Chapter 1 Solutions

Problem 1.3-1

$$L = 14\text{ft} \qquad q_0 = 12\frac{\text{lbf}}{\text{ft}} \qquad P = 50\text{lbf} \qquad M_0 = 300\text{lbf}\cdot\text{ft}$$

Reactions

$$\Sigma F_x = 0 \qquad B_x = \frac{3}{5}\cdot P = 30\cdot\text{lbf}$$

$$\Sigma M_A = 0 \qquad B_y = \frac{1}{L}\cdot\left[-M_0 + \left(\frac{1}{2}\cdot q_0\right)\cdot L\cdot\left(\frac{2\cdot L}{3}\right) + \frac{4}{5}\cdot P\cdot\left(L + \frac{L}{2}\right)\right] = 94.571\cdot\text{lbf}$$

$$\Sigma F_y = 0 \qquad A_y = \left(\frac{1}{2}\cdot q_0\right)\cdot L + \frac{4}{5}\cdot P - B_y = 29.429\cdot\text{lbf}$$

N, V and M at midspan of AB - LHFB is used below

$$N_{mid} = 0$$

$$V_{mid} = A_y - \frac{1}{2}\cdot\frac{q_0}{2}\cdot\frac{L}{2} = 8.429\cdot\text{lbf}$$

$$M_{mid} = -M_0 + A_y\cdot\frac{L}{2} - \frac{1}{2}\cdot\frac{q_0}{2}\cdot\frac{L}{2}\cdot\left(\frac{1}{3}\cdot\frac{L}{2}\right) = -143\cdot\text{lbf}\cdot\text{ft}$$

Problem 1.3-2

$$L = 4m \qquad q_0 = 160\frac{N}{m} \qquad P = 200 \cdot N \qquad M_0 = 380N \cdot m$$

Reactions

$$\Sigma F_x = 0 \qquad B_x = \frac{-3}{5} \cdot P = -120\,N$$

$$\Sigma M_A = 0 \qquad B_y = \frac{1}{L} \cdot \left[M_0 + \left(\frac{1}{2} \cdot q_0\right) \cdot L \cdot \left(\frac{L}{3}\right) - \frac{4}{5} \cdot P \cdot \left(L + \frac{L}{2}\right)\right] = -38.333 \cdot N$$

$$\Sigma F_y = 0 \qquad A_y = \left(\frac{1}{2} \cdot q_0\right) \cdot L - \frac{4}{5} \cdot P - B_y = 198.333 \cdot N$$

N, V and M at midspan of AB - LHFB is used below

$$N_{mid} = 0$$

$$V_{mid} = A_y - \frac{1}{2} \cdot \left(\frac{q_0}{2} + q_0\right) \cdot \frac{L}{2} = -41.667 \cdot N$$

$$M_{mid} = M_0 + A_y \cdot \frac{L}{2} - \frac{1}{2} \cdot q_0 \cdot \frac{L}{2} \cdot \left(\frac{2}{3} \cdot \frac{L}{2}\right) - \frac{1}{2} \cdot \frac{q_0}{2} \cdot \frac{L}{2} \cdot \left(\frac{1}{3} \cdot \frac{L}{2}\right) = 510 \cdot N \cdot m$$

Check using RHFB

$$N_{mid} = B_x + \frac{3}{5} \cdot P = 0\,N \qquad V_{mid} = \frac{1}{2} \cdot \frac{q_0}{2} \cdot \frac{L}{2} - B_y - \frac{4}{5} \cdot P = -41.667\,N$$

$$M_{mid} = \frac{-1}{2} \cdot \frac{q_0}{2} \cdot \frac{L}{2} \cdot \left(\frac{1}{3} \cdot \frac{L}{2}\right) + B_y \cdot \frac{L}{2} + \frac{4}{5} \cdot P \cdot \left(\frac{L}{2} + \frac{L}{2}\right) = 510 \cdot N \cdot m$$

Problem 1.3-3

(a) APPLY LAWS OF STATICS

$\Sigma F_x = 0 \qquad C_x = 100\text{ lb} - 50\text{ lb} = 50\text{ lb}$

FBD of $BC \qquad \Sigma M_B = 0 \qquad C_y = \dfrac{1}{10\text{ ft}}(0) = 0$

Entire FBD $\qquad \Sigma M_A = 0 \qquad B_y = \dfrac{1}{20\text{ ft}}(-100\text{ lb-ft}) = -5\text{ lb}$

$\Sigma F_y = 0 \qquad A_y = -B_y = 5\text{ lb-ft}$

Reactions are $\boxed{A_y = 5\text{ lb}}$ $\boxed{B_y = -5\text{ lb}}$ $\boxed{C_x = 50\text{ lb}}$ $\boxed{C_y = 0}$

(b) INTERNAL STRESS RESULTANTS N, V, AND M AT $x = 15$ ft

Use FBD of segment from A to $x = 15$ ft

$\Sigma F_x = 0 \qquad \boxed{N = 100\text{ lb} - 50\text{ lb} = 50\text{ lb}}$

$\Sigma F_y = 0 \qquad \boxed{V = A_y = 5\text{ lb}}$

$\Sigma M = 0 \qquad \boxed{M = A_y 15\text{ ft} = 75\text{ lb-ft}}$

Problem 1.3-4

(a) Apply laws of statics

$\Sigma F_x = 0 \qquad A_x = 0$

FBD of $AB \qquad \Sigma M_B = 0 \qquad M_A = 0$

Entire FBD $\qquad \Sigma M_C = 0 \qquad D_y = \dfrac{1}{3\text{ m}}\left[200\text{ N}\cdot\text{m} - \dfrac{1}{2}(80\text{ N/m})\,4\text{ m}\left(\dfrac{2}{3}\right)4\text{ m}\right] = -75.556\text{ N}$

$\Sigma F_y = 0 \qquad C_y = \dfrac{1}{2}(80\text{ N/m})\,4\text{ m} - D_y = 235.556\text{ N}$

Reactions are $\boxed{M_A = 0} \qquad A_x = 0 \qquad \boxed{C_y = 236\text{ N}} \qquad \boxed{D_y = -75.6\text{ N}}$

(b) Internal stress resultants N, V, and M at $x = 5$ m

Use FBD of segment from A to $x = 5$ m; ordinate on triangular load at $x = 5$ m is $\dfrac{3}{4}(80\text{ N/m}) = 60\text{ N/m}$.

$\Sigma F_x = 0 \qquad N_x = -A_x = 0$

$\Sigma F_y = 0 \qquad V = \dfrac{-1}{2}[(80\text{ N/m} + 60\text{ N/m})\,1\text{ m}] = -70\text{ N} \qquad \boxed{V = -70\text{ N}}$ Upward

$\Sigma M = 0 \qquad M = -M_A - \dfrac{1}{2}(80\text{ N/m})\,1\text{ m}\left(\dfrac{2}{3}1\text{ m}\right) - \dfrac{1}{2}(60\text{ N/m})\,1\text{ m}\left(\dfrac{1}{3}1\text{ m}\right) = -36.667\text{ N}\cdot\text{m}$

(break trapezoidal load into two triangular loads in moment expression)

$\boxed{M = -36.7\text{ N}\cdot\text{m}}$ CW

(c) Replace roller support at C with spring support

Structure remains statically determinate so all results above in (a) and (b) are unchanged.

Problem 1.3-5

(a) STATICS

FBD of AB (cut through beam at pin): $\Sigma M_B = 0 \qquad A_y = \frac{1}{10\text{ ft}}(-150\text{ lb-ft}) = -15\text{ lb}$

Entire FBD: $\Sigma M_D = 0$

$$C_y = \frac{1}{10\text{ ft}}\left[\frac{4}{5}40\text{ lb}(5\text{ ft}) + \frac{1}{2}(2.5\text{ lb/ft})10\text{ ft}\left(10\text{ ft} + \frac{10\text{ ft}}{3}\right) + \frac{1}{2}(5\text{ lb/ft})10\text{ ft}\left(10\text{ ft} + \frac{2}{3}10\text{ ft}\right)\right.$$
$$\left. - 150\text{ lb-ft} - A_y\,30\text{ ft}\right] = 104.333\text{ lb}$$

$$\Sigma F_y = 0 \quad D_y = \frac{4}{5}40\text{ lb} + \frac{1}{2}(5\text{ lb/ft} + 2.5\text{ lb/ft})\,10\text{ ft} - A_y - C_y = -19.833\text{ lb} \quad \text{so} \quad D_x = \frac{-D_y}{\tan(60^\circ)} = 11.451\text{ lb}$$

$$\Sigma F_x = 0 \quad A_x = \frac{3}{5}40\text{ lb} - D_x = 12.549\text{ lb}$$

$$\boxed{A_x = 12.55\text{ lb}, A_y = -15\text{ lb}, C_y = 104.3\text{ lb}, D_x = 11.45\text{ lb}, D_y = -19.83\text{ lb}}$$

(b) USE FBD OF AB ONLY; MOMENT AT PIN IS ZERO

$$F_{Bx} = -A_x \qquad F_{Bx} = -12.55\text{ lb} \qquad F_{By} = -A_y \qquad F_{By} = 15\text{ lb} \qquad \boxed{\text{Resultant}_B = \sqrt{{F_{Bx}}^2 + {F_{By}}^2} = 19.56\text{ lb}}$$

(c) ADD ROTATIONAL SPRING AT A AND REMOVE ROLLER AT C; APPLY EQUATIONS OF STATICAL EQUILIBRIUM

Use FBD of BCD $\Sigma M_B = 0$

$$D_y = \frac{1}{20\text{ ft}}\left[\frac{1}{2}(2.5\text{ lb/ft})\,10\text{ ft}\left(\frac{2}{3}10\text{ ft}\right) + \frac{1}{2}(5\text{ lb/ft})\,10\text{ ft}\left(\frac{1}{3}10\text{ ft}\right) + \frac{4}{5}40\text{ lb}\,(15\text{ ft})\right] = 32.333\text{ lb}$$

so $$D_x = \frac{-D_y}{\tan(60^\circ)} = -18.668\text{ lb}$$

Use entire FBD $\quad \Sigma F_y = 0 \quad A_y = \frac{1}{2}(5\text{ lb/ft} + 2.5\text{ lb/ft})\,10\text{ ft} + \frac{4}{5}(40\text{ lb}) - D_y = 37.167\text{ lb}$

$$\Sigma F_x = 0 \quad A_x = \frac{3}{5}(40\text{ lb}) - D_x = 42.668\text{ lb}$$

Use FBD of AB $\quad \Sigma M_B = 0 \quad M_A = 150\text{ lb-ft} + A_y\,10\text{ ft} = 521.667\text{ lb-ft}$

SO REACTIONS ARE $\boxed{A_x = 42.7\text{ lb}}$ $\boxed{A_y = 37.2\text{ lb}}$ $\boxed{M_A = 522\text{ lb-ft}}$ $\boxed{D_x = -18.67\text{ lb}}$ $\boxed{D_y = 32.3\text{ lb}}$

RESULTANT FORCE IN PIN CONNECTION AT B

$$F_{Bx} = -A_x \qquad F_{By} = -A_y \qquad \boxed{\text{Resultant}_B = \sqrt{{F_{Bx}}^2 + {F_{By}}^2} = 56.6\text{ lb}}$$

Problem 1.3-6

(a) Statics

$\Sigma F_y = 0 \qquad R_{3y} = 20\text{ N} - 45\text{ N} = -25\text{ N}$

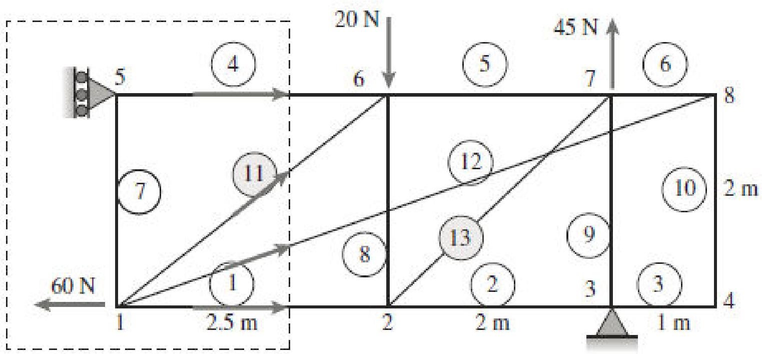
$\Sigma M_3 = 0 \qquad R_{5x} = \dfrac{1}{2\text{ m}}(20\text{ N} \times 2\text{ m}) = 20\text{ N}$

$\Sigma F_x = 0 \qquad R_{3x} = -R_{5x} + 60\text{ N} = 40\text{ N}$

(b) Member forces in members 11 and 13

Number of unknowns: $m = 13 \quad r = 3 \quad m + r = 16$

Number of equations: $j = 8 \quad 2j = 16$ So statically determinate



Section cut for left FBD

Truss analysis

(1) $\Sigma F_V = 0$ at joint 4 so $F_{10} = 0$

(2) $\Sigma F_V = 0$ at joint 8 so $F_{12} = 0$

(3) $\Sigma F_H = 0$ at joint 5 so $F_4 = -R_{5x} = -20\text{ N}$

(4) Cut vertically through 4, 11, 12, and 1; use left FBD; sum moments about joint 2

$F_{11V} = \dfrac{1}{2.5\text{ m}}(R_{5x} - F_4)$ so $\boxed{F_{11} = 0}$

(5) Sum vertical forces at joint 3; $F_9 = R_{3y}$

$F_9 = 25\text{ N}$

(6) Sum vertical forces at joint 7 $\qquad F_{13V} = 45\text{ N} - F_9 = 20\text{ N} \qquad \boxed{F_{13} = \sqrt{2}F_{13V} = 28.3\text{ N}}$

Problem 1.3-7

(a) STATICS

$\Sigma F_x = 0 \qquad A_x = 0$

$\Sigma M_A = 0 \qquad E_y = \dfrac{1}{20\text{ ft}}(3\text{ k} \times 10\text{ ft} + 2\text{ k} \times 20\text{ ft} + 1\text{ k} \times 30\text{ ft}) = 5\text{ k}$

$\Sigma F_y = 0 \qquad A_y = 3\text{ k} + 2\text{ k} + 1\text{ k} - E_y = 1\text{ k}$

(b) MEMBER FORCE IN MEMBER FE

Number of unknowns: $m = 11 \qquad r = 3 \qquad m + r = 14$

Number of equations: $j = 7 \qquad 2j = 14$ So statically determinate

TRUSS ANALYSIS

(1) Cut vertically through AB, GC, and GF; use left FBD; sum moments about C

$$F_{GFx}(15\text{ ft}) - F_{GFy}(20\text{ ft}) = A_y(20\text{ ft}) = 20\text{ ft-k} \qquad F_{GFx} = F_{GF}\frac{10}{\sqrt{2^2 + 10^2}} \qquad F_{GFy} = F_{GF}\frac{2}{\sqrt{2^2 + 10^2}}$$

so $$F_{GF} = \frac{A_y(20\text{ ft})}{15\text{ ft}\dfrac{10}{\sqrt{2^2 + 10^2}} - 20\text{ ft}\dfrac{2}{\sqrt{2^2 + 10^2}}} = 1.854\text{ k} \quad \text{and} \quad F_{GFx} = F_{GF}\frac{10}{\sqrt{2^2 + 10^2}} = 1.818\text{ k}$$

(2) Sum horizontal forces at joint F $\qquad F_{FEx} = F_{GFx} = 1.818\text{ k} \qquad F_{FE} = \dfrac{\sqrt{10^2 + 3^2}}{10}F_{FEx} = 1.898\text{ k}$

$$\boxed{F_{FE} = 1.898\text{ k}}$$

Problem 1.3-8

(a) Statics

$$\Sigma F_x = 0 \qquad F_x = 0$$

$$\Sigma M_F = 0 \qquad D_y = \frac{1}{6\text{ m}}\left[3\text{ kN}(6\text{ m}) + 6\text{ kN}(3\text{ m})\right] = 6\text{ kN}$$

$$\Sigma F_y = 0 \qquad F_y = 9\text{ kN} + 6\text{ kN} + 3\text{ kN} - D_y = 12\text{ kN}$$

(b) Member force in member FE

Number of unknowns: $m = 11$ $\quad r = 3$ $\quad m + r = 14$

Number of equations: $j = 7$ $\quad 2j = 14$ $\quad$ So statically determinate

Truss analysis

(1) Cut vertically through AB, GD, and GF; use left FBD; sum moments about D to get $F_{GF} = 0$

(2) Sum horizontal forces at joint F $\qquad F_{FEx} = -F_x = 0 \qquad$ so $\qquad \boxed{F_{FE} = 0}$

Problem 1.3-9

$c = 8\,ft \quad P = 20\,kip$

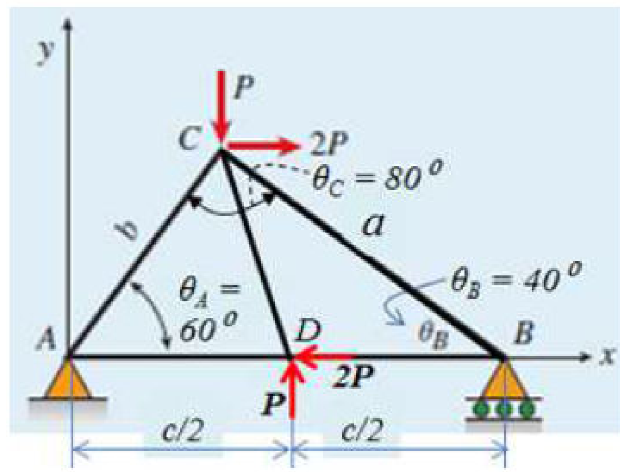
$a = \dfrac{\sin(60deg)}{\sin(80deg)} \cdot c = 7.035 \cdot ft \qquad b = \dfrac{\sin(40deg)}{\sin(80deg)} \cdot c = 5.222 \cdot ft$

$\Sigma M_A = P \cdot \dfrac{c}{2} - P \cdot b \cdot \cos(60deg) - 2P \cdot b \cdot \sin(60deg) + B_y \cdot c = 0$

$B_y = \dfrac{P \cdot b \cdot \cos(60deg) + 2P \cdot b \cdot \sin(60deg) - P \cdot \dfrac{c}{2}}{c} = 19.137 \cdot kip$

$A_y = -B_y = -19.137 \cdot kip$

$A_x = 0$



Joint A

$F_{AC} = \dfrac{-A_y}{\sin(60deg)} = 22.098 \cdot kip$

$F_{AD} = -F_{AC} \cdot \cos(60deg) - A_x = -11.049 \cdot kip$

Joint B

$F_{BC} = \dfrac{-B_y}{\sin(40deg)} = -29.772 \cdot kip$

$F_{BD} = -F_{BC} \cdot \cos(40deg) = 22.807 \cdot kip$

$CD = \sqrt{b^2 + \left(\dfrac{c}{2}\right)^2 - 2 \cdot b \cdot \dfrac{c}{2} \cdot \cos(60deg)} = 4.731 \cdot ft$

$ACD = \operatorname{asin}\left(\dfrac{\sin(60deg)}{CD} \cdot \dfrac{c}{2}\right) = 47.077 \cdot deg$

Joint D

$$F_{DC} = \frac{-P}{\cos(90\text{deg} - 72.923\text{deg})} = -20.922 \cdot \text{kip}$$

$$180\text{deg} - 60\text{deg} - ACD = 72.923 \cdot \text{deg}$$

$$BCD = \text{asin}\left(\frac{\sin(40\text{deg})}{CD} \cdot \frac{c}{2}\right) = 32.923 \cdot \text{deg}$$

$$\frac{a \cdot \sin(BCD)}{\sin(ACD)} = 5.222 \cdot \text{ft}$$

$$ACD + BCD = 80 \cdot \text{deg}$$

Problem 1.3-10

Geometry $b = 3m$ $P = 80kN$

$$a = \sin(60deg)\cdot\left(\frac{b}{\sin(40deg)}\right) = 4.042\,m$$

$$L_{AB} = \sin(80deg)\cdot\left(\frac{b}{\sin(40deg)}\right) = 4.596\,m$$

$$L_{DB} = \sqrt{\left(\frac{b}{2}\right)^2 + L_{AB}{}^2 - 2\cdot\left(\frac{b}{2}\right)\cdot(L_{AB})\cdot\cos(60deg)} = 4.06\,m$$

$$\frac{L_{DB}}{\sin(60deg)} = \frac{\frac{b}{2}}{\sin(DBA)} \quad \text{so} \quad DBA = \operatorname{asin}\left(\frac{\frac{b}{2}}{L_{DB}}\cdot\sin(60deg)\right) = 18.662\cdot deg$$

and $CBD = 40deg - DBA = 21.338\cdot deg$ $ADB = 180deg - 60deg - DBA = 101.338\cdot deg$

$CDB = 180deg - ADB = 78.662\cdot deg$

Reactions

$\Sigma F_x = 0$ $A_x = -2\cdot P + 2\cdot P = 0\,N$

$$\Sigma M_A = 0 \qquad B_y = \frac{1}{L_{AB}}\cdot\left[-2\cdot P\cdot\left(\frac{b}{2}\cdot\sin(60deg)\right) + P\cdot(b\cdot\cos(60deg)) + 2\cdot P\cdot(b\cdot\sin(60deg))\right] = 71.329\cdot kN$$

$\Sigma F_y = 0$ $A_y = P - B_y = 8.671\cdot kN$

MoJ to find member forces

Joint A $AD = \dfrac{-A_y}{\sin(60deg)} = -10.013\cdot kN$ $AB = -A_x - AD\cdot\cos(60deg) = 5.006\cdot kN$

Joint D - sum forces normal to & along line ADC $DB = \dfrac{2\cdot P\cdot\sin(60deg)}{\cos(90deg - CDB)} = 141.322\cdot kN$

$$DC = AD + 2\cdot P\cdot(\cos(60deg)) - DB\cdot\cos(CDB) = 42.204\cdot kN$$

Joint C $CB = \dfrac{1}{\cos(40deg)}\cdot(-2\cdot P + DC\cdot\cos(60deg)) = -181.319\cdot kN$

Joint B check $-AB - DB\cdot\cos(DBA) - CB\cdot\cos(40deg) = 0\,N$

$$DB\cdot\sin(DBA) + CB\cdot\sin(40deg) + B_y = 0\,N$$

Problem 1.3-11

Reactions $c = 8\,\text{ft}$ $P = 20\,\text{kip}$

$A_x = 0$ $A_y = -19.137\,\text{kip}$ $B_y = -A_y$

AC: MoS - cut through AC and AD, use LHFB

$\Sigma M_D = 0$ $-A_y \cdot \frac{c}{2} - AC \cdot \sin(60\,\text{deg}) \cdot \frac{c}{2} = 0$

$$AC = \frac{-A_y}{\sin(60\,\text{deg})} = 22.098 \cdot \text{kip}$$

BD: MoS - cut through BC andf BD, use RHFB $b = \frac{\sin(40\,\text{deg})}{\sin(80\,\text{deg})} \cdot c = 5.222 \cdot \text{ft}$

$\Sigma M_C = 0$ $B_y \cdot (c - b \cdot \cos(60\,\text{deg})) - BD \cdot (b \cdot \sin(60\,\text{deg})) = 0$ $BD = \frac{B_y \cdot (c - b \cdot \cos(60\,\text{deg}))}{b \cdot \sin(60\,\text{deg})} = 22.807 \cdot \text{kip}$

Problem 1.3-12

Reactions $\quad b = 3m \quad P = 80kN$

$A_x = 0 \quad A_y = 8.671kN \quad B_y = 71.329kN$

AB: MoS - cut through AD and AB, use LHFB

$$\Sigma M_D = 0 \qquad AB \cdot \frac{b}{2} \cdot \sin(60deg) + A_x \cdot \frac{b}{2} \cdot \sin(60deg) - A_y \cdot \frac{b}{2} \cdot \cos(60deg) = 0$$

$$AB = \frac{-\left(A_x \cdot \frac{b}{2} \cdot \sin(60deg) - A_y \cdot \frac{b}{2} \cdot \cos(60deg)\right)}{\left(\frac{b}{2} \cdot \sin(60deg)\right)} = 5.006 \cdot kN$$

DC: MoS - cut through DC and CB, use upper FBD $\qquad a = \sin(60deg) \cdot \left(\frac{b}{\sin(40deg)}\right) = 4.042\,m$

$$DC_x = DC \cdot \cos(60deg) \qquad DC_y = DC \cdot \sin(60deg)$$

$$\Sigma M_B = 0 \qquad -\left(-DC_x + 2 \cdot P\right) \cdot (a \cdot \sin(40deg)) + \left(DC_y + P\right) \cdot (a \cdot \cos(40deg)) = 0$$

$$-(-DC \cdot \cos(60deg) + 2 \cdot P) \cdot (a \cdot \sin(40deg)) + (DC \cdot \sin(60deg) + P) \cdot (a \cdot \cos(40deg)) = 0$$

Collect and simplify, solve for DC

$$DC = \frac{1.0 \cdot (80.0 \cdot kN \cdot \cos(40.0 \cdot deg) - 160.0 \cdot kN \cdot \sin(40.0 \cdot deg))}{\cos(60.0 \cdot deg) \cdot \sin(40.0 \cdot deg) + \sin(60.0 \cdot deg) \cdot \cos(40.0 \cdot deg)} = 42.204 \cdot kN$$

Problem 1.3-13

(a) FIND REACTIONS USING STATICS $\quad m = 3 \quad r = 9 \quad m + r = 12 \quad j = 4 \quad 3j = 12$

$m + r = 3j$ So truss is statically determinate

$$r_{AQ} = \begin{pmatrix} 4 \\ -3 \\ 0 \end{pmatrix} \quad r_{OA} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} \quad e_{AQ} = \frac{r_{AQ}}{|r_{AQ}|} = \begin{pmatrix} 0.8 \\ -0.6 \\ 0 \end{pmatrix} \quad P_A = P e_{AQ} = \begin{pmatrix} 0.8P \\ -0.6P \\ 0 \end{pmatrix} \quad r_{OC} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} \quad r_{OB} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$$

$\Sigma M = 0$

$$M_O = r_{OA} \times P_A + r_{OC} \times \begin{pmatrix} C_x \\ C_y \\ C_z \end{pmatrix} + r_{OB} \times \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} = \begin{pmatrix} 4C_z + 3.0P \\ 4.0P - 2B_z \\ 2B_y - 4C_x \end{pmatrix} \quad \text{so} \quad \Sigma M_x = 0 \text{ gives } C_z = \frac{-3}{4}P$$

$\Sigma M_y = 0$ gives $\boxed{B_z = 2P}$

$\Sigma F = 0$

$$R_O = P_A + \begin{pmatrix} O_x \\ O_y \\ O_z \end{pmatrix} + \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} + \begin{pmatrix} C_x \\ C_y \\ C_z \end{pmatrix} = \begin{pmatrix} B_x + C_x + O_x + 0.8P \\ B_y + C_y + O_y + -0.6P \\ O_z + \frac{5P}{4} \end{pmatrix} \quad \text{so} \quad \Sigma M_z = 0 \text{ gives } \boxed{O_z = \frac{-5}{4}P}$$

METHOD OF JOINTS $\quad$ Joint O $\quad \Sigma F_x = 0 \quad O_x = 0 \quad \Sigma F_y = 0 \quad O_y = 0$

Joint B $\quad \Sigma F_y = 0 \quad B_y = 0$

Joint C $\quad \Sigma F_x = 0 \quad C_x = 0$

For entire structure $\Sigma F_x = 0$ gives $\boxed{B_x = -0.8P}$ $\quad \Sigma F_y = 0 \quad C_y = 0.6P - B_y = O_y \quad C_y = 0.6P$

(b) FORCE IN MEMBER AC

$$\Sigma F_z = 0 \text{ at joint } C \quad F_{AC} = \frac{\sqrt{4^2 + 5^2}}{5}|C_Z| = \frac{3\sqrt{41}\,|P|}{20} \quad \boxed{F_{AC} = \frac{3\sqrt{41}}{20}P} \text{ tension} \quad \frac{3\sqrt{41}}{20} = 0.96$$

Problem 1.3-14

(a) FIND REACTIONS USING STATICS $\quad m = 4 \quad r = 8 \quad m + r = 12 \quad j = 4 \quad 3j = 12$

$m + r = 3j$ so truss is statically determinate

$$r_{OA} = \begin{pmatrix} 0 \\ 0 \\ 0.8L \end{pmatrix} \quad r_{OB} = \begin{pmatrix} L \\ 0 \\ 0 \end{pmatrix} \quad r_{OC} = \begin{pmatrix} 0 \\ 0.6L \\ 0 \end{pmatrix} \quad F_A = \begin{pmatrix} A_x \\ A_y \\ P \end{pmatrix} \quad F_B = \begin{pmatrix} 0 \\ B_y \\ B_z \end{pmatrix} \quad F_c = \begin{pmatrix} C_x \\ -2P \\ 0 \end{pmatrix} \quad F_O = \begin{pmatrix} O_x \\ O_y \\ O_z \end{pmatrix}$$

$\Sigma M = 0$

Resultant moment at O

$$M_O = r_{OA} \times F_A + r_{OB} \times F_B + r_{OC} \times F_C = \begin{pmatrix} -0.8A_yL \\ 0.8A_xL - B_zL \\ B_yL - 0.6C_xL \end{pmatrix} \quad so \quad \Sigma M_x = 0 \quad \text{gives} \quad A_y = 0$$

$\Sigma F = 0$

Resultant force at O

$$R_O = F_O + F_A + F_B + F_C = \begin{pmatrix} A_x + C_x + O_x \\ A_y + B_y + O_y - 2P \\ B_z + O_z + P \end{pmatrix}$$

METHOD OF JOINTS $\quad$ Joint O $\quad \Sigma F_z = 0 \quad O_z = 0$

so from $\quad \Sigma F_z = 0 \quad \boxed{B_z = -P}$ and $\quad \Sigma M_y = 0 \quad \boxed{A_x = \dfrac{B_z}{0.8} = -1.25P}$

Joint B $\quad \Sigma F_y = 0 \quad \boxed{B_y = 0}$

Joint C $\quad \Sigma F_x = 0 \quad C_x = 0$

(b) FORCE IN MEMBER AB

$$\Sigma F_z = 0 \quad \text{at joint } B \quad F_{AB} = \frac{\sqrt{(0.8L)^2 + L^2}}{0.8L}|B_z| \quad |B_z| = |P| \quad \frac{\sqrt{(0.8L)^2 + L^2}}{0.8L} = 1.601$$

$\boxed{F_{AB} = 1.601P}$ $\quad$ tension

Problem 1.3-15

(a) FIND REACTIONS USING STATICS $\quad m = 3 \quad r = 6 \quad m + r = 9 \quad j = 3 \quad 3j = 9$

$m + r = 3j$ So truss is statically determinate

$$r_{OA} = \begin{pmatrix} 3L \\ 0 \\ 0 \end{pmatrix} \quad r_{OB} = \begin{pmatrix} 0 \\ 4L \\ 0 \end{pmatrix} \quad r_{OC} = \begin{pmatrix} 0 \\ 2L \\ 4L \end{pmatrix} \quad F_A = \begin{pmatrix} -2P \\ A_y \\ A_z \end{pmatrix} \quad F_B = \begin{pmatrix} B_x \\ B_y \\ 3P \end{pmatrix} \quad F_C = \begin{pmatrix} C_x \\ C_y \\ P \end{pmatrix}$$

$\Sigma M = 0$

Resultant moment at O

$$M_O = r_{OA} \times F_A + r_{OB} \times F_B + r_{OC} \times F_C = \begin{pmatrix} 14LP - 4C_yL \\ 4C_xL - 3A_zL \\ 3A_yL - 4B_xL - 2C_xL \end{pmatrix} \quad \text{so} \quad \Sigma M_x = 0 \quad \text{gives} \quad C_y = \frac{14}{4}P$$

$\Sigma F = 0$

Resultant force at O

$$R_O = F_A + F_B + F_C = \begin{pmatrix} B_x + C_x - 2P \\ A_y + B_y + C_y \\ A_z + 4P \end{pmatrix} \quad \text{so} \quad \Sigma F_z = 0 \quad \text{gives} \quad \boxed{A_z = -4.0P}$$

METHOD OF JOINTS

Joint A $\quad \Sigma F_z = 0 \quad F_{ACz} = -A_z = 4.0P \quad$ so $\quad F_{ACy} = \dfrac{2}{4}F_{ACz} = 2.0P \quad F_{ACx} = \dfrac{3}{4}F_{ACz} = 3.0P$

$$\Sigma F_x = 0 \quad F_{ABx} = -2P - F_{ACx} = -3.0P - 2P \quad \text{so} \quad F_{ABy} = \frac{4}{3}F_{ABx} = -4.0P - \frac{8P}{3}$$

$$\Sigma F_y = 0 \quad A_y = -(F_{ABy} + F_{ACy}) = \frac{8P}{3} + 4.0P + -2.0P \quad \boxed{A_y = 4.67P}$$

(b) FORCE IN MEMBER AB

$$F_{AB} = \sqrt{F_{ABx}^{\,2} + F_{ABy}^{\,2}} \qquad F_{AB} = -\sqrt{5^2 + \left(\frac{20}{3}\right)^2}\,P = -\frac{25P}{3} \qquad \frac{25}{3} = 8.33$$

$\boxed{F_{AB} = -8.33P}$ compression

Problem 1.3-16

(a) FIND REACTIONS USING STATICS $\quad m = 3 \quad r = 6 \quad m + r = 9 \quad j = 3 \quad 3j = 9$

$m + r = 3j$ so truss is statically determinate

$L = 2 \text{ m} \qquad P = 5 \text{ kN}$

$$r_{OA} = \begin{pmatrix} 3L \\ 0 \\ 0 \end{pmatrix} \quad r_{OB} = \begin{pmatrix} 0 \\ 4L \\ 2L \end{pmatrix} \quad r_{OC} = \begin{pmatrix} 0 \\ 0 \\ 4L \end{pmatrix} \quad F_A = \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} \quad F_B = \begin{pmatrix} B_x \\ 0 \\ P \end{pmatrix} \quad F_C = \begin{pmatrix} C_x \\ C_y \\ -P \end{pmatrix}$$

$\Sigma F = 0$

Resultant force at O $\quad R_O = F_A + F_B + F_C = \begin{pmatrix} A_x + B_x + C_x \\ A_y + C_y \\ A_z \end{pmatrix}$ so $\quad \Sigma F_z = 0$ gives $\quad A_z = 0$

RESULTANT MOMENT AT A

$$r_{AC} = \begin{pmatrix} -3L \\ 0 \\ 4L \end{pmatrix} \quad e_{AC} = \frac{r_{AC}}{|r_{AC}|} = \begin{pmatrix} -0.6 \\ 0 \\ 0.8 \end{pmatrix} \quad r_{AB} = \begin{pmatrix} -3L \\ 4L \\ 2L \end{pmatrix}$$

$$M_A = r_{AB} \times F_B + r_{AC} \times F_C = \begin{pmatrix} 120 \text{ kN} - 24C_y \\ 12B_x + 24C_x \\ -24B_x - 18C_y \end{pmatrix} \quad M_A e_{AC} = -19.2B_x - 72.0 \text{ kN} \quad \text{so} \quad \boxed{B_x = \frac{-72}{19.2} \text{kN} = -3.75 \text{ kN}}$$

(b) FORCE IN MEMBER AB

Method of joints at B $\quad \Sigma F_x = 0 \quad F_{ABx} = -B_X \quad \boxed{F_{AB} = \frac{\sqrt{29}}{3} F_{ABx} = 6.73 \text{ kN}}$

Problem 1.3-17

(a) APPLY LAWS OF STATICS $L_1 = 30$ in. $L_2 = 20$ in. $T_1 = 21000$ lb-in. $T_2 = 10000$ lb-in.

$\Sigma M_x = 0$ $\boxed{T_A = T_1 - T_2 = 11{,}000 \text{ lb-in.}}$

(b) INTERNAL STRESS RESULTANT T AT TWO LOCATIONS

Cut shaft at midpoint between A and B at $x = L_1/2$ (use left FBD) $\Sigma M_x = 0$ $\boxed{T_{AB} = -T_A = -11{,}000 \text{ lb-in.}}$

Cut shaft at midpoint between B and C at $x = L_1 + L_2/2$ (use right FBD) $\Sigma M_x = 0$ $\boxed{T_{BC} = T_2 = 10{,}000 \text{ lb-in.}}$

Problem 1.3-18

(a) REACTION TORQUE AT A $L_1 = 0.75\text{ m}$ $L_2 = 0.75\text{ m}$ $t_1 = 3100\text{ N·m/m}$ $T_2 = 1100\text{ N·m}$

Statics $\Sigma M_x = 0$ $T_A = -t_1 L_1 + T_2 = -1225\text{ N·m}$ $\boxed{T_A = -1225\text{ N·m}}$

(b) INTERNAL TORSIONAL MOMENTS AT TWO LOCATIONS

Cut shaft between A and B (use left FBD) $T_1(x) = -T_A - t_1 x$ $\boxed{T_1\left(\frac{L_1}{2}\right) = 62.5\text{ N·m}}$

Cut shaft between B and C (use left FBD) $T_2(x) = -T_A - t_1 L_1$ $\boxed{T_2\left(L_1 + \frac{L_2}{2}\right) = -1100\text{ N·m}}$

Problem 1.3-19

(a) STATICS

$$\Sigma F_H = 0 \qquad A_x = \frac{-1}{2}(90\ \text{lb/ft})\,12\ \text{ft} = -540\ \text{lb}$$

$$\Sigma F_V = 0 \qquad A_y + C_y = 0$$

$$\Sigma M_{\text{FBD}BC} = 0 \qquad C_y = \frac{500\ \text{lb-ft}}{9\ \text{ft}} = 55.6\ \text{lb} \qquad A_y = -C_y = -55.6\ \text{lb}$$

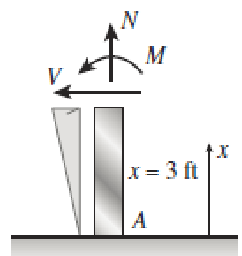
$$\Sigma M_A = 0 \qquad M_A = 500\ \text{lb-ft} + \frac{1}{2}(90\ \text{lb/ft})\,12\ \text{ft}\left(\frac{2}{3}12\ \text{ft}\right) - C_y\,9\ \text{ft} = 4320\ \text{lb-ft}$$

(b) INTERNAL STRESS RESULTANTS

$$N = -A_y = 55.6\ \text{lb}$$

$$V = -A_x - \frac{1}{2}\left(\frac{3}{12}90\ \text{lb/ft}\right)3\ \text{ft} = 506\ \text{lb}$$

$$M = -M_A - A_x 3\ \text{ft} - \frac{1}{2}\left(\frac{3}{12}90\ \text{lb/ft}\right)3\ \text{ft}\left(\frac{1}{3}3\ \text{ft}\right) = -2734\ \text{lb-ft}$$



Problem 1.3-20

(a) STATICS

$$\Sigma F_x = 0 \qquad A_x = \frac{3}{5}(200\text{ N}) + \frac{1}{2}(80\text{ N/m})\,4\text{ m} = 280\text{ N}$$

$$\Sigma M_{BRHFB} = 0 \quad D_y = \frac{1}{3\text{ m}}\left[\frac{4}{5}(200\text{ N})(1.5\text{ m}) + \frac{1}{2}(80\text{ N/m})\,4\text{ m}\left(\frac{1}{3}4\text{ m}\right)\right]$$

$$= 151.1\text{ N} \quad < \text{use right hand FBD } (BCD \text{ only})$$

$$\Sigma F_y = 0 \qquad A_y = -D_y + \frac{4}{5}(200\text{ N}) = 8.89\text{ N}$$

$$\Sigma M_A = 0 \qquad M_A = \frac{4}{5}(200\text{ N})(1.5\text{ m}) - \frac{3}{5}(200\text{ N})(4\text{ m}) - D_y\,3\text{ m} - \frac{1}{2}(80\text{ N/m})\,4\text{ m}\left(\frac{2}{3}4\text{ m}\right) = -1120\text{ N}\cdot\text{m}$$

(b) RESULTANT FORCE IN PIN AT B

LEFT HAND FBD (SEE FIGURE)

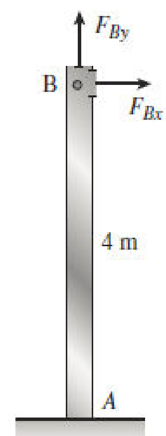
$F_{Bx} = -A_x = -280\text{ N} \qquad F_{By} = -A_y = -8.89\text{ N}$

RIGHT HAND FBD

$$F_{Bx} = \frac{3}{5}(200\text{ N}) + \frac{1}{2}(80\text{ N/m})\,4\text{ m} = 280\text{ N}$$

$$F_{By} = \frac{4}{5}(200\text{ N}) - D_y = 8.89\text{ N}$$

$$\boxed{\text{Resultant}_B = \sqrt{F_{Bx}^2 + F_{By}^2} = 280\text{ N}}$$



Left hand FBD

Problem 1.3-21

$$L = 14\text{ft} \quad q_o = 12\frac{\text{lbf}}{\text{ft}} \quad P = 50\text{lbf} \quad M_o = 300\text{lbf}\cdot\text{ft}$$

$$\Sigma M_D = -M_o + \frac{1}{2}\cdot q_o\cdot L\cdot\frac{L}{3} - \frac{4}{5}\cdot P\cdot\frac{L}{2} + \frac{3}{5}P\cdot L - \frac{1}{2}\cdot q_o\cdot L\cdot\frac{2L}{3} - A_y\cdot L = 0$$

$$A_y = \frac{-M_o + \frac{1}{2}\cdot q_o\cdot L\cdot\frac{L}{3} - \frac{4}{5}\cdot P\cdot\frac{L}{2} + \frac{3}{5}P\cdot L - \frac{1}{2}\cdot q_o\cdot L\cdot\frac{2L}{3}}{L} = -39.429\cdot\text{lbf}$$

$$D_y = -A_y + \frac{1}{2}\cdot q_o\cdot L + \frac{4}{5}\cdot P = 163.429\cdot\text{lbf}$$

$$D_x = \frac{-1}{2}\cdot q_o\cdot L + \frac{3}{5}\cdot P = -54\cdot\text{lbf}$$

$$V_{midAB} = A_y - \frac{1}{2}\cdot\frac{L}{2}\cdot\frac{q_o}{2} = -60.429\cdot\text{lbf} \qquad N_{mid} = 0$$

$$M_{midAB} = M_o + A_y\cdot\frac{L}{2} - \frac{1}{2}\cdot\frac{q_o}{2}\cdot\frac{L}{2}\cdot\frac{\frac{L}{2}}{3} = -25\cdot\text{lbf}\cdot\text{ft}$$

Problem 1.3-22

$$L = 4m \qquad q_0 = 160 \cdot \frac{N}{m} \qquad P = 200N \qquad M_0 = 380 \cdot N \cdot m$$

Reactions

$$\Sigma F_x = 0 \qquad A_x = 2 \cdot \left(\frac{3}{5} \cdot P\right) - \frac{1}{2} \cdot q_0 \cdot L = -80\,N$$

$$\Sigma M_A = 0 \qquad D_y = \frac{1}{L} \cdot \left[M_0 + \frac{4}{5} \cdot P \cdot \frac{L}{2} + \frac{4}{5} \cdot P \cdot \frac{3 \cdot L}{2} - \frac{1}{2} \cdot q_0 \cdot L \cdot \left(\frac{L}{3}\right)\right] = 308.333\,N$$

$$\Sigma F_y = 0 \qquad A_y = -D_y + 2 \cdot \left(\frac{4}{5} \cdot P\right) = 11.667\,N$$

Column BD internal forces and moment at mid-height - cut through column, use lower FBD (D on your left)

$$N_{mid} = -D_y = -308.333\,N \qquad V_{mid} = \frac{-1}{2} \cdot \frac{q_0}{2} \cdot \frac{L}{2} = -80\,N \qquad M_{mid} = -\left(\frac{1}{2} \cdot \frac{q_0}{2} \cdot \frac{L}{2}\right) \cdot \left(\frac{1}{3} \cdot \frac{L}{2}\right) = -53.333 \cdot N \cdot m$$

Problem 1.3-23

$$L_{BC} = \frac{\frac{4}{5}\cdot 30in}{\frac{2}{\sqrt{5}}} = 26.833\cdot in \qquad L_{AC} = \frac{3}{5}\cdot(30in) + \frac{1}{\sqrt{5}}\cdot L_{BC} = 30\cdot in$$

Part (a) - statics

$$\Sigma M_A = 0 \qquad C_y = \frac{1}{L_{AC}}\cdot\left(200lb\cdot\frac{1}{2}\cdot\frac{3}{5}\cdot 30in\right) = 60\,lbf \qquad C_x = \frac{-1}{2}\cdot C_y = -30\,lbf$$

(resultant of C_x and C_y acts along line of strut)

$$\Sigma F_x = 0 \qquad A_x = -C_x = 30\,lbf$$

$$\Sigma F_y = 0 \qquad A_y = 200lb - C_y = 140\,lbf$$

Part (b) - internal stress resultants N, V, M

distributed weight of door in -y dir. $\quad w = \frac{200lb}{30in} = 6.667\cdot\frac{lb}{in}$

components of w along and perpendicular to door

$$w_a = \frac{4}{5}\cdot w = 5.333\cdot\frac{lb}{in} \qquad w_p = \frac{3}{5}\cdot w = 4\cdot\frac{lb}{in}$$

$$N_x = w_a\cdot(20in) - \frac{3}{5}\cdot A_x - \frac{4}{5}\cdot A_y = -23.333\,lbf$$

$$V_x = -w_p\cdot(20in) - \frac{4}{5}\cdot A_x + \frac{3}{5}\cdot A_y = -20\,lbf$$

$$M_x = -w_p\cdot(20in)\cdot\frac{20in}{2} - \frac{4}{5}\cdot A_x\cdot(20in) + \frac{3}{5}\cdot A_y\cdot(20in) = 400\cdot lb\cdot in \qquad M_x = 33.333\,lb\cdot ft$$

$$\boxed{N_x = -23.3\,lbf} \qquad \boxed{V_x = -20\,lbf} \qquad \boxed{M_x = 33.3\cdot lb\cdot ft}$$

Problem 1.3-24

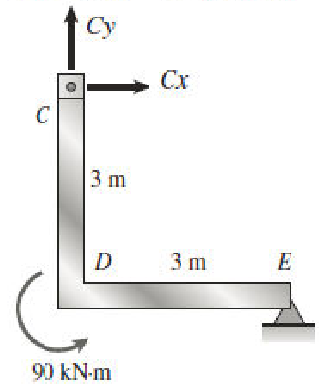
(a) STATICS

$\Sigma M_A = 0$

$$10\text{ kN}(6\text{ m}) - 10\text{ kN}\left(\frac{1}{\sqrt{2}}\right)(6\text{ m}) + 90\text{ kN}\cdot\text{m} + E_y(6\text{ m}) - E_x 3\text{ m} = 6E_y\text{m} - 3E_x\text{m} + 150\text{ kN}\cdot\text{m} - 30\sqrt{2}\text{ kN}\cdot\text{m}$$

so $\quad 6E_y\text{m} - 3E_x\text{m} + 150\text{ kN}\cdot\text{m} - 30\sqrt{2}\text{ kN}\cdot\text{m} = 0$

or $\quad -E_x + 2E_y = \dfrac{-(150\text{ kN}\cdot\text{m} - 30\sqrt{2}\text{ kN}\cdot\text{m})}{3\text{ m}} = -35.858\text{ kN}$



$\Sigma M_{CRHFB} = 0$ < right hand FBD (*CDE*) - see figure.

$(E_x + E_y)3\text{ m} = -90\text{ kN}\cdot\text{m} \qquad E_x + E_y = \dfrac{-90\text{ kN}\cdot\text{m}}{3\text{ m}} = -30\text{ kN}$

Solving $\quad \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ 1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} -35.858\text{ kN} \\ -30\text{ kN} \end{pmatrix} = \begin{pmatrix} -8.05 \\ -21.95 \end{pmatrix}\text{ kN}$

$\boxed{E_x = -8.05\text{ kN}} \quad \boxed{E_y = -22\text{ kN}}$

$\Sigma F_x = 0 \qquad A_x = -E_x + 10\text{ kN} - 10\text{ kN}\left(\dfrac{1}{\sqrt{2}}\right) = 10.98\text{ kN} \qquad \boxed{A_x = 10.98\text{ kN}}$

$\Sigma F_y = 0 \qquad A_y = -E_y + 10\text{ kN}\left(\dfrac{1}{\sqrt{2}}\right) = 29.07\text{ kN} \qquad \boxed{A_y = 29.1\text{ kN}}$

(b) RIGHT HAND FBD $\qquad C_x = -E_x = 8.05\text{ kN} \qquad C_y = -E_y = 22\text{ kN}$

$\boxed{\text{Resultant}_C = \sqrt{C_x^2 + C_y^2} = 23.4\text{ kN}}$

Problem 1.3-25

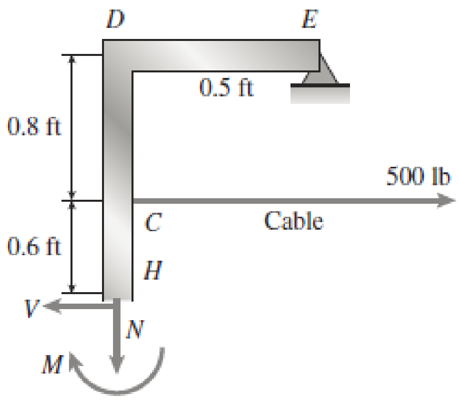
(a) STATICS

$\Sigma F_x = 0 \qquad E_x = 0$

$\Sigma M_E = 0 \qquad A_y = \dfrac{1}{1\text{ ft}}(-500\text{ lb} \times 2.5\text{ ft}) = -1250\text{ lb}$

$\Sigma F_y = 0 \qquad E_y = 500\text{ lb} - A_y = 1750\text{ lb}$

(b) USE UPPER (SEE FIGURE BELOW) OR LOWER FBD TO FIND STRESS RESULTANTS N, V, AND M AT H



$\Sigma F_x = 0 \qquad V = E_x + 500\text{ lb} = 500\text{ lb}$

$\Sigma F_y = 0 \qquad N = E_y = 1750\text{ lb}$

$\Sigma M_H = 0$

$M = -0.6\text{ ft}(500\text{ lb}) - E_x 1.4\text{ ft} + E_y 0.5\text{ ft} = 575\text{ lb-ft}$

Problem 1.3-26

(a) STATICS

$$\Sigma F_x = 0 \qquad A_x = \frac{4}{5}(400\text{ N}) = 320\text{ N} \qquad \boxed{A_x = 320\text{ N}}$$

Use left hand FBD (cut through pin just left of C)

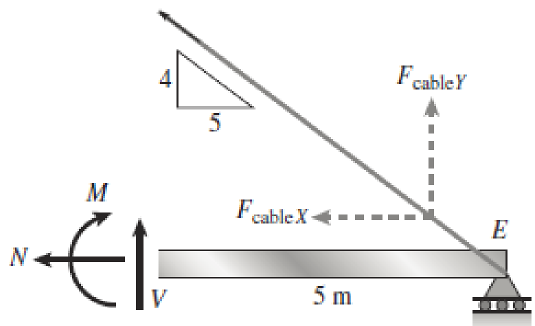
$$\Sigma M_C = 0 \qquad A_y = \frac{1}{7\text{ m}}\left[\left[\frac{-3}{5}(400\text{ N}) - \frac{4}{5}(400\text{ N})\right](3\text{ m})\right] = -240\text{ N} \qquad \boxed{A_y = -240\text{ N}}$$

Use entire FBD $\qquad \Sigma M_C = 0 \qquad E_y = \frac{1}{5\text{ m}}\left[A_y(7\text{ m}) + \left(\frac{3}{5}400\text{ N}\right)(3\text{ m})\right] = -192\text{ N} \qquad \boxed{E_y = -192\text{ N}}$

$$\Sigma F_y = 0 \qquad C_y = -A_y - E_y - \frac{3}{5}(400\text{ N}) = 192\text{ N} \qquad \boxed{C_y = 192\text{ N}}$$

(b) N, V, AND M JUST RIGHT OF C; USE RIGHT HAND FBD

$$F_{\text{cable}X} = 400\text{ N}\left(\frac{5}{\sqrt{4^2+5^2}}\right) = 312.348\text{ N}$$



$$F_{\text{cable}Y} = \frac{4}{5}F_{\text{cable}X} = 249.878\text{ N}$$

$$\Sigma F_x = 0 \qquad \boxed{N_x = -F_{\text{cable}X} = -312\text{ N}}$$

$$\Sigma F_y = 0 \qquad \boxed{V = -F_{\text{cable}Y} - E_y = -57.9\text{ N}}$$

$$\Sigma M_C = 0 \qquad M = \left(F_{\text{cable}Y} + E_y\right)(5\text{ m}) = \boxed{289\text{ N}\cdot\text{m}}$$

(c) RESULTANT FORCE IN PIN JUST LEFT OF C; USE LEFT HAND FBD $\qquad A_x = 320\text{ N}$

$$F_{Cx} = -A_x + \left(\frac{4}{5} - \frac{3}{5}\right)400\text{ N} = -240\text{ N} \qquad F_{Cy} = -A_y - \left(\frac{3}{5} + \frac{4}{5}\right)400\text{ N} = -320\text{ N}$$

$$\boxed{\text{Res}_C = \sqrt{F_{Cx}^{\ 2} + F_{Cy}^{\ 2}} = 400\text{ N}}$$

Problem 1.3-27

(a) STATICS $W = 150$ lb

$$\Sigma M_A = 0 \qquad B_x(4) + W\left(\frac{2\sqrt{3}}{2}\right) = 0 \text{ solve, } B_x = -\frac{75\sqrt{3}}{2}$$

$$\text{so} \qquad B_x = -\frac{75\sqrt{3}}{2} = -64.952$$

$$\Sigma F_x = 0 \qquad -A\sin(30°) + B_x + T\cos(30°) + T\cos\left(\arctan\left(\frac{7}{2\sqrt{3}}\right)\right) = 0$$

$$\Sigma F_y = 0 \qquad A\cos(30°) + T\sin(30°) + T\sin\left(\arctan\left(\frac{7}{2\sqrt{3}}\right)\right) = W$$

$$\begin{pmatrix} A \\ T \end{pmatrix} = \begin{pmatrix} -\sin(30°) & \cos(30°) + \cos\left(\arctan\left(\frac{7}{2\sqrt{3}}\right)\right) \\ \cos(30°) & \sin(30°) + \sin\left(\arctan\left(\frac{7}{2\sqrt{3}}\right)\right) \end{pmatrix}^{-1} \begin{pmatrix} -B_x \\ W \end{pmatrix} \qquad \begin{pmatrix} A \\ T \end{pmatrix} = \begin{pmatrix} 57.713 \\ 71.634 \end{pmatrix} \text{ lb}$$

SUPPORT REACTIONS

$\boxed{B_x = -65}$ $\boxed{A = 57.7}$ Units = lbs

$A_x = -A\sin(30°) = -28.9$ lb $\qquad A_y = A\cos(30°) = 50$ lb

$\sqrt{A_x^2 + A_y^2} = 57.713$

(b) CABLE FORCE IS T (LBS) FROM ABOVE SOLUTION

$\boxed{T = 71.6 \text{ lb}}$

Problem 1.3-28

(a) STATICS

RIGHT-HAND FBD

$$\Sigma M_{pin} = 0 \qquad E_y = \frac{1}{6\text{ m}}\left[\frac{1}{2}(3\text{ kN/m})\,4\text{ m}\left(\frac{1}{3}4\text{ m}\right)\right] = 1.333\text{ kN} \qquad \boxed{E_y = 1.333\text{ kN}}$$

ENTIRE FBD

$$\Sigma M_A = 0 \quad C_y = \frac{1}{6\text{ m}}\left[-E_y 12\text{ m} + (16\text{ kN})4\text{ m} + (1.5\text{ kN/m})\,6\text{ m}(3\text{ m}) - \frac{1}{2}(3\text{ kN/m})\,4\text{ m}\left(\frac{2}{3}4\text{ m}\right)\right] = 9.833\text{ kN}$$

$$\boxed{C_y = 9.83\text{ kN}}$$

$$\Sigma F_y = 0 \qquad A_y = -C_y - E_y + (1.5\text{ kN/m})\,6\text{ m} = -2.167\text{ kN} \qquad \boxed{A_y = -2.17\text{ kN}}$$

$$\Sigma F_x = 0 \qquad A_x = -16\text{ kN} + \frac{1}{2}(3\text{ kN/m})\;4\text{ m} = -10\text{ kN} \qquad \boxed{A_x = -10\text{ kN}}$$

(b) RESULTANT FORCE IN PIN; USE EITHER RIGHT HAND OR LEFT HAND FBD (CUT THROUGH PIN EXPOSING PIN FORCES F_{Dx} AND F_{Dy}) THEN SUM FORCES IN x AND y DIRECTIONS FOR EITHER FBD

LHFB:

$$F_{Dx} = -16\text{ kN} - A_x = -6\text{ kN}$$

$$F_{Dy} = -A_y + (1.5\text{ kN/m})\,6\text{ m} = 11.167\text{ kN}$$

$$\text{Resultant}_D = \sqrt{F_{Dx}^2 + F_{Dy}^2} = 12.68\text{ kN}$$

RHFB:

$$F_{Dx} = \frac{1}{2}(3\text{ kN/m})\,4\text{ m} = 6\text{ kN}$$

$$F_{Dy} = -C_y - E_y = -11.167\text{ kN}$$

$$\boxed{\text{Resultant}_D = 12.68\text{ kN}}$$

Problem 1.3-29

(a) STATICS $\quad P_1 = 50 \text{ lb} \quad P_2 = 40 \text{ lb}$

$\Sigma F_x = 0 \quad O_x = -P_1\cos(15°) = -48.3 \text{ lb} \qquad \Sigma F_y = 0 \quad O_y = P_2 = 40 \text{ lb}$

$\Sigma F_z = 0 \quad O_z = P_1\sin(15°) = 12.94 \text{ lb}$

$\Sigma M_x = 0 \quad M_{Ox} = P_2 6 \text{ in.} + P_1\sin(15°)(7 \text{ in.}) = 331 \text{ lb-in.}$

$\Sigma M_y = 0 \quad M_{Oy} = P_1\sin(15°)(8 \text{ in.} \sin(15°)) + P_1\cos(15°)(6 \text{ in.} + 8 \text{ in.} \cos(15°))$

$M_{Oy} = 690 \text{ lb-in.}$

$\Sigma M_z = 0 \quad M_{Oz} = -P_1\cos(15°)(7 \text{ in.}) = -338 \text{ lb-in.}$

(b) INTERNAL STRESS RESULTANTS AT MIDPOINT OF OA

$N = -O_y = -40 \text{ lb}$

$V_x = -O_x = 48.3 \text{ lb} \qquad V_z = -O_z = -12.94 \text{ lb} \qquad V = \sqrt{V_x^2 + V_z^2} = 50 \text{ lb}$

$T = -M_{Oy} = -690 \text{ lb-in.}$

$M_x = -M_{Ox} = -330.59 \text{ lb-in.} \qquad M_z = -M_{Oz} = 338.07 \text{ lb-in.} \qquad M = \sqrt{M_x^2 + M_z^2} = 473 \text{ lb-in.}$

Problem 1.3-30

FORCES

$P_x = 60\text{ N} \qquad P_z = -45\text{ N} \qquad M_y = 120\text{ N}\cdot\text{m} \qquad q_0 = 75\text{ N/m}$

$$F_C = \begin{pmatrix} P_x \\ 0 \\ P_z \end{pmatrix} = \begin{pmatrix} 60 \\ 0 \\ -45 \end{pmatrix}\text{N} \qquad R_A = \begin{pmatrix} 0 \\ A_y \\ A_z \end{pmatrix}$$

VECTOR ALONG MEMBER CD

$$r_{EC} = \begin{bmatrix} 1.5 - 2.5 \\ 2 - 0 \\ 0 - (-0.5) \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 0.5 \end{bmatrix} \qquad |r_{EC}| = 2.291 \qquad e_{EC} = \frac{r_{EC}}{|r_{EC}|} = \begin{pmatrix} -0.436 \\ 0.873 \\ 0.218 \end{pmatrix}$$

(a) STATICS (FORCE AND MOMENT EQUILIBRIUM)

$$\Sigma F = 0 \qquad \begin{pmatrix} 0 \\ A_y \\ A_z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ R_T \end{pmatrix} + \begin{pmatrix} P_x \\ 0 \\ P_z \end{pmatrix} + \begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} = 0 \qquad \text{resultant of triangular load:} \quad R_T = \frac{1}{2}q_0(2\text{ m}) = 75\text{ N}$$

$$\text{where} \qquad \begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} = D\,e_{EC}$$

SOLVING ABOVE THREE EQUATIONS:

$$\Sigma F_x = 0 \qquad D_x = -P_x \text{ so} \qquad D = \frac{-P_x}{e_{EC_1}} \qquad D = 137.477\text{ N} \qquad \boxed{D_x = -60\text{ N}}$$

$$\Sigma F_y = 0 \qquad D_y = e_{EC_2}D \qquad \boxed{D_y = 120\text{ N}} \qquad \boxed{A_y = -D_y = -120\text{ N}}$$

$$\Sigma F_z = 0 \qquad D_z = e_{EC_3}D \qquad \boxed{D_z = 30\text{ N}} \qquad \sqrt{D_x^2 + D_y^2 + D_z^2} = 137.477\text{ N}$$

$$\text{so} \quad A_z = -D_z - R_T - P_z \qquad \boxed{A_z = -60\text{ N}}$$

$$\Sigma M_A = 0$$

$$\begin{pmatrix} M_{Ax} \\ M_{Ay} \\ M_{Az} \end{pmatrix} + r_{AE} \times D + r_{AC} \times \begin{pmatrix} P_x \\ 0 \\ P_z \end{pmatrix} + \begin{pmatrix} 0 \\ M_y \\ 0 \end{pmatrix} + r_{cg} \times \begin{pmatrix} 0 \\ 0 \\ R_T \end{pmatrix} = 0$$

$$r_{AE} = \begin{pmatrix} 2.5 - 0 \\ 0 - 0 \\ -0.5 - 0 \end{pmatrix}\text{m} \qquad D = \begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} \qquad D = \begin{pmatrix} -60 \\ 120 \\ 30 \end{pmatrix}\text{N} \qquad |D| = 137.477\text{ N} \qquad r_{AE} \times D = \begin{pmatrix} 60 \\ -45 \\ 300 \end{pmatrix}\text{N}\cdot\text{m}$$

$$r_{AC} = \begin{pmatrix} 1.5 - 0 \\ 2 - 0 \\ 0 - 0 \end{pmatrix}\text{m} \qquad r_{AC} \times \begin{pmatrix} P_x \\ 0 \\ P_z \end{pmatrix} = \begin{pmatrix} -90 \\ 67.5 \\ -120 \end{pmatrix}\text{J} \qquad r_{cg} = \begin{pmatrix} 0 \\ \frac{2}{3}(2\text{ m}) \\ 0 \end{pmatrix} \qquad r_{cg} \times \begin{pmatrix} 0 \\ 0 \\ R_T \end{pmatrix} = \begin{pmatrix} 100 \\ 0 \\ 0 \end{pmatrix}\text{N}\cdot\text{m}$$

$$\begin{pmatrix} M_{Ax} \\ M_{Ay} \\ M_{Az} \end{pmatrix} = -\left[r_{AE} \times D + r_{AC} \times \begin{pmatrix} P_x \\ 0 \\ P_z \end{pmatrix} + \begin{pmatrix} 0 \\ M_y \\ 0 \end{pmatrix} + r_{cg} \times \begin{pmatrix} 0 \\ 0 \\ R_T \end{pmatrix} \right] = \begin{pmatrix} -70 \\ -142.5 \\ -180 \end{pmatrix}\text{N}\cdot\text{m} \qquad \boxed{\begin{pmatrix} M_{Ax} \\ M_{Ay} \\ M_{Az} \end{pmatrix} = \begin{pmatrix} -70 \\ -142.5 \\ -80 \end{pmatrix}\text{N}\cdot\text{m}}$$

(b) RESULTANTS AT MID-HEIGHT OF AB (SEE FBD IN FIGURE BELOW)

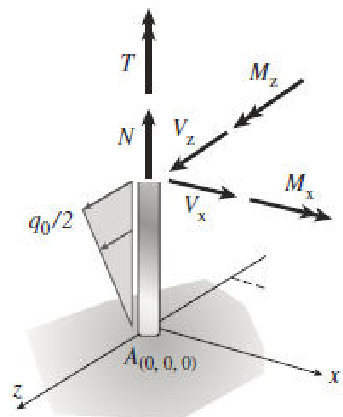
$\boxed{N = -A_y = 120\ \text{N}}$ $\quad V_x = -D_x - P_x = 0\ \text{N} \quad V_z = -A_z - \dfrac{1}{2}\dfrac{q_0}{2}(2\ \text{m})/2 = 41.25\ \text{N}$ $\quad \boxed{V = V_z = 41.3\ \text{N}}$

$\boxed{T = -M_{Ay} = 142.5\ \text{N}\cdot\text{m}}$ $\quad M_x = -M_{Ax} + A_z(1\ \text{m}) + \dfrac{1}{2}\dfrac{q_0}{2}1\ \text{m}\left(\dfrac{1}{3}1\ \text{m}\right) = 16.25\ \text{N}\cdot\text{m}$

$$M_z = -M_{Az} = 180\ \text{N}\cdot\text{m}$$

$$M_{\text{resultant}} = \sqrt{M_x^2 + M_z^2} = 180.732\ \text{N}\cdot\text{m}$$

$$\boxed{M_{\text{resultant}} = 180.7\ \text{N}\cdot\text{m}}$$



Problem 1.3-31

POSITION AND UNIT VECTORS

$$r_{AB} = \begin{pmatrix} 10 \\ 0 \\ 0 \end{pmatrix} \quad r_{AP} = \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} \quad r_{AC} = \begin{pmatrix} 10 \\ 4 \\ -4 \end{pmatrix} \quad r_{CD} = \begin{bmatrix} 0-10 \\ 10-4 \\ -20-(-4) \end{bmatrix} = \begin{pmatrix} -10 \\ 6 \\ -16 \end{pmatrix} \quad e_{CD} = \frac{r_{CD}}{|r_{CD}|} = \begin{pmatrix} -0.505 \\ 0.303 \\ -0.808 \end{pmatrix}$$

APPLIED FORCE AND MOMENT

$$r_{CE} = \begin{bmatrix} 0-10 \\ 8-4 \\ 10-(-4) \end{bmatrix} = \begin{pmatrix} -10 \\ 4 \\ 14 \end{pmatrix} \quad e_{CE} = \frac{r_{CE}}{|r_{CE}|} = \begin{pmatrix} -0.566 \\ 0.226 \\ 0.793 \end{pmatrix}$$

$$P_y = -50 \text{ lb} \qquad M_x = -20 \text{ lb-in.}$$

STATICS FORCE AND MOMENT EQUILIBRIUM

First sum moment about point A

$$\Sigma M_A = 0$$

$$M_A = \begin{pmatrix} 0 \\ 0 \\ M_{Az} \end{pmatrix} + r_{AP} \times \begin{pmatrix} 0 \\ P_y \\ 0 \end{pmatrix} + \begin{pmatrix} M_x \\ 0 \\ 0 \end{pmatrix} + r_{AC} \times (T_D\, e_{CD} + T_E\, e_{CE}) = \begin{pmatrix} -2.0203\, T_D + 4.0762\, T_E - 20.0 \\ 10.102\, T_D + -5.6614\, T_E \\ M_{Az} + 5.0508\, T_D + 4.5291\, T_E - 250.0 \end{pmatrix}$$

Solve moment equilibrium equations for moments about x and y axes to get cable tension forces

$$\begin{pmatrix} T_D \\ T_E \end{pmatrix} = \begin{pmatrix} -2.0203 & 4.0762 \\ 10.102 & -5.6614 \end{pmatrix}^{-1} \begin{pmatrix} 20 \\ 0 \end{pmatrix} = \begin{pmatrix} 3.81 \\ 6.79 \end{pmatrix} \text{lb} \qquad \text{(b)}$$

Next, solve moment equilibrium equation about z axis now that cable forces are known

$$M_{Az} = -(5.0508\, T_D + 4.5291\, T_E - 250.0) = 200 \text{ lb-in.} \qquad \text{(a)}$$

Finally, use force equilibrium to find reaction forces at point A

$$\Sigma F = 0 \qquad \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = -\begin{pmatrix} 0 \\ P_y \\ 0 \end{pmatrix} - (T_D\, e_{CD} + T_E\, e_{CE}) = \begin{pmatrix} 5.77 \\ 47.31 \\ -2.31 \end{pmatrix} \text{lb}$$

Problem 1.3-32

Find member lengths $L_{QS} = 2\cdot(3.65m) = 7.3\,m$ $L_{RS} = \sqrt{(2.44m)^2 + (2.44m - 1.22m)^2} = 2.728\,m$ $L_{PQ} = L_{RS}$

Assume that soccer goal is supported only at points C, H and D (see reaction force components at each loaction in fig.)

Statics - sum moment about each axis and forces in each axis direction $F = 200N$

$\Sigma M_x = 0$ to find reaction component H_y

Find moments about x due to for component F_y and also for distributed weight of each frame component

$$M_{xGP} = \frac{(1.22m)^2}{2}\cdot\left(29\frac{N}{m}\right) \quad M_{xBR} = M_{xGP} \quad M_{xDQ} = \frac{(2.44m)^2}{2}\cdot\left(29\frac{N}{m}\right) \quad M_{xCS} = M_{xDQ}$$

$$M_{xRS} = L_{RS}\cdot\left(29\frac{N}{m}\right)\cdot\left(1.22m + \frac{1.22m}{2}\right) \quad M_{xPQ} = M_{xRS} \quad M_{xQS} = L_{QS}\cdot\left(29\frac{N}{m}\right)\cdot(2.44m)$$

$$H_z = \frac{1}{2.44m}\cdot\left[\frac{4}{5}\cdot F\cdot\left(\frac{2.44m}{2}\right) + 2\cdot M_{xGP} + 2\cdot M_{xDQ} + 2\cdot M_{xPQ} + M_{xQS}\right] = 498.818\,N \quad \boxed{H_z = 499\,N}$$

$\Sigma M_y = 0$ to find reaction force D_z

$$M_{yGD} = 2.44m\cdot\left(73\frac{N}{m}\right)\cdot L_{QS} \quad M_{yGP} = 1.22m\cdot\left(29\frac{N}{m}\right)\cdot L_{QS} \quad M_{yDQ} = 2.44m\cdot\left(29\frac{N}{m}\right)\cdot L_{QS}$$

$$M_{yPQ} = L_{RS}\cdot\left(29\frac{N}{m}\right)\cdot L_{QS} \quad M_{yBG} = L_{QS}\cdot\left(73\frac{N}{m}\right)\cdot\frac{L_{QS}}{2} \quad M_{yQS} = L_{QS}\cdot\left(29\frac{N}{m}\right)\cdot\frac{L_{QS}}{2}$$

$$D_z = \frac{1}{L_{QS}}\cdot\left[M_{yGD} + M_{yGP} + M_{yDQ} + M_{yPQ} + M_{yBG} + M_{yQS} - H_z\cdot\frac{L_{QS}}{2} - \frac{3}{5}\cdot F\cdot\left(\frac{2.44m}{2}\right)\right] = 466.208\,N \quad \boxed{D_z = 466\,N}$$

$\Sigma M_z = 0$ to find reaction force H_y

$$H_y = \frac{1}{3.65m}\cdot\left(\frac{4}{5}\cdot F\cdot L_{QS}\right) = 320\,N \quad \boxed{H_y = 320\,N}$$

$\Sigma F_x = 0$ to find reaction force C_x $\boxed{C_x = \frac{3}{5}\cdot F = 120\,N}$

$\Sigma F_y = 0$ to find reaction force C_y $C_y = -H_y + \frac{4}{5}\cdot F = -160\,N$ $\boxed{C_y = -160\,N}$

$\Sigma F_z = 0$ to find reaction force C_z

$$C_z = -D_z - H_z + \left(29\frac{N}{m}\right)\cdot\left[2\cdot(1.22m) + 2\cdot(2.44m) + 2\cdot L_{RS} + L_{QS}\right] + \left(73\frac{N}{m}\right)\cdot\left[2\cdot(2.44m) + L_{QS}\right] =$$

506.318 $\boxed{C_z = 506\,N}$

Problem 1.3-33

$\alpha = \arcsin\left(\frac{10}{50}\right) = 11.537°$ Analysis pertains to this position of exerciser only

STATICS UFBD (CUT AT AXIAL AND MOMENT RELEASES JUST ABOVE B)

Inclined vertical component of reaction at $C = 0$ (due to axial release)

Sum moments about moment release to get inclined normal reaction at C

$$C = \frac{20\,\text{lb}(34\text{ in.} + 16\text{ in.})}{34\text{ in.}} = 29.412\text{ lb} \qquad \boxed{C_x = C\cos(\alpha) = 28.8\text{ lb}}$$

$$\boxed{C_y = C\sin(\alpha) = 5.88\text{ lb}} \qquad \sqrt{C_x^2 + C_y^2} = 29.412\text{ lb}$$

STATICS LFBD (CUT THROUGH AXIAL AND MOMENT RELEASES)

Sum moments to find reaction A_y $\boxed{A_y = \dfrac{175\text{ lb}(16\text{ in.})}{(34\text{ in.} + 16\text{ in.})\cos(\alpha)} = 57.2\text{ lb}}$

STATICS SUM FORCES FOR ENTIRE FBD TO FIND REACTION AT B

Sum forces in x-direction: $B_x = C_x + 175\,\text{lb}(\sin(\alpha)) - 20\,\text{lb}(\cos(\alpha)) = 44.2\,\text{lb}$ < acts leftward

Sum forces in y-direction: $B_y = -A_y - C_y + 175\text{ lb}(\cos(\alpha)) + 20\text{ lb}(\sin(\alpha)) = 112.4\text{ lb}$

$$\boxed{B_x = 44.2\text{ lb}} \qquad \boxed{B_y = 112.4\text{ lb}}$$

Resultant reaction force at B: $B = \sqrt{B_x^2 + B_y^2} = 120.8\text{ lb}$

Problem 1.3-34

(a) Reactions: Sum moments about rear hub to find vertical reaction at front hub (Fig. 1)

$$\Sigma M_B = 0$$

$$V_F = \frac{1}{1130}[670(241) - 90(\cos(5°))254 + 200\cos(15°)660 + 2(45)\cos(30°)1021 + 2(45)\sin(30°)752]$$

$$V_F = 335.945 \text{ N}$$

Sum forces to get force components at rear hub

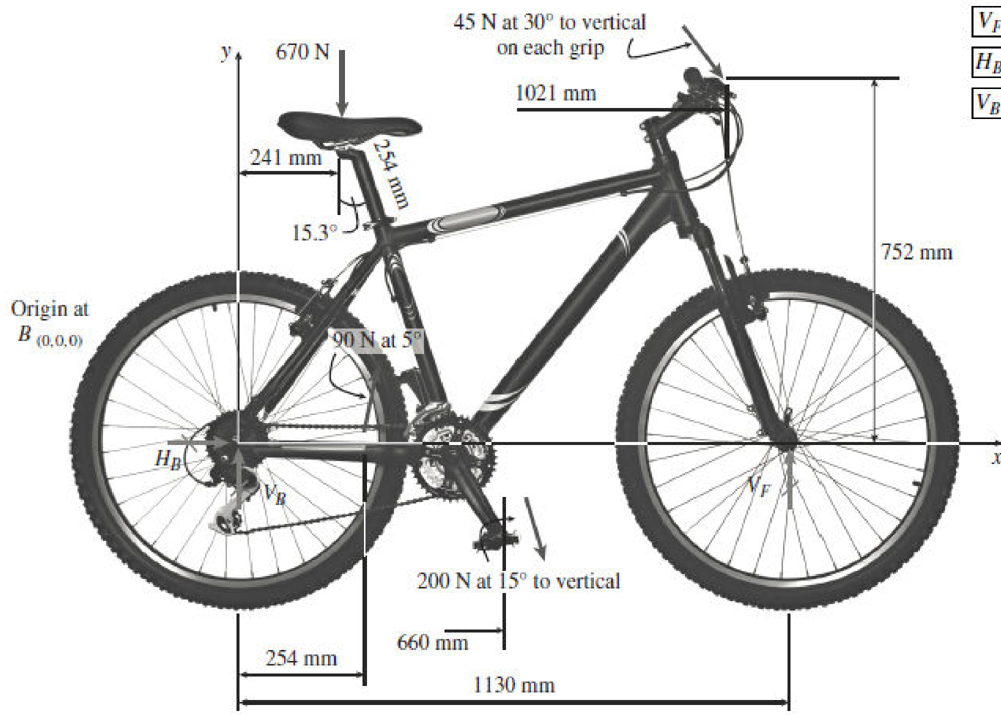
$\Sigma F_{\text{vert}} = 0 \quad V_B = 670 - 90\cos(5°) + 200\cos(15°) + 2(45)\cos(30°) - V_F = 515.525 \text{ N}$

$\Sigma F_{\text{horiz}} = 0 \quad H_B = -90\sin(5°) - 200\sin(15°) - 2(45)\sin(30°) = -104.608 \text{ N}$

$\boxed{V_F = 336 \text{ N}}$

$\boxed{H_B = -104.6 \text{ N}}$

$\boxed{V_B = 516 \text{ N}}$



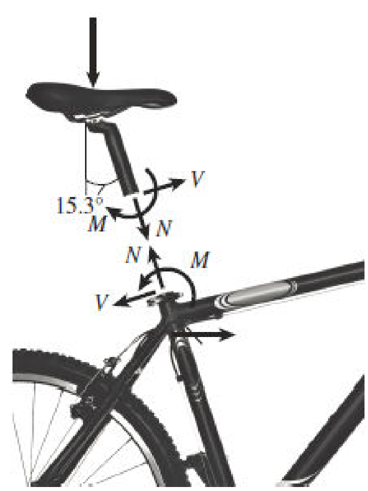
(b) Stress resultants N, V, and M in seat post (Fig. 2)

Seat post resultants (Fig. 2)

$N = -670\cos(15.3°) = -646.253 \text{ N}$ $\quad \boxed{N = -646 \text{ N}}$

$V = 670\sin(15.3°) = 176.795 \text{ N}$ $\quad \boxed{V = 176.8 \text{ N}}$

$M = 670\sin(15.3°)\,254 = 44{,}905.916 \text{ N·mm}$ $\quad \boxed{M = 44.9 \text{ N·m}}$



Problem 1.4-1

PART (a)

$P_1 = 1700$ lb $\quad d_{AB} = 1.25$ in. $\quad t_{AB} = 0.5$ in.

$d_{BC} = 2.25$ in. $\quad t_{BC} = 0.375$ in.

$$A_{AB} = \frac{\pi[\,d_{AB}^2 - (d_{AB} - 2t_{AB})^2]}{4}$$

$$A_{AB} = 1.178 \text{ in.}^2 \qquad \sigma_{AB} = \frac{P_1}{A_{AB}}$$

$\sigma_{AB} = 1443$ psi compression

PART (b)

$$A_{BC} = \frac{\pi[d_{BC}^2 - (d_{BC} - 2t_{BC})^2]}{4}$$

$$A_{BC} = 2.209 \text{ in.}^2 \qquad P_2 = \sigma_{AB}A_{BC} - P_1$$

$$P_2 = 1488 \text{ lbs} \quad \longleftarrow$$

CHECK: $\quad \dfrac{P_1 + P_2}{A_{BC}} = 1443$ psi

PART (c)

$$P_2 = 2260 \quad \frac{P_1 + P_2}{\sigma_{AB}} = A_{BC}$$

$$\frac{P_1 + P_2}{\sigma_{AB}} = 2.744$$

$$(d_{BC} - 2t_{BC})^2 = d_{BC}^2 - \frac{4}{\pi}\left(\frac{P_1 + P_2}{\sigma_{AB}}\right)$$

$$d_{BC} - 2t_{BC} = \sqrt{d_{BC}^2 - \frac{4}{\pi}\left(\frac{P_1 + P_2}{\sigma_{AB}}\right)}$$

$$t_{BC} = \frac{d_{BC} - \sqrt{d_{BC}^2 - \frac{4}{\pi}\left(\frac{P_1 + P_2}{\sigma_{AB}}\right)}}{2}$$

$$t_{BC} = 0.499 \text{ in.} \quad \longleftarrow$$

Problem 1.4-2

$P_A = 10kN \qquad P_B = 20kN$

$d_1 = 50mm \quad d_2 = 60mm \quad d_3 = 55mm \quad d_4 = 65mm$

$L_2 = 400mm \quad L_1 = 300mm \qquad \delta_1 = 3.29mm \qquad \delta_2 = 1.25mm$

$$A_1 = \frac{1}{4}\cdot\pi\cdot\left(d_2^{\,2} - d_1^{\,2}\right) = 863.938\cdot mm^2$$

$$A_2 = \frac{1}{4}\cdot\pi\cdot\left(d_4^{\,2} - d_3^{\,2}\right) = 942.478\cdot mm^2$$

a) axial normal stresses

$$\sigma_1 = \frac{P_B}{A_1} = 23.15\cdot MPa \qquad \sigma_2 = \frac{P_B - P_A}{A_2} = 10.61\cdot MPa$$

b) axial normal strains

$$\varepsilon_1 = \frac{\delta_1 - \delta_2}{L_1} = 6.8\times 10^{-3} \qquad \varepsilon_2 = \frac{\delta_2}{L_2} = 3.125\times 10^{-3}$$

Problem 1.4-3

$$A = \pi\cdot\left[(1.5\text{in})^2 - \left(1.5\text{in} - \frac{3}{4}\text{in}\right)^2\right] = 5.301\cdot\text{in}^2 \qquad \sigma_{max} = \frac{3\text{kip}}{A} = 0.566\cdot\text{ksi}$$

Problem 1.4-4

$P = 70\text{ N} \qquad A_e = 1.075\text{ mm}^2$

$L = 460\text{ mm} \qquad \delta = 0.214\text{ mm}$

Statics: sum moments about A to get $T = 2P$

$$\sigma = \frac{T}{A_e} \qquad \sigma = 103.2\text{ MPa} \qquad \longleftarrow$$

$$\varepsilon = \frac{\delta}{L} \qquad \varepsilon = 4.65 \times 10^{-4} \qquad \longleftarrow$$

$$E = \frac{\sigma}{\varepsilon} = 1.4 \times 10^5\text{ MPa}$$

NOTE: (E for cables is approximately 140 GPa)

Problem 1.4-5

$T = 45 \text{ lbs} \qquad A_{pad} = 0.625 \text{ in.}^2$

$A_{cable} = 0.00167 \text{ in.}^2$

(a) CANTILEVER BRAKES—BRAKING FORCE R_B and PAD PRESSURE

STATICS SUM FORCES AT D TO GET $T_{DCv} = T/2$

$$\sum M_A = 0$$

$$R_B(1) = T_{DCh}(3) + T_{DCv}(1)\text{s}$$

$$T_{DCh} = T_{DCv} \qquad T_{DCh} = T/2$$

$$R_B = 2T \qquad R_B = 90 \text{ lbs} \quad \longleftarrow$$

so $R_B = 2T$ versus $4.25T$ for V-brakes (next)

$$\sigma_{pad} = \frac{R_B}{A_{pad}} \qquad \sigma_{pad} = 144 \text{ psi} \quad \longleftarrow \qquad \frac{4.25}{2} = 2.125$$

$$\sigma_{cable} = \frac{T}{A_{cable}} \qquad \sigma_{cable} = 26{,}946 \text{ psi} \quad \longleftarrow$$

(same for V-brakes (below))

(b) V-BRAKES—BRAKING FORCE R_B AND PAD PRESSURE

$$\sum M_A = 0 \qquad R_B = 4.25T \qquad R_B = 191.3 \text{ lbs} \quad \longleftarrow$$

$$\sigma_{pad} = \frac{R_B}{A_{pad}} \qquad \sigma_{pad} = 306 \text{ psi} \quad \longleftarrow$$

Problem 1.4-6

$L = 420\text{ mm} \qquad d_2 = 60\text{ mm} \qquad d_1 = 35\text{ mm} \qquad \varepsilon_h = 470\left(10^{-6}\right) \qquad \sigma_a = 48\text{ MPa}$

PART (a)

$$A_s = \frac{\pi}{4}{d_2}^2 = 2.827 \times 10^{-3}\text{m}^2 \qquad A_h = \frac{\pi}{4}\left({d_2}^2 - {d_1}^2\right) = 1.865 \times 10^{-3}\text{m}^2$$

$$\varepsilon_s = \frac{A_h}{A_s}\varepsilon_h = 3.101 \times 10^{-4}$$

PART (b)

$$\delta = \varepsilon_h\frac{L}{3} + \varepsilon_s\left(\frac{2L}{3}\right) = 0.1526\text{ mm} \qquad \varepsilon_h\frac{L}{3} = 0.066\text{ mm} \qquad \varepsilon_s\left(\frac{2L}{3}\right) = 0.087\text{ mm}$$

PART (c)

$$P_{\text{maxh}} = \sigma_a A_h = 89.535\text{ kN} \qquad P_{\text{maxs}} = \sigma_a A_s = 135.717\text{ kN} \qquad < \text{lesser value controls}$$

$$\boxed{P_{\max} = P_{\text{maxh}} = 89.5\text{ kN}}$$

Problem 1.4-7

$P = 3500 \text{ kips}$

$$A = (24 + 20)(20 + 16 + 8) - \left(\frac{1}{2}8^2\right) - 20^2 - \frac{\pi}{4}10^2$$

$A = 1425.46 \text{ in.}^2$

(a) Average compressive stress

$$\sigma_c = \frac{P}{A} \qquad \boxed{\sigma_c = 2.46 \text{ ksi}}$$

(b) Centroid

$$x_c = \frac{(24 + 20)^2 \dfrac{(24 + 20)}{2} - \left(20^2\right)(24 + 10) - \dfrac{1}{2}8^2\left(\dfrac{8}{3}\right)}{A} - \frac{\left(\dfrac{\pi}{4}10^2\right)(8 + 5)}{A}$$

$\boxed{x_c = 19.56 \text{ in.}}$

$$y_c = \frac{(24 + 20)^2 \dfrac{(24 + 20)}{2} - \left(20^2\right)(24 + 10) - \dfrac{1}{2}8^2\left(\dfrac{8}{3}\right)}{A} - \frac{\left(\dfrac{\pi}{4}10^2\right)(8 + 5)}{A}$$

$\boxed{y_c = 19.56 \text{ in.}}$

^ x_c and y_c are the same as expected due to symmetry about a diagonal

Problem 1.4-8

$W = 130\text{ kN} \quad \alpha = 30° \quad A = 490\text{ mm}^2 \quad \sigma_a = 150\text{ MPa}$

PART (a)

$$\sigma_t = \frac{W\sin(\alpha)}{A} = 132.7\text{ MPa}$$

PART (b)

$$\alpha_{max} = \arcsin\left(\frac{\sigma_a A}{W}\right) = 34.4°$$

Problem 1.4-9

$d_1 = 30\left(10^{-3}\right)$ in. $\quad d_2 = 35\left(10^{-3}\right)$ in. $\quad A_1 = \dfrac{\pi}{4}d_1^2 = 7.069 \times 10^{-4}$ in.^{2}

$W = 28$ lb $\quad A_2 = \dfrac{\pi}{4}d_2^2 = 9.621 \times 10^{-4}$ in.^{2}

$\alpha = 22°$ $\quad \beta = 40°$

(a) Find normal stress in wires

$$T_2 = \frac{W}{\dfrac{\cos(\beta)}{\cos(\alpha)}\sin(\alpha) + \sin(\beta)} = 29.403 \text{ lb} \qquad \boxed{\sigma_2 = \frac{T_2}{A_2} = 30.6 \text{ ksi}}$$

$$T_1 = T_2\frac{\cos(\beta)}{\cos(\alpha)} = 24.293 \text{ lb} \qquad \boxed{\sigma_1 = \frac{T_1}{A_1} = 34.4 \text{ ksi}}$$

(b) Find new d_1 s.t. normal stresses in wires is the same

$$A_{1\,\text{new}} = \frac{T_1}{\sigma_2} = 7.949 \times 10^{-4} \text{ in.}^2 \qquad \boxed{d_{1\,\text{new}} = \sqrt{\frac{4}{\pi}A_{1\,\text{new}}} = 3.18 \times 10^{-2} \text{ in.}} \quad \text{or} \quad 31.8 \text{ mils}$$

$$\sigma_{1\text{new}} = \frac{T_1}{\dfrac{\pi}{4}{d_{1\text{new}}}^2} = 30.6 \text{ ksi}$$

(c) Now, to stabilize the camera for windy outdoor conditions, a third wire is added (see figure b); assume the 3 wires meet at a common point (coordinates = (0, 0, 0) above the camera at the instant shown in figure b); wire 1 is attached to a support at coordinates (75', 48', 70'); wire 2 is supported at (−70', 55', 80'); and wire 3 is supported at (−10', −85', 75'); assume that all three wires have diameter of 30 mils. Find tensile stresses in wires 1 to 3.

$d = 30\left(10^{-3}\right)$ in. $\quad A\dfrac{\pi}{4}d^2 = 7.069 \times 10^{-4}$ in.^{2}

Position vectors from camera to each support $\quad r_1 = \begin{pmatrix} 75 \\ 48 \\ 70 \end{pmatrix}$ ft $\quad r_2 = \begin{pmatrix} -70 \\ 55 \\ 80 \end{pmatrix}$ ft $\quad r_3 = \begin{pmatrix} -10 \\ -85 \\ 75 \end{pmatrix}$ ft $\quad W = 28\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ lb

$L_1 = |r_1| = 113.265 \qquad L_2 = |r_2| = 119.687 \qquad L_3 = |r_3| = 113.798$

Unit vectors along wires 1 to 3 $\quad e_1 = \dfrac{r_1}{|r_1|} = \begin{pmatrix} 0.662 \\ 0.424 \\ 0.618 \end{pmatrix} \quad e_2 = \dfrac{r_2}{|r_2|} = \begin{pmatrix} -0.585 \\ 0.46 \\ 0.668 \end{pmatrix} \quad e_3 = \dfrac{r_3}{|r_3|} = \begin{pmatrix} -0.088 \\ -0.747 \\ 0.659 \end{pmatrix}$

$T_1 = F_1 e_1 \qquad T_2 = F_2 e_2 \qquad T_3 = F_3 e_3 \qquad i = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad j = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad k = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

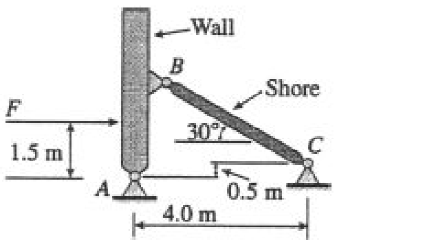
Equilibrium of forces $\quad T_1 + T_2 + T_3 = W$

$$T^{\langle 1\rangle} = e_1 \qquad T^{\langle 2\rangle} = e_2 \qquad T^{\langle 3\rangle} = e_3 \qquad T = \begin{pmatrix} 0.662 & -0.585 & -0.088 \\ 0.424 & 0.46 & -0.747 \\ 0.618 & 0.668 & 0.659 \end{pmatrix}$$

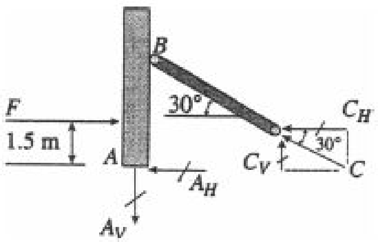
$$F = T^{-1}W = \begin{pmatrix} 13.854 \\ 13.277 \\ 16.028 \end{pmatrix} \text{ lb} \qquad \sigma_1 = \frac{F_1}{A} = 19.6 \text{ ksi} \qquad \sigma_2 = \frac{F_2}{A} = 18.78 \text{ ksi} \qquad \sigma_3 = \frac{F_3}{A} = 22.7 \text{ ksi}$$

$$\boxed{\sigma_1 = 19.6 \text{ ksi}} \qquad \boxed{\sigma_2 = 18.78 \text{ ksi}} \qquad \boxed{\sigma_3 = 22.7 \text{ ksi}}$$

Problem 1.4-10



FREE-BODY DIAGRAM OF WALL AND SHORE



C = compressive force in wood shore

C_H = horizontal component of C

C_V = vertical component of C

$C_H = C\cos 30°$

$C_V = C\sin 30°$

$F = 190\text{ kN}$

A = area of one shore

$$A = (150\text{ mm})(150\text{ mm})$$
$$= 22{,}500\text{ mm}^2$$
$$= 0.0225\text{ m}^2$$

SUMMATION OF MOMENTS ABOUT POINT A

$$\Sigma M_A = 0$$

$$-F(1.5\text{ m}) + C_V(4.0\text{ m}) + C_H(0.5\text{ m}) = 0$$

or

$$-(190\text{ kN})(1.5\text{ m}) + C(\sin 30°)(4.0\text{ m}) + C(\cos 30°)(0.5\text{ m}) = 0$$

$$\therefore C = 117.14\text{ kN}$$

COMPRESSIVE STRESS IN THE SHORES

$$\sigma_c = \frac{C}{A} = \frac{117.14\text{ kN}}{0.0225\text{ m}^2}$$
$$= 5.21\text{ MPa} \quad \leftarrow$$

Problem 1.4-11

$$W_c = 150 \text{ lb}$$
$$A_e = 0.017 \text{ in.}^2$$
$$W_T = 60$$
$$\delta = 0.01$$
$$d_c = 18$$
$$d_T = 14$$
$$H = 12$$
$$L = 16$$
$$L_c = \sqrt{L^2 + H^2} \qquad L_c = 20$$
$$\sum M_{\text{hinge}} = 0 \qquad 2T_vL = W_cd_c + W_Td_T$$
$$T_v = \frac{W_cd_c + W_Td_T}{2L} \qquad T_v = 110.625 \text{ lb}$$
$$T_h = \frac{L}{H}T_v \qquad T_h = 147.5$$

(a) $T = \sqrt{T_v^{\,2} + T_h^{\,2}} \quad T = 184.4 \text{ lb} \quad \leftarrow$

$\sigma_{\text{cable}} = \dfrac{T}{A_e} \quad \sigma_{\text{cable}} = 10.8 \text{ ksi} \quad \leftarrow$

(b) $\varepsilon_{\text{cable}} = \dfrac{\delta}{L_c} \quad \varepsilon_{\text{cable}} = 5 \times 10^{-4} \quad \leftarrow$

Problem 1.4-12

$M_c = 68$

$M_T = 27\ \text{kg} \qquad g = 9.81\ \text{m/s}^2$

$W_c = M_c g \qquad W_T = M_T g$

$W_c = 667.08 \qquad W_T = 264.87$

$N = \text{kg}\cdot\text{m/s}^2$

$A_e = 11.0\ \text{mm}^2 \qquad \delta = 0.25$

$d_c = 460 \qquad d_T = 350$

$H = 305 \qquad L = 406$

$L_c = \sqrt{L^2 + H^2} \qquad L_c = 507.8\ \text{mm}$

$\sum M_{\text{hinge}} = 0 \qquad 2T_v L = W_c d_c + W_T d_T$

$$T_v = \frac{W_c d_c + W_T d_T}{2L} \qquad T_v = 492.071\ \text{N}$$

$$T_h = \frac{L}{H} T_v \qquad T_h = 655.019\ \text{N}$$

(a) $T = \sqrt{T_v^2 + T_h^2} \qquad T = 819\ \text{N} \quad \longleftarrow$

$$\sigma_{\text{cable}} = \frac{T}{A_e} \qquad \sigma_{\text{cable}} = 74.5\ \text{MPa} \quad \longleftarrow$$

(b) $\varepsilon_{\text{cable}} = \dfrac{\delta}{L_c} \qquad \varepsilon_{\text{cable}} = 4.92 \times 10^{-4} \quad \longleftarrow$

Problem 1.4-13

CABLE LENGTHS (FT)

$L_1 = \sqrt{5^2 + 5^2 + 7^2} \quad L_1 = 9.95 \qquad L_2 = \sqrt{5^2 + 7^2 + 7^2} \qquad L_2 = 11.091 \qquad L_3 = \sqrt{7^2 + 7^2} \qquad L_3 = 9.899$

(a) SOLUTION FOR CABLE FORCES USING STATICS (THREE EQUATIONS, THREE UNKNOWNS); UNITS = lb, ft

$$r_{OQ} = \begin{pmatrix} 5 \\ 5 \\ 7 \end{pmatrix} \qquad r_{BQ} = \begin{pmatrix} -7 \\ 5 \\ 7 \end{pmatrix} \qquad r_{DQ} = \begin{pmatrix} 0 \\ -7 \\ 7 \end{pmatrix}$$

$$e_{OQ} = \frac{r_{OQ}}{|r_{OQ}|} = \begin{pmatrix} 0.503 \\ 0.503 \\ 0.704 \end{pmatrix} \qquad e_{BQ} = \frac{r_{BQ}}{|r_{BQ}|} = \begin{pmatrix} -0.631 \\ 0.451 \\ 0.631 \end{pmatrix} \qquad e_{DQ} = \frac{r_{DQ}}{|r_{DQ}|} = \begin{pmatrix} 0 \\ -0.707 \\ 0.707 \end{pmatrix}$$

$$W = 150\left(12^2 - 6^2\right)\frac{9}{12} = 12{,}150 \text{ lbs}$$

STATICS $\quad \Sigma F = 0 \quad T_1 e_{OQ} + T_2 e_{BQ} + T_3 e_{DQ} - \begin{pmatrix} 0 \\ 0 \\ W \end{pmatrix} = \begin{pmatrix} 0.50252\, T_1 - 0.63117\, T_2 \\ 0.50252\, T_1 + 0.45083\, T_2 - 0.70711\, T_3 \\ 0.70353\, T_1 + 0.63117\, T_2 + 0.70711\, T_3 - 12{,}150 \end{pmatrix}$

or in matrix form; solve simultaneous equations to get cable tension forces

$$\begin{pmatrix} T_1 \\ T_2 \\ T_3 \end{pmatrix} = \begin{pmatrix} e_{OQ_{1,1}} & e_{BQ_{1,1}} & e_{DQ_{1,1}} \\ e_{OQ_{2,1}} & e_{BQ_{2,1}} & e_{DQ_{2,1}} \\ e_{OQ_{3,1}} & e_{BQ_{3,1}} & e_{DQ_{3,1}} \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \\ W \end{pmatrix} = \begin{pmatrix} 5877 \\ 4679 \\ 7159 \end{pmatrix} \text{lb} \qquad \boxed{T = \begin{pmatrix} 5877 \\ 4679 \\ 7159 \end{pmatrix} \text{lb}}$$

(b) AVERAGE NORMAL STRESS IN EACH CABLE

$$i = 1 \ldots 3 \qquad \sigma_i = \frac{T_i}{A_e} \qquad \boxed{\sigma = \begin{pmatrix} 48975 \\ 38992 \\ 59658 \end{pmatrix} \text{psi}} \qquad A_e = 0.12 \text{ in.}^2$$

(c) ADD CONTINUOUS CABLE OQA

$$r_{OQ} = \begin{pmatrix} 5 \\ 5 \\ 7 \end{pmatrix} \qquad r_{AQ} = \begin{pmatrix} 5 \\ -7 \\ 7 \end{pmatrix} \qquad r_{BQ} = \begin{pmatrix} -7 \\ 5 \\ 7 \end{pmatrix} \qquad r_{DQ} = \begin{pmatrix} 0 \\ -7 \\ 7 \end{pmatrix} \qquad e_{AQ} = \frac{r_{AQ}}{|r_{AQ}|} = \begin{pmatrix} 0.451 \\ -0.631 \\ 0.631 \end{pmatrix}$$

$$e_{OQ} = \frac{r_{OQ}}{|r_{OQ}|} = \begin{pmatrix} 0.503 \\ 0.503 \\ 0.704 \end{pmatrix} \qquad e_{AQ} = \frac{r_{AQ}}{|r_{AQ}|} = \begin{pmatrix} 0.451 \\ -0631 \\ 0.631 \end{pmatrix} \qquad e_{BQ} = \frac{r_{BQ}}{|r_{BQ}|} = \begin{pmatrix} -0.631 \\ 0.451 \\ 0.631 \end{pmatrix}$$

STATICS $\quad$ Solve simultaneous equations to get cable tension forces

$$\begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{pmatrix} = \begin{pmatrix} e_{OQ_{1,1}} & e_{BQ_{1,1}} & e_{DQ_{1,1}} & e_{AQ_{1,1}} \\ e_{OQ_{2,1}} & e_{BQ_{2,1}} & e_{DQ_{2,1}} & e_{AQ_{2,1}} \\ e_{OQ_{3,1}} & e_{BQ_{3,1}} & e_{DQ_{3,1}} & e_{AQ_{3,1}} \\ 1 & 0 & 0 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \\ W \\ 0 \end{pmatrix} = \begin{pmatrix} 4278 \\ 6461 \\ 3341 \\ 4278 \end{pmatrix} \text{lbs} \qquad \boxed{T = \begin{pmatrix} 4278 \\ 6461 \\ 3341 \\ 4278 \end{pmatrix} \text{lb}}$$

< for case of $T_1 = T_4$

Normal stresses in cables

$$i = 1 \ldots 4 \qquad \sigma_i = \frac{T_i}{A_e} \qquad \boxed{\sigma = \begin{pmatrix} 35650 \\ 53842 \\ 27842 \\ 35650 \end{pmatrix} \text{psi}}$$

Problem 1.4-14

Data $\quad M_{\text{boom}} = 450 \text{ kg}$

$g = 9.81 \text{ m/s}^2 \qquad W_{\text{boom}} = M_{\text{boom}}\, g$

$W_{\text{boom}} = 4415 \text{ N}$

$P = 20 \text{ kN}$

$A_e = 304 \text{ mm}^2$

(a) Symmetry: $T_{AQ} = T_{BQ}$

$$\sum M_x = 0$$

$$2T_{AQZ}(3000) = W_{\text{boom}}(5000) + P(9000)$$

$$T_{AQZ} = \frac{W_{\text{boom}}(5000) + P(9000)}{2(3000)}$$

$$T_{AQ} = \frac{\sqrt{2^2 + 2^2 + 1^2}}{2} T_{AQz}$$

$$T_{AQ} = 50.5 \text{ kN} = T_{BQ} \quad \leftarrow$$

(b) $\sigma = \dfrac{T_{AQ}}{A_e} \qquad \sigma = 166.2 \text{ MPa} \quad \leftarrow$

Problem 1.4-15

$W_B = 450$

$W_C = 650$ lb

$\Delta_B = 3.9$ ft

$\Delta_C = 7.1$ ft

$L = 100$ ft

$D_{AB} = 12$ ft

$D_{BC} = 70$ ft

$D_{CD} = 20$ ft

$D_{AB} + D_{BC} + D_{CD} = 102$ ft

$A_e = 0.12$ in.^{2}

COMPUTE INITIAL VALUES OF THETA ANGLES (RADIANS)

$$\theta_1 = \arcsin\left(\frac{\Delta_B}{D_{AB}}\right) \qquad \theta_1 = 0.331$$

$$\theta_2 = \arcsin\left(\frac{\Delta_C - \Delta_B}{D_{BC}}\right) \qquad \theta_2 = 0.046$$

$$\theta_3 = \arcsin\left(\frac{\Delta_C}{D_{CD}}\right) \qquad \theta_3 = 0.363$$

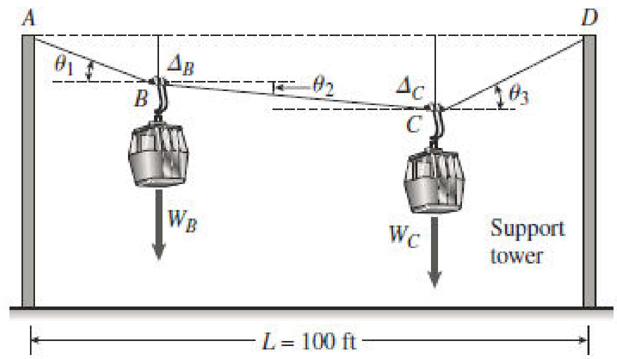
(a) STATICS AT B AND C

$$-T_{AB}\cos(\theta_1) + T_{BC}\cos(\theta_2) = 0$$

$$T_{AB}\sin(\theta_1) - T_{BC}\sin(\theta_2) = W_B$$

$$-T_{BC}\cos(\theta_2) + T_{CD}\cos(\theta_3) = 0$$

$$T_{BC}\sin(\theta_2) + T_{CD}\sin(\theta_3) = W_C$$



CONSTRAINT EQUATIONS

$$D_{AB}\cos(\theta_1) + D_{BC}\cos(\theta_2) + D_{CD}\cos(\theta_3) = L$$

$$D_{AB}\sin(\theta_1) + D_{BC}\sin(\theta_2) = D_{CD}\sin(\theta_3)$$

SOLVE SIMULTANEOUS EQUATIONS NUMERICALLY FOR TENSION FORCE IN EACH CABLE SEGMENT

$T_{AB} = 1620$ lb $\quad T_{CB} = 1536$ lb $\quad T_{CD} = 1640$ lb $\quad \leftarrow$

CHECK EQUILIBRIUM AT B AND C

$$T_{AB}\sin(\theta_1) - T_{BC}\sin(\theta_2) = 450$$

$$T_{BC}\sin(\theta_2) + T_{CD}\sin(\theta_3) = 650$$

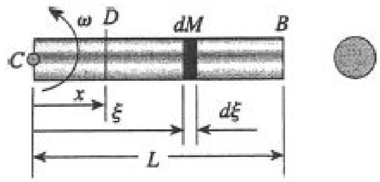
(b) COMPUTE STRESSES IN CABLE SEGMENTS

$$\sigma_{AB} = \frac{T_{AB}}{A_e} \qquad \sigma_{BC} = \frac{T_{BC}}{A_e} \qquad \sigma_{CD} = \frac{T_{CD}}{A_e}$$

$\sigma_{AB} = 13.5$ ksi $\quad \sigma_{BC} = 12.8$ ksi

$\sigma_{CD} = 13.67$ ksi $\quad \leftarrow$

Problem 1.4-16



ω = angular speed (rad/s)

A = cross-sectional area

γ = weight density

$\dfrac{\gamma}{g}$ = mass density

We wish to find the axial force F_x in the bar at Section D, distance x from the midpoint C.

The force F_x equals the inertia force of the part of the rotating bar from D to B.

Consider an element of mass dM at distance ξ from the midpoint C. The variable ξ ranges from x to L.

$$dM = \frac{\gamma}{g} A\ dj$$

dF = Inertia force (centrifugal force) of element of mass dM

$$dF = (dM)(j\omega^2) = \frac{\gamma}{g}\ A\omega^2 jdj$$

$$F_x = \int_D^B dF = \int_x^L \frac{\gamma}{g} A\omega^2 jdj = \frac{\gamma A\omega^2}{2g}(L^2 - x^2)$$

(a) Tensile stress in bar at distance x

$$\sigma_x = \frac{F_x}{A} = \frac{\gamma\omega^2}{2g}(L^2 - x^2) \quad \leftarrow$$

(b) Maximum tensile stress

$$x = 0 \quad \sigma_{max} = \frac{\gamma\omega^2 L^2}{2g} \quad \leftarrow$$

Problem 1.4-17

$W = 1575\text{lbf}$

Position and unit vectors

$$r_{CA} = \begin{pmatrix} 0-6.5 \\ 6.5-0 \\ -4-0 \end{pmatrix} = \begin{pmatrix} -6.5 \\ 6.5 \\ -4 \end{pmatrix} \qquad r_{CB} = \begin{pmatrix} 0-6.5 \\ 3-0 \\ 4-0 \end{pmatrix} = \begin{pmatrix} -6.5 \\ 3 \\ 4 \end{pmatrix} \qquad r_{OB} = \begin{pmatrix} 0 \\ 3 \\ 4 \end{pmatrix} \qquad r_{OA} = \begin{pmatrix} 0 \\ 6.5 \\ -4 \end{pmatrix}$$

$$n_{CA} = \frac{r_{CA}}{|r_{CA}|} = \begin{pmatrix} -0.648 \\ 0.648 \\ -0.399 \end{pmatrix} \qquad n_{CB} = \frac{r_{CB}}{|r_{CB}|} = \begin{pmatrix} -0.793 \\ 0.366 \\ 0.488 \end{pmatrix} \qquad r_{OW} = \begin{pmatrix} 5 \\ -3 \\ 0 \end{pmatrix} \qquad r_{OC} = \begin{pmatrix} 6.5 \\ 0 \\ 0 \end{pmatrix}$$

$$|r_{CA}| = 10.025 \qquad |r_{CB}| = 8.201 \qquad r_{OD} = \begin{pmatrix} 0 \\ -6.5 \\ 0 \end{pmatrix}$$

Sum moments about O

$$M_O = r_{OW} \times \begin{pmatrix} 0 \\ -W \\ 0 \end{pmatrix} + r_{OC} \times \left(T_A \cdot n_{CA} + T_B \cdot n_{CB}\right) + r_{OD} \times \begin{pmatrix} 0 \\ 0 \\ D_z \end{pmatrix}$$

$$M_O \text{ float}, 5 \rightarrow \begin{pmatrix} -6.5 \cdot D_z \\ 2.5935 \cdot T_A + -3.1705 \cdot T_B \\ 4.2145 \cdot T_A + 2.3779 \cdot T_B + -7875.0 \cdot \text{lbf} \end{pmatrix}$$

a) Find tension forces in cables

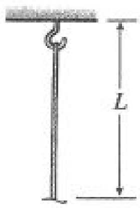
$$M_O = 0 \left| \begin{array}{l} \text{solve}, T_A, T_B, D_z \\ \text{float}, 8 \end{array} \right. \rightarrow (1278.4879 \cdot \text{lbf} \quad 1045.8267 \cdot \text{lbf} \quad 0) \qquad T_A = 1278\text{lbf} \qquad T_B = 1046\text{lbf}$$

b) average stress in each cable

$A_e = 0.471\text{in}^2$

$$\sigma_A = \frac{T_A}{A_e} = 2.713 \cdot \text{ksi} \qquad \sigma_B = \frac{T_B}{A_e} = 2.221 \cdot \text{ksi}$$

Problem 1.5-1



W = total weight of steel wire

γ_S = weight density of steel $= 490\ \text{lb/ft}^3$

γ_w = weight density of sea water $= 63.8\ \text{lb/ft}^3$

A = cross-sectional area of wire

$\sigma_{max} = 40$ ksi (yield strength)

(a) Wire hanging in air

$$W = \gamma_S AL$$

$$\sigma_{max} = \frac{W}{A} = \gamma_S L$$

$$L_{max} = \frac{\sigma_{max}}{\gamma_S} = \frac{40{,}000\ \text{psi}}{490\ \text{lb/ft}^3}\left(144\ \text{in.}^2/\text{ft}^2\right)$$

$$= 11{,}800\ \text{ft} \quad \leftarrow$$

(b) Wire hanging in sea water

F = tensile force at top of wire

$$F = \left(\gamma_S - \gamma_W\right)AL \quad \sigma_{max} = \frac{F}{A} = \left(\gamma_S - \gamma_W\right)L$$

$$L_{max} = \frac{\sigma_{max}}{\gamma_S - \gamma_W}$$

$$= \frac{40{,}000\ \text{psi}}{(490 - 63.8)\ \text{lb/ft}^3}\left(144\ \text{in.}^2/\text{ft}^2\right)$$

$$= 13{,}500\ \text{ft} \quad \leftarrow$$

Problem 1.5-2

(a) PIPE SUSPENDED IN AIR

$$\sigma_U = 550 \text{ MPa}$$

$$\gamma_s = 77 \text{ kN/m}^3$$

$$W = \gamma_s AL$$

$$\boxed{L_{max} = \frac{\sigma_U}{\gamma_s} = 7143 \text{ m}}$$

(b) PIPE SUSPENDED IN SEA WATER

$$\gamma_w = 10 \text{ kN/m}^3$$

Force at top of pipe: $F = (\gamma_s - \gamma_w) A L$

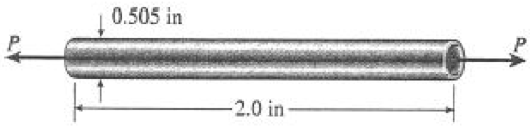
Stress at top of pipe:

$$\sigma_{max} = \frac{F}{A} \qquad \sigma_{max} = (\gamma_s - \gamma_w) L$$

Set max stress equal to ultimate and then solve for L_{max}

$$\boxed{L_{max} = \frac{\sigma_U}{(\gamma_s - \gamma_w)} = 8209 \text{ m}}$$

Problem 1.5-3



Percent elongation $= \frac{L_1 - L_0}{L_0}(100) = \left(\frac{L_1}{L_0} - 1\right)100$

$L_0 = 2.0$ in.

Percent elongation $= \left(\frac{L_1}{2.0} - 1\right)(100)$ (Eq. 1)

where L_1 is in inches.

Percent reduction in area $= \frac{A_0 - A_1}{A_0}(100)$

$= \left(1 - \frac{A_1}{A_0}\right)(100)$

d_0 = initial diameter d_1 = final diameter

$\frac{A_1}{A_0} = \left(\frac{d_1}{d_0}\right)^2$ $d_0 = 0.505$ in.

Percent reduction in area

$= \left[1 - \left(\frac{d_1}{0.505}\right)^2\right](100)$ (Eq. 2)

where d_1 is in inches.

Material	L_1 (in.)	d_1 (in.)	% Elongation (Eq. 1)	% Reduction (Eq. 2)	Brittle or Ductile?
A	2.13	0.484	6.5%	8.1%	Brittle
B	2.48	0.398	24.0%	37.9%	Ductile
C	2.78	0.253	39.0%	74.9%	Ductile

Problem 1.5-4

The ultimate stress σ_U for each material is obtained from Appendix I, Tables I-3, and the weight density γ is obtained from Table I-1.

The strength-to-weight ratio (meters) is

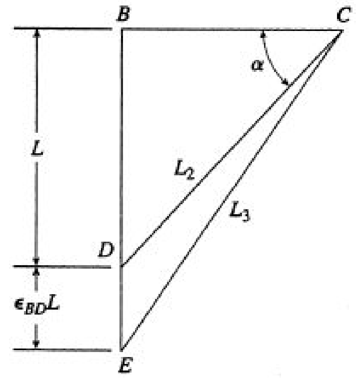
$$R_{S/W} = \frac{\sigma_U\,(\text{MPa})}{\gamma(\text{kN/m}^3)}\,(10^3)$$

Values of σ_U, γ, and $R_{S/W}$ are listed in the table.

	σ_U (MPa)	γ (kN/m^3)	$R_{S/W}$ (m)
Aluminum alloy 6061-T6	310	26.0	11.9×10^3
Douglas fir	65	5.1	12.7×10^3
Nylon	60	9.8	6.1×10^3
Structural steel ASTM-A572	500	77.0	6.5×10^3
Titanium alloy	1050	44.0	23.9×10^3

Titanium has a high strength-to-weight ratio, which is why it is used in space vehicles and high-performance airplanes. Aluminum is higher than steel, which makes it desirable for commercial aircraft. Some woods are also higher than steel, and nylon is about the same as steel.

Problem 1.5-5



Data

$\varepsilon_{BD} = 0.036 \qquad \alpha = 52° \qquad L_{BD} = 1$ < assume unit length to facilitate numerical calculations below

Strain in *CE*

$$\varepsilon_{CE} = \frac{L_3 - L_2}{L_2}$$

$$L_2 = \frac{L_{BD}}{\sin(\alpha)} \qquad L_{BC} = \frac{L_{BD}}{\tan(\alpha)}$$

Increased length of *CE* (see figure)

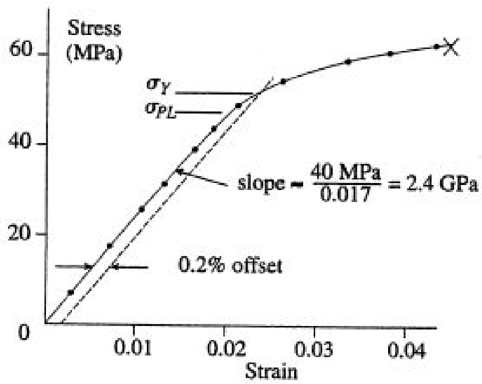
$$L_3 = \sqrt{L_{BC}^2 + (L_{BD} + \varepsilon_{BD} L_{BD})^2} = \sqrt{\frac{1}{\tan(52°)^2} + 1.073296}$$

Compute strain in *CE* then substitute strain value into stress-strain relationship to find tensile stress in outer bars:

$$\varepsilon_{CE} = \frac{L_3 - L_2}{L_2} = 0.023 \qquad \sigma = \frac{18000\,\varepsilon_{CE}}{1 + 300\,\varepsilon_{CE}} \qquad \boxed{\sigma = 52.3 \quad \text{ksi}}$$

Problem 1.5-6

Using the stress-strain data given in the problem statement, plot the stress-strain curve:



σ_{PL} = proportional limit $\quad \sigma_{PL} \approx 47$ MPa $\quad \leftarrow$

Modulus of elasticity (slope) ≈ 2.4 GPa $\quad \leftarrow$

σ_Y = yield stress at 0.2% offset

$\sigma_Y \approx 53$ MPa $\quad \leftarrow$

Material is *brittle*, because the strain after the proportional limit is exceeded is relatively small. $\quad \leftarrow$

Problem 1.5-7

$d_0 = 0.505$ in. $\quad L_0 = 2.00$ in.

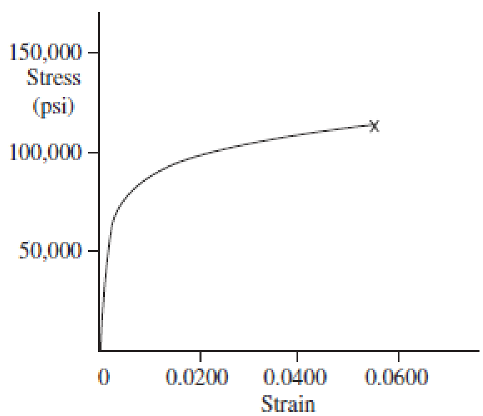
$$A_0 = \frac{\pi d_0^2}{4} = 0.200 \text{ in.}^2$$

Conventional stress and strain

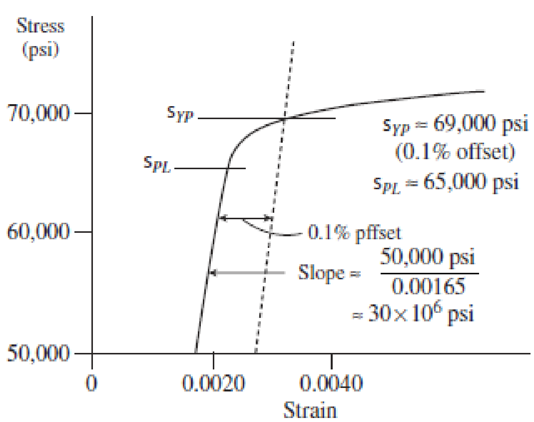
$$\sigma = \frac{P}{A_0} \quad \varepsilon = \frac{\delta}{L_0}$$

Load P (lb)	Elongation δ (in.)	Stress σ (psi)	Strain ε
1,000	0.0002	5,000	0.00010
2,000	0.0006	10,000	0.00030
6,000	0.0019	30,000	0.00100
10,000	0.0033	50,000	0.00165
12,000	0.0039	60,000	0.00195
12,900	0.0043	64,500	0.00215
13,400	0.0047	67,000	0.00235
13,600	0.0054	68,000	0.00270
13,800	0.0063	69,000	0.00315
14,000	0.0090	70,000	0.00450
14,400	0.0102	72,000	0.00510
15,200	0.0130	76,000	0.00650
16,800	0.0230	84,000	0.01150
18,400	0.0336	92,000	0.01680
20,000	0.0507	100,000	0.02535
22,400	0.1108	112,000	0.05540
22,600	Fracture	113,000	

Stress-strain diagram



Enlargement of part of the stress-strain curve



Results

Proportional limit $\approx$ 65,000 psi $\leftarrow$

Modulus of elasticity (slope) $\approx 30 \times 10^6$ psi $\leftarrow$

Yield stress at 0.1% offset $\approx$ 69,000 psi $\leftarrow$

Ultimate stress (maximum stress)

$\approx$ 113,000 psi $\leftarrow$

Percent elongation in 2.00 in.

$$= \frac{L_1 - L_0}{L_0}(100)$$

$$= \frac{0.12 \text{ in.}}{2.00 \text{ in.}}(100) = 6\% \quad \leftarrow$$

Percent reduction in area

$$= \frac{A_0 - A_1}{A_0}(100)$$

$$= \frac{0.200 \text{ in.}^2 - \frac{\pi}{4}(0.42 \text{ in.})^2}{0.200 \text{ in.}^2}(100)$$

$$= 31\% \quad \leftarrow$$

Problem 1.6-1

Part (a)

$\delta = 0.2\text{in}$ $\quad L = 60\text{in}$ $\quad E = 29000\text{ksi}$

$\varepsilon = \dfrac{\delta}{L} = 3.333 \times 10^{-3}$

$\sigma_Y = 50\text{ksi}$ elastic recovery $\varepsilon_E = \dfrac{\sigma_Y}{E} = 1.724 \times 10^{-3}$ $\quad \varepsilon_Y = \dfrac{\sigma_Y}{E} = 1.724 \times 10^{-3}$

residual strain $\varepsilon_R = \varepsilon - \varepsilon_E = 1.609 \times 10^{-3}$

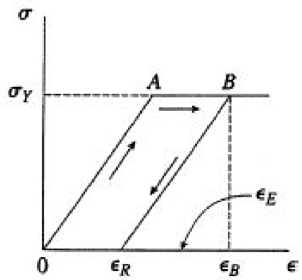
permanent set $\varepsilon_R \cdot L = 0.097 \cdot \text{in}$ < final length of bar is 0.097 in. more than original length

Part (b) $d = 1.5\text{in}$ $\quad P = 80\text{kip}$ $\quad A = \dfrac{\pi}{4} \cdot d^2 = 1.767 \cdot \text{in}^2$ $\quad \sigma = \dfrac{P}{A} = 45.271 \cdot \text{ksi}$ < below yield

stress in bar is 45.3 ksi < σ_Y so no permament set

strain of bar $\varepsilon = \dfrac{\sigma}{E} = 1.561 \times 10^{-3}$ < less than yield strain

Problem 1.6-2



$L = 2.0\text{ m} = 2000\text{ mm}$

Yield stress $\sigma_Y = 250\text{ MPa}$

Slope $= 200\text{ GPa}$

$\delta = 6.5\text{ mm}$

STRESS AND STRAIN AT POINT B

$\sigma_B = \sigma_Y = 250\text{ MPa}$

$$\varepsilon_B = \frac{\delta}{L} = \frac{6.5\text{ mm}}{2000\text{ mm}} = 0.00325$$

ELASTIC RECOVERY ε_E

$$\varepsilon_E = \frac{\sigma_B}{\text{Slope}} = \frac{250\text{ MPa}}{200\text{ GPa}} = 0.00125$$

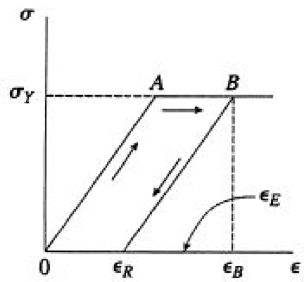
RESIDUAL STRAIN ε_R

$$\varepsilon_R = \varepsilon_B - \varepsilon_E = 0.00325 - 0.00125$$
$$= 0.00200$$

Permanent set $= \varepsilon_R L = (0.00200)(2000\text{ mm})$
$= 4.0\text{ mm}$

Final length of bar is 4.0 mm greater than its original length. ←

Problem 1.6-3



$L = 48$ in.

Yield stress $\sigma_Y = 42$ ksi

Slope $= 30 \times 10^3$ ksi

$\delta = 0.20$ in.

STRESS AND STRAIN AT POINT B

$\sigma_B = \sigma_Y = 42$ ksi

$$\varepsilon_B = \frac{\delta}{L} = \frac{0.20 \text{ in.}}{48 \text{ in.}} = 0.00417$$

ELASTIC RECOVERY ε_E

$$\varepsilon_E = \frac{\sigma_B}{\text{Slope}} = \frac{42 \text{ ksi}}{30 \times 10^3 \text{ ksi}} = 0.00140$$

RESIDUAL STRAIN ε_R

$$\varepsilon_R = \varepsilon_B - \varepsilon_E = 0.00417 - 0.00140$$
$$= 0.00277$$

PERMANENT SET

$$\varepsilon_R L = (0.00277)(48 \text{ in.})$$
$$= 0.13 \text{ in.}$$

Final length of bar is 0.13 in. greater than its original length. ←

Problem 1.6-4

numerical data $L = 750$ mm $\delta = 6$ mm

$\varepsilon_B = \dfrac{\delta}{L} = 8 \times 10^{-3}$ $\sigma_B = 65.6$ MPa < from curve (see figure)

$\varepsilon_E = 0.0023$ < elastic recovery (see figure)

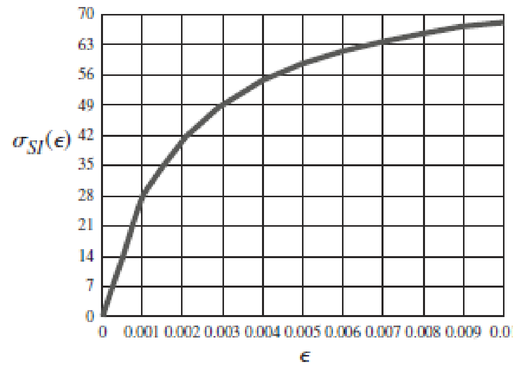
$\varepsilon_R = \varepsilon_B - \varepsilon_E = 5.7 \times 10^{-3}$ < residual strain

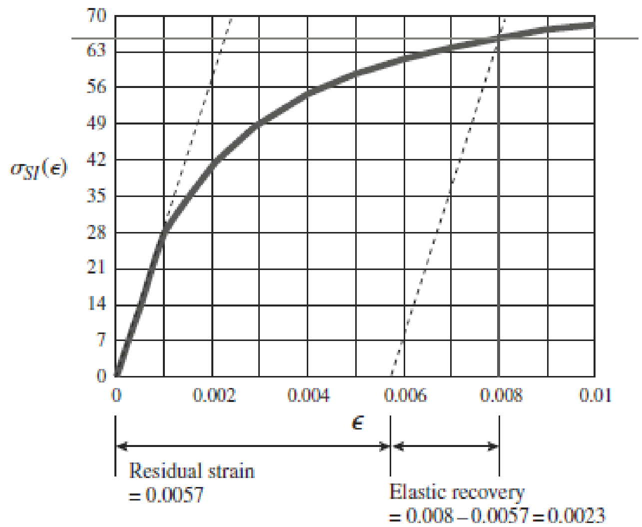
(a) PERMANENT SET

$\delta_{pset} = \varepsilon_R L = 4.275$ $\boxed{\delta_{pset} = 4.28 \text{ mm}}$

(b) PROPORTIONAL LIMIT WHEN RELOADED

$\boxed{\sigma_B = 65.6 \text{ MPa}}$





Problem 1.6-5

DATA

$P = 44.6 \text{ kip} \qquad L = 6 \text{ ft}$

$d = 1.375 \text{ in.} \qquad E = 10.6(10^6) \text{ psi}$

NORMAL STRESS IN BAR

$$\sigma_B = \frac{P}{\frac{\pi}{4}d^2} = 30036 \text{ psi}$$

from curve, say that $\varepsilon_B = 0.025$

ELASTIC RECOVERY unloading parallel to initial straight line

$$\varepsilon_E = \frac{\sigma_B}{E} = 2.834 \times 10^{-3}$$

RESIDUAL STRAIN

$$\varepsilon_R = \varepsilon_B - \varepsilon_E = 0.022$$

(a) PERMANENT SET

$\varepsilon_R L = 1.596 \text{ in.}$

(b) PROPORTIONAL LIMIT WHEN RELOADED IS $\sigma_B = 30$ ksi

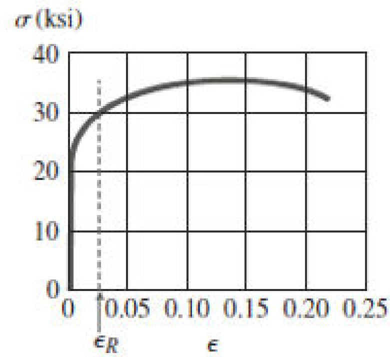


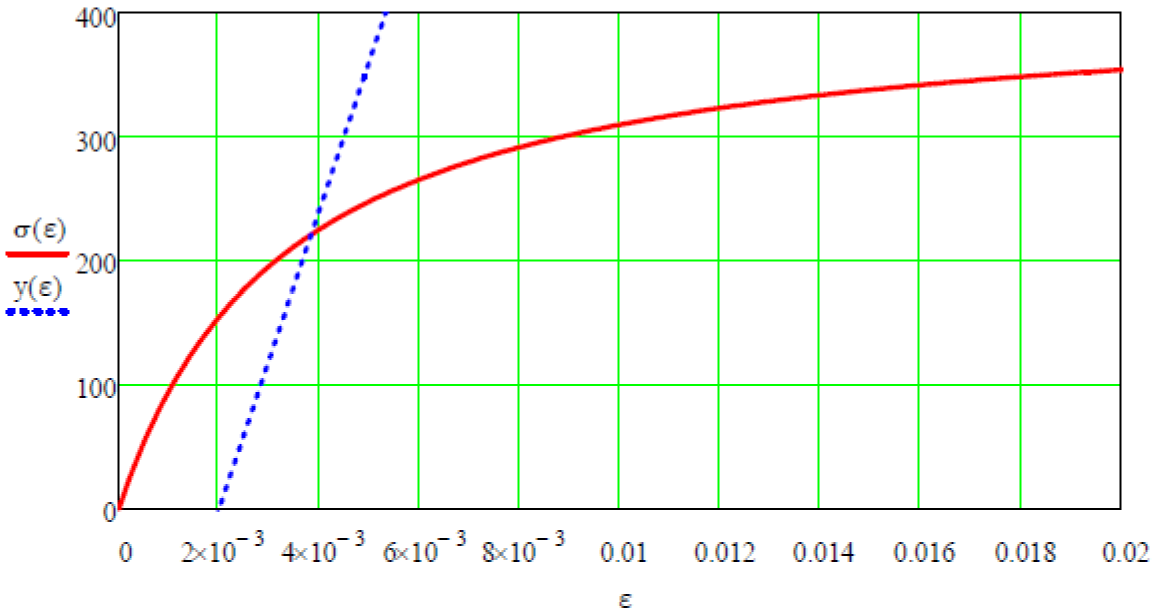
FIG 1-34 Typical stress-strain diagram for an aluminum alloy

Problem 1.6-6

copper wire, d = 6 mm

$\varepsilon = 0, 0.0001 .. 0.03$

$$\sigma(\varepsilon) = \frac{124000 \cdot \varepsilon}{1 + 300 \cdot \varepsilon} \qquad \frac{\sigma(0.0001)}{0.0001} = 1.204 \times 10^5 \qquad y(\varepsilon) = 120400 \cdot (\varepsilon - 0.002)$$



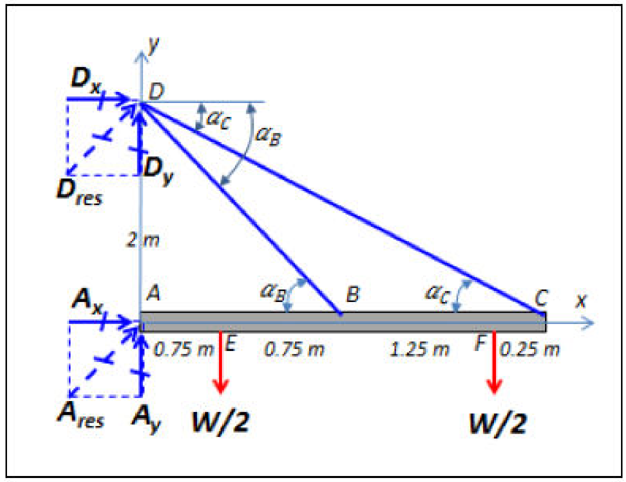
$$E(\varepsilon) = \frac{d}{d\varepsilon}\sigma(\varepsilon) \rightarrow \frac{124000}{300 \cdot \varepsilon + 1} - \frac{37200000 \cdot \varepsilon}{(300 \cdot \varepsilon + 1)^2} \qquad E(0) = 1.24 \times 10^5$$

$E = E(0) = 1.24 \times 10^5$ MPa

$y(\varepsilon) = E \cdot (\varepsilon - 0.002)$

$$\frac{124000 \cdot x}{1 + 300 \cdot x} - E \cdot (x - 0.002) \text{ solve}, x \rightarrow \begin{pmatrix} 0.0037688746209726916175 \\ -0.0017688746209726916175 \end{pmatrix}$$

$\sigma(0.0037688746209726916175) = 219.34 \ \text{MPa}$
< yield stress at 0.2% offset



Find force T in continuous cable $\quad W = 6.8kN$

Summing moments about point D (counterclockwise moments are positive) gives:

$\Sigma M_D = 0 \qquad A_x \cdot (2m) - \frac{W}{2} \cdot (0.75m + 2.75m) = 0 \quad \text{or} \qquad A_x = \frac{6.8kN}{2} \cdot \left(\frac{3.5m}{2m}\right) = 5950 \cdot N$

Next, **sum forces in the x direction**: $\Sigma F_x = 0 \qquad A_x + D_x = 0 \qquad \text{or} \qquad D_x = -A_x \ = \ -5950N$

The minus sign means that D_x acts in the negative x direction.

Summing forces in the x direction at joint D will give us the force in the continuous cable BDC:

First compute angles α_B and α_C (see fig.): $\quad \alpha_B = \text{atan}\left(\frac{2}{1.5}\right) = 53.13 \cdot deg \qquad \alpha_C = \text{atan}\left(\frac{2}{3}\right) = 33.69 \cdot deg$

Now $\Sigma F_x = 0$ at joint D $\qquad D_x + T \cdot (\cos(\alpha_B) + \cos(\alpha_C)) = 0 \qquad \text{so} \qquad T = \frac{-D_x}{(\cos(\alpha_B) + \cos(\alpha_C))}$

$\text{or} \qquad T = \frac{-(-5950N)}{(\cos(\alpha_B) + \cos(\alpha_C))} = 4155 \cdot N$

a) Find the axial normal strain in the cable and its elongation $\qquad$ cable length $\qquad L = 2.5m + \sqrt{13}\,m = 6.106\,m$

$\sigma = \frac{T}{\frac{1}{4} \cdot \pi \cdot (6mm)^2} = 146.953 \cdot MPa$ $\quad$ < less than σ_Y = 219 MPa (at 0.2% offset)

$124000 \cdot x - 146.953 \cdot (1 + 300 \cdot x) \text{ solve}, x \ \rightarrow 0.0018388870049215344977$

$\varepsilon_\sigma = \frac{146.953}{(124000 - 146.953 \cdot 300)} = 1.839 \times 10^{-3} \qquad \delta = \varepsilon_\sigma \cdot (L) = 11.227 \cdot mm$

b) Find the permanent set of the wire if all forces are removed

There is no permanent set since the stress is still below the 0.2% offset yield stress

Problem 1.6-7

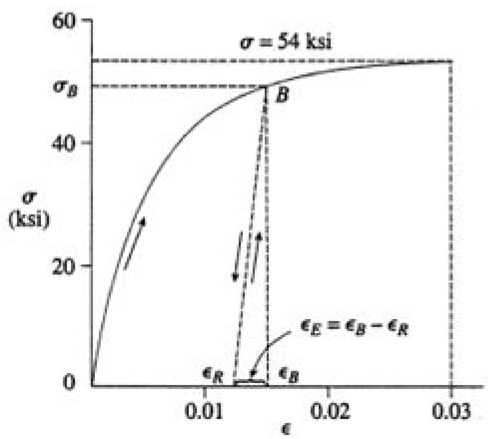
$L = 4\text{ ft} = 48\text{ in.}$ $\quad d = 0.125\text{ in.}$

$P = 600\text{ lb}$

COPPER ALLOY

$$\sigma = \frac{18{,}000\varepsilon}{1 + 300\varepsilon} \quad 0 \le \varepsilon \le 0.03 \; (\sigma = \text{ksi}) \quad \text{(Eq. 1)}$$

(a) STRESS-STRAIN DIAGRAM (From Eq. 1)



INITIAL SLOPE OF STRESS-STRAIN CURVE

Take the derivative of σ with respect to ε:

$$\frac{d\sigma}{d\varepsilon} = \frac{(1 + 300\varepsilon)(18{,}000) - (18{,}000)(300)\sigma}{(1 + 300\varepsilon)^2}$$

$$= \frac{18{,}000}{(1 + 300\varepsilon)^2}$$

At $\varepsilon = 0$, $\quad \frac{d\sigma}{d\varepsilon} = 18{,}000\text{ ksi}$

$\therefore$ Initial slope $= 18{,}000\text{ ksi}$

ALTERNATIVE FORM OF THE STRESS-STRAIN RELATIONSHIP

Solve Eq. (1) for ε in terms of σ:

$$\varepsilon = \frac{\sigma}{18{,}000 - 300\sigma} \quad 0 \le \sigma \le 54\text{ ksi} \; (\sigma = \text{ksi}) \quad \text{(Eq. 2)}$$

This equation may also be used when plotting the stress-strain diagram.

(b) ELONGATION δ OF THE WIRE

$$\sigma = \frac{P}{A} = \frac{600\text{ lb}}{\frac{\pi}{4}(0.125\text{ in.})^2} 48{,}900\text{ psi} = 48.9\text{ ksi}$$

From Eq. (2) or from the stress-strain diagram:

$\varepsilon = 0.0147$

$\delta = \varepsilon L = (0.0147)(48\text{ in.}) = 0.71\text{ in.}$ $\quad\leftarrow$

STRESS AND STRAIN AT POINT B (see diagram)

$\sigma_B = 48.9\text{ ksi} \quad \varepsilon_B = 0.0147$

ELASTIC RECOVERY ε_E

$$\varepsilon_E = \frac{\sigma_B}{\text{Slope}} = \frac{48.9\text{ ksi}}{18{,}000\text{ ksi}} = 0.00272$$

RESIDUAL STRAIN ε_R

$\varepsilon_R = \varepsilon_B - \varepsilon_E = 0.0147 - 0.0027 = 0.0120$

(c) Permanent set $= \varepsilon_R L = (0.0120)(48\text{ in.})$

$= 0.58\text{ in.}$ $\quad\leftarrow$

(d) Proportional limit when reloaded $= \sigma_B$

$\sigma_B = 49\text{ ksi}$ $\quad\leftarrow$

Problem 1.7-1

STEEL BAR

$d = 2.00$ in. Maximum $\Delta d = 0.001$ in.

$E = 29 \times 10^6$ psi $\nu = 0.29$

LATERAL STRAIN

$$\varepsilon' = \frac{\Delta d}{d} = \frac{0.001 \text{ in.}}{2.00 \text{ in.}} = 0.0005$$

AXIAL STRAIN

$$\varepsilon = -\frac{\varepsilon'}{\nu} = -\frac{0.0005}{0.29} = -0.001724$$

(shortening)

AXIAL STRESS

$$\sigma = E\varepsilon = (29 \times 10^6 \text{ psi})(-0.001724)$$
$$= -50.00 \text{ ksi (compression)}$$

Assume that the yield stress for the high-strength steel is greater than 50 ksi. Therefore, Hooke's law is valid.

MAXIMUM COMPRESSIVE LOAD

$$P_{max} = \sigma A = (50.00 \text{ ksi})\left(\frac{\pi}{4}\right)(2.00 \text{ in.})^2$$
$$= 157 \text{ k} \quad \leftarrow$$

Problem 1.7-2

$d = 10$ mm $\quad \Delta d = 0.016$ mm

(Decrease in diameter)

7075-T6

From Table I-2: $E = 72$ GPa $\quad \nu = 0.33$

From Table I-3: Yield stress $\sigma_Y = 480$ MPa

Lateral strain

$$\varepsilon' = \frac{\Delta d}{d} = \frac{-0.016 \text{ mm}}{10 \text{ mm}} = -0.0016$$

Axial strain

$$\varepsilon = -\frac{\varepsilon'}{\nu} = \frac{0.0016}{0.33}$$
$$= 0.004848 \text{ (Elongation)}$$

Axial stress

$$\sigma = E\varepsilon = (72 \text{ GPa})(0.004848)$$
$$= 349.1 \text{ MPa (Tension)}$$

Because $\sigma < \sigma_Y$, Hooke's law is valid.

Load P (tensile force)

$$P = \sigma A = (349.1 \text{ MPa})\left(\frac{\pi}{4}\right)(10 \text{ mm})^2$$
$$= 27.4 \text{ kN} \quad \longleftarrow$$

Problem 1.7-3

NUMERICAL DATA

$d_1 = 4$ in. $\quad d_2 = 4.01$ in. $\quad E = 200$ ksi

$v = 0.4 \quad \Delta d_1 = 0.01$ in.

$A_1 = \frac{\pi}{4} d_1{}^2 \quad A_2 = \frac{\pi}{4} d_2{}^2 \quad A_1 = 12.566$ in.^{2}

$A_2 = 12.629$ in.^{2}

LATERAL STRAIN

$$\varepsilon_p = \frac{\Delta d_1}{d_1} \qquad \varepsilon_p = \frac{0.01}{4} \qquad \varepsilon_p = 2.5 \times 10^{-3}$$

NORMAL STRAIN

$$\varepsilon_1 = \frac{-\varepsilon_p}{v} \qquad \varepsilon_1 = -6.25 \times 10^{-3}$$

AXIAL STRESS

$\sigma_1 = E\,\varepsilon_1 \quad \sigma_1 = -1.25$ ksi

COMPRESSION FORCE

$P = EA_1\varepsilon_1$

$P = -15.71$ kips $\quad \leftarrow$

Problem 1.7-4

$s_p = 193\text{mm} \qquad L_c = 400\text{mm} \qquad s_c = 200\text{mm} \qquad t_c = 3\text{mm} \qquad \nu_p = 0.4$

Part (a)

$gap = s_c - 2\cdot t_c - s_p = 1\cdot\text{mm} \qquad \varepsilon_{lat} = \dfrac{gap}{s_p} = 5.181\times 10^{-3} \qquad \varepsilon = \dfrac{-\varepsilon_{lat}}{\nu_p} = -0.013$

$\delta_p = \varepsilon\cdot L_p \qquad \delta_{p2} = -(L_p - L_c)$ equate δ_p and δ_{p2} then solve for L_p $\qquad L_p = \dfrac{L_c}{1+\varepsilon} = 405.249\cdot\text{mm}$

Part (b)

$V_{ini} = L_p\cdot s_p\cdot s_p = 0.0150951\cdot\text{m}^3 \qquad V_{final} = L_c\cdot(s_c - 2\cdot t_c)^2 = 0.0150544\cdot\text{m}^3 \qquad \dfrac{V_{ini}}{V_{final}} = 1.003$

Problem 1.7-5

$E_p = 200\text{ksi}$ $\qquad \nu_p = 0.4$ $\qquad A_p = 7\text{in}\cdot(7.35\text{in}) = 51.45\cdot\text{in}^2$ $\qquad A_s = (8\text{in})^2 - (8\text{in} - 0.6\text{in})^2 = 9.24\cdot\text{in}^2$

original gap on each side: $\dfrac{[8\text{in} - 2\cdot(0.3\text{in})] - 7.35\text{in}}{2} = 0.025\cdot\text{in}$

original gap at both top & bottom: $\dfrac{[8\text{in} - 2\cdot(0.3\text{in})] - 7\text{in}}{2} = 0.2\cdot\text{in}$

<u>Force required to close gap on left-right sides</u>

$\varepsilon_{lat} = \dfrac{[8\text{in} - 2\cdot(0.3\text{in})] - 7.35\text{in}}{7.35\text{in}} = 6.803\times10^{-3}$ $\qquad \varepsilon_p = \dfrac{-\varepsilon_{lat}}{\nu_p} = -1.701\times10^{-2}$ $\qquad \varepsilon_{lat}\cdot(7.35\text{in}) = 0.05\cdot\text{in}$

$P = E_p\cdot A_p\cdot\varepsilon_p = -175\cdot\text{kip}$

<u>Remaining gap on top-bottom</u>

$\Delta h = \varepsilon_{lat}\cdot(7\text{in}) = 0.048\cdot\text{in}$ $\qquad \text{gap} = \dfrac{[8\text{in} - 2\cdot(0.3\text{in})] - (7\text{in} + \Delta h)}{2} = 0.1762\cdot\text{in}$

Problem 1.7-6

(a) GIVEN STRESS, FIND FORCE P IN BAR FIGURE (A)

$\sigma = 57\text{ MPa} \quad E = 73\text{ GPa} \quad \nu = 0.33$

$L = 600\text{ mm} \quad d_2 = 75\text{ mm} \quad d_1 = 63\text{ mm}$

$A = \frac{\pi}{4}\left(d_2^2 - d_1^2\right) = 1301\text{ mm}^2$

$\boxed{P = \sigma A = 74.1\text{ kN}}$

(b) GIVEN STRAIN, FIND CHANGE IN LENGTH IN BAR FIGURE (A) AND ALSO VOLUME CHANGE

$\varepsilon = -781\left(10^{-6}\right) \qquad t = \frac{d_2 - d_1}{2} = 6\text{ mm}$

$\boxed{\delta = \varepsilon L = -0.469\text{ mm}}$ shortening

$\text{Vol}_1 = L(A) = 7.804 \times 10^5\text{ mm}^3$

$\varepsilon_{\text{lat}} = -\nu\varepsilon = 2.577 \times 10^{-4} \qquad \Delta t = \varepsilon_{\text{lat}}\,t = 1.546 \times 10^{-3}\text{ mm}$

$\Delta d_2 = \varepsilon_{\text{lat}} d_2 = 0.019\text{ mm} \qquad \Delta d_1 = \varepsilon_{\text{lat}} d_1 = 0.016\text{ mm}$

$$A_f = \frac{\pi}{4}\left[\left(d_2 + \Delta d_2\right)^2 - \left(d_1 + \Delta d_1\right)^2\right]$$

$$A_f = 1301.29\text{ mm}^2$$

$$\frac{A_f - A}{A} = \frac{1301.29 - 1300.62}{1300.62} = 0.052\%$$

$$V_{1f} = (L + \delta)\left(A_f\right) = 7.802 \times 10^5\text{ mm}^3$$

$$\Delta V_1 = V_{1f} - \text{Vol}_1 = -207.482\text{ mm}^3$$

$\boxed{\Delta V_1 = -207\text{ mm}^3}$ change

(c) IF THE TUBE HAS CONSTANT OUTER DIAMETER OF $d_2 = 75$ mm ALONG ITS ENTIRE LENGTH L BUT NOW HAS INCREASED INNER DIAMETER d_3 OVER THE MIDDLE THIRD WITH NORMAL STRESS OF 70 MPa, WHILE THE REST OF THE BAR REMAINS AT NORMAL STRESS OF 57 MPa, WHAT IS THE DIAMETER d_3?

$\sigma_{M3} = 70\text{ MPa} \quad P = 74.135\text{ kN} \quad A_{M3} = \frac{P}{\sigma_{M3}} = 1059.076\text{ mm}^2 \qquad d_2 = 75\text{ mm} \qquad d_1 = 63\text{ mm}$

$d_2^2 - d_3^2 = \frac{4}{\pi}A_{M3} \qquad \text{SO} \qquad d_3 = \sqrt{d_2^2 - \frac{4}{\pi}A_{M3}} = 65.4\text{ mm} \qquad t_{M3} = \frac{d_2 - d_3}{2} = 4.802\text{ mm}$

$\boxed{d_3 = 65.4\text{ mm}}$

Problem 1.7-7

NUMERICAL DATA

$E = 25{,}000$ ksi

$\nu = 0.32$

$L = 9$ in.

$\delta = 0.0195$ in.

$d = 0.225$ in.

NORMAL STRAIN

$\varepsilon = \dfrac{\delta}{L} \quad \varepsilon = 2.167 \times 10^{-3}$

LATERAL STRAIN

$\varepsilon_p = -\nu\varepsilon \quad \varepsilon_p = -6.933 \times 10^{-4}$

DECREASE IN DIAMETER

$\Delta d = \varepsilon_p d$

$\Delta d = -1.56 \times 10^{-4}$ in. $\leftarrow$

INITIAL CROSS SECTIONAL AREA

$A_i = \dfrac{\pi}{4} d^2 \quad A_i = 0.04 \text{ in.}^2$

MAGNITUDE OF LOAD P

$P = EA_i\varepsilon$

$P = 2.15$ kips $\leftarrow$

Problem 1.7-8

$d = 10$ mm   Gage length $L = 50$ mm

$P = 20$ kN $\delta = 0.122$ mm $\Delta d = 0.00830$ mm

Axial stress

$$\sigma = \frac{P}{A} = \frac{20\text{ k}}{\frac{\pi}{4}(10\text{ mm})^2} = 254.6\text{ MPa}$$

Assume σ is below the proportional limit so that Hooke's law is valid.

Axial strain

$$\varepsilon = \frac{\delta}{L} = \frac{0.122\text{ mm}}{50\text{ mm}} = 0.002440$$

(a) Modulus of elasticity

$$E = \frac{\sigma}{\varepsilon} = \frac{254.6\text{ MPa}}{0.002440} = 104\text{ GPa} \quad \longleftarrow$$

(b) Poisson's ratio

$$\varepsilon' = \nu\varepsilon$$

$$\Delta d = \varepsilon' d = \nu\varepsilon d$$

$$\nu = \frac{\Delta d}{\varepsilon d} = \frac{0.00830\text{ mm}}{(0.002440)(10\text{ mm})} = 0.34 \quad \longleftarrow$$

Problem 1.7-9

NUMERICAL DATA

$P_1 = 26.5$ k

$P_2 = 22$ k

$d_{AB} = 1.25$ in.

$t_{AB} = 0.5$ in.

$d_{BC} = 2.25$ in.

$t_{BC} = 0.375$ in.

$E = 14000$ ksi

$\Delta t_{BC} = 200 \times 10^{-6}$

(a) INCREASE IN THE INNER DIAMETER OF PIPE SEGMENT BC

$$\varepsilon_{pBC} = \frac{\Delta t_{BC}}{t_{BC}} \quad \varepsilon_{pBC} = 5.333 \times 10^{-4}$$

$$\Delta d_{BCinner} = \varepsilon_{pBC}(d_{BC} - 2t_{BC})$$

$$\Delta\, d_{BCinner} = 8 \times 10^{-4} \text{ in.} \quad \longleftarrow$$

(b) POISSON'S RATIO FOR THE BRASS

$$A_{BC} = \frac{\pi}{4}\left[\, d_{BC}{}^2 - (d_{BC} - 2t_{BC})^2 \right]$$

$$A_{BC} = 2.209 \text{ in.}^2$$

$$\varepsilon_{BC} = \frac{-(P_1 + P_2)}{(EA_{BC})} \quad \varepsilon_{BC} = -1.568 \times 10^{-3}$$

$$\nu_{\text{brass}} = \frac{-\varepsilon_{pBC}}{\varepsilon_{BC}} \qquad \nu_{\text{brass}} = 0.34$$

(c) INCREASE IN THE WALL THICKNESS OF PIPE SEGMENT AB AND THE INCREASE IN THE INNER DIAMETER OF AB

$$A_{AB} = \frac{\pi}{4}\left[d_{AB}{}^2 - (d_{AB} - 2t_{AB})^2 \right]$$

$$\varepsilon_{AB} = \frac{-P_1}{EA_{AB}} \qquad \varepsilon_{AB} = -1.607 \times 10^{-3}$$

$$\varepsilon_{pAB} = -\nu_{\text{brass}}\varepsilon_{AB} \qquad \varepsilon_{pAB} = 5.464 \times 10^{-4}$$

$$\Delta t_{AB} = \varepsilon_{pAB} t_{AB} \quad \Delta t_{AB} = 2.73 \times 10^{-4} \text{ in.} \quad \longleftarrow$$

$$\Delta d_{ABinner} = \varepsilon_{pAB}(d_{AB} - 2t_{AB})$$

$$\Delta d_{ABinner} = 1.366 \times 10^{-4} \text{ in.}$$

Problem 1.7-10

$P = 1400 \text{ kN} \qquad L = 5 \text{ m} \qquad d = 80 \text{ mm} \qquad E = 110 \text{ GPa} \qquad \nu = 0.33$

$$A_d = \frac{\pi}{4}d^2 = 5026.5 \text{ mm}^2 \qquad A_{2d} = \frac{\pi}{4}(2\,d)^2 = 20{,}106.2 \text{ mm}^2$$

(a) FIND CHANGE IN LENGTH OF EACH BAR

BAR #1 $\quad \varepsilon_1 = \dfrac{P}{EA_d} = 2.532 \times 10^{-3} \qquad \sigma_1 = E\varepsilon_1 = 279 \text{ MPa}$

< Appendix I, Table I-3: copper alloys can have yield stress in range 55–760 MPa so assume this is below proportional limit so that Hooke's Law applies

$\boxed{\Delta L_1 = \varepsilon_1 L = 12.66 \text{ mm}} \qquad L_{f1} = L + \Delta L_1 = 5012.66 \text{ mm}$

BAR #2 $\quad \varepsilon_{2a} = \dfrac{P}{EA_d} = 2.532 \times 10^{-3} \qquad \varepsilon_{2b} = \dfrac{P}{EA_{2d}} = 6.33 \times 10^{-4} \qquad \dfrac{\varepsilon_{2a}}{4} = 6.33 \times 10^{-4}$

$$\Delta L_{2a} = \varepsilon_{2a}\frac{L}{5} = 2.532 \text{ mm} \qquad \Delta L_{2b} = \varepsilon_{2b}\left(\frac{4L}{5}\right) = 2.532 \text{ mm}$$

$$\boxed{\Delta L_2 = \Delta L_{2a} + \Delta L_{2b} = 5.06 \text{ mm}} \qquad L_{f2} = L + \Delta L_2 = 5005.06 \text{ mm} \qquad \frac{\Delta L_2}{\Delta L_1} = 0.4$$

BAR #3

$$\Delta L_{2a} = \varepsilon_{2a}\frac{L}{15} = 0.844 \text{ mm} \qquad \Delta L_{2b} = \varepsilon_{2b}\left(\frac{14L}{15}\right) = 2.954 \text{ mm}$$

$$\boxed{\Delta L_3 = \Delta L_{2a} + \Delta L_{2b} = 3.8 \text{ mm}} \qquad L_{f3} = L + \Delta L_3 = 5003.08 \text{ mm} \qquad \frac{\Delta L_3}{\Delta L_1} = 0.3$$

(b) FIND CHANGE IN VOLUME OF EACH BAR

Use lateral strain (ε_p) in each segment to find change in diameter Δd, then find change in cross sectional area, then volume

BAR #1

$$\varepsilon_{p1} = -\nu\varepsilon_1 = -8.356 \times 10^{-4} \quad \Delta d_1 = \varepsilon_{p1} d = -0.067 \text{ mm} \quad A_1 = \frac{\pi}{4}(d + \Delta d_1)^2 = 5018.152 \text{ mm}^2$$

$$\Delta \text{Vol}_1 = A_1 L_{f1} - A_d L = 21548 \text{ mm}^3 \qquad \frac{\Delta Vol_1}{A_d L} = 8.574 \times 10^{-4}$$

BAR #2

$$\varepsilon_{p2a} = \varepsilon_{p1} \quad \varepsilon_{p2b} = -\nu\varepsilon_{2b} = -2.089 \times 10^{-4} \quad \frac{\varepsilon_{p1}}{4} = -2.089 \times 10^{-4}$$

$$\Delta d_{2b} = \varepsilon_{p2b}(2d) = -0.33 \text{ mm} \quad A_{2a} = A_1 \quad A_{2b} = \frac{\pi}{4}(2d + \Delta d_{2b})^2 = 20097.794 \text{ mm}^2$$

$$\Delta L_{2a} = \varepsilon_{2a}\frac{L}{5} = 2.532 \text{ mm} \quad \Delta L_{2b} = \varepsilon_{2b}\left(\frac{4L}{5}\right) = 2.532 \text{ mm}$$

$$\Delta\text{Vol}_2 = \left[A_1\left(\frac{L}{5} + \Delta L_{2a}\right) + A_{2b}\left(\frac{4L}{5} + \Delta L_{2b}\right)\right] - \left[A_{2d}\left(\frac{4L}{5}\right) + A_d\left(\frac{L}{5}\right)\right]$$

$$= 21601 \text{ mm}^3 \qquad \frac{\Delta\text{Vol}_2}{\Delta\text{Vol}_1} = 1.002$$

Bar #3

$$\Delta L_{2a} = \varepsilon_{2a}\frac{L}{15} = 0.844 \text{ mm} \qquad \Delta L_{2b} = \varepsilon_{2b}\left(\frac{14\,L}{15}\right) = 2.954 \text{ mm}$$

$$\Delta \text{Vol}_3 = \left[A_1\left(\frac{L}{15} + \Delta L_{2a}\right) + A_{2b}\left(\frac{14\,L}{15} + \Delta L_{2b}\right)\right] - \left[A_{2d}\left(\frac{14\,L}{15}\right) + A_d\left(\frac{L}{15}\right)\right]$$

$$= 21610 \text{ mm}^3 \qquad \frac{\Delta \text{Vol}_3}{\Delta \text{Vol}_2} = 1.003$$

$$\boxed{\Delta \text{Vol}_1 = 21548 \text{ mm}^3} \quad \boxed{\Delta \text{Vol}_2 = 21601 \text{ mm}^3} \quad \boxed{\Delta \text{Vol}_3 = 21610 \text{ mm}^3}$$

Problem 1.8-1

NUMERICAL DATA

$t = 0.75 \text{ in.} \qquad L = 8 \text{ in.}$

$b = 3. \text{ in.} \qquad p = \dfrac{275}{1000} \text{ ksi} \qquad d = \dfrac{5}{8} \text{ in.}$

BEARING FORCE

$F = pbL \qquad F = 6.6 \text{ k}$

SHEAR AND BEARING AREAS

$A_S = \dfrac{\pi}{4} d^2 \qquad A_S = 0.307 \text{ in.}^2$

$A_b = dt \qquad A_b = 0.469 \text{ in.}^2$

BEARING STRESS

$\sigma_b = \dfrac{F}{2A_b} \qquad \sigma_b = 7.04 \text{ ksi} \quad \leftarrow$

SHEAR STRESS

$\tau_{ave} = \dfrac{F}{2A_S} \qquad \tau_{ave} = 10.76 \text{ ksi} \quad \leftarrow$

Problem 1.8-2

NUMERICAL DATA

$t_{ep} = 14\ \text{mm}$

$t_{gp} = 26\ \text{mm}$

$P = 80\ \text{kN}$

$d_p = 22\ \text{mm}$

$\tau_{\text{ult}} = 190\ \text{MPa}$

(a) BEARING STRESS ON PIN

$\sigma_b = \dfrac{P}{d_p t_{gp}}$ gusset plate is thinner than $(2\ t_{ep})$ so gusset plate controls

$\sigma_b = 139.9\ \text{MPa}$ ←

(b) ULTIMATE FORCE IN SHEAR

Cross sectional area of pin

$$A_p = \frac{\pi d_p^2}{4}$$

$A_p = 380.133\ \text{mm}^2$

$P_{\text{ult}} = 2\tau_{\text{ult}} A_p \qquad P_{\text{ult}} = 144.4\ \text{kN}$ ←

Problem 1.8-3

NUMERICAL DATA

$P = 160$ kips $\quad d_p = 2$ in.

$t_g = 1.5$ in. $\quad t_f = 1$ in.

(a) SHEAR STRESS ON PIN

$$\tau = \frac{V}{\left(\frac{\pi d_p^2}{4}\right)} \qquad \tau = \frac{\frac{P}{4}}{\left(\frac{\pi d_p^2}{4}\right)}$$

$$\tau = 12.73 \text{ ksi} \quad \leftarrow$$

(b) BEARING STRESS ON PIN FROM FLANGE PLATE

$$\sigma_{bf} = \frac{\frac{P}{4}}{d_p t_f} \qquad \sigma_{bf} = 20 \text{ ksi} \quad \leftarrow$$

BEARING STRESS ON PIN FROM GUSSET PLATE

$$\sigma_{bg} = \frac{\frac{P}{2}}{d_p t_g} \qquad \sigma_{bg} = 26.7 \text{ ksi} \quad \leftarrow$$

Problem 1.8-4

numerical data $t_r = 4 \ \ mm$ $t_s = 5 \ mm$ $d_p = 8 \ mm$ $P = 85\cdot(9.81)$ $P = 833.85 \ \ N$

$a = 1.8 \ m$ $b = 0.7 \ m$ $H = 7.5 \ \ m$ $q = 40 \ \frac{N}{m}$

(a) support reactions

$L = \sqrt{(a+b)^2 + H^2}$ $L = 7.906 \ \ m$ $L_{AC} = \frac{a}{a+b}\cdot L$ $L_{AC} = 5.692$

$L_{CB} = \frac{b}{a+b}\cdot L$ $L_{CB} = 2.214$ $L_{AC} + L_{CB} = 7.906$

sum moments about A

$B_x = \dfrac{P\cdot a + q\cdot L\cdot\left(\frac{a+b}{2}\right)}{-H}$ $B_x = -252.829$ N (left) & Ax = -Bx (Ax acts to right) $A_x = -B_x$

$A_y = P + q\cdot L$ $A_y = 1150.078 \ \ N$ $\boxed{B_x = -252.8}$ N $\boxed{A_x = -B_x}$ $\boxed{A_y = 1150.1}$ N

(b) resultant force in shoe bolt at A $A_{resultant} = \sqrt{{A_x}^2 + {A_y}^2}$

$A_{resultant} = 1177.54 \ \ N$ $\boxed{A_{resultant} = 1178}$ N

(c) maximum shear and bearing stresses in shoe bolt at A $d_p = 8 \ \ mm$ $t_s = 5 \ \ mm$ $t_r = 4 \ \ mm$

shear area $A_s = \frac{\pi}{4}\cdot {d_p}^2$ $A_s = 50.265 \ \ mm^2$ shear stress $\tau = \dfrac{\frac{A_{resultant}}{2}}{2\cdot A_s}$ $\boxed{\tau = 5.86}$ MPa

bearing area $A_b = 2\cdot d_p\cdot t_s$ $A_b = 80 \ \ mm^2$ bearing stress $\sigma_{bshoe} = \dfrac{\frac{A_{resultant}}{2}}{A_b}$ $\sigma_{bshoe} = 7.36$ MPa

Check bearing stress from ladder rail $\sigma_{brail} = \dfrac{\frac{A_{resultant}}{2}}{d_p\cdot t_r}$ $\boxed{\sigma_{brail} = 18.4}$ MPa

Problem 1.8-5

NUMERICAL DATA

$d_p = 0.25 \text{ in.} \qquad L = \frac{5}{8} \text{ in.} \qquad CD = 3.25 \text{ in.}$

$BC = 1 \text{ in.} \qquad T = 45 \text{ lb}$

EQUILIBRIUM - FIND HORIZONTAL FORCES AT B AND C [VERTICAL REACTION $V_B = 0$]

$\sum M_B = 0 \qquad H_C = \dfrac{T(BC + CD)}{BC}$

$H_C = 191.25 \text{ lb} \qquad \sum F_H = 0$

$H_B = T - H_C \qquad H_B = -146.25 \text{ lb}$

(a) FIND THE AVE SHEAR STRESS τ_{ave} IN THE PIVOT PIN WHERE IT IS ANCHORED TO THE BICYCLE FRAME AT B:

$A_S = \dfrac{\pi d_p^{\,2}}{4} \qquad A_s = 0.049 \text{ in.}^2$

$\tau_{\text{ave}} = \dfrac{|H_B|}{A_S} \qquad \tau_{\text{ave}} = 2979 \text{ psi} \quad \longleftarrow$

(b) FIND THE AVE BEARING STRESS $\sigma_{b,\text{ave}}$ IN THE PIVOT PIN OVER SEGMENT AB.

$A_b = d_p L \qquad A_b = 0.156 \text{ in.}^2$

$\sigma_{b,\text{ave}} = \dfrac{|H_B|}{A_b} \qquad \sigma_{b,\text{ave}} = 936 \text{ psi} \quad \longleftarrow$

Problem 1.8-6

NUMERICAL DATA

$L_1 = 3.2\text{ m} \quad L_2 = 3.9\text{ m} \quad \alpha = 54.9\left(\frac{\pi}{180}\right)\text{rad}$

$\theta = 94.4\left(\frac{\pi}{180}\right)\text{rad}$

$a = 0.6\text{ m} \quad b = 1\text{ m}$

$W = 77.0(2.5 \times 1.5 \times 0.08) \quad W = 23.1\text{ kN}$

(77 = wt density of steel, kN/m^3)

SOLUTION APPROACH

STEP (1) $d = \sqrt{a^2 + b^2} \quad d = 1.166\text{ m}$

STEP (2) $\theta_1 = \arctan\left(\frac{a}{b}\right) \quad \theta_1\frac{180}{\pi} = 30.964°$

STEP (3)-Law of cosines

$$H = \sqrt{d^2 + L_1^2 - 2dL_1\cos(\theta + \theta_1)}$$

$$H = 3.99\text{ m}$$

STEP (4) $\beta_1 = \arccos\left(\frac{L_1^2 + H^2 - d^2}{2L_1H}\right)$

$$\beta_1\frac{180}{\pi} = 13.789°$$

STEP (5) $\beta_2 = \arccos\left(\frac{L_2^2 + H^2 - d^2}{2L_2H}\right)$

$$\beta_2\frac{180}{\pi} = 16.95°$$

STEP (6)

Check $(\beta_1 + \beta_2 + \theta + \alpha)\frac{180}{\pi}$

$= 180.039°$

STATICS

$T_1\sin(\beta_1) = T_2\sin(\beta_2)$

$T_1 = T_2\left(\frac{\sin(\beta_2)}{\sin(\beta_1)}\right)$

$T_1\cos(\beta_1) + T_2\cos(\beta_2) = W$

$$T_2 = \frac{W}{\cos(\beta_1)\frac{\sin(\beta_2)}{\sin(\beta_1)} + \cos(\beta_2)}$$

$T_2 = 10.77\text{ kN} \quad \leftarrow$

$T_1 = T_2\left(\frac{\sin(\beta_2)}{\sin(\beta_1)}\right) \quad T_1 = 13.18\text{ kN} \quad \leftarrow$

$T_1\cos(\beta_1) + T_2\cos(\beta_2) = 23.1$ < checks

SHEAR & BEARING STRESSES

$d_p = 18\text{ mm} \quad t = 80\text{ mm}$

$A_S = \frac{\pi}{4}d_p^2 \quad A_b = td_p$

$\tau_{1ave} = \frac{\frac{T_1}{2}}{A_S} \quad \tau_{1ave} = 25.9\text{ MPa} \quad \leftarrow$

$\tau_{2ave} = \frac{\frac{T_2}{2}}{A_S} \quad \tau_{2ave} = 21.2\text{ MPa} \quad \leftarrow$

$\sigma_{b1} = \frac{T_1}{A_b} \quad \sigma_{b1} = 9.15\text{ MPa} \quad \leftarrow$

$\sigma_{b2} = \frac{T_2}{A_b} \quad \sigma_{b2} = 7.48\text{ MPa} \quad \leftarrow$

Problem 1.8-7

cable forces

$T_1 = 800 \quad lb \qquad T_2 = 550 \quad lb \qquad T_3 = 1241 \quad lb \qquad d = 0.50 \qquad t_p = 0.75 \qquad t = 0.25 \quad inches$

(a) resultant force on eye bolt from 3 cables

$T_3 \cdot \cos(30 \cdot deg) = 1075 \qquad T_1 + T_2 \cdot \sin(30 \cdot deg) = 1075$ < so resultant has no y-component

$P = T_2 \cdot \frac{\sqrt{3}}{2} + T_3 \cdot 0.5 \qquad P = 1097 \quad lb$ < x-component only

(b) ave. bearing stress between hex nut & plate

$A_{brg} = 0.2194 \, in^2$ < hexagon area (Case 25, App. E) minus bolt x-sec area

$\sigma_b = \frac{P}{A_{brg}} \qquad \sigma_b = 4999 \quad psi$

(c) shear through nut $\qquad d = 0.5 \quad in$ < bolt diameter $\qquad t = 0.25 \quad in$ < nut thickness

$A_{sn} = (\pi \cdot d) \cdot t \qquad A_{sn} = 0.393 \qquad \tau_{nut} = \frac{P}{A_{sn}} \qquad \tau_{nut} = 2793 \quad psi$

shear through plate $\qquad t_p = 0.75 \qquad r = 0.40$ < r = length of side of hexagon (also = b below)

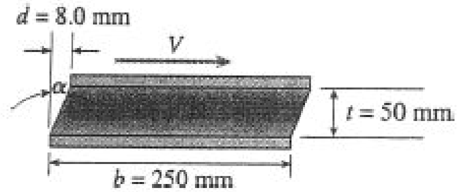
$A_{spl} = (6 \cdot r) \cdot t_p \qquad A_{spl} = 1.8 \quad in^2 \qquad \tau_{pl} = \frac{P}{A_{spl}} \qquad \tau_{pl} = 609 \quad psi$

Problem 1.8-8

NUMERICAL DATA

$V = 12\ \text{kN}$ $\quad a = 125\ \text{mm}$

$b = 240\ \text{mm}$ $\quad t = 50\ \text{mm}$ $\quad d = 8\ \text{mm}$



AVERAGE SHEAR STRESS

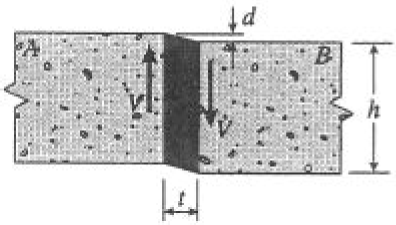
$$\tau_{ave} = \frac{V}{ab} \qquad \tau_{ave} = 0.4\ \text{MPa}$$

AVERAGE SHEAR STRAIN $\quad \gamma_{ave} = \dfrac{d}{t} \qquad \gamma_{ave} = 0.16$

SHEAR MODULUS G $\quad G = \dfrac{\tau_{ave}}{\gamma_{ave}}$

$$G = 2.5\ \text{MPa} \quad \leftarrow$$

Problem 1.8-9



$h = 4.0$ in. $\quad t = 0.5$ in.

$L = 40$ in. $\quad d = 0.002$ in.

$G = 140$ ksi

(a) AVERAGE SHEAR STRAIN

$$\gamma_{aver} = \frac{d}{t} = 0.004 \quad \leftarrow$$

(b) SHEAR FORCES V

Average shear stress: $\tau_{aver} = G\gamma_{aver}$

$$V = \tau_{aver}(hL) = G\gamma_{aver}(hL)$$
$$= (140 \text{ ksi})(0.004)(4.0 \text{ in.})(40 \text{ in.})$$
$$= 89.6 \text{ k} \quad \leftarrow$$

Problem 1.8-10

$d_1 = 24mm \qquad d_2 = 16mm \qquad t = 4mm \qquad P = 70kN$

average shear stress in plate: $\tau_{pl} = \dfrac{P}{\pi \cdot d_2 \cdot t} = 348.151 \cdot MPa$

compressive stresses in punch shaft:

$$\sigma_{upper} = \frac{P}{\frac{\pi}{4} \cdot d_1^{\,2}} = 154.734 \cdot MPa \qquad \sigma_{lower} = \frac{P}{\frac{\pi}{4} \cdot d_2^{\,2}} = 348.151 \cdot MPa$$

Problem 1.8-11

$h = 0.5\text{in} \quad L = 30\text{in} \quad t = 0.5\text{in} \quad V = 25\text{kip} \quad G = 100\text{ksi}$

$\tau = \dfrac{V}{L \cdot h} = 1.667 \cdot \text{ksi} \qquad \gamma = \dfrac{\tau}{G} = 0.0167$

$d = \gamma \cdot t = 8.333 \times 10^{-3} \cdot \text{in} \quad \text{or} \quad d = \tan(\gamma) \cdot t = 8.334 \times 10^{-3} \cdot \text{in}$

Problem 1.8-12

$t = 1mm \qquad P = 10N \qquad L = 2\cdot(12mm) + 2\cdot(1.5mm)\cdot\pi = 33.425\cdot mm$

$$\tau_{ave} = \frac{P}{L\cdot t} = 0.299\cdot MPa$$

Problem 1.8-13

Part (a): pipe suspended in air

$L = 5000ft \qquad \gamma_s = 490 \frac{lbf}{ft^3} \qquad \gamma_w = 63.8 \frac{lbf}{ft^3}$

$d_2 = 16in \qquad d_1 = 15in \qquad t = \frac{d_2 - d_1}{2} = 0.5 \cdot in \qquad t_f = 1.75in \qquad A_{pipe} = \frac{\pi}{4} \cdot \left(d_2^{\ 2} - d_1^{\ 2}\right) = 24.347 \cdot in^2$

$W_{pipe} = \gamma_s \cdot A_{pipe} \cdot L = 414.243 \cdot kip$

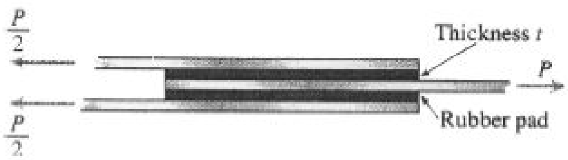
$n = 6 \qquad d_b = 1.125in \qquad d_w = 1.875in \qquad A_b = \frac{\pi}{4} \cdot d_b^{\ 2} = 0.994 \cdot in^2 \qquad A_w = \frac{\pi}{4} \cdot \left(d_w^{\ 2} - d_b^{\ 2}\right) = 1.8 \cdot in^2$

$\boxed{\sigma_b = \frac{W_{pipe}}{n \cdot A_b} = 69.5 \cdot ksi} \qquad \boxed{\sigma_{brg} = \frac{W_{pipe}}{n \cdot A_w} = 39.1 \cdot ksi} \qquad \boxed{\tau_f = \frac{W_{pipe}}{n \cdot \pi \cdot d_w \cdot t_f} = 6.7 \cdot ksi}$

Part (b): pipe suspended in sea water $\qquad W_{inwater} = \left(\gamma_s - \gamma_w\right) \cdot A_{pipe} \cdot L = 360.307 \cdot kip$

$\boxed{\sigma_b = \frac{W_{inwater}}{n \cdot A_b} = 60.4 \cdot ksi} \qquad \boxed{\sigma_{brg} = \frac{W_{inwater}}{n \cdot A_w} = 34 \cdot ksi} \qquad \boxed{\tau_f = \frac{W_{inwater}}{n \cdot \pi \cdot d_w \cdot t_f} = 5.83 \cdot ksi}$

Problem 1.8-14



Rubber pads: $t = 9$ mm

Length $L = 160$ mm

Width $b = 80$ mm

$G = 1250$ kPa

$P = 16$ kN

(a) Shear stress and strain in the rubber pads

$$\tau_{aver} = \frac{P/2}{bL} = \frac{8\text{ kN}}{(80\text{ mm})(160\text{ mm})} = 625\text{ kPa}$$

$$\gamma_{aver} = \frac{\tau_{aver}}{G} = \frac{625\text{ kPa}}{1250\text{ kPa}} = 0.50 \quad \longleftarrow$$

(b) Horizontal displacement

$$\delta = t \times \tan(\gamma_{ave}) = 4.92\text{ mm}$$

Problem 1.8-15

NUMERICAL DATA

$t = \frac{1}{8}$ in. $\quad b = 2$ in.

$h = 7$ in. $\quad W_1 = 10$ lb $\quad W_2 = 40$ lb

$P = 30$ lb $\quad d_B = 0.25$ in. $\quad d_p = \frac{5}{16}$ in.

(a) REACTIONS AT A

$A_x = 0 \quad \leftarrow$

$A_y = W_1 + W_2 + 4P \quad \leftarrow$

$A_y = 170 \text{ lb} \quad \leftarrow$

$L_1 = 17 + 2.125 + 6 \qquad L_1 = 25$ in.

(dist from A to first bike)

$M_A = W_1(9) + W_2(19) + P(4L_1 + 4 + 8 + 12)$

$M_A = 4585$ in.-lb

(b) FORCES IN BOLT AT B AND PIN AT C

$\Sigma F_y = 0 \qquad B_y = W_2 + 4P \qquad B_y = 160 \text{ lb} \quad \leftarrow$

$\Sigma M_B = 0$

Right hand FBD

$$B_x = \frac{[W_2(19 - 17) + P(6 + 2.125) + P(8.125 + 4) + P(8.125 + 8) + P(8.125 + 12)]}{h}$$

$B_x = 254 \text{ lb} \quad \leftarrow \qquad C_x = -B_x$

$B_{res} = \sqrt{B_x^{\,2} + B_y^{\,2}} \qquad B_{res} = 300 \text{ lb} \quad \leftarrow$

(c) AVERAGE SHEAR STRESSES τ_{ave} IN BOTH THE BOLT AT B AND THE PIN AT C

$A_{sB} = 2\frac{\pi d_B^2}{4} \qquad A_{sB} = 0.098 \text{ in.}^2$

$\tau_B = \frac{B_{res}}{A_{sB}} \qquad \tau_B = 3054 \text{ psi} \quad \leftarrow$

$A_{sC} = 2\frac{\pi d_p^2}{4} \qquad A_{sC} = 0.153 \text{ in.}^2$

$\tau_C = \frac{B_x}{A_{sC}} \qquad \tau_C = 1653 \text{ psi} \quad \leftarrow$

(d) BEARING STRESSES σ_B IN THE BOLT AT B AND THE PIN AT C

$t = 0.125$ in.

$A_{bB} = 2td_B \qquad A_{bB} = 0.063 \text{ in.}^2$

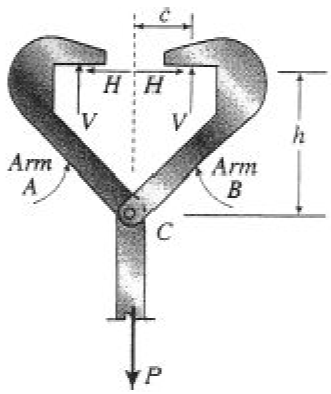
$\sigma_{bB} = \frac{B_{res}}{A_{bB}} \qquad \sigma_{bB} = 4797 \text{ psi} \quad \leftarrow$

$A_{bC} = 2td_p \qquad A_{bC} = 0.078 \text{ in.}^2$

$\sigma_{bC} = \frac{C_x}{A_{bC}} \qquad \sigma_{bC} = 3246 \text{ psi} \quad \leftarrow$

Problem 1.8-16

FREE-BODY DIAGRAM OF CLAMP



$h = 250$ mm

$c = 100$ mm

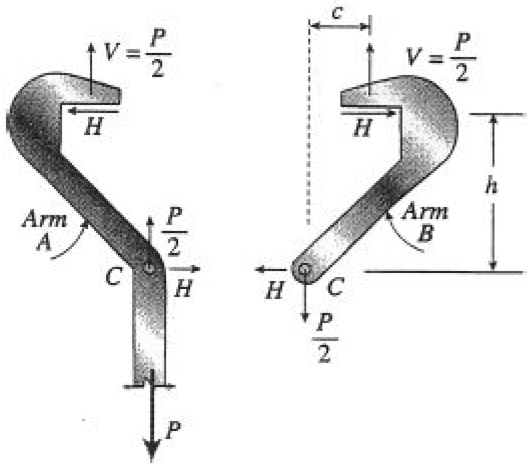
$P = 18$ kN

From vertical equilibrium:

$$V = \frac{P}{2} = 9 \text{ kN}$$

d = diameter of pin at C = 12 mm

FREE-BODY DIAGRAMS OF ARMS A AND B

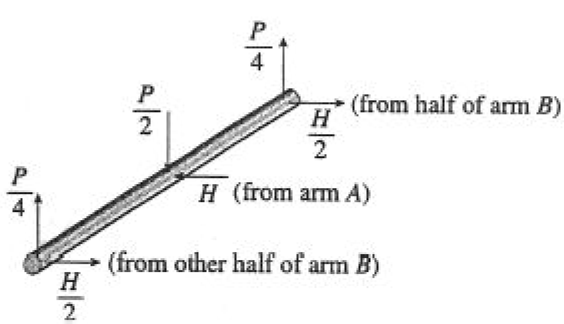


$$\Sigma M_C = 0$$

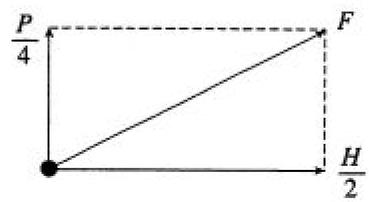
$$V_C - Hh = 0$$

$$H = \frac{V_C}{h} = \frac{P_c}{2h} = 3.6 \text{ kN}$$

FREE-BODY DIAGRAM OF PIN



SHEAR FORCE F IN PIN



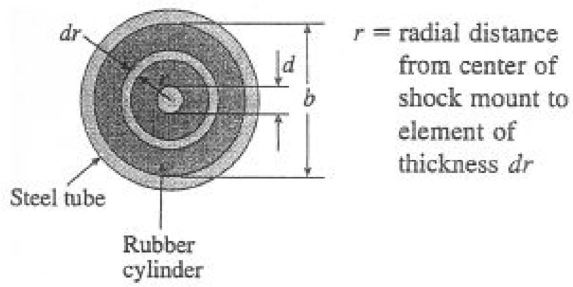
$$F = \sqrt{\left(\frac{P}{4}\right)^2 + \left(\frac{H}{2}\right)^2}$$

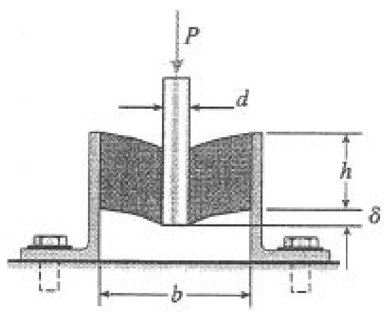
$$= 4.847 \text{ kN}$$

AVERAGE SHEAR STRESS IN THE PIN

$$\tau_{aver} = \frac{F}{A_{pin}} = \frac{F}{\frac{\pi a^2}{4}} = 42.9 \text{ MPa} \quad \leftarrow$$

Problem 1.8-17





r = radial distance from center of shock mount to element of thickness dr

(a) Shear stress τ at radial distance r

A_s = shear area at distance $r = 2\pi rh$

$$\tau = \frac{P}{A_s} = \frac{P}{2\pi rh} \quad \leftarrow$$

(b) Downward displacement δ

γ = shear strain at distance r

$$\gamma = \frac{\tau}{G} = \frac{P}{2\pi rhG}$$

$d\delta$ = downward displacement for element dr

$$d\delta = \gamma dr = \frac{Pdr}{2\pi rhG}$$

$$\delta = \int d\delta = \int_{d/2}^{b/2} \frac{Pdr}{2\pi rhG}$$

$$\delta = \frac{P}{2\pi hG}\int_{d/2}^{b/2}\frac{dr}{r} = \frac{P}{2\pi hG}\,[\ln r]_{d/2}^{b/2}$$

$$\delta = \frac{P}{2\pi hG}\ln\frac{b}{d} \quad \leftarrow$$

Problem 1.8-18

Numerical data $L = 2.75 \cdot m$ $W_y = 667 \cdot N$ $H = 150 \cdot mm$ $h = 108 \cdot mm$ $b = 96 \cdot mm$

$t = 14 \cdot mm$ $d_b = 12 \cdot mm$ $d_w = 22 \cdot mm$

a) Find average **shear stress** (τ, MPa) at **bolt #1** due to the wind force $\boldsymbol{W_y}$; repeat for **bolt #4**

$$A_b = \frac{\pi}{4} \cdot d_b^2 = 113.097 \cdot mm^2 \qquad \tau_1 = \frac{\frac{W_y}{2}}{A_b} = 2.95 \cdot MPa \qquad \boxed{\tau_1 = 2.95 \cdot MPa} \qquad \boxed{\tau_4 = 0} \qquad \frac{W_y}{2} = 333.5\,N$$

^ only bolts 1 & 2 resist wind force shear in +y dir.

b) Find ave. **bearing stress** (σ_b, MPa) between the bolt and the base plate (thickness t) at **bolt** #1; repeat for **bolt #4**

$$A_{brg} = d_b \cdot t = 168 \cdot mm^2$$

$$\sigma_{b1} = \frac{\frac{W_y}{2}}{A_{brg}} = 1.985 \cdot MPa \qquad \boxed{\sigma_{b1} = 1.985 \cdot MPa} \qquad \boxed{\sigma_{b4} = 0}$$

^ only bolts 1 & 2 resist wind force bearing in +y dir.

c) Find ave. **bearing stress** (σ_b, MPa) between base plate and washer at **bolt #4** due to the wind force $\boldsymbol{W_y}$ (assume initial bolt pretension is zero)

Assume wind force creates OTM about x axis = OTM_x $\qquad OTM_x = W_y \cdot L = 1834.25 \cdot N \cdot m$

OTM is resisted by force couples pairs at bolts 1-4 & 2-3; so force in bolt 4 is: $\qquad F_4 = \frac{OTM_x}{2 \cdot h} = 8491.898\,N$

Bearing area is donut shaped area of washer in contact with the plate minus *approx*. rect. cutout for slot

$$A_{brg} = \frac{\pi}{4} \cdot \left(d_w^2 - d_b^2\right) - d_b \cdot \left(\frac{d_w - d_b}{2}\right) = 207.035 \cdot mm^2 \qquad \sigma_{b4} = \frac{F_4}{A_{brg}} = 41 \cdot MPa \qquad \boxed{\sigma_{b4} = 41 \cdot MPa}$$

d) Find ave. **shear stress** (τ, MPa) through the base plate at **bolt #4** due to the wind force $\boldsymbol{W_y}$;

Use force F_4 above; shear stress is on cyl. surface at perimeter of washer; must deduct *approx*. rect. area due to slot

$$A_{sh} = \left(\pi \cdot d_w - d_b\right) \cdot t = 799.611 \cdot mm^2 \qquad \tau = \frac{F_4}{A_{sh}} = 10.62 \cdot MPa \qquad \boxed{\tau = 10.62 \cdot MPa}$$

e) Find an expression for **normal stress** (σ) in **bolt #3** due to the wind force $\boldsymbol{W_y}$.

Force in bolt 3 due to OTM_x is same as that in bolt 4 $\qquad \sigma_3 = \frac{F_4}{A_b} = 75.1 \cdot MPa \qquad \boxed{\sigma_3 = 75.1 \cdot MPa}$

Problem 1.8-19

NUMERICAL DATA

$F = 5\text{ lb} \quad t = \frac{1}{16}\text{ in.} \quad d_p = \frac{1}{8}\text{ in.} \quad d_b = \frac{3}{16}\text{ in.}$

$f_p = 30\text{ lb} \qquad d_N = \frac{5}{8}\text{ in.} \qquad \theta = 15\frac{\pi}{180}\text{ rad}$

$a = 0.75\text{ in.} \qquad b = 1.5\text{ in.} \qquad c = 1.75\text{ in.}$

(a) FIND THE FORCE IN THE PIN AT O DUE TO APPLIED FORCE F

$$\sum M_o = 0$$

$$F_{AB} = \frac{[F\cos(\theta)(b - a)] + F\sin(\theta)(c)}{a}$$

$$F_{AB} = 7.849\text{ lb}$$

$$\sum F_H = 0 \qquad O_x = F_{AB} + F\cos(\theta)$$

$$O_y = F\sin(\theta)$$

$$O_x = 12.68\text{ lb} \qquad O_y = 1.294\text{ lb}$$

$$O_{\text{res}} = \sqrt{O_x^2 + O_y^2} \qquad O_{\text{res}} = 12.74\text{ lb} \quad \leftarrow$$

(b) FIND AVERAGE SHEAR STRESS τ_{ave} AND BEARING STRESS σ_b IN THE PIN AT O

$$A_s = 2\frac{\pi d_p^2}{4} \qquad \tau_O = \frac{O_{\text{res}}}{A_s} \qquad \tau_O = 519\text{ psi} \quad \leftarrow$$

$$A_b = 2td_p \qquad \sigma_{bO} = \frac{O_{\text{res}}}{A_b} \qquad \sigma_{bO} = 816\text{ psi} \quad \leftarrow$$

(c) FIND THE AVERAGE SHEAR STRESS τ_{ave} IN THE BRASS RETAINING BALLS AT B DUE TO WATER PRESSURE FORCE F_p

$$A_s = 3\frac{\pi d_b^2}{4} \qquad \tau_{\text{ave}} = \frac{f_p}{A_s} \qquad \tau_{\text{ave}} = 362\text{ psi} \quad \leftarrow$$

Problem 1.8-20

<u>numerical data</u>

$d_s = 8$ mm $\quad d_b = 10$ mm $\quad m = 20$ kg

$a = 760 \quad b = 254 \quad c = 506 \quad d = 150 \quad h = 660 \quad h_o = 490 \quad H = h\cdot\left(\tan\left(30\cdot\frac{\pi}{180}\right) + \tan\left(45\cdot\frac{\pi}{180}\right)\right)$

$H = 1.041\times 10^3$ mm

$W = m\cdot(9.81) \quad W = 196.2 \quad N = kg\cdot\frac{m}{s^2} \qquad \frac{a+b+c}{2} = 760$

vector r_{AB}

$$r_{AB} = \begin{pmatrix} 0 \\ H \\ c-d \end{pmatrix} \qquad r_{AB} = \begin{pmatrix} 0 \\ 1.041\times 10^3 \\ 356 \end{pmatrix}$$

unit vector e_{AB}

$$e_{AB} = \frac{r_{AB}}{|r_{AB}|} \qquad e_{AB} = \begin{pmatrix} 0 \\ 0.946 \\ 0.324 \end{pmatrix} \qquad |e_{AB}| = 1$$

$$W = \begin{pmatrix} 0 \\ -W \\ 0 \end{pmatrix} \qquad W = \begin{pmatrix} 0 \\ -196.2 \\ 0 \end{pmatrix} \qquad r_{DO} = \begin{pmatrix} h_o \\ h_o \\ b+c \end{pmatrix} \qquad r_{DO} = \begin{pmatrix} 490 \\ 490 \\ 760 \end{pmatrix}$$

$\sum M_D \qquad M_D = r_{DB}\times F_s\cdot e_{AB} + W\times r_{DO}$ < ignore force at hinge C since it will vanish with moment about line DC

$$F_{sx} = 0 \qquad F_{sy} = \frac{H}{\sqrt{H^2+(c-d)^2}}\cdot F_s \qquad F_{sz} = \frac{c-d}{\sqrt{H^2+(c-d)^2}}\cdot F_s$$

$$\frac{H}{\sqrt{H^2+(c-d)^2}} = 0.946 \qquad \frac{c-d}{\sqrt{H^2+(c-d)^2}} = 0.324$$

(a) <u>Find the strut force Fs and average normal stress σ in the strut</u>

$$\sum M_{lineDC} = 0 \qquad F_{sy} = \frac{|W|\cdot h_o}{h} \qquad F_{sy} = 145.664 \qquad F_s = \frac{F_{sy}}{\frac{H}{\sqrt{H^2+(c-d)^2}}} \qquad F_s = 153.9 \text{ N}$$

$$A_{strut} = \frac{\pi}{4}\cdot d_s^2 \qquad A_{strut} = 50.265 \text{ mm}^2 \qquad \sigma = \frac{F_s}{A_{strut}} \qquad \sigma = 3.06 \text{ MPa}$$

(b) <u>Find the average shear stress τ_{ave} in the bolt at A</u> $\quad d_b = 10$ mm

$$A_s = \frac{\pi}{4}\cdot d_b^2 \qquad A_s = 78.54 \qquad \tau = \frac{F_s}{A_s} \qquad \tau = 1.96 \text{ MPa}$$

(c) <u>Find the bearing stress σ_b on the bolt at A</u> $\quad A_b = d_s\cdot d_b \quad A_b = 80$ mm^2

$$\sigma_b = \frac{F_s}{A_b} \qquad \sigma_b = 1.924 \text{ MPa}$$

Problem 1.8-21

NUMERICAL PROPERTIES

$d_p = \frac{1}{8}$ in. $\quad t_b = \frac{3}{32}$ in. $\quad t_c = \frac{3}{8}$ in.

$T = 25$ lb $\quad d_{BC} = 6$ in.

$d_{CD} = 1$ in.

(a) FIND THE CUTTING FORCE P ON THE CUTTING BLADE AT D IF THE TENSION FORCE IN THE ROPE IS $T = 25$ lb:

$\sum M_c = 0$

$$\begin{aligned} M_C = & T(6 \sin(70°)) \\ & + 2T \cos(20°)(6 \sin(70°)) \\ & - 2T \sin(20°)(6 \cos(70°)) \\ & - P \cos(20°)(1) \end{aligned}$$

SOLVE EQUATION FOR P

$$P = \frac{[T(6\sin(70°)) + 2T\cos(20°) \, 6\sin(70°)) - 2T\sin(20°)(6\cos(70°))]}{\cos(20°)}$$

$P = 395$ lbs $\quad \leftarrow$

(b) SOLVE FOR FORCES ON PIN AT C

$\sum F_x = 0 \quad C_x = T + 2T\cos(20°) + P\cos(40°)$

$C_x = 374$ lbs $\quad \leftarrow$

$\sum F_y = 0 \quad C_y = 2T\sin(20°) - P\sin(40°)$

$C_y = -237$ lbs $\quad \leftarrow$

RESULTANT AT C

$C_{res} = \sqrt{C_x^2 + C_y^2} \qquad C_{res} = 443$ lbs $\quad \leftarrow$

(c) FIND MAXIMUM SHEAR AND BEARING STRESSES IN THE SUPPORT PIN AT C (SEE SECTION A–A THROUGH SAW).

SHEAR STRESS—PIN IN DOUBLE SHEAR

$A_s = \frac{\pi}{4} d_p^2 \qquad A_s = 0.012$ in.^{2}

$\tau_{ave} = \frac{C_{res}}{2A_s} \qquad \tau_{ave} = 18.04$ ksi

BEARING STRESSES ON PIN ON EACH SIDE OF COLLAR

$\sigma_{bC} = \frac{\frac{C_{res}}{2}}{d_p t_c} \qquad \sigma_{bC} = 4.72$ ksi $\quad \leftarrow$

BEARING STRESS ON PIN AT CUTTING BLADE

$\sigma_{bcb} = \frac{C_{res}}{d_p t_b} \qquad \sigma_{bcb} = 37.8$ ksi $\quad \leftarrow$

Problem 1.8-22

Cable forces

$$\boldsymbol{F}_1 = F_1\boldsymbol{n}_{A1} = 110kN\left(\frac{3\boldsymbol{i} + (9 - 0.45)\boldsymbol{j}}{\sqrt{3^2 + 8.55^2}}\right) = 36.42\boldsymbol{i} + 103.8\boldsymbol{j}\ kN$$

$$\boldsymbol{F}_2 = F_2\boldsymbol{n}_{A2} = 85kN\left(\frac{6.5\boldsymbol{i} + (8.5 - 0.45)\boldsymbol{j} + 2\boldsymbol{k}}{\sqrt{6.5^2 + 8.05^2 + 2^2}}\right) = 52.43\boldsymbol{i} + 64.93\boldsymbol{j} + 16.13\boldsymbol{k}\ kN$$

$$\boldsymbol{F}_3 = F_3\boldsymbol{n}_{A3} = 90kN\left(\frac{8\boldsymbol{i} + (9 - 0.45)\boldsymbol{j} + 5\boldsymbol{k}}{\sqrt{8^2 + 8.55^2 + 5^2}}\right) = 56.55\boldsymbol{i} + 60.44\boldsymbol{j} + 35.34\boldsymbol{k}\ kN$$

The resultant of cable forces $\boldsymbol{F_1}$, $\boldsymbol{F_2}$ and $\boldsymbol{F_3}$ is easily obtained as:

$$\boldsymbol{Q} = \boldsymbol{F}_1 + \boldsymbol{F}_2 + \boldsymbol{F}_3 = 145.4\boldsymbol{i} + 229.2\boldsymbol{j} + 51.5\boldsymbol{k}\ kN$$

(a) Reactions - use resultant forces Q_x, Q_y, Q_z applied at point A (a distance S = 0.45 m above base)

$$\Sigma F_x = 0: \quad R_x + Q_x = 0 \quad so\ R_x = -Q_x = -145.4kN$$

$$\Sigma F_y = 0: \quad R_y + Q_y = 0 \quad so\ R_y = -Q_y = -229.2kN$$

$$\Sigma F_z = 0: \quad R_z + Q_z = 0 \quad so\ R_z = -Q_z = -51.5kN$$

$$\Sigma M_x = 0: \quad M_x + Q_z(0.45m) = 0 \ \ so \ \ M_x = -(51.5kN)(0.45m) = -23.2kN \cdot m$$

$$\Sigma M_y = 0: \quad M_y = 0$$

$$\Sigma M_z = 0: \quad M_z - Q_x(0.45m) = 0 \ \ so \ \ M_z = (145.4kN)(0.45m) = 65.4kN \cdot m$$

(b) Average shear stress for each of 8 anchor bolts $\quad d_b = 24mm \quad R_x = 145.4kN \quad R_z = 51.5kN$

$$\tau_{ave} = \frac{\sqrt{R_x^{\,2} + R_z^{\,2}}}{8\cdot\left(\frac{\pi}{4}\cdot d_b^{\,2}\right)} = 42.621\cdot MPa$$

Problem 1.8-23

a) Reactions at point O $\quad P_1 = 110\text{lbf} \quad P_2 = P_1$

Force vectors and resultant

$$P_1 = P_1 \cdot \begin{pmatrix} 0 \\ \frac{-2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{pmatrix} = \begin{pmatrix} 0 \\ -98.387 \\ 49.193 \end{pmatrix} \cdot \text{lbf} \qquad P_2 = P_2 \cdot \begin{pmatrix} 0 \\ \frac{-5}{\sqrt{26}} \\ \frac{1}{\sqrt{26}} \end{pmatrix} = \begin{pmatrix} 0 \\ -107.864 \\ 21.573 \end{pmatrix} \cdot \text{lbf} \qquad |P_1| = 110 \cdot \text{lbf}$$

$$|P_2| = 110 \cdot \text{lbf}$$

Resultant

$$R = P_1 + P_2 = \begin{pmatrix} 0 \\ -206.251 \\ 70.766 \end{pmatrix} \cdot \text{lbf} \qquad |R| = 218.053 \cdot \text{lbf}$$

Moment about point O (or Force-couple system at pt. O)

$$r_{OB} = \begin{pmatrix} 14 \\ 2 \\ 9 \end{pmatrix} \cdot \text{in} \qquad r_{OA} = \begin{pmatrix} 23 \\ 2 \\ 0 \end{pmatrix} \cdot \text{in} \qquad M_O = r_{OA} \times P_2 + r_{OB} \times P_1 = \begin{pmatrix} 1027 \\ -1185 \\ -3858 \end{pmatrix} \cdot \text{lbf} \cdot \text{in} \quad \text{or} \quad M_O = \begin{pmatrix} 85.6 \\ -98.7 \\ -321.5 \end{pmatrix} \cdot \text{lbf} \cdot \text{ft}$$

$$|M_O| = 4165 \cdot \text{lbf} \cdot \text{in} \quad \text{or} \quad |M_O| = 347 \cdot \text{lbf} \cdot \text{ft}$$

REACTIONS at point O

$$R_O = -R = \begin{pmatrix} 0 \\ 206.3 \\ -70.8 \end{pmatrix} \cdot \text{lbf} \qquad M_{Oreac} = -M_O = \begin{pmatrix} -1027 \\ 1185 \\ 3858 \end{pmatrix} \cdot \text{lbf} \cdot \text{in}$$

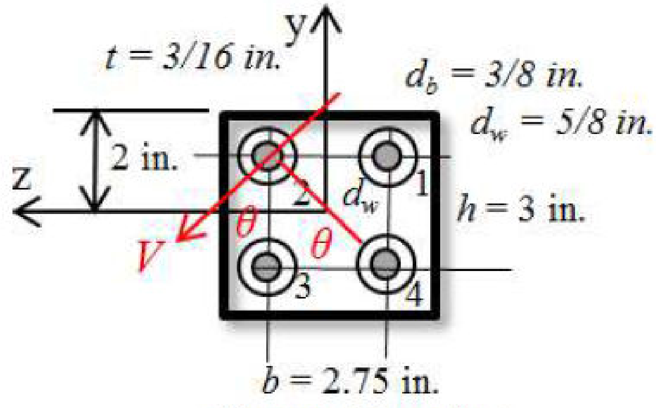
b) Shear stress on bolt 2 $\quad b = 2.75\text{in} \quad h = 3\text{in} \quad d = \sqrt{b^2 + h^2} = 4.07 \cdot \text{in} \quad \theta = \text{atan}\left(\frac{h}{b}\right) = 47.49 \cdot \text{deg}$

BOLT FORCES at bolt 2

$$F_{y2} = \frac{R_2}{4} = -51.563 \cdot \text{lbf} \qquad F_{z2} = \frac{R_3}{4} = 17.692 \cdot \text{lbf}$$

$$V = \frac{M_{O_1}}{2 \cdot d} = 126.178 \cdot \text{lbf}$$ < CCW twisting moment = 2 force couples

$$V_y = -V \cdot \cos(\theta) = -85.262 \cdot \text{lbf} \qquad V_z = V \cdot \sin(\theta) = 93.013 \cdot \text{lbf}$$



Support bracket

In-plane force resultant at bolt 2

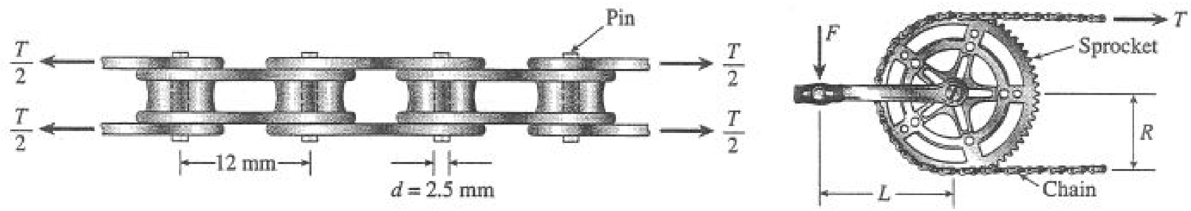
$$R_2 = \sqrt{(F_{y2} + V_y)^2 + (F_{z2} + V_z)^2} = 176.001 \cdot \text{lbf}$$

shear stress on bolt 2

$$d_b = \frac{3}{8} \text{in} \qquad A_b = \frac{\pi}{4} \cdot d_b^{\,2} = 0.11 \cdot \text{in}^2$$

$$\tau = \frac{R_2}{A_b} = 1594 \cdot \text{psi}$$

Problem 1.8-24



F = force applied to pedal = 800 N

L = length of crank arm

R = radius of sprocket

MEASUREMENTS (FOR AUTHOR'S BICYCLE)

(1) $L = 162$ mm (2) $R = 90$ mm

(a) TENSILE FORCE T IN CHAIN

$$\Sigma M_{\text{axle}} = 0 \qquad FL = TR \qquad T = \frac{FL}{R}$$

Substitute numerical values:

$$T = \frac{(800\text{ N})(162\text{ mm})}{90\text{ mm}} = 1440\text{ N} \quad \leftarrow$$

(b) SHEAR STRESS IN PINS

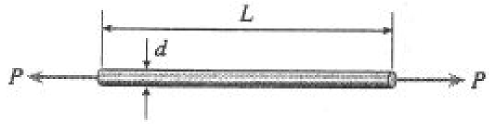
$$\tau_{\text{aver}} = \frac{T/2}{A_{\text{pin}}} = \frac{T}{2\frac{\pi d^2}{(4)}} = \frac{2T}{\pi d^2}$$

$$= \frac{2FL}{\pi d^2 R}$$

Substitute numerical values:

$$\tau_{\text{aver}} = \frac{2(800\text{ N})(162\text{ mm})}{\pi(2.5\text{ mm})^2(90\text{ mm})} = 147\text{ MPa} \quad \leftarrow$$

Problem 1.9-1



$L = 16.0$ in. $\quad d = 0.50$ in.

$E = 6.4 \times 10^6$ psi

$\sigma_{allow} = 17{,}000$ psi $\quad \delta_{max} = 0.04$ in.

MAXIMUM LOAD BASED UPON ELONGATION

$$\varepsilon_{max} = \frac{\delta_{max}}{L} = \frac{0.04 \text{ in.}}{16 \text{ in.}} 0.00250$$

$$\sigma_{max} = E\epsilon_{max} = (6.4 \times 10^6 \text{ psi})(0.00250)$$

$$= 16{,}000 \text{ psi}$$

$$P_{max} = \sigma_{max}A = (16.000 \text{ psi})\left(\frac{\pi}{4}\right)(0.50 \text{ in.})^2$$

$$= 3140 \text{ lb}$$

MAXIMUM LOAD BASED UPON TENSILE STRESS

$$P_{max} = \sigma_{allow}A = (17{,}000 \text{ psi})\left(\frac{\pi}{4}\right)(0.50 \text{ in.})^2$$

$$= 3340 \text{ lb}$$

ALLOWABLE LOAD

Elongation governs.

$P_{allow} = 3140$ lb $\quad \leftarrow$

Problem 1.9-2

NUMERICAL DATA

$r = 10 \quad d = 250 \text{ mm}$

^ bolts ^ flange

$A_s = \pi r^2$

$A_s = 314.159 \text{ m}^2$

$\tau_a = 85 \text{ MPa}$

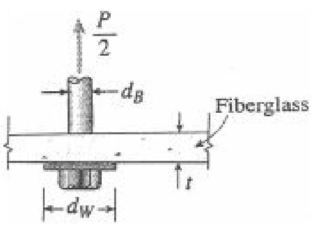
MAXIMUM PERMISSIBLE TORQUE

$$T_{max} = \tau_a A_s \left(r \frac{d}{2} \right)$$

$$T_{max} = 3.338 \times 10^7 \text{ N}\cdot\text{mm}$$

$$T_{max} = 33.4 \text{ kN}\cdot\text{m} \quad \leftarrow$$

Problem 1.9-3



$d_B = \frac{1}{4}$ in.

$d_W = \frac{7}{8}$ in.

$t = \frac{3}{8}$ in.

ALLOWABLE LOAD BASED UPON SHEAR STRESS IN FIBERGLASS

$\tau_{allow} = 300$ psi

Shear area $A_s = \pi d_W t$

$$\frac{P_1}{2} = \tau_{\text{allow}} A_s = \tau_{\text{allow}} (\pi d_W t)$$

$$= (300 \text{ psi})(\pi)\left(\frac{7}{8} \text{ in.}\right)\left(\frac{3}{8} \text{ in.}\right)$$

$$\frac{P_1}{2} = 309.3 \text{ lb}$$

$$P_1 = 619 \text{ lb}$$

ALLOWABLE LOAD BASED UPON BEARING PRESSURE

$\sigma_b = 550$ psi

Bearing area $A_b = \frac{\pi}{4}(d_W^2 - d_B^2)$

$$\frac{P_2}{2} = \sigma_b A_b = (550 \text{ psi})\left(\frac{\pi}{4}\right)\left[\left(\frac{7}{8} \text{ in.}\right)^2 - \left(\frac{1}{4} \text{ in.}\right)^2\right]$$

$$= 303.7 \text{ lb}$$

$$P_2 = 607 \text{ lb}$$

ALLOWABLE LOAD

Bearing pressure governs.

$P_{\text{allow}} = 607$ lb $\leftarrow$

Problem 1.9-4

Yield and ultimate stresses (all in MPa) TUBES AND PIN DIMENSIONS (MM)

TUBES: $\sigma_Y = 200$ $\sigma_u = 340$ $FS_y = 3.5$ $d_{AB} = 41$ $t_{AB} = 6.5$

PIN (SHEAR): $\tau_Y = 8$ $\tau_u = 140$ $FS_u = 4.5$ $d_{BC} = d_{AB} - 2\,t_{AB}$ $d_{BC} = 28$

PIN (BEARING): $\sigma_{bY} = 260$ $\sigma_{bu} = 450$ $t_{BC} = 7.5$ $d_p = 11$

(a) P_{ALLOW} CONSIDERING TENSION IN THE TUBES

$A_{netAB} = \frac{\pi}{4}\left[d_{AB}^2 - (d_{AB} - 2t_{AB})^2\right] - 4d_p t_{AB}$ $A_{netAB} = 418.502\ \text{mm}^2$

$A_{netBC} = \frac{\pi}{4}\left[d_{BC}^2 - (d_{BC} - 2t_{BC})^2\right] - 4d_p t_{BC}$ $A_{netBC} = 153.02$ $<$ use smaller

$P_{aT1} = \frac{\sigma_y}{FS_y} A_{netBC}$ $P_{aT1} = 8743.993\ \text{N}$ $<$ controls $\boxed{P_{allow} = 8.74\ \text{kN}}$

$P_{aT2} = \frac{\sigma_u}{FS_u} A_{netBC}$ $P_{aT2} = 11{,}561.501\ \text{N}$

(b) P_{allow} CONSIDERING SHEAR IN THE PINS $A_s = \frac{\pi}{4} d_p^2$ $A_s = 95.033\ \text{mm}^2$

$P_{aS1} = (4A_s)\frac{\tau_Y}{FS_y}$ $P_{aS1} = 8688.748\ \text{N}$ $<$ controls $\boxed{P_{allow} = 8.69\ \text{kN}}$

$P_{as2} = (4A_s)\frac{\tau_u}{FS_u}$ $P_{as2} = 11{,}826.351\ \text{N}$

(c) P_{allow} CONSIDERING BEARING IN THE PINS

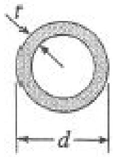
$A_{bAB} = 4\,d_p\,t_{AB}$ $A_{bAB} = 286\ \text{mm}^2$ $<$ smaller controls

$A_{bBC} = 4\,d_p\,t_{BC}$ $A_{bBC} = 330$

$P_{ab1} = A_{bAB}\left(\frac{\sigma_{by}}{FS_y}\right)$ $P_{ab1} = 21{,}245.714\ \text{N}$ $<$ controls $\boxed{P_{allow} = 21.2\ \text{kN}}$

$P_{ab2} = A_{bAB}\left(\frac{\sigma_{bu}}{FS_u}\right)$ $P_{ab2} = 28{,}600\ \text{N}$ $\boxed{\text{Overall, shear controls (Part (b))}}$

Problem 1.9-5



Four piers

$\sigma_U = 50$ ksi

$n = 3.5$

$$\sigma_{\text{allow}} = \frac{\sigma_U}{n} = \frac{50 \text{ ksi}}{3.5} = 14.29 \text{ ksi}$$

$d = 4.5$ in.

$t = 0.4$ in.

$d_0 = d - 2t = 3.7$ in.

$$A = \frac{\pi}{4}(d^2 - d_0^2) = \frac{\pi}{4}[(4.5 \text{ in.})^2 - (3.7 \text{ in.})^2]$$

$$= 5.152 \text{ in.}^2$$

P_1 = allowable load on one pier

$$= \sigma_{\text{allow}} A = (14.29 \text{ ksi})(5.152 \text{ in.}^2)$$

$$= 73.62 \text{ k}$$

Total load $P = 4P_1 = 294$ k $\quad\longleftarrow$

Problem 1.9-6

$\sigma_u = 400MPa \qquad P = 900kN \qquad FS_u = 3 \qquad t = 12mm$

$$\sigma_a = \frac{\sigma_u}{FS_u} = 133.333 \cdot MPa \qquad A_{reqd} = \frac{\frac{P}{4}}{\sigma_a} = 1687.5 \cdot mm^2$$

Set cross-sectional area of one pier equal to required area then solve for d

$$\frac{\pi}{4} \cdot \left[d^2 - (d - 2 \cdot t)^2\right] = A_{reqd} \qquad d = \frac{A_{reqd} + \pi \cdot t^2}{\pi \cdot t} = 56.8 \cdot mm$$

Problem 1.9-7

$L = 50ft \quad d_2 = 14in \quad d_1 = 13in \quad t_f = 1.5in \quad d_b = 1.125in \quad d_w = 1.875in$

$\sigma_{ap} = 50ksi \quad \sigma_{ab} = 120ksi$

From Table I-1

$$A_p = \frac{\pi}{4}\cdot\left(d_2^{\,2} - d_1^{\,2}\right) = 21.206\cdot in^2 \qquad A_b = \frac{\pi}{4}\cdot d_b^{\,2} = 0.994\cdot in^2 \qquad \gamma_s = 490\cdot\frac{lbf}{ft^3} \qquad \gamma_{sea} = 63.8\cdot\frac{lbf}{ft^3}$$

1) Permissible number of pipe segment (n) if hanging in air - max. normal stresses at top of pipe at drill rig

Based on allowable stress in pipe: $W = \gamma_s\cdot A_p\cdot n\cdot L \qquad \dfrac{\gamma_s\cdot A_p\cdot n\cdot L}{A_p} = \sigma_{ap}$ so $n = \dfrac{\sigma_{ap}}{\gamma_s\cdot L} = 293.878$

Based on allowable normal stress in each of 6 bolts: $\dfrac{\gamma_s\cdot A_p\cdot n\cdot L}{6\cdot A_b} = \sigma_{ab}$ so $n = \dfrac{6\cdot A_b\cdot\sigma_{ab}}{\gamma_s\cdot A_p\cdot L} = 198.367$

198 pipe segments controls

2) Permissible number of pipe segment (n) if hanging in sea water - max. normal stresses at top of pipe at drill rig

Based on allowable stress in pipe: $n = \dfrac{\sigma_{ap}}{\left(\gamma_s - \gamma_{sea}\right)\cdot L} = 337.87$

Based on allowable normal stress in each of 6 bolts: $n = \dfrac{6\cdot A_b\cdot\sigma_{ab}}{\left(\gamma_s - \gamma_{sea}\right)\cdot A_p\cdot L} = 228.062$

228 pipe segments controls

Problem 1.9-8

NUMERICAL DATA

$M_h = 43\text{ kg}$ $\sigma_a = 70\text{ MPa}$

$\tau_a = 45\text{ MPa}$ $\sigma_{ba} = 110\text{ MPa}$

$d_s = 10\text{ mm}$ $d_p = 9\text{ mm}$ $t = 8\text{ mm}$

$P = 50\text{ N}$ $g = 9.81\text{ m/s}^2$

(a) FORCE F IN EACH STRUT FROM STATICS (SUM MOMENTS ABOUT B)

$F_V = F\cos(10°)$ $F_H = F\sin(10°)$

$\sum M_B = 0$

$$F_V(127) + F_H(75) = \frac{M_h}{2}g\,(127 + 505) + \frac{P}{2}[127 + 2(505)]$$

$$F(127\cos(10°) + 75\sin(10°))$$

$$= \frac{M_h}{2}g\,(127 + 505) + \frac{P}{2} + 2(505)]$$

$$F = \frac{\frac{M_h}{2}g\,(127 + 505) + \frac{P}{2}[127 + 2(505)]}{(127\cos(10°) + 75\sin(10°))}$$

$F = 1.171\text{ kN}$ $\leftarrow$

(b) MAXIMUM PERMISSIBLE FORCE F IN EACH STRUT F_{max} IS SMALLEST OF THE FOLLOWING

$F_{a1} = \sigma_a \frac{\pi}{4} d_s^2$ $F_{a1} = 5.50\text{ kN}$

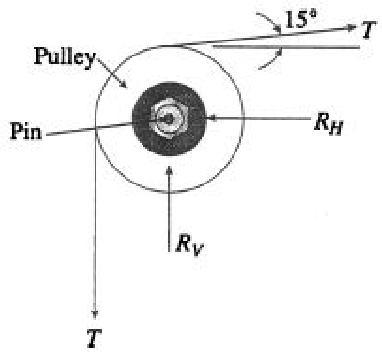
$F_{a2} = \tau_a \frac{\pi}{4} d_p^2$

$F_{a2} = 2.86\text{ kN}$ $\leftarrow$ $\frac{F_{a2}}{F} = 2.445$

$F_{a3} = \sigma_{ba} d_p t$ $F_{a3} = 7.92\text{ kN}$

Problem 1.9-9

FREE-BODY DIAGRAM OF ONE PULLEY



Pin diameter $d = 0.80$ in.

T = tensile force in one cable

$T_{allow} = 1800$ lb

$\tau_{allow} = 4000$ psi

W = weight of lifeboat

$= 1500$ lb

$\Sigma F_{horiz} = 0 \qquad R_H = T\cos 15° = 0.9659T$

$\Sigma F_{vert} = 0 \qquad R_V = T - T\sin 15° = 0.7412T$

V = shear force in pin

$$V = \sqrt{(R_H)^2 + (R_v)^2} = 1.2175T$$

ALLOWABLE TENSILE FORCE IN ONE CABLE BASED UPON SHEAR IN THE PINS

$$V_{allow} = \tau_{allow} A_{pin} = (4000 \text{ psi})\left(\frac{\pi}{4}\right)(0.80 \text{ in.})^2$$
$$= 2011 \text{ lb}$$

$$V = 1.2175T \qquad T_1 = \frac{V_{allow}}{1.2175} = 1652 \text{ lb}$$

ALLOWABLE FORCE IN ONE CABLE BASED UPON TENSION IN THE CABLE

$$T_2 = T_{allow} = 1800 \text{ lb}$$

MAXIMUM WEIGHT

Shear in the pins governs.

$T_{max} = T_1 = 1652$ lb

Total tensile force in four cables

$= 4T_{max} = 6608$ lb

$$W_{max} = 4T_{max} - W$$
$$= 6608 \text{ lb} - 1500 \text{ lb}$$
$$= 5110 \text{ lb} \quad \leftarrow$$

Problem 1.9-10

NUMERICAL DATA

$M = 300\ \text{kg} \qquad g = 9.81\ \text{m/s}^2$

$\tau_a = 50\ \text{MPa} \qquad \sigma_{ba} = 110\ \text{MPa}$

$t_A = 40\ \text{mm} \qquad t_B = 40\ \text{mm}$

$t_C = 50 \qquad d_{pA} = 25\ \text{mm}$

$d_{pB} = 30 \qquad d_{pC} = 22\ \text{mm}$

(a) RESULTANT FORCES F ACTING ON PULLEYS A, B, AND C

$F_A = \sqrt{2}\,T \qquad F_B = 2T$

$F_C = T \qquad T = \frac{Mg}{2} + \frac{W_{max}}{2}$

$W_{max} = 2T - Mg$

From statics at B

(b) MAXIMUM LOAD W THAT CAN BE ADDED AT B DUE TO τ_a AND σ_{ba} IN PINS AT A, B, AND C

PULLEY AT A

$\tau_A = \frac{F_A}{A_s}$

DOUBLE SHEAR

$F_A = \tau_a A_s \qquad \sqrt{2}T = \tau_a A_s$

$$\frac{Mg}{2} + \frac{W_{max}}{2} = \frac{\tau_a A_s}{\sqrt{2}}$$

$$W_{max1} = \frac{2}{\sqrt{2}}\left(\tau_a A_s\right) - Mg$$

$$W_{max1} = \frac{2}{\sqrt{2}}\left(\tau_a 2\frac{\pi}{4} d_p A^2\right) - Mg$$

$$\frac{W_{max1}}{Mg} = 22.6$$

$W_{max1} = 66.5\ \text{kN} \quad \leftarrow$
(shear at A controls)

OR check bearing stress

$$W_{max2} = \frac{2}{\sqrt{2}}\left(\sigma_{ba} A_b\right) - Mg$$

$$W_{max2} = \frac{2}{\sqrt{2}}\left(\sigma_{ba} t_A d_{pA}\right) - Mg$$

$W_{max2} = 152.6\ \text{kN} \quad$ (bearing at A)

PULLEY AT $B \qquad 2T = \tau_a A_s$

$$W_{max3} = \frac{2}{2}(\tau_a A_s) - Mg$$

$$W_{max3} = \left[\tau_a\left(2\frac{\pi}{4} d_{pB}{}^2\right)\right] - Mg$$

$W_{max3} = 67.7\ \text{kN} \quad$ (shear at B)

$$W_{max4} = \frac{2}{2}(\sigma_{ba} A_b) - Mg$$

$$W_{max4} = \sigma_{ba} t_B d_{pB} - Mg$$

$W_{max4} = 129.1\ \text{kN} \quad$ (bearing at B)

PULLEY AT $C \qquad T = \tau_a A_s$

$$W_{max5} = 2(\tau_a A_s) - Mg$$

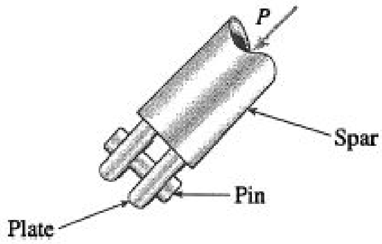
$$W_{max5} = \left[2\tau_a\left(2\frac{\pi}{4} d_{pC}{}^2\right)\right] - Mg$$

$W_{max5} = 7.3 \times 10^4 \qquad W_{max5} = 73.1\ \text{kN} \quad$ (shear at C)

$$W_{max6} = 2\sigma_{ba} t_C d_{pC} - Mg$$

$W_{max6} = 239.1\ \text{kN} \quad$ (bearing at C)

Problem 1.9-11



NUMERICAL DATA

$d_2 = 3.5$ in. $\quad d_1 = 2.8$ in.

$d_p = 1$ in. $\quad t = 0.5$ in.

$\sigma_a = 10$ ksi $\quad \tau_a = 6.5$ ksi $\quad \sigma_{ba} = 16$ ksi

COMPRESSIVE STRESS IN SPAR

$$P_{a1} = \sigma_a \frac{\pi}{4}(d_2{}^2 - d_1{}^2) \qquad P_{a1} = 34.636 \text{ k}$$

SHEAR STRESS IN PIN

$$P_{a2} = \tau_a\left(2\frac{\pi}{4}d_p{}^2\right)$$

$$P_{a2} = 10.21 \text{ kips} < \text{controls} \quad \leftarrow$$

^double shear

BEARING STRESS BETWEEN PIN AND CONECTING PLATES

$$P_{a3} = \sigma_{ba}(2d_p t) \qquad P_{a3} = 16 \text{ k}$$

Problem 1.9-12

NUMERICAL DATA

$FS = 3 \qquad \tau_u = 340\text{ MPa} \qquad \tau_a = \dfrac{\tau_u}{FS}$

$d = 5\text{ mm}$

$$\tau_a = \frac{\sqrt{R_x^{\,2} + R_y^{\,2}}}{A_s} \quad < \text{pin at } C \text{ in single shear}$$

$R_x = -C\cos(40°) \qquad R_y = P + C\sin(40°)$

$a = 50\cos(40°) + 125 \qquad a = 163.302\text{ mm}$

$b = 38\text{ mm}$

STATICS $\quad \sum M_{pin} = 0 \qquad C = \dfrac{P(a)}{b}$

$$R_x = -\frac{P(a)}{b}\cos(40°) \qquad R_y = P\left[1 + \frac{a}{b}\sin(40°)\right]$$

$$P\sqrt{\left[-\frac{a}{b}\cos(40°)\right]^2 + \left[1 + \frac{a}{b}\sin(40°)\right]^2} = \tau_a A_s$$

$$A_s = \frac{\pi}{4}d^2$$

$\tau_a = \dfrac{\tau_u}{FS} \qquad \tau_a = 113.333\text{ MPa}$

Find P_{max}

$$P_{max} = \frac{\tau_a A_s}{\sqrt{\left[-\frac{a}{b}\cos(40°)\right]^2 + \left[1 + \frac{a}{b}\sin(40°)\right]^2}}$$

$P_{max} = 445\text{ N} \qquad \leftarrow$

here $\dfrac{a}{b} = 4.297 \quad < a/b =$ **mechanical advantage**

FIND MAXIMUM CLAMPING FORCE

$C_{ult} = P_{max}FS\left(\dfrac{a}{b}\right) \qquad C_{ult} = 5739\text{ N} \qquad \leftarrow$

$P_{ult} = P_{max}FS \qquad P_{ult} = 1335$

$\dfrac{C_{ult}}{P_{ult}} = 4.297$

Problem 1.9-13

NUMERICAL DATA

$d = \frac{5}{64}$ in. $\quad \sigma_Y = 65$ ksi $\quad \text{FS}_y = 1.9$

$\sigma_a = \frac{\sigma_Y}{\text{FS}_y} \quad \sigma_a = 34.211$ ksi

$$W_{max} = 0.539\left(\frac{\sigma_Y}{\text{FS}_y}\right)\left(\frac{\pi}{4}d^2\right)$$

$$W_{max} = 0.305 \text{ kips} \quad \leftarrow$$

CHECK ALSO FORCE IN WIRE CD

$\sum F_H = 0 \quad$ at C or D

FORCES IN WIRES AC, EC, BD, AND FD

$\sum F_V = 0 \quad$ at A, B, E, or F

$$F_W = \frac{\sqrt{2^2+5^2}}{5} \times \frac{W}{2} \qquad \frac{\sqrt{2^2+5^2}}{10} = 0.539$$

$W_{max} = 0.539\,\sigma_a \times A$

$$F_{CD} = 2\left(\frac{2}{\sqrt{2^2+5^2}}F_w\right)$$

$$F_{CD} = 2\left[\frac{2}{\sqrt{2^2+5^2}}\left(\frac{\sqrt{2^2+5^2}}{5} \times \frac{W}{2}\right)\right]$$

$F_{CD} = \frac{2}{5}W \quad$ less than F_{AC} so AC controls

Problem 1.9-14

NUMERICAL DATA

$A = 2180\ \text{mm}^2$

$t_g = 12\ \text{mm}$ $\quad d_r = 16\ \text{mm}$ $\quad t_{ang} = 6.4\ \text{mm}$

$\sigma_u = 390\ \text{MPa}$ $\quad \tau_u = 190\ \text{MPa}$

$\sigma_{bu} = 550\ \text{MPa}$ $\quad \text{FS} = 2.5$

$$\sigma_a = \frac{\sigma_u}{\text{FS}} \qquad \tau_a = \frac{\tau_u}{\text{FS}} \qquad \sigma_{ba} = \frac{\sigma_{bu}}{\text{FS}}$$

MEMBER FORCES FROM TRUSS ANALYSIS

$$F_{BC} = \frac{5}{3}P \qquad F_{CD} = \frac{4}{3}P \qquad F_{CF} = \frac{\sqrt{2}}{3}P$$

$$\frac{\sqrt{2}}{3} = 0.471 \qquad F_{CG} = \frac{4}{3}P$$

P_{allow} FOR TENSION ON NET SECTION IN TRUSS BARS

$A_{net} = A - 2d_r t_{ang}$ $\qquad A_{net} = 1975\ \text{mm}^2$

$$\frac{A_{net}}{A} = 0.906$$

$F_{allow} = \sigma_a A_{net}$ $\quad$ < allowable force in a member so BC controls since it has the largest member force for this loading

$$P_{allow} = \frac{3}{5}F_{BC\max} \qquad P_{allow} = \frac{3}{5}(\sigma_a A_{net})$$

$P_{allow} = 184.879\ \text{kN}$

Next, P_{allow} for shear in rivets (all are in double shear)

$A_s = 2\frac{\pi}{4}d_r^{\,2}$ $\quad$ < for one rivet in DOUBLE shear

$\dfrac{F_{max}}{N} = \tau_a A_s$ $\quad$ N = number of rivets in a particular member (see drawing of connection detail)

$P_{BC} = 3\left(\frac{3}{5}\right)(\tau_a A_s)$ $\qquad P_{BC} = 55.0\ \text{kN}$

$P_{CF} = 2\left(\frac{3}{\sqrt{2}}\right)(\tau_a A_s)$ $\qquad P_{CF} = 129.7\ \text{kN}$

$P_{CG} = 2\left(\frac{3}{4}\right)(\tau_a A_s)$

$P_{CG} = 45.8\ \text{kN}$ ← $\quad$ < so shear in rivets in CG and CD controls P_{allow} here

$P_{CD} = 2\left(\frac{3}{4}\right)(\tau_a A_s)$ $\qquad P_{CD} = 45.8\ \text{kN}$ ←

Next, P_{allow} for bearing of rivets on truss bars $A_b = 2d_r t_{ang}$ $\quad$ < rivet bears on each angle in two angle pairs

$$\frac{F_{max}}{N} = \sigma_{ba}A_b$$

$P_{BC} = 3\left(\frac{3}{5}\right)(\sigma_{ba}A_b)$ $\qquad P_{BC} = 81.101\ \text{kN}$

$P_{CF} = 2\left(\frac{3}{\sqrt{2}}\right)(\sigma_{ba}A_b)$ $\qquad P_{CF} = 191.156\ \text{kN}$

$P_{CG} = 2\left(\frac{3}{4}\right)(\sigma_{ba}A_b)$ $\qquad P_{CG} = 67.584\ \text{kN}$

$P_{CD} = 2\left(\frac{3}{4}\right)(\sigma_{ba}A_b)$ $\qquad P_{CD} = 67.584\ \text{kN}$

Finally, P_{allow} for bearing of rivets on gusset plate

$A_b = d_r t_g$

(bearing area for each rivert on gusset plate)

$t_g = 12\ \text{mm} < 2t_{ang} = 12.8\ \text{mm}$

so gusset will control over angles

$P_{BC} = 3\left(\frac{3}{5}\right)(\sigma_{ba}A_b)$ $\qquad P_{BC} = 76.032\ \text{kN}$

$P_{CF} = 2\left(\frac{3}{\sqrt{2}}\right)(\sigma_{ba}A_b)$ $\qquad P_{CF} = 179.209\ \text{kN}$

$P_{CG} = 2\left(\frac{3}{4}\right)(\sigma_{ba}A_b)$ $\qquad P_{CG} = 63.36\ \text{kN}$

$P_{CD} = 2\left(\frac{3}{4}\right)(\sigma_{ba}A_b)$ $\qquad P_{CD} = 63.36\ \text{kN}$

So, shear in rivets controls: $P_{allow} = 45.8\ \text{kN}$ ←

Problem 1.9-15

NUMERICAL DATA

$d = 1.75 \text{ in.} \qquad \sigma_a = 12 \text{ ksi}$

(a) FORMULA FOR P_{ALLOW} IN TENSION

From Case 15, Appendix E:

$$A = 2r^2\left(\alpha - \frac{ab}{r^2}\right) \qquad r = \frac{d}{2} \qquad a = \frac{d}{10}$$

$$\alpha = \operatorname{acos}\left(\frac{a}{r}\right) \qquad r = 0.875 \text{ in.} \qquad a = 0.175 \text{ in.}$$

$$\alpha\frac{180}{\pi} = 78.463°$$

$$b = \sqrt{r^2 - a^2}$$

$$b = \sqrt{\left[\left(\frac{d}{2}\right)^2 - \left(\frac{d}{10}\right)^2\right]}$$

$$b = \sqrt{\left(\frac{6}{25}d^2\right)} \qquad b = \frac{d}{5}\sqrt{6}$$

$$P_a = \sigma_a A$$

$$P_a = \sigma_a\left[\frac{1}{2}d^2\left(\arccos\left(\frac{1}{5}\right) - \frac{2}{25}\sqrt{6}\right)\right]$$

$$\frac{\arccos\left(\frac{1}{5}\right) - \frac{2}{25}\sqrt{6}}{2} = 0.587 \qquad \frac{\pi}{4} = 0.785$$

$$P_a = \sigma_a(0.587\,d^2) \quad \leftarrow$$

$$\frac{0.587}{0.785} = 0.748$$

(b) EVALUATE NUMERICAL RESULT

$d = 1.75 \text{ in.} \qquad \sigma_a = 12 \text{ ksi}$

$P_a = 21.6 \text{ k} \quad \leftarrow$

Problem 1.9-16

NUMERICAL DATA

$d_1 = 60\text{ mm} \qquad d_2 = 32\text{ mm}$

$\tau_Y = 120\text{ MPa} \qquad \sigma_Y = 250\text{ MPa}$

$FS_y = 2$

ALLOWABLE STRESSES

$$\tau_a = \frac{\tau_Y}{FS_y} \qquad \tau_a = 60\text{ MPa}$$

$$\sigma_a = \frac{\sigma_Y}{FS_y} \qquad \sigma_a = 125\text{ MPa}$$

From Case 15, Appendix E: $\quad r = \dfrac{d_1}{2}$

$$A = 2r^2\left(\alpha - \frac{ab}{r^2}\right) \qquad \alpha = \text{arc cos}\frac{d_2/2}{d_1/2} = \text{arc cos}\frac{d_2}{d_1}$$

$$a = \frac{d_2}{2} \qquad b = \sqrt{r^2 - a^2}$$

SHEAR AREA (DOUBLE SHEAR)

$$A_s = 2\left(\frac{\pi}{4}d_2{}^2\right) \qquad A_s = 1608\text{ mm}^2$$

NET AREA IN TENSION (FROM APPENDIX E)

$$A_{\text{net}} = 2\left(\frac{d_1}{2}\right)^2 \left[\arccos\left(\frac{d_2}{d_1}\right) - \frac{\frac{d_2}{2}\left[\sqrt{\left(\frac{d_1}{2}\right)^2 - \left(\frac{d_2}{2}\right)^2}\right]}{\left(\frac{d_1}{2}\right)^2}\right]$$

$A_{\text{net}} = 1003\text{ mm}^2$

P_{allow} in tension: smaller of values based on either shear or tension allowable stress x appropriate area

$P_{a1} = \tau_a A_s \quad P_{a1} = 96.5\text{ kN} <$ shear governs $\leftarrow$

$P_{a2} = \sigma_a A_{\text{net}} \qquad P_{a2} = 125.4\text{ kN}$

Problem 1.9-17

NUMERICAL DATA

$\sigma_u = 60$ ksi $\quad \tau_u = 17$ ksi $\quad \tau_{hu} = 25$ ksi

$\sigma_{bu} = 75$ ksi $\quad \sigma_{bw} = 50$ ksi $\quad FS_u = 2.5$

$d_b = \frac{3}{4}$ in. $\quad d_w = 1.5$ in. $\quad t_{bp} = 1$ in.

$h = 14$ in. $\quad b = 12$ in. $\quad d = 6$ in. $\quad t = \frac{3}{8}$ in.

$W = 0.500$ kips $\quad H = 17(12) \quad H = 204$ in.

$L_v = 10(12) \quad L_h = 12(12) \quad L_v = 120$ in.

$L_h = 144$ in.

ALLOWABLE STRESSES (ksi)

$\sigma_a = \frac{\sigma_u}{FS_u} \quad \sigma_a = 24 \quad \tau_a = \frac{\tau_u}{FS_u}$

$\tau_a = 6.8 \quad \tau_{ha} = \frac{\tau_{hu}}{FS_u} \quad \tau_{ha} = 10$

$\sigma_{ba} = \frac{\sigma_{bu}}{FS_u} \quad \sigma_{ba} = 30 \quad \sigma_{bwa} = \frac{\sigma_{bw}}{FS_u}$

$\sigma_{bwa} = 20$

FORCES F AND R IN TERMS OF p_{max}

$F = p_{max} L_v L_h \quad R = \frac{FH}{2h}$

$R = p_{max} \frac{L_v L_h H}{2h}$

(1) COMPUTE p_{max} BASED ON NORMAL STRESS IN EACH BOLT (GREATER AT B AND D)

$$\sigma = \frac{R + \frac{W}{4}}{\frac{\pi}{4} d_b^2} \qquad R_{max} = \sigma_a\left(\frac{\pi}{4} d_b^2\right) - \frac{W}{4}$$

$$p_{max1} = \frac{\sigma_a\left(\frac{\pi}{4} d_b^2\right) - \frac{W}{4}}{\frac{L_v L_h H}{2h}}$$

$p_{max1} = 11.98$ psf $\quad \leftarrow \quad$ controls

(2) COMPUTE p_{max} BASED ON SHEAR THROUGH BASE PLATE (GREATER AT B AND D)

$$\tau = \frac{R + \frac{W}{4}}{\pi d_w t_{bp}}$$

$$R_{max} = \tau_a(\pi d_w t_{bp}) - \frac{W}{4}$$

$$p_{max2} = \frac{\tau_a(\pi d_w t_{bp}) - \frac{W}{4}}{\frac{L_v L_h H}{2h}}$$

$p_{max2} = 36.5$ psf

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(3) COMPUTE p_{max} BASED ON HORIZONTAL SHEAR ON EACH BOLT

$$\tau_h = \frac{\frac{F}{4}}{\left(\frac{\pi}{4} d_b^2\right)} \qquad F_{max} = 4\tau_{ha}\left(\frac{\pi}{4} d_b^2\right)$$

$$p_{max3} = \frac{\tau_{ha}(\pi d_b^2)}{L_v L_h}$$

$p_{max3} = 147.3$ psf

(4) COMPUTE p_{max} BASED ON HORIZONTAL BEARING ON EACH BOLT

$$\sigma_b = \frac{\frac{F}{4}}{(t_{bp} d_b)} \qquad F_{max} = 4\sigma_{ba}(t_{bp} d_b)$$

$$p_{max4} = \frac{4\sigma_{ba}(t_b p d_b)}{L_v L_h}$$

$p_{max4} = 750$ psf

(5) COMPUTE p_{max} BASED ON BEARING UNDER THE TOP WASHER AT A (OR C) AND THE BOTTOM WASHER AT B (OR D)

$$\sigma_{bw} = \frac{R + \frac{W}{4}}{\frac{\pi}{4}\left(d_w^2 - d_b^2\right)}$$

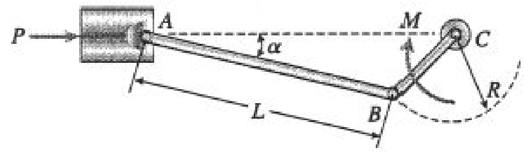
$$R_{max} = \sigma_{bwa}\left[\frac{\pi}{4}\left(d_w^2 - d_b 2\right)\right] - \frac{W}{4}$$

$$p_{max5} = \frac{\sigma_{bwa}\left[\frac{\pi}{4}\left(d_w^2 - d_b^2\right)\right] - \frac{W}{4}}{\frac{L_v L_h H}{2h}}$$

$p_{max5} = 30.2$ psf

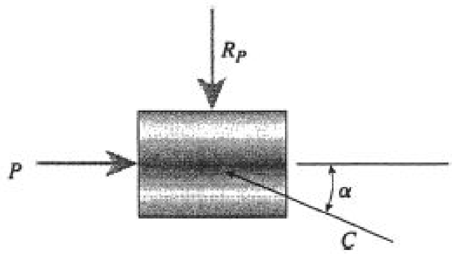
So, normal/stress in bolts controls; $p_{max} = 11.98$ psf

Problem 1.9-18



d = diameter of rod AB

FREE-BODY DIAGRAM OF PISTON



P = applied force (constant)

C = compressive force in connecting rod

RP = resultant of reaction forces between cylinder and piston (no friction)

$$\sum F_{\text{horiz}} = 0 \;\overset{\rightarrow}{+}\;\overset{\leftarrow}{-}$$

$$P - C\cos\alpha = 0$$

$$P = C\cos\alpha$$

MAXIMUM COMPRESSIVE FORCE C IN CONNECTING ROD

$$C_{\max} = \sigma_c A_c$$

in which A_c = area of connecting rod

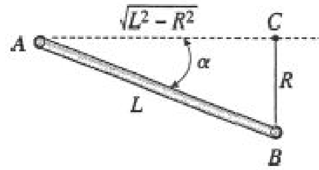
$$A_c = \frac{\pi d^2}{4}$$

MAXIMUM ALLOWABLE FORCE P

$$P = C_{\max}\cos\alpha$$
$$= \sigma_c A_c \cos\alpha$$

The maximun allowable force P occurs when $\cos\alpha$ has its smallest value, which means that α has its largest value.

LARGEST VALUE OF α



The largest value of α occurs when point B is the farthest distance from line AC. The farthest distance is the radius R of the crank arm.

Therefore,

$$\overline{BC} = R$$

Also, $\overline{AC} = \sqrt{L^2 - R^2}$

$$\cos\alpha = \frac{\sqrt{L^2 - R^2}}{L} = \sqrt{1 - \left(\frac{R}{L}\right)^2}$$

(a) MAXIMUM ALLOWABLE FORCE P

$$P_{\text{allow}} = \sigma_c A_c \cos\alpha$$
$$= \sigma_c\left(\frac{\pi d^2}{4}\right)\sqrt{1 - \left(\frac{R}{L}\right)^2} \quad \leftarrow$$

(b) SUBSTITUTE NUMERICAL VALUES

$\sigma_c = 160$ MPa $\quad d = 9.00$ mm

$R = 0.28L \quad R/L = 0.28$

$P_{\text{allow}} = 9.77$ kN $\quad\leftarrow$

Problem 1.10-1

NUMERICAL DATA

$P = 33$ kips $\quad t = 0.25$ in. $\quad \sigma_a = 12$ ksi

(a) MINIMUM DIAMETER OF TUBE (NO HOLES)

$$A_1 = \frac{\pi}{4}\left[d^2 - (d-2t)^2\right] \qquad A_2 = \frac{P}{\sigma_a}$$

$$A_2 = 2.75 \text{ in.}^2$$

Equating A_1 and A_2 and solving for d:

$$d = \frac{P}{\pi \sigma_a t} + t \qquad d = 3.75 \text{ in.} \quad \leftarrow$$

(b) MINIMUM DIAMETER OF TUBE (WITH HOLES)

$$A_1 = \left[\frac{\pi}{4}\left[d^2 - (d-2t)^2\right] - 2\left(\frac{d}{10}\right)t\right]$$

$$A_1 = d\left(\pi t - \frac{t}{5}\right) - \pi t^2$$

Equating A_1 and A_2 and solving for d:

$$d = \frac{\dfrac{P}{\sigma_a} + \pi t^2}{\pi t - \dfrac{t}{5}} \qquad d = 4.01 \text{ in.} \quad \leftarrow$$

Problem 1.10-2

NUMERICAL DATA

$\sigma_Y = 290$ MPa

$P = 1500$ kN

$FS_y = 1.8$

(a) MINIMUM DIAMETER (NO HOLES)

$$A_1 = \frac{\pi}{4}\left[d^2 - \left(d - \frac{d}{4}\right)^2\right]$$

$$A_1 = \frac{7}{64}\pi d^2$$

$$A_2 = \frac{P}{\dfrac{\sigma_Y}{FS_y}} \qquad A_2 = 9.31 \times 10^3 \text{ mm}^2$$

Equate A_1 and A_2 and solve for d:

$$d^2 = \frac{7}{64\pi}\left(\frac{P}{\dfrac{\sigma_Y}{FS_y}}\right)$$

$$d_{min}\sqrt{\frac{7}{64\pi}\left(\frac{P}{\dfrac{\sigma_Y}{FS_y}}\right)}$$

$$d_{min} = 164.6 \text{ mm} \quad \leftarrow$$

(b) MINIMUM DIAMETER (WITH HOLES)

Redefine A_1—subtract area for two holes—then equate to A_2

$$A_1 = \left[\frac{\pi}{4}\left[d^2 - \left(d - \frac{d}{4}\right)^2\right] - 2\left(\frac{d}{10}\right)\left(\frac{d}{8}\right)\right]$$

$$A_1 = \frac{7}{64}\pi d^2 - \frac{1}{40}d^2$$

$$A_1 = d^2\left(\frac{7}{64}\pi - \frac{1}{40}\right) \qquad \frac{7}{64}\pi - \frac{1}{40} = 0.319$$

$$d^2 = \frac{\left(\dfrac{P}{\dfrac{\sigma_Y}{FS_y}}\right)}{\left(\dfrac{7}{64}\pi - \dfrac{1}{40}\right)}$$

$$d_{min} = \sqrt{\frac{\left(\dfrac{P}{\dfrac{\sigma_y}{FS_y}}\right)}{\left(\dfrac{7}{64}\pi - \dfrac{1}{40}\right)}} \qquad d_{min} = 170.9 \text{ mm} \quad \leftarrow$$

Problem 1.10-3

NUMERICAL DATA

$P = 2.7\text{ k} \qquad b = 0.75\text{ in.} \qquad h = 8\text{ in.}$

$\tau_a = 13\text{ ksi} \qquad \sigma_{ba} = 19\text{ ksi}$

(a) d_{min} BASED ON ALLOWABLE SHEAR—DOUBLE SHEAR IN STRUT

$$\tau_a = \frac{F_{DC}}{A_s} \qquad F_{DC} = \frac{15}{4}P$$

$$A_s = 2\left(\frac{\pi}{4}d^2\right)$$

$$d_{min} = \sqrt{\frac{\frac{15}{4}P}{\tau_a\left(\frac{\pi}{2}\right)}} \qquad d_{min} = 0.704\text{ in.} \quad \longleftarrow$$

(b) d_{min} BASED ON ALLOWABLE BEARING AT JT C

Bearing from beam ACB $\qquad \sigma_b = \dfrac{15\,P/4}{bd}$

$$d_{min} = \frac{15\,P/4}{b\sigma_{ba}} \qquad d_{min} = 0.711\text{ in.} \quad \longleftarrow$$

Bearing from strut DC $\quad \sigma_b = \dfrac{\frac{15}{4}P}{2\frac{5}{8}bd}$

$$\sigma_b = 3\frac{P}{bd} \qquad \text{(lower than } ACB\text{)}$$

Problem 1.10-4

NUMERICAL DATA

$F = 190\text{ kN}$ $\quad \tau_a = 90\text{ MPa}$ $\quad \sigma_{ba} = 150\text{ MPa}$

$t_g = 20\text{ mm}$ $\quad t_c = 16\text{ mm}$

(1) $d_{\min}$ BASED ON ALLOW SHEAR—DOUBLE SHEAR IN PIN

$$\tau = \frac{F}{A_s} \qquad A_s = 2\left(\frac{\pi}{4}d^2\right)$$

$$d_{\min} = \sqrt{\frac{F}{\tau_a\left(\frac{\pi}{2}\right)}} \qquad d_{\min} = 36.7\text{ mm}$$

(2) $d_{\min}$ BASED ON ALLOW BEARING IN GUSSET AND CLEVIS PLATES

Bearing on gusset plate

$$\sigma_b = \frac{F}{A_b} \qquad A_b = t_g d \qquad d_{\min} = \frac{F}{t_g \sigma_{ba}}$$

$d_{\min} = 63.3\text{ mm}$ <controls $\leftarrow$

Bearing on clevis $\quad A_b = d(2t_c)$

$$d_{\min} = \frac{F}{2t_c\sigma_{ba}} \qquad d_{\min} = 39.6\text{ mm}$$

Problem 1.10-5

$P = 5200$ lb $\quad F_{BE} = 3.83858\,P = 19{,}960.616$ lb < from plane truss analysis (see Probs. 1.3-6 to 1.3-12) $\quad \tau_a = 12$ ksi

$t_p = \dfrac{5}{8}$ in. $\quad t_g = 1.125$ in. $\quad t_p = 0.625$ in. $\quad 2\,t_p = 1.25$ in. $\quad \sigma_{ba} = 18$ ksi

PIN DIAMETER BASED ON ALLOWABLE SHEAR STRESS (PINS IN DOUBLE SHEAR)

$$d_{p1} = \sqrt{\frac{\frac{F_{BE}}{2}}{\frac{\pi}{4}\tau_a}} = 1.029 \text{ in.} \qquad < \text{controls} \qquad \boxed{d_{\text{pin}} = 1.029 \text{ in.}}$$

PIN DIAMETER BASED ON BEARING BETWEEN PIN AND EACH OF TWO END PLATES $\quad < 2t_p$ is greater than t_g so gusset will control

$$d_{p2} = \frac{F_{BE}}{2\,t_p\sigma_{ba}} = 0.887 \text{ in.}$$

PIN DIAMETER BASED ON BEARING BETWEEN PIN AND GUSSET PLATE

$$d_{p3} = \frac{F_{BE}}{t_g\sigma_{ba}} = 0.986 \text{ in.}$$

Problem 1.10-6

$$\sigma_a = 125MPa \qquad \tau_a = 80MPa \qquad W = 8kN$$

Cut cable - use FBD of OABC to find cable tension T

$$\Sigma M_O = 0 \qquad \frac{2.5}{\sqrt{2.5^2 + 3^2}} T \cdot (3m) = W \cdot \left(\frac{4.5m}{2}\right) \qquad \text{so} \qquad T = \frac{W \cdot \left(\frac{2.25m}{3m}\right)}{\frac{2.5}{\sqrt{2.5^2+3^2}}} = 9.372 \cdot kN$$

Required cross sectional area of cable $\quad A_{reqd} = \frac{T}{\sigma_a} = 74.978 \cdot mm^2$

Reaction force at O - use FBD of OABC

$$\Sigma F_x = 0 \qquad R_{Ox} = \frac{3}{\sqrt{2.5^2 + 3^2}} \cdot T = 7.2 \cdot kN \qquad \Sigma F_y = 0 \qquad R_{Oy} = W - \frac{2.5}{\sqrt{2.5^2 + 3^2}} \cdot T = 2 \cdot kN$$

Resultant $\quad R_{Ores} = \sqrt{R_{Ox}^2 + R_{Oy}^2} = 7.473 \cdot kN$

Diameter of pin at O (in double shear) $\quad d_O = \sqrt{\frac{4}{\pi} \cdot \left(\frac{R_{Ores}}{2 \cdot \tau_a}\right)} = 7.711 \cdot mm$

Diameter of pins at B and D (in double shear) $\quad d_B = d_D \qquad d_B = \sqrt{\frac{4}{\pi} \cdot \left(\frac{T}{2 \cdot \tau_a}\right)} = 8.636 \cdot mm$

Problem 1.10-7

$W = 1700\,lbf \qquad \sigma_a = 18\,ksi \qquad \tau_a = 12\,ksi$ Assume single shear in pins

a) Cut continuous cable - use FBD of OABC to find cable force T then use allowable normal stress to find A_{reqd}

$$\Sigma M_O = 0 \qquad T\cdot\left(\frac{8}{\sqrt{5^2+8^2}}\right)\cdot(5ft) + T\cdot\left(\frac{8}{\sqrt{10^2+8^2}}\right)\cdot(10ft) = W\cdot\left(\frac{15ft}{2}\right)$$

$$T = \frac{W\cdot\left(\frac{15ft}{2}\right)}{\frac{8}{\sqrt{5^2+8^2}}\cdot(5ft) + \left(\frac{8}{\sqrt{10^2+8^2}}\right)\cdot(10ft)} = 1215.798\cdot lbf \qquad \text{so} \qquad A_{reqd} = \frac{T}{\sigma_a} = 0.0675\cdot in^2$$

b) Find resultant force at each pin location, then find reqd pin diameter assuming single shear

Pins at A and B: $d_B = d_A \qquad d_A = \sqrt{\frac{4}{\pi}\cdot\frac{T}{\tau_a}} = 0.359\cdot in$

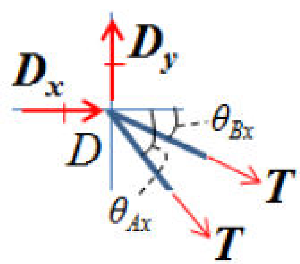
Pin at O - find reactions at O then resultant force - use lower FBD of OABC; assume single shear in pins

$$\Sigma F_x = 0 \qquad R_{Ox} = T\cdot\left(\frac{5}{\sqrt{8^2+5^2}} + \frac{10}{\sqrt{8^2+10^2}}\right) = 1593.75\cdot lbf$$

$$\Sigma F_y = 0 \qquad R_{Oy} = W - T\cdot\left(\frac{8}{\sqrt{8^2+5^2}} + \frac{8}{\sqrt{8^2+10^2}}\right) = -90.497\cdot lbf$$

$$d_O = \sqrt{\frac{4}{\pi}\cdot\frac{\left(\sqrt{R_{Ox}^2 + R_{Oy}^2}\right)}{\tau_a}} = 0.412\cdot in$$

Pin at D - use resultant of continuous cable forces from A and B



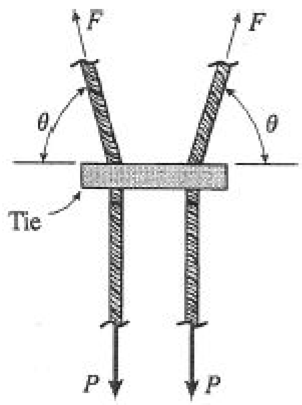
$$\theta_{Ax} = \operatorname{atan}\left(\frac{8}{5}\right) = 57.995\cdot deg \qquad \theta_{Bx} = \operatorname{atan}\left(\frac{8}{10}\right) = 38.66\cdot deg$$

$$D_x = -T\cdot\left(\cos(\theta_{Ax}) + \cos(\theta_{Bx})\right) = -1593.75\cdot lbf$$

$$D_y = T\cdot\left(\sin(\theta_{Ax}) + \sin(\theta_{Bx})\right) = 1790.497\cdot lbf$$

$$D_{res} = \sqrt{D_x^2 + D_y^2} = 2397.065\cdot lbf \qquad \text{so} \qquad d_D = \sqrt{\frac{4}{\pi}\cdot\frac{D_{res}}{\tau_a}} = 0.504\cdot in$$

Problem 1.10-8



F = tensile force in cable above tie

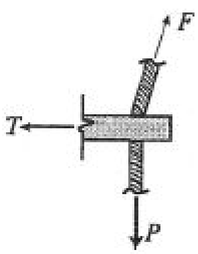
P = tensile force in cable below tie

σ_{allow} = allowable tensile stress in the tie

FREE-BODY DIAGRAM OF HALF THE TIE

Note: Include a small amount of the cable in the free-body diagram

T = tensile force in the tie



FORCE TRIANGLE

$$\cot\theta = \frac{T}{P}$$

$$T = P\cot\theta$$

(a) MINIMUM REQUIRED AREA OF TIE

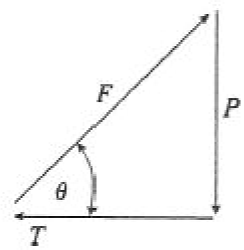
$$A_{min} = \frac{T}{\sigma_{allow}} = \frac{P\cot\theta}{\sigma_{allow}} \quad \longleftarrow$$

(b) SUBSTITUTE NUMERICAL VALUES:

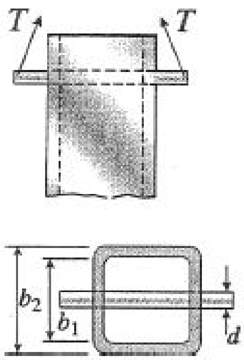
$P = 130\text{ kN} \qquad \theta = 75°$

$\sigma_{allow} = 80\text{ MPa}$

$A_{min} = 435\text{ mm}^2 \quad \longleftarrow$



Problem 1.10-9



T = tensile force in cable

W = weight of steel tube

d = diameter of pin

b_1 = inner dimension of tube = 8.5 in.

b_2 = outer dimension of tube = 10.0 in.

L = length of tube = 20 ft

$\tau_{allow} = 8{,}700$ psi

$\sigma_b = 13{,}000$ psi

Weight of tube

γ_s = weight density of steel = 490 lb/ft^3

A = area of tube

$$= b_2^2 - b_1^2 = (10.0 \text{ in.})^2 - (8.5 \text{ in.})^2 = 27.75 \text{ in.}$$

$$W = \gamma_s AL$$
$$= (490 \text{ lb/ft}^3)(27.75 \text{ in.}^2)\left(\frac{1 \text{ft}^2}{144 \text{ in.}}\right)(20 \text{ ft})$$
$$= 1{,}889 \text{ lb}$$

Diameter of pin based upon shear

Double shear. $2\tau_{allow} A_{pin} = W$

$$2(8{,}700 \text{ psi})\left(\frac{\pi \, d^2}{4}\right) = 1889 \text{ lb}$$

$$d^2 = 0.1382 \text{ in.}^2 \qquad d_1 = 0.372 \text{ in.}$$

Diameter of pin based upon bearing

$$\sigma_b(b_2 - b_1)d = W$$
$$(13{,}000 \text{ psi})(10.0 \text{ in.} - 8.5 \text{ in.})\, d = 1{,}889 \text{ lb}$$
$$d_2 = 0.097 \text{ in.}$$

Minimum diameter of pin

Shear governs. $d_{min} = 0.372$ in.

Problem 1.10-10

ALLOWABLE SHEAR AND BEARING STRESSES

$\tau_a = 60\text{ MPa} \qquad \sigma_{ba} = 90\text{ MPa}$

FIND INCLINATION OF AND FORCE IN CABLE, T

let α = angle between pole and cable at C; use law of cosines

$$DC = \sqrt{5^2 + 4^2 - 2(5)(4)\cos\left(120\,\frac{\pi}{180}\right)}$$

$$DC = 7.81\text{ m} \qquad \alpha = \arccos\left[\frac{5^2 + DC^2 - 4^2}{2DC(5)}\right]$$

$$\alpha = 26.33^\circ \qquad \theta = 60 - \alpha$$

$\theta = 33.67$ <ange between cable and horizontal at D

$$W = 230\text{ kg}(9.81\text{ m/s}^2) \qquad W = 2.256 \times 10^3\text{ N}$$

STATICS TO FIND CABLE FORCE T

$$\sum M_A = 0 \qquad W(3\sin(30^\circ)) - T_X(5\cos(30^\circ)) + T_y(5\sin(30^\circ)) = 0$$

substitute for T_x and T_y in terms of T and solve for T:

$$T = \frac{\frac{3}{2}W}{\frac{-5}{2}\sin(\theta) + \frac{5\sqrt{3}}{2}\cos(\theta)}$$

$$T = 1.53 \times 10^3\text{ N} \qquad T_x = T\cos(\theta)$$

$$T_y = T\sin(\theta) \qquad T_x = 1.27 \times 10^3\text{ N} \qquad T_y = 846.11\text{ N}$$

(1) d_{min} BASED ON ALLOWABLE SHEAR–DOUBLE SHEAR AT A

$$A_x = -T_x \qquad A_y = T_y + W$$

CHECK SHEAR DUE TO RESULTANT FORCE ON PIN AT A

$$R_A = \sqrt{A_x^2 + A_y^2} \qquad R_A = 3.35 \times 10^3\text{ N}$$

$$d_{min} = \sqrt{\frac{\frac{R_A}{2}}{\tau_a\left(\frac{\pi}{4}\right)}}$$

$d_{min} = 5.96\text{ mm}$ <controls $\leftarrow$

(2) d_{min} BASED ON ALLOWABLE BEARING ON PIN

$$d_{pole} = 140\text{ mm} \qquad t_{pole} = 12\text{ mm}$$

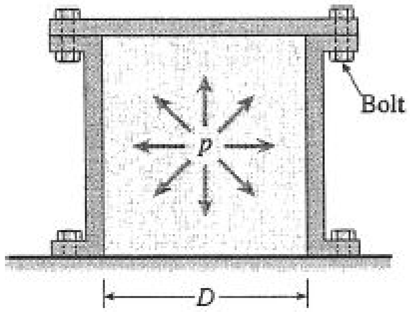
$$L_{pole} = 6000\text{ mm}$$

MEMBER AB BEARING ON PIN

$$\sigma_b = \frac{R_A}{A_b} \qquad A_b = 2t_{pole}d$$

$$d_{min} = \frac{R_A}{2t_{pole}\sigma_{ba}} \qquad d_{min} = 1.55\text{ mm}$$

Problem 1.10-11



$p = 290$ psi $\quad D = 10.0$ in. $\quad d_b = 0.50$ in.

$\sigma_{\text{allow}} = 10{,}000$ psi $\quad n =$ number of bolts

$F =$ total force acting on the cover plate from the internal pressure

$$F = p\left(\frac{\pi D^2}{4}\right)$$

NUMBER OF BOLTS

$P =$ tensile force in one bolt

$$P = \frac{F}{n} = \frac{\pi p D^2}{4n}$$

$$A_b = \text{area of one bolt} = \frac{\pi}{4} d_b^2$$

$$P = \sigma_{\text{allow}} A_b$$

$$\sigma_{\text{allow}} = \frac{P}{A_b} = \frac{\pi p D^2}{(4n)(\frac{\pi}{4})d_b^2} = \frac{pD^2}{nd_b^2}$$

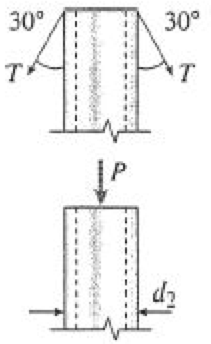
$$n = \frac{pD^2}{d_b^2 \sigma_{\text{allow}}}$$

SUBSTITUTE NUMERICAL VALUES:

$$n = \frac{(290 \text{ psi})(10 \text{ in.})^2}{(0.5 \text{ in.})^2(10{,}000 \text{ psi})} = 11.6$$

Use 12 bolts $\quad \leftarrow$

Problem 1.10-12



d_2 = outer diameter

d_1 = inner diameter

t = wall thickness

$= 15$ mm

T = tensile force in a cable

$= 110$ kN

$\sigma_{allow} = 35$ MPa

P = compressive force in post

$= 2T\cos 30°$

REQUIRED AREA OF POST

$$A = \frac{P}{\sigma_{allow}} = \frac{2T\cos 30°}{\sigma_{allow}}$$

AREA OF POST

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = \frac{\pi}{4}[d_2^2 - (d_2 - 2t)^2]$$

$$= \pi t(d_2 - t)$$

EQUATE AREAS AND SOLVE FOR d_2:

$$\frac{2T\cos 30°}{\sigma_{allow}} = \pi t(d_2 - t)$$

$$d_2 = \frac{2T\cos 30°}{\pi t \sigma_{allow}} + t \quad \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$$(d_2)_{min} = 131 \text{ mm} \quad \leftarrow$$

Problem 1.10-13

$L_1 = 22ft \quad L_2 = 10ft \quad d = 14ft \quad W = 85kip \quad \sigma_u = 91ksi \quad FS_u = 4$

Geometry

$\theta = acos\left(\frac{L_1^{\,2} + d^2 - L_2^{\,2}}{2 \cdot L_1 \cdot d}\right) \qquad \theta = 19.685 \cdot deg \qquad \beta = acos\left(\frac{L_1^{\,2} + L_2^{\,2} - d^2}{2 \cdot L_1 \cdot L_2}\right) \qquad \beta = 28.138 \cdot deg$

$\alpha = asin\left(\frac{L_1}{L_2} \cdot \sin(\theta)\right) = 47.823 \cdot deg \qquad \alpha = 180deg - \alpha = 132.177 \cdot deg$

OR $\quad \beta = asin\left(\frac{d}{L_2} \cdot \sin(\theta)\right) = 28.138 \cdot deg \qquad \alpha = acos\left(\frac{d^2 + L_2^{\,2} - L_1^{\,2}}{2 \cdot d \cdot L_2}\right) = 132.177 \cdot deg$

C.G. of panel $\quad d_2 = \left(\frac{d + \frac{d}{2} + \frac{d}{4}}{2}\right) - \frac{d}{4} = 8.75\,ft \qquad d_1 = \left(\frac{d + \frac{d}{2} + \frac{d}{4}}{2}\right) - \frac{d}{2} = 5.25\,ft$

$HC = \sqrt{d_2^{\,2} + L_2^{\,2} - 2 \cdot L_2 \cdot d_2 \cdot \cos(\alpha)} = 17.148\,ft$ $\qquad$ HC must be vertical line

$\beta_1 = acos\left(\frac{L_1^{\,2} + HC^2 - d_1^{\,2}}{2 \cdot L_1 \cdot HC}\right) = 5.919 \cdot deg \qquad \beta_2 = acos\left(\frac{L_2^{\,2} + HC^2 - d_2^{\,2}}{2 \cdot L_2 \cdot HC}\right) = 22.218 \cdot deg$

$\beta = 28.138 \cdot deg \qquad \beta_1 + \beta_2 = 28.138 \cdot deg$

Solution approach: find cable tensions then A_c = larger $T/(\sigma_u/FS)$

Statics $\quad \sum_H F_x = 0 \qquad T_1 \cdot \sin(\beta_1) = T_2 \cdot \sin(\beta_2)$ $\quad$ SO $\quad T_2 = T_1 \cdot \frac{\sin(\beta_1)}{\sin(\beta_2)}$

$\sum_H F_y = 0 \qquad T_1 \cdot \cos(\beta_1) + T_2 \cdot \cos(\beta_2) = F$ $\qquad$ and F = W/2

$W = 85 \cdot kip$

SO $\quad T_1 \cdot \left(\cos(\beta_1) + \frac{\sin(\beta_1)}{\sin(\beta_2)} \cdot \cos(\beta_2)\right) = F$

$T_1 = \frac{\frac{W}{2}}{\left(\cos(\beta_1) + \frac{\sin(\beta_1)}{\sin(\beta_2)} \cdot \cos(\beta_2)\right)} \qquad T_1 = 34.078 \cdot kip \qquad T_2 = T_1 \cdot \frac{\sin(\beta_1)}{\sin(\beta_2)} \qquad T_2 = 9.294 \cdot kip$

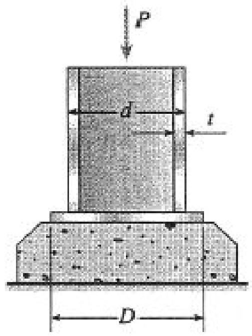
Compute required cross-sectional area

$\sigma_u = 91 \cdot ksi \qquad FS_u = 4 \qquad \frac{\sigma_u}{FS_u} = 22.75 \cdot ksi$

$A_c = \frac{T_1}{\frac{\sigma_u}{FS_u}} \qquad A_c = 1.498 \cdot in^2$

< Table 2-1
use nominal diam. 2.00 in. with area = 1.92 in^2
(or perhaps1.75 in. with area = 1.47 in^2)

Problem 1.10-14



$d = 250$ mm $\qquad P = 750$ kN

$\sigma_{allow} = 55$ MPa (compression in column)

t = thickness of column

D = diameter of base plate

$\sigma_b = 11.5$ MPa (allowable pressure on concrete)

(a) THICKNESS t OF THE COLUMN

$$A = \frac{P}{\sigma_{allow}} \qquad A = \frac{\pi d^2}{4} - \frac{\pi}{4}(d - 2t)^2$$

$$= \frac{\pi}{4}(4t)(d - t) = \pi t(d - t)$$

$$\pi t(d - t) = \frac{P}{\sigma_{allow}}$$

$$\pi t^2 - \pi t d + \frac{P}{\sigma_{allow}} = 0$$

$$t^2 - td + \frac{P}{\pi \sigma_{allow}} = 0 \qquad \text{(Eq. 1)}$$

SUBSTITUTE NUMERICAL VALUES IN EQ. (1):

$$t^2 - 250\,t + \frac{(750 \times 10^3\text{ N})}{\pi(55\text{ N/mm}^2)} = 0$$

(Note: In this eq., t has units of mm.)

$$t^2 - 250t + 4{,}340.6 = 0$$

Solve the quadratic eq. for t:

$t = 18.77$ mm $\qquad t_{min} = 18.8$ mm $\quad \leftarrow$

Use $t = 20$ mm $\quad \leftarrow$

(b) DIAMETER D OF THE BASE PLATE

For the column, $\qquad P_{allow} = \sigma_{allow} A$

where A is the area of the column with $t = 20$ mm.

$A = \pi t(d - t)\; P_{allow} = \sigma_{allow}\, \pi t(d - t)$

$$\text{Area of base plate} = \frac{\pi D^2}{4} = \frac{P_{allow}}{\sigma_b}$$

$$\frac{\pi D^2}{4} = \frac{\sigma_{allow} \pi t(d - t)}{\sigma_b}$$

$$D^2 = \frac{4\sigma_{allow} t(d - t)}{\sigma_b}$$

$$= \frac{4(55\text{ MPa})(20\text{ mm})(230\text{ mm})}{11.5\text{ MPa}}$$

$D^2 = 88{,}000$ mm^2 $\qquad D = 296.6$ mm

$D_{min} = 297$ mm $\quad \leftarrow$

Problem 1.10-15

NUMERICAL DATA

$L = 7.5(12)$ $\quad L = 90$ in. $\quad T_{BC} = 425$ lb

$\sigma_u = 60$ ksi $\quad FS_u = 3 \quad \sigma_{ba} = 0.565$ ksi

$q = \frac{50}{12} \quad q = 4.167$ lb/in. $\quad W_E = 175$ lb

$d_{BC} = \frac{3}{16}$ in. $\quad d_B = 1.0$ in.

(a) FIND FORCE IN ROD DF AND FORCE ON WASHER AT F

$$\Sigma M_H = 0 \qquad T_{DF} = \frac{W_E\frac{L}{2} + qL\frac{L}{2}}{\left(L - \frac{L}{25}\right)}$$

$$T_{DF} = 286.458 \text{ lb}$$

NORMAL STRESS IN ROD DF:

$$\sigma_{DF} = \frac{T_{DF}}{\frac{\pi}{4}d_{BC}^2}$$

$\sigma_{DF} = 10.38$ ksi $\quad$ OK—less than σ_a; rod is acceptable $\leftarrow$

$\sigma_a = \frac{\sigma_u}{FS_u} \quad \sigma_a = 20$ ksi

BEARING STRESS ON WASHER AT F:

$$\sigma_{bF} = \frac{T_{DF}}{\frac{\pi}{4}(d_B^2 - d_{BC}^2)}$$

$\sigma_{bF} = 378$ psi $\quad$ OK—less than σ_{ba}; washer is acceptable $\leftarrow$

(b) FIND NEW FORCE IN ROD BC—SUM MOMENT ABOUT A FOR UPPER FBD—THEN CHECK NORMAL STRESS IN BC and BEARING STRESS AT B

$\Sigma M_A = 0$

$$T_{BC2} = \frac{T_{BC}L + T_{DF}\left(L - \frac{L}{25}\right)}{L}$$

$T_{BC2} = 700$ lb

REVISED NORMAL STRESS IN ROD BC:

$$\sigma_{BC2} = \frac{T_{BC2}}{\left(\frac{\pi}{4}d_{BC}^2\right)}$$

$\sigma_{BC2} = 25.352$ ksi $\quad$ exceeds $\sigma_a = 20$ ksi

SO RE-DESIGN ROD BC:

$$d_{BCreqd} = \sqrt{\frac{T_{BC2}}{\frac{\pi}{4}\sigma_a}}$$

$d_{BCreqd} = 0.211$ in. $\quad d_{BCreqd} \times 16 = 3.38$

^say 4/16 = 1/4 in. $\quad d_{BC2} = \frac{1}{4}$ in.

RE-CHECK BEARING STRESS IN WASHER AT B:

$$\sigma_{bB2} = \frac{T_{BC2}}{\left[\frac{\pi}{4}(d_B^2 - d_{BC}^2)\right]}$$

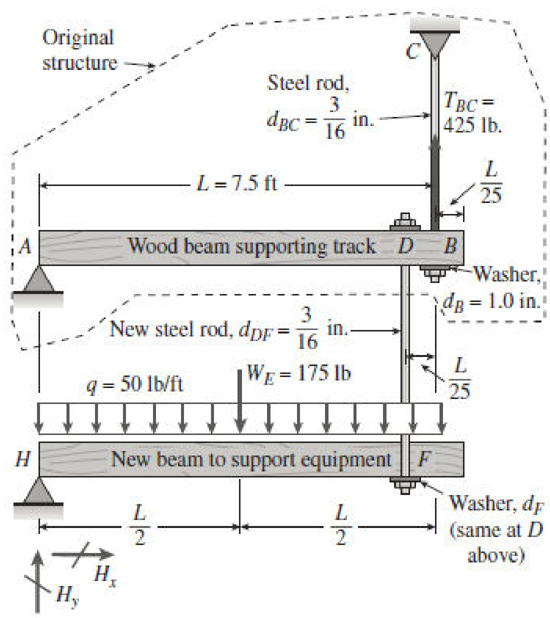
$\sigma_{bB2} = 924$ psi

^ exceeds $\sigma_{ba} = 565$ psi

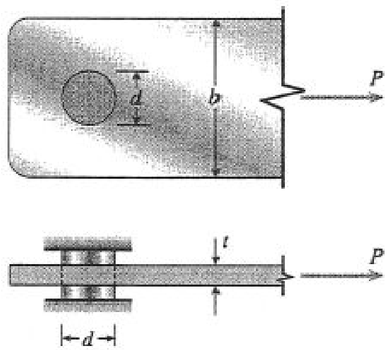
SO RE-DESIGN WASHER AT B:

$$d_{Breqd} = \sqrt{\frac{T_{BC2}}{\frac{\pi}{4}\sigma_{ba}} + d_{BC}^2} \qquad d_{Breqd} = 1.281 \text{ in.}$$

use 1 − 5/16 in washer at B: 1 + 5/16 = 1.312 in. $\leftarrow$



Problem 1.10-16



$b = 60$ mm

$t = 10$ mm

d = diameter of hole and pin

$\sigma_T = 140$ MPa

$\tau_S = 80$ MPa

$\sigma_B = 200$ MPa

UNITS USED IN THE FOLLOWING CALCULATIONS:

P is in kN

σ and τ are in N/mm^2 (same as MPa)

b, t, and d are in mm

TENSION IN THE BAR

$$P_T = \sigma_T(\text{Net area}) = \sigma_t(t)(b - d)$$

$$= (140 \text{ MPa})(10 \text{ mm})(60 \text{ mm} - d)\left(\frac{1}{1000}\right)$$

$$= 1.40\,(60 - d) \qquad \text{(Eq. 1)}$$

SHEAR IN THE PIN

$$P_S = 2\tau_S A_{\text{pin}} = 2\tau_S\left(\frac{\pi d^2}{4}\right)$$

$$= 2(80 \text{ MPa})\left(\frac{\pi}{4}\right)(d^2)\left(\frac{1}{1000}\right)$$

$$= 0.040\,\pi d^2 = 0.12566 d^2 \qquad \text{(Eq. 2)}$$

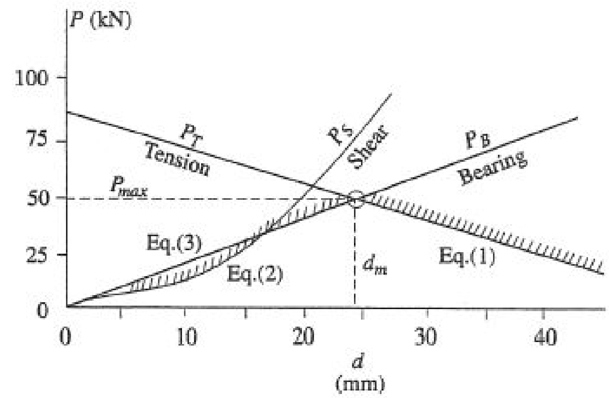
BEARING BETWEEN PIN AND BAR

$$P_B = \sigma_B\, td$$

$$= (200 \text{ MPa})(10 \text{ mm})(d)\left(\frac{1}{1000}\right)$$

$$= 2.0\, d \qquad \text{(Eq. 3)}$$

GRAPH OF EQS. (1), (2), AND (3)



(a) PIN DIAMETER d_m

$P_T = P_B$ or $1.40(60 - d) = 2.0\,d$

Solving, $d_m = \dfrac{84.0}{3.4}$ mm $= 24.7$ mm $\leftarrow$

(b) LOAD $P_{\max}$

Substitute d_m into Eq. (1) or Eq. (3):

$P_{\max} = 49.4$ kN $\leftarrow$

Problem 1.10-17

$W = 1700lbf \qquad T = 4200lbf \qquad \theta_{OBD} = 20deg + atan\left(\frac{8ft - 10ft\cdot sin(20deg)}{10ft\cdot cos(20deg)}\right) = 45.983\cdot deg$

$$\theta_{OAD} = 20deg + atan\left(\frac{8ft - 5ft\cdot sin(20deg)}{5ft\cdot cos(20deg)}\right) = 73.241\cdot deg$$

a) Maximum permissible load P if allowable force in cable is 4200 lb - cut cables, use lower FBD of OABC

$$\Sigma M_O = -W\cdot cos(20deg)\cdot 7.5ft - P\cdot 15\cdot cos(20deg) + T\cdot sin(\theta_{OAD})\cdot 5ft + T\cdot sin(\theta_{OBD})\cdot 10ft = 0$$

$$P_{max} = \frac{T\cdot sin(\theta_{OAD})\cdot 5ft + T\cdot sin(\theta_{OBD})\cdot 10ft - W\cdot cos(20deg)\cdot 7.5ft}{15ft\cdot cos(20deg)} = 2719.38\cdot lbf$$

b) Given P, find cable force T then required pin diameters $\qquad P = 2300lbf \qquad \tau_a = 10ksi$

$$T = \frac{P\cdot 15ft\cdot cos(20deg) + W\cdot cos(20deg)\cdot 7.5ft}{5ft\cdot sin(\theta_{OAD}) + 10ft\cdot sin(\theta_{OBD})} = 3706.526\cdot lbf$$

$$\alpha = (\theta_{OAD} - 20deg) - (\theta_{OBD} - 20deg) = 27.257\cdot deg \qquad R_D = \sqrt{(T^2 + T^2) + 2\cdot T\cdot T\cdot cos(\alpha)} = 7.204\cdot kip$$

Pin at D: $\qquad d_D = \sqrt{\frac{4}{\pi}\cdot\frac{R_D}{2\cdot\tau_a}} = 0.677\cdot in$

Pins at A and B: $\qquad d_B = d_A \qquad d_A = \sqrt{\frac{4}{\pi}\cdot\frac{T}{2\cdot\tau_a}} = 0.486\cdot in$

Problem 1.10-18

$q_0 = 5\frac{kN}{m}$ $\quad W = 8kN$ $\quad A_c = 100mm^2$ $\quad \tau_a = 80MPa$

$\theta_{OBD} = 20deg + atan\left(\frac{2.5 - 3\cdot\sin(20deg)}{3\cdot\cos(20deg)}\right) = 47.603\cdot deg$ $\qquad \theta_{Bx} = \theta_{OBD} - 20deg = 27.603\cdot deg$

$\theta_{OAD} = 20deg + atan\left(\frac{2.5 - 1.5\cdot\sin(20deg)}{1.5\cdot\cos(20deg)}\right) = 74.648\cdot deg$ $\qquad \theta_{Ax} = \theta_{OAD} - 20deg = 54.648\cdot deg$

Cut through cables, use lower FBD to find cable force T

resultant of distributed load = area under load (or load on projected area)

$\Sigma M_O = 0$

$Q = \frac{1}{2}\cdot q_0\cdot(4.5m\cdot\cos(20deg)) = 10.572\cdot kN$

$$T\cdot(\sin(\theta_{OAD}))\cdot(1.5m) + T\cdot(\sin(\theta_{OBD}))\cdot(3m) = W\cdot(2.25m)\cdot\cos(20deg) + Q\cdot\frac{4.5m\cdot\cos(20deg)}{3}$$

$$T = \frac{W\cdot(2.25m)\cdot\cos(20deg) + Q\cdot\frac{4.5m\cdot\cos(20deg)}{3}}{(\sin(\theta_{OAD}))\cdot(1.5m) + (\sin(\theta_{OBD}))\cdot(3m)} = 8.688\cdot kN$$

cable normal stress is

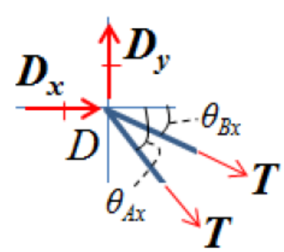
$$\frac{T}{A_c} = 86.882\cdot MPa$$

Pins at A and B: $\quad d_B = d_A \quad d_A = \sqrt{\frac{4}{\pi}\cdot\frac{T}{2\cdot\tau_a}} = 8.315\cdot mm$

Pin at D - use resultant of continuous cable forces from A and B

$$\theta_{Ax} = atan\left(\frac{2.5 - 1.5\cdot\sin(20deg)}{1.5\cdot\cos(20deg)}\right) = 54.648\cdot deg$$

$$\theta_{Bx} = atan\left(\frac{2.5 - 3\cdot\sin(20deg)}{3\cdot\cos(20deg)}\right) = 27.603\cdot deg$$

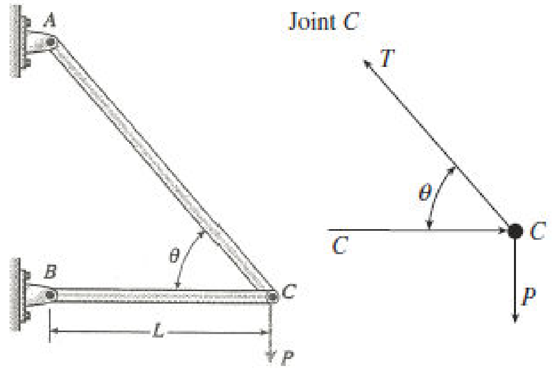


$$D_x = -T \cdot \left(\cos(\theta_{Ax}) + \cos(\theta_{Bx})\right) = -12.726 \cdot kN$$

$$D_y = T \cdot \left(\sin(\theta_{Ax}) + \sin(\theta_{Bx})\right) = 11.112 \cdot kN$$

$$D_{res} = \sqrt{{D_x}^2 + {D_y}^2} = 16.895 \cdot kN$$ so $$d_D = \sqrt{\frac{4}{\pi} \cdot \frac{D_{res}}{2 \cdot \tau_a}} = 11.595 \cdot mm$$

Problem 1.10-19



T = tensile force in bar AC

C = compressive force in bar BC

$$\sum F_{\text{vert}} = 0 \qquad T = \frac{P}{\sin\theta}$$

$$\sum F_{\text{horiz}} = 0 \qquad C = \frac{P}{\tan\theta}$$

AREAS OF BARS

$$A_{AC} = \frac{T}{\sigma_{\text{allow}}} = \frac{P}{\sigma_{\text{allow}}\sin\theta}$$

$$A_{BC} = \frac{C}{\sigma_{\text{allow}}} = \frac{P}{\sigma_{\text{allow}}\tan\theta}$$

LENGTHS OF BARS

$$L_{AC} = \frac{L}{\cos\theta} \qquad L_{BC} = L$$

WEIGHT OF TRUSS

γ = weight density of material

$$W = \gamma(A_{AC}L_{AC} + A_{BC}L_{BC})$$

$$= \frac{\gamma PL}{\sigma_{\text{allow}}}\left(\frac{1}{\sin\theta\cos\theta} + \frac{1}{\tan\theta}\right)$$

$$= \frac{\gamma PL}{\sigma_{\text{allow}}}\left(\frac{1+\cos^2\theta}{\sin\theta\cos\theta}\right) \qquad \text{Eq. (1)}$$

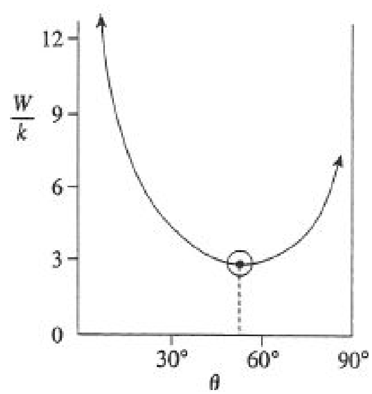
γ, P, L, and σ_{allow} are constants

W varies only with θ

Let $k = \dfrac{\gamma PL}{\sigma_{\text{allow}}}$ (k has unis of force)

$$\frac{W}{k} = \frac{1+\cos^2\theta}{\sin\theta\cos\theta} \text{ (Nondimensional)} \qquad \text{Eq. (2)}$$

GRAPH OF EQ. (2):



ANGLE θ THAT MAKES W_A MINIMUM

Use Eq. (2)

Let $f = \dfrac{1+\cos^2\theta}{\sin\theta\cos\theta}$

$$\frac{df}{d\theta} = 0$$

$$\frac{df}{d\theta} = \frac{(\sin\theta\cos\theta)(2)(\cos\theta)(-\sin\theta) - (1+\cos^2\theta)(-\sin^2\theta+\cos^2\theta)}{\sin^2\theta\cos^2\theta}$$

$$= \frac{-\sin^2\theta\cos^2\theta + \sin^2\theta - \cos^2\theta - \cos^4\theta}{\sin^2\theta\cos^2\theta}$$

SET THE NUMERATOR = 0 AND SOLVE FOR θ:

$$-\sin^2\theta\cos^2\theta + \sin^2\theta - \cos^2\theta - \cos^4\theta = 0$$

Replace $\sin^2\theta$ by $1 - \cos^2\theta$:

$$-(1-\cos^2\theta)(\cos^2\theta) + 1 - \cos^2\theta - \cos^2\theta - \cos^4\theta = 0$$

Combine terms to simplify the equation:

$$1 - 3\cos^2\theta = 0 \qquad \cos\theta = \frac{1}{\sqrt{3}}$$

$$\theta = 54.7° \quad \leftarrow$$