SOLUTIONS MANUAL

MICROWAVE TRANSISTOR AMPLIFIERS

Analysis and Design

SECOND EDITION

Guillermo Gonzalez

PRENTICE HALL
Upper Saddle River, NJ 07458

Acquisitions Editor: *Eric Svendsen*
Project Editor: *Lorena Cerisano*
Production Manager: *Joan Eurell*
Production Coordinator: *Julia Meehan*

©1996 by Prentice-Hall, Inc.
Simon & Schuster / A Viacom Company
Upper Saddle River, New Jersey 07458

All rights reserved. No part of this book may be reproduced, in any form or by any means, without permission in writing from the publisher.

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

ISBN 0-13-271255-5

Prentice-Hall International (UK) Limited, *London*
Prentice-Hall of Australia Pty. Limited, *Sydney*
Prentice-Hall Canada Inc., *Toronto*
Prentice-Hall Hispanoamericana, S.A., *Mexico*
Prentice-Hall of India Private Limited, *New Delhi*
Prentice-Hall of Japan, Inc., *Tokyo*
Simon & Schuster Asia Pte. Ltd., *Singapore*
Editora Prentice-Hall do Brasil, Ltda., *Rio de Janeiro*

CONTENTS

CHAPTER 1

1.1) IN (1.3.41) LET $\Gamma_o = |\Gamma_o| e^{j\psi_o}$:

$$|V(d)| = |A_1| \, |1 + |\Gamma_o| e^{j(\psi_o - 2\beta d)}|$$

THE MAXIMUM VALUE OF $|V(d)|$ OCCURS WHEN $e^{j(\psi_o - 2\beta d)} = 1$, AND THE MINIMUM VALUE OCCURS WHEN $e^{j(\psi_o - 2\beta d)} = -1$.
HENCE,

$$|V(d)|_{max} = |A_1| \, |1 + |\Gamma_o|| = |A_1|(1 + |\Gamma_o|) \quad \text{AND} \quad |V(d)|_{min} = |A_1| \, |1 - |\Gamma_o|| = |A_1|(1 - |\Gamma_o|)$$

1.2) $v(d,t) = \text{Re}[V(d) e^{j\omega t}]$

$$V(d) = 3.95 \, e^{j(\beta d - 63.44°)} + 1.77 \, e^{-j\beta d}$$

$$\therefore \; v(d,t) = 3.95 \cos(\omega t + \beta d - 63.44°) + 1.77 \cos(\omega t - \beta d)$$

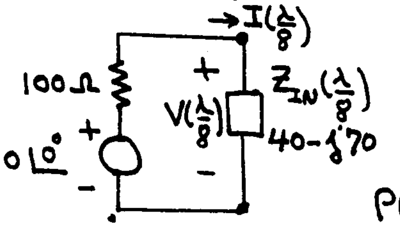
AND $i(d,t) = \dfrac{3.95}{50} \cos(\omega t + \beta d - 63.44°) - \dfrac{1.77}{50} \cos(\omega t - \beta d)$

1.3) (a) $\Gamma_o = \dfrac{Z_L - Z_o}{Z_L + Z_o} = \dfrac{(100 + j100) - 50}{(100 + j100) + 50} = 0.62 \angle 29.7°$

$$Z_{IN}\left(\tfrac{\lambda}{8}\right) = 50 \, \frac{(100 + j100)\cos 45° + j50 \sin 45°}{50 \cos 45° + j(100 + j100) \sin 45°} = 40 - j70 \; \Omega$$

$$VSWR = \frac{1 + |\Gamma_o|}{1 - |\Gamma_o|} = \frac{1 + 0.62}{1 - 0.62} = 4.26$$

(b)



$$V\left(\tfrac{\lambda}{8}\right) = \frac{10\angle 0° \, (40 - j70)}{40 - j70 + 100} = 5.15 \angle -33.7°$$

$$I\left(\tfrac{\lambda}{8}\right) = V\left(\tfrac{\lambda}{8}\right) \Big/ Z_{IN}\left(\tfrac{\lambda}{8}\right) = 0.064 \angle 26.6°$$

$$P\left(\tfrac{\lambda}{8}\right) = \tfrac{1}{2} \text{Re}\left[V\left(\tfrac{\lambda}{8}\right) I^*\left(\tfrac{\lambda}{8}\right)\right] = 82 \; \text{mW}$$

$$V\left(\tfrac{\lambda}{8}\right) = 5.15 \angle -33.7 = A_1 e^{j\frac{\pi}{4}} \left[1 + 0.62 \angle 29.7° \, e^{-j\frac{\pi}{2}}\right]$$

$$\therefore \; A_1 = 3.643 \angle -56.31°$$

$$V(d) = 3.643\angle{-56.31^\circ}\, e^{j\beta d}\left[1 + 0.62\angle{29.7^\circ}\, e^{-j2\beta d}\right]$$

$$V(0) = 3.643\angle{-56.31^\circ}\left[1 + 0.62\angle{29.7^\circ}\right] = 5.72\angle{-45.02^\circ}$$

$$I(0) = \frac{5.72\angle{-45.02^\circ}}{100 + j100} = 0.04\angle{-90^\circ}$$

$$P(0) = \frac{1}{2}\operatorname{Re}\left[V(0) I^*(0)\right] = 82 \text{ mW}$$

AS EXPECTED: $P\left(\frac{\lambda}{8}\right) = P(0)$

1.4) $b_1 = S_{11} a_1 + S_{12} a_2$

$b_2 = S_{21} a_1 + S_{22} a_2$

LETTING $Z_0 = Z_{01} = Z_{02}$, THEN $b_1 = \frac{V_1^-}{\sqrt{Z_0}}$, $b_2 = \frac{V_2^-}{\sqrt{Z_0}}$, $a_1 = \frac{V_1^+}{\sqrt{Z_0}}$, AND $a_2 = \frac{V_2^+}{\sqrt{Z_0}}$.

$$\therefore \frac{V_1^-}{\sqrt{Z_0}} = S_{11}\frac{V_1^+}{\sqrt{Z_0}} + S_{12}\frac{V_2^+}{\sqrt{Z_0}} \Rightarrow V_1^- = S_{11}V_1^+ + S_{12}V_2^+$$

$$\frac{V_2^-}{\sqrt{Z_0}} = S_{21}\frac{V_1^+}{\sqrt{Z_0}} + S_{22}\frac{V_2^+}{\sqrt{Z_0}} \Rightarrow V_2^- = S_{21}V_1^+ + S_{22}V_2^+$$

WITH $b_1 = \sqrt{Z_0}\, I_1^-$, $b_2 = \sqrt{Z_0}\, I_2^-$, $a_1 = \sqrt{Z_0}\, I_1^+$, $a_2 = \sqrt{Z_0}\, I_2^+$

WE OBTAIN: $I_1^- = S_{11} I_1^+ + S_{12} I_2^+$

$I_2^- = S_{21} I_1^+ + S_{22} I_2^+$

1.5) $\begin{cases} a_1 = T_{11} b_2 + T_{12} a_2 & (1) \\ b_1 = T_{21} b_2 + T_{22} a_2 & (2) \end{cases}$ $\begin{cases} b_1 = S_{11} a_1 + S_{12} a_2 & (3) \\ b_2 = S_{21} a_1 + S_{22} a_2 & (4) \end{cases}$

FROM (3) AND (4):

$$a_1 = \frac{1}{S_{11}} b_1 - \frac{S_{12}}{S_{11}} a_2 \quad (5)$$

$$a_1 = \frac{1}{S_{21}} b_2 - \frac{S_{22}}{S_{21}} a_2 \quad (6)$$

EQUATING (5) AND (6):

$$\frac{1}{S_{11}} b_1 - \frac{S_{12}}{S_{11}} a_2 = \frac{1}{S_{21}} b_2 - \frac{S_{22}}{S_{21}} a_2$$

$$\therefore \quad b_1 = \frac{S_{11}}{S_{21}} b_2 + \left[S_{12} - \frac{S_{22} S_{11}}{S_{21}} \right] a_2 \quad (7)$$

COMPARING (2) AND (7):

$$T_{21} = \frac{S_{11}}{S_{21}} \quad \text{AND} \quad T_{22} = S_{12} - \frac{S_{11} S_{22}}{S_{21}}$$

EQUATING (3) AND (7) GIVES

$$S_{11} a_1 + S_{12} a_2 = \frac{S_{11}}{S_{21}} b_2 + \left[S_{12} - \frac{S_{11} S_{22}}{S_{21}} \right] a_2$$

$$\therefore a_1 = \frac{1}{S_{21}} b_2 - \frac{S_{22}}{S_{21}} a_2 \quad (8)$$

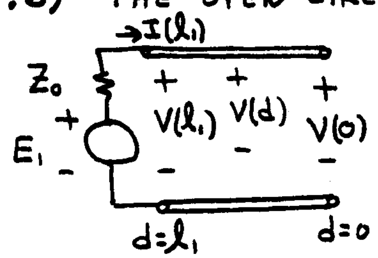
COMPARING (1) AND (8):

$$T_{11} = \frac{1}{S_{21}} \quad \text{AND} \quad T_{12} = -\frac{S_{22}}{S_{21}}$$

SIMILARLY, STARTING WITH (1) AND (2), IT FOLLOWS THAT:

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} \frac{T_{21}}{T_{11}} & T_{22} - \frac{T_{12} T_{21}}{T_{11}} \\ \frac{1}{T_{11}} & -\frac{T_{12}}{T_{11}} \end{bmatrix}$$

1.6) THE OPEN-CIRCUIT VOLTAGE $E_{1,TH}$ IS OBTAINED AS FOLLOWS:



$\Gamma_0 = 1$

$$V(d) = A(e^{j\beta d} + e^{-j\beta d}) = 2A\cos\beta d$$

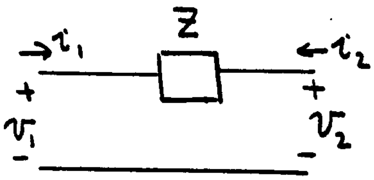
$$I(d) = j2\frac{A}{Z_0}\sin\beta d$$

$$V(\ell_1) = 2A\cos\beta\ell_1 = E_1 - I(\ell_1) Z_0$$
$$= E_1 - j2A\sin\beta d$$

$$\therefore A = \frac{E_1}{2(\cos\beta\ell_1 + j\sin\beta\ell_1)} = \frac{E_1}{2} e^{-j\beta\ell_1}$$

HENCE: $E_{1,TH} = V(0) = 2A = E_1 e^{-j\beta\ell_1}$

1.7) (a)



$$v_1 = A v_2 - B i_2$$
$$i_1 = C v_2 - D i_2$$

$A = \left.\dfrac{v_1}{v_2}\right|_{i_2=0}$ WITH $i_2 = 0$: $v_1 = v_2$

$\therefore A = 1$

$B = -\left.\dfrac{v_1}{i_2}\right|_{v_2=0}$ WITH $v_2 = 0$: $v_1 = i_1 Z = -i_2 Z$

$\therefore B = -\dfrac{(-i_2 Z)}{i_2} = Z$

$C = \left.\dfrac{i_1}{v_2}\right|_{i_2=0}$ WITH $i_2 = 0$: $i_1 = -i_2 = 0$

$\therefore C = 0$

$D = \left.\dfrac{i_1}{-i_2}\right|_{v_2=0}$ WITH $v_2 = 0$: $i_1 = -i_2$

$\therefore D = 1$

(b) FROM FIG. 1.8.1:

$$S_{11} = \frac{A'+B'-C'-D'}{A'+B'+C'+D'} = \frac{1+\frac{Z}{Z_0}-0-1}{1+\frac{Z}{Z_0}+0+1} = \frac{Z}{Z+2Z_0}$$

$$S_{12} = \frac{2(A'D'-B'C')}{A'+B'+C'+D'} = \frac{2(1-0)}{1+\frac{Z}{Z_0}+0+1} = \frac{2Z_0}{Z+2Z_0}$$

$$S_{21} = \frac{2}{A'+B'+C'+D'} = \frac{2}{1+\frac{Z}{Z_0}+0+1} = \frac{2Z_0}{Z+2Z_0}$$

$$S_{22} = \frac{-A'+B'-C'+D'}{A'+B'+C'+D'} = \frac{-1+\frac{Z}{Z_0}-0+1}{1+\frac{Z}{Z_0}+0+1} = \frac{Z}{Z+2Z_0}$$

FOR THE SHUNT ADMITTANCE Y THE ABCD MATRIX IS :

$A = 1$, $B = 0$, $C = Y$, AND $D = 1$

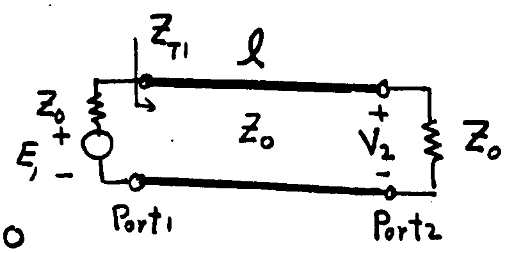
AND

$$S_{11} = \frac{A'+B'-C'-D'}{A'+B'+C'+D'} = \frac{1+0-YZ_0-1}{1+0+YZ_0+1} = \frac{-YZ_0}{2+YZ_0}$$

$$S_{12} = \frac{2(A'D'-B'C')}{A'+B'+C'+D'} = \frac{2(1-0)}{1+0+YZ_0+1} = \frac{2}{2+YZ_0}$$

ALSO: $S_{22} = S_{11}$ AND $S_{21} = S_{12}$

1.8) IN A Z_0 SYSTEM:



$$Z_{T1} = Z_0$$

$$S_{11} = \frac{Z_{T1} - Z_0}{Z_{T1} + Z_0} = \frac{Z_0 - Z_0}{Z_0 + Z_0} = 0$$

ALSO (FROM SYMMETRY): $S_{22} = 0$

$$S_{21} = 2\frac{\sqrt{Z_0}}{\sqrt{Z_0}}\frac{V_2}{E_1} = 2\frac{V_2}{E_1}$$

. SINCE $V(x) = \frac{E_1}{2} e^{-j\beta x}$

$V_2 = V(\ell) = \frac{E_1}{2} e^{-j\beta \ell}$

$$\therefore \; S_{21} = \frac{2\left(\frac{E_1}{2} e^{-j\beta\ell}\right)}{E_1} = e^{-j\beta\ell}$$

ALSO (FROM SYMMETRY): $S_{12} = e^{-j\beta\ell}$

AN ALTERNATE WAY OF DERIVING S_{21} IS:

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$$

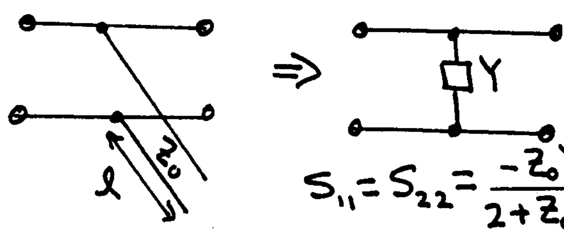
SINCE b_2 IS THE WAVE AT PORT2 AND a_1 IS THE WAVE AT PORT1, WE HAVE:

$$b_2 = a_1 e^{-j\beta\ell} \quad (a_2 = 0)$$

$$\therefore \; S_{21} = \frac{b_2}{a_1} = e^{-j\beta\ell}$$

$$T = \begin{bmatrix} \frac{1}{S_{21}} & -\frac{S_{22}}{S_{21}} \\ \frac{S_{11}}{S_{21}} & S_{12} - \frac{S_{11}S_{22}}{S_{21}} \end{bmatrix} = \begin{bmatrix} e^{j\beta\ell} & 0 \\ 0 & e^{-j\beta\ell} \end{bmatrix}$$

1.9)

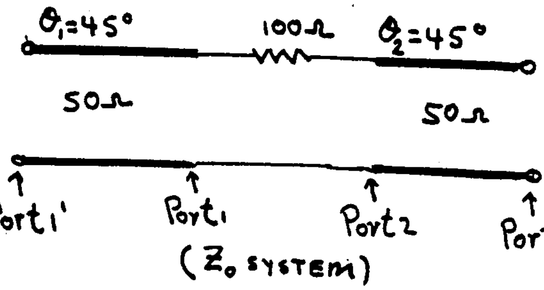


$$Y = \frac{1}{Z_{oc}} = \frac{1}{-jZ_0 \cot\beta\ell} = jY_0 \tan\beta\ell$$

$$S_{11} = S_{22} = \frac{-Z_0 Y}{2 + Z_0 Y} = \frac{-1}{1 - j2\cot\beta\ell}$$

$$S_{12} = S_{21} = \frac{2}{2 + Z_0 Y} = \frac{2}{2 + j\tan\beta\ell}$$

USE (1.4.11) TO EVALUATE THE T PARAMETERS.

1.10) 

THE S PARAMETERS AT PORTS 1 AND 2 ARE:

$$S_{11} = \frac{Z}{Z+2Z_o} = \frac{100}{100+2(50)} = 0.5$$

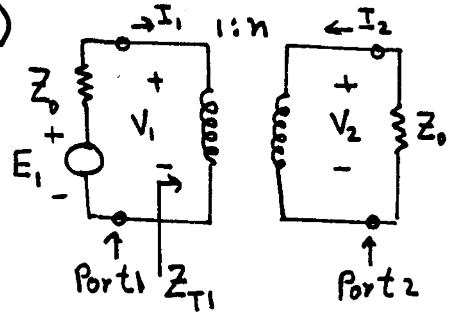
$$S_{22} = S_{11} = 0.5$$

$$S_{12} = S_{21} = \frac{2Z_o}{Z+2Z_o} = 0.5$$

THE S PARAMETERS AT PORTS 1' AND 2' ARE OBTAINED USING (1.S.4):

$$S'_{11} = S'_{22} = S_{11} e^{-j2\theta_1} = 0.5 \angle{-90°} = -j0.5$$

$$S'_{21} = S'_{12} = S_{21} e^{-j(\theta_1+\theta_2)} = 0.5 \angle{-90°} = -j0.5$$

1.11) 

IN THE Z_o SYSTEM SHOWN, THE PARAMETERS S_{11} AND S_{21} ARE CALCULATED AS FOLLOWS:

$$I_2 = \frac{I_1}{n}, \quad V_2 = V_1 n$$

$$Z_{T1} = \frac{V_1}{I_1} = \frac{V_2}{I_2 n^2} = \frac{Z_o}{n^2}$$

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} = \frac{Z_{T1} - Z_o}{Z_{T1} + Z_o} = \frac{Z_o/n^2 - Z_o}{\frac{Z_o}{n^2} + Z_o} = \frac{1-n^2}{1+n^2}$$

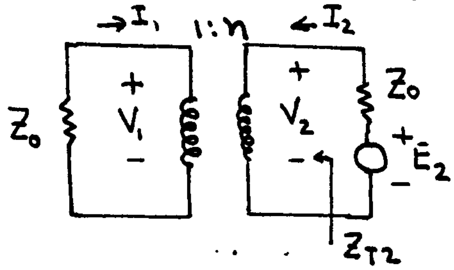
$$S_{21} = 2\sqrt{\frac{Z_{o2}}{Z_{o1}}}\,\frac{V_2}{E_1} = 2\frac{V_2}{E_1} \quad (1)$$

$$V_1 = \frac{E_1 Z_{T1}}{Z_{T1} + Z_o} = \frac{E_1\, Z_o/n^2}{\frac{Z_o}{n^2} + Z_o} = \frac{E_1}{n^2+1}$$

$$V_2 = nV_1 = \frac{nE_1}{n^2+1} \quad \text{OR} \quad \frac{V_2}{E_1} = \frac{n}{n^2+1} \quad (2)$$

(2) INTO (1):

$$S_{21} = \frac{2n}{n^2+1}$$



THE PARAMETERS S_{22} AND S_{12} ARE CALCULATED AS FOLLOWS:

$$Z_{T2} = Z_o n^2$$

$$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0} = \frac{Z_{T2}-Z_0}{Z_{T2}+Z_0} = \frac{n^2 Z_0 - Z_0}{n^2 Z_0 + Z_0} = \frac{n^2-1}{n^2+1}$$

$$S_{12} = 2\frac{V_1}{E_2} \quad (3)$$

$$V_2 = \frac{E_2 Z_{T2}}{Z_{T2}+Z_0} = \frac{E_2 Z_0 n^2}{Z_0 n^2 + Z_0} = \frac{E_2 n^2}{n^2+1}$$

$$V_1 = \frac{V_2}{n} = \frac{E_2 n}{n^2+1} \quad \text{OR} \quad \frac{V_1}{E_2} = \frac{n}{n^2+1} \quad (4)$$

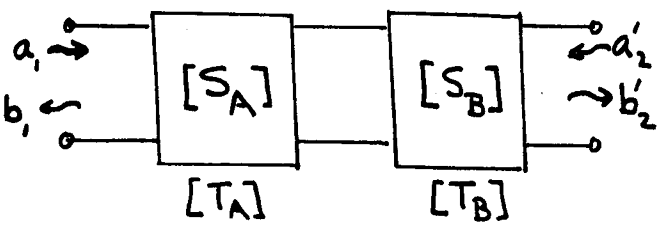
(4) INTO (3):

$$S_{12} = \frac{2n}{n^2+1}$$

AT PORTS 1' AND 2', WITH $\theta = \theta_1 = \theta_2$, WE OBTAIN:

$$[S'] = \begin{bmatrix} S_{11}e^{-j2\theta} & S_{12}e^{-j2\theta} \\ S_{21}e^{-j2\theta} & S_{22}e^{-j2\theta} \end{bmatrix} = e^{-j2\theta}\begin{bmatrix} \frac{1-n^2}{1+n^2} & \frac{2n}{n^2+1} \\ \frac{2n}{n^2+1} & \frac{n^2-1}{n^2+1} \end{bmatrix}$$

1.12)



[T] AND [S] PARAMETERS ARE RELATED BY (1.4.11)

FROM (1.4.13):

$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} T_{11}^A & T_{12}^A \\ T_{21}^A & T_{22}^A \end{bmatrix}\begin{bmatrix} T_{11}^B & T_{12}^B \\ T_{21}^B & T_{22}^B \end{bmatrix}\begin{bmatrix} b_2' \\ a_2' \end{bmatrix}$$

OVERALL T_{11} IS: $T_{11} = T_{11}^A T_{11}^B + T_{12}^A T_{21}^B$

$$T_{11} = \frac{1}{S_{21}^A}\frac{1}{S_{21}^B} + \left(-\frac{S_{22}^A}{S_{21}^A}\right)\left(\frac{S_{11}^B}{S_{21}^B}\right)$$

SINCE $T_{11} = \dfrac{1}{S_{21}} = \dfrac{1-S_{22}^A S_{11}^B}{S_{21}^A S_{21}^B} \Rightarrow S_{21} = \dfrac{S_{21}^A S_{21}^B}{1-S_{22}^A S_{11}^B}$