

Preface

I once facilitated a mandatory workshop on the teaching of calculus that was attended by a diverse mixture of professors and teaching assistants. Before the seminar began, I asked for written answers to the following question: “Ideally, what would you like to get out of the next two days’ activities?”

Their responses formed a collection of contradictory expectations. Some were cautionary: “You can tell me *what* to teach, but don’t tell me *how*.” Some wanted help with group work and cooperative learning, while others just wanted a general idea of what it meant to teach calculus “with modern pedagogy.” And many wanted specifics: “How can you teach the Chain Rule ‘reform-style’?” “How much homework should I assign?”

This Instructor’s Guide tries to address the issues brought out by the above comments. The overall goal was not to write a reference book for a shelf, but to provide a user-friendly source of suggestions and activities for any teacher of calculus within a typical calculus curriculum. Instructors that have used previous editions of this book have said that it saved them a great deal of time, and helped them to teach a more student-oriented course. They have also reported that their classes have become more “fun,” but agreed that this unfortunate by-product of an engaged student population can’t always be avoided.

This guide should be used together with *Calculus, Eighth Edition* as a source of both supplementary and complementary material. Depending on individual preference, instructors can choose from occasionally glancing through the Guide for content ideas and alternate approaches, or using the material from the Instructor’s Guide as a major component in planning their day-to-day classes as well as to set homework assignments and reading quizzes. There are student activities and worksheets, sample exam questions, and examples for every section.

Some of the continuing debates about changes in calculus content and pedagogy are rendered moot by adopting the principle that the instruction of any topic in calculus can be enhanced by using a wider range of approaches. This guide includes some conceptual and geometric problems in topics as mundane as rules for differentiation, and as traditional as ε - δ limits. Whether a class consists of a straight lecture or an hour of group work, the materials provided are meant to help.

I value reactions from all my colleagues who are teaching calculus from this guide, both to correct any errors and to suggest additional material for future editions. I am especially interested in which particular parts of the guide are the most and the least useful. Please email any feedback to calculus@dougshaw.com.

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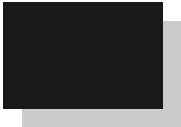
PREFACE

This guide could not have been completed without the help of many people. I especially want to thank Jim Stewart for his continuing belief in this project and trust in me. My editor, Sam Lugtu, has been wonderful. Previous versions of this guide have benefitted from the input of Virge Cornelius, Tom Hull, Joe Mercer, Melissa Pfohl, Michael Prophet, Suzanne Riehl, and John Samons. Over the years, I've had students read through the guide and offer suggestions from their perspective. Thanks go to Kate Degner, Ken Doss, Job Evers, Slade Hovick, Patricia Kloeckner, Jordan Meyer, Ben Nicholson, Paul Schou, Laura Waechter, and Cody Wilson. Further thanks go to James Stewart, John Hall, Robert Hesse, Harvey Keynes, Michael Lawler, and Dan O'Loughlin for their contributions to earlier incarnations of this guide. The book's typesetter and proofreader, Andy Bulman-Fleming, again went above and beyond the call of duty, both in his work on the book, and in keeping me humble by regularly trouncing me at online Scrabble as we produced it. The talents of these people and others at Cengage have truly helped to make writing this guide a learning experience.

This guide is dedicated to Russ Campbell.

Doug Shaw

INSTRUCTOR USE ONLY



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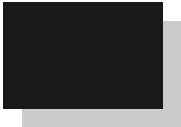
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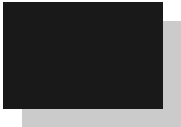


How to Use the Instructor's Guide

For each section of *Calculus, Eighth Edition*, this *Instructor's Guide* provides information on the items listed below.

1. **Suggested Time and Emphasis** Here are suggestions for the amount of time to spend in a class of “average” students, and whether or not the material is essential to the rest of the course. If a section is labeled optional, the time range given is the amount of time for the material in the event that it is covered.
2. **Points to Stress** This is a short summary of the main topics to be covered. The stress is on the big ideas, rather than specific details.
3. **Quiz Questions** Some instructors have reported that they like to open or close class by handing out a single question, either as a quiz or to start a discussion. Two types are included:
 - **Text Question** This question is designed for students who have done the reading, but haven't yet seen the material in class. These questions can be used to help ensure that the students are reading the textbook carefully.
 - **Drill Question** These questions are designed to be straightforward “right down the middle” questions for students who have tried, but not necessarily mastered, the material.
4. **Materials for Lecture** These suggestions are meant to work along with the text to create a classroom atmosphere of experimentation and inquiry. They have a theoretical bent to help the students understand the material at a deep conceptual level. In a course with a “lecture and discussion” format, these ideas can be used during the lectures.
5. **Workshop/Discussion** These suggestions are interesting examples and applications aimed at motivating the material and helping the students master it. In a course with a “lecture and discussion” format, these ideas can be used during the discussions.
6. **Group Work** One of the main difficulties instructors have in presenting group work to their classes is that of choosing an appropriate group task. Suggestions for implementation and answers to the group activities are provided first, followed by photocopy-ready handouts on separate pages. The guide's main philosophy of group work is that there should be a solid introduction to each exercise (“What are we supposed to do?”) and good closure before class is dismissed (“Why did we just do that?”)
7.  **Tools for Enriching Calculus** is a companion to the text, intended to enrich and complement its contents. Marginal notes in the main text direct students to TEC modules where appropriate. When a TEC module relates to an *Instructor's Guide* item, it is referenced there as well.
8. **Homework Problems** For each section, a set of essential **Core Exercises** (a bare minimum set of homework problems) is provided. Using this core set as a base, a **Sample Assignment** is suggested, and each exercise in that assignment is classified as **Descriptive**, **Algebraic**, **Numeric**, and/or **Graphic**.
 - **Descriptive:** The student is required to translate mathematical concepts into everyday terms, or vice-versa.
 - **Algebraic:** The student is required to use algebraic and/or symbolic manipulation and computation.

- **Numeric:** The student is required to work with numerical data, or provide a numerical approximation.
 - **Graphic:** The student is required to provide or receive information presented in the form of a graph. Also available is a *Student Solutions Manual* which presents complete solutions to all of the odd-numbered exercises in the text.
- 9. Sample Exam Questions** I recommend that tests in a calculus course have a mix of routine and non-routine questions. The sample exam questions provided are meant to inspire the “non-routine” portion of a calculus test. We do not recommend that calculus tests be composed entirely of questions from this section. One strategy is to announce to the students that one-third of the text questions will be based on homework, one-third will be based on in-class group work, and one-third will not be immediately familiar.
- 10. Web Resources** Useful resources can be found on the website for *Calculus, Eighth Edition* (<http://www.stewartcalculus.com>).



How to Implement the Projects

One exciting yet intimidating aspect of teaching a calculus course is projects. An extended assignment gives students the chance to take a focused problem or project and explore it in-depth — making conjectures, discussing them, eventually drawing conclusions and writing them up in a clear, precise format. *Calculus, Eighth Edition* has many possible projects throughout its chapters. Here are some tips on ensuring that your students have a successful experience.

Time Students should have two to three weeks to work on any extended out-of-class assignment. This is not because they will need all this time to complete them! But a fifteen-to-twenty-day deadline allows the students to be flexible in structuring their time wisely, and allows the instructors to apply fairly strict standards in grading the work.

Groups Students usually work in teams and are expected to have team meetings. The main problem students have in setting up these meetings is scheduling. Four randomly selected undergraduates will probably find it very hard to get together for more than a few hours, which may not be sufficient. One way to help your students is to clearly specify a minimum number of meetings, and have one or all group members turn in summaries of what was accomplished at each meeting. On a commuter campus, a good first grouping might be by location.

Studies have shown that the optimal group size is three people, followed by four, then two. I advocate groups of four whenever possible. That way, if someone doesn't show up to a team meeting, there are still three people there to discuss the problems.

Before the first project, students should discuss the different roles that are assumed in a team. Who will be responsible for keeping people informed of where and when they meet? Who will be responsible for making sure that the final copy of the report is all together when it is supposed to be? These types of jobs can be assigned within the team, or by the teacher at the outset.

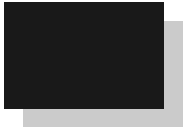
Tell the students that you will be grading on both content and presentation. They should gear their work toward an audience that is bright, but not necessarily up-to-speed on this problem. For example, they can think of themselves as professional mathematicians writing for a manager, or as research assistants writing for a professor who is not necessarily a mathematician.

If the students are expected to put some effort into the project, it is important to let them know that some effort was put into the grading. Both form and content should be commented on, and recognition of good aspects of their work should be included along with criticism.

One way to help ensure cooperation is to let the students know that there will be an exam question based on the project. If every member of the group does well on that particular question, then they can all get a bonus, either on the exam or on the project grade.

Providing assistance Make sure that the students know when you are available to help them, and what kind of help you are willing to provide. Students may be required to hand in a rough draft ten days before the due date, to give them a little more structure and to make sure they have a solid week to write up the assignment.

Individual Accountability It is important that the students are individually accountable for the output of their group. Giving each student a different grade is a dangerous solution, because it does not necessarily encourage the students to discuss the material, and may actually discourage their working together. A better alternative might be to create a feedback form. If the students are given a copy of the feedback form ahead of time, and they know that their future group placement will be based on what they do in their present group, then they are given an incentive to work hard. One surprising result is that when a group consists of students who were previously slackers, that group often does quite well. The exam question idea discussed earlier also gives individuals an incentive to keep up with their colleagues.

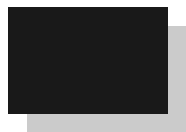


How to Use the Review Sections

Review sections for chapters of a calculus book are often assigned to students the weekend before a test, but never graded. Students realize that they won't be evaluated on this work and often skip the exercises, instead reworking previously done homework problems or quizzes, if they study at all. A more useful activity for students is to use the review sections in *Calculus, Eighth Edition* to discover their precise areas of difficulty. Implemented carefully, these are a useful resource for the students, particularly for helping them to retain the skills and concepts they've learned. To encourage more student usage, try the following alternatives:

1. Instead of giving a review session where you reiterate previous lectures, make notes of the types of problems students had difficulty with during the quarter and assign students to work on these exercises in the review sections and go over them at the end of class.
2. Use the review section problems to create a game. For instance, break students into groups and have a contest where the group that correctly answers the most randomly picked review questions "wins". One fun technique is to create a math "bingo" game. Give each group a 5×5 grid with answers to review problems. If you laminate the cards and give the students dry-erase markers, then you can use them year after year. Randomly pick review problems, and write the questions on the board. Make sure that for a group to win, they must not only have the correct answers to the problems, but be able to give sound explanations as to how they got the answers.
3. A simple way to give students an incentive to look at a review section is to use one of the problems, verbatim, for an exam question, and make no secret of your intention to do so. It is important that students have an opportunity to get answers to any questions they have on the review problems before the exam is given; otherwise, this technique loses a great deal of its value.

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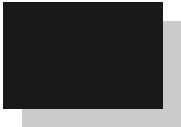
How to Use the Problem-Solving Sections

Principles of Problem Solving (after Chapter 1) and Problems Plus (after subsequent chapters) are designed to help students build their critical thinking skills on calculus problems with significant depth. Some of these problems are more open-ended than the typical problems calculus students usually solve, while others are more challenging computationally. Many can be used as small-scale projects, suitable for an individual or a group to chew over at home or in an extended classroom session. If you are looking for problems to create your own projects, or to assign to students who would benefit from a challenge, then these sections provide excellent choices.

Because of the depth of these questions, students who come up with interesting partial solutions, or approaches leading to a possible solution, should be rewarded.

Before the students start one of these problems, or if they reach an impasse, it may be helpful for them to look at the Principles of Problem Solving section for ideas.

INSTRUCTOR USE ONLY



Tips on In-Class Group Work

This *Instructor's Guide* gives classroom-tested group work activities for every section of *Calculus, Eighth Edition*. One reason for the popularity of in-class group work is that *it is effective*. When students are engaged in doing mathematics, and talking about mathematics with others, they tend to learn better and retain the material longer. Think back to your own career: didn't you learn a lot of mathematics when you began teaching it to other people? Many skeptics experiment by trying group work for one semester, and then they get hooked. Pick a group activity from the guide that you like, make some photocopies, and dive in!

1. Mechanics Books and seminars on in-class group work abound. I have conducted many such seminars myself. What follows are some tips to give you a good start:

(a) Do it on the first day.

The sources all agree on this one. If you want your students to believe that group work is an important part of the course, you have to start them on the first day. My rule of thumb is “at least three times the first week, and then at least once a week thereafter.” I mention this first because it is the most important.

(b) Make them move.

Ideally, students should be eye-to-eye and knee-to-knee. If this isn't possible, do the best you can. But it is important to have them move. If your groups are randomly selected, then they will have to get up and sit in a different chair. If your groups are organized by where they are seated in the classroom, make them move their chairs so they face each other. There needs to be a “break” between sitting-and-writing mode and talking-to-colleagues mode.

(c) Use the ideal group size.

Research has shown that the ideal group size is three students, with four-student groups next. I like to use groups of four: if one of them is absent (physically or otherwise), the group still has three participating members.

(d) Fixed versus random groups.

There is a lot of disagreement here. Fixed groups allow each student to find her or his niche, and allow you to be thoughtful when you assign groups or reassign them after exams. Random groups allow students to have the experience of working with a variety of people. I believe the best thing to do is to try both methods, and see which works best for you and your students.

(e) Should students hand in their work?

The advantage of handing in group works is accountability. My philosophy is that I want the group work to have obvious, intrinsic benefit. I try to make the experience such that it is obvious to the student that they get a lot out of participating, so I don't need the threat of “I'm grading this” to get them to focus. I sometimes have the students hand in the group work, but only as a last resort.

2. Closure As stated above, I want my students to understand the value of working together actively in their groups. Once you win this battle, you will find that a lot of motivation and discipline problems simply go away. I've found the best way to ensure that the students understand why they've done an activity is to tell them. The students should leave the room having seen the solutions and knowing why they did that particular activity. You can have the students present answers or present them yourself, whatever suits your teaching style. I've had success with having groups write their results on transparencies and present them to the class (after I've checked their accuracy).

Here is another way to think about closure: Once in a while, give a future homework problem out as a group work. When the students realize that participating fully in the group work helps them in the homework, they get a solid feeling about the whole process.

3. Introduction The most important part of a group activity, in my opinion, is closure. The second most important is the introduction. A big killer of group work is that awful time between you telling your students they can start, and the first move of pencil on paper—the “what on earth do we do now?” moment. A good introduction should be focused on getting them past that moment. You don't want to give too much away, but you also don't want to throw them into the deep end of the swimming pool. In some classes, you will have to say very little, and in some you may have to do the first problem with them. Experiment with your introductions, but never neglect them.

4. Help when you are needed Some group work methods involve giving absolutely no help when the students are working. Again, you will have to find what is best for you. If you give help too freely, the students have no incentive to talk to each other. If you are too stingy, the students can wind up frustrated. When a student asks me for help, I first ask the group what they think, and if it is clear they are all stuck at the same point, I give a hint.

5. Make understanding a goal in itself Convey to the students (again, directness is a virtue here) that their goal is not just to get the answer written down, but to ensure that every student in their group understands the answer. Their work is not done until they are sure that every one of their colleagues can leave the room knowing how to do the problem. You don't have to sell every single student on this idea for it to work.

6. Bring it back when you can Many of the group works in this guide foreshadow future material. When you are lecturing, try to make reference to past group works when it is appropriate. You will find that your students more easily recall a particular problem they discussed with their friends than a particular statement that you made during a lecture.

The above is just the tip of the iceberg. There are plenty of resources available, both online and in print. Don't be intimidated by the literature—start it on the first day of the next semester, and once you are into it, you may naturally want to read what other people have to say!

1 Functions and Limits

1.1 Four Ways to Represent a Function

SUGGESTED TIME AND EMPHASIS

1 class Essential material

POINTS TO STRESS

1. Definition of function, including piecewise functions.
2. Understanding the interplay between the four ways of representing a function (verbally, numerically, visually, algebraically) perhaps using the concepts of increasing and decreasing functions as an example.
3. Finding the domain and range of a function, regardless of representation.
4. Investigating even and odd functions.

QUIZ QUESTIONS

TEXT QUESTIONS

- Why does the author assert that “the $\sqrt{x}$ key on your calculator is not quite the same as the exact mathematical function f defined by $f(x) = \sqrt{x}$ ”?

ANSWER The calculator gives an approximation to the square root.

- Fill in the blanks: $|x| = \begin{cases} \text{---} & \text{if } x \geq 0 \\ \text{---} & \text{if } x < 0 \end{cases}$

ANSWER $x, -x$

- **DRILL QUESTION** What is the domain of the function $f(x) = \sqrt{1 - \sqrt{x}}$?

ANSWER $0 \leq x \leq 1$

MATERIALS FOR LECTURE

- Draw a graph of electrical power consumption in the classroom versus time on a typical weekday, pointing out important features throughout, and using the vocabulary of this section as much as possible.
- In 1984, United States President Ronald Reagan proposed a plan to change the United States personal income tax system. According to his plan, the income tax would be 15% on the first \$19,300 earned, 25% on the next \$18,800, and 35% on all income above and beyond that. Describe this situation to the class, and have them graph (marginal) tax rate and tax owed versus income for incomes ranging from \$0 to \$80,000. Then have them try to come up with equations describing this situation.
- In the year 2000, Presidential candidate Steve Forbes proposed a “flat tax” model: 0% on the first \$36,000 and 17% on the rest. Have the students do the same analysis, and compare the two models. As an extension, perhaps have the students look at a current tax table and draw similar graphs.
- Let $f(x)$ be the leftmost nonzero digit of x . So $f(386.6) = 3$ and $f(0.000451) = 4$. Have the students try to find the domain and range of f .

ANSWER The domain seems to be all real numbers except zero, and the range seems to be the set of integers from 1 through 9. Although the graph of this “function” cannot be drawn, ask the students to verify that it passes the Vertical Line Test. It turns out that it does not (and in fact is not even a function), for a subtle reason. For example, let $x = \frac{1}{5}$. If we write x as 0.2, then $f(x) = 2$, but if we write x as $0.1\bar{9}$, then $f(x) = 1$. Therefore, f is not a function.

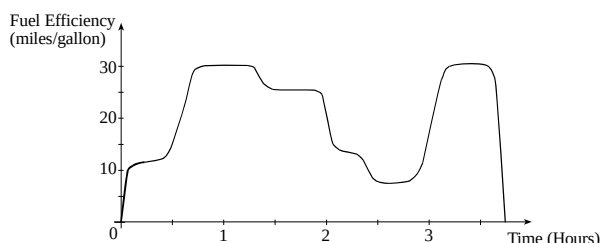
WORKSHOP/DISCUSSION

- Present graphs of even and odd functions, such as $\sin x$, $\cos x + x^2$, and $\cos(\sin x)$, and check with the standard algebraic tests.
- Start with a table of values for the function $f(x) = \frac{1}{4}x^2 + x$:

x	0	1	2	3	4
$f(x)$	0	1.25	3	5.25	8

First, have the class describe the behavior of the function in words, trying to elicit the information that the function is increasing, and that its rate of increase is also increasing. Then, have them try to extrapolate the function in both directions, debating whether or not the function is always positive and increasing. Plot the points and connect the dots, then have them try to concoct a formula (not necessarily expecting them to succeed).

- Draw a graph of fuel efficiency versus time on a trip, such as the one below. Lead a discussion of what could have happened on the trip.



- Discuss the domain and range of a function such as $f(x) = \begin{cases} \sqrt{x} & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$

Also talk about why f is neither increasing nor decreasing for $x > 0$. Stress that when dealing with new sorts of functions, it becomes important to know the precise mathematical definitions of such terms.

- Define “difference quotient” as done in the text. Define $f(x) = x^3$, and show that $\frac{f(a+h) - f(a)}{h} = 3a^2 + 3ah + h^2$. This example both reviews algebra skills and foreshadows future calculations.

GROUP WORK 1: EVERY PICTURE TELLS A STORY

Put the students in groups of four, and have them work on the exercise. If there are questions, encourage them to ask each other before asking you. After going through the correct matching with them, have each group tell their story to the class and see if it fits the remaining graph.

ANSWERS

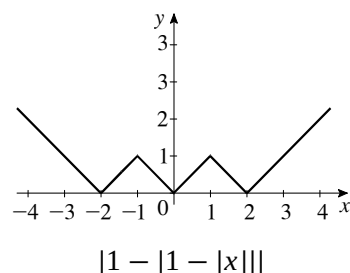
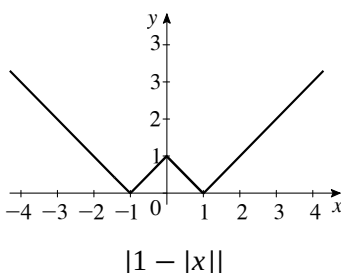
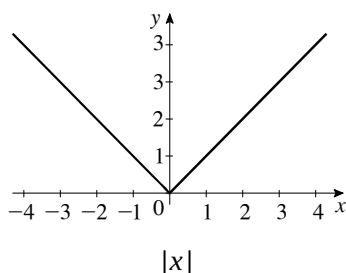
1. (b) 2. (a) 3. (c) 4. The roast beef was cooked in the morning and put in the refrigerator in the afternoon.

GROUP WORK 2: A CHAIN OF FUNCTIONS

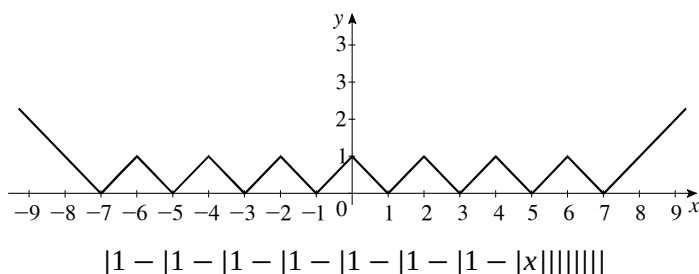
It is recommended that students not be allowed to use graphing technology to do this activity. The intention is to give them an opportunity to practice working with absolute values and order of operations, and to reinforce the idea of looking for mathematical patterns.

ANSWERS

1.



2. (a) 1 (b)



GROUP WORK 3: FINDING A FORMULA

Make sure that the students know the equation of a circle with radius r , and that they remember the notation for piecewise-defined functions. Split the students into groups of four. In each group, have half of the students work on each problem first, and then have them check each other's work. If the students find these problems difficult, have them work together on each problem.

ANSWERS

$$1. f(x) = \begin{cases} -x - 2 & \text{if } x \leq -2 \\ x + 2 & \text{if } -2 < x \leq 0 \\ 2 & \text{if } x > 0 \end{cases} \quad 2. g(x) = \begin{cases} x + 4 & \text{if } x \leq -2 \\ 2 & \text{if } -2 < x \leq 0 \\ \sqrt{4 - x^2} & \text{if } 0 < x \leq 2 \\ x - 2 & \text{if } x > 2 \end{cases}$$

HOMEWORK PROBLEMS

CORE EXERCISES 3, 10, 13, 21, 32, 43, 56, 72, 79

SAMPLE ASSIGNMENT 3, 8, 10, 13, 21, 23, 25, 32, 34, 43, 56, 64, 72, 75, 79

EXERCISE	D	A	N	G
3				×
8				×
10				×
13	×			
21				×
23			×	×
25		×		
32		×		

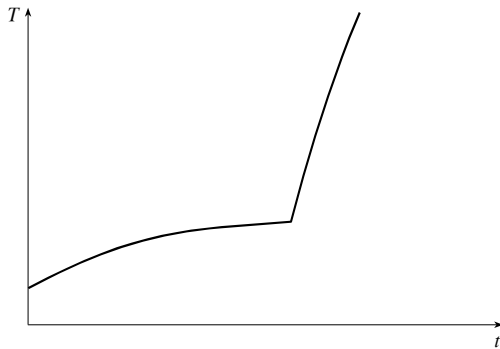
EXERCISE	D	A	N	G
34		×		
43		×		×
56		×		×
64		×		
72				×
75		×		
79				×

NOT FOR SALE

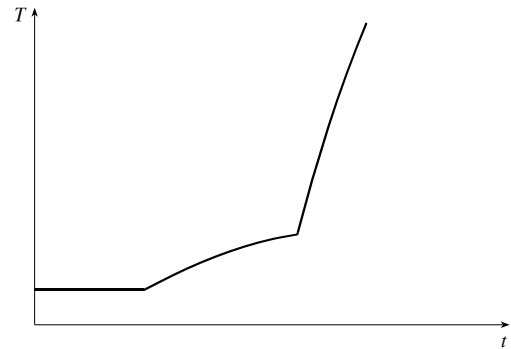
GROUP WORK 1, SECTION 1.1

Every Picture Tells a Story

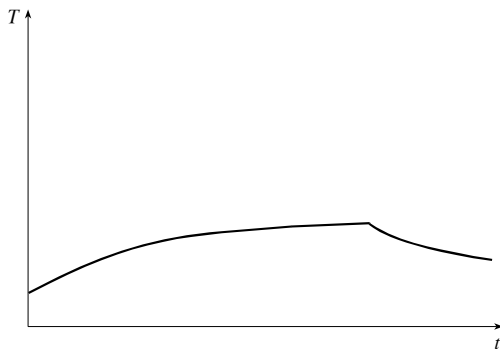
One of the skills you will be learning in this course is the ability to take a description of a real-world occurrence, and translate it into mathematics. Conversely, given a mathematical description of a phenomenon, you will learn how to describe what is happening in plain language. Here follow four graphs of temperature versus time and three stories. Match the stories with the graphs. When finished, write a similar story that would correspond to the final graph.



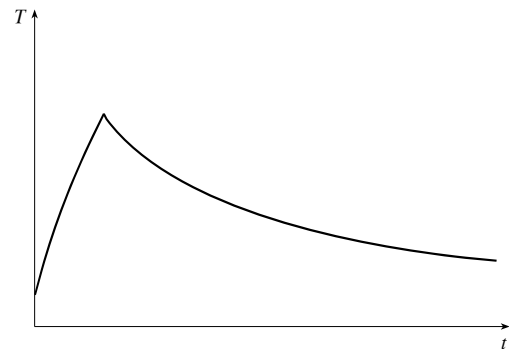
Graph 1



Graph 2



Graph 3



Graph 4

- (a) I took my roast beef out of the freezer at noon, and left it on the counter to thaw. Then I cooked it in the oven when I got home.
- (b) I took my roast beef out of the freezer this morning, and left it on the counter to thaw. Then I cooked it in the oven when I got home.
- (c) I took my roast beef out of the freezer this morning, and left it on the counter to thaw. I forgot about it, and went out for Chinese food on my way home from work. I put it in the refrigerator when I finally got home.

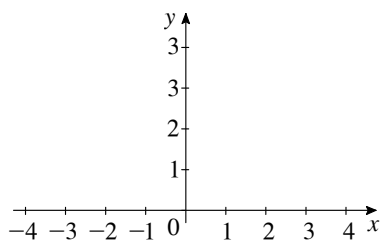
INSTRUCTOR USE ONLY

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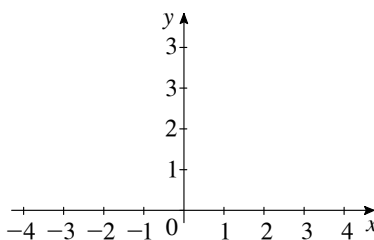
GROUP WORK 2, SECTION 1.1

A Chain of Functions

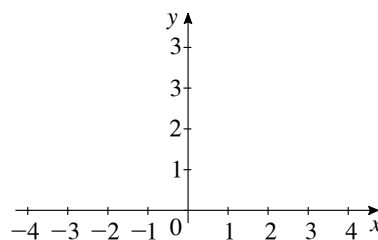
- 1. Sketch graphs of the following three functions.**



$$f_1(x) = |x|$$



$$f_2(x) = |1 - |x||$$

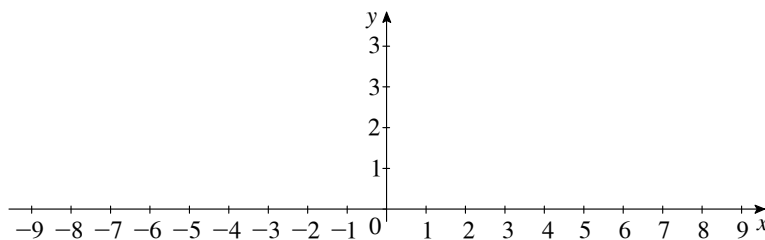


$$f_3(x) = |1 - |1 - |x|||$$

2. Continuing with the pattern, we get $f_8(x) = |1 - |1 - |1 - |1 - |1 - |1 - |1 - |x||| | | | | | |$.

- (a) Compute $f'(0)$.

- (b) Using your graphs from Part 1 as a guide, sketch the graph of $f_8(x)$.



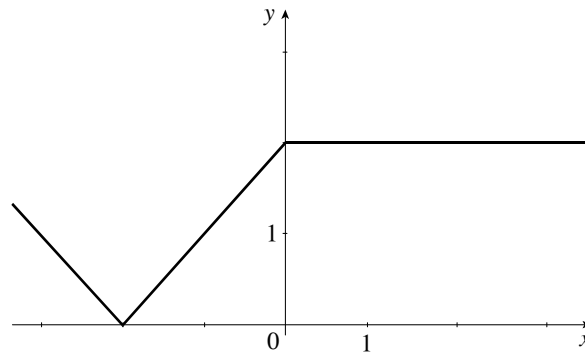
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GROUP WORK 3, SECTION 1.1

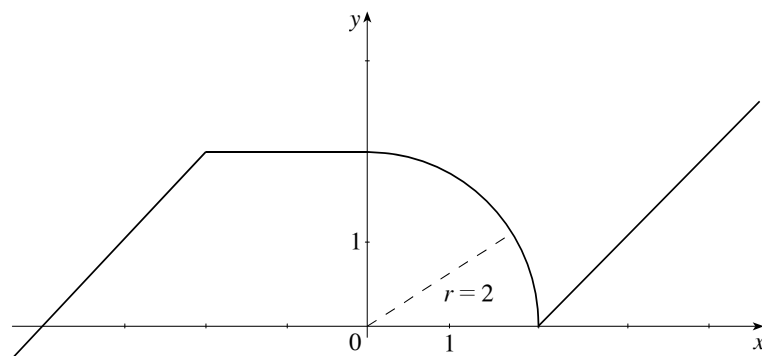
Finding a Formula

Find formulas for the following functions:

1.



2.



INSTRUCTOR USE ONLY

1.2 Mathematical Models: A Catalog of Essential Functions

SUGGESTED TIME AND EMPHASIS

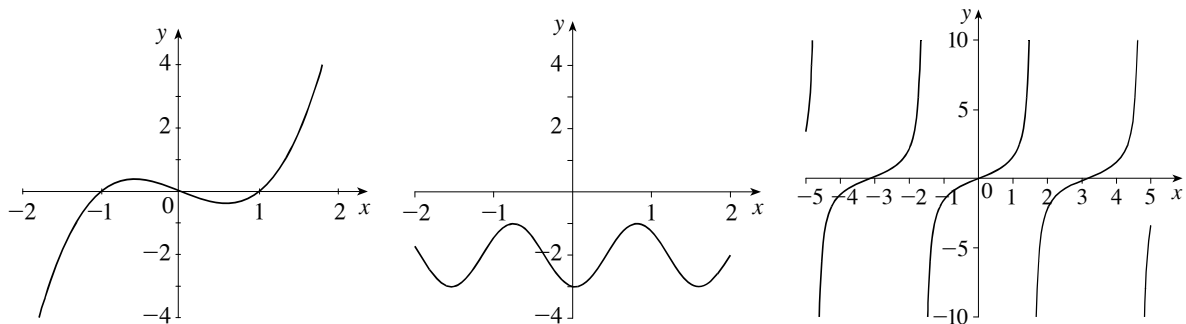
1 class Recommended material

POINTS TO STRESS

1. The modeling process: developing, analyzing, and interpreting a mathematical model.
2. Classes of functions: linear, power, rational, algebraic, and trigonometric functions. Include the special characteristics of each class of functions.

QUIZ QUESTIONS

- **TEXT QUESTION** What is the difference between a power function x^n with $n = 3$ and a cubic function?
ANSWER A cubic function can have lower order terms, whereas a power function has just one term.
- **DRILL QUESTION** Classify each function graphed below as a power function, root function, polynomial, rational function, algebraic function, or trigonometric function. Explain your reasoning.



ANSWER Polynomial, trigonometric, trigonometric

MATERIALS FOR LECTURE

- Show that linear functions have constant differences in y-values for equally spaced x-values. This example illustrates the point:

Linear function (difference=1.2)

x	$f(x)$
-2	-2.0
0	-0.8
2	0.4
4	1.6

- Discuss the shape, symmetries, and general “flatness” near 0 of the power functions x^n for various values of n . Similarly discuss $\sqrt[n]{x}$ for n even and n odd. A blackline master is provided at the end of this section, before the group work handouts.
- If Exercises 23–28 are to be assigned, Exercise 23 can be done in class, discussing part (c) in the context of the technology available to the students.

WORKSHOP/DISCUSSION

- Have the students use technology to graph 2^x , $\sin x$, $\sin 2^x$, and $2^{\sin x}$. Discuss why the latter two look the way that they do. Notice that for this discussion, students don't need to know anything about exponential functions; the emphasis is on how functions combine.
- Figure 17 shows examples of a noncontinuous function and a nondifferentiable function, both expressible as simple formulas. Discuss these curves with the students, trying to get them to describe the ideas of a break in a graph and a cusp.

GROUP WORK 1: THE SMALL SHALL GROW LARGE

If a group finishes early, ask them to similarly compare x^3 and x^4 .

ANSWERS 1. $x^6 \geq x^8$ for $-1 \leq x \leq 1$ 2. $x^3 \geq x^5$ for $-\infty < x \leq -1, 0 \leq x \leq 1$ 3. $x^3 \geq x^{105}$ for $-\infty < x \leq -1, 0 \leq x \leq 1$. If the exponents are both even, the answer is the same as for Problem 1, if the exponents are both odd, the answer is the same as for Problem 2.

GROUP WORK 2: FUN WITH FOURIER

This activity will get students looking at combinations of sine curves, while at the same time foreshadowing the concepts of infinite series and Fourier series.

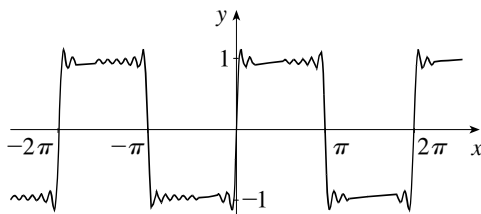
ANSWERS

1. No

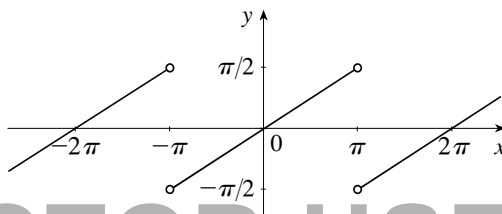
2. $\frac{4}{\pi} \left(\sin x + \frac{1}{3} \sin 3x \right)$

3. $\frac{4}{\pi} \left(\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \frac{1}{7} \sin 7x + \frac{1}{9} \sin 9x \right)$

4. $\frac{4}{\pi} \left(\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \frac{\sin 7x}{7} + \frac{\sin 9x}{9} + \frac{\sin 11x}{11} + \frac{\sin 13x}{13} + \frac{\sin 15x}{15} + \frac{\sin 17x}{17} + \frac{\sin 19x}{19} \right)$



5.

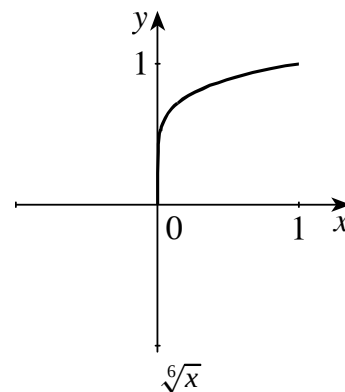
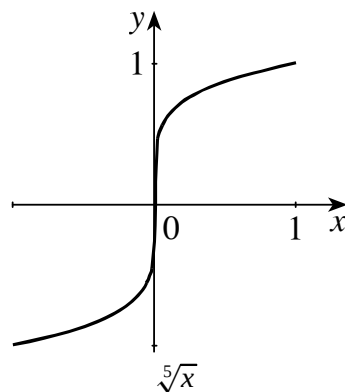
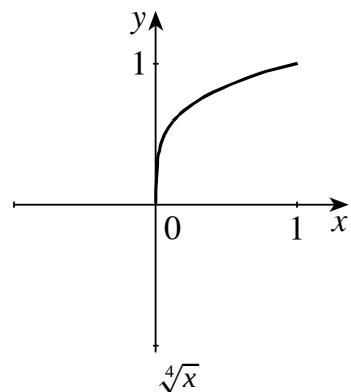
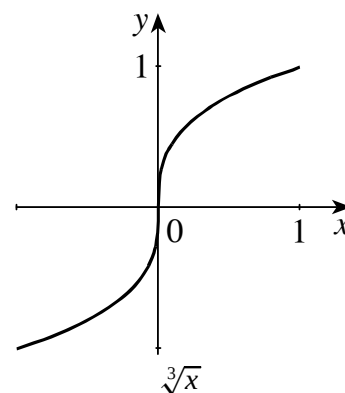
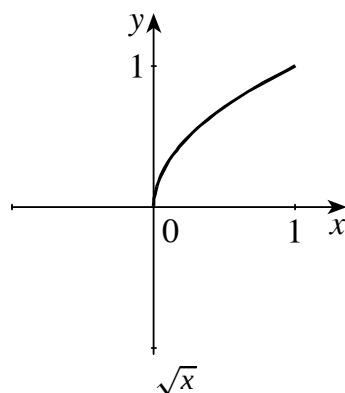
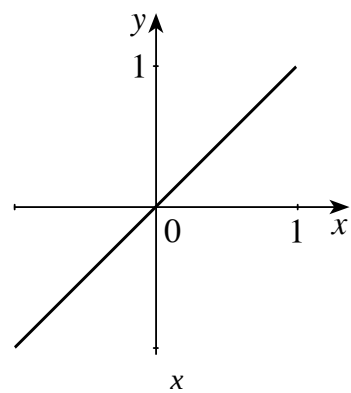
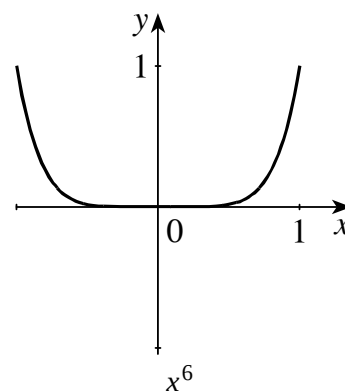
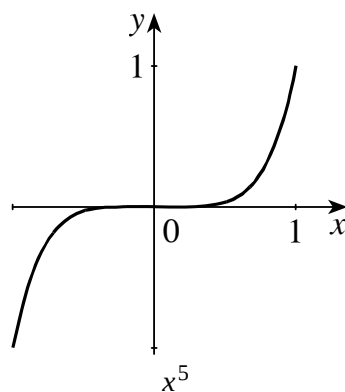
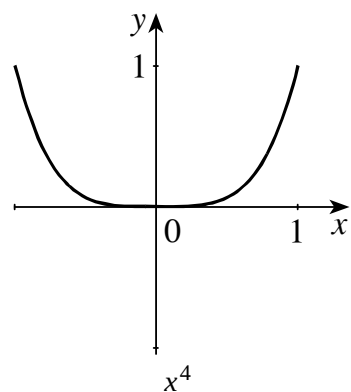
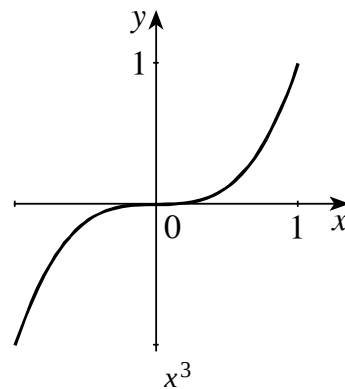
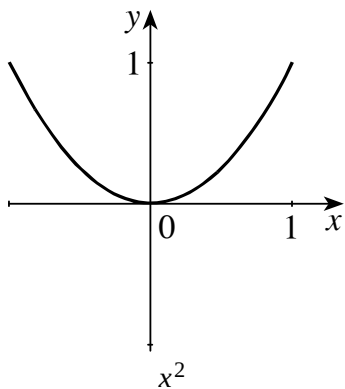
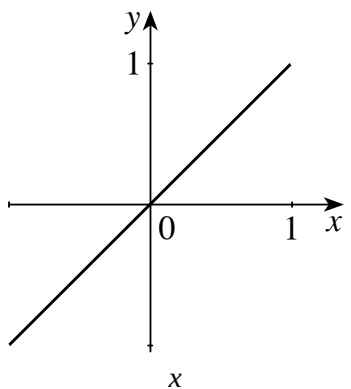


HOMEWORK PROBLEMS

CORE EXERCISES 2, 4, 5, 15, 21, 27, 30

SAMPLE ASSIGNMENT 2, 4, 5, 8, 11, 15, 20, 21, 27, 30, 31

EXERCISE	D	A	N	G
2	×			
4	×		×	
5	×			
8	×		×	
11		×		
15	×		×	
20	×	×		×
21				×
27		×	×	×
30		×		
31		×	×	



NOT FOR SALE

GROUP WORK 1, SECTION 1.2

The Small Shall Grow Large

1. For what values of x is $x^6 \geq x^8$? For what values is $x^6 \leq x^8$?

2. For what values of x is $x^3 \geq x^5$?

3. For what values of x is $x^3 \geq x^{105}$? Can you generalize your results?

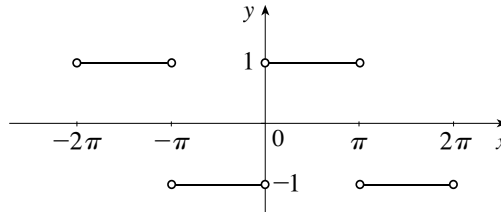
INSTRUCTOR USE ONLY

NOT FOR SALE

GROUP WORK 2, SECTION 1.2

Fun with Fourier

The following function $S(x)$ is called a “square wave”.



1. Can you find a function on your calculator which has the given graph?
2. Select the function from among the following which gives the best approximation to the square wave:
 $\frac{4}{\pi} \sin x$, $\frac{4}{\pi} (\sin x + \sin 3x)$, and $\frac{4}{\pi} \left(\sin x + \frac{1}{3} \sin 3x \right)$.
3. Select the function from among the following which gives the best approximation to the square wave:
 $\frac{4}{\pi} \left(\sin x + \frac{1}{3} \sin 3x \right)$, $\frac{4}{\pi} \left(\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x \right)$, $\frac{4}{\pi} \left(\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \frac{1}{7} \sin 7x \right)$, and $\frac{4}{\pi} \left(\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \frac{1}{7} \sin 7x + \frac{1}{9} \sin 9x \right)$.

INSTRUCTOR USE ONLY

4. A *Fourier approximation* of a function is an approximation of the form

$$F(x) = a_0 + a_1 \cos x + b_1 \sin x + a_2 \cos 2x + b_2 \sin 2x + \cdots + a_n \cos nx + b_n \sin nx$$

You have just discovered the Fourier approximation to $S(x)$ with five terms. Find the Fourier approximation to $S(x)$ with ten terms, and sketch its graph.

5. The following expressions are Fourier approximations to a different function, $T(x)$:

$$T(x) \approx \sin x$$

$$T(x) \approx \sin x - \frac{1}{2} \sin 2x$$

$$T(x) \approx \sin x - \frac{1}{2} \sin 2x + \frac{1}{3} \sin 3x$$

$$T(x) \approx \sin x - \frac{1}{2} \sin 2x + \frac{1}{3} \sin 3x - \frac{1}{4} \sin 4x$$

$$T(x) \approx \sin x - \frac{1}{2} \sin 2x + \frac{1}{3} \sin 3x - \frac{1}{4} \sin 4x + \frac{1}{5} \sin 5x$$

Sketch $T(x)$.

1.3 New Functions from Old Functions

SUGGESTED TIME AND EMPHASIS

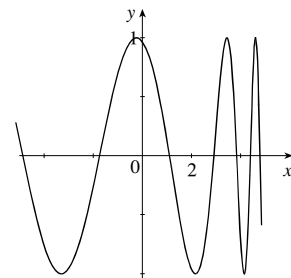
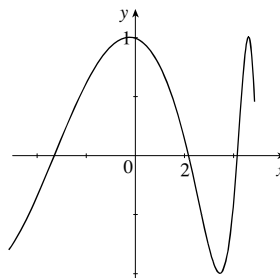
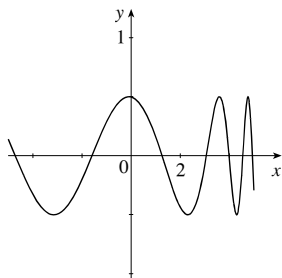
1 class Essential material

POINTS TO STRESS

1. The mechanics and geometry of transforming functions.
2. The mechanics and geometry of adding, subtracting, multiplying, and dividing functions.
3. The mechanics and geometry of composing functions.

QUIZ QUESTIONS

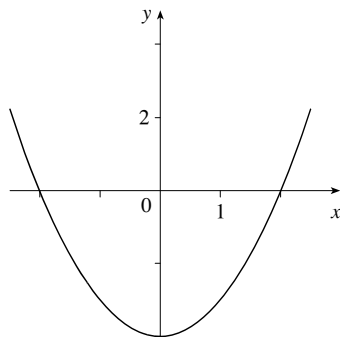
- **TEXT QUESTION** Label the following graphs: $f(x)$, $\frac{1}{2}f(x)$, $f\left(\frac{1}{2}x\right)$.



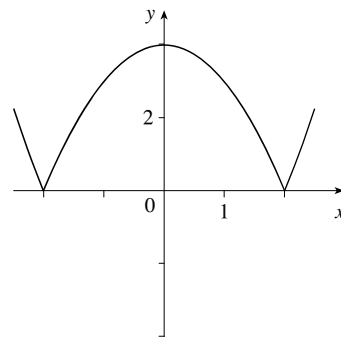
ANSWER $\frac{1}{2}f(x)$, $f\left(\frac{1}{2}x\right)$, $f(x)$

- **DRILL QUESTION** How can we construct the graph of $y = |f(x)|$ from the graph of $y = f(x)$? Explain in words, and demonstrate with the graph of $y = x^2 - 4$.

ANSWER We leave the positive values of $f(x)$ alone, and reflect the negative values about the x -axis.



$$y = x^2 - 4$$

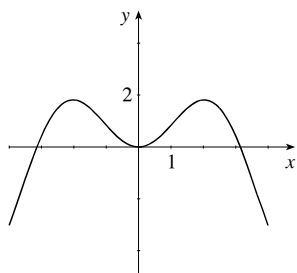


$$y = |x^2 - 4|$$

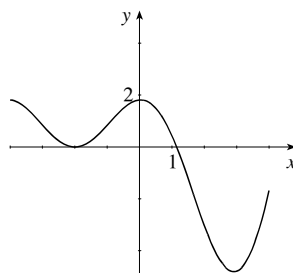
MATERIALS FOR LECTURE

- Using $f(x) = x \sin x$, explore graphs of $f(x+2)$, $f(x) + 2$, $-f(x)$, $f(-x)$, $|f(x)|$. Note why $f(-x) = f(x)$.

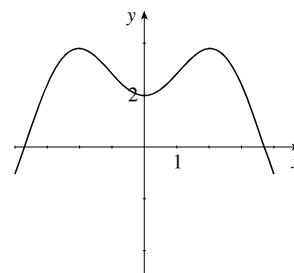
ANSWER



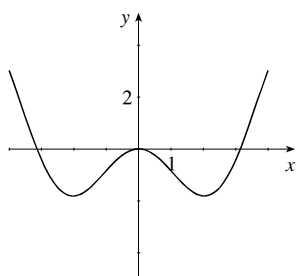
$$y = f(x)$$



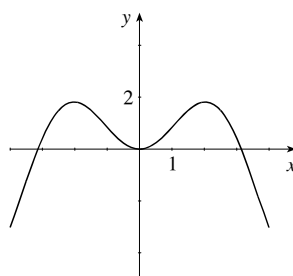
$$y = f(x+2)$$



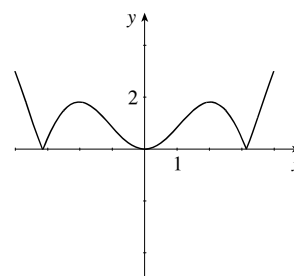
$$y = f(x) + 2$$



$$y = -f(x)$$



$$y = f(-x)$$



$$y = |f(x)|$$

$f(-x) = f(x)$ because f is even.

- Graph $f(x) = \sin(\sqrt{x})$ and $g(x) = \sqrt{\sin x}$. Draw the relevant “arrow diagrams” and then write them in the forms $l \circ k$ and $k \circ l$. Then discuss reasons for the differences in their graphs.

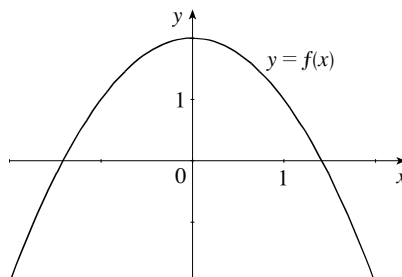
ANSWER See the text for sample arrow diagrams. f is a sine function whose argument grows larger more and more slowly as we move away from the origin. g is a root function whose argument oscillates, causing g to oscillate as well.

WORKSHOP/DISCUSSION

- Using $f(x) = 1/x^2$ and $g(x) = \cos x$, compute the domains of $f + g$, f/g , g/f , $f \circ g$, and $g \circ f$, and the range of $g \circ f$. Pay particular attention to the domain of g/f , as many students will think it is $\mathbb{R}$.

ANSWER $f + g$ has domain $\{x \mid x \neq 0\}$, f/g has domain $\{x \mid x \neq 0, x \neq \frac{\pi}{2} + k\pi\}$, g/f has domain $\{x \mid x \neq 0\}$, $f \circ g$ has domain $\{x \mid x \neq 0, x \neq \frac{\pi}{2} + k\pi\}$, $g \circ f$ has domain $\{x \mid x \neq 0\}$, and $g \circ f$ has range $[-1, 1]$.

- Do the following problem with the students:



From the graph of $y = f(x) = -x^2 + 2$ shown above, compute $f \circ f$ at $x = -1, 0$, and 1 . First do it graphically (as in Exercises 53 and 54), then algebraically.

- After doing a few basic examples of composition, it is possible to foreshadow the idea of inverses, which is covered in Chapter 6. Let $f(x) = 2x^3 + 3$ and $g(x) = x^2 - x$. Compute $f \circ g$ and $g \circ f$ for your students. Then ask them to come up with a function $h(x)$ with the property that $(f \circ h)(x) = x$. They may not be used to the idea of finding examples by themselves; important hints they might need are “Don’t give up,” “When in doubt, just try something and see what happens,” and “I’m not expecting you to get it in fifteen seconds.” If the class is really stuck, have them try $f(x) = 2x^3$ to get a feel for how the game is played. Once they have determined that $h(x) = \sqrt[3]{\frac{x-3}{2}}$, have them first compute $(h \circ f)(x)$, then conjecture whether $(f \circ g)(x) = x$ implies $(g \circ f)(x) = x$ in general.

GROUP WORK 1: WHICH IS THE ORIGINAL?

ANSWERS 1. $2f(x+2)$, $2f(x)$, $f(2x)$, $f(x+2)$, $f(x)$ 2. $2f(x)$, $f(x)$, $f(x+2)$, $f(2x)$, $2f(x+2)$

GROUP WORK 2: LABEL LABEL LABEL, I MADE IT OUT OF CLAY

Some of these transformations were not covered directly in the book. If the students are urged not to give up, and to use the process of elimination and testing individual points, they should be able to complete this activity.

ANSWERS 1. (d) 2. (a) 3. (f) 4. (e) 5. (i) 6. (j) 7. (b) 8. (c) 9. (g) 10. (h)

GROUP WORK 3: IT’S MORE FUN TO COMPUTE

Each group gets one copy of the graph. During each round, one representative from each group stands, and one of the questions below is asked. The representatives write their answer down, and all display their answers at the same time. Each representative has the choice of consulting with their group or not. A correct solo answer is worth two points, and a correct answer after a consult is worth one point.

ANSWERS 1. 0 2. 0 3. 1 4. 5 5. 1 6. 1 7. 1 8. 0 9. 2 10. 1 11. 1 12. 1

HOMEWORK PROBLEMS

CORE EXERCISES 2, 3, 4, 15, 26, 31, 52, 62

SAMPLE ASSIGNMENT 2, 3, 4, 7, 15, 26, 29, 31, 41, 52, 56, 62, 66

EXERCISE	D	A	N	G
2	×			
3	×			
4				×
7		×		
15				×
26	×			
29	×			

EXERCISE	D	A	N	G
31		×		
41		×		
52			×	×
56	×	×		
62		×		
66	×			

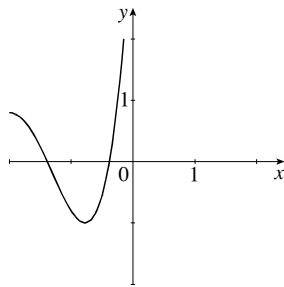
NOT FOR SALE

GROUP WORK 1, SECTION 1.3

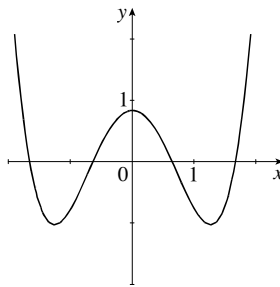
Which is the Original?

Below are five graphs. One is the graph of a function $f(x)$ and the others include the graphs of $2f(x)$, $f(2x)$, $f(x+2)$, and $2f(x+2)$. Determine which is the graph of $f(x)$ and match the other functions with their graphs.

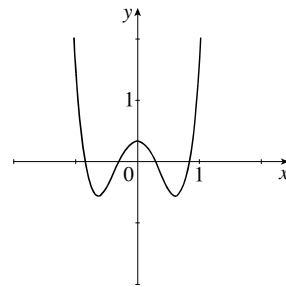
1.



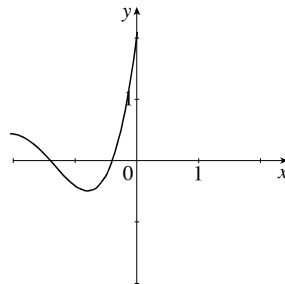
Graph 1



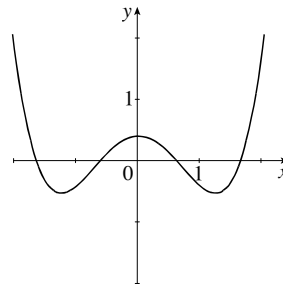
Graph 2



Graph 3

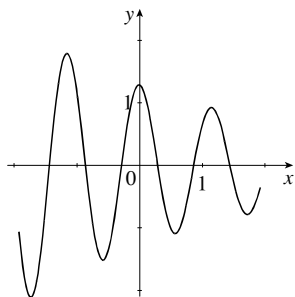


Graph 4

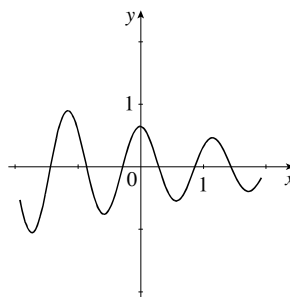


Graph 5

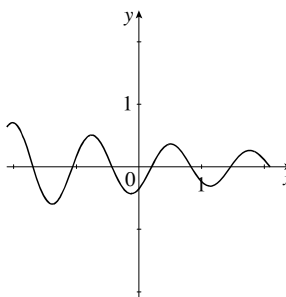
2.



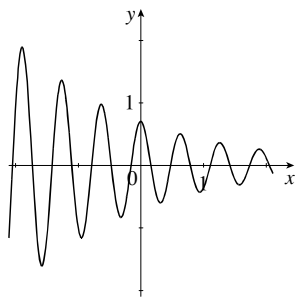
Graph 1



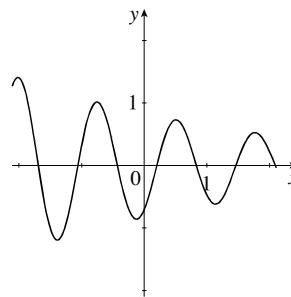
Graph 2



Graph 3



Graph 4



Graph 5

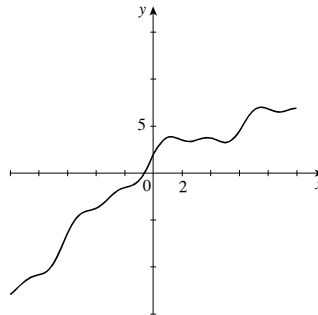
INSTRUCTOR USE ONLY

NOT FOR SALE

GROUP WORK 2, SECTION 1.3

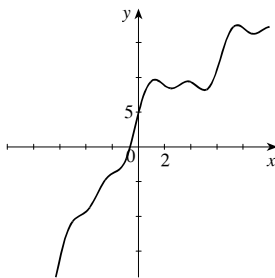
Label Label Label, I Made it Out of Clay

This is a graph of the function $f(x)$:

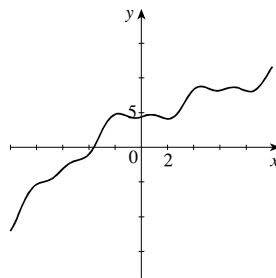


Give each graph below the correct label from the following:

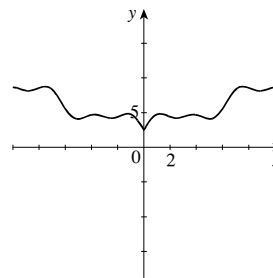
- (a) $f(x+3)$ (b) $f(x-3)$ (c) $f(2x)$ (d) $2f(x)$ (e) $|f(x)|$
(f) $f(|x|)$ (g) $2f(x)-1$ (h) $f(2x)+2$ (i) $f(x)-x$ (j) $1/f(x)$



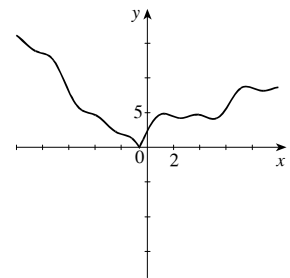
Graph 1



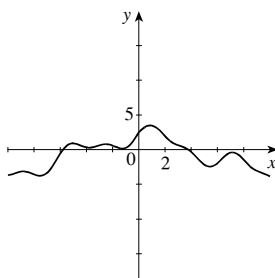
Graph 2



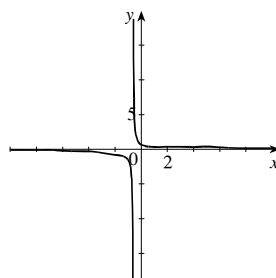
Graph 3



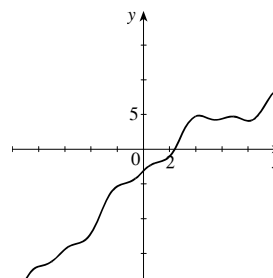
Graph 4



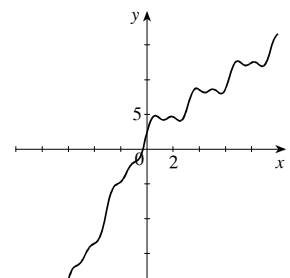
Graph 5



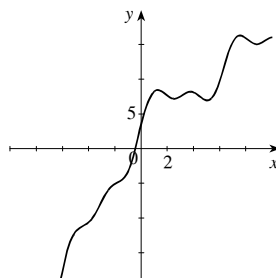
Graph 6



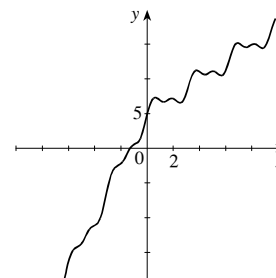
Graph 7



Graph 8



Graph 9



Graph 10

INSTRUCTOR USE ONLY

NOT FOR SALE

GROUP WORK 3, SECTION 1.3

It's More Fun to Compute

Using the graph below, find the following quantities.

1. $(f \circ g)(5)$

5. $(g \circ g)(5)$

9. $(g \circ f)(1)$

2. $(g \circ f)(5)$

6. $(g \circ g)(-3)$

10. $(f \circ f \circ g)(4)$

3. $(f \circ g)(0)$

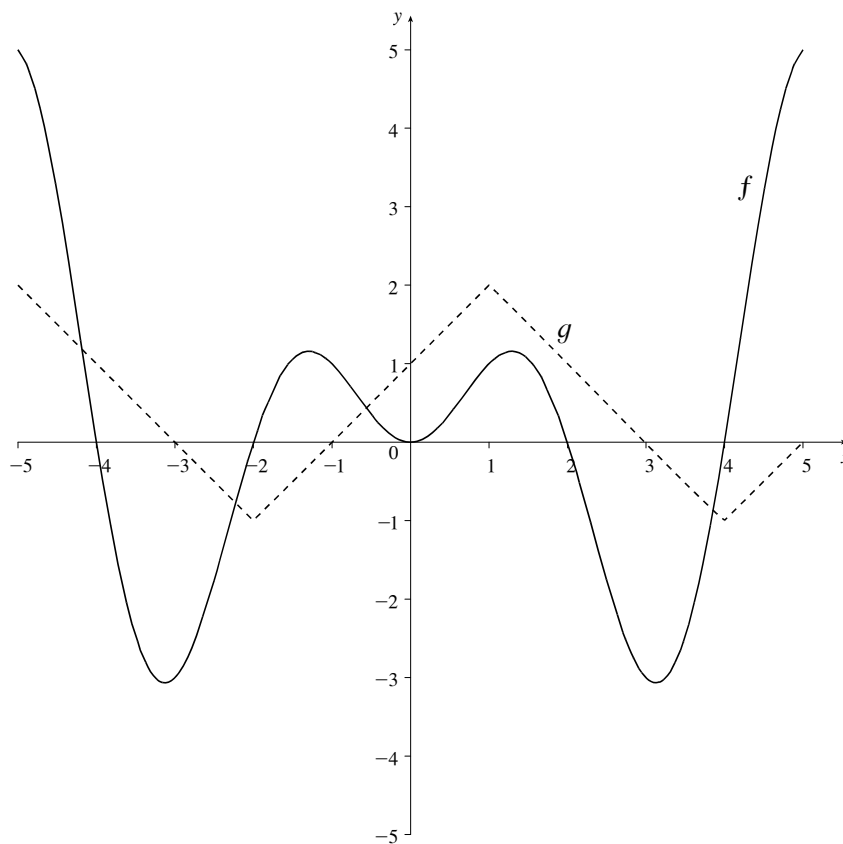
7. $(g \circ g)(-1)$

11. $(g \circ f \circ f)(4)$

4. $(f \circ f)(5)$

8. $(f \circ g)(1)$

12. $(f \circ g \circ f)(4)$



INSTRUCTOR USE ONLY

1.4 The Tangent and Velocity Problems

SUGGESTED TIME AND EMPHASIS

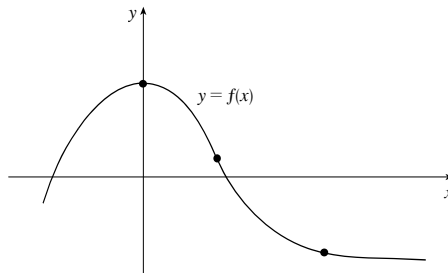
$\frac{1}{2}$ –1 class Essential material

POINTS TO STRESS

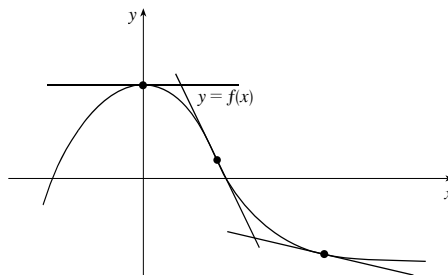
1. The tangent line viewed as the limit of secant lines.
2. The concepts of average versus instantaneous velocity, described numerically, visually, and in physical terms.
3. The tangent line as the line obtained by “zooming in” on a smooth function; local linearity.
4. Approximating the slope of the tangent line using slopes of secant lines.

QUIZ QUESTIONS

- **TEXT QUESTION** Geometrically, what is “the line tangent to a curve” at a particular point?
ANSWER There are different correct ones. Examples include the best linear approximation to a curve at a point, or the result of repeated “zooming in” on a curve.
- **DRILL QUESTION** Draw the line tangent to the following curve at each of the indicated points:



ANSWER



MATERIALS FOR LECTURE

- Point out that if a car is driving along a curve, the headlights will point along the direction of the tangent line.
- Discuss the phrase “instantaneous velocity.” Ask the class for a definition, such as, “It is the limit of average velocities.” Use this discussion to shape a more precise definition of a limit.
- Illustrate that many functions such as x^2 and $x - 2 \sin x$ look locally linear, and discuss the relationship of this property to the concept of the tangent line. Then pose the question, “What does a secant line to a linear function look like?”

- Show that the slopes of the tangent lines to $f(x) = \sqrt[3]{x}$ and $g(x) = |x|$ are not defined at $x = 0$. Note that f has a tangent line (which is vertical), but g does not (it has a cusp). The absolute value function can be explored graphically.

WORKSHOP/DISCUSSION

- Estimate slopes from discrete data, as in Exercises 2 and 7.
- Estimate the slope of $y = \frac{3}{1+x^2}$ at the point $(1, \frac{3}{2})$ using the graph, and then numerically. Draw the tangent line to this curve at the indicated point. Do the same for the points $(0, 3)$ and $(2, \frac{3}{5})$.
ANSWER $-1.5, 0, -0.48$
- Draw tangent lines to the curve $y = \sin\left(\frac{1}{x}\right)$ at $x = \frac{1}{2\pi}$ and $x = \frac{1}{\pi/2}$. Notice the difference in the quality of the tangent line approximations.

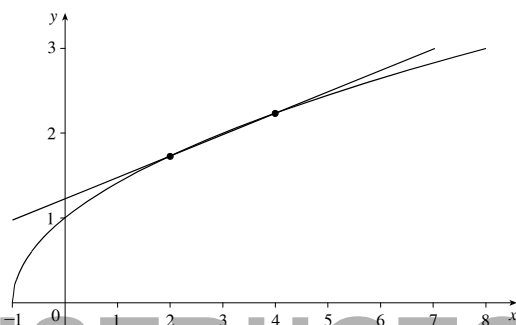
GROUP WORK 1: WHAT'S THE PATTERN?

The students will not be able to do Problem 3 from the graph alone, although some will try. After a majority of them are working on Problem 3, announce that they can do this numerically.

If they are unable to get Problem 6, have them repeat Problem 4 for $x = 15$, and again for $x = 0$.

ANSWERS

1, 2.



- $2 - \sqrt{3} \approx 0.268, \sqrt{5} - 2 \approx 0.236,$
 $\sqrt{4.5} - \sqrt{3.5} \approx 0.250, \frac{\sqrt{4.2} - \sqrt{3.8}}{0.4} \approx 0.250$
- $\frac{1}{4}$ is a good estimate.
- $\frac{1}{6}$ is a good estimate.
- $\frac{1}{2\sqrt{a+1}}$

GROUP WORK 2: SLOPE PATTERNS

When introducing this activity, it may be best to fill out the first line of the table with your students, or to estimate the slope at $x = -1$. If a group finishes early, have them try to justify the observations made in the last part of Problem 2.

ANSWERS

1. (a) 0, 0.2, 0.4, 0.6 (b) 11.5
2. (a) Estimating from the graph gives that the function is increasing for $x < -3.2$, decreasing for $-3.2 < x < 3.2$, and increasing for $x > 3.2$.
 - (b) The slope of the tangent line is positive when the function is increasing, and the slope of the tangent line is negative when the function is decreasing.
 - (c) The slope of the tangent line is zero somewhere between $x = -3.2$ and -3.1 , and somewhere between $x = 3.1$ and 3.2 . The graph has a local maximum at the first point and a local minimum at the second.
 - (d) The tangent line approximates the curve worst at the maximum and the minimum. It approximates best at $x = 0$, where the curve is “straightest,” that is, at the point of inflection.

HOMEWORK PROBLEMS

CORE EXERCISES 1, 5, 7

SAMPLE ASSIGNMENT 1, 2, 5, 7

EXERCISE	D	A	N	G
1		×	×	
2			×	×
5	×	×		
7			×	×