Solutions Manual

for

Microwave Engineering
4^{th} edition

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Chapter 1

1.1 This is an open-ended question where the focus of the answer may be largely chosen by the student or the instructor. Some of the relevant historical developments related to the early days of radio are listed here (as cited from T. S. Sarkar, R. J. Mailloux, A. A. Oliner, M. Salazar-Palma, and D. Sengupta, *History of Wireless*, Wiley, N.J., 2006):

1865: James Clerk Maxwell published his work on the unification of electric and magnetic phenomenon, including the introduction of the displacement current and the theoretical prediction of EM wave propagation.

1872: Mahlon Loomis, a dentist, was issued US Patent 129,971 for "aerial telegraphy by employing an 'aerial' used to radiate or receive pulsations caused by producing a disturbance in the electrical equilibrium of the atmosphere". This sounds a lot like radio, but in fact Loomis was not using an RF source, instead relying on static electricity in the atmosphere. Strictly speaking this method does not involve a propagating EM wave. It was not a practical system.

1887-1888: Heinrich Hertz studied Maxwell's equations and experimentally verified EM wave propagation using spark gap sources with dipole and loop antennas.

1893: Nikola Tesla demonstrated a wireless system with tuned circuits in the transmitter and receiver, with a spark gap source.

1895: Marconi transmitted and received a coded message over a distance of 1.75 miles in Italy.

1894: Oliver Lodge demonstrated wireless transmission of Morse code over a distance of 60 m, using coupled induction coils. This method relied on the inductive coupling between the two coils, and did not involve a propagating EM wave.

1897: Marconi was issued a British Patent 12,039 for wireless telegraphy.

1901: Marconi achieved the first trans-Atlantic wireless transmission.

1943: The US Supreme Court invalidated Marconi's 1904 US patent on tuning using resonant circuits as being superseded by prior art of Tesla, Lodge, and Braun.

So it is clear that many workers contributed to the development of wireless technology during this time period, and that Marconi was not the first to develop a wireless system that relied on the propagation of electromagnetic waves. On the other hand, Marconi was very successful at making radio practical and commercially viable, for both shipping and land-based services.

1.2

$$E_y = E_0 \cos(\omega t - kx)\ ,\ E_0 = 5\ \text{V/m},\ f = 2.4\ \text{GHz}.$$
$$\epsilon_r = 2.54\ ,\ x_1 = 0.1\ ,\ x_2 = 0.15$$

a) $\eta = \eta_0/\sqrt{\epsilon_r} = 236.6\,\Omega$

$H_z = E_y/\eta = 0.0211 \cos(\omega t - kz)$

b) $v_p = c/\sqrt{\epsilon_r} = 1.88 \times 10^8$ m/sec

c) $\lambda = v_p/f = 0.0784$ m , $k = 2\pi/\lambda = 80.11\ \text{m}^{-1}$

d) $\Delta\phi = k(x_2 - x_1) = 80.11(.15 - .1) = 4.00$ rad $= 229.5^\circ$

1.3

$\bar{E} = E_0(a\hat{x} + b\hat{y})\,e^{-jk_0 z}$; a, b real

Let $\bar{E} = A(\hat{x} - j\hat{y})e^{-jk_0 z} + B(\hat{x} + j\hat{y})\,e^{-jk_0 z}$

where A, B are the amplitudes of the RHCP and LHCP components. Equating vector components gives

$\hat{x}$: $A + B = aE_0$

$\hat{y}$: $-jA + jB = bE_0$, or $A - B = jbE_0$

so

$A = E_0(a + jb)/2$

$B = E_0(a - jb)/2$

check: if $a=1$, $b=2$ then $A = (\frac{1}{2} + j)E_0$, $B = (\frac{1}{2} - j)E_0$

(agrees with Problem 1.5 from 3rd ed.)

1.4 From eq. (1.76),

$$\bar{H} = \frac{1}{\eta_0}\hat{n}\times\bar{E} \quad , \quad \bar{E} = \bar{E}_0 e^{-j\bar{k}\cdot\bar{r}}$$

$$\bar{S} = \bar{E}\times\bar{H}^* = \frac{1}{\eta_0}\bar{E}\times\hat{n}\times\bar{E}^*$$

$$= \frac{1}{\eta_0}\left[(\bar{E}\cdot\bar{E}^*)\hat{n} - (\bar{E}\cdot\hat{n})\bar{E}^*\right] \qquad \text{(from B.5)}$$

Since $\bar{k}\cdot\bar{E}_0 = k_0\hat{n}\cdot\bar{E}_0 = 0$ from (1.69) and (1.74), we have

$$\bar{S} = \frac{\hat{n}}{\eta_0}\bar{E}\cdot\bar{E}^* = \frac{\hat{n}}{\eta_0}|E_0|^2 \quad W/m^2 \quad \checkmark$$

1.5 Writing general plane wave fields in each region:

$$\bar{E}^i = \hat{x}e^{-jk_0z} \qquad \bar{H}^i = \frac{\hat{y}}{\eta_0}e^{-jk_0z} \qquad \text{for } z<0$$

$$\bar{E}^r = \hat{x}\Gamma e^{jk_0z} \qquad \bar{H}^r = \frac{-\hat{y}}{\eta_0}\Gamma e^{jk_0z} \qquad \text{for } z<0$$

$$\bar{E}^s = \hat{x}(Ae^{-jkz}+Be^{jkz}) \qquad \bar{H}^s = \frac{\hat{y}}{\eta}(Ae^{-jkz}-Be^{jkz}) \qquad \text{for } 0<z<d$$

$$\bar{E}^t = \hat{x}Te^{-jk_0(z-d)} \qquad \bar{H}^t = \frac{\hat{y}}{\eta_0}Te^{-jk_0(z-d)} \qquad \text{for } z>d$$

Now match E_x and H_y at $z=0$ and $z=d$ to obtain four equations for Γ, T, A, B:

$$z=0: \qquad 1+\Gamma = A+B \qquad \frac{1}{\eta_0}(1-\Gamma) = \frac{1}{\eta}(A-B)$$

$$z=d: \qquad j(-A+B) = T \qquad \frac{j}{\eta}(-A-B) = \frac{T}{\eta_0} \qquad (\text{since } d=\lambda_0/4\sqrt{\epsilon_r})$$

Solving for Γ gives

$$\Gamma = \frac{\eta^2-\eta_0^2}{\eta^2+\eta_0^2} \quad \checkmark$$

CHECK:

$\lambda/4$ TRANSFORMER $\Rightarrow Z_{in} = \eta^2/\eta_0$, $\Gamma = \dfrac{\eta^2/\eta_0 - \eta_0}{\eta^2/\eta_0+\eta_0} = \dfrac{\eta^2-\eta_0^2}{\eta^2+\eta_0^2}$.

1.6 The incident, reflected, and transmitted fields can be written as,

$$\bar{E}^i = E_o(\hat{x} - j\hat{y})e^{-jk_o z} \qquad \bar{H}^i = j\frac{E_o}{\eta_o}(\hat{x} - j\hat{y})e^{-jk_o z} \qquad (RHCP)$$

$$\bar{E}^r = E_o\Gamma(\hat{x} - j\hat{y})e^{jk_o z} \qquad \bar{H}^r = -j\frac{E_o}{\eta_o}\Gamma(\hat{x} - j\hat{y})e^{jk_o z} \qquad (LHCP)$$

$$\bar{E}^t = E_o T(\hat{x} - j\hat{y})e^{-\gamma z} \qquad \bar{H}^t = j\frac{E_o}{\eta}T(\hat{x} - j\hat{y})e^{-\gamma z} \qquad (RHCP)$$

Matching fields at $z=0$ gives

$$\Gamma = \frac{\eta - \eta_o}{\eta + \eta_o}, \qquad T = \frac{2\eta}{\eta + \eta_o}$$

The Poynting vectors are: $\quad (\hat{x} - j\hat{y}) \times (\hat{x} - j\hat{y})^* = 2j\hat{z}$

For $z<0$: $\quad \bar{S}^- = (\bar{E}^i + \bar{E}^r) \times (\bar{H}^i + \bar{H}^r)^* = \dfrac{2\hat{z}|E_o|^2}{\eta_o}\left(1 - |\Gamma|^2 + \Gamma e^{2jk_o z} + \Gamma^* e^{-2jk_o z}\right)$ ✓

For $z>0$: $\quad \bar{S}^+ = \bar{E}^t \times \bar{H}^{t*} = \dfrac{2\hat{z}|E_o|^2|T|^2}{\eta^*}e^{-2\alpha z}$ ✓

at $z=0$,

$$\bar{S}^- = \frac{2\hat{z}|E_o|^2}{\eta_o}\left(1 - |\Gamma|^2 + \Gamma - \Gamma^*\right) = \frac{2\hat{z}|E_o|^2}{\eta_o}(1+\Gamma)(1-\Gamma^*) \checkmark$$

$$\bar{S}^+ = 2\hat{z}|E_o|^2 \frac{4\eta}{|\eta + \eta_o|^2} \qquad \left(\text{using } T = \frac{2\eta}{\eta + \eta_o}\right)$$

$$= \frac{2\hat{z}|E_o|^2}{\eta_o}\left(\frac{2\eta}{\eta + \eta_o}\right)\left(\frac{2\eta_o}{\eta + \eta_o}\right)^* = \frac{2\hat{z}|E_o|^2}{\eta_o}(1+\Gamma)(1-\Gamma^*) \checkmark$$

Thus $\bar{S}^- = \bar{S}^+$ at $z=0$, and power is conserved.

1.7 From Table 1.1,

$$\gamma = j\omega\sqrt{\mu_0 \epsilon} = 2\pi j f \sqrt{\mu_0 \epsilon_0}\,\sqrt{5-j2} = j\,\frac{2\pi(1000)}{300}\sqrt{5.385\angle{-22^\circ}}$$

$$= 48.5\angle{79^\circ} = 9.25 + j\,47.6 = \alpha + j\beta \quad (\text{neper/m, rad/m})$$

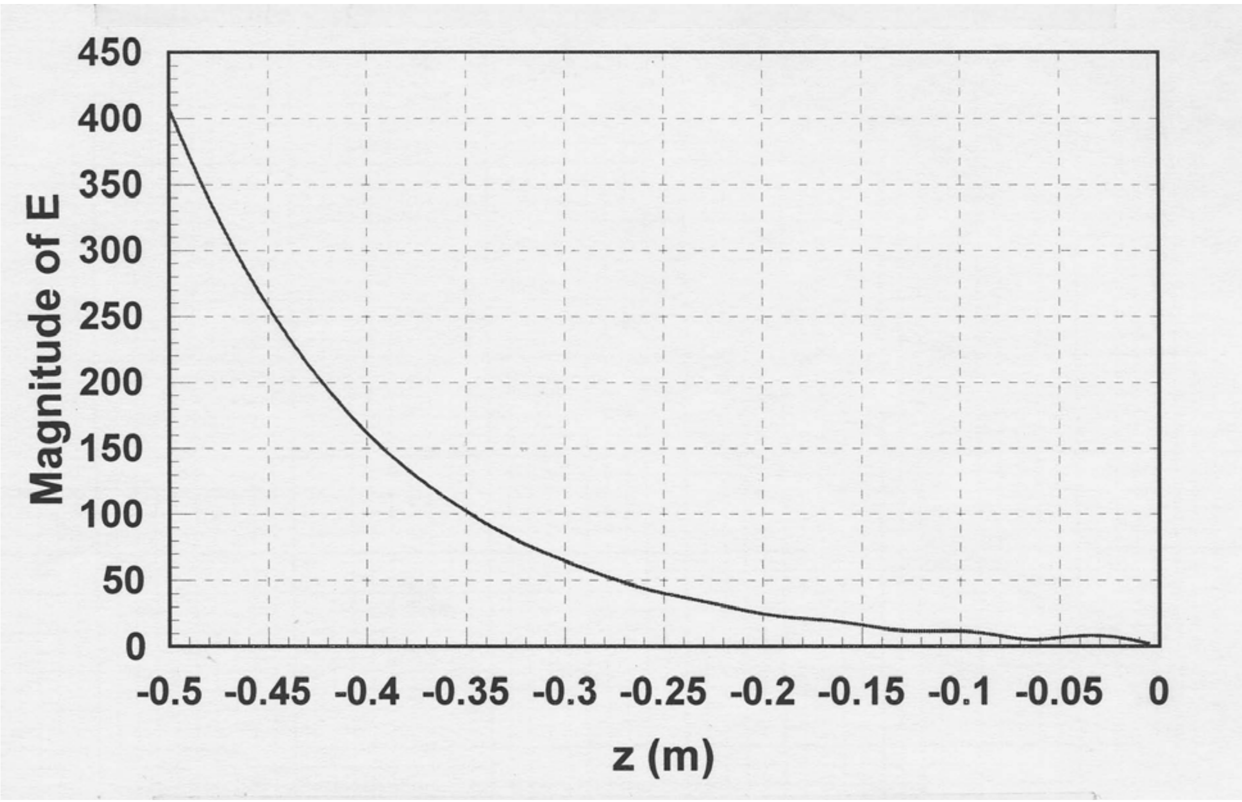
$$\eta = \frac{j\omega\mu}{\gamma} = \frac{j\omega\sqrt{\mu_0\epsilon_0}}{j\omega\sqrt{\mu_0\epsilon}}\sqrt{\frac{\mu_0}{\epsilon_0}} = \frac{\eta_0}{\sqrt{5-j2}} = \frac{377}{2.32\angle{-11^\circ}} = 163\angle{11^\circ}\ \Omega$$

$$\Gamma = -1$$

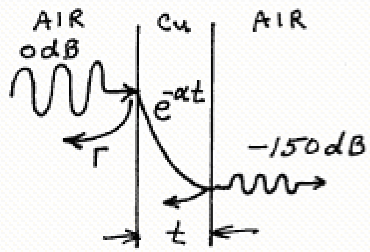
For $z<0$, $\bar{E} = \bar{E}^i + \bar{E}^r = 4\hat{x}\left(e^{-\gamma z} - e^{\gamma z}\right)$

$$|\bar{E}| = 4\left|e^{-\alpha z}e^{-j\beta z} - e^{\alpha z}e^{j\beta z}\right|$$

$|\bar{E}|$ vs z is plotted below.



1.8 The total loss through the sheet is the product of the transmission losses at the air-copper and copper-air interfaces, and the exponential loss through the sheet.



$$\delta_s = \sqrt{\frac{2}{\omega\mu\sigma}} = 2.09 \times 10^{-6}\,m = \frac{1}{\alpha}$$

$$\eta_c = \frac{(1+j)}{\sigma\delta_s} = 8.2\times10^{-3}(1+j)\,\Omega$$

a) Power transfer from air into copper is given by,

$$1-|\Gamma|^2 \quad , \quad \Gamma = \frac{\eta_c-\eta_o}{\eta_c+\eta_o} \simeq \frac{8.2\times10^{-3}(1+j)-377}{377} = -0.999956+j\,4.35E\text{-}5$$

This yields a power transfer of -40.6 dB into the copper. By symmetry, the same transfer occurs for the copper-air interface.

b) the attenuation within the copper sheet is,

$$\text{copper att.} = 150\,dB - 40.6\,dB - 40.6\,dB = 68.8\,dB$$

$$= -20\log e^{-t/\delta_s} \Rightarrow t = \underline{0.017\text{ mm}}$$ ✓

(J. Mead provided this correction on 9/04)

1.9 From Table 1.1,

$$\gamma = j\omega\sqrt{\mu_0\epsilon} = j\frac{2\pi(3000)}{300}\sqrt{3(1-j.1)} = 5.435 + j108.964 = \alpha + j\beta \ \mathrm{m}^{-1}$$

$$\eta = \frac{\eta_0}{\sqrt{\epsilon_r(1-j.1)}} = 217.121 \angle 2.855°$$

a) $$S_i = \mathrm{Re}\left\{\frac{|\bar{E}_i(z=0)|^2}{\eta^*}\right\} = 46.000 \ \mathrm{W/m^2} \ ✓$$

$$\Gamma = -1 \ \text{at } z = \ell = 20 \text{ cm}$$

$$\bar{E}_r = \Gamma\bar{E}_i(z=\ell)e^{\gamma(z-\ell)} = -100\hat{x}e^{-2\gamma\ell}e^{\gamma z}$$

$$S_r = \mathrm{Re}\left\{\frac{|\bar{E}_r(z=0)|^2}{\eta^*}\right\} = 0.595 \ \mathrm{W/m^2} \ ✓$$

b) $$\bar{E}_t = \bar{E}_i + \bar{E}_r$$

$$\bar{E}_t(z=0) = 100\hat{x}(1-e^{-2\gamma\ell}), \quad \bar{H}_t(z=0) = \frac{100\hat{y}}{\eta}(1+e^{-2\gamma\ell})$$

$$S_{in} = \mathrm{Re}\left\{\bar{E}_t \times \bar{H}_t^* \cdot \hat{z}\right\} = 45.584 \ \mathrm{W/m^2}$$

But $S_i - S_r = 45.405 \ \mathrm{W/m^2} \neq S_{in}$. This is because S_i and S_r individually are not physically meaningful in a lossy medium.

(The above were computed using a FORTRAN program, with 6 digit precision. The error between $S_i - S_r$ and S_{in} is only about 0.4% - this would be larger if the loss were greater.)

1.10 As in Example 1.3, assume outgoing plane wave fields in each region. To get J_{sx}, we need H_y, since $\hat{n}\times(\bar{H}_2-\bar{H}_1)=\bar{J}_s$ ($\hat{n}=\hat{z}$). Then we must have E_x to get $\bar{S}=\bar{E}\times\bar{H}^*=\pm S\hat{z}$. So the form of the fields must be,

for $z<0$, $\bar{E}_1=\hat{x}Ae^{jk_0z}$ $\qquad$ for $z>0$, $\bar{E}_2=\hat{x}Be^{-jkz}$

$\bar{H}_1=\frac{-\hat{y}}{\eta_0}Ae^{jk_0z}$ $\qquad$ $\bar{H}_2=\hat{y}\frac{B}{\eta}e^{-jkz}$

with $k_0=\omega\sqrt{\mu_0\epsilon_0}$, $k=\omega\sqrt{\mu_0\epsilon_0\epsilon_r}$, $\eta_0=\sqrt{\mu_0/\epsilon_0}$, $\eta=\sqrt{\mu_0/\epsilon_0\epsilon_r}$, and A and B are unknown amplitudes to be determined.

The boundary conditions at $z=0$ are, from (1.36) and (1.37),

$$(\bar{E}_2-\bar{E}_1)\times\hat{n}=0 \quad\Rightarrow\quad A=B$$

$$\hat{z}\times(\bar{H}_2-\bar{H}_1)=\bar{J}_s \quad\Rightarrow\quad -\left(\frac{B}{\eta}+\frac{A}{\eta_0}\right)=J_0$$

$$\therefore\quad A=B=\frac{-J_0\eta\eta_0}{\eta+\eta_0}$$

1.11 This current sheet will generate obliquely propagating plane waves. From (1.132)–(1.133), assume

$$\left.\begin{aligned}\bar{E}_1 &= A(\hat{x}\cos\theta_1 + \hat{z}\sin\theta_1)\,e^{-jk_o(x\sin\theta_1 - z\cos\theta_1)} \\ \bar{H}_1 &= \frac{-A}{\eta_o}\hat{y}\,e^{-jk_o(x\sin\theta_1 - z\cos\theta_1)}\end{aligned}\right\}\text{ for } z<0$$

$$\left.\begin{aligned}\bar{E}_2 &= B(\hat{x}\cos\theta_2 - \hat{z}\sin\theta_2)\,e^{-jk(x\sin\theta_2 + z\cos\theta_2)} \\ \bar{H}_2 &= \frac{B}{\eta}\hat{y}\,e^{-jk(x\sin\theta_2 + z\cos\theta_2)}\end{aligned}\right\}\text{ for } z>0$$

with $k_o = \omega\sqrt{\mu_o\epsilon_o}$, $k = \sqrt{\epsilon_r}\,k_o$, $\eta_o = \sqrt{\mu_o/\epsilon_o}$, $\eta = \eta_o/\sqrt{\epsilon_r}$.

Apply boundary conditions at $z=0$:

$$\hat{z}\times(\bar{E}_2 - \bar{E}_1) = 0 \Rightarrow A\cos\theta_1 e^{-jk_o x\sin\theta_1} - B\cos\theta_2 e^{-jkx\sin\theta_2} = 0$$

$$\hat{z}\times(\bar{H}_2 - \bar{H}_1) = J_s \Rightarrow \frac{A}{\eta_o}e^{-jk_o x\sin\theta_1} + \frac{B}{\eta}e^{-jkx\sin\theta_2} = -J_o e^{-j\beta x}$$

For phase matching we must have $k_o\sin\theta_1 = k\sin\theta_2 = \beta$

$$\therefore\ \theta_1 = \sin^{-1}\beta/k_o \checkmark \qquad \theta_2 = \sin^{-1}\beta/k \quad (\text{must have } \beta < k_o)$$

Then,

$$A\cos\theta_1 = B\cos\theta_2, \qquad \frac{A}{\eta_o} + \frac{B}{\eta} = -J_o$$

$$A = \frac{-J_o\eta\eta_o\cos\theta_2}{\eta\cos\theta_2 + \eta_o\cos\theta_1}, \qquad B = \frac{-J_o\eta\eta_o\cos\theta_1}{\eta\cos\theta_2 + \eta_o\cos\theta_1}$$

Check: If $\beta = 0$, then $\theta_1 = \theta_2 = 0$, and $A = B = \dfrac{-J_o\eta\eta_o}{\eta + \eta_o}$,

which agrees with Problem 1.10 ✓

1.12 This solution is identical to the parallel polarized dielectric case of Section 1.8, except for the definitions of k_1, k_2, η_1, and η_2. Thus,

$$k_o \sin\theta_i = k_o \sin\theta_r = k \sin\theta_t \quad ; \; k = k_o\sqrt{\mu_r}$$

$$\Gamma = \frac{\eta\cos\theta_t - \eta_o\cos\theta_i}{\eta\cos\theta_t + \eta_o\cos\theta_i} \qquad T = \frac{2\eta\cos\theta_i}{\eta\cos\theta_t + \eta_o\cos\theta_i}$$

$$\eta = \eta_o\sqrt{\mu_r}$$

There will be a Brewster angle if $\Gamma = 0$. This requires that,

$$\eta\cos\theta_t = \eta_o\cos\theta_i$$

$$\sqrt{\mu_r}\sqrt{1-\left(\frac{k_o}{k}\right)^2\sin^2\theta_i} = \cos\theta_i = \sqrt{1-\sin^2\theta_i}$$

$$\mu_r\left(1-\frac{1}{\mu_r}\cancel{\sin^2\theta_i}\right) = 1-\cancel{\sin^2\theta_i}$$

or, $\mu_r = 1$. This implies a uniform region, so there is no Brewster angle for $\mu_r \neq 1$. ✓

1.13 Again, this solution is similar to the perpendicular polarized case of Section 1.8, except for the definition of k_1, k_2, η_1, η_2. Thus,

$$\Gamma = \frac{\eta \cos\theta_i - \eta_0 \cos\theta_t}{\eta \cos\theta_i + \eta_0 \cos\theta_t} \quad , \quad T = \frac{2\eta \cos\theta_i}{\eta \cos\theta_i + \eta_0 \cos\theta_t}$$

A Brewster angle exists if

$$\eta \cos\theta_i = \eta_0 \cos\theta_t$$

$$\sqrt{\mu_r}\sqrt{1-\sin^2\theta_i} = \sqrt{1-\frac{1}{\mu_r}\sin^2\theta_i}$$

$$\mu_r^2 - \mu_r^2 \sin^2\theta_i = \mu_r - \sin^2\theta_i$$

$$\mu_r = (\mu_r + 1)\sin^2\theta_i$$

$$\sin\theta_i = \sin\theta_b = \sqrt{\frac{\mu_r}{1+\mu_r}} < 1 \quad \checkmark$$

Thus, a Brewster angle does exist for this case.

1.14 $\bar{E} = 3\hat{x} - 2\hat{y} + 5\hat{z}$

$$\bar{D} = [\epsilon]\bar{E} = \begin{bmatrix} 1 & 3j & 0 \\ -3j & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}\begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix} = \begin{bmatrix} 3-6j \\ -4-9j \\ 20 \end{bmatrix} = (3-6j)\hat{x} + (-4-9j)\hat{y} + 20\hat{z}$$

1.15

$$D_x = \epsilon_0(\epsilon_r E_x + j\kappa E_y)$$
$$D_y = \epsilon_0(-j\kappa E_x + \epsilon_r E_y)$$
$$D_z = \epsilon_0 E_z$$

Then,
$$D_+ = D_x - jD_y = \epsilon_0(\epsilon_r - \kappa)E_x - j\epsilon_0(\epsilon_r - \kappa)E_y = \epsilon_0(\epsilon_r - \kappa)E_+$$
$$D_- = D_x + jD_y = \epsilon_0(\epsilon_r + \kappa)E_x + j\epsilon_0(\epsilon_r + \kappa)E_y = \epsilon_0(\epsilon_r + \kappa)E_-$$

OR,
$$\begin{bmatrix} D_+ \\ D_- \\ D_z \end{bmatrix} = \begin{bmatrix} (\epsilon_r - \kappa) & 0 & 0 \\ 0 & (\epsilon_r + \kappa) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} E_+ \\ E_- \\ E_z \end{bmatrix}$$

From Maxwell's equations,
$$\left.\begin{aligned} \nabla \times \bar{E} &= -j\omega\mu \bar{H} \\ \nabla \times \bar{H} &= j\omega[\epsilon]\bar{E} \end{aligned}\right\} \quad \begin{aligned} &\nabla \times \nabla \times \bar{E} = -j\omega\mu \nabla \times \bar{H} = \omega^2\mu[\epsilon]\bar{E} \\ &\nabla^2 \bar{E} + \omega^2\mu[\epsilon]\bar{E} = 0 \quad \text{(CARTESIAN)} \end{aligned}$$

Expanding this wave equation gives,
$$\nabla^2 E_x + \omega^2\mu\epsilon_0(\epsilon_r E_x + j\kappa E_y) = 0 \qquad (1)$$
$$\nabla^2 E_y + \omega^2\mu\epsilon_0(-j\kappa E_x + \epsilon_r E_y) = 0 \qquad (2)$$
$$\nabla^2 E_z + k_0^2 E_z = 0 \qquad (3)$$

adding (1) $+j$(2) gives $\nabla^2(E_x + jE_y) + \omega^2\mu\epsilon_0[(\epsilon_r + \kappa)E_x + j(\epsilon_r + \kappa)E_y] = 0$
$$\nabla^2 E^+ + \omega^2\mu\epsilon_0(\epsilon_r + \kappa)E^+ = 0$$
$$\therefore \beta_+ = k_0\sqrt{\epsilon_r + \kappa} \checkmark$$

adding (1) $-j$(2) gives $\nabla^2(E_x - jE_y) + \omega^2\mu\epsilon_0[(\epsilon_r - \kappa)E_x - j(\epsilon_r - \kappa)E_y] = 0$
$$\nabla^2 E^- + \omega^2\mu\epsilon_0(\epsilon_r - \kappa)E^- = 0$$
$$\therefore \beta_- = k_0\sqrt{\epsilon_r - \kappa} \checkmark$$

Note that the wave equations for E^+, E^- must be satisfied simultaneously. Thus, for E^+ we must have $E^- = 0$. This implies that $E_y = jE_x = jE_0$. The actual electric field is then,
$$\bar{E}^+ = \hat{x}E_x + \hat{y}E_y = E_0(\hat{x} + j\hat{y})e^{-j\beta_+ z} \qquad \text{(LHCP)}$$
This is a LHCP wave. Similarly for $\bar{E}^-$ we must have $E^+ = 0$:
$$\bar{E}^- = \hat{x}E_x + \hat{y}E_y = E_0(\hat{x} - j\hat{y})e^{-j\beta_- z} \qquad \text{(RHCP)}$$

1.16 Comparing (1.118), (1.125), and (1.129) shows that

$$E_t = \frac{J_t}{\sigma} = \frac{J_s}{\sigma \delta_s} = R_s J_s .$$

Thus $\bar{E}_t = R_s \bar{J}_s = R_s \hat{n} \times \bar{H}$ is the desired surface impedance relation. Applying this to the surface integral of (1.155) gives, on S,

$$\left[(\bar{E}_1 \times \bar{H}_2) - (\bar{E}_2 \times \bar{H}_1) \right] \cdot \hat{n} = R_s \left[(\hat{n} \times \bar{H}_{1t}) \times \bar{H}_{2t} - (\hat{n} \times \bar{H}_{2t}) \times \bar{H}_{1t} \right]$$

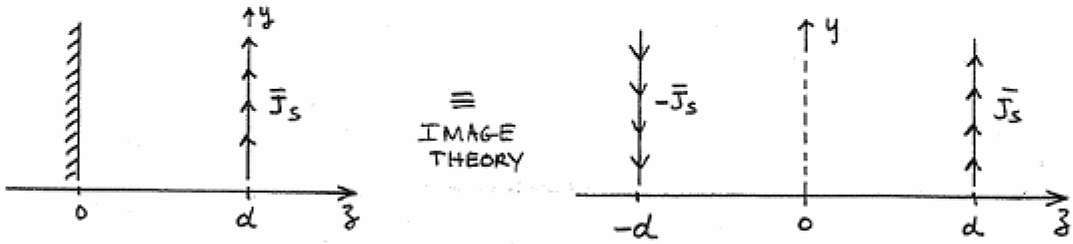
(USING B.5)

$$= R_s \left[\cancelto{0}{(\bar{H}_{2t} \cdot \hat{n})} \bar{H}_{1t} - \cancel{(\bar{H}_{2t} \cdot \bar{H}_{1t})} \hat{n} - \cancelto{0}{(\bar{H}_{1t} \cdot \hat{n})} H_{2t} + \cancel{(\bar{H}_{1t} \cdot \bar{H}_{2t})} \hat{n} \right]$$

$$= 0$$

So (1.157) is obtained.

1.17



First find the fields due to the source at $z=d$. From (1.139)–(1.140),

For $z<d$,
$$\bar{E}_1 = A\hat{y}\, e^{-jk_o(x\sin\theta - z\cos\theta)}$$
$$\bar{H}_1 = \frac{A}{\eta_o}(\hat{x}\cos\theta + \hat{z}\sin\theta)\, e^{-jk_o(x\sin\theta - z\cos\theta)}$$

For $z>d$,
$$\bar{E}_2 = B\hat{y}\, e^{-jk_o(x\sin\theta + z\cos\theta)}$$
$$\bar{H}_2 = \frac{B}{\eta_o}(-\hat{x}\cos\theta + \hat{z}\sin\theta)\, e^{-jk_o(x\sin\theta + z\cos\theta)}$$

Apply boundary conditions at $z=d$:
$$\hat{z}\times[\bar{E}(d^+) - \bar{E}(d^-)] = 0 \Rightarrow A e^{jk_o d\cos\theta} = B e^{-jk_o d\cos\theta}$$
$$\hat{z}\times[\bar{H}(d^+) - \bar{H}(d^-)] = \bar{J}_s \Rightarrow [-B\cos\theta\, e^{-jk_o d\cos\theta} - A\cos\theta\, e^{jk_o d\cos\theta}]\cdot e^{-jk_o x\sin\theta} = \eta_o J_o e^{-j\beta x}$$

For phase matching, $k_o\sin\theta = \beta$

Then, $A = \dfrac{-\eta_o J_o}{2\cos\theta} e^{-jk_o d\cos\theta}$ $\qquad B = \dfrac{-\eta_o J_o}{2\cos\theta} e^{jk_o d\cos\theta}$

$$\bar{E} = \frac{-\eta_o J_o\hat{y}}{2\cos\theta}\begin{cases} e^{-jk_o[x\sin\theta - (z-d)\cos\theta]} & z<d \\ e^{-jk_o[x\sin\theta + (z-d)\cos\theta]} & z>d \end{cases}$$

The fields due to the source at $z=-d$ can then be found by replacing d with $-d$, and J_o with $-J_o$:

$$\bar{E} = \frac{\eta_0 J_0 \hat{y}}{2\cos\theta} \begin{cases} e^{-jk_0[x\sin\theta - (z+d)\cos\theta]} & z < -d \\ e^{-jk_0[x\sin\theta + (z+d)\cos\theta]} & z > -d \end{cases}$$

Combining these results gives the total fields:

$$\bar{E} = \frac{-j\eta_0 J_0 \hat{y}}{\cos\theta} \begin{cases} e^{-jk_0 x\sin\theta}\, e^{-jk_0 d} \sin(k_0 z\cos\theta) & 0 < z < d \\ e^{-jk_0 x\sin\theta}\, e^{-jk_0 z} \sin(k_0 d\cos\theta) & z > d \end{cases}$$

CHECK: If $\beta = 0$, then $\theta = 0$ and we have,

$$\bar{E} = -j\eta_0 J_0 \hat{y} \begin{cases} e^{-jk_0 d} \sin k_0 z & \text{for } 0 < z < d \\ e^{-jk_0 z} \sin k_0 d & \text{for } z > d \end{cases}$$

This agrees with the results in (1.161)–(1.162).

1.18

$$\nabla\times\bar{E} = \hat{\rho}\left(\frac{1}{\rho}\frac{\partial E_z}{\partial\phi} - \frac{\partial E_\phi}{\partial z}\right) + \hat{\phi}\left(\frac{\partial E_\rho}{\partial z} - \frac{\partial E_z}{\partial\rho}\right) + \hat{z}\,\frac{1}{\rho}\left(\frac{\partial(\rho E_\phi)}{\partial\rho} - \frac{\partial E_\rho}{\partial\phi}\right)$$

$$\nabla\times\nabla\times\bar{E} = \hat{\rho}\left[\frac{-1}{\rho^2}\frac{\partial^2 E_\rho}{\partial\phi^2} - \frac{\partial^2 E_\rho}{\partial z^2} + \frac{\partial^2 E_z}{\partial\rho\partial z} + \frac{1}{\rho}\frac{\partial^2 E_\phi}{\partial\rho\partial\phi} + \frac{1}{\rho^2}\frac{\partial E_\phi}{\partial\phi}\right]$$

$$+\hat{\phi}\left[-\frac{\partial^2 E_\phi}{\partial z^2} + \frac{1}{\rho}\frac{\partial^2 E_z}{\partial\phi\partial z} - \frac{\partial^2 E_\phi}{\partial\rho^2} - \frac{1}{\rho}\frac{\partial E_\phi}{\partial\rho} + \frac{E_\phi}{\rho^2} - \frac{1}{\rho^2}\frac{\partial E_\rho}{\partial\phi} + \frac{1}{\rho}\frac{\partial^2 E_\rho}{\partial\phi\partial\rho}\right]$$

$$+\hat{z}\left[\frac{-\partial^2 E_z}{\partial\rho^2} - \frac{1}{\rho^2}\frac{\partial^2 E_z}{\partial\phi^2} + \frac{\partial^2 E_\rho}{\partial\rho\partial z} + \frac{1}{\rho}\frac{\partial^2 E_\phi}{\partial\phi\partial z} + \frac{1}{\rho}\frac{\partial E_\rho}{\partial z} - \frac{1}{\rho}\frac{\partial E_z}{\partial\rho}\right]$$

$$\nabla(\nabla\cdot\bar{E}) = \hat{\rho}\left[\frac{\partial^2 E_\rho}{\partial\rho^2} + \frac{\partial^2 E_z}{\partial\rho\partial z} + \frac{1}{\rho}\frac{\partial^2 E_\phi}{\partial\rho\partial\phi} + \frac{1}{\rho}\frac{\partial E_\rho}{\partial\rho} - \frac{1}{\rho^2}\frac{\partial E_\phi}{\partial\phi} - \frac{E_\rho}{\rho^2}\right]$$

$$+\hat{\phi}\left[\frac{1}{\rho}\frac{\partial^2 E_z}{\partial\phi\partial z} + \frac{1}{\rho^2}\frac{\partial^2 E_\phi}{\partial\phi^2} + \frac{1}{\rho}\frac{\partial^2 E_\rho}{\partial\rho\partial\phi} + \frac{1}{\rho^2}\frac{\partial E_\rho}{\partial\phi}\right]$$

$$+\hat{z}\left[\frac{\partial^2 E_z}{\partial z^2} + \frac{1}{\rho}\frac{\partial^2 E_\phi}{\partial\phi\partial z} + \frac{\partial^2 E_\rho}{\partial\rho\partial z} + \frac{1}{\rho}\frac{\partial E_\rho}{\partial z}\right]$$

If we apply ∇^2 to the cylindrical components of $\bar{E}$ we get:

$$\nabla^2\bar{E} \overset{?}{=} \hat{\rho}\,\nabla^2 E_\rho + \hat{\phi}\,\nabla^2 E_\phi + \hat{z}\,\nabla^2 E_z \qquad \text{(THIS IS NOT A VALID STEP!)}$$

$$= \hat{\rho}\left[\frac{1}{\rho}\frac{\partial E_\rho}{\partial\rho} + \frac{\partial^2 E_\rho}{\partial\rho^2} + \frac{1}{\rho^2}\frac{\partial^2 E_\rho}{\partial\phi^2} + \frac{\partial^2 E_\rho}{\partial z^2}\right]$$

$$+\hat{\phi}\left[\frac{1}{\rho}\frac{\partial E_\phi}{\partial\rho} + \frac{\partial^2 E_\phi}{\partial\rho^2} + \frac{1}{\rho^2}\frac{\partial^2 E_\phi}{\partial\phi^2} + \frac{\partial^2 E_\phi}{\partial z^2}\right]$$

$$+\hat{z}\left[\frac{1}{\rho}\frac{\partial E_z}{\partial\rho} + \frac{\partial^2 E_z}{\partial\rho^2} + \frac{1}{\rho^2}\frac{\partial^2 E_z}{\partial\phi^2} + \frac{\partial^2 E_z}{\partial z^2}\right]$$

Note that the $\hat{\rho}$ and $\hat{\phi}$ components of $\nabla\times\nabla\times\bar{E}$ and $\nabla(\nabla\cdot\bar{E}) - \nabla^2\bar{E}$ do not agree. This is because $\hat{\rho}$ and $\hat{\phi}$ are not constant vectors, so $\nabla^2\bar{E} \neq \hat{\rho}\,\nabla^2 E_\rho + \hat{\phi}\,\nabla^2 E_\phi + \hat{z}\,\nabla^2 E_z$. The $\hat{z}$ components are equal.

Chapter 2

2.1

$$i(t,z) = 1.8 \cos(3.77\times10^9 t - 18.13z) \text{ mA}$$

$$\omega = 3.77\times10^9 \text{ rad/sec}, \quad \beta = 18.13 \text{ m}^{-1}, \quad Z_0 = 75\,\Omega$$

a) $f = \omega/2\pi = 3.77\times10^9/2\pi = 600 \text{ MHz}$

b) $v_p = \omega/\beta = 2.08\times10^8 \text{ m/sec}$

c) $\lambda = 2\pi/\beta = 0.346 \text{ m}$

d) $\epsilon_r = (c/v_p)^2 = 2.08$ (Teflon)

e) $I(z) = 1.8\, e^{-j\beta z}$ (mA)

f) $v(t,z) = 0.135 \cos(\omega t - \beta z)$ V.

2.2

$R = 4.0\ \Omega/\text{m}$, $G = 0.02$ S/m, $L = 0.5\ \mu$H/m, $C = 200$ pF/m

$f = 800$ MHz, $\ell = 30$ cm

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = 0.540 + j\,50.268 \quad /\text{m}$$

$$Z_0 = \sqrt{(R+j\omega L)/(G+j\omega C)} = 49.99 + j\,0.46\ \Omega$$

with $R = G = 0$, $\beta = \omega\sqrt{LC} = 50.265$ rad/m

$$Z_0 = \sqrt{L/C} = 50.0\ \Omega$$

Note that β, Z_0 w/o loss are very close to values with loss.

For $\ell = 30$ cm, att $= \alpha\ell = 0.162\,\text{np}\left(\frac{8.686\text{dB}}{\text{np}}\right) = 1.4\text{dB}$

2.3 From Table 2.1:

$$R_s = \sqrt{\frac{\omega\mu}{2\sigma}} = 0.00825\,\Omega$$

$$L = \frac{\mu_0}{2\pi}\ln\frac{b}{a} = 2.40\times10^{-7}\ \text{H/m}$$

$$C = \frac{2\pi\epsilon_0\epsilon_r}{\ln b/a} = 9.64\times10^{-11}\ \text{Fd/m}$$

$$R = \frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right) = 3.76\ \Omega/\text{m}$$

$$G = \frac{2\pi\omega\epsilon_0\epsilon_r\tan\delta}{\ln b/a} = 2.42\times10^{-4}\ \text{S/m}$$

For small loss, $Z_0 = \sqrt{L/C} = 49.9\,\Omega$ ✓

From (2.85a), $\alpha \simeq \frac{1}{2}\left(\frac{R}{Z_0} + GZ_0\right) = 0.044\ \text{np/m} = 0.38\ \text{dB/m}$ ✓

From RG-402U cable data: $Z_0 = 50\,\Omega$, $\alpha = 13\,\text{dB}/100\,\text{ft}$
$= 0.426\ \text{dB/m}$

Check using formulas from Example 2.7: (difference due to braided outer conductor, not solid)

$$Z_0 = \sqrt{\frac{\mu}{\epsilon}}\,\frac{\ln b/a}{2\pi} = 49.9\,\Omega\ ✓$$

$$\alpha_c = \frac{R_s}{2\eta\ln b/a}\left(\frac{1}{a}+\frac{1}{b}\right) = 0.0376\ \text{np/m} = 0.326\ \text{dB/m}\ ✓$$

$$\alpha_d = \frac{\omega\epsilon_0\epsilon_r}{2}\eta\tan\delta = 0.00605\ \text{np/m} = 0.052\ \text{dB/m}$$

$$\alpha_T = 0.378\ \text{dB/m}\ ✓$$

also verified with Serenade.

2.4 Using the formulas of Problem 2.3, with $\alpha \simeq \frac{1}{2}(R/Z_0 + G Z_0)$:

f	$R_s (\Omega)$	$R (\Omega)$	G (S)	α (Np/m)	α (dB/m)
1 MHz	2.6×10^{-4}	0.118	2.42×10^{-7}	1.19×10^{-3}	0.0103
10 MHz	8.25×10^{-4}	0.376	2.42×10^{-6}	3.82×10^{-3}	0.0332
100 MHz	2.6×10^{-3}	1.18	2.42×10^{-5}	1.24×10^{-3}	0.1078
1 GHz	8.25×10^{-3}	3.76	2.42×10^{-4}	4.365×10^{-2}	0.379
10 GHz	2.6×10^{-2}	11.8	2.42×10^{-3}	1.785×10^{-1}	1.55
100 GHz	8.25×10^{-3}	37.6	2.42×10^{-2}	1.96	17.0

Results are plotted below (with additional data points). Note that the frequency dependence is between $\sqrt{f}$ ($R \sim \sqrt{f}$), and f ($G \sim f$), at low and high frequencies.