O N E

Introduction

ANSWERS TO REVIEW QUESTIONS

1. Guided missiles, automatic gain control in radio receivers, satellite tracking antenna

2. Yes - power gain, remote control, parameter conversion; No - Expense, complexity

3. Motor, low pass filter, inertia supported between two bearings

4. Closed-loop systems compensate for disturbances by measuring the response, comparing it to the input response (the desired output), and then correcting the output response.

5. Under the condition that the feedback element is other than unity

6. Actuating signal

7. Multiple subsystems can time share the controller. Any adjustments to the controller can be implemented with simply software changes.

8. Stability, transient response, and steady-state error

9. Steady-state, transient

10. It follows a growing transient response until the steady-state response is no longer visible. The system will either destroy itself, reach an equilibrium state because of saturation in driving amplifiers, or hit limit stops.

11. Natural response

12. Determine the transient response performance of the system.

13. Determine system parameters to meet the transient response specifications for the system.

14. True

15. Transfer function, state-space, differential equations

16. Transfer function - the Laplace transform of the differential equation

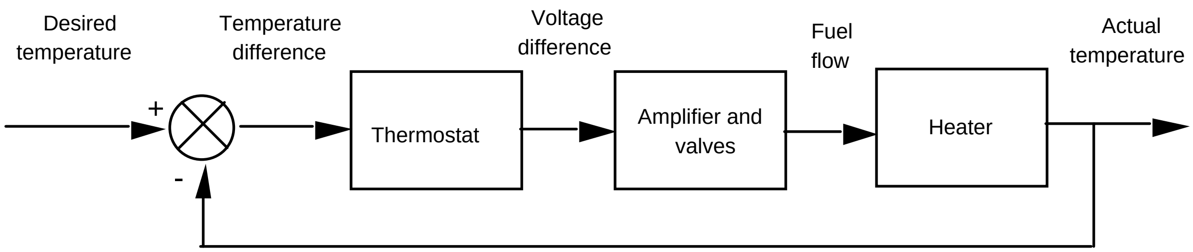
State-space - representation of an nth order differential equation as n simultaneous first-order differential equations

Differential equation - Modeling a system with its differential equation

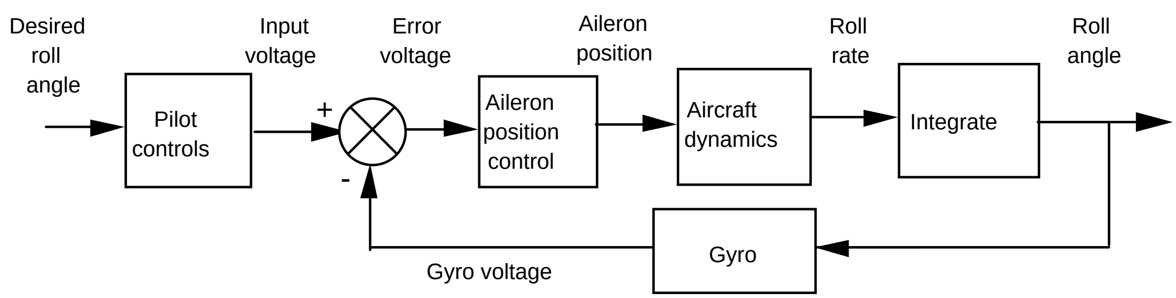
SOLUTIONS TO PROBLEMS

1. Five turns yields 50 v. Therefore K = $\dfrac{50 \text{ volts}}{5 \text{ x } 2\pi \text{ rad}}$ **= 1.59**

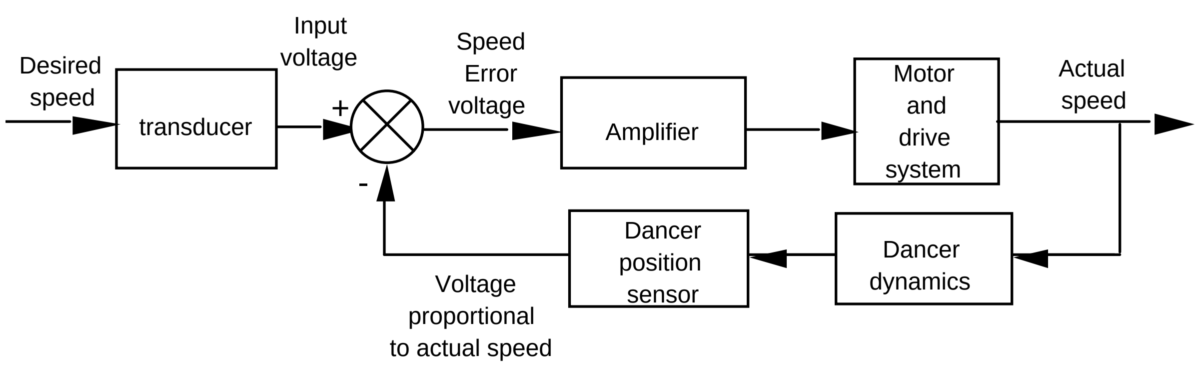
2.


Desired
temperature
Temperature
difference
Voltage
difference
Fuel
flow
Actual
temperature
+
-
Thermostat
Amplifier and
valves
Heater

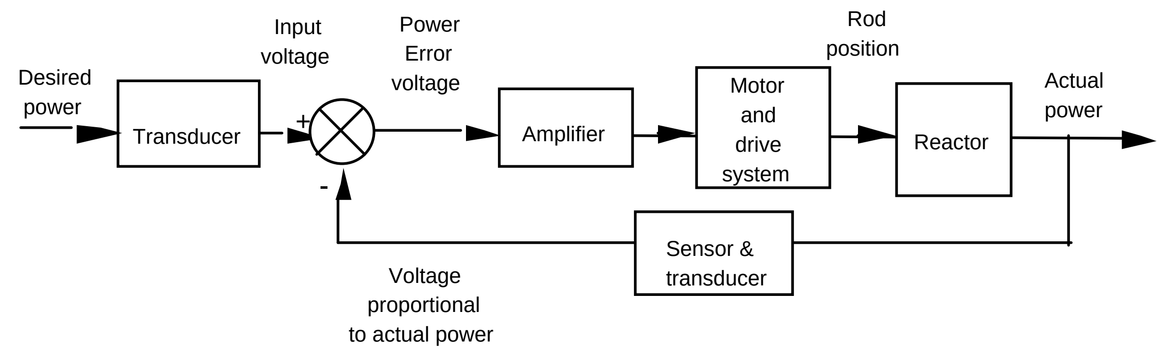
3.


Desired
roll
angle
Input
voltage
Error
voltage
Aileron
position
Roll
rate
Roll
angle
Pilot
controls
+
-
Aileron
position
control
Aircraft
dynamics
Integrate
Gyro
Gyro voltage

4.

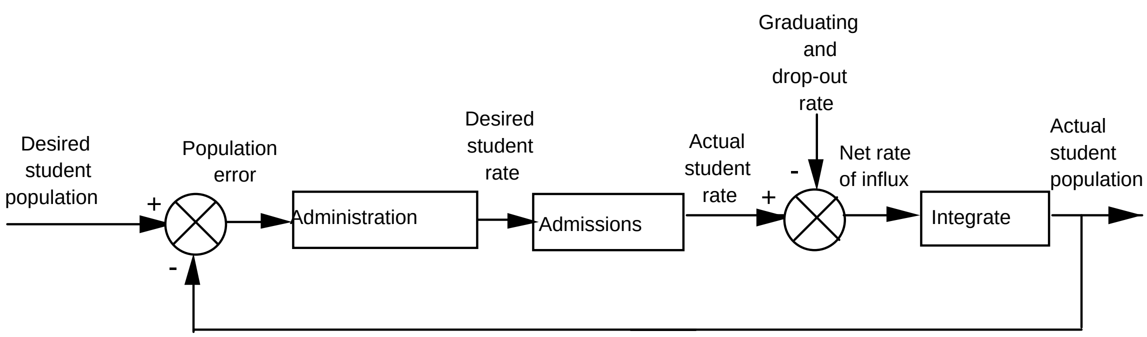

Desired
speed
Input
voltage
Speed
Error
voltage
Actual
speed
transducer
+
-
Amplifier
Motor
and
drive
system
Dancer
position
sensor
Dancer
dynamics
Voltage
proportional
to actual speed

5.



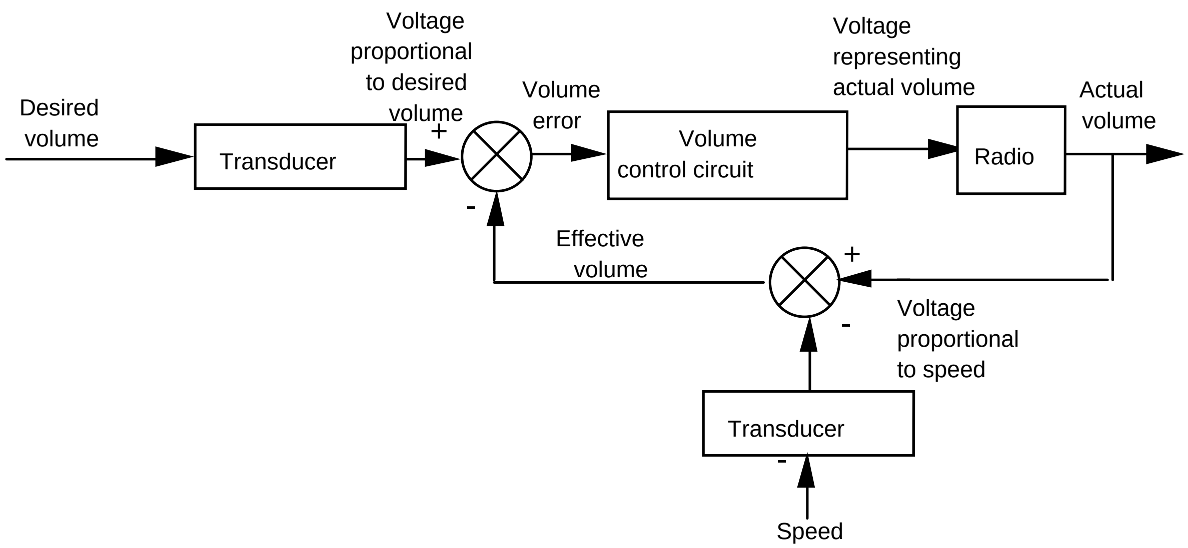
Input voltage
Power Error voltage
Rod position
Desired power
Transducer
+
Amplifier
Motor and drive system
Reactor
Actual power
-
Sensor & transducer
Voltage proportional to actual power

6.



Graduating and drop-out rate
Desired student rate
Actual student rate
Net rate of influx
Actual student population
Desired student population
Population error
+
-
Administration
Admissions
Integrate

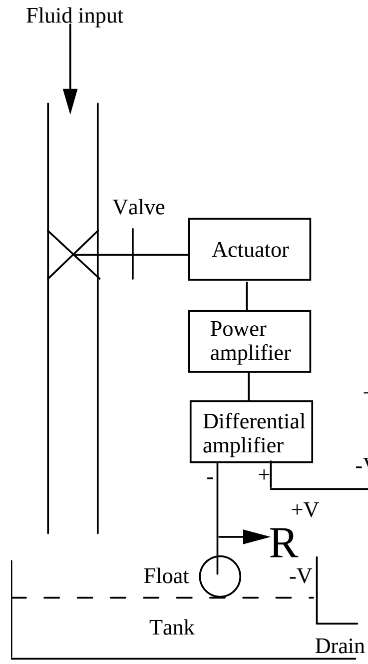
7.



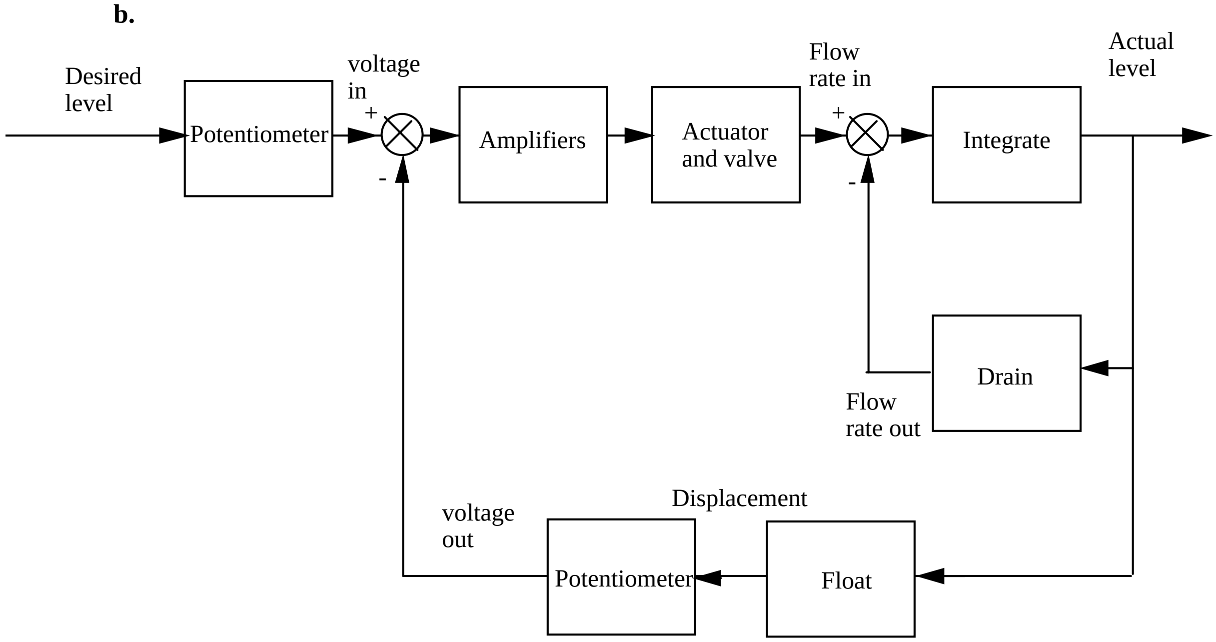
Voltage proportional to desired volume
Voltage representing actual volume
Volume error
Desired volume
Transducer
+
-
Volume control circuit
Radio
Actual volume
Effective volume
Voltage proportional to speed
Transducer
Speed

8.

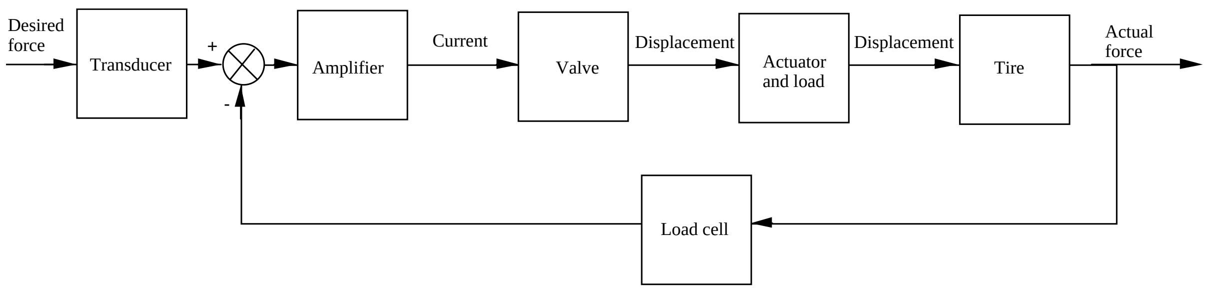
a.


Fluid input
Valve
Actuator
Power amplifier
Differential amplifier
+V
R
-V
Desired level
-
+
+V
R
-V
Float
Tank
Drain

b.

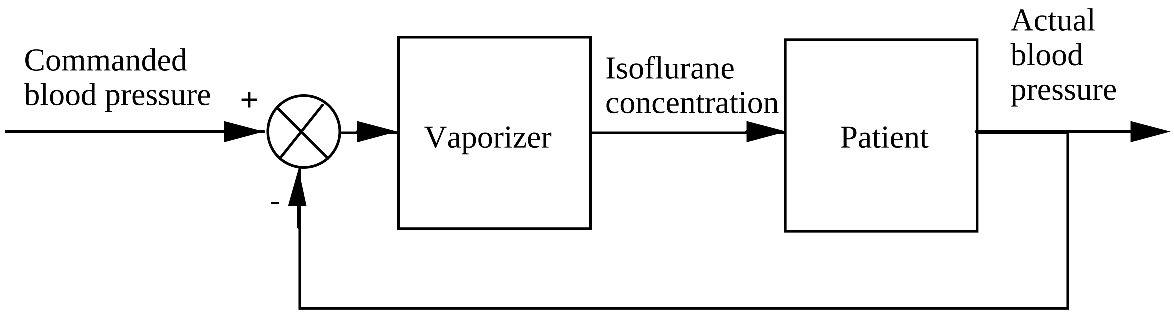

Desired level
Potentiometer
voltage in
+
-
Amplifiers
Actuator and valve
Flow rate in
+
-
Integrate
Actual level
Drain
Flow rate out
Displacement
voltage out
Potentiometer
Float

9.



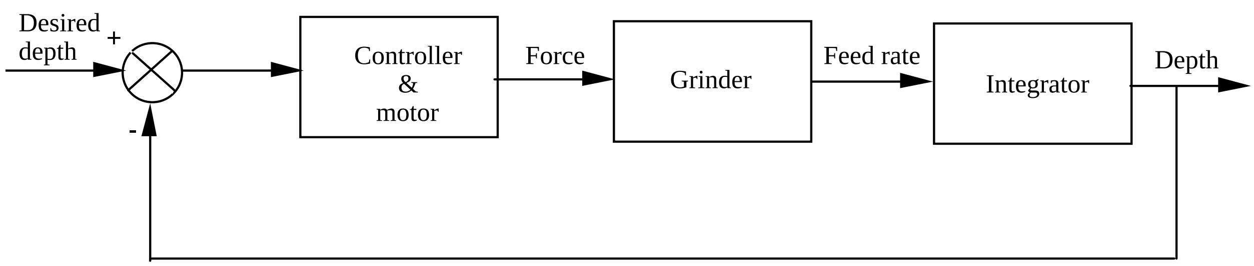
Desired force
Transducer
+
-
Amplifier
Current
Valve
Displacement
Actuator and load
Displacement
Tire
Actual force
Load cell

10.



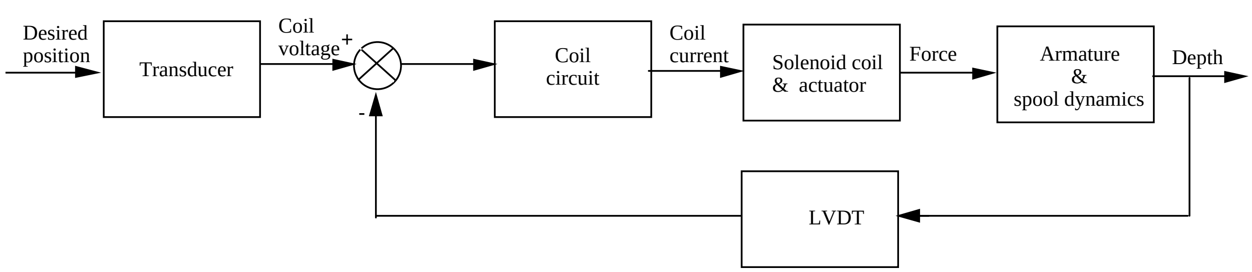
Commanded blood pressure
+
-
Vaporizer
Isoflurane concentration
Patient
Actual blood pressure

11.



Desired depth
+
-
Controller & motor
Force
Grinder
Feed rate
Integrator
Depth

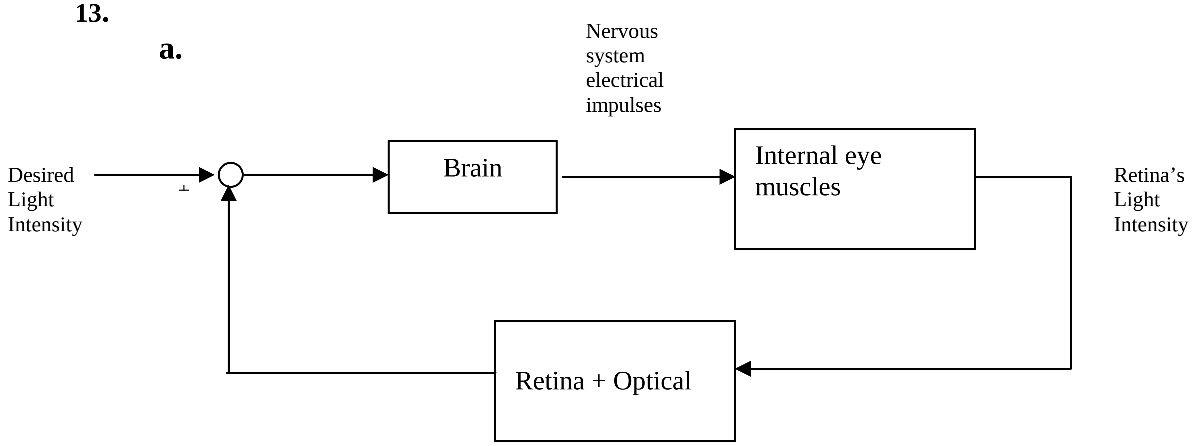
12.



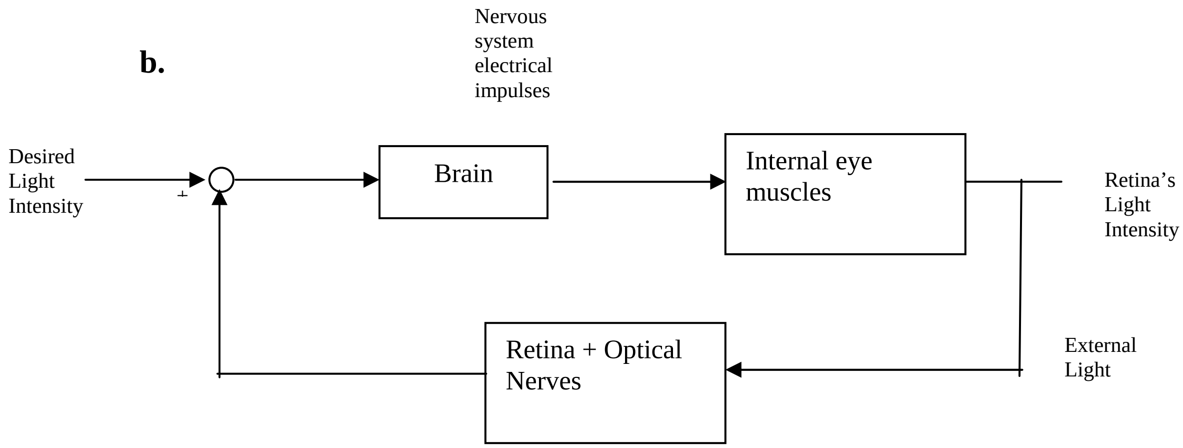
Desired position
Transducer
Coil voltage
+
-
Coil circuit
Coil current
Solenoid coil & actuator
Force
Armature & spool dynamics
Depth
LVDT

13.

a.



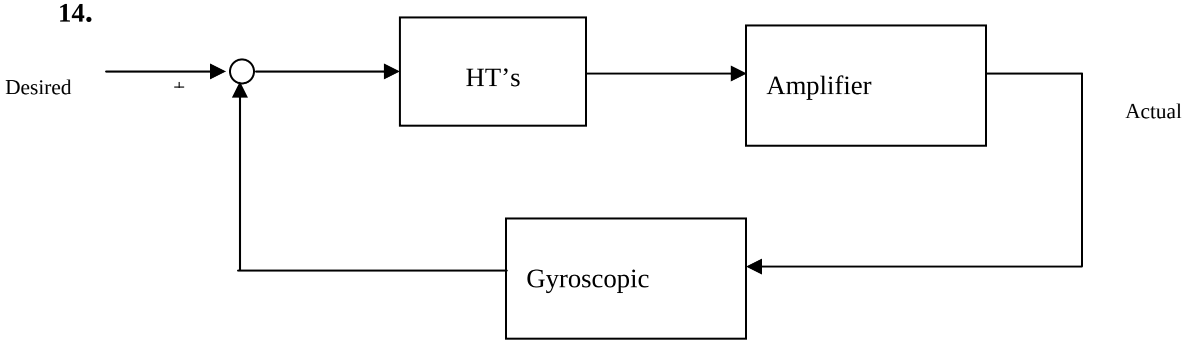
b.



If the narrow light beam is modulated sinusoidally the pupil's diameter will also vary sinusoidally (with a delay see part c) in problem)

c. If the pupil responded with no time delay the pupil would contract only to the point where a small amount of light goes in. Then the pupil would stop contracting and would remain with a fixed diameter.

14.



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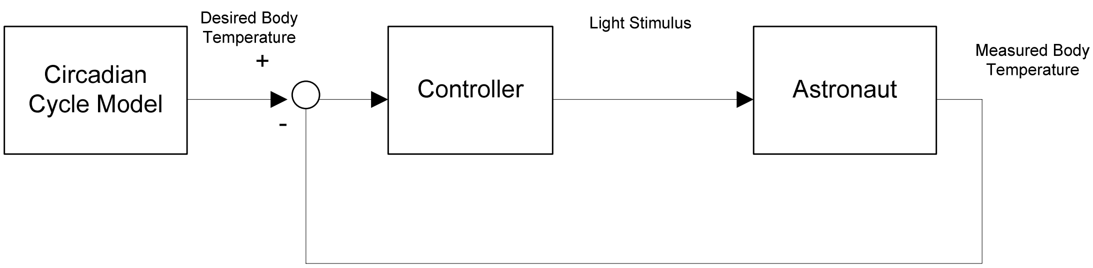
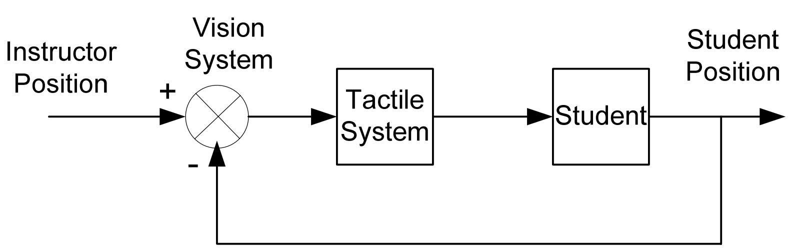


16.



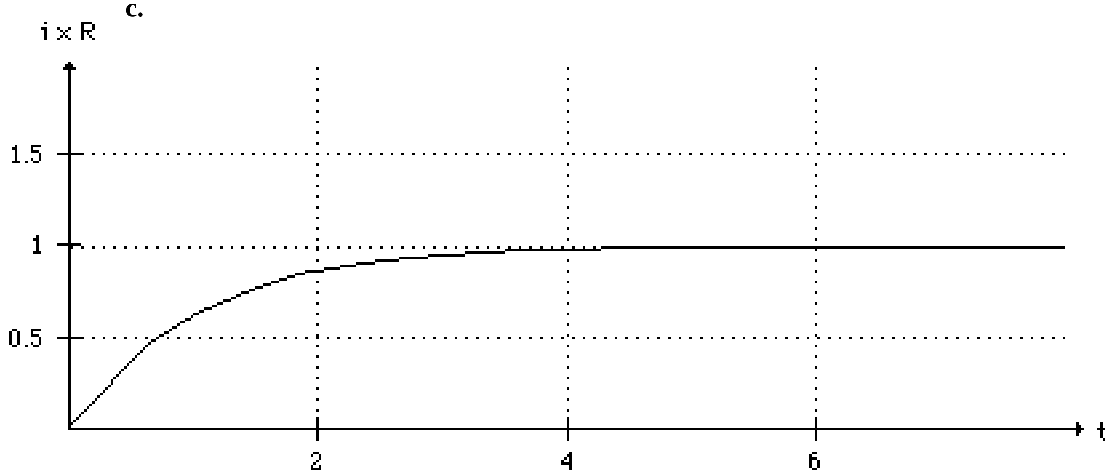
17.

a. $L\dfrac{di}{dt} + Ri = u(t)$

b. Assume a steady-state solution $i_{SS} = B$. Substituting this into the differential equation yields RB = 1,

from which $B = \dfrac{1}{R}$. The characteristic equation is LM + R = 0, from which $M = -\dfrac{R}{L}$. Thus, the total solution is $i(t) = Ae^{-(R/L)t} + \frac{1}{R}$. Solving for the arbitrary constants, $i(0) = A + \frac{1}{R} = 0$. Thus, $A = -\frac{1}{R}$. The final solution is $i(t) = \frac{1}{R} - \frac{1}{R}e^{-(R/L)t} = \frac{1}{R}(1 - e^{-(R/L)t})$.

c.



18.

a. Writing the loop equation, $Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt + v_C(0) = v(t)$

b. Differentiating and substituting values, $\frac{d^2i}{dt^2} + 2\frac{di}{dt} + 25i = 0$

Writing the characteristic equation and factoring,

$M^2 + 2M + 25 = (M + 1 + \sqrt{24}i)(M + 1 - \sqrt{24}i)$.

The general form of the solution and its derivative is

$$i = Ae^{-t}\cos(\sqrt{24}t) + Be^{-t}\sin(\sqrt{24}t)$$

$$\frac{di}{dt} = (-A + \sqrt{24}B)e^{-t}\cos(\sqrt{24}t) - (\sqrt{24}A + B)e^{-t}\sin(\sqrt{24}t)$$

Using $i(0) = 0; \frac{di}{dt}(0) = \frac{v_L(0)}{L} = \frac{1}{L} = 1$

$i(0) = A = 0$

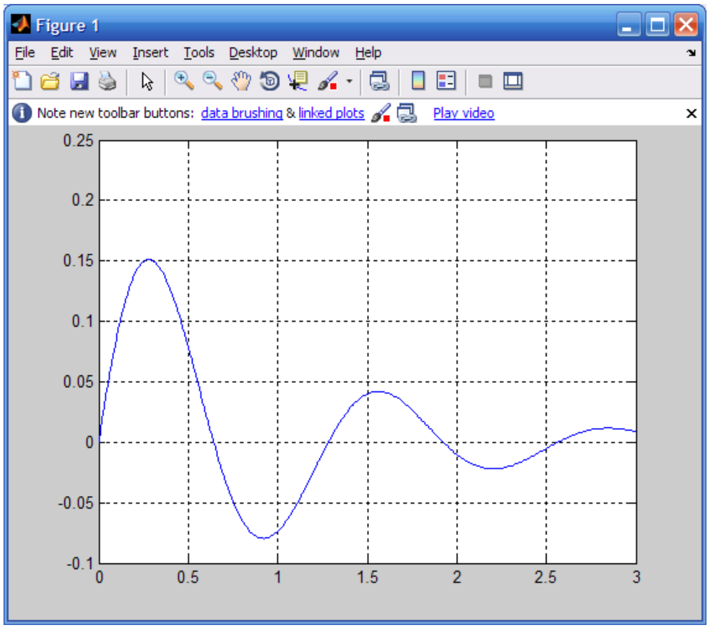
$\frac{di}{dt}(0) = -A + \sqrt{24}B = 1$

Thus, $A = 0$ and $B = \frac{1}{\sqrt{24}}$.

The solution is

$$i = \frac{1}{\sqrt{24}}e^{-t}\sin(\sqrt{24}t)$$

c.



19.

a. Assume a particular solution of

Substitute into the differential equation and obtain

$$(7C + 2D)\cos(2t) + (-2C + 7D)\sin(2t) = 5\cos(2t)$$

Equating like coefficients,

$$7C + 2D = 5$$

$$-2C + 7D = 0$$

From which, C = $\frac{35}{53}$ and D = $\frac{10}{53}$.

The characteristic polynomial is

$$M + 7 = 0$$

Thus, the total solution is

$$x(t) = A\,e^{-7t} + \left(\frac{35}{53}\cos[2t] + \frac{10}{53}\sin[2t]\right)$$

Solving for the arbitrary constants, x(0) = A + $\frac{35}{53}$ = 0. Therefore, A = - $\frac{35}{53}$. The final solution is

$$x(t) = \left(-\frac{35}{53}\right)e^{-7t} + \left(\frac{35}{53}\cos[2t] + \frac{10}{53}\sin[2t]\right)$$

b. Assume a particular solution of

$$x_p = A\sin 3t + B\cos 3t$$

Substitute into the differential equation and obtain

$$(18A - B)\cos(3t) - (A + 18B)\sin(3t) = 5\sin(3t)$$

Therefore, 18A – B = 0 and –(A + 18B) = 5. Solving for A and B we obtain

$$x_p = (-1/65)\sin 3t + (-18/65)\cos 3t$$

The characteristic polynomial is

$$M^2 + 6\,M + 8 = (M + 4)\,(M + 2)$$

Thus, the total solution is

$$x = C\,e^{-4\,t} + D\,e^{-2\,t} + \left(-\frac{18}{65}\cos(3\,t) - \frac{1}{65}\sin(3\,t)\right)$$

Solving for the arbitrary constants, $x(0) = C + D - \frac{18}{65} = 0$.

Also, the derivative of the solution is

$$\frac{dx}{dt} = -\frac{3}{65}\cos(3\,t) + \frac{54}{65}\sin(3\,t) - 4\,C\,e^{-4\,t} - 2\,D\,e^{-2\,t}$$

Solving for the arbitrary constants, $\dot{x}(0)\ -\frac{3}{65} - 4C - 2D = 0$, or $C = -\frac{3}{10}$ and $D = \frac{15}{26}$.

The final solution is

$$x = -\frac{18}{65}\cos(3\,t) - \frac{1}{65}\sin(3\,t) - \frac{3}{10}\,e^{-4\,t} + \frac{15}{26}\,e^{-2\,t}$$

c. Assume a particular solution of

$$x_p = A$$

Substitute into the differential equation and obtain 25A = 10, or A = 2/5.

The characteristic polynomial is

$$M^2 + 8\,M + 25 = (M + 4 + 3\,i)\,(M + 4 - 3\,i)$$

Thus, the total solution is

$$x = \frac{2}{5} + e^{-4\,t}\,(B\sin(3\,t) + C\cos(3\,t))$$

Solving for the arbitrary constants, x(0) = C + 2/5 = 0. Therefore, C = -2/5. Also, the derivative of the solution is

$$\frac{dx}{dt} = ((3B - 4C)\cos(3t) - (4B + 3C)\sin(3t))\,e^{-4t}$$

Solving for the arbitrary constants, $\dot{x}(0) = 3B - 4C = 0$. Therefore, B = -8/15. The final solution is

$$x(t) = \frac{2}{5} - e^{-4t}\left(\frac{8}{15}\sin(3t) + \frac{2}{5}\cos(3t)\right)$$

20.

a. Assume a particular solution of

$$x_p(t) = C\cos(2t) + D\sin(2t)$$

Substitute into the differential equation and obtain

$$-2(C - 2D)\cos(2t) - 4\left(C + \frac{1}{2}D\right)\sin(2t) = \sin(2t)$$

Equating like coefficients,

$$-2(C - 2D) = 0$$
$$-4\left(C + \frac{1}{2}D\right) = 1$$

From which, $C = -\frac{1}{5}$ and $D = -\frac{1}{10}$.

The characteristic polynomial is

$$M^2 + 2M + 2 = (M + 1 + i)(M + 1 - i)$$

Thus, the total solution is

$$x = -\frac{1}{5}\cos(2t) - \frac{1}{10}\sin(2t) + e^{-t}(A\cos[t] + B\sin[t])$$

Solving for the arbitrary constants, $x(0) = A - \frac{1}{5} = 2$. Therefore, $A = \frac{11}{5}$. Also, the derivative of the solution is

$$\frac{dx}{dt} = -\frac{1}{5}\cos(2t) + \frac{2}{5}\sin(2t) + (-A + B)e^{-t}\cos(t) - (A + B)e^{-t}\sin(t)$$

Solving for the arbitrary constants, $\dot{x}(0) = -A + B - 0.2 = -3$. Therefore, $B = -\frac{3}{5}$. The final solution is

$$x(t) = -\frac{1}{5}\cos(2t) - \frac{1}{10}\sin(2t) + e^{-t}\left(\frac{11}{5}\cos(t) - \frac{3}{5}\sin(t)\right)$$

b. Assume a particular solution of

$$x_p = Ce^{-2t} + Dt + E$$

Substitute into the differential equation and obtain

$$Ce^{-2t} + Dt + 2D + E = 5e^{-2t} + t$$

Equating like coefficients, C = 5, D = 1, and 2D + E = 0.

From which, C = 5, D = 1, and E = - 2.

The characteristic polynomial is

$$M^2 + 2M + 1 = (M + 1)^2$$

Thus, the total solution is

$$x(t) = Ae^{-t} + Be^{-t}t + 5e^{-2t} + t - 2$$

Solving for the arbitrary constants, x(0) = A + 5 - 2 = 2 Therefore, A = -1. Also, the derivative of the solution is

$$\frac{dx}{dt} = (-A + B)e^{-t} - Bte^{-t} - 10e^{-2t} + 1$$

Solving for the arbitrary constants, $\dot{x}(0)$ = B - 8 = 1. Therefore, B = 9. The final solution is

$$x(t) = -e^{-t} + 9te^{-t} + 5e^{-2t} + t - 2$$

c. Assume a particular solution of

$$x_p = Ct^2 + Dt + E$$

Substitute into the differential equation and obtain

$$4Ct^2 + 4Dt + 2C + 4E = t^2$$

Equating like coefficients, C = $\frac{1}{4}$, D = 0, and 2C + 4E = 0.

From which, C = $\frac{1}{4}$, D = 0, and E = - $\frac{1}{8}$.

The characteristic polynomial is

$$M^2 + 4 = (M + 2i)(M - 2i)$$

Thus, the total solution is

$$x(t) = A\cos(2t) + B\sin(2t) + \frac{1}{4}t^2 - \frac{1}{8}$$

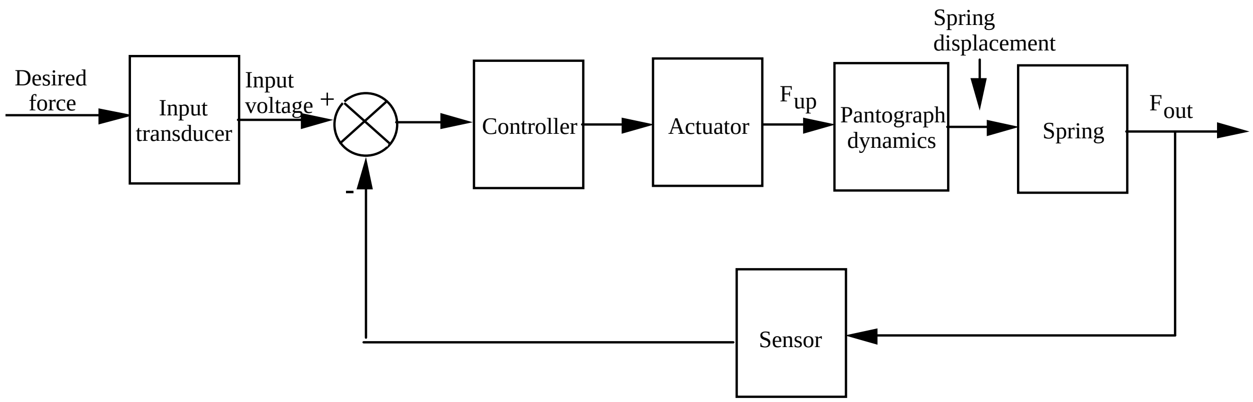
Solving for the arbitrary constants, x(0) = A - $\frac{1}{8}$ = 1 Therefore, A = $\frac{9}{8}$. Also, the derivative of the solution is

$$\frac{dx}{dt} = 2B\cos(2t) - 2A\sin(2t) + \frac{1}{2}t$$

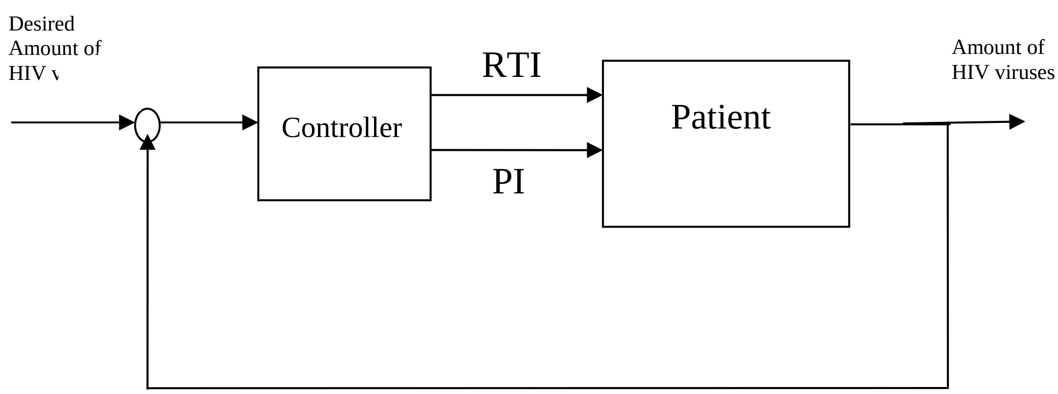
Solving for the arbitrary constants, $\dot{x}(0)$ = 2B = 2. Therefore, B = 1. The final solution is

$$x(t) = \frac{9}{8}\cos(2t) + \sin(2t) + \frac{1}{4}t^2 - \frac{1}{8}$$

21.

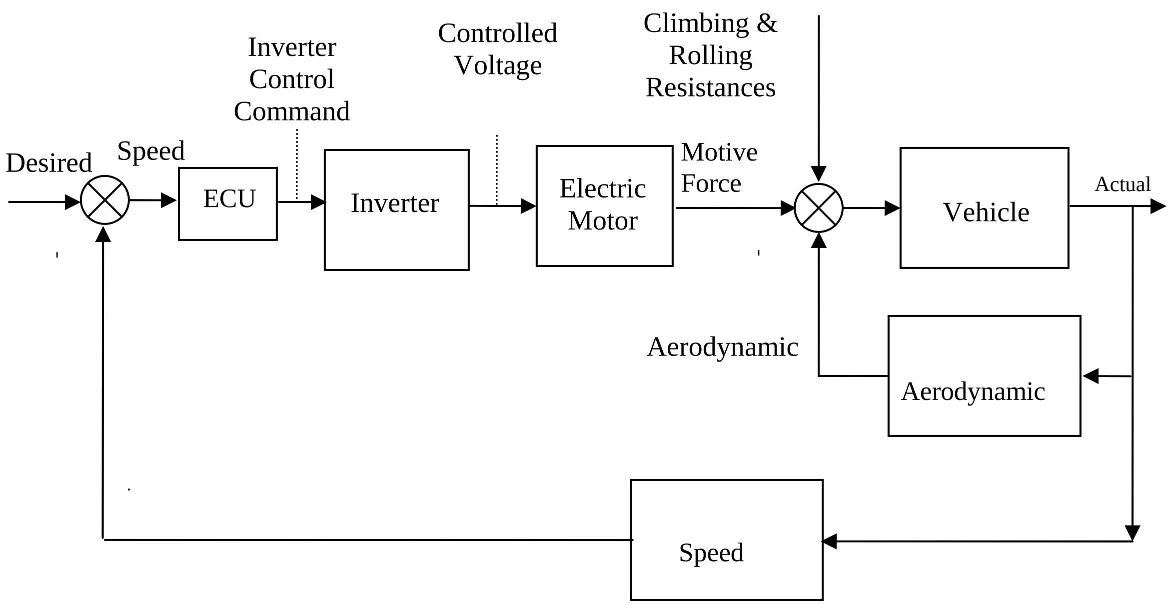


22.

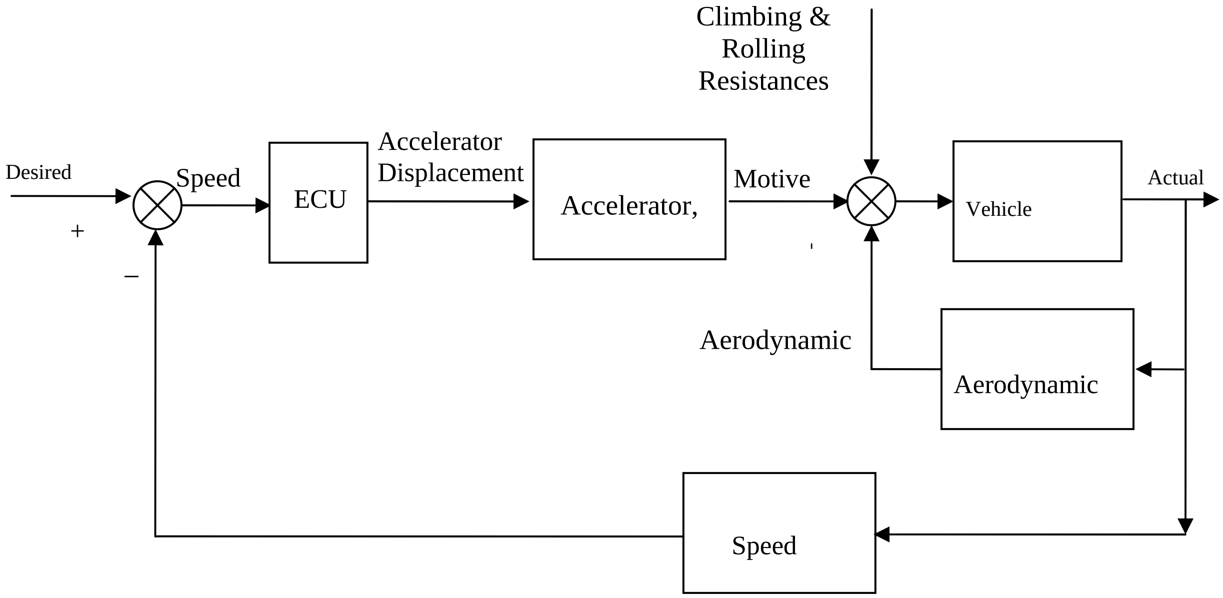


23.

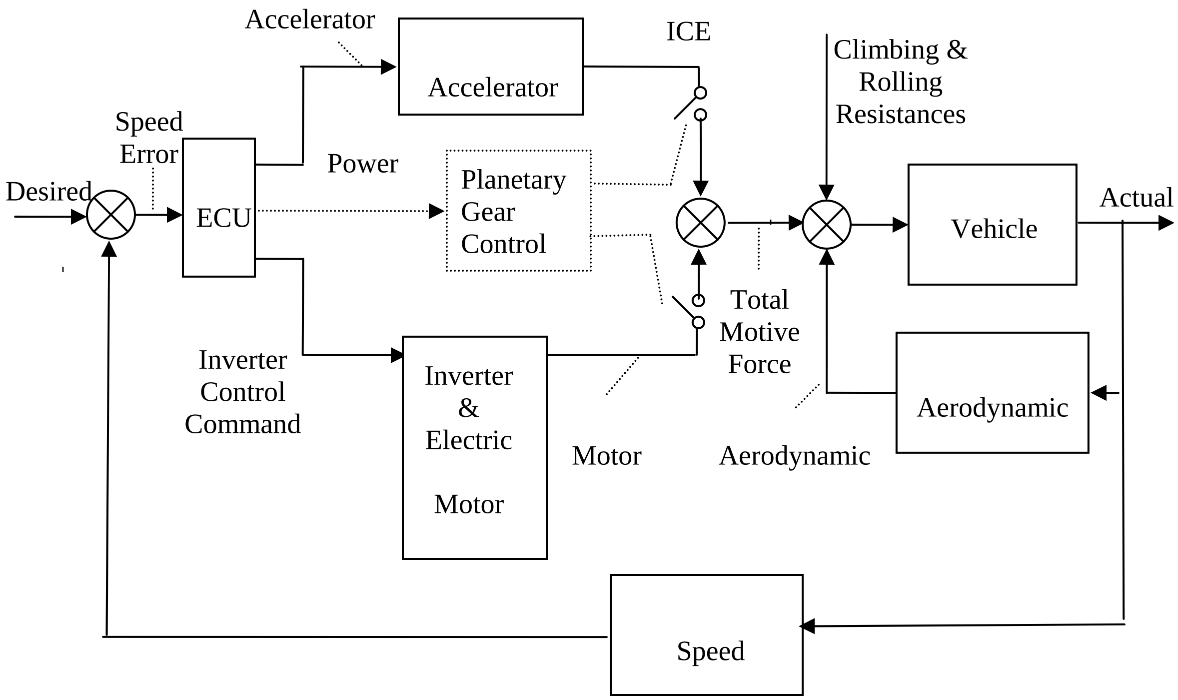
a.


Desired
Speed
ECU
Inverter
Control
Command
Inverter
Controlled
Voltage
Electric
Motor
Motive
Force
Climbing &
Rolling
Resistances
Vehicle
Actual
Aerodynamic
Aerodynamic
Speed

b.


Climbing &
Rolling
Resistances
Desired
Speed
ECU
Accelerator
Displacement
Accelerator,
Motive
Vehicle
Actual
+
–
Aerodynamic
Aerodynamic
Speed

c.



Accelerator
Accelerator
ICE
Climbing & Rolling Resistances
Speed Error
Power
Desired
ECU
Planetary Gear Control
Vehicle
Actual
Total Motive Force
Inverter Control Command
Inverter & Electric Motor
Aerodynamic
Motor
Aerodynamic
Speed