

Chapter 2 Solutions

$$\begin{aligned}
 2.1 \quad \frac{\partial^2 \psi}{\partial z^2} &= \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} \\
 \frac{\partial \psi}{\partial z} &= 2(z + vt) \\
 \frac{\partial^2 \psi}{\partial z^2} &= 2 \\
 \frac{\partial \psi}{\partial t} &= 2v(z + vt) \\
 \frac{\partial^2 \psi}{\partial t^2} &= 2v^2
 \end{aligned}$$

It's a twice differentiable function of $(z - vt)$, where v is in the negative z direction.

$$\begin{aligned}
 2.2 \quad \frac{\partial^2 \psi}{\partial y^2} &= \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} \\
 \psi(y, t) &= (y - 4t)^2 \\
 \frac{\partial \psi}{\partial y} &= 2(y - 4t) \\
 \frac{\partial^2 \psi}{\partial y^2} &= 2 \\
 \frac{\partial \psi}{\partial t} &= -8(y - 4t) \\
 \frac{\partial^2 \psi}{\partial t^2} &= 32
 \end{aligned}$$

Thus, $v = 4$, $v^2 = 16$, and,

$$\frac{\partial^2 \psi}{\partial y^2} = 2 = \frac{1}{16} \frac{\partial^2 \psi}{\partial t^2}$$

The velocity is $v = 4$ in the positive y direction.

2.3 Starting with:

$$\begin{aligned}
 \psi(z, t) &= \frac{A}{(z - vt)^2 + 1} \\
 \frac{\partial^2 \psi}{\partial z^2} &= \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} \\
 \frac{\partial \psi}{\partial z} &= -2A \frac{(z - vt)}{[(z - vt)^2 + 1]} \\
 \frac{\partial^2 \psi}{\partial z^2} &= -2A \left[\frac{-2(z - vt)}{[(z - vt)^2 + 1]^3} + \frac{1}{[(z - vt)^2 + 1]^2} \right] \\
 &= -2A \left[\frac{-4(z - vt)}{[(z - vt)^2 + 1]^3} + \frac{(z - vt)^2 + 1}{[(z - vt)^2 + 1]^3} \right] \\
 &= 2A \frac{3(z - vt)^2 - 1}{[(z - vt)^2 + 1]^3}
 \end{aligned}$$

$$\begin{aligned}
\frac{\partial \psi}{\partial t} &= 2Av \frac{(z-vt)}{[(z-vt)^2 + 1]^2} \\
\frac{\partial^2 \psi}{\partial t^2} &= 2Av \frac{\partial}{\partial t} \left(\frac{(z-vt)}{[(z-vt)^2 + 1]^2} \right) \\
&= 2Av \left[\frac{-v}{[(z-vt)^2 + 1]^2} + (z-vt) \frac{4v(z-vt)}{[(z-vt)^2 + 1]^3} \right] \\
&= 2Av \left[\frac{-v[(z-vt)^2 + 1]}{[(z-vt)^2 + 1]^2} + \frac{4v(z-vt)^2}{[(z-vt)^2 + 1]^3} \right] \\
&= 2Av^2 \frac{3(z-vt)^2 - 1}{[(z-vt)^2 + 1]^3}
\end{aligned}$$

Thus since

$$\frac{\partial^2 \psi}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$$

The wave moves with velocity v in the positive z direction.

2.4 $c = v\lambda$

$$v = \frac{c}{\lambda} = \frac{3 \times 10^8 \text{ m/s}}{5.145 \times 10^{-7} \text{ m}} = 5.831 \times 10^{14} \text{ Hz}$$

2.5 Starting with:

$$\psi(y, t) = A \exp[-a(by-ct)^2]$$

$$\psi(y, t) = A \exp[a(by-ct)^2] = A \exp[a(by-ct)^2]$$

$$\frac{\partial \psi}{\partial t} = -\frac{2Aa}{b^2} \frac{c}{b} \left(y - \frac{c}{b}t \right) \exp[a(by-ct)^2]$$

$$\frac{\partial^2 \psi}{\partial t^2} = \frac{4Aa^2}{b^4} \frac{c^2}{b^2} \left(y - \frac{c}{b}t \right)^2 \exp[a(by-ct)^2]$$

$$\frac{\partial \psi}{\partial y} = -\frac{2Aa}{b^2} \left(y - \frac{c}{b}t \right) \exp[a(by-ct)^2]$$

$$\frac{\partial^2 \psi}{\partial y^2} = \frac{4Aa^2}{b^4} \left(y - \frac{c}{b}t \right)^2 \exp[a(by-ct)^2]$$

Thus $\psi(y, t) = A \exp[-a(by-ct)^2]$ is a solution of the wave equation with $v = c/b$ in the $+y$ direction.

2.6 $(0.003) (2.54 \times 10^{-2} / 580 \times 10^{-9}) = \text{number of waves} = 131, c = v\lambda,$

$$\lambda = c/v = 3 \times 10^8 / 10^{10}, \lambda = 3 \text{ cm. Waves extend } 3.9 \text{ m.}$$

2.7 $\lambda = c/v = 3 \times 10^8 / 5 \times 10^{14} = 6 \times 10^{-7} \text{ m} = 0.6 \mu \text{ m.}$

$$\lambda = 3 \times 10^8 / 60 = 5 \times 10^6 \text{ m} = 5 \times 10^3 \text{ km.}$$

2.8 $v = \lambda \nu = 5 \times 10^{-7} \times 6 \times 10^8 = 300 \text{ m/s.}$

2.9 The time between the crests is the period, so $\tau = 1/2$ s; hence
 $\nu = 1/\tau = 2.0$ Hz. As for the speed $v = L/t = 4.5 \text{ m}/1.5 \text{ s} = 3.0 \text{ m/s}$. We
 now know τ , ν , and v and must determine λ . Thus,
 $\lambda = v/\nu = 3.0 \text{ m/s}/2.0 \text{ Hz} = 1.5 \text{ m}$.

2.10 $v = \nu\lambda = 3.5 \times 10^3 \text{ m/s} = \nu(4.3 \text{ m})$; $\nu = 0.81 \text{ kHz}$.

2.11 $v = \nu\lambda = 1498 \text{ m/s} = (440 \text{ Hz}) \lambda$; $\lambda = 3.40 \text{ m}$.

2.12 $v = (10 \text{ m})/2.0 \text{ s} = 5.0 \text{ m/s}$; $\nu = v/\lambda = (5.0 \text{ m/s})/(0.50 \text{ m}) = 10 \text{ Hz}$.

2.13 $v = \nu\lambda = (\omega/2\pi) \lambda$ and so $\omega = (2\pi/\lambda) v$.

2.14

θ	$-\pi/2$	$-\pi/4$	0	$\pi/4$	$\pi/2$	$3\pi/4$
$\sin \theta$	-1	$-\sqrt{2}/2$	0	$\sqrt{2}/2$	1	$\sqrt{2}/2$
$\cos \theta$	0	$\sqrt{2}/2$	1	$\sqrt{2}/2$	0	$-\sqrt{2}/2$
$\sin(\theta - \pi/4)$	$-\sqrt{2}/2$	-1	$-\sqrt{2}/2$	0	$\sqrt{2}/2$	1
$\sin(\theta - \pi/2)$	0	$-\sqrt{2}/2$	-1	$-\sqrt{2}/2$	0	$\sqrt{2}/2$
$\sin(\theta - 3\pi/4)$	$\sqrt{2}/2$	0	$-\sqrt{2}/2$	-1	$-\sqrt{2}/2$	0
$\sin(\theta + \pi/2)$	0	$\sqrt{2}/2$	1	$\sqrt{2}/2$	0	$-\sqrt{2}/2$

θ	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
$\sin \theta$	0	$-\sqrt{2}/2$	-1	$-\sqrt{2}/2$	0
$\cos \theta$	-1	$-\sqrt{2}/2$	0	$\sqrt{2}/2$	1
$\sin(\theta - \pi/4)$	$\sqrt{2}/2$	0	$-\sqrt{2}/2$	-1	$-\sqrt{2}/2$
$\sin(\theta - \pi/2)$	1	$\sqrt{2}/2$	0	$-\sqrt{2}/2$	-1
$\sin(\theta - 3\pi/4)$	$\sqrt{2}/2$	1	$\sqrt{2}/2$	0	$-\sqrt{2}/2$
$\sin(\theta + \pi/2)$	-1	$-\sqrt{2}/2$	0	$\sqrt{2}/2$	1

$\sin \theta$ leads $\sin(\theta - \pi/2)$.

2.15

x	$-\lambda/2$	$-\lambda/4$	0	$\lambda/4$	$\lambda/2$	$3\lambda/4$	λ
$kx = \frac{2\pi x}{\lambda}$	$-\pi$	$-\pi/2$	0	$\pi/2$	π	$3\pi/2$	2π
$\cos(kx - \pi/4)$	$-\sqrt{2}/2$	$-\sqrt{2}/2$	$\sqrt{2}/2$	$\sqrt{2}/2$	$-\sqrt{2}/2$	$-\sqrt{2}/2$	$\sqrt{2}/2$
$\cos(kx + 3\pi/4)$	$\sqrt{2}/2$	$\sqrt{2}/2$	$-\sqrt{2}/2$	$-\sqrt{2}/2$	$\sqrt{2}/2$	$\sqrt{2}/2$	$-\sqrt{2}/2$

2.16

t	$-\tau/2$	$-\tau/4$	0	$\tau/4$	$\tau/2$	$3\tau/4$	τ
$\omega t = (2\pi/\tau)t$	$-\pi$	$-\pi/2$	0	$\pi/2$	π	$3\pi/2$	2π
$\sin(\omega t + \pi/4)$	$-\sqrt{2}/2$	$-\sqrt{2}/2$	$\sqrt{2}/2$	$\sqrt{2}/2$	$-\sqrt{2}/2$	$-\sqrt{2}/2$	$\sqrt{2}/2$
$\sin(\pi/4 - \omega t)$	$-\sqrt{2}/2$	$\sqrt{2}/2$	$\sqrt{2}/2$	$-\sqrt{2}/2$	$-\sqrt{2}/2$	$\sqrt{2}/2$	$\sqrt{2}/2$

- 2.17** Comparing y with Eq. (2.13) tells us that $A = 0.02$ m. Moreover, $2\pi/\lambda = 157 \text{ m}^{-1}$ and so $\lambda = 2\pi/(157 \text{ m}^{-1}) = 0.0400$ m. The relationship between frequency and wavelength is $v = \nu\lambda$, and so $\nu = v/\lambda = (1.2 \text{ m/s})/0.0400 \text{ m} = 30 \text{ Hz}$. The period is the inverse of the frequency, and therefore $\tau = 1/\nu = 0.033$ s.
- 2.18** (a) $\lambda = (4.0 - 0.0) \text{ m} = 4.0 \text{ m}$
 (b) $v = \nu\lambda$, so

$$\nu = \frac{v}{\lambda} = \frac{20 \text{ m/s}}{4.0 \text{ m}} = 5.0 \text{ Hz}$$

 (c) $\psi(x, t) = A \sin(kx - \omega t + \varepsilon)$
 From the figure, $A = 0.020$ m
 $k = \frac{2\pi}{\lambda} = \frac{2\pi}{4.0 \text{ m}} = 0.5\pi \text{ m}^{-1}$; $\omega = 2\pi\nu = 2\pi(5.0 \text{ Hz}) = 10\pi \text{ rad/s}$

$$\psi(x, t) = [0.020 \text{ m}] \sin\left(\frac{\pi}{2}x - 10\pi t - \frac{\pi}{2}\right) = 0.020 \cos\left(\frac{\pi}{2}x - 10\pi t\right)$$
- 2.19** (a) $\lambda = (30.0 - 0.0) \text{ cm} = 30.0 \text{ cm}$. (c) $v = \nu\lambda$, so
 $\nu = v/\lambda = (100 \text{ cm/s})/(30.0 \text{ cm}) = 3.33 \text{ Hz}$
- 2.20** (a) $\tau = (0.20 - 0.00) \text{ s} = 0.20 \text{ s}$. (b) $\nu = 1/\tau = 1/(0.20 \text{ s}) = 5.00 \text{ Hz}$.
 (c) $v = \nu\lambda$, so $\lambda = v/\nu = (40.0 \text{ cm/s})/(5.00 \text{ s}^{-1}) = 8.00 \text{ cm}$.
- 2.21** $\psi = A \sin 2\pi(kx - \nu t)$, $\psi_1 = 4\sin 2\pi(0.2x - 3t)$. (a) $\nu = 3$, (b) $\lambda = 1/0.2$,
 (c) $\tau = 1/3$, (d) $A = 4$, (e) $v = 15$, (f) positive x
 $\psi = A \sin(kx + \omega t)$, $\psi_2 = (1/2.5) \sin(7x + 3.5t)$. (a) $\nu = 3.5/2\pi$,
 (b) $\lambda = 2\pi/7$, (c) $\tau = 2\pi/3.5$, (d) $A = 1/2.5$, (e) $v = 1/2$, (f) negative x
- 2.22** From of Eq. (2.26) $\psi(x, t) = A \sin(kx - \omega t)$ (a) $\omega = 2\pi\nu$, so
 $\nu = \omega/2\pi = (20.0 \text{ rad/s})/2\pi$, (b) $k = 2\pi/\lambda$, so
 $\lambda = 2\pi/k = 2\pi/(6.28 \text{ rad/m}) = 1.00 \text{ m}$, (c) $\nu = 1/\tau$, so
 $\tau = 1/\nu = 1/(10.0/\pi \text{ Hz}) = 0.10\pi \text{ s}$, (d) From the form of ψ , $A = 30.0 \text{ cm}$,
 (e) $v = \omega/k = (20.0 \text{ rad/s})/(6.28 \text{ rad/m}) = 3.18 \text{ m/s}$, (f) Negative sign indicates motion in $+x$ direction.
- 2.23** (a) 10, (b) $5.0 \times 10^{14} \text{ Hz}$, (c) $\lambda = \frac{c}{\nu} = \frac{3.0 \times 10^8}{5.0 \times 10^{14}} = 6.0 \times 10^{-7} \text{ m}$, (d) $3.0 \times 10^8 \text{ m/s}$,
 (e) $\frac{1}{\nu} = \tau = 2.0 \times 10^{-15} \text{ s}$, (f) $-y$ direction
- 2.24** $\partial^2\psi/\partial x^2 = -k^2\psi$ and $\partial^2\psi/\partial t^2 = -k^2v^2\psi$. Therefore
 $\partial^2\psi/\partial x^2 - (1/v^2)\partial^2\psi/\partial t^2 = (-k^2 + k^2)\psi = 0$.
- 2.25** $\partial^2\psi/\partial x^2 = -k^2\psi$; $\partial^2\psi/\partial t^2 = -\omega^2\psi$; $\omega^2/v^2 = (2\pi\nu)^2/v^2 = (2\pi/\lambda)^2 = k^2$;
 therefore, $\partial^2\psi/\partial x^2 - (1/v^2)\partial^2\psi/\partial t^2 = (-k^2 + k^2)\psi = 0$.
- 2.26** $\psi(x, t) = A \cos(kx - \omega t - (\pi/2)) =$
 $A\{\cos(kx - \omega t) \cos(-\pi/2) - \sin(kx - \omega t) \sin(-\pi/2)\} = A \sin(kx - \omega t)$
- 2.27** $u_y = -\omega A \cos(kx - \omega t + \varepsilon)$, $a_y = -\omega^2 y$. Simple harmonic motion since $a_y \propto y$.

- 2.28** $\tau = 2.2 \times 10^{-15}$ s; therefore $\nu = 1/\tau = 4.5 \times 10^{14}$ Hz; $\nu = \nu\lambda$,
 $\lambda = \nu/\nu = 6.7 \times 10^{-7}$ m and $k = 2\pi/\lambda = 9.4 \times 10^6$ m⁻¹.
 $\psi(x, t) = (10^3 \text{ V/m}) \cos[9.4 \times 10^6 \text{ m}^{-1}(x + 3 \times 10^8 \text{ (m/s)}t)]$. It's cosine
because $\cos 0 = 1$.
- 2.29** $y(x, t) = C/[2 + (x + \nu t)^2]$.
- 2.30** $\psi(0, t) = A \cos(k\nu t + \pi) = -A \cos(k\nu t) = -A \cos(\omega t)$, then
 $\psi(0, \pi/2) = -A \cos(\omega\pi/2) = -A \cos(\pi) = A$,
 $\psi(0, 3\pi/4) = -A \cos(3\omega\pi/4) = -A \cos(3\pi/2) = 0$.
- 2.31** Since $\psi(y, t) = (y - \nu t)A$ is only a function of $(y - \nu t)$, it does satisfy the
conditions set down for a wave. Since $\partial^2 \psi / \partial y^2 = \partial^2 \psi / \partial t^2 = 0$, this
function is a solution of the wave equation. However, $\psi(y, 0) = Ay$ is
unbounded, so cannot represent a localized wave profile.
- 2.32** $k = \pi 3 \times 10^6$ m⁻¹, $\omega = \pi 9 \times 10^{14}$ Hz, $\nu = \omega/k = 3 \times 10^8$ m/s.
- 2.33** $\nu = \nu\lambda = \lambda/\tau$
 $\lambda = \nu\tau = (2.0 \text{ m/s})(1/4 \text{ s}) = 0.5 \text{ m}$
 $\psi(z, t) = (0.020 \text{ m}) \sin 2\pi \left(\frac{z}{0.50 \text{ m}} + \frac{t}{1/4 \text{ s}} \right)$
 $\psi(z, t) = (0.020 \text{ m}) \sin 2\pi \left(\frac{1.5 \text{ m}}{0.50 \text{ m}} + \frac{2.2 \text{ s}}{1/4 \text{ s}} \right)$
 $\psi(z, t) = (0.020 \text{ m}) \sin 2\pi(3.0 + 8.8)$
 $\psi(z, t) = (0.020 \text{ m}) \sin 2\pi(11.8)$
 $\psi(z, t) = (0.020 \text{ m}) \sin 23.6\pi$
 $\psi(z, t) = (0.020 \text{ m}) (-0.9511)$
 $\psi(z, t) = -0.019 \text{ m}$
- 2.34** $d\psi/dt = (\partial\psi/\partial x)(dx/dt) + (\partial\psi/\partial y)(dy/dt)$ and let $y = t$ whereupon
 $d\psi/dt = \partial\psi/\partial x(\pm\nu) + \partial\psi/\partial t = 0$ and the desired result follows immediately.
- 2.35** $d\phi/dt = (\partial\phi/\partial x)(dx/dt) + \partial\phi/\partial t = 0 = k(dx/dt) - k\nu$ and this is zero
provided $dx/dt = \pm\nu$, as it should be. For the particular wave of
Problem 2.32, $\frac{d\phi}{dt} = \partial\phi/\partial y(\pm\nu) + \partial\phi/\partial t = \pi 3 \times 10^6 (\pm\nu) + \pi 9 \times 10^{14} = 0$
and the speed is -3×10^8 m/s.
- 2.36** $-a(bx + ct)^2 = -ab^2(x + ct/b)^2 = g(x + \nu t)$ and so $\nu = c/b$ and the wave
travels in the negative x -direction. Using Eq. (2.34) $(\partial\psi/\partial t)_x / (\partial\psi/\partial x)_t =$
 $-[A(-2a)(bx + ct)c \exp[-a(bx + ct)^2]] / [A(-2a)(bx + ct)b \exp[-a(bx + ct)^2]] = -c/b$;
the minus sign tells us that the motion is in the negative x -direction.
- 2.37** $\psi(z, 0) = A \sin(kz + \epsilon)$; $\psi(-\lambda/12, 0) = A \sin(-\pi/6 + \epsilon) = 0.866$;
 $\psi(\lambda/6, 0) = A \sin(\pi/3 + \epsilon) = 1/2$; $\psi(\lambda/4, 0) = A \sin(\pi/2 + \epsilon) = 0$.
 $A \sin(\pi/2 + \epsilon) = A(\sin \pi/2 \cos \epsilon + \cos \pi/2 \sin \epsilon) = A \cos \epsilon = 0$, $\epsilon = \pi/2$.
 $A \sin(\pi/3 + \pi/2) = A \sin(5\pi/6) = 1/2$; therefore $A = 1$, hence
 $\psi(z, 0) = \sin(kz + \pi/2)$.
- 2.38** Both (a) and (b) are waves since they are twice differentiable functions of
 $z - \nu t$ and $x + \nu t$, respectively. Thus for (a) $\psi = a^2(z - bt/a)^2$ and the
velocity is b/a in the positive z -direction. For (b) $\psi = a^2(x + bt/a + c/a)^2$
and the velocity is b/a in the negative x -direction.

- 2.39** (a) $\psi(y, t) = \exp[-(ay - bt)^2]$, a traveling wave in the +y direction, with speed $v = \omega/k = b/a$. (b) not a traveling wave. (c) traveling wave in the -x direction, $v = a/b$, (d) traveling wave in the +x direction, $v = 1$.
- 2.40** $\psi(x, t) = 5.0 \exp[-a(x + \sqrt{b/at})^2]$, the propagation direction is negative x; $v = \sqrt{b/a} = 0.6$ m/s. $\psi(x, 0) = 5.0 \exp(-25x^2)$.
- 2.41** $\lambda = v/\nu = 0.300$ m; 10.0 cm is a fraction of a wavelength viz. $(0.100 \text{ m})/(0.300 \text{ m}) = 1/3$; hence $2\pi/3 = 2.09$ rad.
- 2.42** 30° corresponds to $\lambda/12$ or $\left(\frac{1}{12}\right)\left(\frac{3 \times 10^8}{6 \times 10^{14}}\right) = 42 \text{ nm}$.
- 2.43** $\psi(x, t) = A \sin 2\pi(x/\lambda \pm t/\tau)$, $\psi = 60 \sin 2\pi(x/400 \times 10^{-9} - t/1.33 \times 10^{-15})$, $\lambda = 400$ nm, $v = 400 \times 10^{-9}/1.33 \times 10^{-15} = 3 \times 10^8$ m/s. $\nu = (1/1.33) \times 10^{15}$ Hz, $\tau = 1.33 \times 10^{-15}$ s.
- 2.44** $\exp[i\alpha]\exp[i\beta] = (\cos \alpha + i \sin \alpha)(\cos \beta + i \sin \beta) = (\cos \alpha \cos \beta - \sin \alpha \sin \beta) + i(\sin \alpha \cos \beta + \cos \alpha \sin \beta) = \cos(\alpha + \beta) + i \sin(\alpha + \beta) = \exp[i(\alpha + \beta)]$
 $\psi\psi^* = A \exp[i\omega t] A \exp[-i\omega t] = A^2$; $\sqrt{\psi\psi^*} = A$. In terms of Euler's formula
 $\psi\psi^* = A^2(\cos \omega t + i \sin \omega t)(\cos \omega t - i \sin \omega t) = A^2(\cos^2 \omega t + \sin^2 \omega t) = A^2$.
- 2.45** If $z = x + iy$, then $z^* = x - iy$ and $z - z^* = 2yi$.
- 2.46** $\tilde{z}_1 = x_1 + iy_1$
 $\tilde{z}_2 = x_2 + iy_2$
 $\tilde{z}_1 + \tilde{z}_2 = x_1 + x_2 + iy_1 + iy_2$
 $\text{Re}(\tilde{z}_1 + \tilde{z}_2) = x_1 + x_2$
 $\text{Re}(\tilde{z}_1) + \text{Re}(\tilde{z}_2) = x_1 + x_2$
- 2.47** $\tilde{z}_1 = x_1 + iy_1$
 $\tilde{z}_2 = x_2 + iy_2$
 $\text{Re}(\tilde{z}_1) \times \text{Re}(\tilde{z}_2) = x_1 x_2$
 $\text{Re}(\tilde{z}_1 \times \tilde{z}_2) = \text{Re}(x_1 x_2 + ix_1 y_2 + ix_2 y_1 - y_1 y_2) = x_1 x_2 - y_1 y_2$
 Thus $\text{Re}(\tilde{z}_1) \times \text{Re}(\tilde{z}_2) \neq \text{Re}(\tilde{z}_1 \times \tilde{z}_2)$.
- 2.48** $\psi = A \exp i(k_x x + k_y y + k_z z)$, $k_x = k\alpha$, $k_y = k\beta$, $k_z = k\gamma$,
 $|\vec{k}| = [(k\alpha)^2 + (k\beta)^2 + (k\gamma)^2]^{1/2} = k(\alpha^2 + \beta^2 + \gamma^2)^{1/2}$.
- 2.49** Consider Eq. (2.64), with $\partial^2 \psi / \partial x^2 = \alpha^2 f''$, $\partial^2 \psi / \partial y^2 = \beta^2 f''$,
 $\partial^2 \psi / \partial z^2 = \gamma^2 f''$, $\partial^2 \psi / \partial t^2 = v^2 f''$. Then
 $\nabla^2 \psi - (1/v^2) \partial^2 \psi / \partial t^2 = (\alpha^2 + \beta^2 + \gamma^2 - 1) f'' = 0$ whenever
 $\alpha^2 + \beta^2 + \gamma^2 = 1$.

*****<<INSERT MATTER OF 2.50 IS MISSING>>*****

- 2.51** Consider the function: $\psi(z, t) = A \exp[-(a^2 z^2 + b^2 t^2 + 2abzt)]$.
Where A , a , and b are all constants. First factor the exponent:

$$(a^2 z^2 + b^2 t^2 + 2abzt) = (az + bt)^2 = \frac{1}{a^2} \left(z + \frac{b}{a} t \right)^2.$$

Thus,

$$\psi(z, t) = A \exp \left[-\frac{1}{a^2} \left(z + \frac{b}{a} t \right)^2 \right].$$

This is a twice differentiable function of $(z - vt)$, where $v = -b/a$, and travels in the $-z$ direction.

- 2.52** $\lambda = (h/m)v = 6.6 \times 10^{-34} / 6(1) = 1.1 \times 10^{-34} \text{ m}.$

- 2.53** $\vec{k}$ can be constructed by forming a unit vector in the proper direction and multiplying it by k . The unit vector is

$$[(4-0)\hat{i} + (2-0)\hat{j} + (1-0)\hat{k}] / \sqrt{4^2 + 2^2 + 1^2} = (4\hat{i} + 2\hat{j} + \hat{k}) / \sqrt{21} \text{ and}$$

$$\vec{k} = k(4\hat{i} + 2\hat{j} + \hat{k}) / \sqrt{21}. \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}, \text{ hence}$$

$$\psi(x, y, z, t) = A \sin[(4k/\sqrt{21})x + (2k/\sqrt{21})y + (k/\sqrt{21})z - \omega t].$$

- 2.54** $\vec{k} = (1\hat{i} + 0\hat{j} + 0\hat{k})$, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, so,

$$\psi = A \sin(\vec{k} \cdot \vec{r} - \omega t + \varepsilon) = A \sin(kx - \omega t + \varepsilon) \text{ where } k = 2\pi/\lambda \text{ (could use cos instead of sin).}$$

- 2.55** $\psi(\vec{r}_1, t) = \psi[\vec{r}_2 - (\vec{r}_2 - \vec{r}_1), t] = \psi(\vec{k} \cdot \vec{r}_1, t) = \psi[\vec{k} \cdot \vec{r}_2 - \vec{k} \cdot (\vec{r}_2 - \vec{r}_1), t] =$
 $\psi(\vec{k} \cdot \vec{r}_2, t) = \psi(\vec{r}_2, t) \text{ since } \vec{k} \cdot (\vec{r}_2 - \vec{r}_1) = 0$

- 2.56** $\psi = A \exp[i(\vec{k} \cdot \vec{r} + \omega t + \varepsilon)]$
 $= A \exp[i(k_x x + k_y y + k_z z + \omega t + \varepsilon)]$

The wave equation is:

$$\nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$$

where,

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\nabla^2 \psi = -(k_x^2 + k_y^2 + k_z^2) A \exp[i(k_x x + k_y y + k_z z + \omega t + \varepsilon)]$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\omega^2 A \exp[i(k_x x + k_y y + k_z z + \omega t + \varepsilon)]$$

where

$$|k| = \sqrt{k_x^2 + k_y^2 + k_z^2}$$

$$k^2 = k_x^2 + k_y^2 + k_z^2$$

then,

$$\nabla^2 \psi = -k^2 A \exp[i(k_x x + k_y y + k_z z + \omega t + \varepsilon)]$$

This means that ψ is a solution of the wave equation if $v^2 = \omega^2 / k^2 \rightarrow v = \omega / k$.

2.57

θ	$-\pi/2$	$-\pi/4$	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
$\sin \theta$	-1	$-1/\sqrt{2}$	0	$1/\sqrt{2}$	1	$1/\sqrt{2}$	0	$-1/\sqrt{2}$	-1	$-1/\sqrt{2}$	0
$2 \sin \theta$	-2	$-\sqrt{2}$	0	$\sqrt{2}$	2	$\sqrt{2}$	0	$-\sqrt{2}$	-2	$-\sqrt{2}$	0
$3 \sin \theta$	-3	$-3/\sqrt{2}$	0	$3/\sqrt{2}$	3	$3/\sqrt{2}$	0	$-3/\sqrt{2}$	-3	$-3/\sqrt{2}$	0

2.58

θ	$-\pi/2$	$-\pi/4$	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
$\sin \theta$	-1	$-1/\sqrt{2}$	0	$1/\sqrt{2}$	1	$1/\sqrt{2}$	0	$-1/\sqrt{2}$	-1	$-1/\sqrt{2}$	0
$\sin(\theta - \pi/2)$	0	$-1/\sqrt{2}$	-1	$-1/\sqrt{2}$	0	$1/\sqrt{2}$	1	$1/\sqrt{2}$	0	$-1/\sqrt{2}$	-1
$\sin \theta + \sin(\theta - \pi/2)$	-1	$-\sqrt{2}$	-1	0	1	$\sqrt{2}$	1	0	-1	$-\sqrt{2}$	-1

2.59 Note that the amplitude of $\{\sin(\theta) + \sin(\theta - \pi/2)\}$ is greater than 1, while the amplitude of $\{\sin(\theta) + \sin(\theta - 3\pi/4)\}$ is less than 1. The phase difference is $\pi/8$.

2.60

x	$-\lambda/2$	$-\lambda/4$	0	$\lambda/4$	$\lambda/2$	$3\lambda/4$	λ
kx	$-\pi$	$-\pi/2$	0	$\pi/2$	π	$3\pi/2$	2π
$\cos kx$	-1	0	1	0	-1	0	1
$\cos(kx + \pi)$	1	0	-1	0	1	0	-1
$\cos kx + \cos(kx + \pi)$	0	0	0	0	0	0	0