CHAPTER 2
Propagation and Noise

Problem 2.1

Early satellite communications systems often used large 20-m diameter parabolic dishes with an efficiency of approximately 60% to receive a signal at 4 GHz. What is the gain of one of these dishes in dB?

Solution

Let D = 20m, η = 60%, and f = 4 GHz. Then from Eq. (2.9)

$$\begin{aligned} G &= \eta\left(\frac{\pi D}{\lambda}\right)^2 \\ &= \eta\left(\frac{\pi D f}{c}\right)^2 \\ &= 0.6\left(\frac{\pi\times 20\times 4\times 10^9}{3\times 10^8}\right) \\ &= 4.21\times 10^5 \end{aligned}$$

Converting this to decibels

$$\begin{aligned} G_{dB} &= 10\log_{10}\left(4.21\times 10^5\right) \\ &= 56.2 \text{ dB} \end{aligned}$$

Problem 2.2

In terrestrial microwave links, line-of-sight transmission limits the separation of transmitters and receivers to about 40 km. If a 100-milliwatt transmitter at 4-GHz is used with transmitting and receiving antennas of 0.5 m^2 effective area, what is the received power level in dBm? If the receiving antenna terminals are matched to a 50 ohm impedance, what voltage would be induced across these terminals by the transmitted signal?

Solution

By the Friis equation Eq.(2.11)

$$P_R = \frac{G_T G_R P_T}{L_p}$$

where

$$G_T = G_R = \frac{A_e}{A_{isotropic}}$$

$$= \frac{A_e}{\left(\lambda^2 / 4\pi\right)}$$

$$= 1.12\times10^3 \sim 30.5 \text{ dB}$$

$$L_p = \left(\frac{4\pi R}{\lambda}\right)^2 = \left(\frac{4\pi Rf}{c}\right)^2$$

$$= 4.50\times10^{15} \sim 156.5 \text{ dB}$$

$$P_T = 0.10 \text{ W} \sim 20 \text{ dBm}$$

So using the decibel version of the Friis equation

$$\begin{aligned} P_R(\text{dBm}) &= G_T(\text{dB}) + G_R(\text{dB}) + P_T(\text{dBm}) - L_p(\text{dB}) \\ &= 30.5 + 30.5 + 20 - 156.5 \\ &= -75.5 \text{ dBm} \end{aligned}$$

This is equivalent to 28 picowatts. The corresponding rms voltage across a 50-ohm resistor is

$$P_R = \frac{V_{rms}^2}{R} \Rightarrow V_{rms} = \sqrt{P_R R} = 37\ \mu\text{V}$$

Problem 2.3

Plot and compare the path loss (dB) for the free-space and plane earth models at 800 MHz versus distance on a logarithmic scale for distances from 1 m to 40 km. Assume the antennas are isotropic and have a height of 10 m.

Solution

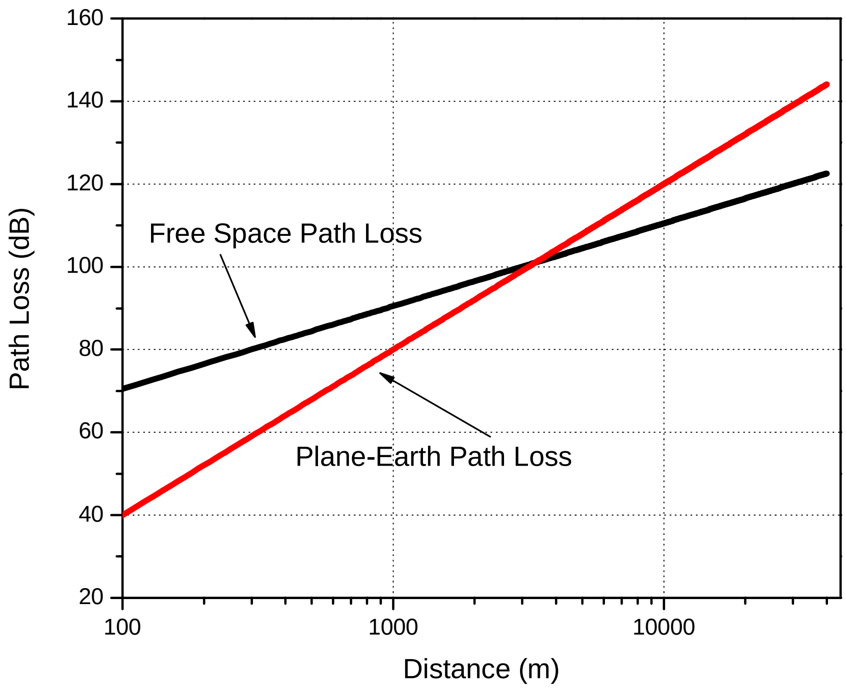
The free-space path loss is

$$L_{space} = \left(\frac{4\pi R}{\lambda}\right)^2$$

and the plane-earth path loss is

$$L_{plane} = \left(\frac{R^2}{h_T h_R}\right)^2$$

These are plotted in the following figure. Note the significantly faster attenuation with the plane-earth model. Note that the plane-earth model applies only for $R >> h_R$, h_T. The plane-earth model shows less loss than free-space at distances less than a kilometer, is this reasonable? How large should R be to apply the plane-Earth model?



Comparison of path losses for Problem 2.3.

Problem 2.4

A company owns two office towers in a city and wants to set up a 4-GHz microwave link between the two towers. The two towers have heights of 100 m and 50 m, respectively, and are separated by 3 km. In the line of sight (LOS) and midway between the two towers is a third tower of height 70 meters. Will line-of-sight transmission be possible between the two towers? Justify your answer. Describe an engineering solution to obtain line-of-sight transmission.

Solution

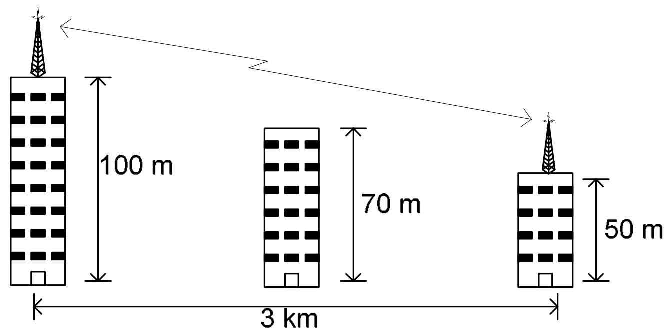
The situation is shown conceptually in the following figure. The radius of the first Fresnel zone is given by Eq. (2.38)

$$r_1 = \sqrt{\frac{\lambda d_1 d_2}{d_1 + d_2}}$$
$$= 7.5 \text{ m}$$

where

$$\lambda = \frac{c}{f} = 0.075 \text{ m}$$

Since the spacing between the centre tower and the line of sight is only 5m (prove using similar triangles), the path does not have a clear first Fresnel zone and some non-line-of-sight effects will be expected. A practical engineering solution would be to raise the height of the antennas on both towers.



Conceptualization of three towers of Problem 2.4.

Problem 2.5

In Problem 2.4, suppose the middle tower was 80 m and the shorter tower was only 30 m. The separation between the two communicating towers is 2 km. What would the increase in path loss be in this case relative to free-space loss? How would the diffraction loss be affected if the transmission frequency is decreased from 4 GHz to 400 MHz?

Solution

Referring to the following figure, the centre office tower now extends 15m above the line of sight. The Fresnel-Kirchhoff diffraction parameter is thus given by

$$\begin{aligned} \nu &= h\sqrt{\frac{2(d_1+d_2)}{\lambda d_1 d_2}} \\ &= 2.8 \end{aligned}$$

Then from Fig. 2.10 of the text, the corresponding diffraction loss is 22 dB.

If we repeat the calculation at 400 MHz,

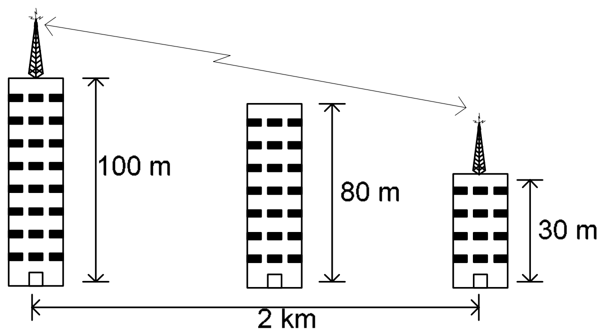
$$\lambda = \frac{3x10^8}{400x10^6} = 0.75 \text{ m}$$

then the Fresnel-Kirchhoff diffraction parameter is

$$\nu = 15\sqrt{\frac{4}{0.75(1500)}} = 0.89\,.$$

and the corresponding diffraction loss is approximately 13 dB.

One must realize that the analysis applies to knife-edge diffraction; in the above example there is liable to be some diffraction of the signal around the sides of the office tower so the above calculations might be treated with some scepticism in practice.



Conceptualization of three towers of Problem 2.5.

Problem 2.6

A brief measurement campaign indicates that the median propagation loss at 420 MHz in a midsize North American city can be modeled with n=2.8 and a fixed loss (β) of 25 dB; that is,

$$L_p = 25\,\mathrm{dB} + 10\log_{10}(r^{2.8})$$

Assuming a cell phone receiver sensitivity of –95 dBm, what transmitter power is required to service a circular area of radius 10 km? Suppose the measurements were optimistic and $n = 3.1$ is more appropriate, what is the corresponding increase in transmit power that would be required?

Solution

For isotropic antennas, the relationship between transmitted and receive power is

$$\begin{aligned} P_T(dBm) &= P_R(dBm) + L_p(dB) \\ &= -95 + L_p(dB) \end{aligned}$$

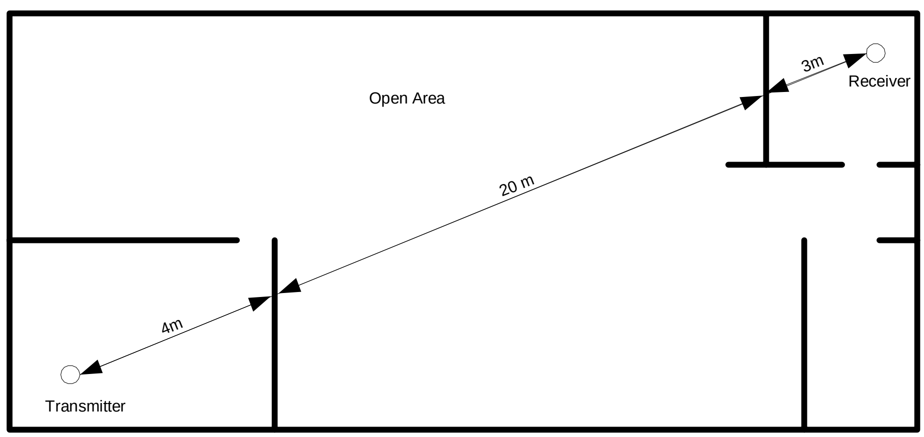
where the second line applies for the above receiver at the edge of coverage (sensitivity threshold). The path loss is

$$\begin{aligned} L_p &= 25 + 28\log_{10} r \\ &= 25 + 28\log_{10}(10^4) \\ &= 137\ \mathrm{dB} \end{aligned}$$

at a distance of 10 km. Consequently, the transmitted power must be 42 dBm or equivalently, 12 dBW. In the second case with n=3.1, the path loss is 149 dB and the transmitted power must by 24 dBW.

Problem 2.7

Using the same model as Example 2.5, predict the path loss for the site geometry shown in the following figure. Assume that the walls cause an attenuation of 5 dB, and floors 10 dB.



Site geometry for Problem 2.7.

Solution

For this scenario we make a link budget as shown below. The total path loss is 87.2 dB and the received power is -67.2 dBm.

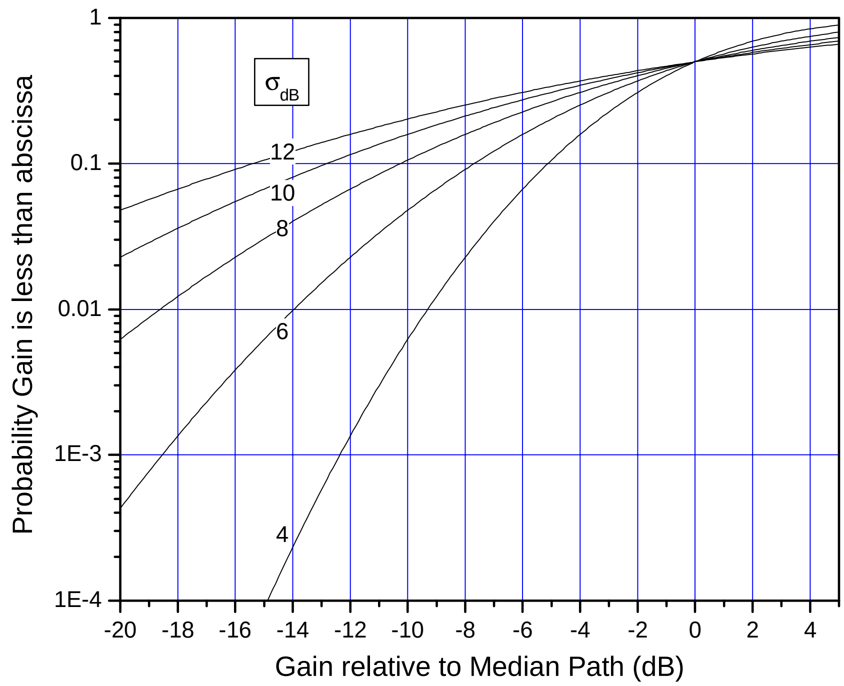
Parameter	**Value**	**Comment**
Transmit Power	20 dBm	
Free Space Loss	52.1 dB	$L_p = \left(\frac{4\pi R}{\lambda}\right)^2$
Wall attenuation	5 dB	
Open Area Loss	24.1 dB	$L_p = \left(\frac{24}{4}\right)^{3.1}$
Wall attenuation	5 dB	
Free space loss	1 dB	$L_p = \left(\frac{27}{24}\right)^{2.0}$
Receive Power	-67.2 dBm	

Link budget for Problem 2.7.

An appropriate general question for this type of scenario is when can we assume free space propagation and when should a different model be used? The answer comes from our study of diffraction and Fresnel zones; whenever the first Fresnel zone along the line of sight is unobstructed, it is reasonable to assume free-space propagation.

Problem 2.8
What are the required margins for lognormal and Rayleigh fading in Example 2.6 if the availability requirement is only 90%?

Solution
Due to an unfortunate oversight in the first printing of the text, Fig. 2.11 was wrong. It should have been the following:



Revised Figure 2.11. The lognormal distribution.

With this revised Fig. 2.11 the required margin for log-normal shadowing with $\sigma_{\mathrm{dB}} = 6$ is 7.7 dB, and from Fig. 2.14, the required margin for Rayleigh fading is 10 dB.

Problem 2.9 Suppose that the aircraft in Example 2.7 has a satellite receiver operating in the aeronautical mobile-satellite band at 1.5 GHz. What is the Doppler shift observed at this receiver? Assume the geostationary satellite has a 45° elevation with respect to the airport.

Solution
Using Eq. (2.68), the Doppler frequency is

$$
\begin{aligned}
f_D &= -\frac{f_0}{c} v \cos\alpha \\
&= -\left(\frac{1.5\times10^9}{3\times10^8}\right)\left(\frac{-500}{3.6}\right)\left(\cos 45^\circ\right) \\
&= 491\,\mathrm{Hz}
\end{aligned}
$$

where v = 500 km/hr.

Problem 2.10

A data signal with a bandwidth of 100 Hz is transmitted at a carrier frequency of 800 MHz. The signal is to be reliably received in vehicles travelling at speeds up to 100 km/hour. What can we say about the minimum bandwidth of the filter at the receiver input?

Solution

The object of this problem is to compute the maximum Doppler shift on the received signal. From Eq. (2.66), this is given by

$$|f_D| = \frac{v}{c} f_0$$

where $f_0 = 800$ MHz. Therefore,

$$\begin{aligned} |f_D| &= \frac{\left(\frac{100}{3.6}\right)}{3\times10^8}\left(8\times10^8\right) \\ &= 74.1\,\text{Hz} \end{aligned}$$

Rounding up, the total bandwidth is the signal bandwidth (100 Hz) plus the maximum Doppler shift (75 Hz). These are the one-sided bandwidths (i.e. the baseband equivalent bandwidth). The RF bandwidth would be twice this.

Problem 2.11

Measurements of a radio channel in the 800 MHz frequency band indicate that the coherence bandwidth is approximately 100 kHz. What is the maximum symbol rate that can be transmitted over this channel that will suffer minimal intersymbol interference?

Solution

From Eq.(2.116), the multipath spread of the channel is approximately

$$\begin{aligned} T_M &\approx \frac{1}{BW_{\text{coh}}} \\ &\approx 10\mu\text{s} \end{aligned}$$

If we assume that spreading of symbol by 10% causes negligible interference into the adjacent symbol, then the maximum symbol period is 100 microseconds. This corresponds to a symbol rate of 10 kHz.

Problem 2.12

Calculate the *rms* delay spread for a HF radio channel for which

$$P(\tau) = .6\delta(\tau) + 0.3\delta(\tau - 0.2) + 0.1\delta(\tau - 0.4)$$

where τ is measured in milliseconds. Assume that signaling with a 5-kHz bandwidth is to use the channel. Will delay spread be a problem, that is, is it likely that some form of compensation (an equalizer) will be necessary?

Solution

From Eq. (2.109) the rms delay spread is given by the square root of μ_2 where

$$\mu_2 = \frac{1}{P_m}\int_0^\infty (\tau - T_D)^2 P_h(\tau)d\tau \tag{1}$$

where the received power is given by Eq. (2.108)

$$\begin{aligned} P_m &= \int_0^\infty P_h(\tau)d\tau \\ &= 0.6 + 0.3 + 0.1 \\ &= 1.0 \end{aligned} \tag{2}$$

and mean delay is given by Eq. (2.107)

$$\begin{aligned} T_D &= \frac{1}{P_m}\int_0^\infty \tau P_h(\tau)d\tau \\ &= 0.6(0) + 0.3(0.2) + 0.1(0.4) \\ &= 0.1\,\text{ms} \end{aligned} \tag{3}$$

Substituting these results in (1), the mean square delay is

$$\begin{aligned} \mu_2 &= (0-0.1)^2 0.6 + (0.2-0.1)^2 0.3 + (0.4-0.1)^2 0.1 \\ &= 0.018\,(\text{ms})^2 \end{aligned} \tag{4}$$

and the corresponding rms delay spread is

$$\begin{aligned} S &= \sqrt{\mu_2} \\ &= 0.134\,\text{ms} \end{aligned} \tag{5}$$

The approximate coherence bandwidth, from Eq. (2.116), is

$$BW_{coh} = \frac{1}{T_m} = \frac{1}{2\sqrt{\mu_2}} = 3.8\,\text{kHz} \tag{6}$$

So some form of equalization will be necessary.

Problem 2.13

Show that the time-varying impulse response of Eq.(2.84) and time-invariant impulse response are related by

$$\tilde{h}_{\text{time-invariant}}(t) = \tilde{h}_{\text{time-varying}}(t,t)$$

Explain, in words, what the preceding equation mean.

Solution

For a time-varying system, the impulse response $h(t,\tau)$ represents the response at time t to an impulse applied at time $t-\tau$. (See Appendix A.2.) The response $h(t,t)$ is therefore the response at time t to an impulse response applied at time zero. Thus, $h(t,t)$ is equivalent to the definition of a *time-invariant* impulse response.

Problem 2.14

What would be the rms voltage observed across a 10-MΩ metallic resistor at room temperature? Suppose the measuring apparatus has a bandwidth of 1 GHz, with an input impedance of 10 MΩ, what voltage would be measured then? Compare your answer with the voltage generated across the antenna terminals by the signal defined in Problem 2.5. Why is the avoidance of large resistors recommended for circuit design? What is the maximum power density (W/Hz) that a thermal resistor therefore delivers to a load?

Solution

Over an infinite bandwidth the rms voltage is given by Eq. (2.117) at 290° K is

$$\begin{aligned}\overline{v}^2 &= \frac{2\pi^2 k^2 T^2}{3h} R \\ &= 1.57\ \text{V}^2 \\ v_{rms} &= 1.3\ \text{volts}\end{aligned}$$

Into a matched load, the noise density due to the resistor is

$$kT = 4\times 10^{-21}\ \text{W/Hz}$$

Over a 1GHz bandwidth, the assicuated power is

$$\begin{aligned}P &= kTB \\ &= 4\times 10^{-12}\ \text{W}\end{aligned}$$

The corresponding rms voltage across a 10 MΩ resistor is

$$\begin{aligned}V &= \sqrt{PR} \\ &= 6\ \text{mV}\end{aligned}$$

In problem 2.5, the voltage across the antenna terminals due to the received signal was 37 μV. Clearly, large resistors have the potential to introduce a lot of noise into the cicuit.

Problem 2.15

The noise figure of a cell phone receiver is specified as 16 dB. What is the equivalent noise temperature? Assume that reliable detection of a 30-kHz FM signal by this receiver requires an SNR of 13 dB. What is the receiver sensitivity in dBm?

Solution

The noise temperature is given by Eq. (2.124)

$$\begin{aligned} T_e &= (F-1)T_o \\ &= 11255^{\circ} K \end{aligned}$$

for nominal temperature T_o=290°K. The receiver sensitivity is (see Example 2.14)

$$\begin{aligned} S &= SNR \times N \\ &= SNR \times kT_e B \\ &\approx 13dB + 10\log(kT_e) + 10\log_{10} B \\ &= -130\ \text{dBW} \\ &= -100\ \text{dBm} \end{aligned}$$

The receiver sensitivity is -100 dBm.

Problem 2.16

Show that the system temperature, in general, is given by Eq.(2.129).

Solution

We may derive Eq. (2.129) from Eq. (2.127) as follows. Let F_1 represents the combined noise figure of the first amplifier and antenna, then Eq. (2.129) is

$$F_{sys} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \ldots$$

We subtract 1 from both sides and multiply by T_0 to obtain

$$(F_{sys} - 1)T_0 = (F_1 - 1)T_0 + \left(\frac{F_2 - 1}{G_1}\right)T_0 + \left(\frac{F_3 - 1}{G_1 G_2}\right)T_0 + \ldots$$

From the relationship of noise temperature and noise figure given by Eq. (2.124) we have the desired result.

$$T_{sys} = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \ldots$$

where T_1 represents the combined noise temperature of the first amplifier and antenna.

Problem 2.17

Microwave ovens operate at a natural frequency of the water molecule at approximately 2.45 GHz. This frequency falls in the middle of a band from 2.41 to 2.48 GHz that has been allocated for low-power unlicensed radio use, including wireless local area networks (sometimes referred to as Wi-Fi – see Section 5.16). The oscillators used in some microwave ovens have poor stability and have been observed to vary ±10 MHz around their nominal frequency. Discuss how this would affect the signal design for use of this band.

Solution

Microwave ovens are well shielded such that very little radiation escapes. However, small amounts do escape; not enough to be a biological hazard but enough that it could interfere with radio systems operating on the same frequencies. Consequently, local WiFi systems should avoid operating in the vicinity. Typically these systems have multiple channels and operation in a channel at the same frequency as the interference should be avoided.

Problem 2.18

The minimum tolerable *C*/*I* ratio depends on the modulation and coding strategy and the quality of service required. Suppose a narrowband digital system requires a *C*/*I* of 12 dB. What would be the maximum frequency reuse factor? If the addition of forward error-correction coding would reduce this to 9 dB without increasing the signal bandwidth, what would be the relative improvement in reuse factor. Assume the propagation loss exponent is 2.6.

Solution

From Eq. (2.135), a lower bound on the frequency reuse factor in a cellular FDMA system is

$$N \geq \frac{1}{3}\left[6\left(\frac{C}{I}\right)_{\min}\right]^{2/n}$$

Substituting the given values we obtain

$$\begin{aligned} N &\geq \frac{1}{3}\left[6\times 10^{12/10}\right]^{2/2.6} \\ &= 11.06 \end{aligned}$$

A reuse factor of $N = 12$ is the smallest one that satisfies this.

Similarly for $C/I = 9\,\text{dB}$

$$\begin{aligned} N &\geq \frac{1}{3}\left[6\times 10^{9/10}\right]^{2/2.6} \\ &= 6.5 \end{aligned}$$

A reuse factor of $N = 7$ satisfies this. Thus, forward error correction coding (see Chapter 4) can significantly improve frequency reuse if the coding can be applied without increasing signal bandwidth.

Problem 2.19

A fixed satellite terminal has a 10-m parabolic dish with 60% efficiency and a system noise temperature of 70°K. Find the G/T ratio of this terminal at 4 GHz. Suppose it was a terrestrial mobile radio using an omni-directional antenna. What would you expect the equivalent noise temperature of the mobile antenna to be?

Solution

For the satellite terminal the antenna gain is

$$
\begin{aligned}
G &= \eta\left(\frac{\pi D}{\lambda}\right)^2 \\
&= \eta\left(\frac{\pi D}{c/f}\right)^2 \\
&= 0.6\left(\frac{\pi 10}{c/4\times 10^9}\right)^2 \\
&= 1.05\times 10^5
\end{aligned}
$$

The corresponding G/T is $10\log_{10}(G/T) = 31.8$ dBK^{-1} for a system noise temperature of 70°K. A satellite antenna points skyward ideally and is only marginally influenced by the Earth. The majority of the noise is caused by the receiver. The noise temperature of the sky is typically a few degrees Kelvin, if the antenna is not looking directly at the sun. On the other hand, an omni-directional antenna will "see" the Earth and have a noise temperature that is at least that of the Earth (290°K); this is noise introduced by the environment.

Problem 2.20

For a geostationary satellite at altitude h (36000 km), determine a formula relating the range r from the satellite to an earth station to the satellite elevation ϕ relative to the earth station. (Let R_e =6400 km be the radius of the Earth.)

Solution

Using the cosine law for triangles as applied to the diagram below, we have

$$(h+R_e)^2 = r^2 + R_e^{\,2} - 2rR_e\cos(90° + \phi)$$

Rearranging this as a quadratic equation in r gives

$$r^2 + 2rR_e\sin\phi + \left(R_e^2 - (h+R_e)^2\right) = 0$$

and solving for r, one obtains

$$r = \frac{-2R_e \sin\phi \pm \sqrt{4R_e^2 \sin^2\phi - 4\left(R_e^2 - (h+R_e)^2\right)}}{2}$$
$$= -R_e \sin\phi \pm \sqrt{R_e^2 \sin^2\phi - R_e^2 + (h+R_e)^2}$$
$$= -R_e \sin\phi \pm \sqrt{(h+R_e)^2 - R_e^2 \cos^2\phi}$$

Taking the positive square root gives the range[1]

$$r = -R_e \sin\phi + \sqrt{(h+R_e)^2 - R_e{}^2 \cos^2\phi}$$

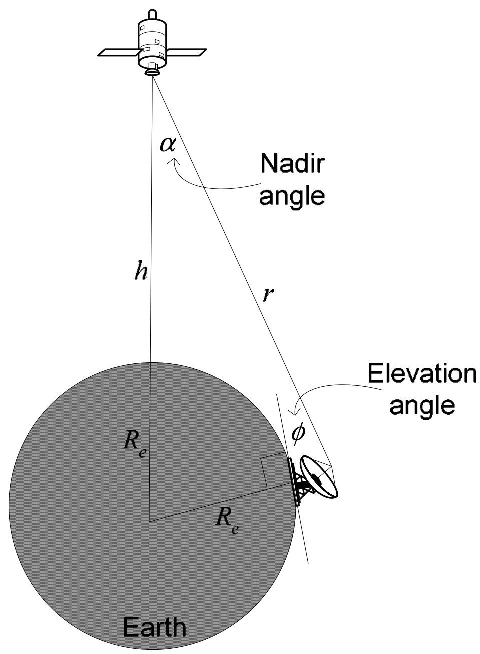


Diagram for Problem 2.20.

Problem 2.21

Numerous other quantities may be included in the satellite link budget. For example if the satellite amplifier (which is typically nonlinear) is shared with a number of carriers, then *intermodulation distortion* will be generated. When there are a large number of equal-power carriers present, intermodulation distortion can be modeled as white noise with spectral density I_o. This will produce a term C/I_o at the satellite that must be combined with the uplink and downlink C/N_o to produce the overall $C/(N_o+I_o)$. Given the uplink and downlink C/N_o of Examples 2.18 and 2.19, what is the overall $C/(N_o+I_o)$ if C/I_o = 50 dB-Hz?

[1] Note that the answer given in the textbook is in error.

Solution
The initial objective is to compute the combination of the thermal noise N_0 and the intermodulation noise I_0, given by $N_0 + I_0$. Since these are not known directly, we compute the scaled equivalent.

$$\frac{N_0 + I_0}{C} = \frac{N_0}{C} + \frac{I_0}{C}$$

Inverting this equation gives

$$\frac{C}{N_0 + I_0} = \left[\left(\frac{C}{N_0}\right)^{-1} + \left(\frac{C}{I_0}\right)^{-1}\right]^{-1}$$

From example 2.19, the overall C/N_o is 47.1 dBHz. Consequently,

$$\begin{aligned}\frac{C}{N_0 + I_0} &= \left[\left(\frac{C}{N_0}\right)^{-1} + \left(\frac{C}{I_0}\right)^{-1}\right]^{-1} \\ &= \left[10^{-4.71} + 10^{-5}\right]^{-1} \\ &= 3.39\times 10^{4}\end{aligned}$$

where the second line is a conversion from decibels to absolute. Converting the answer to decibels we obtain $C/(N_0+I_0) = 45.3$ dBHz.

Problem 2.22
Repeat the link budget of Table 2.5, analyzing the performance in the city core. Assume that the maximum range within the city core is 2 km, but that the path-loss exponent is 3.5 and the log-normal shadowing deviation is 10 dB. Is this service limited by the receiver sensitivity? What is the expected service availability for the city core?

Solution[2]
In the following link budget, we compute the received signal power based on the propagation losses and the margin required for shadowing. When we compared the received signal power to the signal power required by the modem, we see there is a shortfall of 14.2 dB. This means the system is unable to provide the full 16.5 dB shadowing margin. The shadowing margin is only 2.3 dB. Looking at the revised Fig. 2.11 (see Problem 2.8), this implies that the availability is only 60% in the city core.

The required C/N_0 of the modem and the noise figure of the receiver shown in this table are parameters provided to the propagation analysis.

[2] There is an error in the answer given in the text.

Parameter	Units		Comments
Base station transmitter			
Transmit Frequency	MHz	705	Mobile public safety band
Tx Power	dBW	15	
Tx Antenna Gain	dBi	2	Uniform radiation in azimuth
Tx EIRP	dBW	17	Maximum EIRP of 30 dBW
Power at 1m (P_0)	dBm	17.6	$P_o = P_T/(4\pi/\lambda)^2$
Losses			
Path loss exponent		3.5	Applicable at edge of coverage
Range (r)	km	2	Range at edge of coverage
Median Path Loss	dB	115	$3.5\text{x}10\log_{10}(r/r_o)$
Log-normal shadowing	dB	10	standard deviation of log-normal shadowing
Shadowing margin	dB	16.5	for 95% availability ($1.65\sigma_{dB}$)
Rx Signal			
Rx Antenna Gain (G_n)	dBi	1.5	Vertically polarized whip antenna
Rx signal strength	dBm	-112.4	$P_R = P_o + G_R - 21\log_{10}(r/r_o)$ - M_{shadow}
Receiver characteristics			
Required C/N_o	dB-Hz	69.8	from modem characteristics
Boltzmann's constant	dBm-K	-198.6	
Rx Noise Figure	dB	6.0	provided
Rx sensitivity	dBm	-98.2	$S = C/N_o + NF + kT_o$
Margin	dB	-14.2	$Margin = P_R - S$

Link budget for Problem 2.22

Problem 2.23

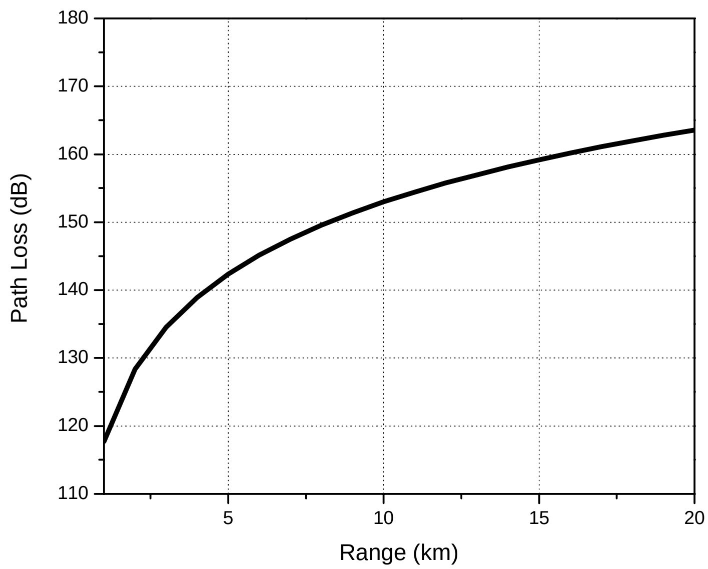
Evaluate the path loss at 900 MHz using the Okumura-Hata model for the suburban environment. Assume that the base and mobile station antenna heights are 30 m and 1 m, respectively.

Solution

From Eq. (2.141), the path loss for a suburban environment is given by

$$L_p = A + B\log_{10} r - C \qquad (1)$$

Where A, B and C are given by Eq.(2.142). For a medium-sized city the correction factor for mobile antenna height is given by Eq. (2.145). Substituting these in Eq. (1) and plotting the result we obtain the following figure.



Path loss at 900 MHz in a suburban environment for Problem 2.23

Problem 2.24

The 54 Mbps service of IEEE 802.11a uses 64-quadrature-amplitude-modulation (QAM). (This method of modulation is considered in Chapter 3) Suppose the practical E_b/N_o required to achieve a bit error rate of 10^{-6} with 64 QAM is 30 dB. For this data rate, what is the sensitivity of the receiver just discussed?

Solution

Using Eq. (2.149), the sensitivity is given by

$$\begin{aligned} S &= \frac{E_s}{N_0} + R_b + N_0 \\ &= 30\text{ dB} + 10\log\left(54\times10^6\right) + (-164) \\ &= \text{-}\,56.7\text{ dBm} \end{aligned}$$

With a 3 dB implementation loss, the sensitivity decreases to -53.7 dBm.

Problem 2.25

From the results of Problem 2.24, what is the maximum range expected with the 54-Mbps service?

Solution

At the receiver threshold, with a 200 milliwatt transmitter (23 dBm), the range is determined from Eq. (2.151)

$$\begin{aligned} 31\log_{10} r &= P_T - P_R - 41 \\ &= 23 - (-53.7) - 41 \\ &= 35.7 \end{aligned}$$

Solving this for r, one obtains a range of 14 m.

Problem 2.26

Consider a communications link with a geostationary satellite such that the transmitter-receiver separation is 40,000 km. Assume the same transmitter and receiver characteristics as described in Problem 2.2. What is the received power level in dBm? What implications does this power level have on the receiver design? With a land-mobile satellite terminal, the typical antenna gain is 10 dB or less. What does this imply about the data rates that may be supported by such a link?

Solution

The received power is given by Eq. (2.12)

$$P_R(dB) = P_T(dB) + G_R(dB) + G_T(dB) - L_p(dB) \tag{1}$$

with a transmit power of 100 milliwatts (20dBm), a frequency of 4 GHz, and an antenna G_R of 30.5 dB as determined from Problem 2.2. The path loss is

$$\begin{aligned} L_p &= \left(\frac{4\pi r}{\lambda}\right)^2 \\ &= \left(\frac{4\pi 4\times10^6}{c/f}\right)^2 \\ &= 4.5\times10^{19} \quad (196.5\,\text{dB}) \end{aligned} \tag{2}$$

Combining these observations in Eq.(1), the received power is

$$\begin{aligned} P_R(\text{dBm}) &= 20\,\text{dBm} + 30.5\,\text{dB} + 30.5\,\text{dB} - 196.5\,\text{dB} \\ &= -115.5\,\text{dBm} \end{aligned} \tag{3}$$

For the land-mobile satellite terminal, if the receiver antenna has a G_R of 10 dB then the received power will be

$$\begin{aligned} P_R(\text{dBm}) &= 20\,\text{dBm} + 10\,\text{dB} + 30.5\,\text{dB} - 196.5\,\text{dB} \\ &= -136\,\text{dBm} \end{aligned} \tag{4}$$

Therefore the receiver has to be very sensitive with very little noise. With a low gain antenna, the data rates would also have to be low.

Problem 2.27

Consider a 10-watt transmitter communicating with a mobile receiver having a sensitivity of –100 dBm. Assume that the receiver antenna height is 2 m, and the transmitter and receiver antenna gains are 1 dB. What height of base station antenna would be necessary to provide a service area of radius 10 km? If the receiver is mobile, and the maximum radiated power is restricted by regulation to be 10 watts or less, what realistic options are there for increasing the service area?

Solution

Using the plane-earth model, Eq. (2.30)

$$P_R = P_T G_T G_R \left(\frac{h_T h_R}{R^2} \right)^2$$

and isolating h_T,

$$\begin{aligned} h_T &= \left(\frac{R^2}{h_R} \right) \sqrt{\left(\frac{P_R}{P_T G_T G_R} \right)} \\ &= \left(\frac{10000^2}{2} \right) \sqrt{\left(\frac{1 \times 10^{-13}}{(10)(1.26)(1.26)} \right)} \\ &= 4 \text{ m} \end{aligned}$$

where -100 dBm is equivalent to 10^{-13} watts.

Therefore, the plane earth model indicates the base station antenna would have to be 4 meters in height. Service area may be increased either by improving receiver sensitivity, or boosting the transmitter antenna height, or increasing antenna gain.

Realistically, a 4-meter antenna would be unlikely to provide a line of sight path over a distance of 10 km, thus the plane-earth model would not be applicable. The student is invited to solve this problem using the Okamura-Hata model assuming a transmission frequency of 400 MHz.

<u>Problem 2.28</u>

In Problem 2.26, the satellite-receiver separation was 40,000 km. Assume the altitude of a geostationary satellite was said to be 36,000 km. What is the elevation angle from the receiver to the satellite in Problem 2.26? What was the increased path loss, in decibels, relative to a receiver where the satellite is in the *zenith* position (directly overhead)? If the transmitter-receiver separation in Problem 2.2 had been 20 km, what would the path loss have been? What can be said about comparing the dB path losses in satellite and terrestrial scenarios as a function of absolute distance?

Solution

From Problem 2.20 the range is given by

$$r = -R_e \sin\phi + \sqrt{(R_e+h)^2 - R_e^2\cos^2\phi}$$

$$(r + R_e \sin\phi)^2 = (R_e+h)^2 - R_e^2\cos^2\phi$$

$$r^2 + 2rR_e\sin\phi + R_e^2\sin^2\phi + R_e^2\cos^2\phi = (R_e+h)^2$$

$$\sin\phi = \frac{h^2 + 2hR_e - r^2}{2rR_e}$$

$$\phi = \sin^{-1}\left(\frac{36000^2 + 2(36000)(6400) - 40000^2}{2(40000)(6400)}\right)$$

$$\phi = 17.8^\circ$$

Therefore the elevation angle is 17.8 degrees.

The increase in path loss relative to the zenith position is

$$\frac{R_{\text{elevation}}^2}{R_{\text{zenith}}^2} = \frac{L_{p\,(\text{elevation})}}{L_{p\,(\text{zenith})}}$$

Expressed in dB, the ratio becomes

$$\begin{aligned}\Delta L_p &= 20\log_{10}\left(\frac{R_{\text{elevation}}}{R_{\text{zenith}}}\right)\\ &= 20\log_{10}\left(\frac{40000}{36000}\right)\\ &= 0.92\text{ dB}\end{aligned}$$

With the free-space model for propagation, the difference between 40000 km and 20 km is

$$\begin{aligned}&= 20\log_{10}\left(\frac{40000}{20}\right)\\ &= 33\text{ dB}\end{aligned}$$

The propagation losses are greater in satellite scenarios, but the variation in propagation losses are significantly less.

Problem 2.29

Suppose that, by law, a service operator is not allowed to radiate more than 30 watts of power. From the plane-Earth model, what antenna height is required for a service radius of 1 km? 10 km? Assume the receiver sensitivity is -100 dBm.

Solution

Using the plane earth model for propagation from Eq. (2.30)