

Solutions manual

Operations Research: An Introduction

Tenth Edition

Hamdy A. Taha

University of Arkansas/Fayetteville

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Chapter 1

What is Operations Research?

Chapter 1

1

Weeks 1-4: 2 weekend trips FYV-DEN-FYV and 2 weekend trips DEN-FYV-DEN. Week 5: 1 regular trip.

Cost: $4 \times 320 + 1 \times 400 = \2200

2

(a) Given a fence of length L :

$$(1) h = .3L, w = .2L, \text{Area} = .06L^2$$

$$(2) h = .1L, w = .4L, \text{Area} = .04L^2$$

Solution (2) is better because the area is larger

(b) $L = 2(w + h)$, $w = L/2 - h$

$$z = wh = h(L/2 - h) = Lh/2 - h^2$$

$$\delta z / \delta h = L/2 - 2h = 0$$

Thus, $h = L/4$ and $w = L/4$.

Solution is optimal because z is a concave function

3

x = cumulative number of drops of balls #1 and #2 at any floor (problem unknown)

y_i = floor from which i th drop of ball #1 occurs.

Step 0: Set $y_0 = 0$, $y_1 = x$, and $i = 1$.

General step i : Drop ball #1 from floor y_i . If it is dented, use ball #2 to check floors $y_{i-1} + 1$ to $y_i - 1$, in that order. Else, if #1 is not dented, set $i = i + 1$, and repeat step i .

Formula for determining y_i :

y_i must include the (cumulative) i #1-drops from floor y_1 to floor y_i . To maintain the same number of drops at any floor y_i , #2-drops cannot exceed $x - i$.

Thus,

$$y_i = y_{i-1} + (x - i + 1)$$

$$= x + (x-1) + (x-2) + \dots + (x-i+1)$$

$$= ix - (1 + 2 + \dots + i-1) = ix - (i-1)i/2$$

Maximum number of #1-drops is x (else $y_i \leq y_{i-1}$ for $i > x$). Hence the highest floor from which #1 can be dropped is

$$y_x = x^2 - (x-1)x/2 = x^2 - x^2/2 + x/2 = (x^2 + x)/2$$

For a 100-storey building, $y_x \geq 100$, or $x^2 + x - 200 \geq 0$. The associated quadratic equation yields $x = 13.64$ and -14.64 . The rounded positive value $x = 14$ is the smallest integer that satisfies the inequality.

4

(a) Let T = total time to move all four individuals to the other side of the river. The objective is to determine the transfer schedule that minimizes T .

(b) Let t = crossing time from one side to the other. Use codes 1, 2, 5, and 10 to represent Amy, Jim, John, and Kelly.

East	Crossing	West
5,10	$(1,2) \rightarrow (t=2)$	1,2
1,5,10	$(t=1) \leftarrow (1)$	2
1	$(5,10) \rightarrow (t=10)$	2,5,10
1,2	$(t=2) \leftarrow (2)$	5,10
none	$(1,2) \rightarrow (t=2)$	1,2,5,10
Total = $2 + 1 + 10 + 2 + 2 = 17$ minutes		

5

		Jim	
Joe	Curve	.500	.200
	Fast	.100	.300

(a) Alternatives:

Jim: Throw curve or fast ball.

Joe: Prepare for curve or fast ball.

(b) Joe tries to improve his batting score and Jim tries to counter Joe's action by selecting a less favorable strategy. This means that neither player will be satisfied with a single (pure) strategy.

The problem is not an optimization situation in the familiar sense in which the objective is maximized or minimized. Instead, the conflicting situation requires a compromise solution in which neither player is tempted to change strategy. Game theory (Chapter 14) provides such a solution.

6

\L L Let L=ops. 1 and 2=20 sec, C=ops. 3 and 4=25 sec, U=op. 5=20 sec

Gant chart: L1=load horse 1, L2=load horse 2, etc.

one joist: 0---L1---20---C1---45---U1+L1---85---U2+L2---125---U1+L1---165---U2+L2---205

20-L2-40 45---C2---70 85---C1---110 125---C2---140

165-C1-190

205---C2---230---U2---250

Total = 250

Loaders utilization=[$250-(5+25)$]/250=88%

Cutter utilization=[$250-(20+15+15+15+15)$]/250=68%

two joists: 0---2L1---40---2C1---90---2(U1+L1)---170---2C1---220---2U1--
-260

40---2L2---80 90---2C2---140 170---2U2---210

Total =260

Loaders utilization=[$260-(10+10)$]/260=92%

Cutter utilization=[$260-(40+30+40)$]/250=58%

three joists: 0---3L1---60---3C1---135---3C2---210---3U2---270

60---3L2---120 135---3U1---195

Total =270

Loaders utilization=[$270-(15+15)$]/270=89%

Cutter utilization=[$270-(60+60)$]/270=56%

Recommendation: One joist at time gives the smallest time. The problem has other alternatives that combine 1, 2, and 3 joists. Cutter utilization indicates that cutter represents the bottleneck.

7

Note that all 'dots' are indistinguishable even if they are designated as 1, 2, 3, ..., 10.

- (a) Alternative 1: Move dots 5, 6, and 7 below bottom row, move dots 8 and 9 below new 5, 6, and 7. Move 10 to the bottom. Number of moves = 6. Alternative 2: See part (b).
- (b) Three moves: Move dot 1 up to the left of dot 8, dot 4 to the right of dot 9, and dot 10 below dots 2 and 3.

```

      10
     8  9
    5  6  7
   1  2  3  4

```

Chapter 1

8

- (a) Alternative 1: Break one end link of each chain and connect to another chain. Four breaks and re-solders, cost = $4 \times (2 + 3) = 20$ cents. Alternative 2: See Part (b)
- (b) Break three links in one chain and use them to connect the remaining three chains: Three breaks and re-solder, cost = $3 \times (2 + 3) = 15$ cents.

9

Represent the selected 2-digit number as $10x+y$. The corresponding square number is $10x+y-(x+y)=9x$. This means that the selected square will always be 9, 18, 27, ..., or 81. By assigning zero dollars to these squares, the reward is always zero regardless of the rewards assigned to the remaining squares or the number of times the game is repeated.

10

Assign a sequential number x to each cartons, $x \in X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

- (a) Let Y be the set of cartons so far weighed (initially $Y = \emptyset$).

General Step: Randomly select a carton $y \in X - Y$. If y weighs 90 oz, stop. Else, augment y to Y and repeat the General Step. $1 \leq \text{number of times scale is used} \leq 10$.

- (b) Exactly once! Take x bottles from carton $x \in X$ to end up with $(1+2+\dots+10) = (10+1)/2 = 55$ bottles. Weigh the 55 bottles. If the weight = $550 - x$, carton x is the defective one.

CHAPTER 2

Modeling with Linear Programming

Chapter 2

- (a) $x_2 - x_1 \geq 1$ or $-x_1 + x_2 \geq 1$
 (b) $x_1 + 2x_2 \geq 3$ and $x_1 + 2x_2 \leq 6$
 (c) $x_2 \geq x_1$ or $x_1 - x_2 \leq 0$
 (d) $x_1 + x_2 \geq 3$
 (e) $\frac{x_2}{x_1 + x_2} \leq .5$ or $.5x_1 - .5x_2 \geq 0$

1

Quantity discount results in the following nonlinear objective function:

4

$$Z = \begin{cases} 5x_1 + 4x_2, & 0 \leq x_1 \leq 2 \\ 4.5x_1 + 4x_2, & x_1 > 2 \end{cases}$$

The situation cannot be treated as a linear program. Nonlinearity can be accounted for in this case using mixed integer programming (chapter 9).

(a) $(x_1, x_2) = (1, 4)$
 $(x_1, x_2) \geq 0$
 $6x_1 + 4x_2 = 22 < 24$
 $1x_1 + 2x_2 = 9 \neq 6$ infeasible

2

(b) $(x_1, x_2) = (2, 2)$
 $(x_1, x_2) \geq 0$
 $6x_1 + 4x_2 = 20 < 24$
 $1x_1 + 2x_2 = 6 = 6$
 $-1x_1 + 1x_2 = 0 < 1$
 $1x_2 = 2 = 2$ } feasible

$$Z = 5x_2 + 4x_2 = \$18$$

(c) $(x_1, x_2) = (3, 1.5)$
 $x_1, x_2 \geq 0$
 $6x_1 + 4x_2 = 24 = 24$
 $1x_1 + 2x_2 = 6 = 6$
 $-1x_1 + 1x_2 = -1.5 < 1$
 $1x_2 = 1.5 < 2$ } feasible

$$Z = 5x_3 + 4x_{1.5} = \$21$$

(d) $(x_1, x_2) = (2, 1)$
 $x_1, x_2 \geq 0$
 $6x_2 + 4x_1 = 16 < 24$
 $1x_2 + 2x_1 = 4 < 6$
 $-1x_2 + 1x_1 = -1 < 1$
 $1x_1 = 1 < 2$ } feasible

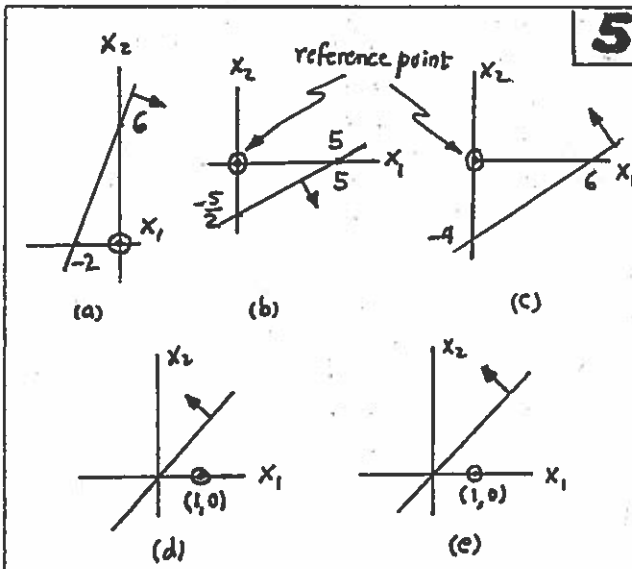
$$Z = 5x_2 + 4x_1 = \$14$$

(e) $(x_1, x_2) = (2, -1)$
 $x_1 \geq 0, x_2 < 0$, infeasible

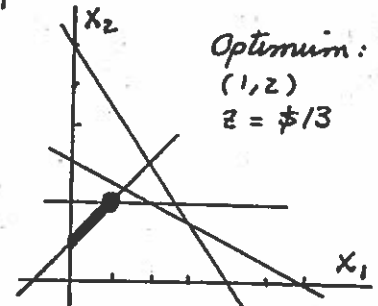
Conclusion: (c) gives the best feasible solution

3

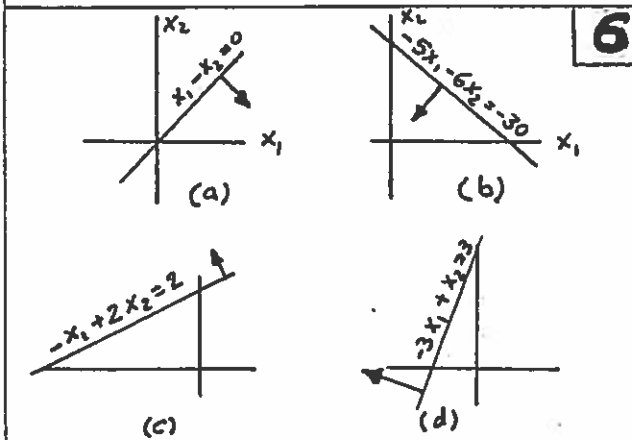
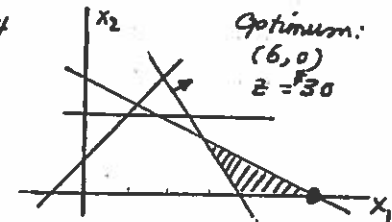
$(x_1, x_2) = (2, 2)$
 Let S_1 and S_2 be the unused daily amounts of M1 and M2.
 For M1: $S_1 = 24 - (6x_1 + 4x_2) = 4$ tons/day
 For M2: $S_2 = 6 - (x_1 + 2x_2)$
 $= 6 - (2 + 2 \times 2) = 0$ tons/day



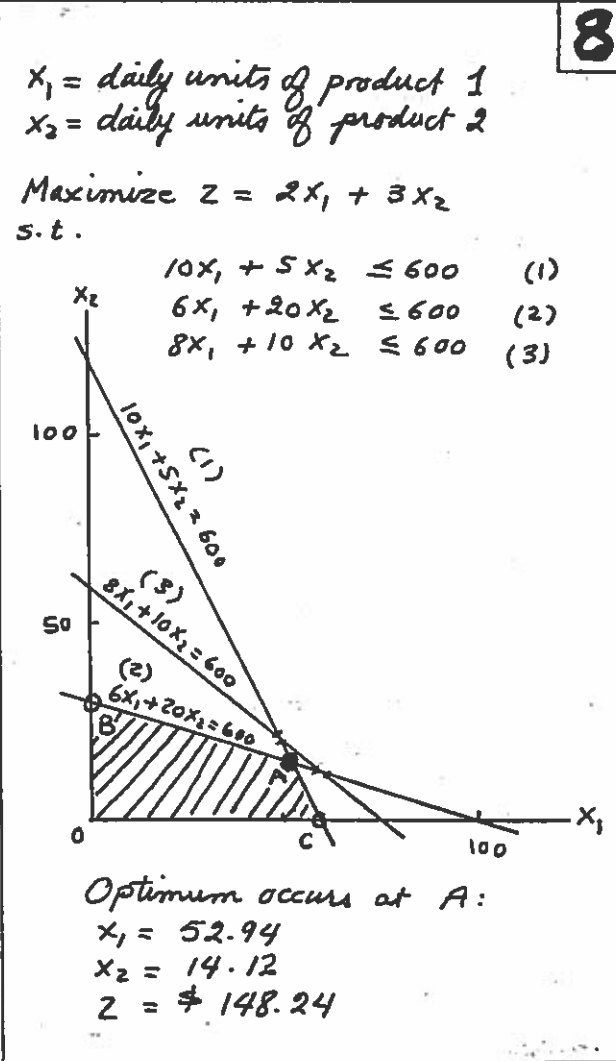
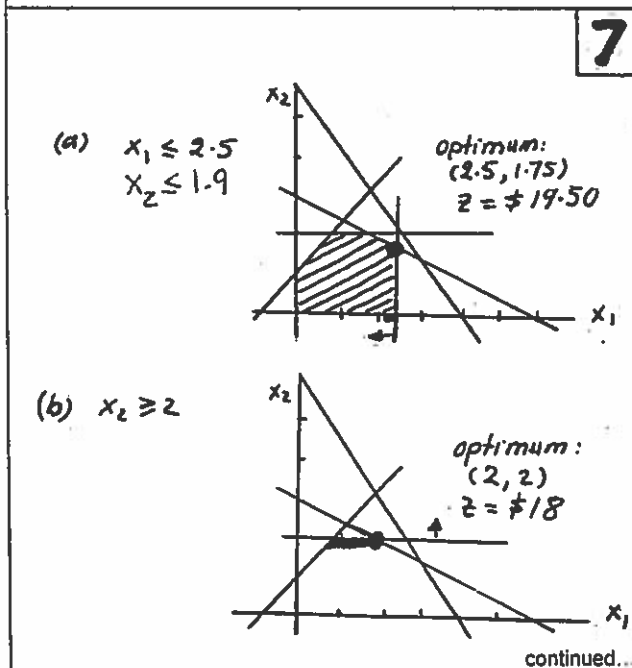
(c) $-x_1 + x_2 = 1$



(d) $6x_1 + 4x_2 \geq 24$



(e) No feasible space



Chapter 2

x_1 = number of units of A
 x_2 = number of units of B

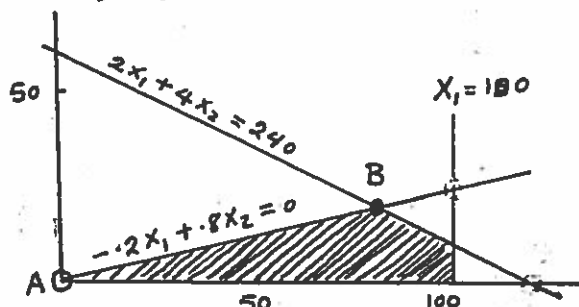
Maximize $Z = 20x_1 + 50x_2$

$$\frac{x_1}{x_1 + x_2} \geq .8 \quad \text{or} \quad -.2x_1 + .8x_2 \leq 0$$

$$x_1 \leq 100$$

$$2x_1 + 4x_2 \leq 240$$

$$x_1, x_2 \geq 0$$



Optimal occurs at B:

$$x_1 = 80 \text{ units}$$

$$x_2 = 20 \text{ units}$$

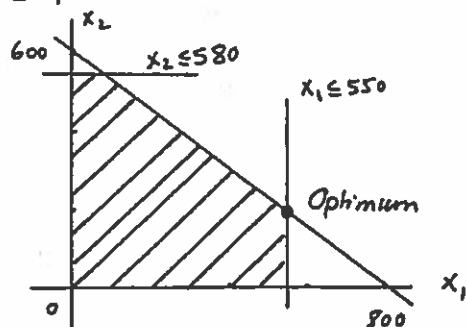
$$Z = \$2,600$$

x_1 = number of sheets/day
 x_2 = number of bars/day

Maximize $Z = 40x_1 + 35x_2$

$$\text{s.t.} \quad \frac{x_1}{800} + \frac{x_2}{600} \leq 1$$

$$0 \leq x_1 \leq 550, \quad 0 \leq x_2 \leq 580$$



Optimum solution:

$$x_1 = 550 \text{ sheets}$$

$$x_2 = 187.13 \text{ bars}$$

$$Z = \$28,549.40$$

9

x_1 = \$ invested in A

x_2 = \$ invested in B

Maximize $Z = .05x_1 + .08x_2$

s.t.

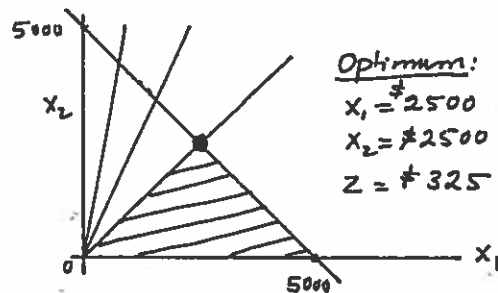
$$x_1 \geq .25(x_1 + x_2)$$

$$x_2 \leq .5(x_1 + x_2)$$

$$x_1 \geq .5x_2$$

$$x_1 + x_2 \leq 5000$$

$$x_1, x_2 \geq 0$$



Optimum:

$$x_1 = \$2500$$

$$x_2 = \$2500$$

$$Z = \$325$$

x_1 = number of practical courses

x_2 = number of humanistic courses

Maximize $Z = 1500x_1 + 1000x_2$

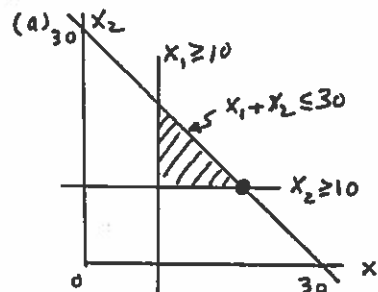
s.t.

$$x_1 + x_2 \leq 30$$

$$x_1 \geq 10$$

$$x_2 \geq 10$$

$$x_1, x_2 \geq 0$$



Optimum:

$$x_1 = 20$$

$$x_2 = 10$$

$$Z = \$40,000$$

(b) Change $x_1 + x_2 \leq 30$ to $x_1 + x_2 \leq 31$

$$\text{Optimum } Z = \$41,500$$

$$\Delta Z = \$41,500 - 40,000 = \$1,500$$

Conclusion: Any additional course will be of the practical type.

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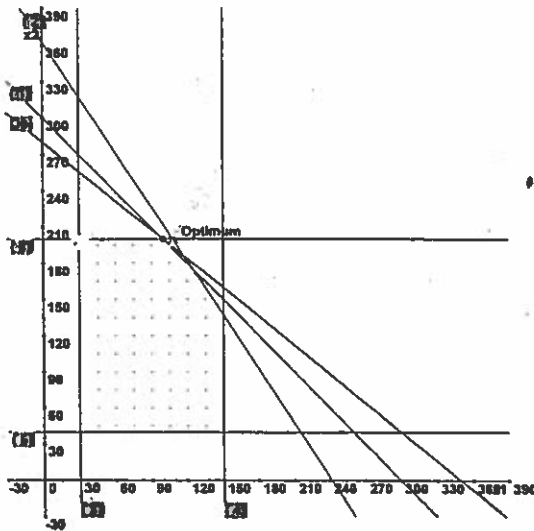
12

x_1 = units of solution A
 x_2 = units of solution B

Maximize $Z = 8x_1 + 10x_2$
 Subject to

$$\begin{aligned} .5x_1 + .5x_2 &\leq 150 \\ .6x_1 + .4x_2 &\leq 145 \\ x_1 &\geq 30 \\ x_1 &\leq 150 \\ x_2 &\geq 40 \\ x_2 &\leq 200 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Summary of Optimal Solution:
 Objective Value = 2800.00
 $x_1 = 100.00$
 $x_2 = 200.00$



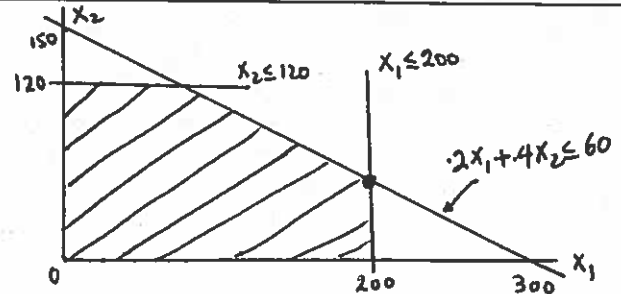
x_1 = nbr. of grano boxes
 x_2 = nbr. of wheatie boxes

Maximize $Z = x_1 + 1.35x_2$

s.t.

$$\begin{aligned} .2x_1 + .4x_2 &\leq 60 \\ x_1 &\leq 200 \\ x_2 &\leq 120 \\ x_1, x_2 &\geq 0 \end{aligned}$$

13



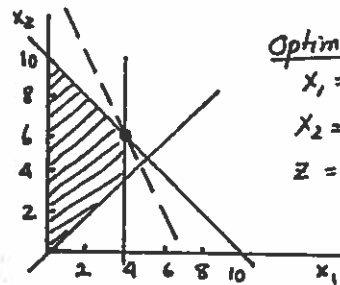
Optimum: $x_1 = 200, x_2 = 50, Z = \267.50
 Area allocation: 67% grano, 33% wheatie

x_1 = play hours per day
 x_2 = work hours per day

Maximize $Z = 2x_1 + x_2$
 s.t.

$$\begin{aligned} x_1 + x_2 &\leq 10 \\ x_1 - x_2 &\leq 0 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Optimum solution:
 $x_1 = 4$ hours
 $x_2 = 6$ hours
 $Z = 14$ "pleasurits"

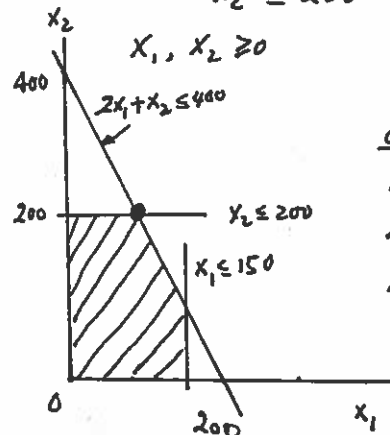


15

x_1 = Daily nbr. of type 1 rat
 x_2 = Daily nbr. of type 2 rat

Maximize $Z = 8x_1 + 5x_2$
 s.t.

$$\begin{aligned} 2x_1 + x_2 &\leq 400 \\ x_1 &\leq 150 \\ x_2 &\leq 200 \\ x_1, x_2 &\geq 0 \end{aligned}$$



Optimum:
 $x_1 = 100$ type 1
 $x_2 = 200$ type 2
 $Z = \$1800$

16

14

continued...

continued...

Chapter 2

x_1 = radio minutes

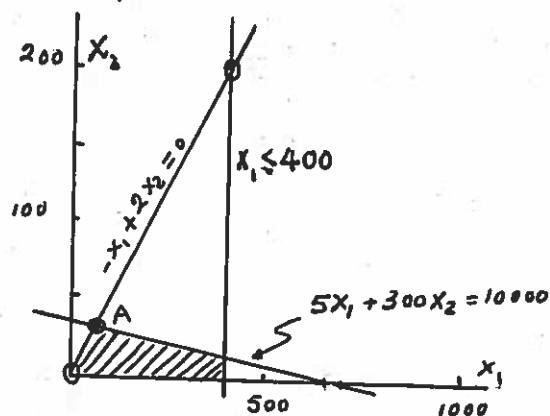
x_2 = TV minutes

Maximize $Z = x_1 + 25x_2$

s.t. $15x_1 + 300x_2 \leq 10,000$

$\frac{x_1}{x_2} \geq 2$ or $-x_1 + 2x_2 \leq 0$

$x_1 \leq 400, x_1, x_2 \geq 0$



Optimum occurs at A:

$x_1 = 60.61$ minutes

$x_2 = 30.3$ minutes

$Z = 818.18$

x_1 = tons of C_1 consumed per hour

x_2 = tons of C_2 consumed per hour

Maximize $Z = 12000x_1 + 9000x_2$

s.t.

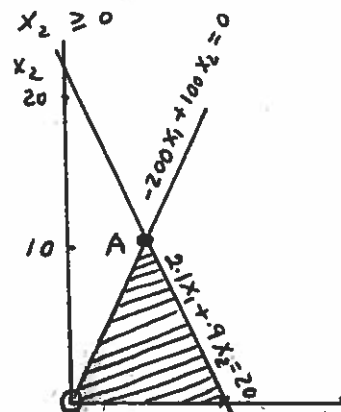
$1800x_1 + 2100x_2 \leq 2000(x_1 + x_2)$

or

$-200x_1 + 100x_2 \leq 0$

$2.1x_1 + .9x_2 \leq 20$

$x_1, x_2 \geq 0$



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(a) Optimum occurs at A:

$x_1 = 5.128$ tons per hour

$x_2 = 10.256$ tons per hour

$Z = 153,846$ lb of steam

Optimal ratio = $\frac{5.128}{10.256} = .5$

(b) $2.1x_1 + .9x_2 \leq (20+1) = 21$

Optimum $Z = 161,538$ lb of steam

$\Delta Z = 161,538 - 153,846 = 7,692$ lb

x_1 = Nbr. of radio commercials beyond the first

x_2 = Nbr. of TV ads beyond the first

Maximize $Z = 2000x_1 + 3000x_2 + 5000 + 2000$

s.t. $300(x_1+1) + 2000(x_2+1) \leq 20,000$

$300(x_1+1) \leq .8 \times 20,000$

$2000(x_2+1) \leq .8 \times 20,000$

$x_1, x_2 \geq 0$

or

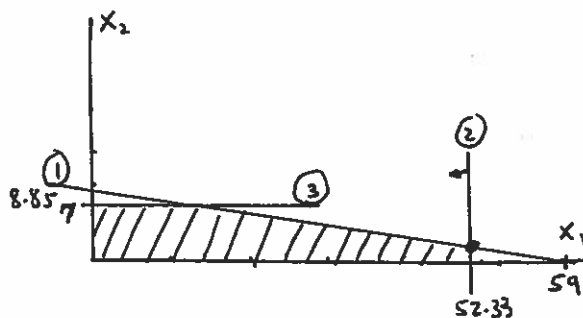
Maximize $Z = 2000x_1 + 3000x_2 + 7000$

s.t. $300x_1 + 2000x_2 \leq 17700$ (1)

$300x_1 \leq 15700$ (2)

$2000x_2 \leq 14000$ (3)

$x_1, x_2 \geq 0$



Optimum solution:

Radio commercials = $52.33 + 1 = 53.33$

TV ads = $1 + 1 = 2$

$Z = 107666.67 + 7000 = 114666.67$

18

x_1 = number of shirts per hour
 x_2 = number of blouses per hour

Maximize $Z = 8x_1 + 12x_2$

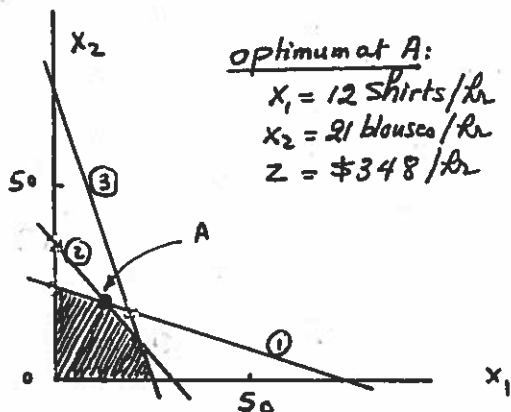
s.t.

$$20x_1 + 60x_2 \leq 25 \times 60 = 1500 \quad (1)$$

$$70x_1 + 60x_2 \leq 35 \times 60 = 2100 \quad (2)$$

$$12x_1 + 4x_2 \leq 5 \times 60 = 300 \quad (3)$$

$$x_1, x_2 \geq 0$$



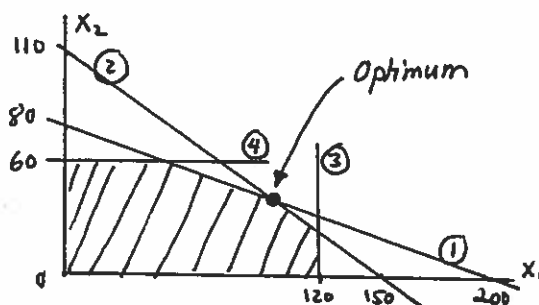
x_1 = Nbr. of desks per day
 x_2 = Nbr. of chairs per day

Maximize $Z = 50x_1 + 100x_2$

$$\frac{x_1}{200} + \frac{x_2}{80} \leq 1 \quad (1)$$

$$\frac{x_1}{150} + \frac{x_2}{110} \leq 1 \quad (2)$$

$$x_1 \leq 120, x_2 \leq 60 \quad (3,4)$$



Optimum:

$$x_1 = 90 \text{ desks}$$

$$x_2 = 44 \text{ chairs}$$

$$Z = \$8900$$

20

x_1 = number of Hi-Fi 1 units
 x_2 = number of Hi-Fi 2 units

Constraints:

$$6x_1 + 4x_2 \leq 480 \times 9 = 432$$

$$5x_1 + 5x_2 \leq 480 \times 86 = 412.8$$

$$4x_1 + 6x_2 \leq 480 \times 88 = 422.4$$

or

$$6x_1 + 4x_2 + s_1 = 432$$

$$5x_1 + 5x_2 + s_2 = 412.8$$

$$4x_1 + 6x_2 + s_3 = 422.4$$

Objective function:

$$\text{Minimize } s_1 + s_2 + s_3 = 1267.2 - 15x_1 - 15x_2$$

$$\text{Thus, } \min s_1 + s_2 + s_3 \equiv \max 15x_1 + 15x_2$$

Maximize $Z = 15x_1 + 15x_2$

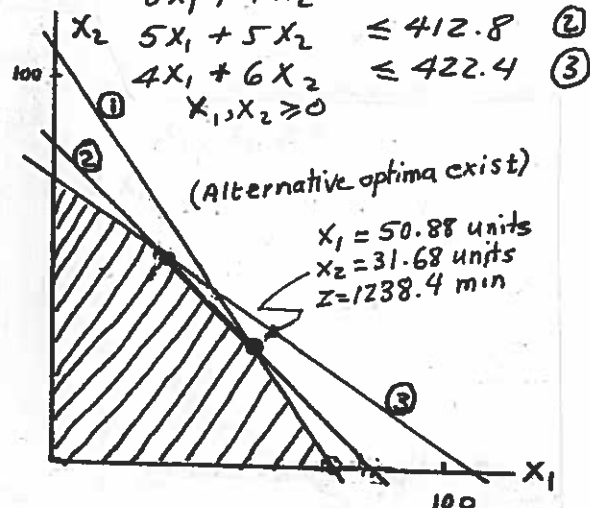
s.t.

$$6x_1 + 4x_2 \leq 432 \quad (1)$$

$$5x_1 + 5x_2 \leq 412.8 \quad (2)$$

$$4x_1 + 6x_2 \leq 422.4 \quad (3)$$

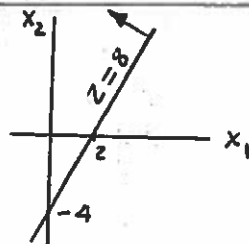
$$x_1, x_2 \geq 0$$



23

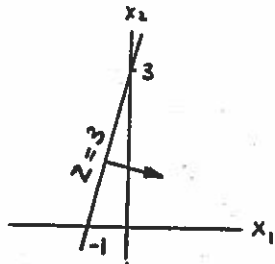
Corner point	(x_1, x_2)	z
A	(0,0)	0
B	(4,0)	20
C	(3,1.5)	21 (OPTIMUM)
D	(2,2)	18
E	(1,2)	13
F	(0,1)	4

(a)

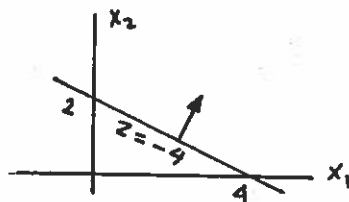


28

(b)

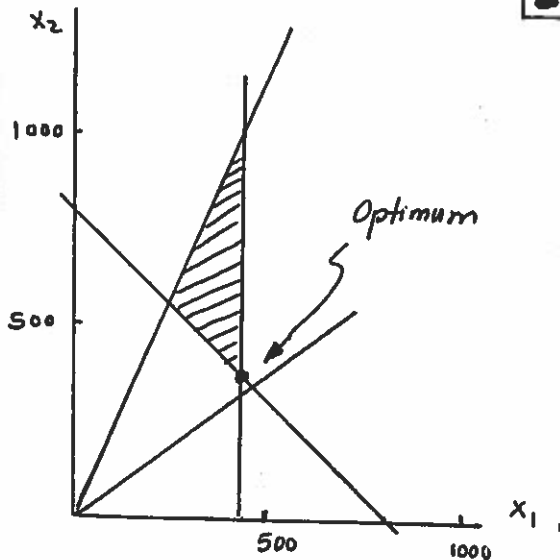


(c)



Additional constraint: $x_1 \leq 450$

29



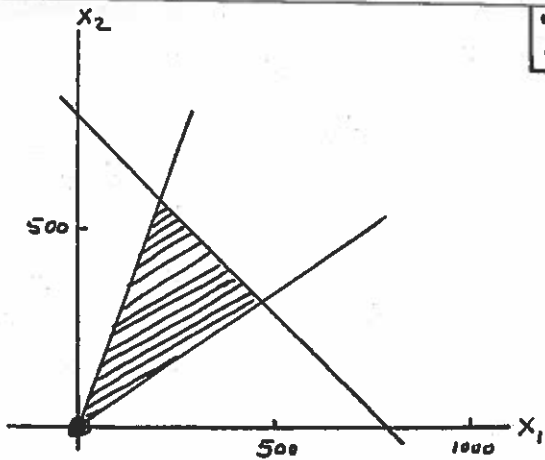
Optimum Solution:

$$x_1 = 450 \text{ lb}$$

$$x_2 = 350 \text{ lb}$$

$$Z = \$450$$

continued...



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Optimum: $x_1 = 0, x_2 = 0, Z = 0$, which is nonsensical

31

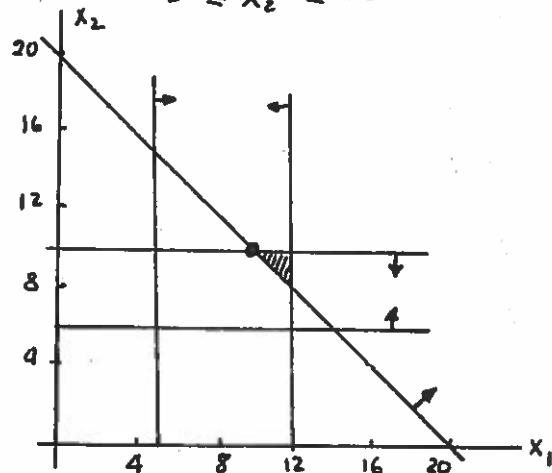
x_1 = number of hours/week in store 1
 x_2 = number of hours/week in store 2

Minimize $Z = 8x_1 + 6x_2$
 s.t.

$$x_1 + x_2 \geq 20$$

$$5 \leq x_1 \leq 12$$

$$6 \leq x_2 \leq 10$$



Optimum:

$$x_1 = 10 \text{ hours}$$

$$x_2 = 10 \text{ hours}$$

$$Z = 140 \text{ stress index}$$

continued...

32

Let

$$x_1 = 10^3 \text{ bbl/day from Iran}$$

$$x_2 = 10^3 \text{ bbl/day from Dubai}$$

$$\text{Refinery capacity} = x_1 + x_2 \leq 10^3 \text{ bbl/day}$$

$$\text{Minimize } Z = x_1 + x_2$$

Subject to

$$x_1 \geq .4(x_1 + x_2)$$

$$\text{or } -.6x_1 + .4x_2 \leq 0$$

$$.2x_1 + .1x_2 \geq 14$$

$$.25x_1 + .6x_2 \geq 30$$

$$.1x_1 + .15x_2 \geq 10$$

$$.15x_1 + .1x_2 \geq 8$$

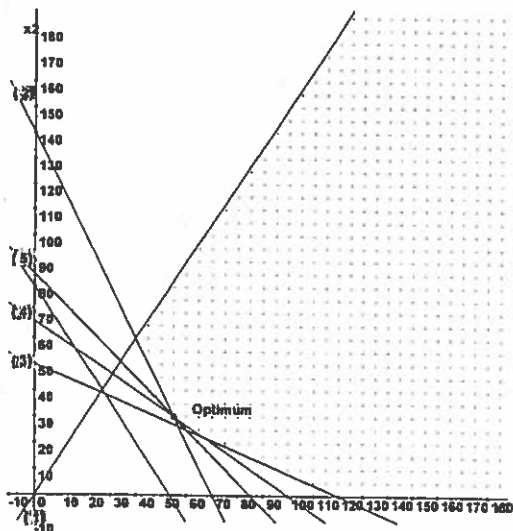
$$x_1, x_2 \geq 0$$

Optimum solution from TORA:

LINEAR PROGRAMMING - GRAPHICAL SOLUTION

Title: diet problem

Summary of Optimal Solution:
Objective Value = 53.00
x1 = 53.00
x2 = 30.00



33

Let

$$x_1 = 10^3 \$ \text{ invested in blue chip stock}$$

$$x_2 = 10^3 \$ \text{ invested in high-tech stocks}$$

$$\text{Minimize } Z = x_1 + x_2$$

Subject to

$$.1x_1 + .25x_2 \geq 10$$

$$.6x_1 - .4x_2 \geq 0$$

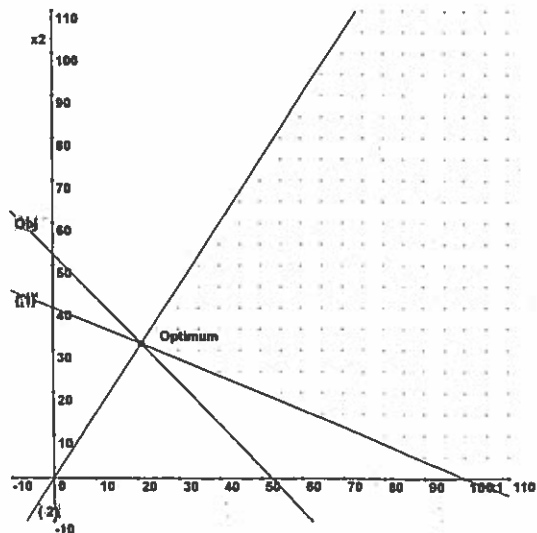
$$x_1, x_2 \geq 0$$

TORA optimum solution:

LINEAR PROGRAMMING - GRAPHICAL SOLUTION

Title: diet problem

Summary of Optimal Solution:
Objective Value = 52.63
x1 = 21.09
x2 = 31.58



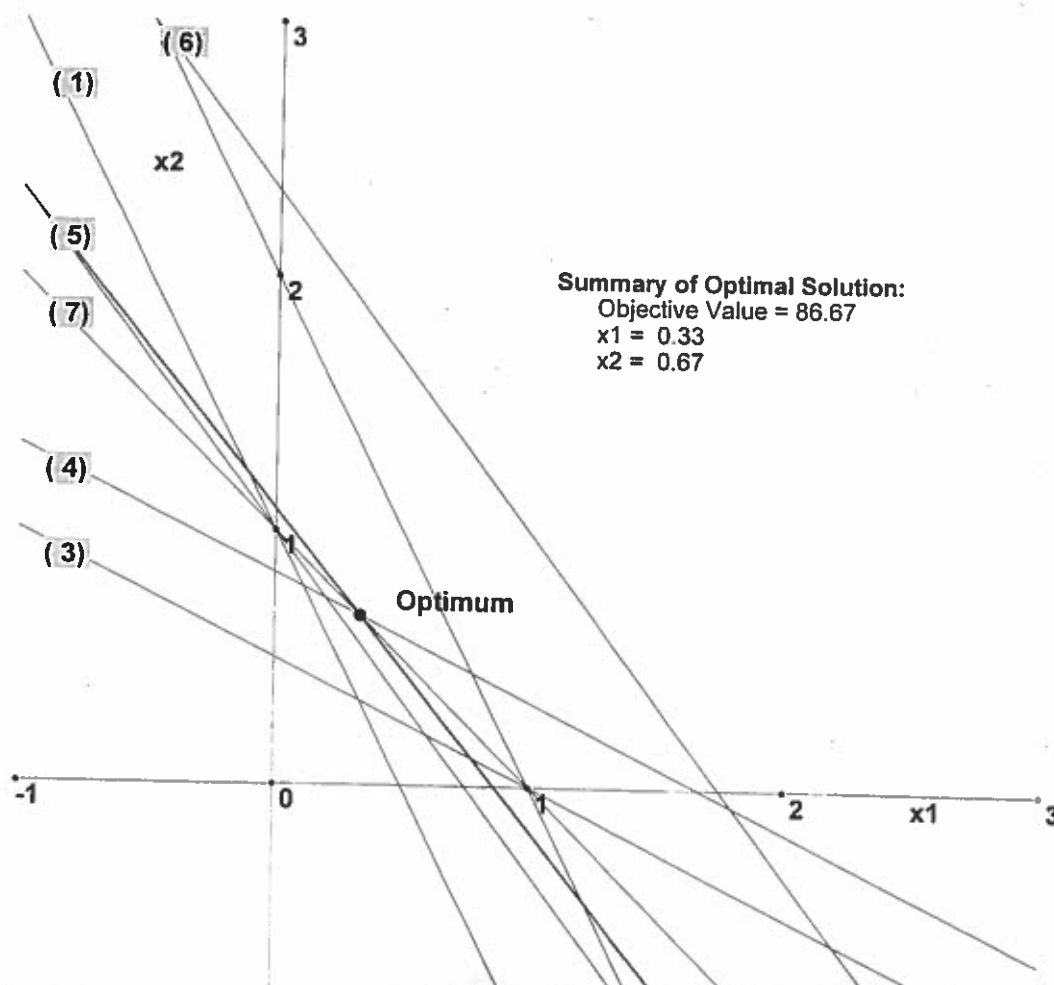
Chapter 2

34

x_1 = Ratio of scrap A in alloy

x_2 = Ratio of scrap B in alloy

	x_1	x_2		
Minimize	100.00	80.00		
Subject to				
(1)	0.06	0.03	$\geq$	0.03
(2)	0.06	0.03	$\leq$	0.06
(3)	0.03	0.06	$\geq$	0.03
(4)	0.03	0.06	$\leq$	0.05
(5)	0.04	0.03	$\geq$	0.03
(6)	0.04	0.03	$\leq$	0.07
(7)	1.00	1.00	$=$	1.00



2-10

(a) x_i = Undertaken portion of project i 40

Maximize

$$Z = 32.4x_1 + 35.8x_2 + 17.75x_3 + 14.8x_4 + 18.2x_5 + 12.35x_6$$

Subject to

$$10.5x_1 + 8.3x_2 + 10.2x_3 + 7.2x_4 + 12.3x_5 + 9.2x_6 \leq 60$$

$$14.4x_1 + 12.6x_2 + 14.2x_3 + 10.5x_4 + 10.1x_5 + 7.8x_6 \leq 70$$

$$2.2x_1 + 9.5x_2 + 5.6x_3 + 7.5x_4 + 8.3x_5 + 6.9x_6 \leq 35$$

$$2.4x_1 + 3.1x_2 + 9.2x_3 + 5.0x_4 + 6.3x_5 + 5.1x_6 \leq 20$$

$$0 \leq x_j \leq 1, j = 1, 2, \dots, 6$$

TORA optimum solution:

$$x_1 = x_2 = x_3 = x_4 = 1, x_5 = .84, x_6 = 0, Z = 116.06$$

(b) Add the constraint $x_2 \leq x_6$

TORA optimum solution:

$$x_1 = x_2 = x_3 = x_4 = x_6 = 1, x_5 = .03, Z = 113.68$$

(c) Let S_i be the unused funds at the end of year i and change the right-hand sides of constraints 2, 3, and 4 to $70 + S_1$, $35 + S_2$, and $20 + S_3$, respectively.

TORA optimum solution:

$$x_1 = x_2 = x_3 = x_4 = x_5 = 1, x_6 = .71$$

$$Z = \$127.72 \text{ (thousand)}$$

The solution is interpreted as follows:

i	S_i	$S_i - S_{i-1}$	Decision
1	4.96	—	—
2	7.62	+2.66	Don't borrow from yr 1
3	4.62	-3.00	Borrow \$3 from year 2
4	0	-4.62	Borrow \$4.62 from yr 2

The effect of availing excess money for use in later years is that the first five projects are completed and 71% of project 6 is undertaken.

The total revenue increases from \$116,060 to 127,720.

continued...

(d) The slack S_i in period i is treated as an unrestricted variable.

TORA optimum solution: $Z = \$131.30$

$$S_1 = 2.3, S_2 = -.4, S_3 = -.5, S_4 = -6.1$$

This means that additional funds are needed in years 3 and 4.

$$\text{Increase in return} = 131.30 - 116.06 = \$15.24$$

Ignoring the time value of money, the amount borrowed $5 + 6.1 - (2.3 + .4) = \$8.4$. Thus,

$$\text{rate of return} = \frac{15.24 - 8.4}{8.4} \approx 81\%$$

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x_i = dollar investment in project i , $i = 1, 2, 3, 4$

y_j = dollar investment in bank in year j , $j = 1, 2, 3, 4, 5$

Maximize $Z = y_5$

Subject to

$$x_1 + x_2 + x_4 + y_1 \leq 10,000$$

$$.5x_1 + .6x_2 - x_3 + .4x_4 + 1.065y_1 - y_2 = 0$$

$$.3x_1 + .2x_2 + .8x_3 + .6x_4 + 1.065y_2 - y_3 = 0$$

$$1.8x_1 + 1.5x_2 + 1.9x_3 + 1.8x_4 + 1.065y_3 - y_4 = 0$$

$$1.2x_1 + 1.3x_2 + .8x_3 + .95x_4 + 1.065y_4 - y_5 = 0$$

All variables ≥ 0

TORA optimal solution:

$$x_1 = 0, x_2 = \$10,000, x_3 = \$6,000, x_4 = 0$$

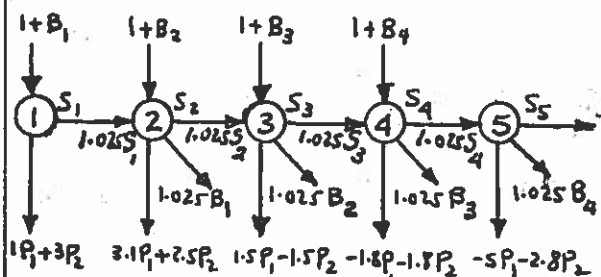
$$y_1 = 0, y_2 = 0, y_3 = \$6,800, y_4 = \$33,642$$

$$Z = \$53,628.73 \text{ at the start of year 5}$$

P_i = fraction undertaken of project i , $i = 1, 2$

B_j = million dollars borrowed in quarter j , $j = 1, 2, 3, 4$

S_j = surplus million dollars at the start of quarter j , $j = 1, 2, 3, 4, 5$



(a) Maximize $Z = S_5$

subject to

$$\begin{aligned} P_1 + 3P_2 + S_1 - B_1 &= 1 \\ 3.1P_1 + 2.5P_2 - 1.025S_1 + S_2 + 1.025B_1 - B_2 &= 1 \\ 1.5P_1 - 1.5P_2 - 1.025S_2 + S_3 + 1.025B_2 - B_3 &= 1 \\ -1.8P_1 - 1.8P_2 - 1.025S_3 + S_4 + 1.025B_3 - B_4 &= 1 \\ -5P_1 - 2.8P_2 - 1.025S_4 + S_5 + 1.025B_4 &= 1 \\ 0 \leq P_i \leq 1, \quad 0 \leq P_2 \leq 1 \\ 0 \leq B_j \leq 1, \quad j = 1, 2, 3, 4 \end{aligned}$$

Optimum Solution:

$$P_1 = .7113 \quad P_2 = 0$$

$$Z = 5.8366 \text{ million dollars}$$

$$B_1 = 0, \quad B_2 = .9104 \text{ million dollars}$$

$$B_3 = 1 \text{ million dollars}, \quad B_4 = 0$$

(b) $B_1 = 0, \quad S_1 = .2887 \text{ million \$}$

$$B_2 = .9104, \quad S_2 = 0$$

$$B_3 = 1, \quad S_3 = 0$$

$$B_4 = 0, \quad S_4 = 1.2553$$

The solution shows that $B_i S_i = 0$, meaning that you can't borrow and also end up with surplus in any quarter. The result makes sense because the cost of borrowing (2.5%) is higher than the return on surplus funds (2%).

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Assume that the investment program ends at the start of year 11. Thus, the 6-year bond option can be exercised in years 1, 2, 3, 4, and 5 only. Similarly, the 9-year bond can be used in years 1 and 2 only. Hence, from year 6 on, the only option available is insured savings at 7.5%.

Let

I_i = insured savings investments in year i , $i = 1, 2, \dots, 10$

G_i = 6-year bond investment in year i , $i = 1, 2, \dots, 5$

M_i = 9-year bond investment in year i , $i = 1, 2$

The objective is to maximize total accumulation at the end of year 10; that is,

$$\text{maximize } Z = 1.075 I_{10} + 1.079 G_5 + 1.085 M_2$$

The constraints represent the balance equation for each year's cash flow.

$$I_1 + .98 G_1 + 1.02 M_1 = 2$$

$$I_2 + .98 G_2 + 1.02 M_2$$

$$= 2 + 1.075 I_1 + .079 G_1 + .085 M_1$$

$$I_3 + .98 G_3$$

$$= 2.5 + 1.075 I_2 + .079 (G_1 + G_2)$$

$$+ .085 (M_1 + M_2)$$

$$I_4 + .98 G_4 = 2.5 + 1.075 I_3 +$$

$$.079 (G_1 + G_2 + G_3) +$$

$$.085 (M_1 + M_2)$$

$$I_5 + .98 G_5 = 3 + 1.075 I_4 +$$

$$.079 (G_1 + G_2 + G_3 + G_4) +$$

$$.085 (M_1 + M_2)$$

$$I_6 = 3.5 + 1.075 I_5$$

$$+ .079 (G_1 + G_2 + G_3 + G_4 + G_5)$$

$$+ .085 (M_1 + M_2)$$

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continued...

$$\begin{aligned}
 I_7 &= 3.5 + 1.075 I_6 + 1.079 G_1 \\
 &\quad + 0.079 (G_2 + G_3 + G_4 + G_5) \\
 &\quad + 0.085 (M_1 + M_2) \\
 I_8 &= 4 + 1.075 I_7 + 1.079 G_2 \\
 &\quad + 0.079 (G_3 + G_4 + G_5) \\
 &\quad + 0.085 (M_1 + M_2) \\
 I_9 &= 4 + 1.075 I_8 + 1.079 G_3 \\
 &\quad + 0.079 (G_4 + G_5) \\
 &\quad + 0.085 (M_1 + M_2) \\
 I_{10} &= 5 + 1.075 I_9 + 1.079 G_4 \\
 &\quad + 0.079 G_5 + 1.085 M_1 + 0.085 M_2 \\
 \text{all variables} &\geq 0
 \end{aligned}$$

*** OPTIMUM SOLUTION SUMMARY ***

Title: Problem 26a-14
Final Iteration No: 14
Objective value (max) = 44.8500

Variable	Value	Obj Coeff	Obj Val Contrib
x1 I1	0.0000	0.0000	0.0000
x2 I2	0.0000	0.0000	0.0000
x3 I3	0.0000	0.0000	0.0000
x4 I4	0.0000	0.0000	0.0000
x5 I5	0.0000	0.0000	0.0000
x6 I6	0.0000	0.0000	0.0000
x7 I7	4.6331	0.0000	0.0000
x8 I8	9.8137	0.0000	0.0000
x9 I9	15.4678	0.0000	0.0000
x10 I10	24.6663	0.0000	0.0000
x11 G1	37.5201	1.0750	40.3341
x12 G2	0.0000	0.0000	0.0000
x13 G3	0.0000	0.0000	0.0000
x14 G4	2.9053	0.0000	0.0000
x15 G5	3.1395	0.0000	0.0000
x16 M1	3.9028	1.0790	4.2111
x17 M2	1.9608	0.0000	0.0000
	2.1242	1.0850	2.3047

Constraint	RHS	Slack(-)/Surplus(+)
1 (=)	2.0000	0.0000
2 (=)	2.0000	0.0000
3 (=)	2.5000	0.0000
4 (=)	2.5000	0.0000
5 (=)	3.0000	0.0000
6 (=)	3.5000	0.0000
7 (=)	3.5000	0.0000
8 (=)	4.0000	0.0000
9 (=)	4.0000	0.0000
10 (=)	5.0000	0.0000

Year	Recommendation
1	Invest all in 9-yr bond
2	Invest all in 9-yr bond
3	Invest all in 6-yr bond
4	Invest all in 6-yr bond
5	Invest all in 6-yr bond
7	Invest all in insured savings
8	Invest all in insured savings
9	Invest all in insured savings
10	Invest all in insured savings

x_{iA} = amount invested in year i ,
plan A (1000\$) **44**

x_{iB} = amount invested in year i ,
plan B (1000\$)

$$\text{Maximize } Z = 3x_{2B} + 1.7x_{3A}$$

Subject to

$$x_{1A} + x_{1B} \leq 100$$

$$-1.7x_{1A} + x_{2A} + x_{2B} = 0$$

$$-3x_{1B} - 1.7x_{2A} + x_{3A} = 0$$

$$x_{iA}, x_{iB} \geq 0 \text{ for } i = 1, 2, 3$$

*** OPTIMUM SOLUTION SUMMARY ***

Title: Problem 2.6a-15
Final Iteration No: 4
Objective value (max) = \$10,000
=> ALTERNATIVE solution detected at x2

Variable	Value	Obj Coeff	Obj Val Contrib
x1 x1A	100.0000	0.0000	0.0000
x2 x1B	0.0000	0.0000	0.0000
x3 x2A	0.0000	0.0000	0.0000
x4 x2B	170.0000	3.0000	510.0000
x5 x3A	0.0000	1.7000	0.0000

Constraint	RHS	Slack(-)/Surplus(+)
1 (=)	100.0000	0.0000
2 (=)	0.0000	0.0000
3 (=)	0.0000	0.0000

Optimum solution: Invest \$100,000 in A in yr 1 and
\$170,000 in B in yr 2.

Alternative optimum: Invest \$100,000 in B in yr 1 and
\$300,000 in A in yr 3.

x_i = dollars allocated to choice i ,
 $i = 1, 2, 3, 4$ **45**

r = minimum return

$$\begin{aligned}
 \text{Maximize } Z &= \min \begin{cases} -3x_1 + 4x_2 - 7x_3 + 15x_4 \\ 5x_1 - 3x_2 + 9x_3 + 4x_4 \\ 3x_1 - 9x_2 + 10x_3 - 8x_4 \end{cases} \\
 \text{Subject to } &
 \end{aligned}$$

$$x_1 + x_2 + x_3 + x_4 \leq 500$$

$$x_1, x_2, x_3, x_4 \geq 0$$

The problem can be converted to
a linear program as

continued...

Chapter 2

Maximize $Z = y$
 subject to
 $-3x_1 + 4x_2 - 7x_3 + 15x_4 \geq y$
 $5x_1 - 3x_2 + 9x_3 + 4x_4 \geq y$
 $3x_1 - 9x_2 + 10x_3 - 8x_4 \geq y$
 $x_1 + x_2 + x_3 + x_4 \leq 500$
 $x_1, x_2, x_3, x_4 \geq 0$
 y unrestricted

*** OPTIMUM SOLUTION SUMMARY ***

Title:			
Final iteration No: 5			
Objective value (max) = 1175.0000			
Variable	Value	Obj Coeff	Obj Val Contrib
x1	0.0000	0.0000	0.0000
x2	0.0000	0.0000	0.0000
x3	287.5000	0.0000	0.0000
x4	212.5000	0.0000	0.0000
x5 y	1175.0000	1.0000	1175.0000
Constraint	RHS	Slack(-)/Surplus(+)	
1 (>)	0.0000	0.0000+	
2 (>)	0.0000	2262.5000+	
3 (>)	0.0000	0.0000+	
4 (<)	500.0000	0.0000-	

Allocate \$287.50 to choice 3
 and \$212.50 to choice 4. Return =
 \$1175.00

$i = \begin{cases} 1, & \text{regular savings} \\ 2, & \text{3-month CD} \\ 3, & \text{6-month CD} \end{cases}$

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x_{it} = Deposit in plan i at start of month t

$t = \begin{cases} 1, 2, \dots, 12 & \text{if } i = 1 \\ 1, 2, \dots, 10 & \text{if } i = 2 \\ 1, 2, \dots, 7 & \text{if } i = 3 \end{cases}$

y_1 = initial amount on hand to insure a feasible solution

r_i = interest rate for plan $i = 1, 2, 3$

$\bar{t}_i = \begin{cases} 12, & i = 1 \\ 10, & i = 2 \\ 7, & i = 3 \end{cases}$

continued...

$P_i = \begin{cases} 1, & i = 1 \\ 3, & i = 2 \\ 6, & i = 3 \end{cases} \quad d_t = \$ \text{demand for period } t$

Maximize $Z = \sum_{t=1}^{12} \sum_{i=1}^3 r_i x_{i,t} - y_1$
 $t - P_i > 0$

s.t.

$y_1 - x_{11} - x_{21} - x_{31} \geq d_1$

$1000 + \sum_{i=1}^3 (1+r_i) x_{i,t-P_i} - \sum_{i=1}^3 x_{i,t} \geq d_t, t=2, \dots, 12$
 $t - P_i > 0 \quad t \leq \bar{t}_i$

$x_{it}, y_1 \geq 0$

Solution: (see file ampl.2-46.txt)

$y_1 = \$1200, Z = -1136.29$

Interest amount = $1200 - 1136.29 = \$63.71$

Deposits:

t	x_{1t}	x_{2t}	x_{3t}
1	0	0	0
2	0	200	0
3	286.48	313.53	0
4	0	587.43	0
5	314.37	289.30	0
6	0	734.69	0
7	0	98.20	0
8	0	294.60	0
9	0	848.16	0
10	0	0	0
11	0	0	0
12	0	0	0

x_{w1} = # wrenches/wk using regular time
 x_{w2} = # wrenches/wk using overtime
 x_{w3} = # wrenches/wk using subcontracting
 x_{c1} = # chisels/wk using regular time
 x_{c2} = # chisels/wk using overtime
 x_{c3} = # chisels/wk using subcontracting

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$$\text{Minimize } Z = 2x_{w1} + 2.8x_{w2} + 3x_{w3} + 2.1x_{c1}$$

$$\text{Subject to } + 3.2x_{c2} + 4.2x_{c3}$$

$$x_{w1} \leq 550, x_{w2} \leq 250$$

$$x_{c1} \leq 620, x_{c2} \leq 280$$

$$\frac{x_{c1} + x_{c2} + x_{c3}}{x_{w1} + x_{w2} + x_{w3}} \geq 2$$

$$x_{w1} + x_{w2} + x_{w3}$$

or

$$2x_{w1} + 2x_{w2} + 2x_{w3} - x_{c1} - x_{c2} - x_{c3} \leq 0$$

$$x_{w1} + x_{w2} + x_{w3} \geq 1500$$

$$x_{c1} + x_{c2} + x_{c3} \geq 1200$$

$$\text{all variables} \geq 0$$

(a) Optimum from TORA:

$$x_{w1} = 550, x_{w2} = 250, x_{w3} = 700$$

$$x_{c1} = 620, x_{c2} = 280, x_{c3} = 2100$$

$$Z = \$14,918$$

(b) Increasing marginal cost ensures that regular time capacity is used before that of overtime, and that overtime capacity is used before that of subcontracting. If the marginal cost function is not monotonically increasing, additional constraints are needed to ensure that the capacity restriction is satisfied.

continued...

x_j = number of units produced of product j , $j = 1, 2, 3, 4$

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Profit per unit:

$$\text{Product 1} = 75 - 2 \times 10 - 3 \times 5 - 7 \times 4 = \$12$$

$$\text{Product 2} = 70 - 3 \times 10 - 2 \times 5 - 3 \times 4 = \$18$$

$$\text{Product 3} = 55 - 4 \times 10 - 1 \times 5 - 2 \times 4 = \$2$$

$$\text{Product 4} = 45 - 2 \times 10 - 2 \times 5 - 1 \times 4 = \$11$$

$$\text{Maximize } Z = 12x_1 + 18x_2 + 2x_3 + 11x_4$$

s.t.

$$2x_1 + 3x_2 + 4x_3 + 2x_4 \leq 500$$

$$3x_1 + 2x_2 + x_3 + 2x_4 \leq 380$$

$$7x_1 + 3x_2 + 2x_3 + x_4 \leq 450$$

$$x_1, x_2, x_3, x_4 \geq 0$$

TORA Solution:

$$x_1 = 0, x_2 = 133.33, x_3 = 0, x_4 = 50$$

$$Z = \$2950$$

x_j = number of units of model j

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$$\text{Maximize } Z = 30x_1 + 20x_2 + 50x_3$$

Subject to

$$\textcircled{1} \quad 2x_1 + 3x_2 + 5x_3 \leq 4000$$

$$\textcircled{2} \quad 4x_1 + 2x_2 + 7x_3 \leq 6000$$

$$\textcircled{3} \quad x_1 + 5x_2 + \frac{1}{3}x_3 \leq 1500$$

$$\textcircled{4} \quad \frac{x_1}{3} = \frac{x_2}{2}, \text{ or } 2x_1 - 3x_2 = 0$$

$$\textcircled{5} \quad \frac{x_2}{2} = \frac{x_3}{5}, \text{ or } 5x_2 - 2x_3 = 0$$

$$x_1 \geq 200, x_2 \geq 200, x_3 \geq 150$$

*** OPTIMUM SOLUTION SUMMARY ***

Title: Problem 2.6a-12

Final Iteration No: 4

Objective value (max) = 41061.0820

Variable	Value	Obj Coeff	Obj Val Contrib
x1	324.3243	30.0000	9729.7305
x2	216.2162	20.0000	4324.3242
x3	540.5405	50.0000	27027.0273

Constraint	RHS	Slack(-)/Surplus(+)
1 (<=)	4000.0000	0.0000-
2 (<=)	6000.0000	456.4865-
3 (<=)	1500.0000	887.3875-
4 (=)	0.0000	0.0000
5 (=)	0.0000	0.0000
LB-x1	200.0000	124.3243+
LB-x2	200.0000	16.2162+
LB-x3	150.0000	390.5405+

Chapter 2

x_{ij} = Nbr. cartons in month i from supplier j

I_i = End inventory in period i , $I_0 = 0$

C_{ij} = Price per unit of x_{ij}

h = Holding cost/unit/month

C = Supplier capacity/month

d_i = Demand for month i

$i = 1, 2, 3, j = 1, 2$

$$\text{Minimize } Z = \sum_{i=1}^3 \sum_{j=1}^2 C_{ij} x_{ij} + \frac{h}{2} \left(\sum_{i=1}^3 \left(\sum_{j=1}^2 x_{ij} + I_{i-1} + I_i \right) \right)$$

s.t. $x_{ij} \leq C$, all i and j

$$\sum_{j=1}^2 x_{ij} + I_{i-1} - I_i = d_i, \text{ all } i$$

Optimum solution:

i	x_{i1}	x_{i2}	I
1	400	100	0
2	400	400	200
3	200	0	0

Total cost = \$167,450.

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x_{ij} = Qty of product i in month j ,
 $i = 1, 2, j = 1, 2, 3$

I_{ij} = End inventory of product i in month j

$$\text{Minimize } Z = 30(x_{11} + x_{12} + x_{13}) + 28(x_{21} + x_{22} + x_{23}) + 9(I_{11} + I_{12} + I_{13}) + 75(I_{21} + I_{22} + I_{23})$$

s.t.

$$(x_{1j}/1.25) + x_{2j} \leq \begin{cases} 3000, & j=1 \\ 3500, & j=2 \\ 3000, & j=3 \end{cases}$$

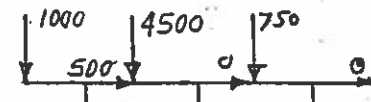
$$I_{1,j-1} + x_{1j} - I_{1j} = \begin{cases} 500, & j=1 \\ 5000, & j=2 \\ 750, & j=3 \end{cases}$$

$$I_{2,j-1} + x_{2j} - I_{2j} = \begin{cases} 1000, & j=1 \\ 1200, & j=2 \\ 1200, & j=3 \end{cases}$$

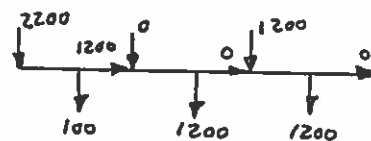
$$x_{ij}, I_{ij} \geq 0$$

Optimum solution: Cost = \$284,050

Product 1:



Product 2:



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x_i = Production amount in quarter i

I_i = End inventory for quarter i

$$\text{Minimize } Z = 20x_1 + 22x_2 + 24x_3 + 26x_4 + 3.5(I_1 + I_2 + I_3)$$

s.t.

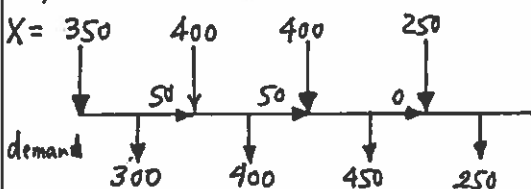
$$x_i = 300 + I_i, \quad x_i \leq 400, \quad i=1,2,3,4$$

$$I_1 + x_2 = 400 + I_2, \quad I_i \leq 100, \quad i=1,2,3$$

$$I_2 + x_3 = 450 + I_3, \quad I_0 = I_4 = 0$$

$$I_3 + x_4 = 250$$

Optimum solution:



Total cost = \$39,250

51

x_{ij} = Qty by operation i in month j

$i = 1, 2, j = 1, 2, 3$

$$\text{Minimize } Z = 2 \sum_{j=1}^3 I_{1j} + 4 \sum_{j=1}^3 I_{2j} + 10x_{11} + 12x_{12} + 11x_{13} + 15x_{21} + 18x_{22} + 16x_{23}$$

$$6x_{11} \leq 800, 6x_{12} \leq 700, 6x_{13} \leq 550$$

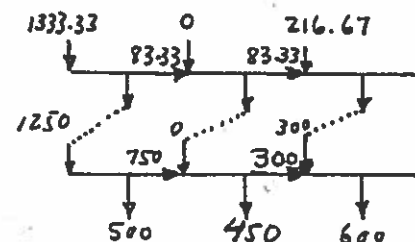
$$8x_{21} \leq 1000, 8x_{22} \leq 850, 8x_{23} \leq 700$$

$$x_{1j} + I_{1,j-1} = x_{2j} + I_{1j}, \quad j=1,2,3$$

$$x_{2j} + I_{2,j-1} = I_{2j} + d_j, \quad j=1,2,3$$

$$I_{i0} = 0, \quad i=1,2$$

Solution: Cost = \$39,720



I_{ij} = Entering inv. of op. i in month j

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x_j = Units of product j , $j=1, 2$

y_i^- = Unused hours of machine i
 y_i^+ = Overtime hours of machine i } $i=1, 2$

Maximize $Z = 110x_1 + 118x_2 - 100(y_1^+ + y_2^+)$

s.t.

$$\frac{x_1}{5} + \frac{x_2}{5} + y_1^- - y_1^+ = 8$$

$$\frac{x_1}{8} + \frac{x_2}{4} + y_2^- - y_2^+ = 8$$

$$y_1^+ \leq 4, \quad y_2^+ \leq 4$$

$$x_1, x_2, y_1^-, y_1^+, y_2^-, y_2^+ \geq 0$$

Solution:

$$\text{Revenue} = \$6,232$$

$$x_1 = 56, \quad y_1^+ = 4 \text{ hrs}$$

$$x_2 = 4, \quad y_2^+ = 0$$

$$y_1^-, y_2^- = 0$$

h = Regular pay hour

8-hr pay = $8h$

12-hr pay = $12h + \frac{4h}{2} = 14h$

x_i = Nbr 8-hr buses starting in period i

y_i = Nbr. of 12-hr buses starting in period i

Minimize $Z = h(8 \sum_{i=1}^6 x_i + 14 \sum_{i=1}^6 y_i)$

s.t.

x_1	x_2	x_3	x_4	x_5	x_6	y_1	y_2	y_3	y_4	y_5	y_6	
1						1	1					≥ 4
1	1						1	1				≥ 8
	1	1					1	1	1			≥ 10
		1	1					1	1	1		≥ 7
			1	1					1	1	1	≥ 12
				1	1					1	1	≥ 4

Solution: $Z = 196h$

$x_1 = 4, x_2 = 4, x_4 = 2, x_5 = 4, x_3 = x_6 = 0$

$y_3 = 6, y_1 = y_2 = y_4 = y_5 = y_6 = 0$

For 8-hr only buses, solution is

$Z = 208h$

$x_1 = x_2 = 4, x_3 = 6, x_4 = 1, x_5 = 11, x_6 = 0$

(8-hr + 12-hr) buses is cheaper.

x_i = Nbr. of volunteers starting in hour i

Minimize $Z = \sum_{i=1}^{14} x_i$

s.t.

(8:00)	x_1	≥ 4
(9:00)	$x_1 + x_2$	≥ 4
(10:00)	$x_1 + x_2 + x_3$	≥ 6
(11:00)	$x_2 + x_3 + x_4$	≥ 6
(12:00)	$x_3 + x_4 + x_5$	≥ 8
(1:00)	$x_4 + x_5 + x_6$	≥ 8
(2:00)	$x_5 + x_6 + x_7$	≥ 6
(3:00)	$x_6 + x_7 + x_8$	≥ 6
(4:00)	$x_7 + x_8 + x_9$	≥ 4
(5:00)	$x_8 + x_9 + x_{10}$	≥ 4
(6:00)	$x_9 + x_{10} + x_{11}$	≥ 6
(7:00)	$x_{10} + x_{11} + x_{12}$	≥ 6
(8:00)	$x_{11} + x_{12} + x_{13}$	≥ 8
(9:00)	$x_{12} + x_{13}$	≥ 8

All $x_i \geq 0$

continued...

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Solution: $Z = 32$ volunteers

$x_1 = 4, x_3 = 2, x_4 = 6, x_6 = 2, x_7 = 4, x_{10} = 6, x_{12} = 8$
all other $x_i = 0$

Same formulation as in Problem 2 with the added constraints $x_5 = 0, x_{11} = 0$
Optimum solution remains the same

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x_i = Nbr. of caucals starting on day i
($i=1$: Monday, $i=7$: Sunday)

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Minimize $Z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7$

s.t.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
M	1			1	1	1	1	≥ 20
T	1	1			1	1	1	≥ 14
W	1	1	1			1	1	≥ 10
Th	1	1	1	1			1	≥ 15
F	1	1	1	1	1			≥ 18
Sat		1	1	1	1	1		≥ 10
Sun			1	1	1	1	1	≥ 12

Solution: $Z = 20$ workers

$x_1 = 8, x_4 = 6, x_5 = 4, x_6 = 1, x_7 = 1$

x_i = Nbr. Students starting at hour i
 $i=1$ (8:01), $i=9$ (4:01), $x_5 = 0$

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Minimize $Z = x_1 + x_2 + x_3 + x_4 + x_6 + x_7 + x_8 + x_9$

s.t.

	x_1	x_2	x_3	x_4	x_6	x_7	x_8	x_9	
8:01	1								≥ 2
9:01	1	1							≥ 2
10:01	1	1	1						≥ 3
11:01		1	1	1					≥ 4
12:01			1	1					≥ 4
1:01				1	1				≥ 3
2:01					1	1			≥ 3
3:01					1	1	1		≥ 3
4:01						1	1	1	≥ 3

Solution: $Z = 9$ students

$x_1 = 2, x_3 = 1, x_4 = 3, x_7 = 3$

Let x_i = Nbr. starting on day i and lasting for 7 days

y_{ij} = Nbr. starting shift on day i and starting their 2 days off on day j , $i \neq j$

Thus, of the x_1 workers who start on Monday, y_{12} will take T and W off, y_{13} will take W and Th off, and so on, as the following table shows.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7
1	start on Mon	y_{12}	$y_{12}+y_{13}$	$y_{13}+y_{14}$	$y_{14}+y_{15}$	$y_{15}+y_{16}$	y_{16}
2	y_{27}	Tue	y_{23}	$y_{23}+y_{24}$	$y_{24}+y_{25}$	$y_{25}+y_{26}$	$y_{26}+y_{27}$
3	$y_{31}+y_{37}$	y_{31}	Wed	y_{34}	$y_{34}+y_{35}$	$y_{35}+y_{36}$	$y_{36}+y_{37}$
4	$y_{41}+y_{47}$	$y_{41}+y_{42}$	y_{42}	Th	y_{45}	$y_{45}+y_{46}$	$y_{46}+y_{47}$
5	$y_{51}+y_{57}$	$y_{51}+y_{52}$	$y_{52}+y_{53}$	y_{53}	Fri	y_{56}	$y_{56}+y_{57}$
6	$y_{61}+y_{67}$	$y_{61}+y_{62}$	$y_{62}+y_{63}$	$y_{63}+y_{64}$	y_{64}	Sat	y_{67}
7	y_{71}	$y_{71}+y_{72}$	$y_{72}+y_{73}$	$y_{73}+y_{74}$	$y_{74}+y_{75}$	y_{75}	Su

$$\text{Minimize } z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7$$

Each employee has 2 days off: $x_i = \sum_{j \in \{1..7, j \neq i\}} y_{ij}$

Mon (1) constraint: $s - (y_{27} + y_{31} + y_{37} + y_{41} + y_{47} + y_{51} + y_{57} + y_{61} + y_{67} + y_{71}) \geq 12$

Tue (2) constraint: $s - (y_{12} + y_{31} + y_{41} + y_{42} + y_{51} + y_{52} + y_{61} + y_{62} + y_{71} + y_{72}) \geq 18$

Wed (3) constraint: $s - (y_{12} + y_{13} + y_{23} + y_{42} + y_{52} + y_{53} + y_{62} + y_{63} + y_{72} + y_{73}) \geq 20$

Th (4) constraint: $s - (y_{13} + y_{14} + y_{23} + y_{24} + y_{24} + y_{53} + y_{63} + y_{64} + y_{73} + y_{74}) \geq 28$

Fri (5) constraint: $s - (y_{14} + y_{15} + y_{24} + y_{25} + y_{34} + y_{35} + y_{45} + y_{64} + y_{74} + y_{75}) \geq 32$

Sat(6) constraint: $s - (y_{15} + y_{16} + y_{25} + y_{26} + y_{35} + y_{36} + y_{45} + y_{46} + y_{56} + y_{75}) \geq 40$

Sun(7) constraint: $s - (y_{16} + y_{26} + y_{27} + y_{36} + y_{37} + y_{46} + y_{47} + y_{56} + y_{57} + y_{67}) \geq 40$

continued

Chapter 2

Solution: 42 employees

Starting		Nbr off						
On	Nbr	M	Tu	Wed	Th	Fri	Sat	Sun
M	16		16	16				
Tu	8				8	8		
Wed	8	8	8					
Th	0							
Fri	6			6	6			
Sat	2	2						2
Sun	2					2	2	
Nbr off		10	24	22	14	10	2	2
Nbr at work		32	18	20	28	32	40	40
Surplus above minimum		20	0	0	0	0	0	0

x_e = Nbr. of efficiency apartments x_d = Nbr. of duplexes x_s = Nbr. of single-family homes x_r = Retail space in ft² Maximize $Z = 600x_e + 750x_d + 1200x_s + 100x_r$ s.t. $x_e \leq 500, x_d \leq 300, x_s \leq 250$ $x_r \geq 10x_e + 15x_d + 18x_s$ $x_r \leq 10000$ $x_d \geq \frac{x_e + x_s}{2}$ $x_e, x_d, x_s, x_r \geq 0$ <u>Optimal Solution:</u> $Z = 1,595,714.29$ $x_e = 207.14, x_d = 228.57$ $x_s = 250, x_r = 10,000$ LP does not guarantee integer solution. Use rounded solution or apply integer LP algorithm (Chapter 9).	x_{ij} = portion of project i completed in year j Maximize $Z = .05(4x_{11} + 3x_{12} + 2x_{13}) + .07(3x_{22} + 2x_{23} + x_{24}) + .15(4x_{31} + 3x_{32} + 2x_{33} + x_{34}) + .02(2x_{43} + x_{44})$ s.t. $\sum_{j=1}^3 x_{1j} = 1, \sum_{j=3}^4 x_{4j} = 1$ $.25 \leq \sum_{j=2}^5 x_{2j} \leq 1, .25 \leq \sum_{j=1}^5 x_{3j} \leq 1$ $5x_{11} + 15x_{31} \leq 3$ $5x_{12} + 8x_{22} + 15x_{32} \leq 6$ $5x_{13} + 8x_{23} + 15x_{33} + 1.2x_{43} \leq 7$ $8x_{24} + 15x_{34} + 1.2x_{44} \leq 7$ $8x_{25} + 15x_{35} \leq 7$ <u>Optimum:</u> $Z = \\$523,750$ $x_{11} = .6, x_{12} = .4$ $x_{24} = .225, x_{25} = .025$ $x_{32} = .267, x_{33} = .387, x_{34} = .346$ $x_{43} = 1$
x_i = Acquired portion of property i Each site is represented by a separate LP. The site that yields the smaller objective value is selected. <u>Site 1 LP:</u> Minimize $Z = 25 + x_1 + 2.1x_2 + 2.35x_3 + 1.85x_4 + 2.95x_5$ s.t. $x_4 \geq .75$, all $x_i \leq 1, i = 1, 2, \dots, 5$ $20x_1 + 50x_2 + 50x_3 + 30x_4 + 60x_5 \geq 200$ <u>Optimum:</u> $Z = 34.6625$ million \$ $x_1 = .875, x_2 = x_3 = 1, x_4 = .75, x_5 = 1$ <u>Site 2 LP:</u> Minimize $Z = 27 + 2.8x_1 + 1.9x_2 + 2.8x_3 + 2.5x_4$ s.t. $x_3 \geq .5, x_1, x_2, x_3, x_4 \leq 1$ $80x_1 + 60x_2 + 50x_3 + 70x_4 \geq 200$ <u>Optimum:</u> $Z = 34.35$ million \$ $x_1 = x_2 = 1, x_3 = x_4 = .5$ Select Site 2.	x_l = Nbr. of low income units x_m = Nbr. of middle income units x_u = Nbr. of upper income units x_p = Nbr. of public housing units x_s = Nbr. of school rooms x_r = Nbr. of retail units x_c = Nbr. of condemned homes Maximize $Z = 7x_l + 12x_m + 20x_u + 5x_p + 15x_r - 10x_s - 7x_c$ s.t. $100 \leq x_l \leq 200, 125 \leq x_m \leq 190$ $75 \leq x_u \leq 260, 300 \leq x_p \leq 600$ $0 \leq x_s \leq 2/.045$ $.05x_l + .07x_m + .03x_u + .025x_p + .045x_s + 1x_r \leq .85(50 + .25x_c)$ $x_r \geq .023x_l + .034x_m + .046x_u + .023x_p + .034x_s$

continued...

Chapter 2

$$25x_5 \geq 1.3x_l + 1.2x_m + .5x_u + 1.4x_p$$

Optimum: $Z = 8290.30$ thousand \$

$$x_l = 100, x_m = 125, x_u = 227.04$$

$$x_p = 300, x_s = 32.54, x_n = 25$$

$$x_c = 0$$

x_1 = Nbr. of single-family homes

x_2 = Nbr. of double-family homes

x_3 = Nbr. of triple-family homes

x_4 = Nbr. of recreation areas

$$\text{Maximize } Z = 10,000x_1 + 12,000x_2 + 15,000x_3$$

s.t.

$$2x_1 + 3x_2 + 4x_3 + x_4 \leq .85 \times 800$$

$$\frac{x_1}{x_1 + x_2 + x_3} \geq .5 \text{ or } .5x_1 - .5x_2 - .5x_3 \geq 0$$

$$x_4 \geq \frac{x_1 + 2x_3 + 3x_3}{200} \text{ or } 200x_4 - x_1 - 2x_3 - 3x_3 \geq 0$$

$$1000x_1 + 1200x_2 + 1400x_3 + 800x_4 \geq 100,000$$

$$400x_1 + 600x_2 + 800x_3 + 450x_4 \leq 200,000$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Optimum solution:

$$x_1 = 339.15 \text{ homes}$$

$$x_2 = 0$$

$$x_3 = 0$$

$$x_4 = 1.69 \text{ areas}$$

$$Z = \$339,521.20$$

New land use constraint:

$$2x_1 + 3x_2 + 4x_3 + x_4 \leq .85(800 + 100)$$

New Optimum solution:

$$Z = \$3,815,461.35$$

$$x_1 = 381.54 \text{ homes}$$

$$x_2 = x_3 = 0$$

$$x_4 = 1.91 \text{ areas}$$

$$\Delta Z = \$3,815,461.35 - 3,391,521.20$$

$$= \$423,940.35$$

$\Delta Z < \$450,000$, the purchasing cost of 100 acres. Hence, the purchase of the new acreage is not recommended.

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x_s = tons of strawberry / day
 x_g = tons of grapes / day
 x_a = tons of apples / day
 x_A = cans of drink A / day
 x_B = cans of drink B / day
 x_C = cans of drink C / day

Each can holds one lb

x_{sA} = lb of strawberry used in drink A / day
 x_{sB} = lb of strawberry used in drink B / day
 x_{gA} = lb of grapes used in drink A / day
 x_{gB} = lb of grapes used in drink B / day
 x_{gC} = lb of grapes used in drink C / day
 x_{aB} = lb of apples used in drink B / day
 x_{aC} = lb of apples used in drink C / day

Maximize $Z = 1.15x_A + 1.25x_B + 1.2x_C - 200x_s - 100x_g - 90x_a$

s.t.

$$x_s \leq 200, x_g \leq 100, x_a \leq 150$$

$$x_{sA} + x_{sB} = 1500x_s$$

$$x_{gA} + x_{gB} + x_{gC} = 1200x_g$$

$$x_{aB} + x_{aC} = 1000x_a$$

$$x_A = x_{sA} + x_{gA}$$

$$x_B = x_{sB} + x_{gB} + x_{aB}$$

$$x_C = x_{gC} + x_{aC}$$

$$x_{sA} = x_{gA}$$

$$x_{sB} = x_{gB}, x_{sB} = .5x_{aB}$$

$$3x_{gC} = 2x_{aC}$$

all variables ≥ 0

Optimum solution:

$x_A = 90,000$ cans, $x_B = 300,000$ cans, $x_C = 0$

X_{ij}	j		
i	A	B	C
s	45,000	75,000	0
g	45,000	75,000	0
a	0	150,000	0
	90,000	300,000	0

$x_s = 80$ tons, $x_g = 100$ tons, $x_a = 150$ tons
 $Z = \$439,000/\text{day}$

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x_s = lb of screws per package
 x_b = lb of bolts per package
 x_n = lb of nuts per package
 x_w = lb of washers per package

Minimize $Z = 1.1x_s + 1.5x_b + \frac{70}{80}x_n + \frac{20}{30}x_w$

s.t. $Y = x_s + x_b + x_n + x_w$

$$x_s \geq .1Y$$

$$x_b \geq .25Y, \frac{x_b}{50} \leq x_w, \frac{x_b}{10} \leq x_n$$

$$x_n \leq .15Y$$

$$x_w \leq .1Y$$

$$Y \geq 1$$

All variables are nonnegative

Optimum solution:

$Y = 1, x_s = .5, x_b = .25, x_n = .15, x_w = .1$

Cost = \$1.12

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$x_{O(A,B,C)}$ = lb of oats in cereals A, B, C
 $x_{r(A,C)}$ = lb of raisins in cereals A, C
 $x_{c(B,C)}$ = lb of coconuts in cereals B, C
 $x_{a(A,B,C)}$ = lb of almond in cereals A, B, C

$Y_O = x_{OA} + x_{OB} + x_{OC}$
 $Y_r = x_{rA} + x_{rC}$
 $Y_c = x_{cB} + x_{cC}$
 $Y_a = x_{aA} + x_{aB} + x_{aC}$

$W_A = x_{OA} + x_{rA} + x_{aA}$
 $W_B = x_{OB} + x_{cB} + x_{aB}$
 $W_C = x_{OC} + x_{rC} + x_{cC} + x_{aC}$

Maximize $Z = \frac{1}{5}(2W_A + 2.5W_B + 3W_C) - \frac{1}{2000}(100Y_O + 120Y_r + 110Y_c + 200Y_a)$

s.t.

$$W_A \leq 500 \times 5 = 2500$$

$$W_B \leq 600 \times 5 = 3000$$

$$W_C \leq 500 \times 5 = 4000$$

continued...

Chapter 2

$$Y_D \leq 5 \times 2000 = 10,000$$

$$Y_r \leq 2 \times 2000 = 4,000$$

$$Y_c \leq 1 \times 2000 = 2,000$$

$$Y_a \leq 1 \times 2000 = 2,000$$

$$X_{0A} = \frac{50}{5} X_{rA}, X_{0A} = \frac{50}{2} X_{aA}$$

$$X_{0B} = \frac{60}{2} X_{cB}, X_{0B} = \frac{60}{3} X_{aB}$$

$$X_{0C} = \frac{60}{3} X_{rC}, X_{0C} = \frac{60}{4} X_{cC}, X_{0C} = \frac{60}{2} X_{aC}$$

all variables are nonnegative.

Optimum solution: $Z = \$5384.84/\text{day}$

$$W_A = 2500 \text{ lb or } 500 \text{ boxes/day}$$

$$W_B = 3000 \text{ lb or } 600 \text{ boxes}$$

$$W_C = 5793.45 \text{ lb or } \approx 1158 \text{ boxes}$$

$$X_0 = 10,000 \text{ lb or } 5 \text{ tons/day}$$

$$X_r = 471.19 \text{ lb or } .236 \text{ ton}$$

$$X_c = 428.16 \text{ lb or } .214 \text{ ton}$$

$$X_a = 394.11 \text{ lb or } .197 \text{ ton}$$

$$\left. \begin{array}{l} X_{Ai} = \text{bbl of gasoline A in fuel } i \\ X_{Bi} = \text{bbl of gasoline B in fuel } i \\ X_{Ci} = \text{bbl of gasoline C in fuel } i \\ X_{Di} = \text{bbl of gasoline D in fuel } i \end{array} \right\} i = 1, 2$$

$$Y_A = X_{A1} + X_{A2}$$

$$Y_B = X_{B1} + X_{B2}$$

$$Y_C = X_{C1} + X_{C2}$$

$$Y_D = X_{D1} + X_{D2}$$

$$F_1 = X_{A1} + X_{B1} + X_{C1} + X_{D1}$$

$$F_2 = X_{A2} + X_{B2} + X_{C2} + X_{D2}$$

$$\text{Maximize } Z = 200F_1 + 250F_2$$

$$- (120Y_A + 90Y_B + 100Y_C + 150Y_D)$$

continued...

s.t.

$$X_{A1} = X_{B1}, X_{A1} = .5X_{C1}, X_{A1} = .25X_{D1}$$

$$X_{A2} = X_{B2}, X_{A2} = 2X_{C2}, X_{A2} = \frac{2}{3}X_{D2}$$

$$Y_A \leq 1000, Y_B \leq 1200, Y_C \leq 900, Y_D \leq 1500$$

$$F_1 \geq 200, F_2 \geq 400$$

Optimum solution: $Z = \$495,416.67$

$$Y_A = 958.33 \text{ bbl/day}$$

$$Y_B = 958.33 \text{ bbl/day}$$

$$Y_C = 516.67 \text{ bbl/day}$$

$$Y_D = 1500 \text{ bbl/day}$$

$$F_1 = 200 \text{ bbl/day}$$

$$F_2 = 3733.33 \text{ bbl/day}$$

A = bbl of crude A/day

B = bbl of crude B/day

R = bbl of regular gasoline/day

P = bbl of premium gasoline/day

J = bbl of jet gasoline/day

$$\begin{aligned} \text{Maximize } Z = & 50(R - R^+) + 70(P - P^+) \\ & + 120(J - J^+) - (10R^- + 15P^- + 20J^-) \\ & - (2R^+ + 3P^+ + 4J^+) - (30A + 40B) \end{aligned}$$

s.t.

$$A \leq 2500, B \leq 3000$$

$$R = .2A + .25B, R + R^- - R^+ = 500$$

$$P = .1A + .3B, P + P^- - P^+ = 700$$

$$J = .25A + .1B, J + J^- - J^+ = 400$$

All variables ≥ 0

Optimum solution:

$$Z = \$21,852.94$$

$$A = 1176.47 \text{ bbl/day}$$

$$B = 1058.82 \text{ bbl/day}$$

$$R = 500 \text{ bbl/day}$$

$$P = 435.29 \text{ bbl/day}$$

$$J = 400 \text{ bbl/day}$$

NR = bbl/day of naphta used in regular
 NP = bbl/day of naphta used in premium
 NJ = bbl/day of naphta used in jet
 LR = bbl/day of light used in regular
 LP = bbl/day of light used in premium
 LJ = bbl/day of light used in jet
 Using the other notation in Problem 5,
 Maximize $Z = 50(R - R^+) + 70(P - P^+) + 12(J - J^+) - (10R^- + 15P^- + 20J^-) - (2R^+ + 3P^+ + 4J^+) - (30A + 40B)$

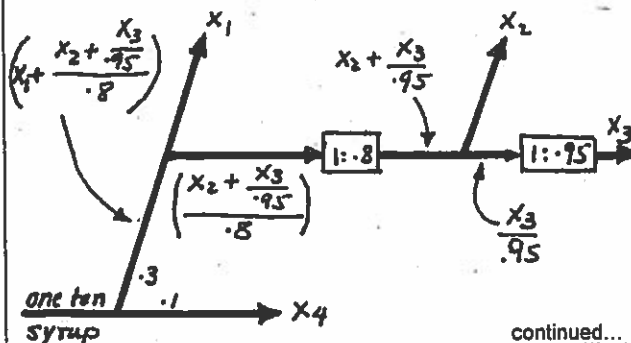
s.t.
 $A \leq 2500, B \leq 3000$
 $R + R^- - R^+ = 500$
 $P + P^- - P^+ = 700$
 $J + J^- - J^+ = 400$
 $.35A + .45B = NR + NP + NJ$
 $.6A + .5B = LR + LP + LJ$
 $R = NR + LR$
 $P = NP + LP$
 $J = NJ + LJ$

all variables are nonnegative

Optimum solution: $Z = \$71,473.68$

$A = 1684.21, B = 0$
 $R = 500, P = 700, J = 400$

x_1 = tons of brown sugar per week
 x_2 = tons of white sugar per week
 x_3 = tons of powdered sugar per week
 x_4 = tons of molasses per week



continued...

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Maximize $Z = 150x_1 + 200x_2 + 230x_3 + 35x_4$
 s.t.

$x_4 \leq 4000 \times .1$
 $x_4 \leq 400$
 $x_1 + \frac{x_2 + \frac{x_3}{.95}}{.8} \leq .3 \times 4000$
 $.76x_1 + .95x_2 + x_3 \leq 912$
 $x_1 \geq 25, x_2 \geq 25$
 $x_3 \geq 25, x_4 \geq 0$

Optimum solution from TORA:

$x_1 = 25$ tons per week
 $x_2 = 25$ tons per week
 $x_3 = 869.25$ tons per week
 $x_4 = 400$ tons per week
 $Z = \$22,677.50$

A = bbl/hr of stock A
 B = bbl/hr of stock B
 Y_{Ai} = bbl/hr of A used in gasoline i
 Y_{Bi} = bbl/hr of B used in gasoline i } $i = 1, 2$

Maximize $Z = 7(Y_{A1} + Y_{B1}) + 10(Y_{A2} + Y_{B2})$

s.t.
 $A = Y_{A1} + Y_{A2}, A \leq 450$
 $B = Y_{B1} + Y_{B2}, B \leq 700$
 $98Y_{A1} + 89Y_{B1} \geq 91(Y_{A1} + Y_{B1})$
 $98Y_{A2} + 89Y_{B2} \geq 93(Y_{A2} + Y_{B2})$
 $10Y_{A1} + 8Y_{B1} \leq 12(Y_{A1} + Y_{B1})$
 $10Y_{A2} + 8Y_{B2} \leq 12(Y_{A2} + Y_{B2})$

all variables are nonnegative

Optimum solution:

$Z = \$10,675$

$A = 450$ bbl/hr

$B = 700$ bbl/hr

Gasoline 1 production = $Y_{A1} + Y_{B1}$
 $= 61.11 + 213.89 = 275$ bbl/hr

Gasoline 2 production = $Y_{A2} + Y_{B2}$
 $= 388.89 + 486.11 = 875$ bbl/hr

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S = tons of steel scrap / day
 A = tons of alum. scrap / day
 C = tons of cast iron scrap / day
 A_b = tons of alum. briquettes / day
 S_b = tons silicon briquettes / day
 a = tons of alum. / day
 g = tons of graphite / day
 s = tons of silicon / day
 aI = tons of alum. in ingot I / day
 aII = tons of alum. in ingot II / day
 gI = tons of graphite in ingot I / day
 gII = tons of graphite in ingot II / day
 sI = tons of silicon in ingot I / day
 sII = tons of silicon in ingot II / day
 I_1 = tons of ingot I / day
 I_2 = tons of ingot II / day
 Minimize $Z = 100S + 150A + 75C + 900A_b + 380S_b$
 s.t. $S \leq 1000, A \leq 500, C \leq 2500$
 $a = .1S + .95A + A_b$
 $g = .05S + .01A + .15C$
 $s = .04S + .02A + .08C + S_b$
 $I_1 = aI + gI + sI$
 $I_2 = aII + gII + sII$
 $aI + aII \leq 8, sI + sII \leq 8, gI + gII \leq 8$
 $.081I_1 \leq aI \leq .108I_1$
 $.015I_1 \leq gI \leq .03I_1$
 $.025I_1 \leq sI \leq .04I_1$
 $.062I_2 \leq aII \leq .089I_2$
 $.041I_2 \leq gII \leq .06I_2$
 $.028I_2 \leq sII \leq .041I_2$
 $I_1 \geq 130, I_2 \geq 250$

Optimum solution :

$$Z = \$117,435.65$$

$$S = 0, A = 38.2, C = 1489.41$$

$$A_b = S_b = 0$$

$$I_1 = 130, I_2 = 250$$

$$a = 36.29, g = 223.79, s = 119.92$$

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x_{ij} = tons of ore i allocated to alloy k
 w_k = tons of alloy k produced

$$\text{Maximize } Z = 200w_A + 300w_B - 30(x_{1A} + x_{1B}) - 40(x_{2A} + x_{2B}) - 50(x_{3A} + x_{3B})$$

Subject to

Specification constraints:

$$.2x_{1A} + .1x_{2A} + .05x_{3A} \leq .8w_A \quad (1)$$

$$.1x_{1A} + .2x_{2A} + .05x_{3A} \leq .3w_A \quad (2)$$

$$.3x_{1A} + .3x_{2A} + .2x_{3A} \geq .5w_A \quad (3)$$

$$.1x_{1B} + .2x_{2B} + .05x_{3B} \geq .4w_B \quad (4)$$

$$.1x_{1B} + .2x_{2B} + .05x_{3B} \leq .6w_B \quad (5)$$

$$.3x_{1B} + .3x_{2B} + .7x_{3B} \geq .3w_B \quad (6)$$

$$.3x_{1B} + .3x_{2B} + .2x_{3B} \leq .7w_B \quad (7)$$

Ore constraints:

$$x_{1A} + x_{1B} \leq 1000$$

$$x_{2A} + x_{2B} \leq 2000$$

$$x_{3A} + x_{3B} \leq 3000$$

*** OPTIMUM SOLUTION SUMMARY ***

Title: Problem 26a-17
 Final Iteration No: 12
 Objective value (max) = 400000.0000

Variable	Value	Obj Coeff	Obj Val Contrib
x1 wA	1799.9999	200.0000	359999.9600
x2 wB	1000.0001	300.0000	300000.0312
x3 x1A	1000.0000	-30.0000	-30000.0000
x4 x1B	0.0000	-30.0000	-0.0000
x5 x2A	0.0000	-40.0000	-0.0000
x6 x2B	2000.0001	-40.0000	-80000.0078
x7 x3A	3000.0000	-50.0000	-150000.0000
x8 x3B	0.0000	-50.0000	-0.0000

Constraint	RHS	Slack(-)/Surplus(+)
1 (<=)	0.0000	1090.0000-
2 (<=)	0.0000	290.0000-
3 (>=)	0.0000	0.0000+
4 (>=)	0.0000	0.0000+
5 (<=)	0.0000	200.0000-
6 (>=)	0.0000	300.0002+
7 (<=)	0.0000	100.0000-
8 (<=)	1000.0000	0.0000-
9 (<=)	2000.0000	0.0000-
10 (<=)	3000.0000	0.0000-

Solution:

Produce 1800 tons of alloy A and 1000 tons of alloy B.

	X_i = Nbr. of ads for issue i, $i=1,2,3,4$ 78 Minimize $Z = S_1^- + S_2^- + S_3^- + S_4^-$ s.t. $(-30,000 + 60,000 + 30,000)X_1 + S_1^- - S_1^+ = .51 \times 400,000$ $(80,000 + 30,000 - 45,000)X_2 + S_2^- - S_2^+ = .51 \times 400,000$ $(40,000 + 10,000)X_3 + S_3^- - S_3^+ = .51 \times 400,000$ $(90,000 - 25,000)X_4 + S_4^- - S_4^+ = .51 \times 400,000$ $1500(X_1 + X_2 + X_3 + X_4) \leq 100,000$ $X_1, X_2, X_3, X_4 \geq 0$ <u>Solution:</u> $X_1 = 3.4, X_2 = 3.14, X_3 = 4.08, X_4 = 3.14$
X_i = Space(in²) allocated to cereal i 77 Maximize $Z = 1.1X_1 + 1.3X_2 + 1.08X_3 + 1.25X_4 + 1.2X_5$ s.t. $16X_1 + 24X_2 + 18X_3 + 22X_4 + 20X_5 \leq 5000$ $X_1 \leq 100, X_2 \leq 85, X_3 \leq 140, X_4 \leq 80, X_5 \leq 90$ $X_i \geq 0 \text{ for all } i=1,2,\dots,5$ <u>Solution:</u> $Z = \$314/\text{day}$ $X_1 = 100, X_3 = 140, X_5 = 44$ $X_2 = X_4 = 0$	X_{ij} = Units of part j produced by department i, $i=1,2$ $j=1,2,3$ 79 Maximize $Z = \min\{X_{11} + X_{21}, X_{12} + X_{22}, X_{13} + X_{23}\}$ or Maximize $Z = Y$ s.t. $Y \leq X_{11} + X_{21}$ $Y \leq X_{12} + X_{22}$ $Y \leq X_{13} + X_{23}$ $\frac{X_{11}}{8} + \frac{X_{12}}{5} + \frac{X_{13}}{10} \leq 100$ $\frac{X_{21}}{6} + \frac{X_{22}}{12} + \frac{X_{23}}{4} \leq 80$ $\text{all } X_{ij} \geq 0$ <u>Solution:</u> $\text{Nbr. of assembly units} = Y = 556.2 \approx 557$ $X_{11} = 354.78, X_{21} = 201.79$ $X_{12} = 0, X_{22} = 556.52$ $X_{13} = 556.52, X_{23} = 0$ X_i = tons of coal i, $i=1,2,3$ 80 Minimize $Z = 30X_1 + 35X_2 + 33X_3$ s.t. $2500X_1 + 1500X_2 + 1600X_3 \leq 2000(X_1 + X_2 + X_3)$ $X_1 \leq 30, X_2 \leq 30, X_3 \leq 30$ $X_1 + X_2 + X_3 \geq 50$ <u>Solution:</u> $Z = \\$1361.11$ $X_1 = 22.22 \text{ tons}, X_2 = 0, X_3 = 27.78 \text{ tons.}$

t_i = Green time in secs for highway i ,
 $i = 1, 2, 3$

Maximize $Z = 3\left(\frac{500}{3600}\right)t_1 + 4\left(\frac{600}{3600}\right)t_2 + 5\left(\frac{400}{3600}\right)t_3$

s.t.

$$\left(\frac{500}{3600}\right)t_1 + \left(\frac{600}{3600}\right)t_2 + \left(\frac{400}{3600}\right)t_3 \leq \frac{510}{3600} (2.2 \times 60 - 3 \times 10)$$

$$t_1 + t_2 + t_3 + 3 \times 10 \leq 2.2 \times 60, t_1 \geq 25, t_2 \geq 25, t_3 \geq 25$$

Solution: $Z = \$58.04/\text{hr}$

$$t_1 = 25, t_2 = 43.6, t_3 = 33.4 \text{ Sec}$$

y_i = observation i

Define straight line as

$$\hat{y}_i = a + b, a, b \text{ unrestricted}$$

Minimize $Z = \sum_{i=1}^{10} |y_i - \hat{y}_i|$

$$= \sum_{i=1}^{10} |y_i - a_i - b|$$

Let $d_i = |y_i - a_i - b|$

Minimize $Z = d_1 + d_2 + \dots + d_{10}$

s.t.

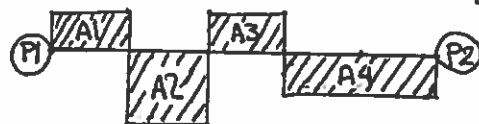
$$y_i - a_i - b \leq d_i$$

$$y_i - a_i - b \geq -d_i$$

$$a, b, \text{ unrestricted}$$

$$d_i \geq 0$$

Solution: $\hat{y}_i = 2.95714i + 6.42857$



$A1 = 2 \times 1760 \times 10 \times 50 = 1760 \text{ (thousand) Yd}^3$

$A2 = 3520, A3 = 1760, A4 = 3520$

Distances (center to center) in miles:

	A2	A4
A1	2	7
A3	2	3
P1	3	8
P2	7	2

continued...

Cost (\$) per cubic yd:

	(5) A2	(6) A4
(1) A1	$.2 + 2 \times .15 = .50$	$.20 + 7 \times .15 = 1.25$
(2) A3	$.20 + 2 \times .15 = .50$	$.20 + 3 \times .15 = .65$
(3) P1	$1.70 + 3 \times .15 = 2.15$	$1.70 + 8 \times .15 = 2.90$
(4) P3	$2.10 + 7 \times .15 = 3.15$	$2.10 + 2 \times .15 = 2.40$

Using the code $A1 \equiv 1, A3 \equiv 2, P1 \equiv 3, P2 \equiv 4, A2 \equiv 5, A4 \equiv 6$, let

$x_{ij} = 10^3 \text{ Yd}^3$ from source i to destination j
 $i = 1, 2, 3, 4, j = 5, 6$

Minimize $Z = 1000(.5x_{15} + 1.25x_{16} + .5x_{25} + .65x_{26} + 2.15x_{35} + 2.9x_{36} + 3.15x_{45} + 2.4x_{46})$

s.t.

$$x_{15} + x_{16} \leq 1760 \quad x_{35} + x_{36} \leq 20,000$$

$$x_{25} + x_{26} \leq 1760 \quad x_{45} + x_{46} \leq 15,000$$

$$x_{15} + x_{25} + x_{35} + x_{45} \geq 3520$$

$$x_{16} + x_{26} + x_{36} + x_{46} \geq 3520$$

Solution:

$$A1 \rightarrow A2: x_{15} = 1760 \text{ (1000 Cu Yd)}$$

$$A1 \rightarrow A4: x_{16} = 0$$

$$A3 \rightarrow A2: x_{25} = 0$$

$$A3 \rightarrow A4: x_{26} = 1760$$

$$P1 \rightarrow A2: x_{35} = 1760$$

$$P1 \rightarrow A4: x_{36} = 0$$

$$P2 \rightarrow A2: x_{45} = 0$$

$$P2 \rightarrow A4: x_{46} = 1760$$

$$\text{Cost} = \$10,032,000$$

x_{ij} = Blue regulars on front i in defense line $j, i =$

y_{ij} = Blue reserves on front i in defense line j .

t_{ij} = Delay days on front i in defense line j .

Maximize $Z = \min \{t_{11} + t_{12} + t_{13}, t_{21} + t_{22} + t_{23}\}$

or

continued...

← I →
← II →
← III →

Maximize $Z = T$
s.t.

$$T \leq t_{11} + t_{12} + t_{13}$$

$$T \leq t_{21} + t_{22} + t_{23}$$

$$x_{11} + x_{12} + x_{13} + x_{21} + x_{22} + x_{23} \leq 200$$

$$y_{12} + y_{13} + y_{22} + y_{23} \leq 200$$

$$t_{11} = .5 + 8.8 \frac{x_{11}}{30}$$

$$t_{12} = .75 + 7.9 \frac{x_{12} + y_{12}}{60}$$

$$t_{13} = .55 + 10.2 \frac{x_{13} + y_{13}}{20}$$

$$t_{21} = 1.1 + 10.5 \frac{x_{21}}{30}$$

$$t_{22} = 1.3 + 8.1 \frac{x_{22} + y_{22}}{40}$$

$$t_{23} = 1.5 + 9.2 \frac{x_{23} + y_{23}}{20}$$

Solution: Battle duration = 87.65 days

continued...

 $x_i = \text{Efficiency of plant } i$

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**Minimize $Z = .2(500)x_1 + .25(3000)x_2$
 $+ .15(6000)x_3 + .18(10000)x_4$**

s.t.

$$500(1-x_1) \leq .00085 \times 215,000$$

$$.94(500)(1-x_1) + 3000(1-x_2) \leq .0009 \times 220,000$$

$$.94^2(500)(1-x_1) + .94(3000)(1-x_2) + 6000(1-x_3) \leq .0008 \times 200,000$$

$$.94^3(500)(1-x_1) + .94^2(3000)(1-x_2) + .94(6000)(1-x_3) + 10000(1-x_4) \leq .0008 \times 210,000$$

$$0 \leq x_1 \leq .99$$

$$0 \leq x_2 \leq .99$$

$$0 \leq x_3 \leq .99$$

$$0 \leq x_4 \leq .99$$

Solution

$$\text{Cost per ton} = \$1891.41$$

$$\text{Plant 1 efficiency} = .99$$

$$\text{Plant 2 efficiency} = .9661$$

$$\text{Plant 3 efficiency} = .99$$

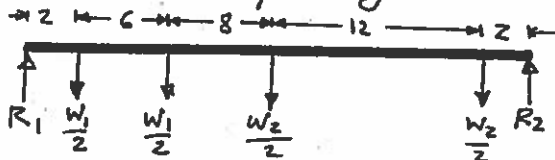
$$\text{Plant 4 efficiency} = .9824$$

Chapter 2

W_i = Capacity of Yoke i (Kips)

R_1 = Reaction in Kips at left end

R_2 = Reaction in Kips at right end



Maximize $Z = W_1 + W_2$

s.t.

$$R_1 + R_2 = W_1 + W_2$$

$$2\left(\frac{W_1}{2}\right) + 8\left(\frac{W_1}{2}\right) + 16\left(\frac{W_2}{2}\right) + 28\left(\frac{W_2}{2}\right) = 30 R_2$$

$$R_1 \leq 25, \quad R_2 \leq 25$$

$$\frac{W_1}{2} \leq 20, \quad \frac{W_2}{2} \leq 20$$

Solution:

$$W_1 = 20.59 \text{ Kips}$$

$$W_2 = 29.41 \text{ Kips}$$

X_{ij} = Nbr. of aircraft of type i
allocated to route j
($i = 1, 2, 3, 4$, $j = 1, 2, 3, 4$)

S_j = Nbr. of passengers not served on
route j , $j = 1, 2, 3, 4$

$$\begin{aligned} \text{Minimize } Z = & 1000(3X_{11}) + 1100(2X_{12}) \\ & + 1200(2X_{13}) + 1500(X_{14}) \\ & + 800(4X_{21}) + 900(3X_{22}) \\ & + 1000(3X_{23}) + 1000(2X_{24}) \\ & + 600(5X_{31}) + 800(5X_{32}) \\ & + 800(4X_{33}) + 900(2X_{34}) \\ & + 40S_1 + 50S_2 + 45S_3 + 70S_4 \end{aligned}$$

subject to

$$\sum_{j=1}^4 X_{1j} \leq 5, \quad \sum_{j=1}^4 X_{2j} \leq 8, \quad \sum_{j=1}^4 X_{3j} \leq 10$$

$$50(3X_{11}) + 30(4X_{21}) + 20(5X_{31}) + S_1 = 1000$$

$$50(2X_{12}) + 30(3X_{22}) + 20(5X_{32}) + S_2 = 2000$$

$$50(2X_{13}) + 30(3X_{23}) + 20(4X_{33}) + S_3 = 900$$

$$50(X_{14}) + 30(2X_{24}) + 20(2X_{34}) + S_4 = 1200$$

$$\text{All } X_{ij} \text{ and } S_j \geq 0$$

continued...

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*** OPTIMUM SOLUTION SUMMARY ***

Titiles Problem 26a-16

Final Iteration No: 16

Objective value (min) = 221900.0000

*** ALTERNATIVE solution detected at x13

Variable	Value	Obj Coeff	Obj Val Contrib
x1 x11	5.0000	3000.0000	14999.9990
x2 x12	0.0000	2200.0000	0.0000
x3 x13	0.0000	2400.0000	0.0000
x4 x14	0.0000	1500.0000	0.0000
x5 x21	0.0000	3200.0000	0.0000
x6 x22	0.0000	2700.0000	0.0000
x7 x23	0.0000	3000.0000	0.0000
x8 x24	8.0000	2000.0000	15999.9990
x9 x31	2.5000	3000.0000	7500.0015
x10 x32	7.5000	4000.0000	29999.9980
x11 x33	0.0000	3200.0000	0.0000
x12 x34	0.0000	1800.0000	0.0000
x13 s1	0.0000	40.0000	0.0000
x14 s2	1250.0000	50.0000	62500.0000
x15 s3	899.9998	45.0000	40499.9922
x16 s4	720.0001	70.0000	50400.0078

Constraint	RHS	Slack(-)/Surplus(+)
1 (=)	5.0000	0.0000-
2 (=)	8.0000	0.0000-
3 (=)	10.0000	0.0000-
4 (=)	1000.0000	0.0000
5 (=)	2000.0000	0.0000
6 (=)	900.0000	0.0000
7 (=)	1200.0000	0.0000

Solution:

Aircraft Type	Route	Nbr. aircraft
1	1	5
2	4	8
3	1	2.5
3	2	7.5

Fractional solution must be rounded.

Cost = \$ 221,900

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