top face of the bracket has length $L = 8$ in. and width $b = 3.0$ in.

Determine the average bearing pressure σ_b between the angle bracket and the bolts and the average shear stress τ_{aver} in the bolts. (Disregard friction between the bracket and the column.)



Solution 1.6-1

NUMERICAL DATA

$t = 0.75$ in. $L = 8$ in.

$b = 3.$ in. $p = \frac{275}{1000}$ ksi $d = \frac{5}{8}$ in.

BEARING FORCE

$F = pbL$ $F = 6.6$ kips

SHEAR AND BEARING AREAS

$A_S = \frac{\pi}{4}d^2$ $A_S = 0.307$ in.^{2}

$A_b = dt$ $A_b = 0.469$ in.^{2}

BEARING STRESS

$\sigma_b = \frac{F}{2A_b}$ $\sigma_b = 7.04$ ksi $\leftarrow$

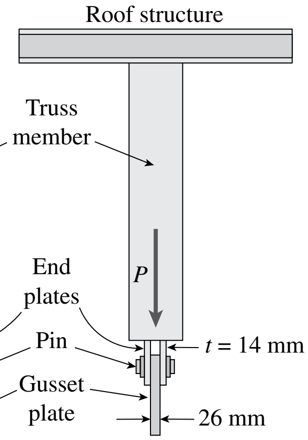
SHEAR STRESS

$\tau_{ave} = \frac{F}{2A_S}$ $\tau_{ave} = 10.76$ ksi $\leftarrow$

Problem 1.6-2 Truss members supporting a roof are connected to a 26-mm-thick gusset plate by a 22 mm diameter pin as shown in the figure and photo. The two end plates on the truss members are each 14 mm thick.

(a) If the load $P = 80$ kN, what is the largest bearing stress acting on the pin?
(b) If the ultimate shear stress for the pin is 190 MPa, what force P_{ult} is required to cause the pin to fail in shear?

(Disregard friction between the plates.)



Solution 1.6-2

NUMERICAL DATA

$t_{ep} = 14$ mm

$t_{gp} = 26$ mm

$P = 80$ kN

$d_p = 22$ mm

$\tau_{ult} = 190$ MPa

(a) BEARING STRESS ON PIN

$\sigma_b = \frac{P}{d_p t_{gp}}$ gusset plate is thinner than ($2\ t_{ep}$) so gusset plate controls

$\sigma_b = 139.9$ MPa $\leftarrow$

(b) ULTIMATE FORCE IN SHEAR

Cross sectional area of pin

$$A_p = \frac{\pi d_p^2}{4}$$

$A_p = 380.133$ mm^2

$P_{ult} = 2\tau_{ult}A_p$ $P_{ult} = 144.4$ kN $\leftarrow$

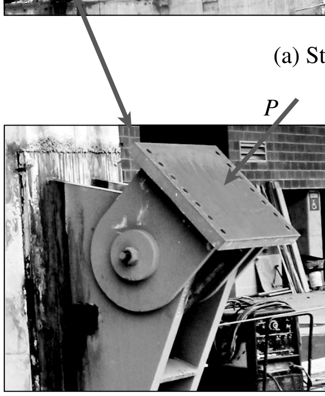
Problem 1.6-3 The upper deck of a football stadium is supported by braces each of which transfers a load $P = 160$ kips to the base of a column [see figure part (a)]. A cap plate at the bottom of the brace distributes the load P to four flange plates ($t_f = 1$ in.) through a pin ($d_p = 2$ in.) to two gusset plates ($t_g = 1.5$ in.) [see figure parts (b) and (c)].

Determine the following quantities.

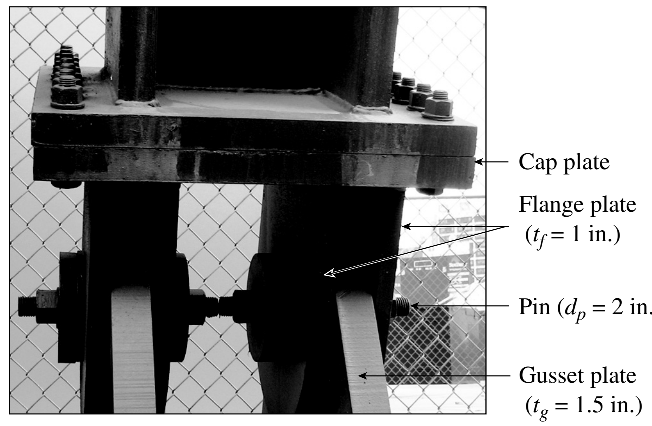
(a) The average shear stress τ_{aver} in the pin.

(b) The average bearing stress between the flange plates and the pin (σ_{bf}), and also between the gusset plates and the pin (σ_{bg}).

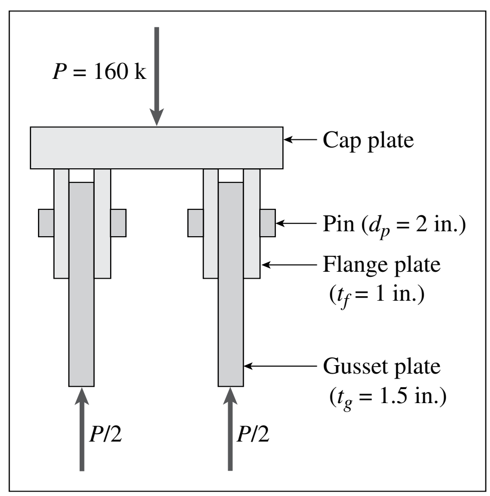
(Disregard friction between the plates.)



(a) Stadium brace



(b) Detail at bottom of brace



(c) Section through bottom of brace

Solution 1.6-3

NUMERICAL DATA

$P = 160 \text{ kips} \qquad d_p = 2 \text{ in.}$

$t_g = 1.5 \text{ in.} \qquad t_f = 1 \text{ in.}$

(a) SHEAR STRESS ON PIN

$$\tau = \frac{V}{\left(\frac{\pi d_p^2}{4}\right)} \qquad \tau = \frac{\frac{P}{4}}{\left(\frac{\pi d_p^2}{4}\right)}$$

$$\tau = 12.73 \text{ ksi} \quad \leftarrow$$

(b) BEARING STRESS ON PIN FROM FLANGE PLATE

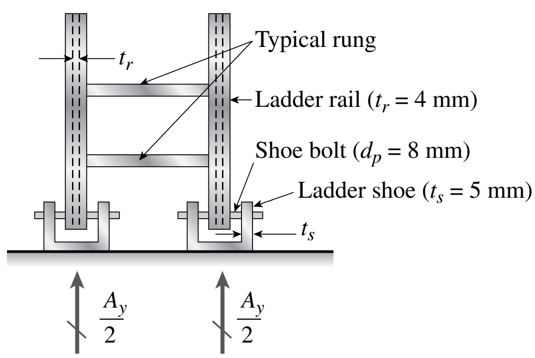
$$\sigma_{bf} = \frac{\frac{P}{4}}{d_p t_f} \qquad \sigma_{bf} = 20 \text{ ksi} \quad \leftarrow$$

BEARING STRESS ON PIN FROM GUSSET PLATE

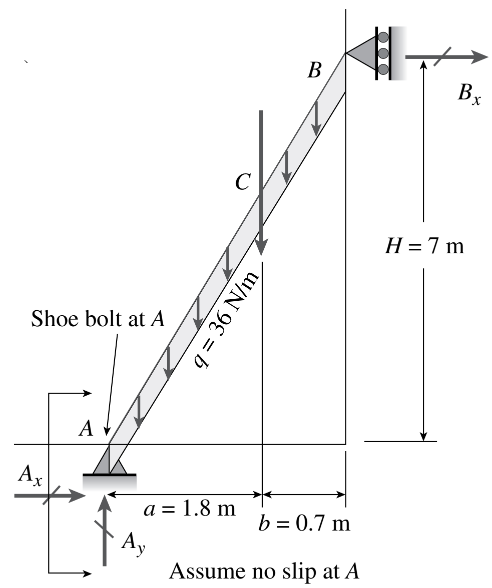
$$\sigma_{bg} = \frac{\frac{P}{2}}{d_p t_g} \qquad \sigma_{bg} = 26.7 \text{ ksi} \quad \leftarrow$$

Problem 1.6-4 The inclined ladder *AB* supports a house painter (82 kg) at *C* and the self weight (q = 36 N/m) of the ladder itself. Each ladder rail (t_r = 4 mm) is supported by a shoe (t_s = 5 mm) which is attached to the ladder rail by a bolt of diameter d_p = 8 mm.

(a) Find support reactions at *A* and *B*.
(b) Find the resultant force in the shoe bolt at *A*.
(c) Find maximum average shear (τ) and bearing (σ_b) stresses in the shoe bolt at *A*.



Section at base



Assume no slip at *A*

Solution 1.6-4

NUMERICAL DATA

$t_r = 4 \text{ mm} \qquad t_s = 5 \text{ mm}$

$d_p = 8 \text{ mm} \qquad P = 82 \text{ kg } (9.81 \text{ m/s}^2)$

$P = 804.42 \text{ N}$

$a = 1.8 \text{ m} \qquad b = 0.7 \text{ m} \qquad H = 7 \text{ m} \qquad q = 36 \frac{\text{N}}{\text{m}}$

(a) SUPPORT REACTIONS

$$L = \sqrt{(a + b)^2 + H^2} \qquad L = 7.433 \text{ m}$$

$$L_{AC} = \frac{a}{a + b} L \qquad L_{AC} = 5.352 \text{ m}$$

$$L_{CB} = \frac{b}{a + b} L \qquad L_{CB} = 2.081 \text{ m}$$

$$L_{AC} + L_{CB} = 7.433 \text{ m}$$

SUM MOMENTS ABOUT A

$$B_x = \frac{Pa + qL\left(\frac{a+b}{2}\right)}{-H}$$

$$B_x = -255\text{ N} \quad \leftarrow$$

$$A_x = -B_x \qquad A_y = P + qL$$

$$A_y = 1072\text{ N} \quad \leftarrow$$

(b) RESULTANT FORCE IN SHOE BOLT AT A

$$A_{resultant} = \sqrt{A_x^2 + A_y^2}$$

$$A_{resultant} = 1102\text{ N} \quad \leftarrow$$

(c) MAXIMUM SHEAR AND BEARING STRESSES IN SHOE BOLT AT A

$$d_p = 8\text{ mm} \qquad t_s = 5\text{ mm} \qquad t_r = 4\text{ mm}$$

SHEAR AREA $A_s = \frac{\pi}{4} d_p^2 \qquad A_s = 50.265\text{ mm}^2$

SHEAR STRESS $\tau = \frac{\frac{A_{resultant}}{2}}{2A_s} \qquad \tau = 5.48\text{ MPa} \quad \leftarrow$

BEARING AREA $A_b = 2d_pt_s \qquad A_b = 80\text{ mm}^2$

BEARING STRESS $\sigma_{bshoe} = \frac{\frac{A_{resultant}}{2}}{A_b}$

$$\sigma_{bshoe} = 6.89\text{ MPa} \quad \leftarrow$$

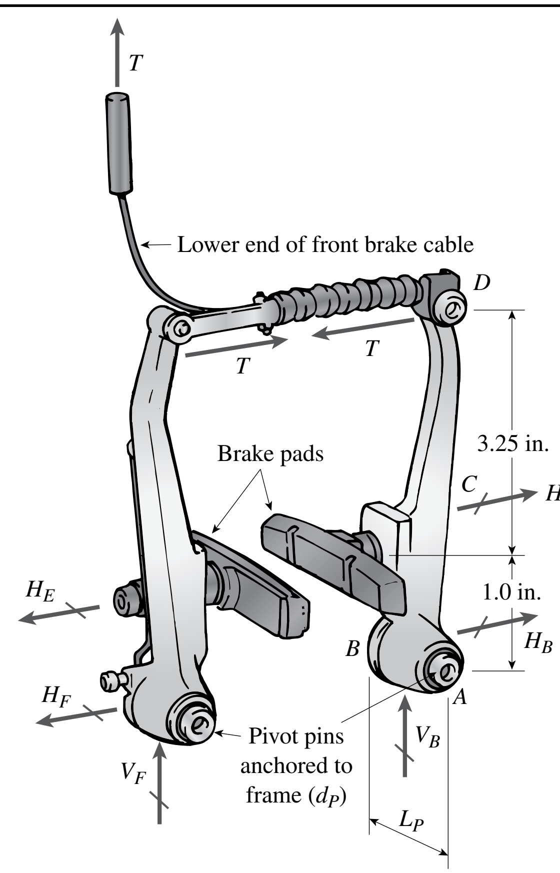
CHECK BEARING STRESS IN LADDER RAIL

$$\sigma_{brail} = \frac{\frac{A_{resultant}}{2}}{d_p\,t_r} \qquad \sigma_{brail} = 17.22\text{ MPa}$$

Problem 1.6-5 The force in the brake cable of the V-brake system shown in the figure is $T = 45$ lb. The pivot pin at A has diameter $d_p = 0.25$ in. and length $L_p = 5/8$ in.

Use dimensions show in the figure. Neglect the weight of the brake system.

(a) Find the average shear stress τ_{aver} in the pivot pin where it is anchored to the bicycle frame at B.
(b) Find the average bearing stress $\sigma_{b,aver}$ in the pivot pin over segment AB.



Solution 1.6-5

NUMERICAL DATA

$d_p = 0.25$ in. $\quad L = \dfrac{5}{8}$ in. $\quad CD = 3.25$ in.

$BC = 1$ in. $\quad T = 45$ lb

EQUILIBRIUM - FIND HORIZONTAL FORCES AT B AND C [VERTICAL REACTION $V_B = 0$]

$\sum M_B = 0 \qquad H_C = \dfrac{T(BC + CD)}{BC}$

$H_C = 191.25$ lb $\qquad \sum F_H = 0$

$H_B = T - H_C \qquad H_B = -146.25$ lb

(a) FIND THE AVE SHEAR STRESS τ_{ave} IN THE PIVOT PIN WHERE IT IS ANCHORED TO THE BICYCLE FRAME AT B:

$A_S = \dfrac{\pi d_p^2}{4} \qquad A_s = 0.049 \text{ in.}^2$

$\tau_{ave} = \dfrac{|H_B|}{A_S} \qquad \tau_{ave} = 2979 \text{ psi} \quad \leftarrow$

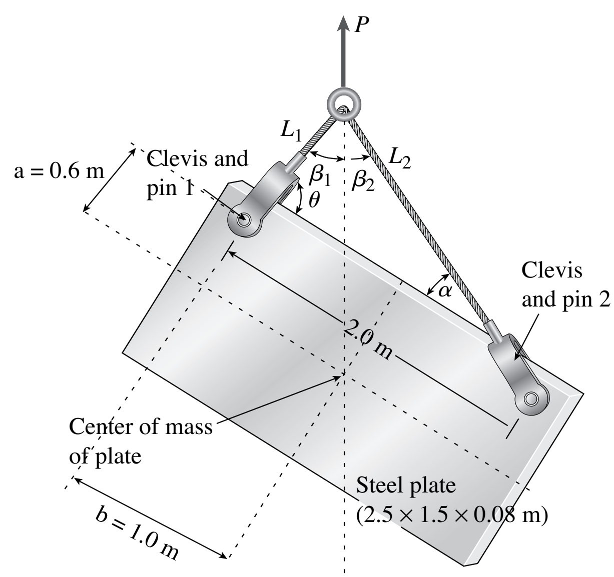
(b) FIND THE AVE BEARING STRESS $\sigma_{b,ave}$ IN THE PIVOT PIN OVER SEGMENT AB.

$A_b = d_p L \qquad A_b = 0.156 \text{ in.}^2$

$\sigma_{b,ave} = \dfrac{|H_B|}{A_b} \qquad \sigma_{b,ave} = 936 \text{ psi} \quad \leftarrow$

Problem 1.6-6 A steel plate of dimensions 2.5 × 1.5 × 0.08 m and weighing 23.1kN is hoisted by steel cables with lengths $L_1 = 3.2$ m and $L_2 = 3.9$ m that are each attached to the plate by a clevis and pin (see figure). The pins through the clevises are 18 mm in diameter and are located 2.0 m apart. The orientation angles are measured to be $\theta = 94.4°$ and $\alpha = 54.9°$.

For these conditions, first determine the cable forces T_1 and T_2, then find the average shear stress τ_{aver} in both pin 1 and pin 2, and then the average bearing stress σ_b between the steel plate and each pin. Ignore the mass of the cables.



Solution 1.6-6

NUMERICAL DATA

$L_1 = 3.2$ m $\quad L_2 = 3.9$ m $\quad \alpha = 54.9\left(\dfrac{\pi}{180}\right)$ rad.

$\theta = 94.4\left(\dfrac{\pi}{180}\right)$ rad.

$a = 0.6$ m $\quad b = 1$ m

$W = 77.0(2.5 \times 1.5 \times 0.08) \qquad W = 23.1$ kN

(77 = wt density of steel, kN/m^3)

SOLUTION APPROACH

STEP (1) $d = \sqrt{a^2 + b^2} \qquad d = 1.166$ m

STEP (2) $\theta_1 = \operatorname{atan}\left(\dfrac{a}{b}\right) \qquad \theta_1 \dfrac{180}{\pi} = 30.964$ degrees

STEP (3)-Law of cosines

$H = \sqrt{d^2 + L_1{}^2 - 2dL_1\cos(\theta + \theta_1)}$

$H = 3.99$ m

STEP (4) $\beta_1 = \mathrm{acos}\left(\dfrac{L_2^2 + H^2 - d^2}{2L_1H}\right)$

$$\beta_1 \frac{180}{\pi} = 13.789 \text{ degrees}$$

STEP (5) $\beta_2 = \mathrm{acos}\left(\dfrac{L_2^2 + H^2 - d^2}{2L_2H}\right)$

$$\beta_2 \frac{180}{\pi} = 16.95 \text{ degrees}$$

STEP (6)

Check $(\beta_1 + \beta_2 + \theta + \alpha)\dfrac{180}{\pi}$

$= 180.039$ degrees

Statics

$$T_1 \sin(\beta_1) = T_2 \sin(\beta_2)$$

$$T_1 = T_2\left(\frac{\sin(\beta_2)}{\sin(\beta_1)}\right)$$

$$T_1\cos(\beta_1) + T_2\cos(\beta_2) = W$$

$$T_2 = \frac{W}{\cos(\beta_1)\dfrac{\sin(\beta_2)}{\sin(\beta_1)} + \cos(\beta_2)}$$

$T_2 = 10.77$ kN ←

$T_1 = T_2\left(\dfrac{\sin(\beta_2)}{\sin(\beta_1)}\right)$ $T_1 = 13.18$ kN ←

$T_1\cos(\beta_1) + T_2\cos(\beta_2) = 23.1$ < checks

SHEAR & BEARING STRESSES

$d_p = 18$ mm $t = 100$ mm

$A_S = \dfrac{\pi}{4} d_p^2$ $A_b = t d_p$

$\tau_{1ave} = \dfrac{\frac{T_1}{2}}{A_S}$ $\tau_{1ave} = 25.9$ MPa ←

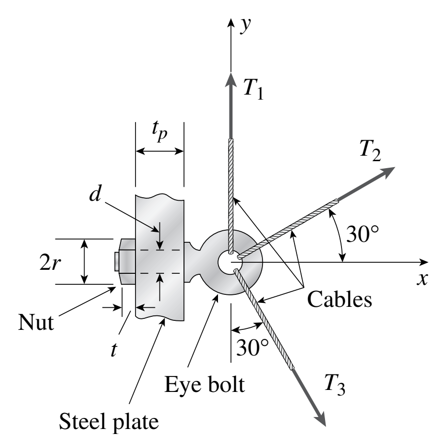
$\tau_{2ave} = \dfrac{\frac{T_2}{2}}{A_S}$ $\tau_{2ave} = 21.2$ MPa ←

$\sigma_{b1} = \dfrac{T_1}{A_b}$ $\sigma_{b1} = 7.32$ MPa ←

$\sigma_{b2} = \dfrac{T_2}{A_b}$ $\sigma_{b2} = 5.99$ MPa ←

Problem 1.6-7 A special-purpose eye bolt of shank diameter $d = 0.50$ in. passes through a hole in a steel plate of thickness $t_p = 0.75$ in. (see figure) and is secured by a nut with thickness $t = 0.25$ in. The hexagonal nut bears directly against the steel plate. The radius of the circumscribed circle for the hexagon is $r = 0.40$ in. (which means that each side of the hexagon has length 0.40 in.). The tensile forces in three cables attached to the eye bolt are $T_1 = 800$ lb., $T_2 = 550$ lb., and $T_3 = 1241$ lb.

(a) Find the resultant force acting on the eye bolt.
(b) Determine the average bearing stress σ_b between the hexagonal nut on the eye bolt and the plate.
(c) Determine the average shear stress τ_{aver} in the nut and also in the steel plate.



Solution 1.6-7

CABLE FORCES

$T_1 = 800$ lb $\quad T_2 = 550$ lb $\quad T_3 = 1241$ lb

(a) RESULTANT

$$P = T_2\frac{\sqrt{3}}{2} + T_3 0.5 \qquad P = 1097 \text{ lb} \quad \leftarrow$$

(b) AVE. BEARING STRESS

$A_b = 0.2194$ in.^{2} $\quad$ hexagon (Case 25, App. D)

$$\sigma_b = \frac{P}{A_b} \qquad \sigma_b = 4999 \text{ psi} \quad \leftarrow$$

(c) AVE. SHEAR THROUGH NUT

$d = 0.5$ in. $\quad t = 0.25$ in.

$$A_{sn} = \pi dt \qquad A_{sn} = 0 \qquad \tau_{nut} = \frac{P}{A_{sn}}$$

$$\tau_{nut} = 2793 \text{ psi} \quad \leftarrow$$

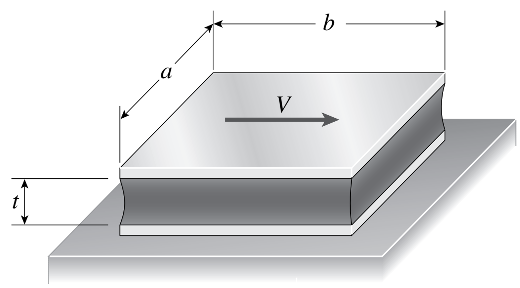
SHEAR THROUGH PLATE $\quad t_p = 0.75 \quad r = 0.40$

$A_{spl} = 6rt_p \qquad A_{spl} = 2$

$$\tau_{pl} = \frac{P}{A_{spl}} \qquad \tau_{pl} = 609 \text{ psi} \quad \leftarrow$$

Problem 1.6-8 An elastomeric bearing pad consisting of two steel plates bonded to a chloroprene elastomer (an artificial rubber) is subjected to a shear force V during a static loading test (see figure). The pad has dimensions $a = 125$ mm and $b = 240$ mm, and the elastomer has thickness $t = 50$ mm. When the force V equals 12 kN, the top plate is found to have displaced laterally 8.0 mm with respect to the bottom plate.

What is the shear modulus of elasticity G of the chloroprene?

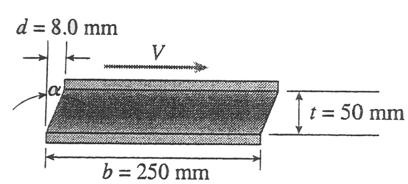


Solution 1.6-8

NUMERICAL DATA

$V = 12$ kN $\quad a = 125$ mm

$b = 240$ mm $\quad t = 50$ mm $\quad d = 8$ mm



AVERAGE SHEAR STRESS

$$\tau_{ave} = \frac{V}{ab} \qquad \tau_{ave} = 0.4 \text{ MPa}$$

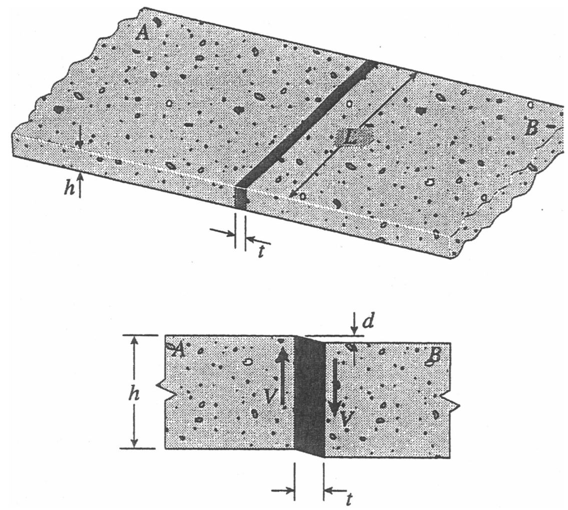
AVERAGE SHEAR STRAIN $\quad \gamma_{ave} = \dfrac{d}{t} \qquad \gamma_{ave} = 0.16$

SHEAR MODULUS G $\quad G = \dfrac{\tau_{ave}}{\gamma_{ave}}$

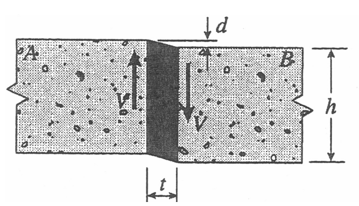
$$G = 2.5 \text{ MPa} \quad \leftarrow$$

Problem 1.6-9 A joint between two concrete slabs A and B is filled with a flexible epoxy that bonds securely to the concrete (see figure). The height of the joint is $h = 4.0$ in., its length is $L = 40$ in., and its thickness is $t = 0.5$ in. Under the action of shear forces V, the slabs displace vertically through the distance $d = 0.002$ in. relative to each other.

(a) What is the average shear strain γ_{aver} in the epoxy?
(b) What is the magnitude of the forces V if the shear modulus of elasticity G for the epoxy is 140 ksi?



Solution 1.6-9 Epoxy joint between concrete slabs



$h = 4.0$ in. $\quad t = 0.5$ in.

$L = 40$ in. $\quad d = 0.002$ in.

$G = 140$ ksi

(a) Average shear strain

$$\gamma_{aver} = \frac{d}{t} = 0.004 \quad \leftarrow$$

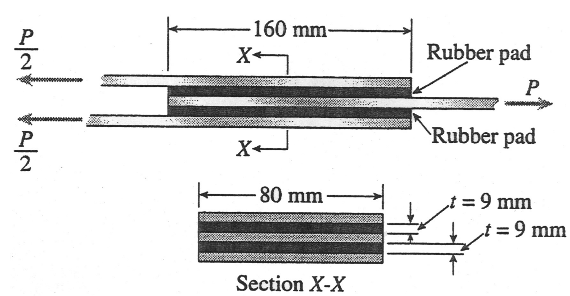
(b) Shear forces V

Average shear stress: $\tau_{aver} = G\gamma_{aver}$

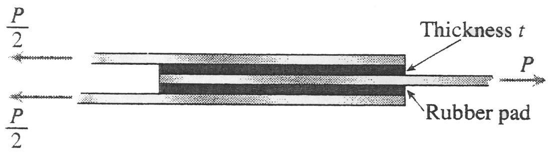
$$V = \tau_{aver}(hL) = G\gamma_{aver}(hL)$$
$$= (140 \text{ ksi})(0.004)(4.0 \text{ in.})(40 \text{ in.})$$
$$= 89.6 \text{ k} \quad \leftarrow$$

Problem 1.6-10 A flexible connection consisting of rubber pads (thickness $t = 9$ mm) bonded to steel plates is shown in the figure. The pads are 160 mm long and 80 mm wide.

(a) Find the average shear strain γ_{aver} in the rubber if the force $P = 16$ kN and the shear modulus for the rubber is $G = 1250$ kPa.
(b) Find the relative horizontal displacement δ between the interior plate and the outer plates.



Solution 1.6-10 Rubber pads bonded to steel plates



Rubber pads: $t = 9$ mm

Length $L = 160$ mm

Width $b = 80$ mm

$G = 1250$ kPa

$P = 16$ kN

(a) SHEAR STRESS AND STRAIN IN THE RUBBER PADS

$$\tau_{aver} = \frac{P/2}{bL} = \frac{8\text{kN}}{(80\text{ mm})(160\text{ mm})} = 625\text{ kPa}$$

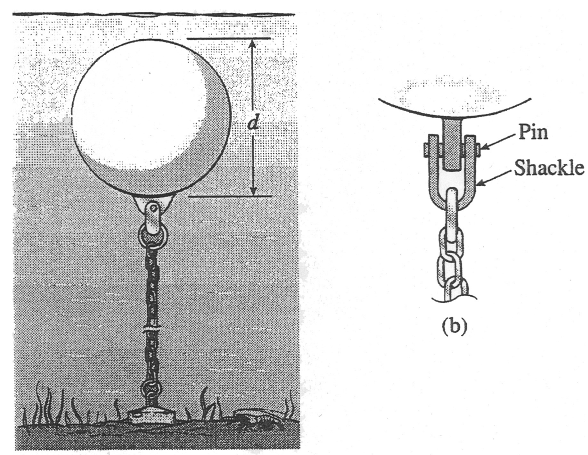
$$\gamma_{aver} = \frac{\tau_{aver}}{G} = \frac{625\text{ kPa}}{1250\text{ kPa}} = 0.50 \quad \leftarrow$$

(b) HORIZONTAL DISPLACEMENT

$$\delta = \gamma_{aver} t = (0.50)(9\text{ mm}) = 4.50\text{ mm} \quad \leftarrow$$

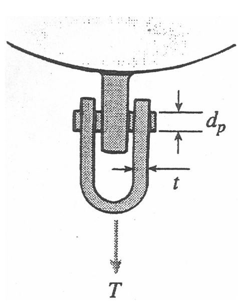
Problem 1.6-11 A spherical fiberglass buoy used in an underwater experiment is anchored in shallow water by a chain [see part (a) of the figure]. Because the buoy is positioned just below the surface of the water, it is not expected to collapse from the water pressure. The chain is attached to the buoy by a shackle and pin [see part (b) of the figure]. The diameter of the pin is 0.5 in. and the thickness of the shackle is 0.25 in. The buoy has a diameter of 60 in. and weighs 1800 lb on land (not including the weight of the chain).

(a) Determine the average shear stress τ_{aver} in the pin.
(b) Determine the average bearing stress σ_b between the pin and the shackle.



(a) (b)

Solution 1.6-11 Submerged buoy



d = diameter of buoy

= 60 in.

T = tensile force in chain

d_p = diameter of pin

= 0.5 in.

t = thickness of shackle

= 0.25 in.

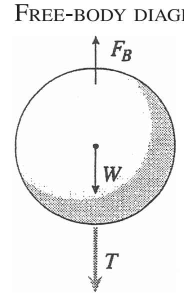
W = weight of buoy

= 1800 lb

γ_W = weight density of sea water

$= 63.8\text{ lb/ft}^3$

FREE-BODY DIAGRAM OF BUOY



F_B = buoyant force of water pressure (equals the weight of the displaced sea water)

V = volume of buoy

$$= \frac{\pi d^3}{6} = 65.45\text{ ft}^3$$

$$F_B = \gamma_W V = 4176\text{ lb}$$

EQUILIBRIUM

$T = F_B - W = 2376$ lb

(a) AVERAGE SHEAR STRESS IN PIN

A_p = area of pin

$$A_p = \frac{\pi}{4} d_p^2 = 0.1963 \text{ in.}^2$$

$$\tau_{aver} = \frac{T}{2A_p} = 6050 \text{ psi} \quad \leftarrow$$

(b) BEARING STRESS BETWEEN PIN AND SHACKLE

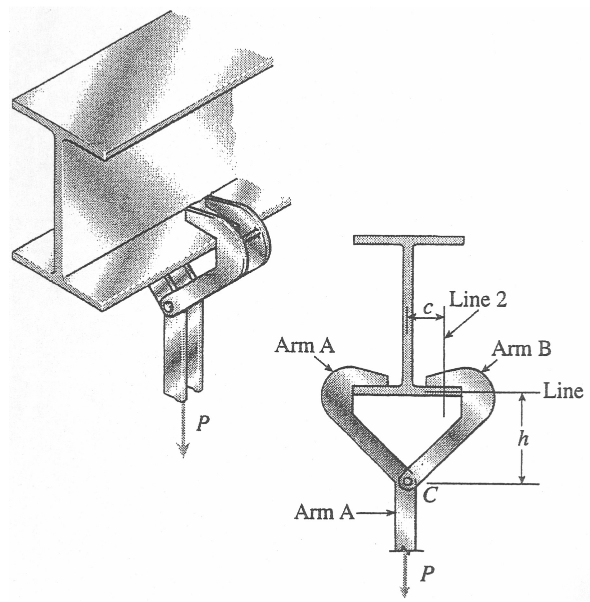
$A_b = 2d_p t = 0.2500 \text{ in.}^2$

$$\sigma_b = \frac{T}{A_b} = 9500 \text{ psi} \quad \leftarrow$$

Problem 1.6-12 The clamp shown in the figure is used to support a load hanging from the lower flange of a steel beam. The clamp consists of two arms (A and B) joined by a pin at C. The pin has diameter $d = 12$ mm. Because arm B straddles arm A, the pin is in double shear.

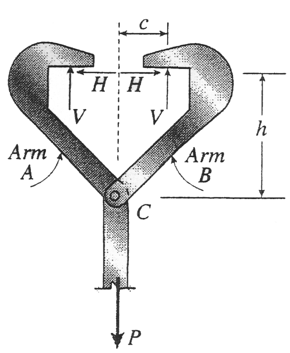
Line 1 in the figure defines the line of action of the resultant horizontal force H acting between the lower flange of the beam and arm B. The vertical distance from this line to the pin is $h = 250$ mm. Line 2 defines the line of action of the resultant vertical force V acting between the flange and arm B. The horizontal distance from this line to the centerline of the beam is $c = 100$ mm. The force conditions between arm A and the lower flange are symmetrical with those given for arm B.

Determine the average shear stress in the pin at C when the load $P = 18$ kN.



Solution 1.6-12 Clamp supporting a load *P*

FREE-BODY DIAGRAM OF CLAMP



$h = 250$ mm

$c = 100$ mm

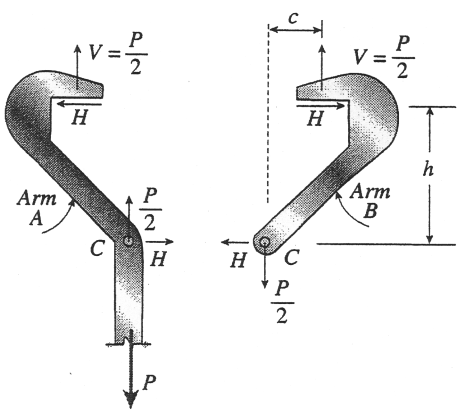
$P = 18$ kN

From vertical equilibrium:

$$V = \frac{P}{2} = 9 \text{ kN}$$

d = diameter of pin at C = 12 mm

FREE-BODY DIAGRAMS OF ARMS A AND B

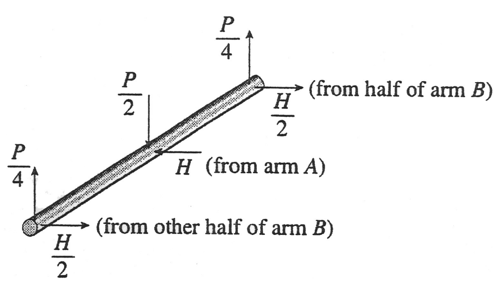


$$\Sigma M_C = 0$$

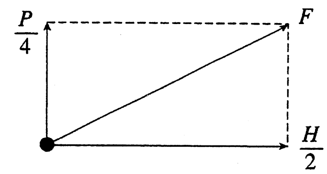
$$V_C - Hh = 0$$

$$H = \frac{V_C}{h} = \frac{P_c}{2h} = 3.6 \text{ kN}$$

FREE-BODY DIAGRAM OF PIN



SHEAR FORCE F IN PIN



$$F = \sqrt{\left(\frac{P}{4}\right)^2 + \left(\frac{H}{2}\right)^2}$$

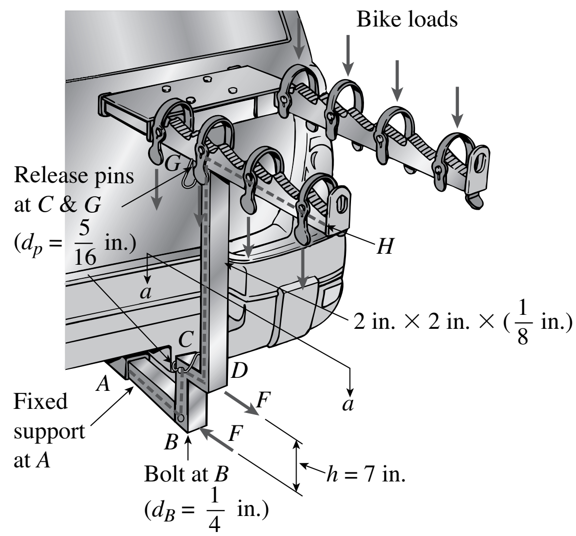
$$= 4.847 \text{ kN}$$

AVERAGE SHEAR STRESS IN THE PIN

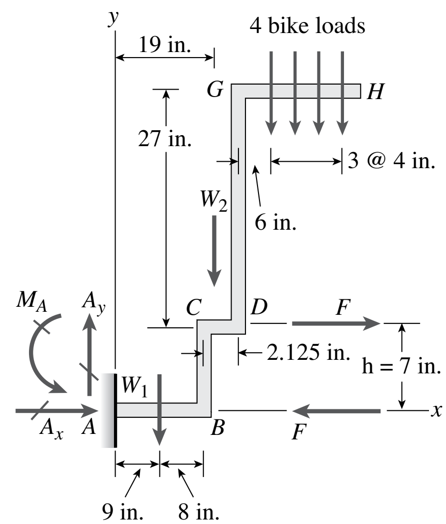
$$\tau_{\text{aver}} = \frac{F}{A_{\text{pin}}} = \frac{F}{\frac{\pi a^2}{4}} = 42.9 \text{ MPa} \quad \leftarrow$$

Problem ★1.6-13 A hitch-mounted bicycle rack is designed to carry up to four 30-lb. bikes mounted on and strapped to two arms *GH* [see bike loads in the figure part (a)]. The rack is attached to the vehicle at *A* and is assumed to be like a cantilever beam *ABCDGH* [figure part (b)]. The weight of fixed segment *AB* is $W_1 = 10$ lb, centered 9 in. from *A* [see the figure part (b)] and the rest of the rack weighs $W_2 = 40$ lb, centered 19 in. from *A*. Segment *ABCDG* is a steel tube, 2×2 in., of thickness $t = 1/8$ in. Segment *BCDGH* pivots about a bolt at *B* of diameter $d_B = 0.25$ in. to allow access to the rear of the vehicle without removing the hitch rack. When in use, the rack is secured in an upright position by a pin at *C* (diameter of pin $d_p = 5/16$ in.) [see photo and figure part (c)]. The overturning effect of the bikes on the rack is resisted by a force couple *Fh* at *BC*.

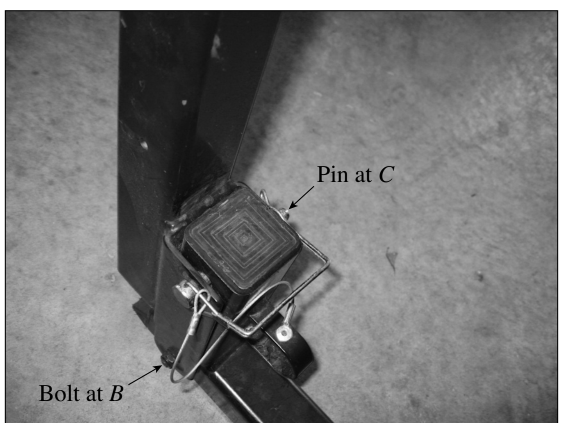
(a) Find the support reactions at A for the fully loaded rack;
(b) Find forces in the bolt at *B* and the pin at *C*.
(c) Find average shear stresses τ_{aver} in both the bolt at *B* and the pin at *C*.
(d) Find average bearing stresses σ_b in the bolt at B and the pin at *C*.

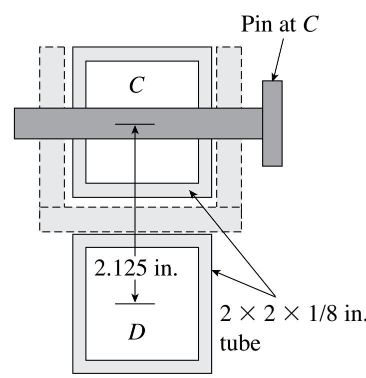


(a)



(b)





(c) Section a–a

Solution *1.6-13

NUMERICAL DATA

$t = \frac{1}{8}$ in. $\quad$ b = 2 in.

h = 7 in. $\quad W_1 = 10$ lb $\quad W_2 = 40$ lb

P = 30 lb $\quad d_B = 0.25$ in. $\quad d_p = \frac{5}{16}$ in.

(a) REACTIONS AT A

$A_x = 0 \quad \leftarrow$

$A_y = W_1 + W_2 + 4P \quad \leftarrow$

$A_y = 170$ lb $\quad \leftarrow$

$L_1 = 17 + 2.125 + 6 \qquad L_1 = 25$ in.

(dist from A to 1st bike)

$M_A = W_1(9) + W_2(19) + P(4L_1 + 4 + 8 + 12)$

$M_A = 4585$ in.-lb

(b) FORCES IN BOLT AT B & PIN AT C

$\Sigma F_y = 0 \qquad B_y = W_2 + 4P \qquad B_y = 160$ lb $\quad \leftarrow$

$\Sigma M_B = 0$

RHFB

$$B_x = \frac{[W_2(19 - 17) + P(6 + 2.125) + P(8.125 + 4) + P(8.125 + 8) + P(8.125 + 12)]}{h}$$

$B_x = 254$ lb ← $\qquad C_x = -B_x$

$B_{res} = \sqrt{B_x^2 + B_y^2} \qquad B_{res} = 300$ lb ←

(c) Average shear stresses τ_{ave} in both the bolt at B and the pin at C

$A_{sB} = 2\dfrac{\pi d_B^2}{4} \qquad A_{sB} = 0.098\ \text{in}^2$

$\tau_B = \dfrac{B_{res}}{A_{sB}} \qquad \tau_B = 3054$ psi ←

$A_{sC} = 2\dfrac{\pi d_p^2}{4} \qquad A_{sC} = 0.153\ \text{in}^2$

$\tau_C = \dfrac{B_x}{A_{sC}} \qquad \tau_C = 1653$ psi ←

(d) Bearing stresses σ_B in the bolt at B and the pin at C

$t = 0.125$ in

$A_{bB} = 2td_B \qquad A_{bB} = 0.063\ \text{in}^2$

$\sigma_{bB} = \dfrac{B_{res}}{A_{bB}} \qquad \sigma_{bB} = 4797$ psi ←

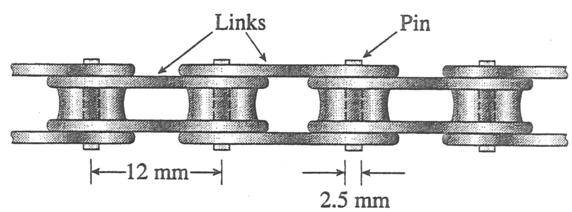
$A_{bC} = 2td_p \qquad A_{bC} = 0.078\ \text{in}^2$

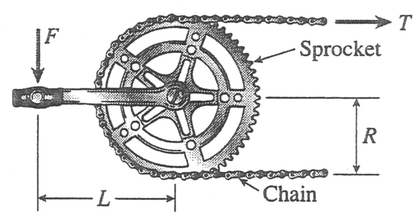
$\sigma_{bC} = \dfrac{C_x}{A_{bC}} \qquad \sigma_{bC} = 3246$ psi ←

Problem 1.6-14 A bicycle chain consists of a series of small links, each 12 mm long between the centers of the pins (see figure). You might wish to examine a bicycle chain and observe its construction. Note particularly the pins, which we will assume to have a diameter of 2.5 mm.

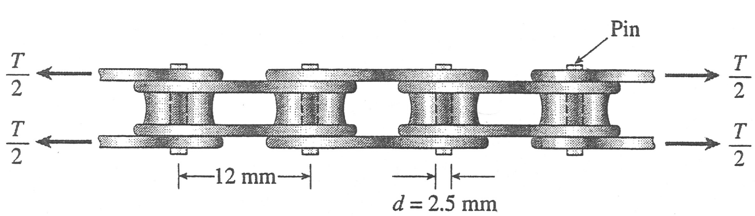
In order to solve this problem, you must now make two measurements on a bicycle (see figure): (1) the length L of the crank arm from main axle to pedal axle, and (2) the radius R of the sprocket (the toothed wheel, sometimes called the chainring).

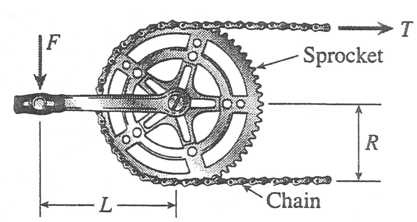
(a) Using your measured dimensions, calculate the tensile force T in the chain due to a force $F = 800$ N applied to one of the pedals.
(b) Calculate the average shear stress τ_{aver} in the pins.





Solution 1.6-14 Bicycle chain





F = force applied to pedal = 800 N

L = length of crank arm

R = radius of sprocket

MEASUREMENTS (FOR AUTHOR'S BICYCLE)

(1) $L = 162$ mm (2) $R = 90$ mm

(a) TENSILE FORCE T IN CHAIN

$$\Sigma M_{\text{axle}} = 0 \qquad FL = TR \qquad T = \frac{FL}{R}$$

Substitute numerical values:

$$T = \frac{(800\ N)(162\text{ mm})}{90\text{ mm}} = 1440\text{ N} \quad \leftarrow$$

(b) SHEAR STRESS IN PINS

$$\tau_{\text{aver}} = \frac{T/2}{A_{\text{pin}}} = \frac{T}{2\dfrac{\pi d^2}{(4)}} = \frac{2T}{\pi d^2}$$

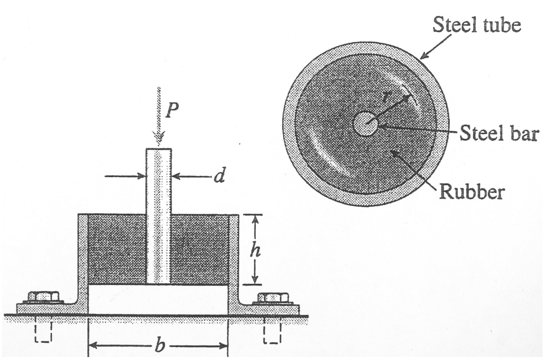
$$= \frac{2FL}{\pi d^2 R}$$

Substitute numerical values:

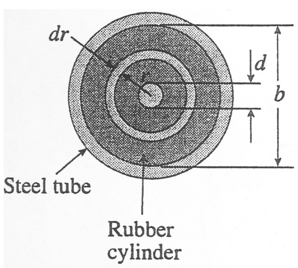
$$\tau_{\text{aver}} = \frac{2(800\text{ N})(162\text{ mm})}{\pi(2.5\text{ mm})^2(90\text{ mm})} = 147\text{ MPa} \quad \leftarrow$$

Problem 1.6-15 A shock mount constructed as shown in the figure is used to support a delicate instrument. The mount consists of an outer steel tube with inside diameter b, a central steel bar of diameter d that supports the load P, and a hollow rubber cylinder (height h) bonded to the tube and bar.

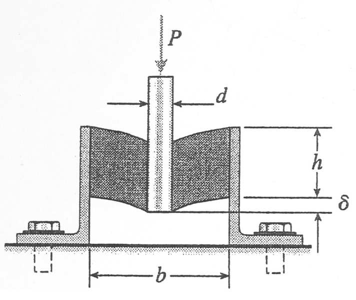
(a) Obtain a formula for the shear τ in the rubber at a radial distance r from the center of the shock mount.
(b) Obtain a formula for the downward displacement δ of the central bar due to the load P, assuming that G is the shear modulus of elasticity of the rubber and that the steel tube and bar are rigid.



Solution 1.6-15 Shock mount



r = radial distance from center of shock mount to element of thickness dr



r = radial distance from center of shock mount to element of thickness dr

(a) SHEAR STRESS τ AT RADIAL DISTANCE r

A_s = shear area at distance r $= 2\pi rh$

$$\tau = \frac{P}{A_s} = \frac{P}{2\pi rh} \quad \leftarrow$$

(b) DOWNWARD DISPLACEMENT δ

γ = shear strain at distance r

$$\gamma = \frac{\tau}{G} = \frac{P}{2\pi rhG}$$

$d\delta$ = downward displacement for element dr

$$d\delta = \gamma dr = \frac{Pdr}{2\pi rhG}$$

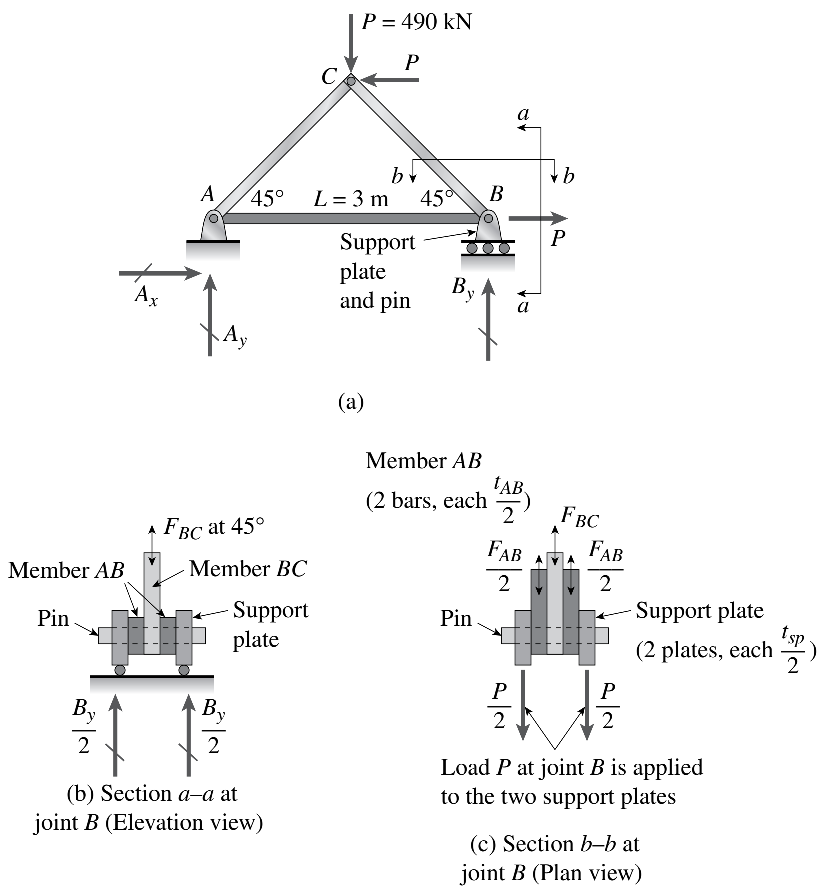
$$\delta = \int d\delta = \int_{d/2}^{b/2} \frac{Pdr}{2\pi rhG}$$

$$\delta = \frac{P}{2\pi hG}\int_{d/2}^{b/2}\frac{dr}{r} = \frac{P}{2\pi hG}[\text{In } r]_{d/2}^{b/2}$$

$$\delta = \frac{P}{2\pi hG}\ln\frac{b}{d} \quad \leftarrow$$

Problem 1.6-16 The steel plane truss shown in the figure is loaded by three forces P, each of which is 490 kN. The truss members each have a cross-sectional area of 3900 mm^2 and are connected by pins each with a diameter of $d_p = 18$ mm. Members AC and BC each consist of one bar with thickness of $t_{AC} = t_{BC} = 19$ mm. Member AB is composed of two bars [see figure part (b)] each having thickness $t_{AB}/2 = 10$ mm and length $L = 3$ m. The roller support at B, is made up of two support plates, each having thickness $t_{sp}/2 = 12$ mm.

(a) Find support reactions at joints A and B and forces in members AB, BC, and AB.
(b) Calculate the largest average shear stress $\tau_{p,\max}$ in the pin at joint B, disregarding friction between the members; see figures parts (b) and (c) for sectional views of the joint.
(c) Calculate the largest average bearing stress $\sigma_{b,\max}$ acting against the pin at joint B.



Solution 1.6-16

NUMERICAL DATA

$L = 3000\text{ mm} \qquad P = 490\text{ kN}$

$d_p = 18\text{ mm} \qquad A = 3900\text{ mm}^2$

$t_{AC} = 19\text{ mm} \qquad t_{BC} = t_{AC}$

$t_{AB} = 20\text{ mm} \qquad t_{sp} = 24\text{ mm}$

(a) SUPPORT REACTIONS AND MEMBER FORCES

$\sum F_x = 0 \qquad A_x = 0 \quad \leftarrow$

$\sum M_A = 0 \qquad B_y = \frac{1}{L}\left(P\frac{L}{2} - P\frac{L}{2}\right)$

$B_y = 0 \quad \leftarrow$

$\sum F_y = 0 \qquad A_y = P$

$A_y = 490\text{ kN} \quad \leftarrow$

METHOD OF JOINTS

$F_{AB} = P \qquad F_{BC} = 0 \quad \leftarrow$

$F_{AC} = -\sqrt{2}P$

$F_{AB} = 490\text{ kN} \quad \leftarrow$

$F_{AC} = -693\text{ kN} \quad \leftarrow$

(b) MAX. SHEAR STRESS IN PIN AT B

$A_s = \frac{\pi d_p^2}{4} \qquad A_s = 254.469\text{ mm}^2$

$\tau_{p\max} = \frac{\frac{F_{AB}}{2}}{A_s} \qquad \tau_{p\max} = 963\text{ MPa} \quad \leftarrow$

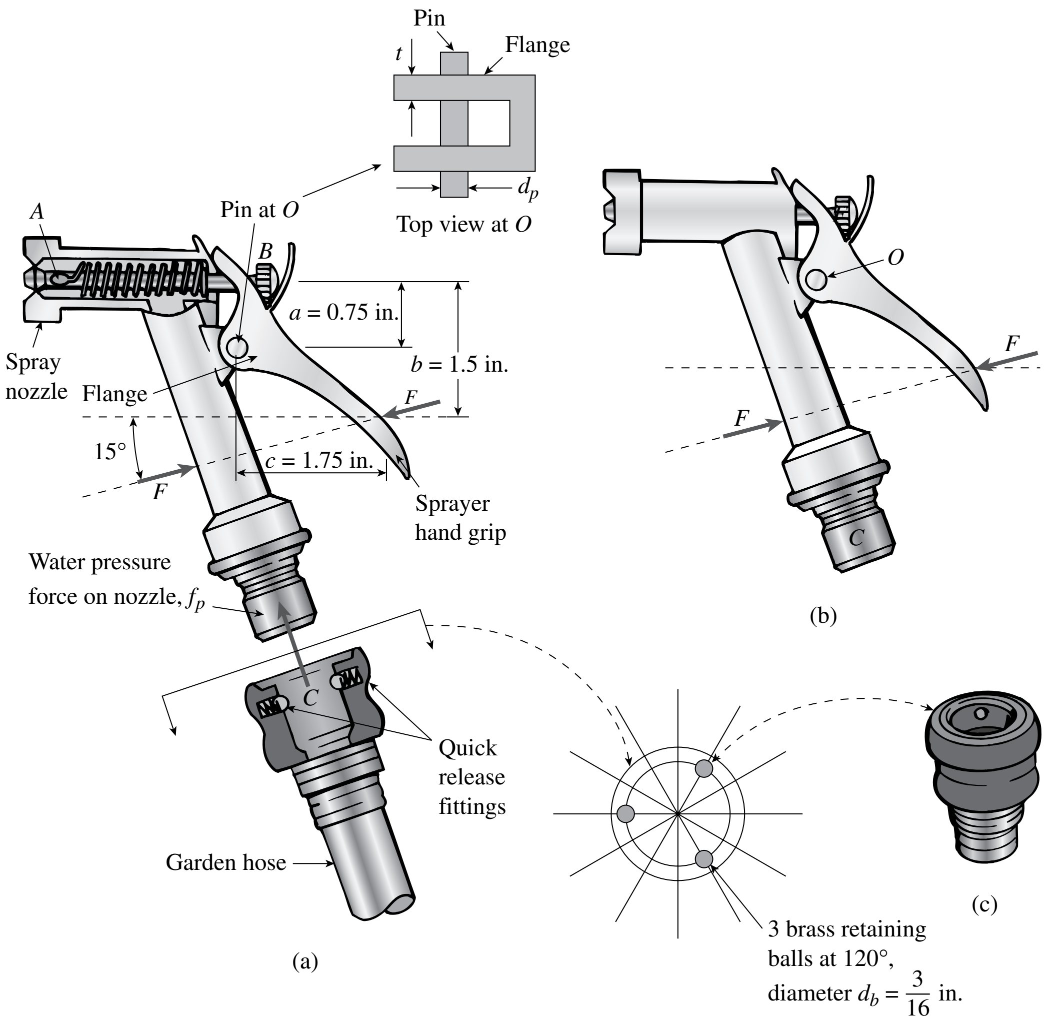
(c) MAX. BEARING STRESS IN PIN AT B ($t_{ab} < t_{sp}$ SO BEARING STRESS ON AB WILL BE GREATER)

$A_b = d_p\frac{t_{AB}}{2}$

$\sigma_{b\max} = \frac{\frac{F_{AB}}{2}}{A_b} \qquad \sigma_{b\max} = 1361\text{ MPa} \quad \leftarrow$

Problem 1.6-17 A spray nozzle for a garden hose requires a force $F = 5$ lb. to open the spring-loaded spray chamber AB. The nozzle hand grip pivots about a pin through a flange at O. Each of the two flanges has thickness $t = 1/16$ in., and the pin has diameter $d_p = 1/8$ in. [see figure part (a)]. The spray nozzle is attached to the garden hose with a quick release fitting at B [see figure part (b)]. Three brass balls (diameter $d_b = 3/16$ in.) hold the spray head in place under water pressure force $f_p = 30$ lb. at C [see figure part (c)]. Use dimensions given in figure part (a).

(a) Find the force in the pin at O due to applied force F.
(b) Find average shear stress τ_{aver} and bearing stress σ_b in the pin at O.



Solution 1.6-17

NUMERICAL DATA

$F = 5\text{ lb} \quad t = \frac{1}{16}\text{ in.} \quad d_p = \frac{1}{8}\text{ in.} \quad d_b = \frac{3}{16}\text{ in.}$

$f_p = 30\text{ lb} \quad d_N = \frac{5}{8}\text{ in.} \quad \theta = 15\frac{\pi}{180}\text{ rad.}$

$a = 0.75\text{ in} \quad b = 1.5\text{ in} \quad c = 1.75\text{ in}$

(a) FIND THE FORCE IN THE PIN AT O DUE TO APPLIED FORCE F

$\sum M_o = 0$

$$F_{AB} = \frac{[F\cos(\theta)(b - a)] + F\sin(\theta)(c)}{a}$$

$F_{AB} = 7.849\text{ lb}$

$\sum F_H = 0 \qquad O_x = F_{AB} + F\cos(\theta)$

$O_y = F\sin(\theta)$

$O_x = 12.68\text{ lb} \quad O_y = 1.294\text{ lb}$

$O_{res} = \sqrt{O_x^2 + O_y^2} \qquad O_{res} = 12.74\text{ lb} \quad \leftarrow$

(b) FIND AVERAGE SHEAR STRESS τ_{ave} AND BEARING STRESS σ_b IN THE PIN AT O

$A_s = 2\frac{\pi d_p^2}{4} \quad \tau_O = \frac{O_{res}}{A_s} \quad \tau_O = 519\text{ psi} \quad \leftarrow$

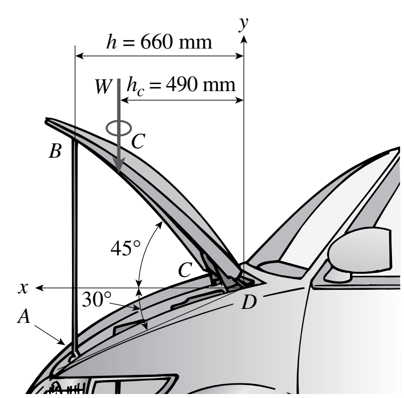
$A_b = 2td_p \quad \sigma_{bO} = \frac{O_res}{A_b} \quad \sigma_{bO} = 816\text{ psi} \quad \leftarrow$

(c) FIND THE AVERAGE SHEAR STRESS τ_{ave} IN THE BRASS RETAINING BALLS AT B DUE TO WATER PRESSURE FORCE F_p

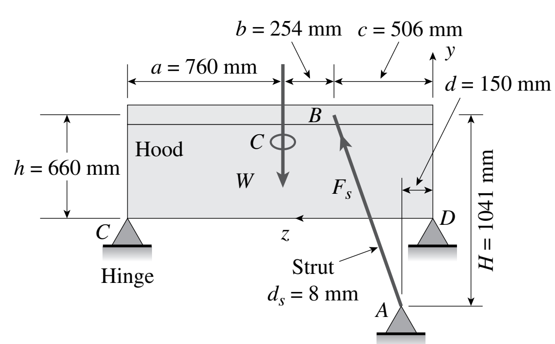
$A_s = 3\frac{\pi d_b^2}{4} \quad \tau_{ave} = \frac{f_p}{A_s} \quad \tau_{ave} = 362\text{ psi} \quad \leftarrow$

Problem 1.6-18 A single steel strut AB with diameter $d_s = 8$ mm. supports the vehicle engine hood of mass 20 kg which pivots about hinges at C and D [see figures (a) and (b)]. The strut is bent into a loop at its end and then attached to a bolt at A with diameter $d_b = 10$ mm. Strut AB lies in a vertical plane.

(a) Find the strut force F_s and average normal stress σ in the strut.
(b) Find the average shear stress τ_{aver} in the bolt at A.
(c) Find the average bearing stress σ_b on the bolt at A.



(a)



(b)

Solution 1.6-18

NUMERICAL DATA

$d_s = 8$ mm $d_b = 10$ mm $m = 20$ kg

$a = 760$ mm $b = 254$ mm

$c = 506$ mm $d = 150$ mm

$h = 660$ mm $h_c = 490$ mm

$$H = h\left(\tan\left(30\frac{\pi}{180}\right) + \tan\left(45\frac{\pi}{180}\right)\right)$$

$H = 1041$ mm

$W = m\,(9.81\text{m/s}^2)$ $W = 196.2$ N

$$\frac{a + b + c}{2} = 760 \text{ mm}$$

VECTOR r_{AB}

$$r_{AB} = \begin{pmatrix} 0 \\ H \\ c - d \end{pmatrix} \qquad r_{AB} = \begin{pmatrix} 0 \\ 1.041 \times 10^3 \\ 356 \end{pmatrix}$$

UNIT VECTOR e_{AB}

$$e_{AB} = \frac{r_{AB}}{|r_{AB}|} \qquad e_AB = \begin{pmatrix} 0 \\ 0.946 \\ 0.324 \end{pmatrix} \qquad |e_{AB}| = 1$$

$$W = \begin{pmatrix} 0 \\ -W \\ 0 \end{pmatrix} \qquad W = \begin{pmatrix} 0 \\ -196.2 \\ 0 \end{pmatrix}$$

$$r_{DC} = \begin{pmatrix} h_c \\ h_c \\ b + c \end{pmatrix} \qquad r_{DC} = \begin{pmatrix} 490 \\ 490 \\ 760 \end{pmatrix}$$

$$\sum M_D \qquad M_D = r_{DB} \times F_s e_{AB} + W \times r_{DC}$$

(ignore force at hinge C since it will vanish with moment about line DC)

$$F_{sx} = 0 \qquad F_{sy} = \frac{H}{\sqrt{H^2 + (c - d)^2}} F_s$$

$$F_{sz} = \frac{c - d}{\sqrt{H^2 + (c - d)^2}} F_s$$

where

$$\frac{H}{\sqrt{H^2 + (c - d)^2}} = 0.946$$

$$\frac{c - d}{\sqrt{H^2 + (c - d)^2}} = 0.324$$

(a) FIND THE STRUT FORCE F_S AND AVERAGE NORMAL STRESS σ IN THE STRUT

$$\sum M_{lineDC} = 0 \qquad F_{sy} = \frac{|W|h_c}{h}$$

$F_{sy} = 145.664$

$$F_s = \frac{F_{sy}}{\dfrac{H}{\sqrt{H^2 + (c - d)^2}}} \qquad F_s = 153.9 \text{ N} \quad \leftarrow$$

$$A_{strut} = \frac{\pi}{4} d_s^2 \qquad A_{strut} = 50.265 \text{ mm}^2$$

$$\sigma = \frac{F_s}{A_{strut}} \qquad \sigma = 3.06 \text{ MPa} \quad \leftarrow$$

(b) FIND THE AVERAGE SHEAR STRESS τ_{ave} IN THE BOLT AT A

$d_b = 10$ mm

$$A_s = \frac{\pi}{4} d_b^2 \qquad A_s = 78.54 \text{ mm}^2$$

$$\tau_{ave} = \frac{F_s}{A_s} \qquad \tau_{ave} = 1.96 \text{ Mpa} \quad \leftarrow$$

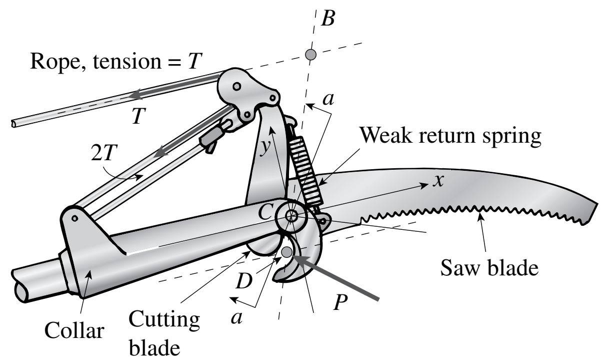
(c) FIND THE BEARING STRESS σ_b ON THE BOLT AT A

$A_b = d_s d_b$ $A_b = 80$ mm^2

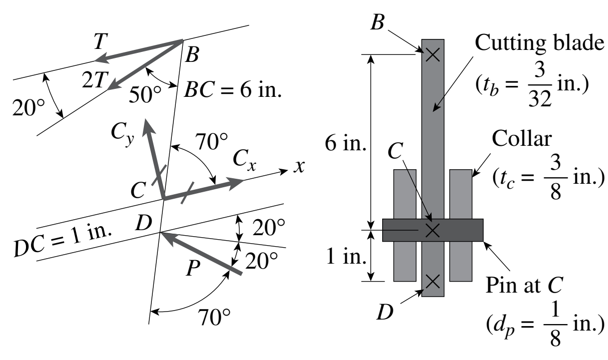
$$\sigma_b = \frac{F_s}{A_b} \qquad \sigma_b = 1.924 \text{ MPa} \quad \leftarrow$$

Problem 1.6-19 The top portion of a pole saw used to trim small branches from trees is shown in the figure part (a). The cutting blade *BCD* [see figure parts (a) and (c)] applies a force *P* at point *D*. Ignore the effect of the weak return spring attached to the cutting blade below *B*. Use properties and dimensions given in the figure.

(a) Find the force *P* on the cutting blade at *D* if the tension force in the rope is $T = 25$ lb (see free body diagram in part (b)].
(b) Find force in the pin at *C*.
(c) Find average shear stress τ_{ave} and bearing stress σ_b in the support pin at *C* [see Section *a–a* through cutting blade in figure part (c)].



(a) Top part of pole saw



(b) Free-body diagram (c) Section *a–a*

Solution 1.6-19

NUMERICAL PROPERTIES

$d_p = \frac{1}{8}$ in $\quad t_b = \frac{3}{32}$ in $\quad t_c = \frac{3}{8}$ in

$T = 25$ lb $\quad d_{BC} = 6$ in

$d_{CD} = 1$ in $\quad \alpha = \frac{\pi}{180}$ rad/deg

(a) Find the cutting force P on the cutting blade at D if the tension force in the rope is T = 25 lb:

$$\sum M_c = 0$$

$$\begin{aligned} M_C = {} & T(6\sin(70\alpha)) \\ & + 2T\cos(20\alpha)(6\sin(70\alpha)) \\ & - 2T\sin(20\alpha)(6\cos(70\alpha)) \\ & - P\cos(20\alpha)(1) \end{aligned}$$

SOLVE ABOVE EQUATION FOR P

$$P = \frac{[T(6\sin(70\alpha)) + 2T\cos(20\alpha) 6\sin(70\alpha)) - 2T\sin(20\alpha)(6\cos(70\alpha))]}{\cos(20\alpha)}$$

$P = 395$ lbs $\leftarrow$

(b) Find force in the pin at *C*

SOLVE FOR FORCES ON PIN AT C

$\sum F_x = 0 \quad C_x = T + 2T\cos(20\alpha) + P\cos(40\alpha)$

$C_x = 374$ lbs $\leftarrow$

$\sum F_y = 0 \quad C_y = 2T\sin(20\alpha) - P\sin(40\alpha)$

$C_y = -237$ lbs $\leftarrow$

RESULTANT AT C

$$C_{res} = \sqrt{C_x^2 + C_y^2} \qquad C_{res} = 443 \text{ lbs} \quad \leftarrow$$

(c) Find maximum shear and bearing stresses in the support pin at *C* (see section a-a through saw).

SHEAR STRESS - PIN IN DOUBLE SHEAR

$$A_s = \frac{\pi}{4} d_p^2 \qquad A_s = 0.012 \text{ in}^2$$

$$\tau_{ave} = \frac{C_{res}}{2A_s} \qquad \tau_{ave} = 18.04 \text{ ksi}$$

BEARING STRESSES ON PIN ON EACH SIDE OF COLLAR

$$\sigma_{bC} = \frac{\frac{C_{res}}{2}}{d_p t_c} \qquad \sigma_{bC} = 4.72 \text{ ksi} \quad \leftarrow$$

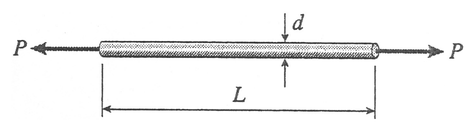
BEARING STRESS ON PIN AT CUTTING BLADE

$$\sigma_{bcb} = \frac{C_{res}}{d_p t_b} \qquad \sigma_{bcb} = 37.8 \text{ ksi} \quad \leftarrow$$

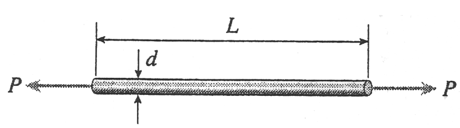
Allowable Stresses and Allowable Loads

Problem 1.7.1 A bar of solid circular cross section is loaded in tension by forces *P* (see figure). The bar has length $L = 16.0$ in. and diameter $d = 0.50$ in. The material is a magnesium alloy having modulus of elasticity $E = 6.4 \times 10^6$ psi. The allowable stress in tension is $\sigma_{allow} = 17{,}000$ psi, and the elongation of the bar must not exceed 0.04 in.

What is the allowable value of the forces *P*?



Solution 1.7-1 Magnesium bar in tension



$L = 16.0$ in. $\qquad d = 0.50$ in.

$E = 6.4 \times 10^6$ psi

$\sigma_{allow} = 17{,}000$ psi $\qquad \delta_{max} = 0.04$ in.

MAXIMUM LOAD BASED UPON ELONGATION

$$\varepsilon_{max} = \frac{\delta_{max}}{L} = \frac{0.04\text{in.}}{16 \text{ in.}} 0.00250$$

$$\sigma_{max} = E\varepsilon_{max} = (6.4 \times 10^6 \text{ psi})(0.00250)$$

$$= 16{,}000 \text{ psi}$$

$$P_{max} = \sigma_{max} A = (16.000 \text{ psi})\left(\frac{\pi}{4}\right)(0.50 \text{ in.})^2$$

$$= 3140 \text{ lb}$$

MAXIMUM LOAD BASED UPON TENSILE STRESS

$$P_{max} = \sigma_{allow} A = (17{,}000 \text{ psi})\left(\frac{\pi}{4}\right)(0.50 \text{ in.})^2$$

$$= 3340 \text{ Ib}$$

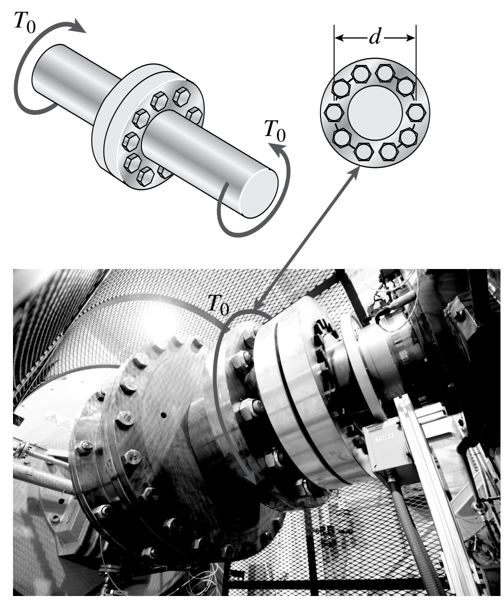
ALLOWABLE LOAD

Elongation governs.

$P_{allow} = 3140$ lb $\quad \leftarrow$

Problem 1.7-2 A torque T_0 is transmitted between two flanged shafts by means of ten 20-mm bolts (see figure and photo). The diameter of the bolt circle is $d = 250$ mm.

If the allowable shear stress in the bolts is 90 MPa, what is the maximum permissible torque? (Disregard friction between the flanges.)



Solution 1.7-2 Shafts with flanges

Numerical data

r = 10 d = 250 mm

^ bolts ^ flange

$$A_s = \pi r^2$$

$$A_s = 314.159 \text{ m}^2$$

$$\tau_a = 85 \text{ MPa}$$

Max. permissible torque

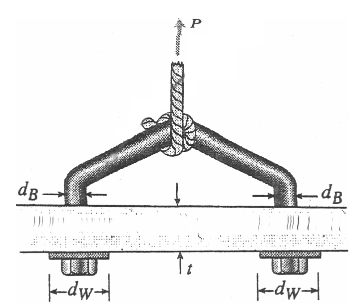
$$T_{max} = \tau_a A_s \left(r \frac{d}{2} \right)$$

$$T_{max} = 3.338 \times 10^7 \text{ N} \cdot \text{mm}$$

$$T_{max} = 33.4 \text{ kN} \cdot \text{m} \quad \leftarrow$$

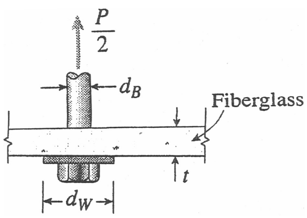
Problem 1.7-3 A tie-down on the deck of a sailboat consists of a bent bar bolted at both ends, as shown in the figure. The diameter d_B of the bar is $^1/_4$ in., the diameter d_W of the washers is $^7/_8$ in., and the thickness t of the fiberglass deck is $^3/_8$ in.

If the allowable shear stress in the fiberglass is 300 psi, and the allowable bearing pressure between the washer and the fiberglass is 550 psi, what is the allowable load P_{allow} on the tie-down?



01Ch01.qxd 9/25/08 12:18 PM Page 53

Solution 1.7-3 Bolts through fiberglass



$d_B = \frac{1}{4}$ in.

$d_W = \frac{7}{8}$ in.

$t = \frac{3}{8}$ in.

ALLOWABLE LOAD BASED UPON SHEAR STRESS IN FIBERGLASS

$\tau_{\text{allow}} = 300$ psi

Shear area $A_s = \pi d_W t$

$$\frac{P_1}{2} = \tau_{\text{allow}} A_s = \tau_{\text{allow}}(\pi d_W t)$$

$$= (300 \text{ psi})(\pi)\left(\frac{7}{8}\text{ in.}\right)\left(\frac{3}{8}\text{ in.}\right)$$

$$\frac{P_1}{2} = 309.3 \text{ lb}$$

$P_1 = 619$ lb

ALLOWABLE LOAD BASED UPON BEARING PRESSURE

$\sigma_b = 550$ psi

Bearing area $A_b = \frac{\pi}{4}(d_W^2 - d_B^2)$

$$\frac{P_2}{2} = \sigma_b A_b = (550 \text{ psi})\left(\frac{\pi}{4}\right)\left[\left(\frac{7}{8}\text{ in.}\right)^2 - \left(\frac{1}{4}\text{ in.}\right)^2\right]$$

$$= 303.7 \text{ lb}$$

$P_2 = 607$ lb

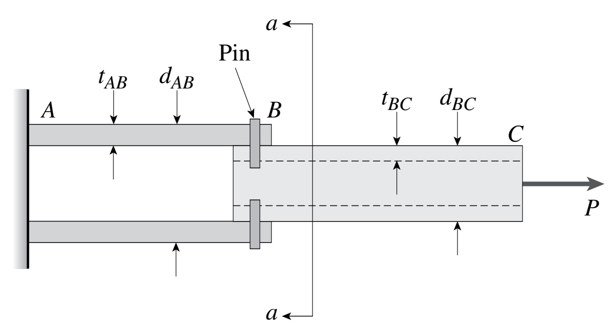
ALLOWABLE LOAD

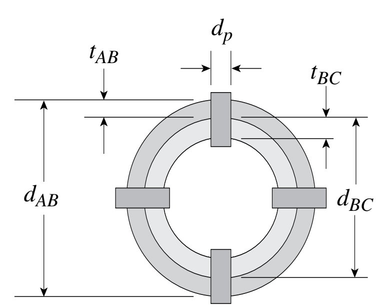
Bearing pressure governs.

$P_{\text{allow}} = 607$ lb ←

Problem 1.7-4 Two steel tubes are joined at B by four pins ($d_p = 11$ mm), as shown in the cross section *a–a* in the figure. The outer diameters of the tubes are $d_{AB} = 40$ mm and $d_{BC} = 28$ mm. The wall thicknesses are $t_{AB} = 6$ mm and $t_{BC} = 7$ mm. The yield stress in tension for the steel is $\sigma_Y = 200$ MPa and the ultimate stress in *tension* is $\sigma_U = 340$ MPa. The corresponding yield and ultimate values in *shear* for the pin are 80 MPa and 140 MPa, respectively. Finally, the yield and ultimate values in *bearing* between the pins and the tubes are 260 MPa and 450 MPa, respectively. Assume that the factors of safety with respect to yield stress and ultimate stress are 4 and 5, respectively.

(a) Calculate the allowable tensile force P_{allow} considering tension in the tubes.
(b Recompute P_{allow} for shear in the pins.
(c) Finally, recompute P_{allow} for bearing between the pins and the tubes. Which is the controlling value of P?





Section a–a

Solution 1.7-4

Yield and ultimate stresses (all in MPa)

TUBES:

$\sigma_Y = 200 \qquad \sigma_u = 340 \qquad FSy = 4$

PIN (SHEAR):

$\tau_Y = 80 \qquad \tau_u = 140 \qquad FSu = 5$

PIN (BEARING):

$\sigma_{bY} = 260 \qquad \sigma_{bu} = 450$

tubes and pin dimensions (mm)

$d_{AB} = 40 \qquad t_{AB} = 6$

$d_{BC} = d_{AB} - 2t_{AB} \qquad d_{BC} = 28$

$t_{BC} = 7 \qquad d_p = 11$

(a) P_{allow} CONSIDERING TENSION IN THE TUBES

$$A_{netAB} = \frac{\pi}{4}\left[d_{AB}^2 - (d_{AB} - 2t_{AB})^2 - 4\,d_p t_{AB}\right]$$

$$A_{netAB} = 433.45 \text{ mm}^2$$

$$A_{netBC} = \frac{\pi}{4}\left[d_{BC}^2 - (d_{BC} - 2t_{BC})^2 - 4\,d_p t_{BC}\right]$$

$$A_{netAB} = 219.911 \text{ mm}^2 \quad \text{use smaller}$$

$$P_{aT1} = \frac{\sigma_Y}{FSy}A_{netBC} \qquad P_{aT1} = 1.1 \times 10^4 \text{ N}$$

$$P_{aT1} = 11.0 \text{ kN} \quad \leftarrow$$

$$P_{aT2} = \frac{\sigma_u}{FSu}A_{netBC} \qquad P_{aT2} = 1.495 \times 10^4$$

(b) P_{allow} CONSIDERING SHEAR IN THE PINS

$$A_s = \frac{\pi}{4}d_p^2 \qquad A_s = 95.033 \text{ mm}^2 \text{ (one pin)}$$

$$P_{aS1} = (4A_s)\frac{\tau_Y}{FSy}$$

$$P_{aS1} = 7.60 \text{ kN} \quad \leftarrow$$

$$P_{aS2} = (4A_s)\frac{\tau_u}{FSu} \qquad P_{aS2} = 10.64 \text{ kN}$$

(c) P_{allow} CONSIDERING BEARING IN THE PINS

$$A_{bAB} = 4d_p t_{AB}$$

$$A_{bAB} = 264 \text{ mm}^2 \qquad < \text{smaller controls}$$

$$A_{bBC} = 4d_p t_{BC} \qquad A_{bBC} = 308 \text{ mm}^2$$

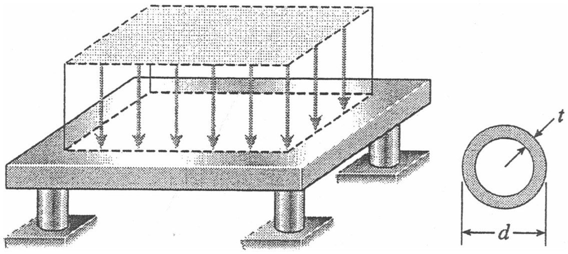
$$P_{ab1} = A_{bAB}\left(\frac{\sigma_{bY}}{FSy}\right) \qquad P_{ab1} = 1.716 \times 10^4$$

$$P_{ab1} = 17.16 \text{ kN} \quad \leftarrow$$

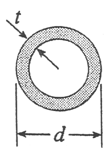
$$P_{ab2} = A_{bAB}\left(\frac{\sigma_{bu}}{FSu}\right) \qquad P_{ab2} = 23.8 \text{ kN}$$

Problem 1.7-5 A steel pad supporting heavy machinery rests on four short, hollow, cast iron piers (see figure). The ultimate strength of the cast iron in compression is 50 ksi. The outer diameter of the piers is $d = 4.5$ in. and the wall thickness is $t = 0.40$ in.

Using a factor of safety of 3.5 with respect to the ultimate strength, determine the total load P that may be supported by the pad.



Solution 1.7-5 Cast iron piers in compression



Four piers

$\sigma_U = 50$ ksi

$n = 3.5$

$$\sigma_{\text{allow}} = \frac{\sigma_U}{n} = \frac{50 \text{ ksi}}{3.5} = 14.29 \text{ ksi}$$

$d = 4.5$ in.

$t = 0.4$ in.

$d_0 = d - 2t = 3.7$ in.

$$A = \frac{\pi}{4}(d^2 - d_0^2) = \frac{\pi}{4}[(4.5 \text{ in.})^2 - (3.7 \text{ in.})^2]$$

$$= 5.152 \text{ in.}^2$$

P_1 = allowable load on one pier

$$= \sigma_{\text{allow}} A = (14.29 \text{ ksi})(5.152 \text{ in.}^2)$$

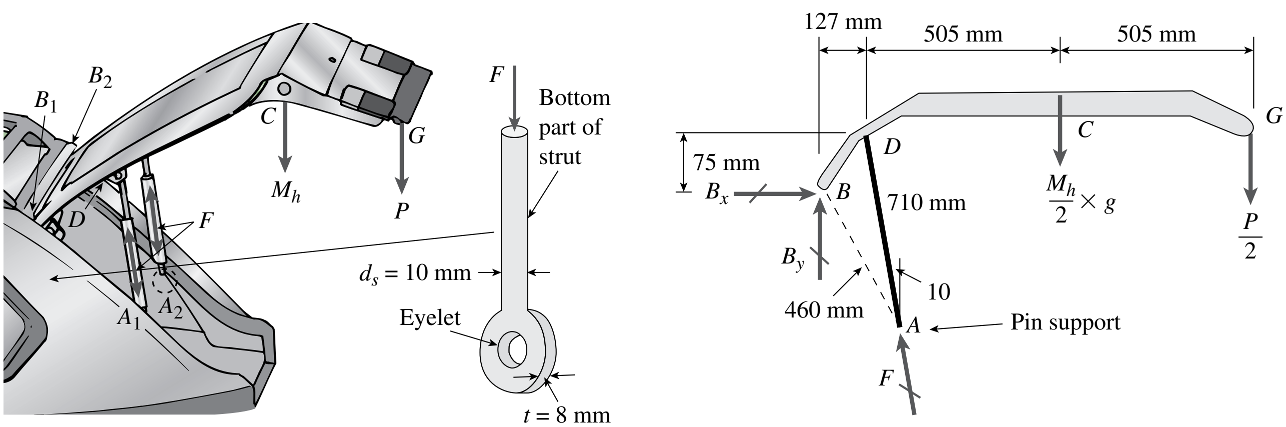
$$= 73.62 \text{ k}$$

Total load $P = 4P_1 = 294$ k ←

Problem 1.7-6 The rear hatch of a van [$BDCF$ in figure part (a)] is supported by two hinges at B_1 and B_2 and by two struts A_1B_1 and A_2B_2 (diameter $d_s = 10$ mm) as shown in figure part (b). The struts are supported at A_1 and A_2 by pins, each with diameter $d_p = 9$ mm and passing through an eyelet of thickness $t = 8$ mm at the end of the strut [figure part (b)]. If a closing force $P = 50$ N is applied at G and the mass of the hatch $M_h = 43$ kg is concentrated at C:

(a) What is the force F in each strut? [Use the free-body diagram of one half of the hatch in the figure part (c)]

(b) What is the maximum permissible force in the strut, F_{allow}, if the allowable stresses are as follows: compressive stress in the strut, 70 MPa; shear stress in the pin, 45 MPa; and bearing stress between the pin and the end of the strut, 110 MPa.



(a) (b) (c)

Solution 1.7-6

NUMERICAL DATA

$M_h = 43$ kg $\quad \sigma_a = 70$ MPa

$\tau_a = 45$ MPa $\quad \sigma_{ba} = 110$ MPa

$d_s = 10$ mm $\quad d_p = 9$ mm $\quad t = 8$ mm

$P = 50$ N $\quad g = 9.81 \dfrac{\text{m}}{\text{s}^2}$

(a) FORCE F IN EACH STRUT FROM STATICS (SUM MOMENTS ABOUT B)

$$\alpha = 10\frac{\pi}{180} \quad F_V = F\cos(\alpha) \quad F_H = F\sin(\alpha)$$

$$\sum M_B = 0$$

$$F_V(127) + F_H(75)$$

$$= \frac{M_h}{2} g\,(127 + 505) + \frac{P}{2}[127 + 2(505)]$$

$$F(127\cos(\alpha) + 75\sin(\alpha))$$

$$= \frac{M_h}{2} g\,(127 + 505) + \frac{P}{2}[127 + 2(505)]$$

$$F = \frac{\frac{M_h}{2} g\,(127 + 505) + \frac{P}{2}[127 + 2(505)]}{(127\cos(\alpha) + 75\sin(\alpha))}$$

$$F = 1.171 \text{ kN} \quad \leftarrow$$

(b) MAX. PERMISSIBLE FORCE F IN EACH STRUT
F_{max} IS SMALLEST OF THE FOLLOWING

$$F_{a1} = \sigma_a \frac{\pi}{4} d_s^{\,2} \qquad F_{a1} = 5.50 \text{ kN}$$

$$F_{a2} = \tau_a \frac{\pi}{4} d_p^{\,2}$$

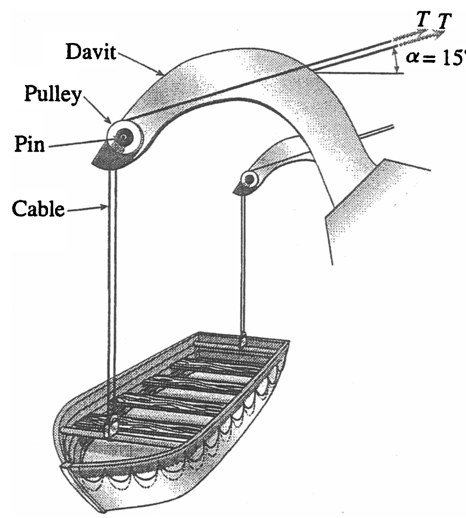
$$F_{a2} = 2.86 \text{ kN} \qquad \leftarrow \qquad \frac{F_{a2}}{F} = 2.445$$

$$F_{a3} = \sigma_{ba} d_p t \qquad F_{a3} = 7.92 \text{ kN}$$

Problem 1.7-7 A lifeboat hangs from two ship's davits, as shown in the figure. A pin of diameter $d = 0.80$ in. passes through each davit and supports two pulleys, one on each side of the davit.

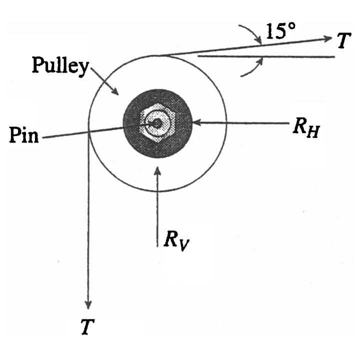
Cables attached to the lifeboat pass over the pulleys and wind around winches that raise and lower the lifeboat. The lower parts of the cables are vertical and the upper parts make an angle $\alpha = 15°$ with the horizontal. The allowable tensile force in each cable is 1800 lb, and the allowable shear stress in the pins is 4000 psi.

If the lifeboat weighs 1500 lb, what is the maximum weight that should be carried in the lifeboat?



Solution 1.7-7 Lifeboat supported by four cables

FREE-BODY DIAGRAM OF ONE PULLEY



Pin diameter $d = 0.80$ in.

T = tensile force in one cable

$T_{allow} = 1800$ lb

$\tau_{allow} = 4000$ psi

W = weight of lifeboat

$= 1500$ lb

$\Sigma F_{horiz} = 0 \qquad R_H = T\cos 15° = 0.9659T$

$\Sigma F_{vert} = 0 \qquad R_V = T - T\sin 15° = 0.7412T$

V = shear force in pin

$V = \sqrt{(R_H)^2 + (R_v)^2} = 1.2175T$

Allowable tensile force in one cable based upon shear in the pins

$$V_{allow} = \tau_{allow} A_{pin} = (4000 \text{ psi})\left(\frac{\pi}{4}\right)(0.80 \text{ in.})^2$$

$$= 2011 \text{ lb}$$

$$V = 1.2175T \qquad T_1 = \frac{V_{allow}}{1.2175} = 1652 \text{ lb}$$

Allowable force in one cable based upon tension in the cable

$$T_2 = T_{allow} = 1800 \text{ lb}$$

Maximum weight

Shear in the pins governs.

$$T_{max} = T_1 = 1652 \text{ lb}$$

Total tensile force in four cables

$$= 4T_{max} = 6608 \text{ lb}$$

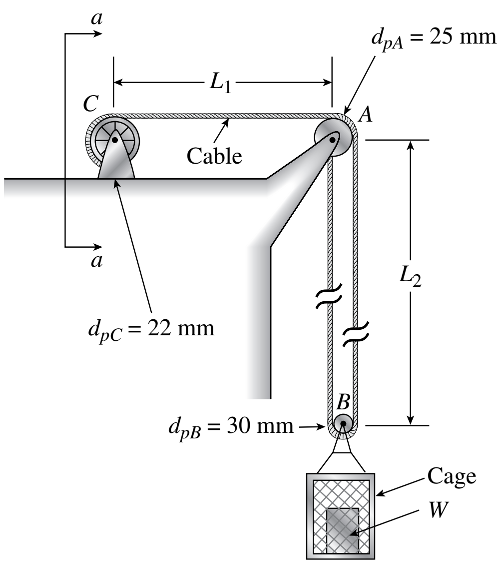
$$W_{max} = 4T_{max} - W$$

$$= 6608 \text{ lb} - 1500 \text{ lb}$$

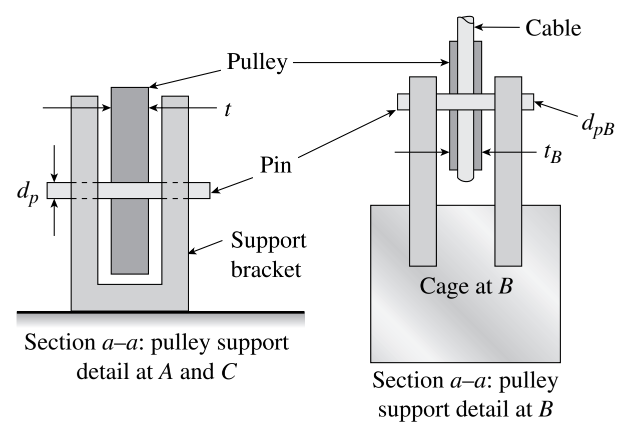
$$= 5110 \text{ lb} \quad \leftarrow$$

Problem 1.7-8 A cable and pulley system in figure part (a) supports a cage of mass 300 kg at B. Assume that this includes the mass of the cables as well. The thickness of each the three steel pulleys is $t = 40$ mm. The pin diameters are $d_{pA} = 25$ mm, $d_{pB} = 30$ mm and $d_{pC} = 22$ mm [see figure, parts (a) and part (b)].

(a) Find expressions for the resultant forces acting on the pulleys at A, B, and C in terms of cable tension T.
(b) What is the maximum weight W that can be added to the cage at B based on the following allowable stresses? Shear stress in the pins is 50 MPa; bearing stress between the pin and the pulley is 110 MPa.



(a)



(b)

Solution 1.7-8

NUMERICAL DATA

$M = 300\ \text{kg} \qquad g = 9.81\dfrac{m}{s^2}$

$\tau_a = 50\ \text{MPa} \qquad \sigma_{ba} = 110\ \text{MPa}$

$t_A = 40\ \text{mm} \qquad t_B = 40\ \text{mm}$

$t_C = 50 \qquad d_{pA} = 25\ \text{mm}$

$d_{pB} = 30 \qquad d_{pC} = 22\ \text{mm}$

(a) RESULTANT FORCES F ACTING ON PULLEYS A, B & C

$F_A = \sqrt{2}\,T \qquad F_B = 2T$

$F_C = T \qquad T = \dfrac{Mg}{2} + \dfrac{W_{max}}{2}$

$W_{max} = 2T - Mg$

From statics at B

(b) MAX. LOAD W THAT CAN BE ADDED AT B DUE TO τ_a & σ_{ba} IN PINS AT A, B & C

PULLEY AT A

$\tau_A = \dfrac{F_A}{A_s}$

DOUBLE SHEAR

$F_A = \tau_a A_s \qquad \sqrt{2}\,T = \tau_a A_s$

$$\frac{Mg}{2} + \frac{W_{max}}{2} = \frac{\tau_a A_s}{\sqrt{2}}$$

$$W_{max1} = \frac{2}{\sqrt{2}}\left(\tau_a A_s\right) - Mg$$

$$W_{max1} = \frac{2}{\sqrt{2}}\left(\tau_a 2\frac{\pi}{4} d_p A^2\right) - Mg$$

$$\frac{W_{max1}}{Mg} = 22.6$$

$W_{max1} = 66.5\ \text{kN} \quad \leftarrow$

(shear at A controls)

OR check bearing stress

$$W_{max2} = \frac{2}{\sqrt{2}}\left(\sigma_{ba} A_b\right) - Mg$$

$$W_{max2} = \frac{2}{\sqrt{2}}\left(\sigma_{ba} t_A d_{pA}\right) - Mg$$

$W_{max2} = 152.6\ \text{kN}$ (bearing at A)

PULLEY AT B $\qquad 2T = \tau_a A_s$

$$W_{max3} = \frac{2}{2}(\tau_a A_s) - Mg$$

$$W_{max3} = \left[\tau_a\left(2\frac{\pi}{4} d_{pB}^2\right)\right] - Mg$$

$W_{max3} = 67.7\ \text{kN}$ (shear at B)

$$W_{max4} = \frac{2}{2}(\sigma_{ba} A_b) - Mg$$

$$W_{max4} = \sigma_{ba} t_B d_{pB} - Mg$$

$W_{max4} = 129.1\ \text{kN}$ (bearing at B)

PULLEY AT C $\qquad T = \tau_a A_s$

$$W_{max5} = 2(\tau_a A_s) - Mg$$

$$W_{max5} = \left[2\tau_a\left(2\frac{\pi}{4} d_{pC}^2\right)\right] - Mg$$

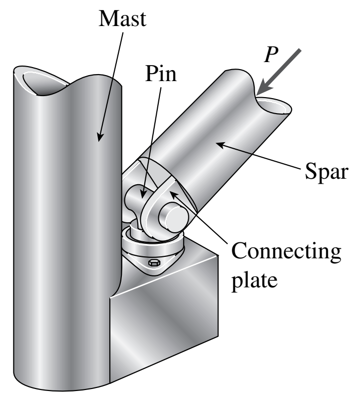
$W_{max5} = 7.3 \times 10^4 \qquad W_{max5} = 73.1\ \text{kN}$ (shear at C)

$$W_{max6} = 2\sigma_{ba} t_C d_{pC} - Mg$$

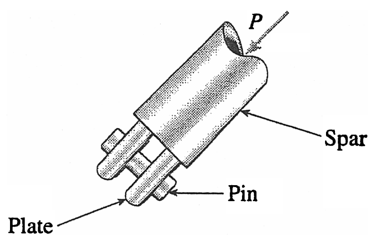
$W_{max6} = 239.1\ \text{kN}$ (bearing at C)

Problem 1.7-9 A ship's spar is attached at the base of a mast by a pin connection (see figure). The spar is a steel tube of outer diameter $d_2 = 3.5$ in. and inner diameter $d_1 = 2.8$ in. The steel pin has diameter $d = 1$ in., and the two plates connecting the spar to the pin have thickness $t = 0.5$ in. The allowable stresses are as follows: compressive stress in the spar, 10 ksi; shear stress in the pin, 6.5 ksi; and bearing stress between the pin and the connecting plates, 16 ksi.

Determine the allowable compressive force P_{allow} in the spar.



Solution 1.7-9



NUMERICAL DATA

$d_2 = 3.5$ in. $d_1 = 2.8$ in.

$d_p = 1$ in. $t = 0.5$ in.

$\sigma_a = 10$ ksi $\tau_a = 6.5$ ksi $\sigma_{ba} = 16$ ksi

COMPRESSIVE STRESS IN SPAR

$$P_{a1} = \sigma_a \frac{\pi}{4}(d_2^2 - d_1^2) \qquad P_{a1} = 34.636 \text{ kips}$$

SHEAR STRESS IN PIN

$$P_{a2} = \tau_a \left(2 \frac{\pi}{4} d_p^2\right)$$

$$P_{a2} = 10.21 \text{ kips} < \text{controls} \quad \leftarrow$$

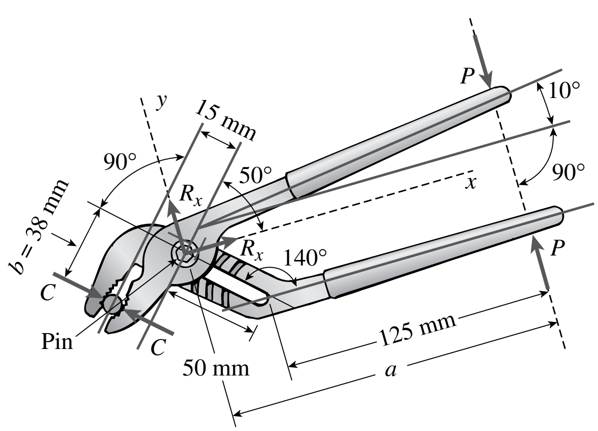
^double shear

BEARING STRESS BETWEEN PIN & CONECTING PLATES

$$P_{a3} = \sigma_{ba}(2d_p t) \qquad P_{a3} = 16 \text{ kips}$$

Problem 1.7-10 What is the maximum possible value of the clamping force C in the jaws of the pliers shown in the figure if the ultimate shear stress in the 5-mm diameter pin is 340 MPa?

What is the maximum permissible value of the applied load P if a factor of safety of 3.0 with respect to failure of the pin is to be maintained?



Solution 1.7-10

NUMERICAL DATA

$FS = 3 \qquad \tau_u = 340\ \text{MPa} \qquad \tau_a = \dfrac{\tau_u}{FS}$

$\alpha = 40\dfrac{\pi}{180}\ \text{rad} \qquad d = 5\ \text{mm}$

$$\tau_a = \frac{\sqrt{R_x{}^2 + R_y{}^2}}{A_s} \quad < \text{pin at C in single shear}$$

$R_x = -C\cos(\alpha) \qquad R_y = P + C\sin(\alpha)$

$a = 50\cos(\alpha) + 125 \qquad a = 163.302\ \text{mm}$

$b = 38\ \text{mm}$

STATICS $\quad \sum M_{pin} = 0 \qquad C = \dfrac{P(a)}{b}$

$$R_x = -\frac{P(a)}{b}\cos(\alpha) \qquad R_y = P\left[1 + \frac{a}{b}\sin(\alpha)\right]$$

$$P\sqrt{\left[-\frac{a}{b}\cos(\alpha)\right]^2 + \left[1 + \frac{a}{b}\sin(\alpha)\right]^2} = \tau_a A_s$$

$$A_s = \frac{\pi}{4}d^2$$

$\tau_a = \dfrac{\tau_u}{FS} \qquad \tau_a = 113.333\ \text{MPa}$

Find P_{max}

$$P_{max} = \frac{\tau_a A_s}{\sqrt{\left[-\frac{a}{b}\cos(\alpha)\right]^2 + \left[1 + \frac{a}{b}\sin(\alpha)\right]^2}}$$

$P_{max} = 445\ \text{N} \quad \leftarrow$

here $\dfrac{a}{b} = 4.297 \quad < a/b =$ **mechanical advantage**

FIND MAX. CLAMPING FORCE

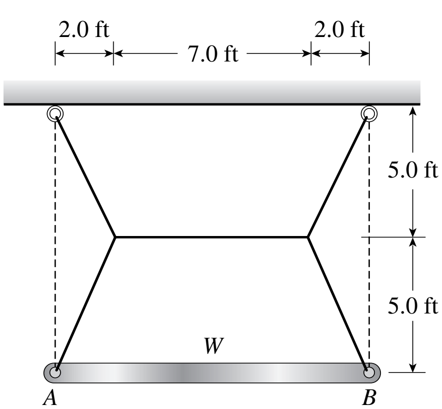
$C_{ult} = P_{max}FS\left(\dfrac{a}{b}\right) \qquad C_{ult} = 5739\ \text{N} \quad \leftarrow$

$P_{ult} = P_{max}FS \qquad P_{ult} = 1335$

$$\frac{C_{ult}}{P_{ult}} = 4.297$$

Problem 1.7-11 A metal bar *AB* of weight *W* is suspended by a system of steel wires arranged as shown in the figure. The diameter of the wires is 5/64 in., and the yield stress of the steel is 65 ksi.

Determine the maximum permissible weight W_{max} for a factor of safety of 1.9 with respect to yielding.



Solution 1.7-11

NUMERICAL DATA

$d = \dfrac{5}{64}\ \text{in.} \qquad \sigma_Y = 65\ \text{ksi} \qquad FS_y = 1.9$

$\sigma_a = \dfrac{\sigma_Y}{FS_y} \qquad \sigma_a = 34.211\ \text{ksi}$

FORCES IN WIRES AC, EC, BD, FD

$\sum F_V = 0 \quad$ at A, B, E or F

$$F_W = \frac{\sqrt{2^2 + 5^2}}{5} \times \frac{W}{2} \qquad \frac{\sqrt{2^2 + 5^2}}{10} = 0.539$$

$W_{max} = 0.539\ \sigma_a \times A$

$$W_{max} = 0.539\left(\frac{\sigma_Y}{FS_y}\right)\left(\frac{\pi}{4}d^2\right)$$

$$W_{max} = 0.305 \text{ kips} \quad \leftarrow$$

CHECK ALSO FORCE IN WIRE CD

$$\sum F_H = 0 \qquad \text{at C or D}$$

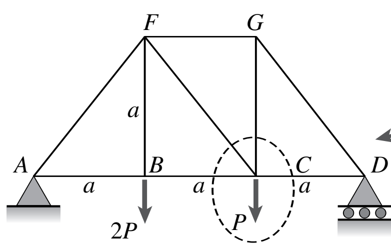
$$F_{CD} = 2\left(\frac{2}{\sqrt{2^2 + 5^2}}F_w\right)$$

$$F_{CD} = 2\left[\frac{2}{\sqrt{2^2 + 5^2}}\left(\frac{\sqrt{2^2 + 5^2}}{5} \times \frac{W}{2}\right)\right]$$

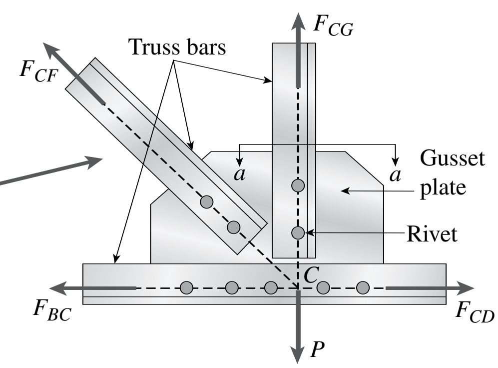
$$F_{CD} = \frac{2}{5}W \qquad \text{less than } F_{AC} \text{ so AC controls}$$

Problem 1.7-12 A plane truss is subjected to loads $2P$ and P at joints B and C, respectively, as shown in the figure part (a). The truss bars are made of two L102 × 76 × 6.4 steel angles [see Table E-5(b): cross sectional area of the two angles, $A = 2180 \text{ mm}^2$, figure part (b)] having an ultimate stress in tension equal to 390 MPa. The angles are connected to an 12 mm-thick gusset plate at C [figure part (c)] with 16-mm diameter rivets; assume each rivet transfers an equal share of the member force to the gusset plate. The ultimate stresses in shear and bearing for the rivet steel are 190 MPa and 550 MPa, respectively.

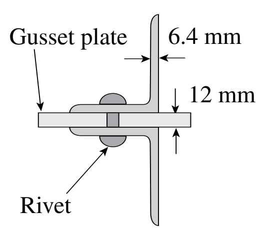
Determine the allowable load P_{allow} if a safety factor of 2.5 is desired with respect to the ultimate load that can be carried. (Consider tension in the bars, shear in the rivets, bearing between the rivets and the bars, and also bearing between the rivets and the gusset plate. Disregard friction between the plates and the weight of the truss itself.)



(a)



(c)



(b) Section a–a

Solution 1.7-12

NUMERICAL DATA

$A = 2180\ \text{mm}^2$

$t_g = 12\ \text{mm}$ $\quad d_r = 16\ \text{mm}$ $\quad t_{ang} = 6.4\ \text{mm}$

$\sigma_u = 390\ \text{MPa}$ $\quad \tau_u = 190\ \text{MPa}$

$\sigma_{bu} = 550\ \text{MPa}$ $\quad FS = 2.5$

$\sigma_a = \dfrac{\sigma_u}{FS}$ $\quad \tau_a = \dfrac{\tau_u}{FS}$ $\quad \sigma_{ba} = \dfrac{\sigma_{bu}}{FS}$

MEMBER FORCES FROM TRUSS ANALYSIS

$F_{BC} = \dfrac{5}{3}P$ $\quad F_{CD} = \dfrac{4}{3}P$ $\quad F_{CF} = \dfrac{\sqrt{2}}{3}P$

$\dfrac{\sqrt{2}}{3} = 0.471$ $\quad F_{CG} = \dfrac{4}{3}P$

P_{allow} FOR TENSION ON NET SECTION IN TRUSS BARS

$A_{net} = A - 2d_r t_{ang}$ $\quad A_{net} = 1975\ \text{mm}^2$

$\dfrac{A_{net}}{A} = 0.906$

$F_{allow} = \sigma_a A_{net}$ < allowable force in a member so BC controls since it has the largest member force for this loading

$P_{allow} = \dfrac{3}{5}F_{BCmax}$ $\quad P_{allow} = \dfrac{3}{5}(\sigma_a A_{net})$

$P_{allow} = 184.879\ \text{kN}$

Next, P_{allow} for shear in rivets (all are in double shear)

$A_s = 2\dfrac{\pi}{4}d_r^2$ < for one rivet in DOUBLE shear

$\dfrac{F_{max}}{N} = \tau_a A_s$ $\quad N$ = number of rivets in a particular member (see drawing of conn. detail)

$P_{BC} = 3\left(\dfrac{3}{5}\right)(\tau_a A_s)$ $\quad P_{BC} = 55.0\ \text{kN}$

$P_{CF} = 2\left(\dfrac{3}{\sqrt{2}}\right)(\tau_a A_s)$ $\quad P_{CF} = 129.7\ \text{kN}$

$P_{CG} = 2\left(\dfrac{3}{4}\right)(\tau_a A_s)$

$P_{CG} = 45.8\ \text{kN}$ $\longleftarrow$ < so shear in rivets in CG & CD controls P_{allow} here

$P_{CD} = 2\left(\dfrac{3}{4}\right)(\tau_a A_s)$ $\quad P_{CD} = 45.8\ \text{kN}$ $\longleftarrow$

Next, P_{allow} for bearing of rivets on truss bars $A_b = 2d_r t_{ang}$ < rivet bears on each angle in two angle pairs

$\dfrac{F_{max}}{N} = \sigma_{ba} A_b$

$P_{BC} = 3\left(\dfrac{3}{5}\right)(\sigma_{ba} A_b)$ $\quad P_{BC} = 81.101\ \text{kN}$

$P_{CF} = 2\left(\dfrac{3}{\sqrt{2}}\right)(\sigma_{ba} A_b)$ $\quad P_{CF} = 191.156\ \text{kN}$

$P_{CG} = 2\left(\dfrac{3}{4}\right)(\sigma_{ba} A_b)$ $\quad P_{CG} = 67.584\ \text{kN}$

$P_{CD} = 2\left(\dfrac{3}{4}\right)(\sigma_{ba} A_b)$ $\quad P_{CD} = 67.584\ \text{kN}$

Finally, P_{allow} for bearing of rivets on gusset plate

$A_b = d_r t_g$

(bearing area for each rivert on gusset plate)

$t_g = 12\ \text{mm} < 2t_{ang} = 12.8\ \text{mm}$

so gusset will control over angles

$P_{BC} = 3\left(\dfrac{3}{5}\right)(\sigma_{ba} A_b)$ $\quad P_{BC} = 76.032\ \text{kN}$

$P_{CF} = 2\left(\dfrac{3}{\sqrt{2}}\right)(\sigma_{ba} A_b)$ $\quad P_{CF} = 179.209\ \text{kN}$

$P_{CG} = 2\left(\dfrac{3}{4}\right)(\sigma_{ba} A_b)$ $\quad P_{CG} = 63.36\ \text{kN}$

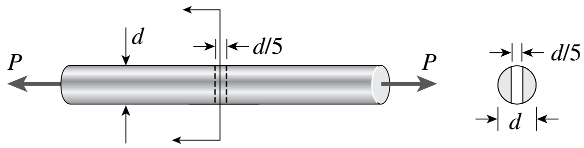
$P_{CD} = 2\left(\dfrac{3}{4}\right)(\sigma_{ba} A_b)$ $\quad P_{CD} = 63.36\ \text{kN}$

So, shear in rivets controls: $P_{allow} = 45.8\ \text{kN}$ $\longleftarrow$

Problem 1.7-13 A solid bar of circular cross section (diameter d) has a hole of diameter $d/5$ drilled laterally through the center of the bar (see figure). The allowable average tensile stress on the net cross section of the bar is σ_{allow}.

(a) Obtain a formula for the allowable load P_{allow} that the bar can carry in tension.

(b) Calculate the value of P_{allow} if the bar is made of brass with diameter $d = 1.75$ in. and $\sigma_{allow} = 12$ ksi.
(*Hint*: Use the formulas of Case 15 Appendix D.)



Solution 1.7-13

NUMERICAL DATA

$d = 1.75$ in $\qquad \sigma_a = 12$ ksi

(a) FORMULA FOR P_{ALLOW} IN TENSION

From Case 15, Appendix D:

$$A = 2r^2\left(\alpha - \frac{ab}{r^2}\right) \qquad r = \frac{d}{2} \qquad a = \frac{d}{10}$$

$$\alpha = \operatorname{acos}\left(\frac{a}{r}\right) \qquad r = 0.875 \text{ in.} \qquad a = 0.175 \text{ in.}$$

$$\alpha\frac{180}{\pi} = 78.463 \text{ degrees}$$

$$b = \sqrt{r^2 - a^2}$$

$$b = \sqrt{\left[\left(\frac{d}{2}\right)^2 - \left(\frac{d}{10}\right)^2\right]}$$

$$b = \sqrt{\left(\frac{6}{25}d^2\right)} \qquad b = \frac{d}{5}\sqrt{6}$$

$$P_a = \sigma_a A$$

$$P_a = \sigma_a\left[\frac{1}{2}d^2\left(\operatorname{acos}\left(\frac{1}{5}\right) - \frac{2}{25}\sqrt{6}\right)\right]$$

$$\frac{\operatorname{acos}\left(\frac{1}{5}\right) - \frac{2}{25}\sqrt{6}}{2} = 0.587 \qquad \frac{\pi}{4} = 0.785$$

$$P_a = \sigma_a(0.587\,d^2) \quad \leftarrow$$

$$\frac{0.587}{0.785} = 0.748$$

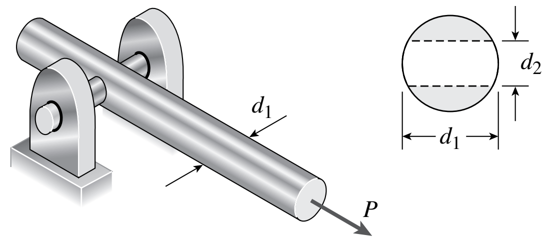
(b) EVALUATE NUMERICAL RESULT

$d = 1.75$ in. $\qquad \sigma_a = 12$ ksi

$P_a = 21.6$ kips $\quad \leftarrow$

Problem 1.7-14 A solid steel bar of diameter $d_1 = 60$ mm has a hole of diameter $d_2 = 32$ mm drilled through it (see figure). A steel pin of diameter d_2 passes through the hole and is attached to supports.

Determine the maximum permissible tensile load P_{allow} in the bar if the yield stress for shear in the pin is $\tau_Y = 120$ MPa, the yield stress for tension in the bar is $\sigma_Y = 250$ MPa and a factor of safety of 2.0 with respect to yielding is required. (*Hint*: Use the formulas of Case 15, Appendix D.)



Solution 1.7-14

NUMERICAL DATA

$d_1 = 60$ mm $\quad d_2 = 32$ mm

$\tau_Y = 120$ MPa $\quad \sigma_Y = 250$ MPa

$FS_y = 2$

ALLOWABLE STRESSES

$$\tau_a = \frac{\tau_Y}{FS_y} \qquad \tau_a = 60 \text{ MPa}$$

$$\sigma_a = \frac{\sigma_Y}{FS_y} \qquad \sigma_a = 125 \text{ MPa}$$

From Case 15, Appendix D: $\quad r = \dfrac{d_1}{2}$

$$A = 2r^2\left(\alpha - \frac{ab}{r^2}\right) \qquad \alpha = \arccos\frac{d_2/2}{d_1/2} = \arccos\frac{d_2}{d_1}$$

$$a = \frac{d_2}{2} \qquad b = \sqrt{r^2 - a^2}$$

SHEAR AREA (DOUBLE SHEAR)

$$A_s = 2\left(\frac{\pi}{4}d_2^2\right) \qquad A_s = 1608 \text{ mm}^2$$

NET AREA IN TENSION (FROM CASE 15, APP. D)

$$A_{net} = 2\left(\frac{d_1}{2}\right)^2 \left[\operatorname{acos}\left(\frac{d_2}{d_1}\right) - \frac{\frac{d_2}{2}\left[\sqrt{\left(\frac{d_1}{2}\right)^2 - \left(\frac{d_2}{2}\right)^2}\right]}{\left(\frac{d_1}{2}\right)^2}\right]$$

$$A_{net} = 1003 \text{ mm}^2$$

P_{allow} in tension: smaller of values based on either shear or tension allowable stress x appropriate area

$P_{a1} = \tau_a A_s \quad P_{a1} = 96.5\text{kN} <$ shear governs $\quad \leftarrow$

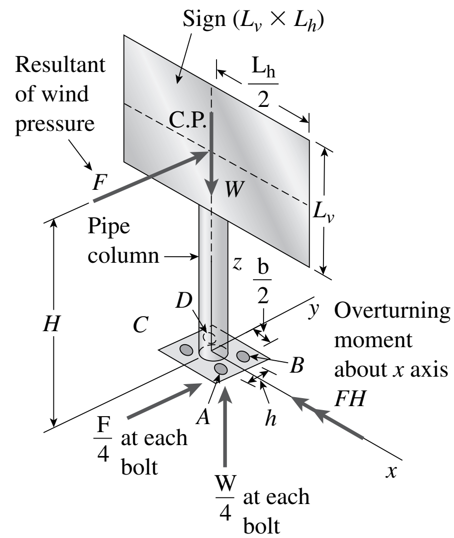
$P_{a2} = \sigma_a A_{net} \quad P_{a2} = 125.4\text{kN}$

01Ch01.qxd 9/25/08 12:18 PM Page 65

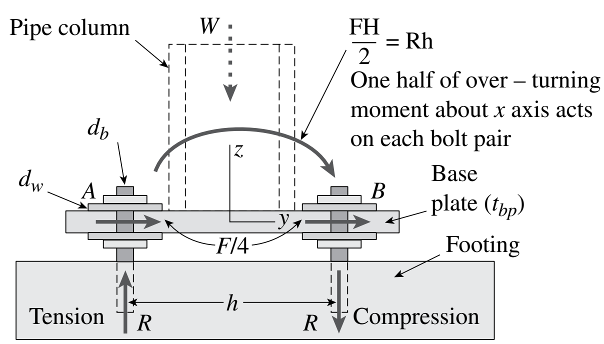
Problem 1.7-15 A sign of weight W is supported at its base by four bolts anchored in a concrete footing. Wind pressure p acts normal to the surface of the sign; the resultant of the uniform wind pressure is force F at the center of pressure. The wind force is assumed to create equal shear forces $F/4$ in the y-direction at each bolt [see figure parts (a) and (c)]. The overturning effect of the wind force also causes an uplift force R at bolts A and C and a downward force $(-R)$ at bolts B and D [see figure part (b)]. The resulting effects of the wind, and the associated ultimate stresses for each stress condition, are: normal stress in each bolt ($\sigma_u = 60$ ksi); shear through the base plate ($\tau_u = 17$ ksi); horizontal shear and bearing on each bolt ($\tau_{hu} = 25$ ksi and $\sigma_{bu} = 75$ ksi); and bearing on the bottom washer at B (or D) ($\sigma_{bw} = 50$ ksi).

Find the maximum wind pressure p_{max} (psf) that can be carried by the bolted support system for the sign if a safety factor of 2.5 is desired with respect to the ultimate wind load that can be carried.

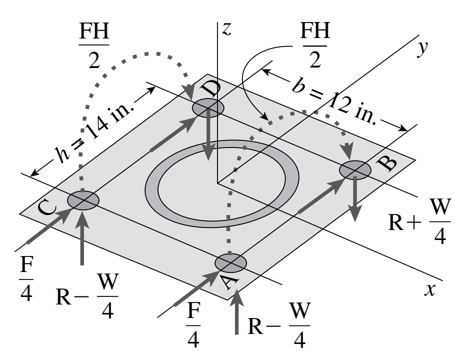
Use the following numerical data: bolt $d_b = 3/4$ in.; washer $d_w = 1.5$ in.; base plate $t_{bp} = 1$ in.; base plate dimensions $h = 14$ in. and $b = 12$ in.; $W = 500$ lb; $H = 17$ ft; sign dimensions ($L_v = 10$ ft. $\times$ $L_h = 12$ ft.); pipe column diameter $d = 6$ in., and pipe column thickness $t = 3/8$ in.



(a)



(b)



(c)

Solution 1.7-15

NUMERICAL DATA

$\sigma_u = 60 \text{ ksi} \quad \tau_u = 17 \text{ ksi} \quad \tau_{hu} = 25 \text{ ksi}$

$\sigma_{bu} = 75 \text{ ksi} \quad \sigma_{bw} = 50 \text{ ksi} \quad FS_u = 2.5$

$d_b = \frac{3}{4} \text{ in.} \quad d_w = 1.5 \text{ in.} \quad t_{bp} = 1 \text{ in.}$

$h = 14 \text{ in.} \quad b = 12 \text{ in.} \quad d = 6 \text{ in.} \quad t = \frac{3}{8} \text{ in.}$

$W = 0.500 \text{ kips} \quad H = 17(12) \quad H = 204 \text{ in.}$

$L_v = 10(12) \quad L_h = 12(12) \quad L_v = 120 \text{ in.}$

$L_h = 144 \text{ in.}$

ALLOWABLE STRESSES (ksi)

$\sigma_a = \frac{\sigma_u}{FS_u} \quad \sigma_a = 24 \quad \tau_a = \frac{\tau_u}{FS_u}$

$\tau_a = 6.8 \quad \tau_{ha} = \frac{\tau_{hu}}{FS_u} \quad \tau_{ha} = 10$

$\sigma_{ba} = \frac{\sigma_{bu}}{FS_u} \quad \sigma_{ba} = 30 \quad \sigma_{bwa} = \frac{\sigma_{bw}}{FS_u}$

$\sigma_{bwa} = 20$

FORCES F AND R IN TERMS OF p_{max}

$F = p_{max} L_v L_h \quad R = \frac{FH}{2h}$

$R = p_{max} \frac{L_v L_h H}{2h}$

(1) COMPUTE p_{max} BASED ON NORMAL STRESS IN EACH BOLT (GREATER AT B & D)

$$\sigma = \frac{R + \frac{W}{4}}{\frac{\pi}{4} d_b^2} \qquad R_{max} = \sigma_a\left(\frac{\pi}{4} d_b^2\right) - \frac{W}{4}$$

$$p_{max1} = \frac{\sigma_a\left(\frac{\pi}{4} d_b^2\right) - \frac{W}{4}}{\frac{L_v L_h H}{2h}}$$

$p_{max1} = 11.98 \text{ psf} \quad \leftarrow \quad$ controls

(2) COMPUTE p_{max} BASED ON SHEAR THROUGH BASE PLATE (GREATER AT B & D)

$$\tau = \frac{R + \frac{W}{4}}{\pi d_w t_{bp}}$$

$$R_{max} = \tau_a(\pi d_w t_{bp}) - \frac{W}{4}$$

$$p_{max2} = \frac{\tau_a\left(\pi d_w t_{bp}\right) - \frac{W}{4}}{\frac{L_v L_h H}{2h}}$$

$p_{max2} = 36.5 \text{ psf}$

(3) COMPUTE p_{max} BASED ON HORIZONTAL SHEAR ON EACH BOLT

$$\tau_h = \frac{\frac{F}{4}}{\left(\frac{\pi}{4} d_b^2\right)} \qquad F_{max} = 4\tau_{ha}\left(\frac{\pi}{4} d_b^2\right)$$

$$p_{max3} = \frac{\tau_{ha}(\pi d_b^2)}{L_v L_h}$$

$$p_{max3} = 147.3 \text{ psf}$$

(4) COMPUTE p_{max} BASED ON HORIZONTAL BEARING ON EACH BOLT

$$\sigma_b = \frac{\frac{F}{4}}{(t_{bp} d_b)} \qquad F_{max} = 4\sigma_{ba}(t_{bp} d_b)$$

$$p_{max4} = \frac{4\sigma_{ba}(t_b p d_b)}{L_v L_h}$$

$$p_{max4} = 750 \text{ psf}$$

(5) COMPUTE p_{max} BASED ON BEARING UNDER THE TOP WASHER AT A (OR C) AND THE BOTTOM WASHER AT B (OR D)

$$\sigma_{bw} = \frac{R + \frac{W}{4}}{\frac{\pi}{4}\left(d_w^2 - d_b^2\right)}$$

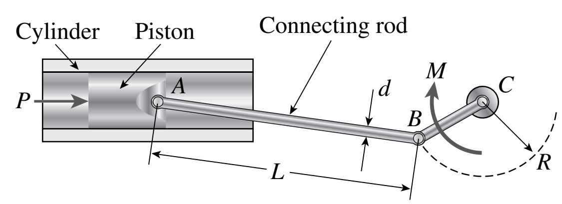
$$R_{max} = \sigma_{bwa}\left[\frac{\pi}{4}\left(d_w^2 - d_b^2\right)\right] - \frac{W}{4}$$

$$p_{max5} = \frac{\sigma_{bwa}\left[\frac{\pi}{4}(d_w^2 - d_b^2)\right] - \frac{W}{4}}{\frac{L_v L_h H}{2h}}$$

$$p_{max5} = 30.2 \text{ psf}$$

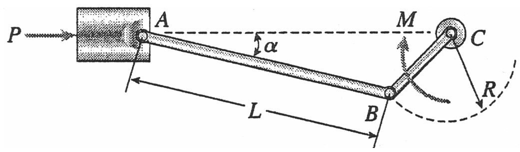
So, normal/stress in bolts controls; $p_{max} = 11.98$ psf

Problem 1.7-16 The piston in an engine is attached to a connecting rod *AB*, which in turn is connected to a crank arm *BC* (see figure). The piston slides without friction in a cylinder and is subjected to a force *P* (assumed to be constant) while moving to the right in the figure. The connecting rod, which has diameter *d* and length *L*, is attached at both ends by pins. The crank arm rotates about the axle at *C* with the pin at *B* moving in a circle of radius *R*. The axle at *C*, which is supported by bearings, exerts a resisting moment *M* against the crank arm.



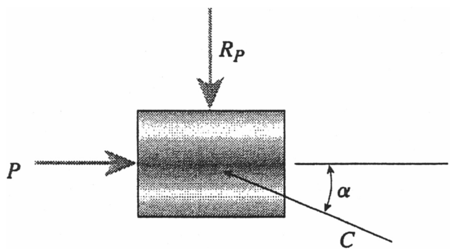
(a) Obtain a formula for the maximum permissible force P_{allow} based upon an allowable compressive stress σ_c in the connecting rod.
(b) Calculate the force P_{allow} for the following data: $\sigma_c = 160$ MPa, $d = 9.00$ mm, and $R = 0.28L$.

Solution 1.7-16



d = diameter of rod AB

FREE-BODY DIAGRAM OF PISTON



P = applied force (constant)

C = compressive force in connecting rod

RP = resultant of reaction forces between cylinder and piston (no friction)

$$\sum F_{\text{horiz}} = 0 \underset{+}{\rightarrow} \underset{-}{\leftarrow}$$

$$P - C\cos\alpha = 0$$

$$P = C\cos\alpha$$

MAXIMUM COMPRESSIVE FORCE C IN CONNECTING ROD

$$C_{\max} = \sigma_c A_c$$

in which A_c = area of connecting rod

$$A_c = \frac{\pi d^2}{4}$$

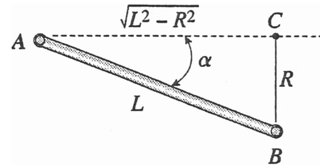
MAXIMUM ALLOWABLE FORCE P

$$P = C_{\max}\cos\alpha$$

$$= \sigma_c A_c \cos\alpha$$

The maximun allowable force P occurs when cos α has its smallest value, which means that α has its largest value.

LARGEST VALUE OF α



The largest value of α occurs when point B is the farthest distance from line AC. The farthest distance is the radius R of the crank arm.

Therefore,

$$\overline{BC} = R$$

Also, $\overline{AC} = \sqrt{L^2 - R^2}$

$$\cos\alpha = \frac{\sqrt{L^2 - R^2}}{L} = \sqrt{1 - \left(\frac{R}{L}\right)^2}$$

(a) MAXIMUM ALLOWABLE FORCE P

$$P_{\text{allow}} = \sigma_c A_c \cos\alpha$$

$$= \sigma_c\left(\frac{\pi d^2}{4}\right)\sqrt{1 - \left(\frac{R}{L}\right)^2} \quad \leftarrow$$

(b) SUBSTITUTE NUMERICAL VALUES

$\sigma_c = 160$ MPa $\qquad d = 9.00$ mm

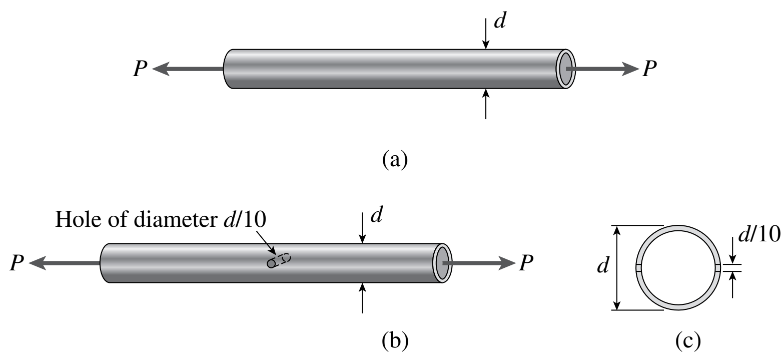
$R = 0.28L \qquad R/L = 0.28$

$P_{\text{allow}} = 9.77$ kN $\quad \leftarrow$

Design for Axial Loads and Direct Shear

Problem 1.8-1 An aluminum tube is required to transmit an axial tensile force $P = 33$ k [see figure part (a)]. The thickness of the wall of the tube is to be 0.25 in.

(a) What is the minimum required outer diameter d_{min} if the allowable tensile stress is 12,000 psi?
(b) Repeat part (a) if the tube will have a hole of diameter $d/10$ at mid-length [see figure parts (b) and (c)].



Solution 1.8-1

NUMERICAL DATA

$P = 33$ kips $\quad t = 0.25$ in. $\quad \sigma_a = 12$ ksi

(a) MIN. DIAMETER OF TUBE (NO HOLES)

$$A_1 = \frac{\pi}{4}\left[d^2 - (d-2t)^2\right] \qquad A_2 = \frac{P}{\sigma_a}$$

$$A_2 = 2.75 \text{ in}^2$$

equating A_1 & A_2 and solving for d:

$$d = \frac{P}{\pi\sigma_a t} + t \qquad d = 3.75 \text{ in.} \quad \leftarrow$$

(b) MIN. DIAMETER OF TUBE (WITH HOLES)

$$A_1 = \left[\frac{\pi}{4}\left[d^2 - (d-2t)^2\right] - 2\left(\frac{d}{10}\right)t\right]$$

$$A_1 = d\left(\pi t - \frac{t}{5}\right) - \pi t^2$$

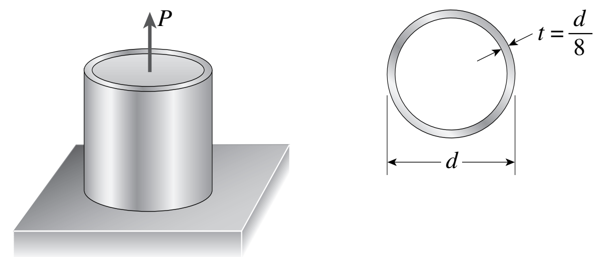
equating A_1 & A_2 and solving for d:

$$d = \frac{\frac{P}{\sigma_a} + \pi t^2}{\pi t - \frac{t}{5}} \qquad d = 4.01 \text{ in.} \quad \leftarrow$$

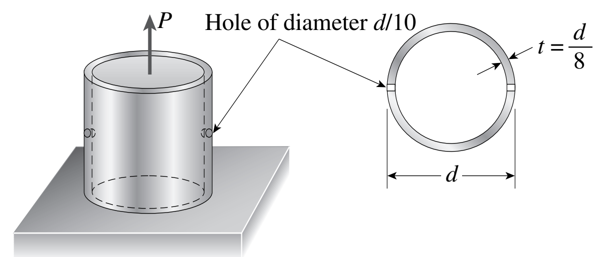
Problem 1.8-2 A copper alloy pipe having yield stress $\sigma_Y = 290$ MPa is to carry an axial tensile load $P = 1500$ kN [see figure part (a)]. A factor of safety of 1.8 against yielding is to be used.

(a) If the thickness t of the pipe is to be one-eighth of its outer diameter, what is the minimum required outer diameter d_{min}?

(b) Repeat part (a) if the tube has a hole of diameter $d/10$ drilled through the entire tube as shown in the figure [part (b)].



(a)



(b)

Solution 1.8-2

NUMERICAL DATA

$\sigma_Y = 290$ MPa

$P = 1500$ kN

$FS_y = 1.8$

(a) MIN. DIAMETER (NO HOLES)

$$A_1 = \frac{\pi}{4}\left[d^2 - \left(d - \frac{d}{8}\right)^2\right]$$

$$A_1 = \frac{\pi}{4}\left(\frac{15}{64}d^2\right) \qquad A_1 = \frac{15}{256}\pi d^2$$

$$A_2 = \frac{P}{\frac{\sigma_Y}{FS_y}} \qquad A_2 = 9.31 \times 10^3\ \text{mm}^2$$

equate A_1 & A_2 and solve for d:

$$d^2 = \frac{256}{15\pi}\left(\frac{P}{\frac{\sigma_Y}{FS_y}}\right)$$

$$d_{min} = \sqrt{\frac{256}{15\pi}\left(\frac{P}{\frac{\sigma_Y}{FS_y}}\right)}$$

$$d_{min} = 225\text{mm} \qquad \leftarrow$$

(b) MIN. DIAMETER (WITH HOLES)

Redefine A_1 - subtract area for two holes - then equate to A_2

$$A_1 = \left[\frac{\pi}{4}\left[d^2 - \left(d - \frac{d}{8}\right)^2\right] - 2\left(\frac{d}{10}\right)\left(\frac{d}{8}\right)\right]$$

$$A_1 = \frac{15}{256}\pi d^2 - \frac{1}{40}d^2$$

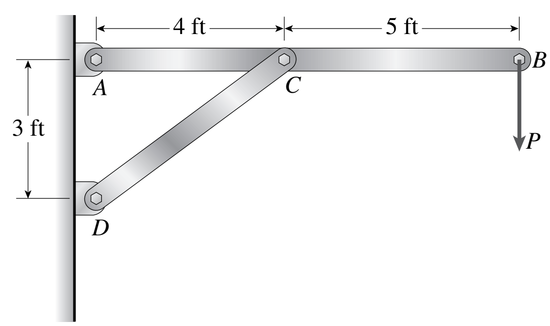
$$A_1 = d^2\left(\frac{15}{256}\pi - \frac{1}{40}\right) \qquad \frac{15}{256}\pi - \frac{1}{40} = 0.159$$

$$d^2 = \frac{\left(\dfrac{P}{\dfrac{\sigma y}{FS_y}}\right)}{\left(\dfrac{15}{256}\pi - \dfrac{1}{40}\right)}$$

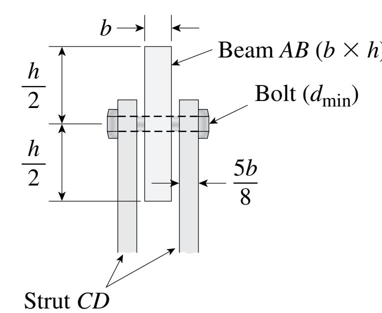
$$d_{min} = \sqrt{\frac{\left(\dfrac{P}{\dfrac{\sigma_y}{FS_y}}\right)}{\left(\dfrac{15}{256}\pi - \dfrac{1}{40}\right)}} \qquad d_{min} = 242 \text{ mm} \quad \longleftarrow$$

Problem 1.8-3 A horizontal beam *AB* with cross-sectional dimensions (b = 0.75 in.) × (h = 8.0 in.) is supported by an inclined strut *CD* and carries a load P = 2700 lb at joint *B* [see figure part (a)]. The strut, which consists of two bars each of thickness $5b/8$, is connected to the beam by a bolt passing through the three bars meeting at joint *C* [see figure part (b)].

(a) If the allowable shear stress in the bolt is 13,000 psi, what is the minimum required diameter d_{min} of the bolt at *C*?
(b) If the allowable bearing stress in the bolt is 19,000 psi, what is the minimum required diameter d_{min} of the bolt at *C*?



(a)



(b)

Solution 1.8-3

NUMERICAL DATA

$P = 2.7$ kips $\quad b = 0.75$ in. $\quad h = 8$ in.

$\tau_a = 13$ ksi $\quad \sigma_{ba} = 19$ ksi

(a) d_{min} BASED ON ALLOWABLE SHEAR - DOUBLE SHEAR IN STRUT

$$\tau_a = \frac{F_{DC}}{A_s} \qquad F_{DC} = \frac{15}{4}P$$

$$A_s = 2\left(\frac{\pi}{4}d^2\right)$$

$$d_{min} = \sqrt{\frac{\frac{15}{4}P}{\tau_a\left(\frac{\pi}{2}\right)}} \qquad d_{min} = 0.704 \text{ inches} \quad \leftarrow$$

(b) d_{min} BASED ON ALLOWABLE BEARING AT JT C

Bearing from beam ACB $\quad \sigma_b = \dfrac{15\,P/4}{b\,d}$

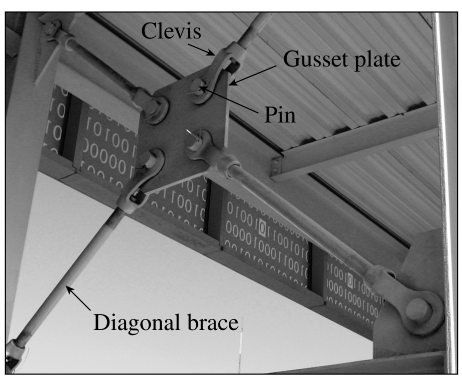
$$d_{min} = \frac{15\,P/4}{b\,\sigma_{ba}} \qquad d_{min} = 0.711 \text{ inches} \quad \leftarrow$$

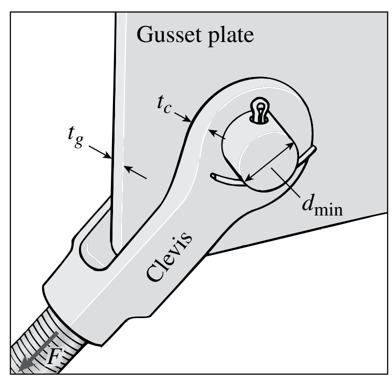
Bearing from strut DC $\quad \sigma_b = \dfrac{\frac{15}{4}P}{2\frac{5}{8}b\,d}$

$$\sigma_b = 3\frac{P}{b\,d} \qquad \text{(lower than ACB)}$$

Problem 1.8-4 Lateral bracing for an elevated pedestrian walkway is shown in the figure part (a). The thickness of the clevis plate $t_c = 16$ mm and the thickness of the gusset plate $t_g = 20$ mm [see figure part (b)]. The maximum force in the diagonal bracing is expected to be $F = 190$ kN.

If the allowable shear stress in the pin is 90 MPa and the allowable bearing stress between the pin and both the clevis and gusset plates is 150 MPa, what is the minimum required diameter d_{min} of the pin?





Solution 1.8-4

NUMERICAL DATA

$F = 190 \text{ kN} \qquad \tau_a = 90 \text{ MPa} \qquad \sigma_{ba} = 150 \text{ MPa}$

$t_g = 20 \text{ mm} \qquad t_c = 16 \text{ mm}$

(1) d_{min} BASED ON ALLOW SHEAR - DOUBLE SHEAR IN STRUT

$$\tau = \frac{F}{A_s} \qquad A_s = 2\left(\frac{\pi}{4}d^2\right)$$

$$d_{min} = \sqrt{\frac{F}{\tau_a\left(\frac{\pi}{2}\right)}} \qquad d_{min} = 36.7 \text{ mm}$$

(2) d_{min} BASED ON ALLOW BEARING IN GUSSET & CLEVIS PLATES

Bearing on gusset plate

$$\sigma_b = \frac{F}{A_b} \qquad A_b = t_g d \qquad d_{min} = \frac{F}{t_g \sigma_{ba}}$$

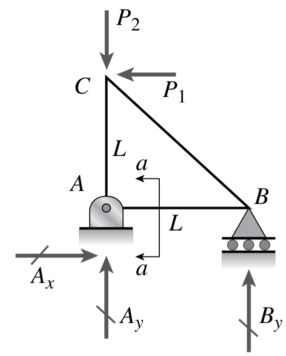
$d_{min} = 63.3 \text{ mm}$ <controls $\leftarrow$

Bearing on clevis $\qquad A_b = d(2t_c)$

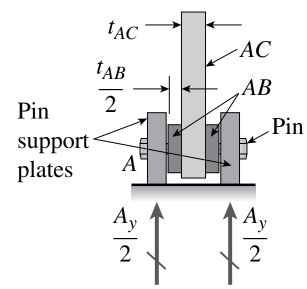
$$d_{min} = \frac{F}{2t_c\sigma_{ba}} \qquad d_{min} = 39.6 \text{ mm}$$

Problem 1.8-5 Forces $P_1 = 1500$ lb and $P_2 = 2500$ lb are applied at joint C of plane truss ABC shown in the figure part (a). Member AC has thickness $t_{AC} = 5/16$ in. and member AB is composed of two bars each having thickness $t_{AB}/2 = 3/16$ in. [see figure part (b)]. Ignore the effect of the two plates which make up the pin support at A.

If the allowable shear stress in the pin is 12,000 psi and the allowable bearing stress in the pin is 20,000 psi, what is the minimum required diameter d_{min} of the pin?



(a)



Section a–a

(b)

Solution 1.8-5

NUMERICAL DATA

$P_1 = 1.5$ kips $\quad P_2 = 2.5$ kips

$t_{AC} = \frac{5}{16}$ in. $\quad t_{AB} = 2\left(\frac{3}{16}\right)$ in.

$\tau_a = 12$ ksi $\quad \sigma_{ba} = 20$ ksi

(1) d_{min} BASED ON ALLOWABLE SHEAR - DOUBLE SHEAR IN STRUT; FIRST CHECK AB (SINGLE SHEAR IN EACH BAR HALF)

Force in each bar of AB is $P_1/2$

$$\tau = \frac{\frac{P_1}{2}}{A_S} \qquad A_s = \left(\frac{\pi}{4}d^2\right)$$

$$d_{min} = \sqrt{\frac{\frac{P_1}{2}}{\tau_2\left(\frac{\pi}{4}\right)}} \qquad d_{min} = 0.282 \text{ in.}$$

Next check double shear to AC; force in AC is $(P_1 + P_2)/2$

$$d_{min} = \sqrt{\frac{(P_1 + P_2)/2}{\tau_a\left(\frac{\pi}{4}\right)}} \qquad d_{min} = 0.461 \text{ inches} \quad \leftarrow$$

Finally check RESULTANT force on pin at A

$$R = \sqrt{\left(\frac{P_1}{2}\right)^2 + \left(\frac{P_1 + P_2}{2}\right)^2} \qquad R = 2.136 \text{ kips}$$

$$d_{min} = \sqrt{\frac{\frac{R}{2}}{\tau_a\left(\frac{\pi}{4}\right)}} \qquad d_{min} = 0.476 \text{ in.}$$

(2) d_{min} BASED ON ALLOWABLE BEARING ON PIN

member AB bearing on pin $\quad \sigma_b = \dfrac{P_1}{A_b} \qquad A_b = t_{AB}d$

$$d_{min} = \frac{P_1}{t_{AB}\sigma_{ba}} \qquad d_{min} = 0.2 \text{ in.}$$

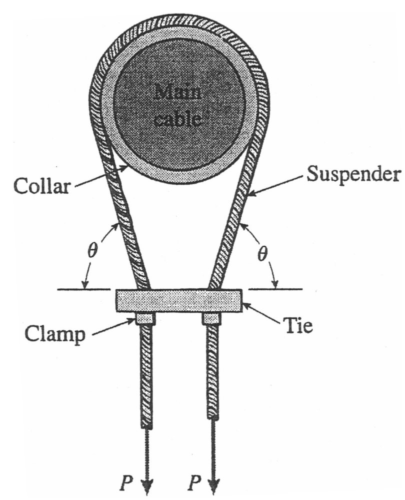
member AC bearing on pin $\qquad A_b = d(t_{AC})$

$$d_{min} = \frac{P_1 + P_2}{t_{AC}\sigma_{ba}} \qquad d_{min} = 0.64 \text{ in.} \qquad \text{controls} \qquad \leftarrow$$

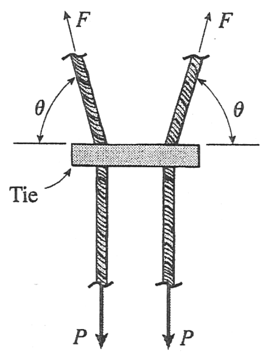
Problem 1.8-6 A suspender on a suspension bridge consists of a cable that passes over the main cable (see figure) and supports the bridge deck, which is far below. The suspender is held in position by a metal tie that is prevented from sliding downward by clamps around the suspender cable.

Let P represent the load in each part of the suspender cable, and let θ represent the angle of the suspender cable just above the tie. Finally, let σ_{allow} represent the allowable tensile stress in the metal tie.

(a) Obtain a formula for the minimum required cross-sectional area of the tie.
(b) Calculate the minimum area if $P = 130$ kN, $\theta = 75°$, and $\sigma_{allow} = 80$ MPa.



Solution 1.8-6 Suspender tie on a suspension bridge



F = tensile force in cable above tie

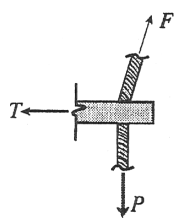
P = tensile force in cable below tie

σ_{allow} = allowable tensile stress in the tie

FREE-BODY DIAGRAM OF HALF THE TIE

Note: Include a small amount of the cable in the free-body diagram

T = tensile force in the tie



FORCE TRIANGLE

$$\cot\theta = \frac{T}{P}$$

$$T = P\cot\theta$$

(a) MINIMUM REQUIRED AREA OF TIE

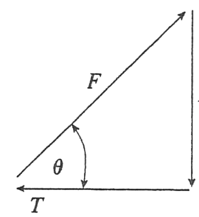
$$A_{min} = \frac{T}{\sigma_{allow}} = \frac{P\cot\theta}{\sigma_{allow}} \quad \leftarrow$$

(b) SUBSTITUTE NUMERICAL VALUES:

$P = 130$ kN $\qquad \theta = 75°$

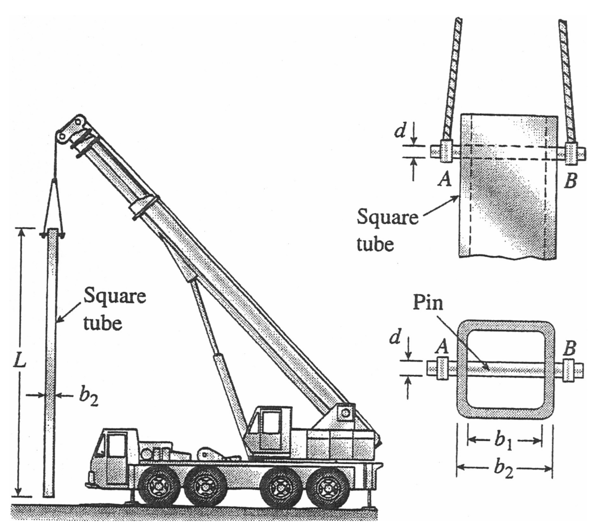
$\sigma_{allow} = 80$ MPa

$A_{min} = 435\ \text{mm}^2 \quad \leftarrow$

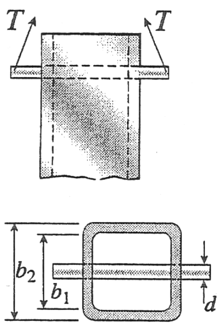


Problem 1.8-7 A square steel tube of length $L = 20$ ft and width $b_2 = 10.0$ in. is hoisted by a crane (see figure). The tube hangs from a pin of diameter d that is held by the cables at points A and B. The cross section is a hollow square with inner dimension $b_1 = 8.5$ in. and outer dimension $b_2 = 10.0$ in. The allowable shear stress in the pin is 8,700 psi, and the allowable bearing stress between the pin and the tube is 13,000 psi.

Determine the minimum diameter of the pin in order to support the weight of the tube. (*Note*: Disregard the rounded corners of the tube when calculating its weight.)



Solution 1.8-7 Tube hoisted by a crane



T = tensile force in cable

W = weight of steel tube

d = diameter of pin

b_1 = inner dimension of tube $= 8.5$ in.

b_2 = outer dimension of tube $= 10.0$ in.

L = length of tube = 20 ft

$\tau_{\text{allow}} = 8{,}700$ psi

$\sigma_b = 13{,}000$ psi

WEIGHT OF TUBE

γ_s = weight density of steel $= 490\ \text{lb/ft}^3$

A = area of tube

$$= b_2^2 - b_1^2 = (10.0\ \text{in.})^2 - (8.5\ \text{in.})^2 = 27.75\ \text{in.}$$

$$W = \gamma_s AL$$

$$= (490\ \text{lb/ft}^3)(27.75\ \text{in.}^2)\left(\frac{1\ \text{ft}^2}{144\ \text{in.}}\right)(20\ \text{ft})$$

$$= 1{,}889\ \text{lb}$$

DIAMETER OF PIN BASED UPON SHEAR

Double shear. $2\tau_{\text{allow}} A_{\text{pin}} = W$

$$2(8{,}700\ \text{psi})\left(\frac{\pi d^2}{4}\right) = 1889\ \text{lb}$$

$d^2 = 0.1382\ \text{in.}^2$ $\quad d_1 = 0.372$ in.

DIAMETER OF PIN BASED UPON BEARING

$$\sigma_b(b_2 - b_1)d = W$$

$$(13{,}000\ \text{psi})(10.0\ \text{in.} - 8.5\ \text{in.})\ d = 1{,}889\ \text{lb}$$

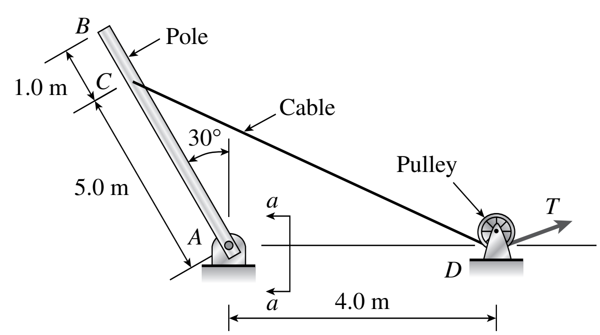
$d_2 = 0.097$ in.

MINIMUM DIAMETER OF PIN

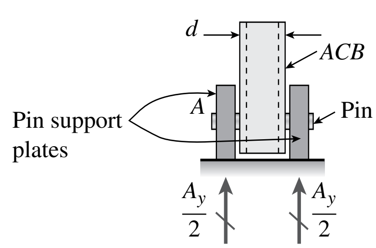
Shear governs. $\quad d_{\min} = 0.372$ in.

Problem 1.8-8 A cable and pulley system at D is used to bring a 230-kg pole (ACB) to a vertical position as shown in the figure part (a). The cable has tensile force T and is attached at C. The length L of the pole is 6.0 m, the outer diameter is $d = 140$ mm, and the wall thickness $t = 12$ mm. The pole pivots about a pin at A in figure part (b). The allowable shear stress in the pin is 60 MPa and the allowable bearing stress is 90 MPa.

Find the minimum diameter of the pin at A in order to support the weight of the pole in the position shown in the figure part (a).



(a)



(b)

Solution 1.8-8

ALLOWABLE SHEAR & BEARING STRESSES

$\tau_a = 60\text{ MPa} \qquad \sigma_{ba} = 90\text{ MPa}$

FIND INCLINATION OF & FORCE IN CABLE, T

let α = angle between pole & cable at C; use Law of Cosines

$$DC = \sqrt{5^2 + 4^2 - 2(5)(4)\cos\left(120\frac{\pi}{180}\right)}$$

$$DC = 7.81\text{ m} \qquad \alpha = \operatorname{acos}\left[\frac{5^2 + DC^2 - 4^2}{2DC(5)}\right]$$

$$\alpha\frac{180}{\pi} = 26.33\ \text{degrees} \qquad \theta = 60\left(\frac{\pi}{180}\right) - \alpha$$

$$\theta\frac{180}{\pi} = 33.67 \quad <\text{ange between cable \& horiz. at D}$$

$$W = 230\text{ kg}(9.81\text{ m/s}^2) \qquad W = 2.256 \times 10^3\text{ N}$$

STATICS TO FIND CABLE FORCE T

$$\sum M_A = 0 \qquad W(3\sin(30\deg)) - T_X(5\cos(30\deg)) + T_y(5\sin(30\deg)) = 0$$

substitute for T_x & T_y in terms of T & solve for T:

$$T = \frac{\frac{3}{2}W}{\frac{-5}{2}\sin(\theta) + \frac{5\sqrt{3}}{2}\cos(\theta)}$$

$$T = 1.53 \times 10^3\text{ N} \qquad T_x = T\cos(\theta)$$

$$T_y = T\sin(\theta) \qquad T_x = 1.27 \times 10^3\text{ N} \qquad T_y = 846.11\text{ N}$$

(1) d_{min} BASED ON ALLOWABLE SHEAR - DOUBLE SHEAR AT A

$$A_x = -T_x \qquad A_y = T_y + W$$

CHECK SHEAR DUE TO RESULTANT FORCE ON PIN AT A

$$R_A = \sqrt{A_x^2 + A_y^2} \qquad R_A = 3.35 \times 10^3\text{ N}$$

$$d_{min} = \sqrt{\frac{\frac{R_A}{2}}{\tau_a\left(\frac{\pi}{4}\right)}}$$

$$d_{min} = 5.96\text{ mm} \quad <\text{controls} \quad \leftarrow$$

(2) d_{min} BASED ON ALLOWABLE BEARING ON PIN

$$d_{pole} = 140\text{ mm} \qquad t_{pole} = 12\text{ mm} \qquad L_{pole} = 6000\text{ mm}$$

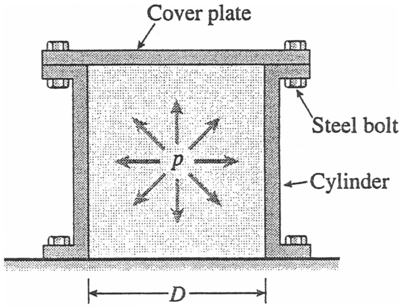
member AB BEARING ON PIN

$$\sigma_b = \frac{R_A}{A_b} \qquad A_b = 2t_{pole}d$$

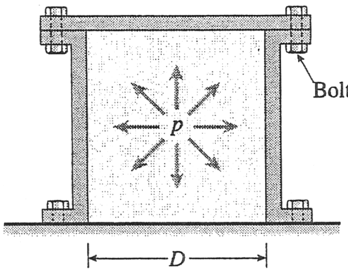
$$d_{min} = \frac{R_A}{2t_{pole}\sigma_{ba}} \qquad d_{min} = 1.55\text{ mm}$$

Problem 1.8-9 A pressurized circular cylinder has a sealed cover plate fastened with steel bolts (see figure). The pressure p of the gas in the cylinder is 290 psi, the inside diameter D of the cylinder is 10.0 in., and the diameter d_B of the bolts is 0.50 in.

If the allowable tensile stress in the bolts is 10,000 psi, find the number n of bolts needed to fasten the cover.



Solution 1.8-9 Pressurized cylinder



$p = 290$ psi $\quad D = 10.0$ in. $\quad d_b = 0.50$ in.

$\sigma_{\text{allow}} = 10{,}000$ psi $\quad n =$ number of bolts

F = total force acting on the cover plate from the internal pressure

$$F = p\left(\frac{\pi D^2}{4}\right)$$

NUMBER OF BOLTS

P = tensile force in one bolt

$$P = \frac{F}{n} = \frac{\pi p D^2}{4n}$$

$$A_b = \text{area of one bolt} = \frac{\pi}{4} d_b^2$$

$$P = \sigma_{\text{allow}} A_b$$

$$\sigma_{\text{allow}} = \frac{P}{A_b} = \frac{\pi p D^2}{(4n)(\frac{\pi}{4}) d_b^2} = \frac{pD^2}{n d_b^2}$$

$$n = \frac{pD^2}{d_b^2 \sigma_{\text{allow}}}$$

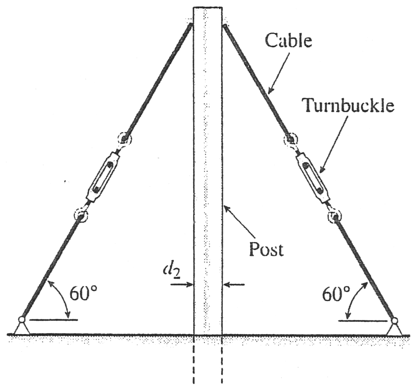
SUBSTITUTE NUMERICAL VALUES:

$$n = \frac{(290 \text{ psi})(10 \text{ in.})^2}{(0.5 \text{ in.})^2 (10{,}000 \text{ psi})} = 11.6$$

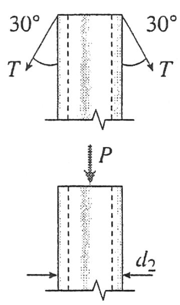
Use 12 bolts $\leftarrow$

Problem 1.8-10 A tubular post of outer diameter d_2 is guyed by two cables fitted with turnbuckles (see figure). The cables are tightened by rotating the turnbuckles, thus producing tension in the cables and compression in the post. Both cables are tightened to a tensile force of 110 kN. Also, the angle between the cables and the ground is 60°, and the allowable compressive stress in the post is $\sigma_c = 35$ MPa.

If the wall thickness of the post is 15 mm, what is the minimum permissible value of the outer diameter d_2?



Solution 1.8-10 Tubular post with guy cables



d_2 = outer diameter

d_1 = inner diameter

t = wall thickness = 15 mm

T = tensile force in a cable = 110 kN

$\sigma_{\text{allow}} = 35$ MPa

P = compressive force in post = $2T\cos 30°$

REQUIRED AREA OF POST

$$A = \frac{P}{\sigma_{\text{allow}}} = \frac{2T\cos 30°}{\sigma_{\text{allow}}}$$

AREA OF POST

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = \frac{\pi}{4}[d_2^2 - (d_2 - 2t)^2]$$

$$= \pi t(d_2 - t)$$

EQUATE AREAS AND SOLVE FOR d_2:

$$\frac{2T\cos 30°}{\sigma_{\text{allow}}} = \pi t(d_2 - t)$$

$$d_2 = \frac{2T\cos 30°}{\pi t \sigma_{\text{allow}}} + t \quad \leftarrow$$

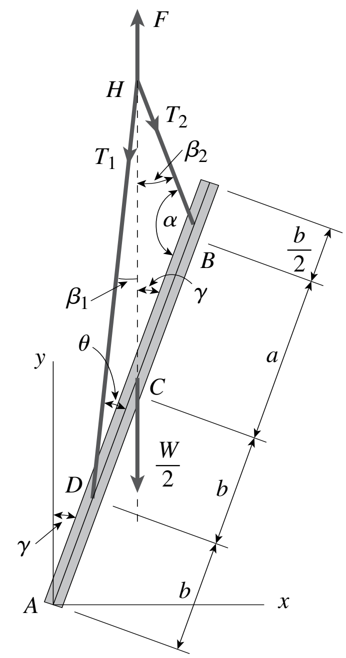
SUBSTITUTE NUMERICAL VALUES:

$(d_2)_{\min} = 131$ mm $\quad \leftarrow$

Problem 1.8-11 A large precast concrete panel for a warehouse is being raised to a vertical position using two sets of cables at two lift lines as shown in the figure part (a). Cable 1 has length $L_1 = 22$ ft and distances along the panel (see figure part (b)) are $a = L_1/2$ and $b = L_1/4$. The cables are attached at lift points B and D and the panel is rotated about its base at A. However, as a worst case, assume that the panel is momentarily lifted off the ground and its total weight must be supported by the cables. Assuming the cable lift forces F at each lift line are about equal, use the simplified model of one half of the panel in figure part (b) to perform your analysis for the lift position shown. The total weight of the panel is $W = 85$ kips. The orientation of the panel is defined by the following angles: $\gamma = 20°$ and $\theta = 10°$.

Find the required cross-sectional area A_C of the cable if its breaking stress is 91 ksi and a factor of safety of 4 with respect to failure is desired.

(a)



(b)

Solution 1.8-11

GEOMETRY

$L_1 = 22$ ft $\quad a = \frac{1}{2}L_1 \quad b = \frac{1}{4}L_1$

$\theta = 10$ deg $\quad a + 2.5b = 24.75$ ft

$\gamma = 20$ deg

Using Law of cosines

$$L_2 = \sqrt{(a + b)^2 + L_1^2 - 2(a + b)L_1\cos(\theta)}$$

$L_2 = 6.425$ ft

$$\beta = \operatorname{acos}\left[\frac{L_1^2 + L_2^2 - (a + b)^2}{2L_1L_2}\right]$$

$\beta = 26.484$ degrees

$\beta_1 = \pi - (\theta + \pi - \gamma) \qquad \beta_1 = 10$ deg

$\beta_2 = \beta - \beta_1 \qquad \beta_2 = 16.484$ deg

SOLUTION APPROACH: FIND T THEN $A_c = T/(\sigma_U/FS)$

STATICS at point H

$$\sum_{H} F_x = 0 \qquad T_1 \sin(\beta_1) = T_2 \sin(\beta_2)$$

SO $\quad T_2 = T_1 \dfrac{\sin(\beta_1)}{\sin(\beta_2)}$

$$\sum_{H} F_Y = 0 \qquad T_1 \cos(\beta_1) + T_2 \cos(\beta_2) = F$$

and $\quad F = W/2, \qquad W = 85 \text{ kips}$

SO $\quad T_1\left(\cos(\beta_1) + \dfrac{\sin(\beta_1)}{\sin(\beta_2)}\cos(\beta_2)\right) = F$

$$T_1 = \frac{\dfrac{W}{2}}{\left(\cos(\beta_1) + \dfrac{\sin(\beta_1)}{\sin(\beta_2)}\cos(\beta_2)\right)}$$

$T_1 = 27.042$ kips

$$T_2 = T_1 \frac{\sin(\beta_1)}{\sin(\beta_2)} \qquad T_2 = 16.549 \text{ kips}$$

COMPUTE REQUIRED CROSS-SECTIONAL AREA

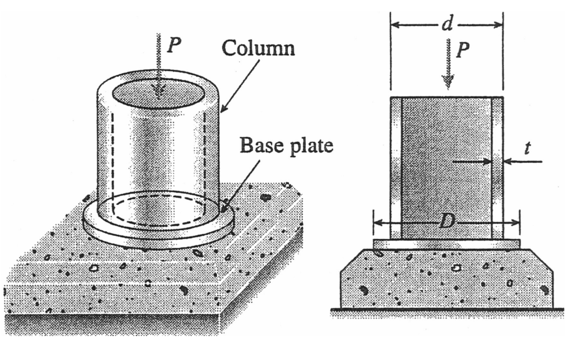
$$\sigma_u = 91 \text{ ksi} \qquad FS = 4 \qquad \frac{\sigma_u}{FS} = 22.75 \text{ ksi}$$

$$A_c = \frac{T_1}{\dfrac{\sigma_u}{FS}} \qquad A_c = 1.189 \text{ in}^2 \quad \leftarrow$$

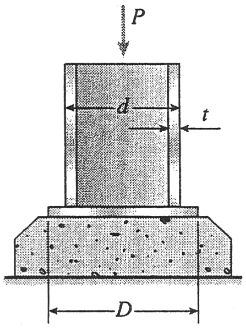
Problem 1.8-12 A steel column of hollow circular cross section is supported on a circular steel base plate and a concrete pedestal (see figure). The column has outside diameter $d = 250$ mm and supports a load $P = 750$ kN.

(a) If the allowable stress in the column is 55 MPa, what is the minimum required thickness t? Based upon your result, select a thickness for the column. (Select a thickness that is an even integer, such as 10, 12, 14, . . . , in units of millimeters.)

(b) If the allowable bearing stress on the concrete pedestal is 11.5 MPa, what is the minimum required diameter D of the base plate if it is designed for the allowable load P_{allow} that the column with the selected thickness can support?



Solution 1.8-12 Hollow circular column



$d = 250$ mm $\quad P = 750$ kN

$\sigma_{\text{allow}} = 55$ MPa (compression in column)

t = thickness of column

D = diameter of base plate

$\sigma_b = 11.5$ MPa (allowable pressure on concrete)

(a) THICKNESS t OF THE COLUMN

$$A = \frac{P}{\sigma_{\text{allow}}} \qquad A = \frac{\pi d^2}{4} - \frac{\pi}{4}(d - 2t)^2$$

$$= \frac{\pi}{4}(4t)(d - t) = \pi t(d - t)$$

$$\pi t(d - t) = \frac{P}{\sigma_{\text{allow}}}$$

$$\pi t^2 - \pi t d + \frac{P}{\sigma_{\text{allow}}} = 0$$

$$t^2 - td + \frac{P}{\pi \sigma_{\text{allow}}} = 0 \qquad \text{(Eq. 1)}$$

SUBSTITUTE NUMERICAL VALUES IN EQ. (1):

$$t^2 - 250\,t + \frac{(750 \times 10^3 \text{ N})}{\pi(55 \text{ N/mm}^2)} = 0$$

(Note: In this eq., t has units of mm.)

$$t^2 - 250t + 4{,}340.6 = 0$$

Solve the quadratic eq. for t:

$t = 18.77$ mm $\qquad t_{\min} = 18.8$ mm $\quad \leftarrow$

Use $t = 20$ mm $\quad \leftarrow$

(b) DIAMETER D OF THE BASE PLATE

For the column, $\qquad P_{\text{allow}} = \sigma_{\text{allow}} A$

where A is the area of the column with $t = 20$ mm.

$$A = \pi t(d - t) \; P_{\text{allow}} = \sigma_{\text{allow}}\, \pi t(d - t)$$

$$\text{Area of base plate} = \frac{\pi D^2}{4} = \frac{P_{\text{allow}}}{\sigma_b}$$

$$\frac{\pi D^2}{4} = \frac{\sigma_{\text{allow}} \pi t(d - t)}{\sigma_b}$$

$$D^2 = \frac{4\sigma_{\text{allow}} t(d - t)}{\sigma_b}$$

$$= \frac{4(55 \text{ MPa})(20 \text{ mm})(230 \text{ mm})}{11.5 \text{ MPa}}$$

$D^2 = 88{,}000$ mm^2 $\qquad D = 296.6$ mm

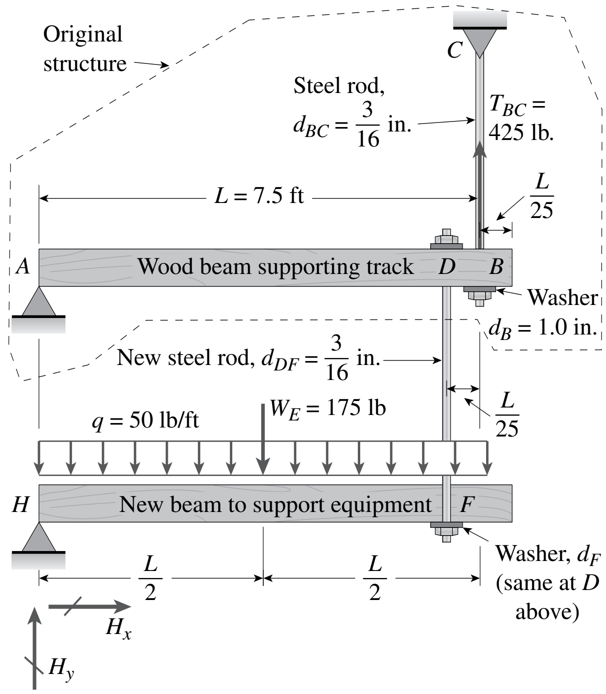
$D_{\min} = 297$ mm $\quad \leftarrow$

Problem 1.8-13 An elevated jogging track is supported at intervals by a wood beam *AB* (L = 7.5 ft) which is pinned at *A* and supported by steel rod *BC* and a steel washer at *B*. Both the rod (d_{BC} = 3/16 in.) and the washer (d_B = 1.0 in.) were designed using a rod tension force of T_{BC} = 425 lb. The rod was sized using a factor of safety of 3 against reaching the ultimate stress σ_u = 60 ksi. An allowable bearing stress σ_{ba} = 565 psi was used to size the washer at *B*.

Now, a small platform *HF* is to be suspended below a section of the elevated track to support some mechanical and electrical equipment. The equipment load is uniform load q = 50 lb/ft and concentrated load W_E = 175 lb at mid-span of beam *HF*. The plan is to drill a hole through beam *AB* at *D* and install the same rod (d_{BC}) and washer (d_B) at both *D* and *F* to support beam *HF*.

(a) Use σ_u and σ_{ba} to check the proposed design for rod *DF* and washer d_F; are they acceptable?

(b) Also re-check the normal tensile stress in rod *BC* and bearing stress at *B*; if either is inadequate under the additional load from platform *HF*, redesign them to meet the original design criteria.



Solution 1.8-13

NUMERICAL DATA

$L = 7.5(12)$ $\quad L = 90$ in. $\quad T_{BC} = 425$ lb

$\sigma_u = 60$ ksi $\quad FS_u = 3$ $\quad \sigma_{ba} = 0.565$ ksi

$q = \frac{50}{12}$ $\quad q = 4.167\frac{\text{lb}}{\text{in}}$ $\quad W_E = 175$ lb

$d_{BC} = \frac{3}{16}$ in. $\quad d_B = 1.0$ in

(a) FIND FORCE IN ROD DF AND FORCE ON WASHER AT F

$$\Sigma M_H = 0 \qquad T_{DF} = \frac{W_E\frac{L}{2} + qL\frac{L}{2}}{\left(L - \frac{L}{25}\right)}$$

$$T_{DF} = 286.458 \text{ lb}$$

NORMAL STRESS IN ROD DF:

$$\sigma_{DF} = \frac{T_{DF}}{\frac{\pi}{4}d_{BC}^2}$$

$\sigma_{DF} = 10.38$ ksi $\quad$ OK - less than σ_a; rod is acceptable ←

$\sigma_a = \frac{\sigma_u}{FS_u}$ $\quad \sigma_a = 20$ ksi

BEARING STRESS ON WASHER AT F:

$$\sigma_{bF} = \frac{T_{DF}}{\frac{\pi}{4}(d_B^2 - d_{BC}^2)}$$

$\sigma_{bF} = 378$ psi $\quad$ OK - less than σ_{ba}; washer is acceptable ←

(b) FIND NEW FORCE IN ROD BC - SUM MOMENT ABOUT A FOR UPPER FBD - THEN CHECK NORMAL STRESS IN BC & BEARING STRESS AT B

$\Sigma M_A = 0$

$$T_{BC2} = \frac{T_{BC}L + T_{DF}\left(L - \frac{L}{25}\right)}{L}$$

$$T_{BC2} = 700 \text{ lb}$$

REVISED NORMAL STRESS IN ROD BC:

$$\sigma_{BC2} = \frac{T_{BC2}}{\left(\frac{\pi}{4} d_{BC}^2\right)}$$

$\sigma_{BC2} = 25.352$ ksi exceeds $\sigma_a = 20$ ksi

SO RE-DESIGN ROD BC:

$$d_{BCreqd} = \sqrt{\frac{T_{BC2}}{\frac{\pi}{4}\sigma_a}}$$

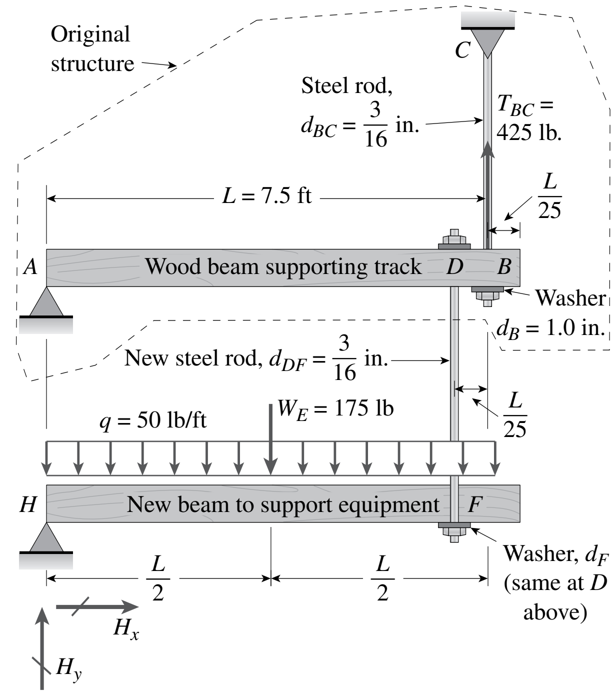
$d_{BCreqd} = 0.211$ in. $d_{BCreqd} \cdot 16 = 3.38$ in.

^say $4/16 = 1/4$ in. $d_{BC2} = \frac{1}{4}$ in.

RE-CHECK BEARING STRESS IN WASHER AT B:

$$\sigma_{bB2} = \frac{T_{BC2}}{\left[\frac{\pi}{4}\left(d_B^2 - d_{BC}^2\right)\right]}$$

$\sigma_{bB2} = 924$ psi
^ exceeds
$\sigma_{ba} = 565$ psi



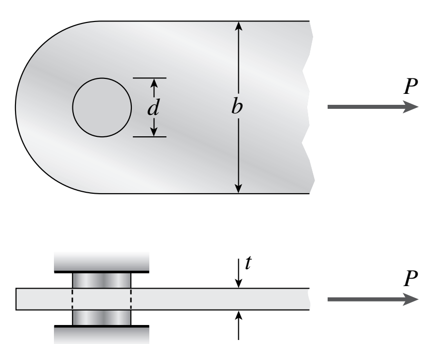
SO RE-DESIGN WASHER AT B:

$$d_{Breqd} = \sqrt{\frac{T_{BC2}}{\frac{\pi}{4}\sigma_{ba}} + d_{BC}^2} \qquad d_{Breqd} = 1.281 \text{ in.}$$

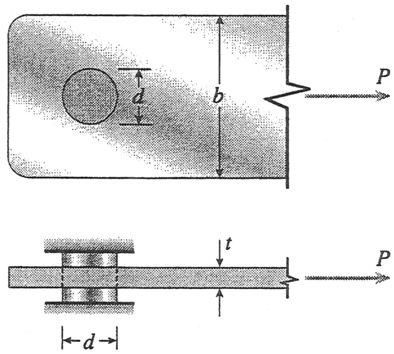
use 1 − 5/16 in washer at B: 1 + 5/16 = 1.312 in. ←

Problem 1.8-14 A flat bar of width $b = 60$ mm and thickness $t = 10$ mm is loaded in tension by a force P (see figure). The bar is attached to a support by a pin of diameter d that passes through a hole of the same size in the bar. The allowable tensile stress on the net cross section of the bar is $\sigma_T = 140$ MPa, the allowable shear stress in the pin is $\tau_S = 80$ MPa, and the allowable bearing stress between the pin and the bar is $\sigma_B = 200$ MPa.

(a) Determine the pin diameter d_m for which the load P will be a maximum.
(b) Determine the corresponding value P_{max} of the load.



Solution 1.8-14 Bar with a pin connection



$b = 60$ mm

$t = 10$ mm

$d =$ diameter of hole and pin

$\sigma_T = 140$ MPa

$\tau_S = 80$ MPa

$\sigma_B = 200$ MPa

UNITS USED IN THE FOLLOWING CALCULATIONS:

P is in kN

σ and τ are in N/mm^2 (same as MPa)

b, t, and d are in mm

TENSION IN THE BAR

$$P_T = \sigma_T(\text{Net area}) = \sigma_t(t)(b - d)$$

$$= (140\text{ MPa})(10\text{ mm})(60\text{ mm} - \text{d})\left(\frac{1}{1000}\right)$$

$$= 1.40\,(60 - d) \qquad \text{(Eq. 1)}$$

SHEAR IN THE PIN

$$P_S = 2\tau_S A_{\text{pin}} = 2\tau_S\left(\frac{\pi d^2}{4}\right)$$

$$= 2(80\text{ MPa})\left(\frac{\pi}{4}\right)(d^2)\left(\frac{1}{1000}\right)$$

$$= 0.040\,\pi d^2 = 0.12566 d^2 \qquad \text{(Eq. 2)}$$

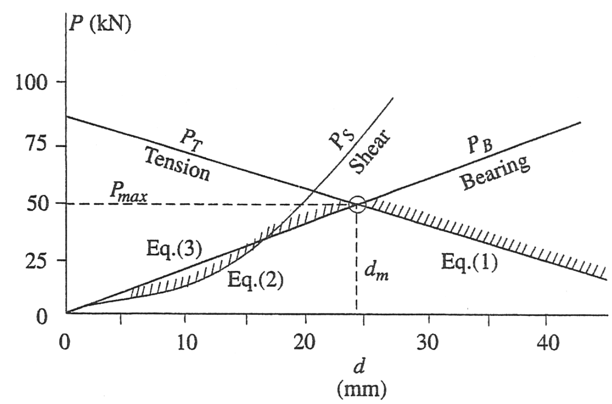
BEARING BETWEEN PIN AND BAR

$$P_B = \sigma_B\,td$$

$$= (200\text{ MPa})(10\text{ mm})(d)\left(\frac{1}{1000}\right)$$

$$= 2.0\,d \qquad \text{(Eq. 3)}$$

GRAPH OF EQS. (1), (2), AND (3)



(a) PIN DIAMETER d_m

$P_T = P_B$ or $1.40(60 - d) = 2.0\,d$

Solving, $d_m = \dfrac{84.0}{3.4}$ mm $= 24.7$ mm $\leftarrow$

(b) LOAD $P_{\max}$

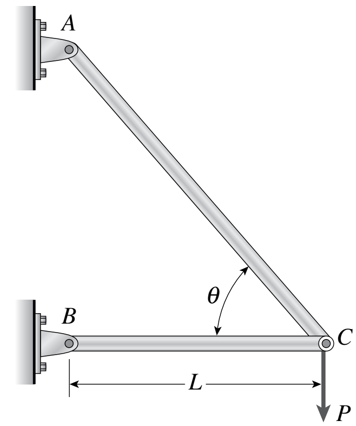
Substitute d_m into Eq. (1) or Eq. (3):

$P_{\max} = 49.4$ kN $\leftarrow$

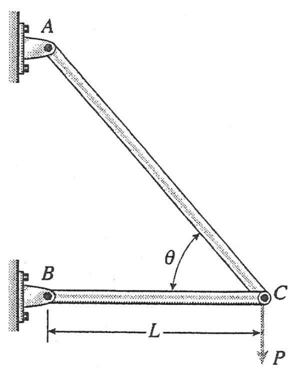
Problem 1.8-15 Two bars AC and BC of the same material support a vertical load P (see figure). The length L of the horizontal bar is fixed, but the angle θ can be varied by moving support A vertically and changing the length of bar AC to correspond with the new position of support A. The allowable stresses in the bars are the same in tension and compression.

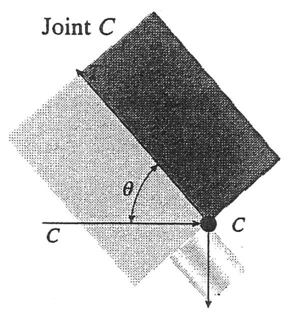
We observe that when the angle θ is reduced, bar AC becomes shorter but the cross-sectional areas of both bars increase (because the axial forces are larger). The opposite effects occur if the angle θ is increased. Thus, we see that the weight of the structure (which is proportional to the volume) depends upon the angle θ.

Determine the angle θ so that the structure has minimum weight without exceeding the allowable stresses in the bars. (*Note*: The weights of the bars are very small compared to the force P and may be disregarded.)



Solution 1.8-15 Two bars supporting a load P





T = tensile force in bar AC

C = compressive force in bar BC

$$\sum F_{\text{vert}} = 0 \qquad T = \frac{P}{\sin\theta}$$

$$\sum F_{\text{horiz}} = 0 \qquad C = \frac{P}{\tan\theta}$$

AREAS OF BARS

$$A_{AC} = \frac{T}{\sigma_{\text{allow}}} = \frac{P}{\sigma_{\text{allow}}\sin\theta}$$

$$A_{BC} = \frac{C}{\sigma_{\text{allow}}} = \frac{P}{\sigma_{\text{allow}}\tan\theta}$$

LENGTHS OF BARS

$$L_{AC} = \frac{L}{\cos\theta} \qquad L_{BC} = L$$

WEIGHT OF TRUSS

γ = weight density of material

$$W = \gamma(A_{AC}L_{AC} + A_{BC}L_{BC})$$

$$= \frac{\gamma PL}{\sigma_{\text{allow}}}\left(\frac{1}{\sin\theta\cos\theta} + \frac{1}{\tan\theta}\right)$$

$$= \frac{\gamma PL}{\sigma_{\text{allow}}}\left(\frac{1+\cos^2\theta}{\sin\theta\cos\theta}\right) \qquad \text{Eq. (1)}$$

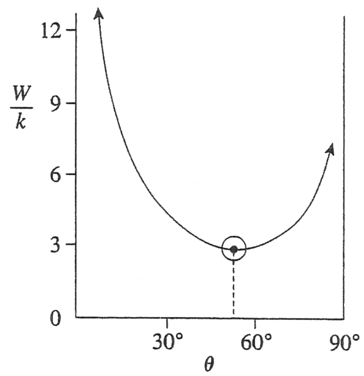
γ, P, L, and σ_{allow} are constants

W varies only with θ

Let $k = \dfrac{\gamma PL}{\sigma_{\text{allow}}}$ (k has unis of force)

$$\frac{W}{k} = \frac{1+\cos^2\theta}{\sin\theta\cos\theta} \text{ (Nondimensional)} \qquad \text{Eq. (2)}$$

GRAPH OF EQ. (2):



ANGLE θ THAT MAKES W A MINIMUM

Use Eq. (2)

Let $f = \dfrac{1 + \cos^2\theta}{\sin\theta\cos\theta}$

$$\frac{df}{d\theta} = 0$$

$$\frac{df}{d\theta} = \frac{(\sin\theta\cos\theta)(2)(\cos\theta)(-\sin\theta) - (1 + \cos^2\theta)(-\sin^2\theta + \cos^2\theta)}{\sin^2\theta\cos^2\theta}$$

$$= \frac{-\sin^2\theta\cos^2\theta + \sin^2\theta - \cos^2\theta - \cos^4\theta}{\sin^2\theta\cos^2\theta}$$

SET THE NUMERATOR = 0 AND SOLVE FOR θ:

$$-\sin^2\theta\cos^2\theta + \sin^2\theta - \cos^2\theta - \cos^4\theta = 0$$

Replace $\sin^2\theta$ by $1 - \cos^2\theta$:

$$-(1 - \cos^2\theta)(\cos^2\theta) + 1 - \cos^2\theta - \cos^2\theta - \cos^4\theta = 0$$

Combine terms to simplify the equation:

$$1 - 3\cos^2\theta = 0 \qquad \cos\theta = \frac{1}{\sqrt{3}}$$

$$\theta = 54.7^\circ \quad \leftarrow$$