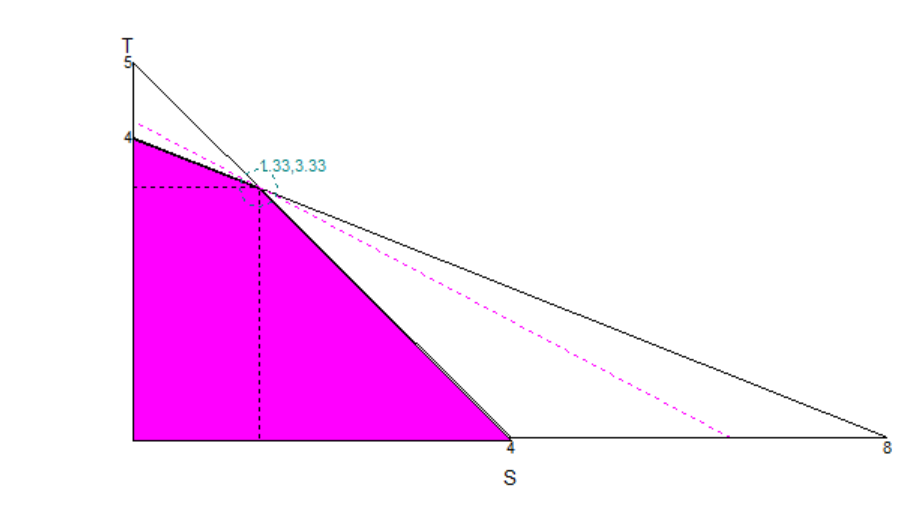
Solutions Manual

# Module A: Linear Programming

1. Maximum occurs at S = 1.333, T = 3.333, Profit = $50.67



Cognitive Domain: Knowledge

Difficulty Level: Easy

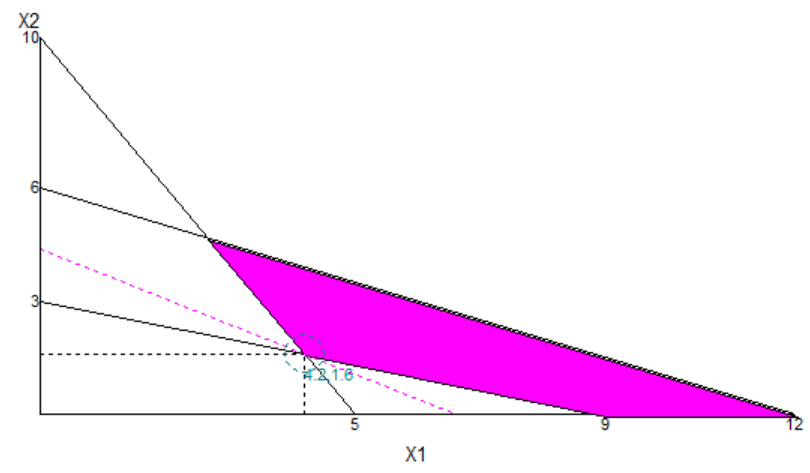
2a. The optimum solution occurs at X = 2, Y = 9 for Profit = $57.

2b. The constraints have zero slack; the solution lies at the intersection of these two constraints. This means all 24 hours of fabrication time and 40 hours of assembly time are used up when producing 2 of Y and 9 of Y to achieve a maximum profit of $57.

Cognitive Domain: Knowledge

Difficulty Level: Easy

3.



The optimal solution is X1 = 4.2, X2 = 1.6 for Cost = $26.4.

Cognitive Domain: Knowledge

Difficulty Level: Easy

4a. Costs are minimized at 76 when P = 8 and R = 20.

4b. The third constraint has a surplus of 46, and the fourth constraint has a surplus of 15. This means mixing 8 ounces of P and 20 ounces of R to obtain the lowest costs of $76; the resulting mixture exceeds the protein requirement by 46 units and the amount of R required by 15 units. Similarly, surplus is the amount by which the left-hand side values are greater than the right-hand side values. So for the fourth constraint, R must be greater than or equal to 5, but at the optimal solution R = 20, which is 15 greater than the constraint’s RHS.

Cognitive Domain: Comprehension

Difficulty Level: Medium

5a. The optimal product mix is 925 soft, 150 hardwood, 200 tropical, and 300 aircraft for a total profit of $21,800.

5b. The allowance increase and allowance decrease values for the objective function coefficients define the range within which the unit profit of each types of plywood can change without affecting the optimal profit of $21,800. For example, the allowance increase and allowance decrease for soft plywood is infinity and 2, respectively. This means as long as the unit profit for plywood is within the range of infinity to ($8 – $2) $6, the optimal profit for Diamond Plywood stays at $21,800. Similarly, the optimal profit will stay at $21,800 as long as the unit profit for hardwood is within the range of $0 to $20 ($12 + 8); for tropical is $0 to $28 ($18 + 10); and for aircraft is $0 to $40 ($30 + 10).

5c. The shadow price of a constraint defines the increase/decrease in profit per unit increase/decrease of that constraint’s RHS, as long as the change in the constraint’s RHS is within the allowable increase/decrease. For example, the shadow price for grading time is $8 with an allowance increase/decrease of 275 and 805. This means there will be an increase of profit by $8 with an additional hour of grading time up to 275 hours (i.e., from 3,500 to 3,775 hours), and there will be a decrease of profit by $8, with each reduction of grading time down to 805 hours (i.e., from 3,500 to 2,695 hours).

Cognitive Domain: Analysis

Difficulty Level: Medium

6a. Regular Blend should be made from 36 pounds of Hawaiian and 84 pounds of Ethiopian, and Premium Blend should be made of 45 pounds Hawaiian, 25 pounds Ethiopian, and 30 pounds Columbian.

6b. The coefficients for Ethiopian Premium, Columbian Premium, and Columbian Regular all show allowable decreases of infinity, meaning that their profit contributions could become negative, yet they would still appear in the solution. While unprofitable, they are required in order to achieve the required percentages in the various blends. The remainder of the allowable increases and decreases show the range in which the coefficients must remain in order to retain this optimal solution.

6c. Shadow prices are the marginal values for one additional unit of the right-hand side. The shadow price for Columbian Premium is -2; if the RHS of the original constraint is increased by 1 unit, the value of the objective function at optimality drops by 2 units (in this case, profits) as long as that increase or decrease is within plus 20 or minus 30 (the allowable increase and decrease respectively). Changes in RHS values outside the allowable increases and decreases will not be a direct multiplier of the shadow prices.

Cognitive Domain: Analysis

Difficulty Level: Medium

7a. The optimal schedule is as follows:

|  |  |
| --- | --- |
| *Time Interval* | *Workers Starting This Shift* |
| 00–04 | 7 |
| 04–08 | 1 |
| 08–12 | 21 |
| 12–16 | 6 |
| 16–20 | 12 |
| 20–24 | 0 |

Based on this system of equations,



7b. The allowable increase and decrease values for the objective function coefficients provide a valid range for the reduced cost entries. In the objective function, all coefficients are 1; counting the number of workers for any shift more than twice will result in a different schedule but identical number of workers.

7c. Shadow prices are the marginal values for one additional unit of the right-hand side. If the RHS of the original constraint is increased by 1 unit, the value of the objective function at optimality changes by the value of the shadow price as long as that increase or decrease is within the allowable increase and decrease. Changes in RHS values outside the allowable increases and decreases will not be a direct multiplier of the shadow prices.

Cognitive Domain: Application

Difficulty Level: Hard

8.

Max Profit = $800*W* + $600*R*

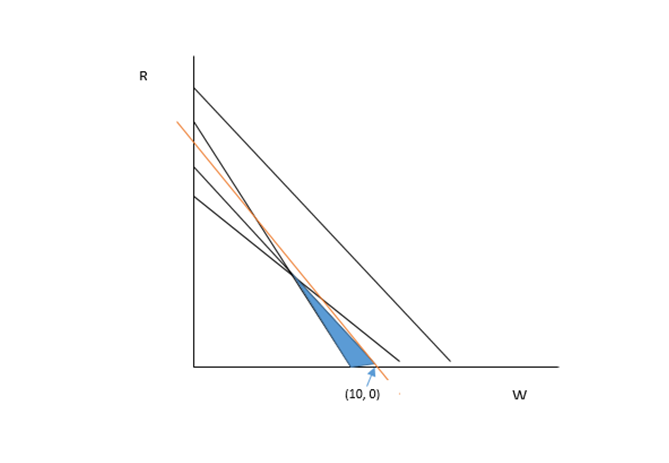
*subject to:*

400W + 200 R ≤ 3200

2W + 3R ≤ 24

*W* + *R* ≤ 20

*W* + *R* ≥ 10



The optimal solution is 10W and 0R.

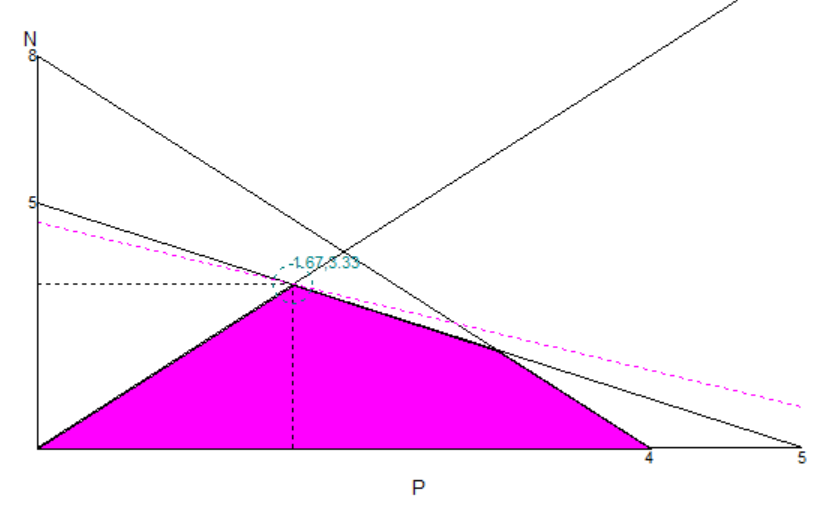
Cognitive Domain: Application

Difficulty Level: Hard

9a.



9b.



The optimal extraction solution is to mine 1.6666 tons of P and 3.3333 tons of N.

Cognitive Domain: Application  
Difficulty Level: Hard

10a.



10b. The optimal plan is to ship 300 from Chicago to Springfield, 600 from Chicago to Madison, and 600 from Omaha to Madison.

Cognitive Domain: Application

Difficulty Level: Hard

11a.



11b. 1.8 units of B and 4.266 units of C make a feed that meets these requirements while costing $41.067 per pound

Cognitive Domain: Application

Difficulty Level: Hard

12.



The optimal production mix from a profit perspective is to make 25 tables and no chairs. If you intend to invite guests for dinner, this may not be the best plan.

Cognitive Domain: Analysis

Difficulty Level: Medium

13. Let B be the number of baseballs produced per week; let C be the number of cricket balls produced per week.

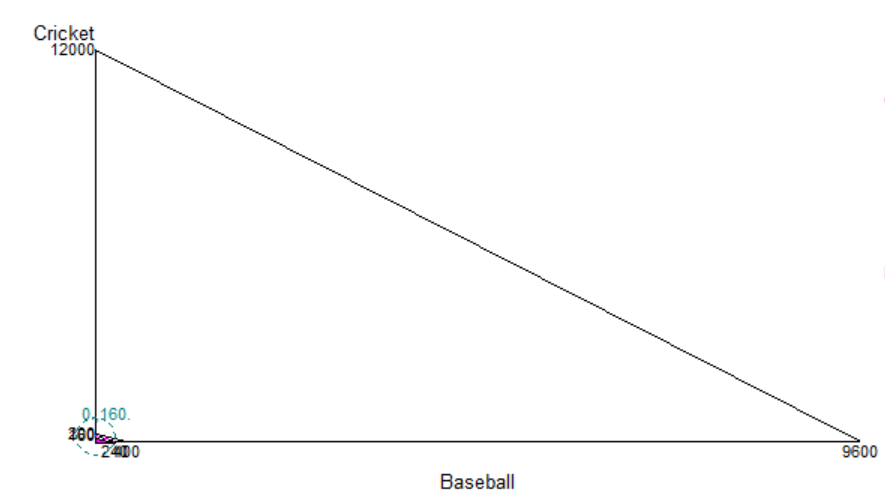
Maximize profit = 8B + 12C subject to

Leather 0.5B + 0.4C <= 4800

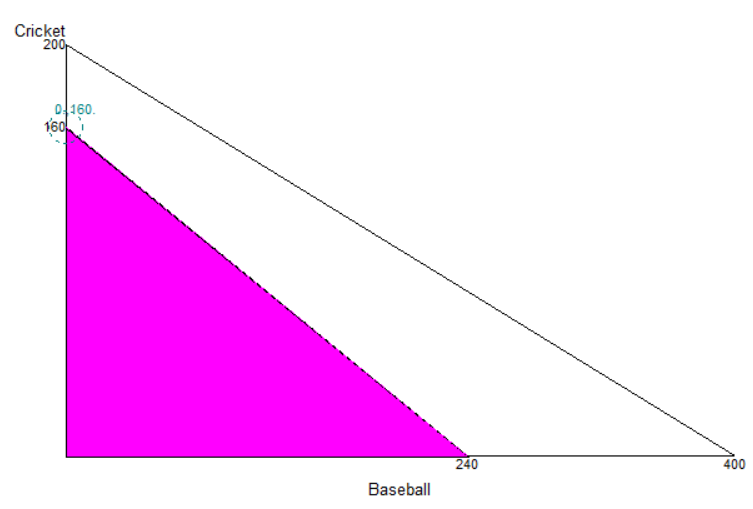
Cutting & dyeing B + 2C <= 400

Sewing & packaging 2B + 3C <= 480

Non-negative B, C >= 0



Producing only 160 cricket balls maximizes profit at $1,920. The leather constraint is nonbinding in a big way. The same graph without the nonbinding leather constraint is as follows:



The cutting and dyeing constraint is also nonbinding, but is less so than the leather constraint.

Cognitive Domain: Analysis

Difficulty Level: Medium

14. This problem cannot be formulated without a per-quart revenue and expenses provided for the mixed juice. Elixer should make only the minimum required amount of mango juice because their per-quart profit is $0. Both the orange and pineapple juice products are profitable.

Let M be the number of quarts of mango juice produced per day

O be the number of quarts of orange juice produced per day

P be the number of quarts of pineapple juice produced per day

F be the number of quarts of mixed fruit juice produced per day

Maximize Profit = 0.08M + 0.095O + 0.21P + 0.72F subject to:

Mango concentrate: 0.3M + 0.25F < = 10,000

Orange concentrate: 0.4O + 0.25F < = 6,000

Pineapple concentrate: 0.45P + 0.25F < = 8,000

Water: 0.7M + 0.6O + 0.55P + 0.25F < = 28,000

Sugar: 2.5M + 1.5 + 2P +F < = 80,000

Capacity: M + P + O + F < = 43,000

Demand: M > = 8,000

O > = 5,000

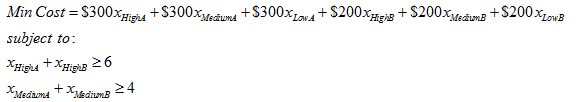
P > = 7,000

F > = 6,000

Cognitive Domain: Analysis

Difficulty Level: Medium

15.



XlowA + XlowB > = 10

XHighA < = 4

XHighB < = 3

XMediumA < = 2

XMediumB < = 2

XLowA < = 5

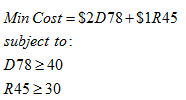
XLowB < = 5

Three tons of XHighA and XHighB, 2 tons of XMediumA, XMediumB, 5 tons of XLowA and XLowB should be extracted each day. The total cost is $5,000.

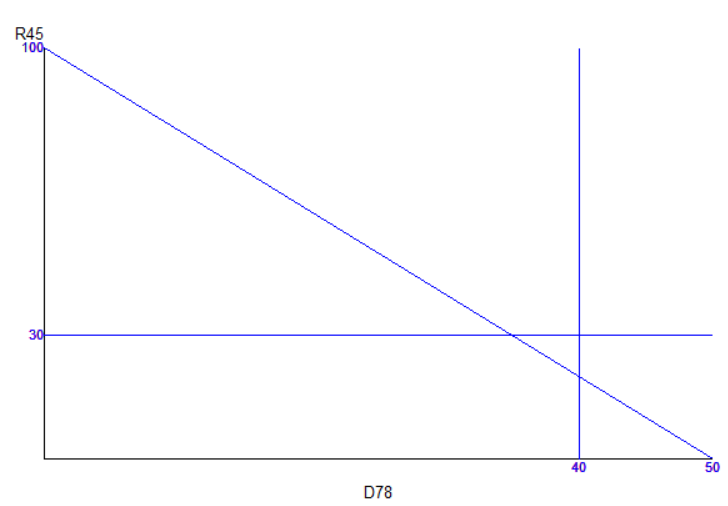
Cognitive Domain: Analysis

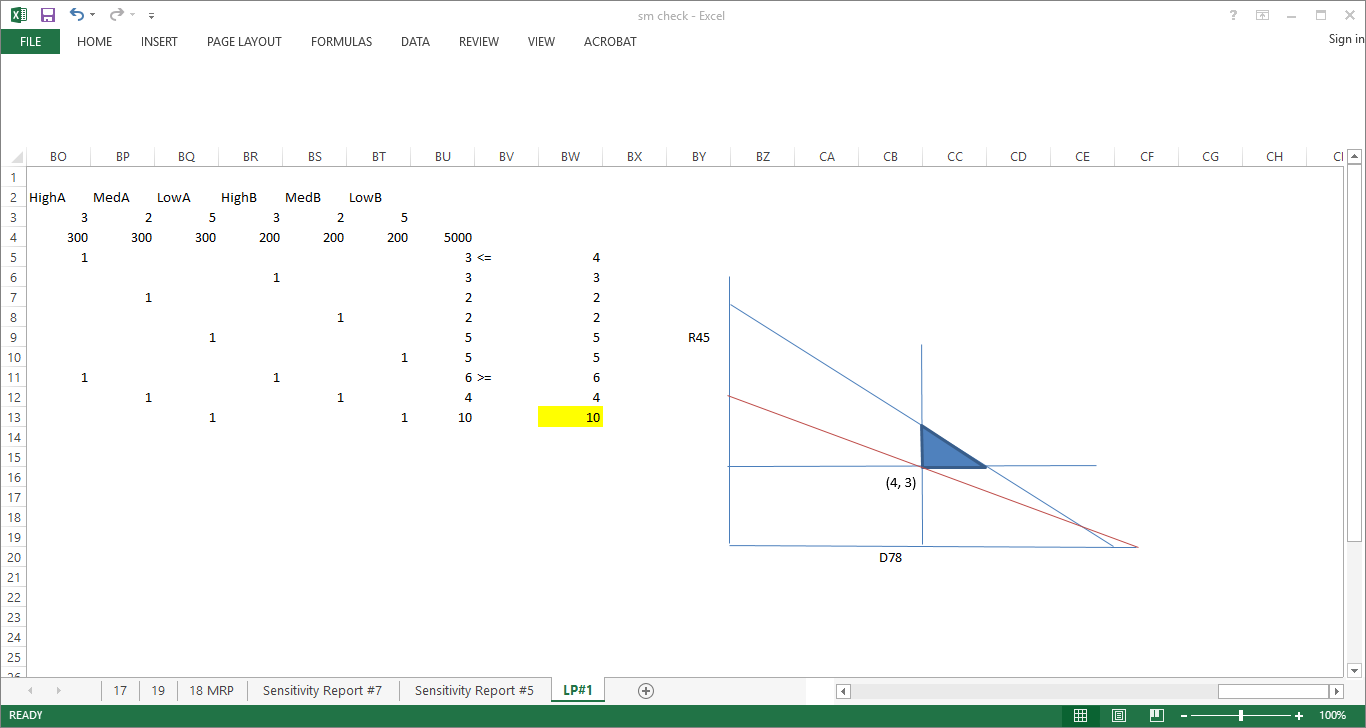
Difficulty Level: Medium

16.



2D78 + R45 <= 120





The optimal solution is 40 gallons of D78 and 30 gallons of R45.

Cognitive Domain: Analysis

Difficulty Level: Medium

17. This objective function and constraint seeks to assign all students to the three high schools in such a way as to minimize total distance traveled.



930

930

930

The optimal solution is to assign 230 students from Erie, 700 from Millcreek to Millcreek High School; 330 students from Erie, 600 from Fairview to Fairview High School; 40 students from Erie, 500 from Harborcreek, 300 from Northeast to Harborcreek High School. The total miles traveled by bus is 3,890.

Cognitive Domain: Analysis

Difficulty Level: Medium

18.



The optimal shipping solution costs $920 and sends 900 gallons from Trichy to Chennai, 300 gallons from Vijayawada to Chennai, 100 gallons from Trichy to Cochin, 500 gallons from Vijayawada to Vizag, and 700 gallons from Mysore to Cochin.

Cognitive Domain: Analysis

Difficulty Level: Medium

19ab.



Invest 5555.6 stocks and 7407.4 in bonds. The annual income generated is 50,000.

19c.



Invest $10,000 in stocks and $4,444.4 in bonds. The annual income generated is $66,000.

Cognitive Domain: Application

Difficulty Level: Hard

20.



The optimal schedule requires 34 workers to start in each of these shift segments:

|  |  |
| --- | --- |
| *Workers Starting* | *Time Interval* |
| 4 | 2 a.m.–6 a.m. |
| 6 | 6 a.m.–10 a.m. |
| 10 | 10 a.m.–2 p.m. |
| 1 | 2 p.m.–6 p.m. |
| 13 | 6 p.m.–10 p.m. |
| 0 | 10 p.m.–2 a.m. |

Cognitive Domain: Application

Difficulty Level: Hard