STUDENT SOLUTIONS MANUAL

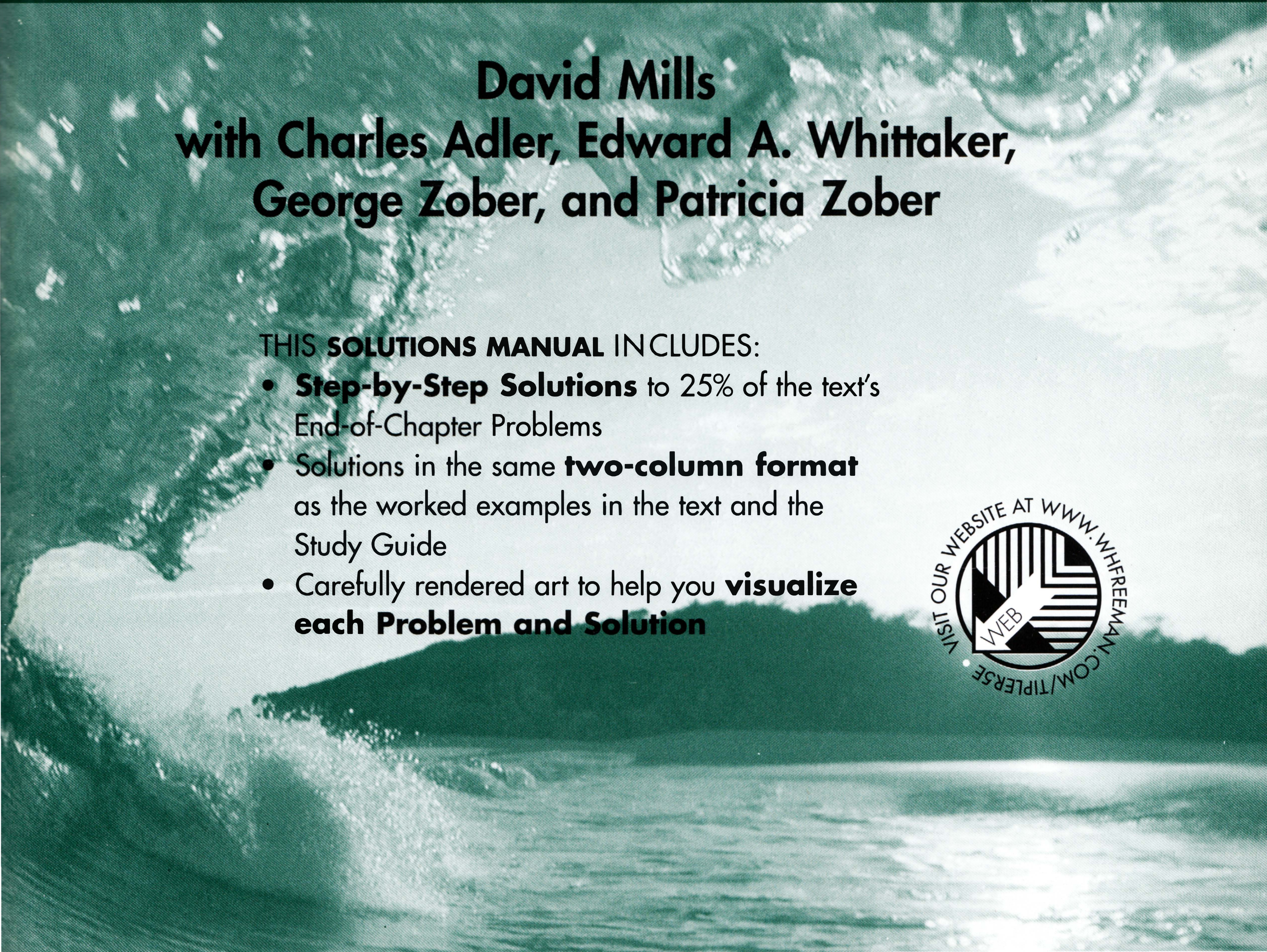
Volume 2

TO ACCOMPANY

Paul A. Tipler and Gene Mosca's

PHYSICS FOR SCIENTISTS AND ENGINEERS

FIFTH EDITION



David Mills

with Charles Adler, Edward A. Whittaker, George Zober, and Patricia Zober

THIS **SOLUTIONS MANUAL** INCLUDES:

- **Step-by-Step Solutions** to 25% of the text's End-of-Chapter Problems
- Solutions in the same **two-column format** as the worked examples in the text and the Study Guide
- Carefully rendered art to help you **visualize each Problem and Solution**

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This solutions manual doesn't just give you the answers. It shows you how to work your way through the problems. The Solutions Manual includes:

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Other great resources to help you with your course work:

Study Guide
Gene Mosca, United States Naval Academy
Todd Ruskell, Colorado School of Mines
Vol. 1, 0-7167-8332-0, Vol. 2, 0-7167-8331-2

- Begins with review of **Key Ideas and Equations** for each chapter
- Tests your knowledge of the chapter's material with **True and False Exercises** and **Short Questions and Answers**
- Provides additional **Problems and Solutions** to help you master your understanding of the chapter content

Be sure to visit the **Tipler/Mosca Student Companion Web site** at:
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Student Solutions Manual
for
Tipler and Mosca's

Physics for Scientists and Engineers

Fifth Edition

Volume 2

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Contents

To the Student

This solution manual accompanies *Physics for Scientists and Engineers, 5e*, by Paul Tipler and Gene Mosca. Following the structure of the solutions to the Worked Examples in the text, we begin the solutions to the back-of-the-chapter numerical problems with a brief discussion of the physics of the problem, represent the problem pictorially whenever appropriate, express the physics of the solution in the form of a mathematical model, fill in any intermediate steps as needed, make the appropriate substitutions and algebraic simplifications, and complete the solution with the substitution of numerical values (including their units) and the evaluation of whatever physical quantity is called for in the problem. This is the problem-solving strategy used by experienced learners of physics, and it is our hope that you will see the value in such an approach to problem solving and learn to use it consistently.

Believing that it will maximize your learning of physics, we encourage you to create your own solution before referring to the solutions in this manual. You may find that, by following this approach, you will find different, but equally valid, solutions to some of the problems. In any event, studying the solutions contained herein without having first attempted the problems will do little to help you learn physics.

You'll find that nearly all problems with numerical answers have their answers given to three significant figures. Most of the exceptions to this rule are in the solutions to the problems on Significant Figures and Order of Magnitude and the problems dealing with nuclear physics. When the nature of the problem makes it desirable to do so, we keep more than three significant figures in the answers to intermediate steps and then round to three significant figures for the final answer. Some of the Estimation and Approximation Problems have answers to fewer than three significant figures.

Physics for Scientists and Engineers, 5e includes numerous spreadsheet problems. Most of them call for the plotting of one or more graphs. The solutions to these problems were generated using Microsoft Excel and its "paste special" feature, so that you can easily make changes to the graphical parts of the solutions.

Acknowledgments

Charles L. Adler (Saint Mary's College of Maryland), Ed Whittaker (Stevens Institute of Technology, George Zober (Yough Senior High School) and Patricia Zober (Ringgold High School) are the authors of the new problems appearing in the Fifth Edition. Chuck, Ed, George, and Patricia saved me (dm) many hours of work by providing rough-draft solutions to these new problems, and I thank them for their help. Gene Mosca (United States Naval Academy and the co-author of the Fifth Edition) helped me tremendously by reviewing my work, helping me clarify many of my solutions, and providing solutions when I was unsure how best to proceed. It was a pleasure to collaborate with Gene in the creation of this solutions manual. All of us who were involved in the creation of this solutions manual hope that you will find the solutions useful in learning physics.

We want to thank Lay Nam Chang (Virginia Polytechnic Institute), Brent A. Corbin (UCLA), Alan Cresswell (Shippensburg University), Ricardo S. Decca (Indiana University–Purdue University), Michael Dubson (The University of Colorado at Boulder), David Faust (Mount Hood Community College), Philip Fraundorf (The University of Missouri–Saint Louis), Clint Harper (Moorpark College), Kristi R. G. Hendrickson (University of Puget Sound), Michael Hildreth (The University of Notre Dame), David Ingram (Ohio University), James J. Kolata (The University of Notre Dame), Eric Lane (The University of Tennessee–Chattanooga), Jerome Licini (Lehigh University), Laura McCullough (The University of Wisconsin–Stout), Carl Mungan (United States Naval Academy), Jeffrey S. Olafsen (University of Kansas), Robert Pompi (The State University of New York at Binghamton), R. J. Rollefson (Wesleyan University), Andrew Scherbakov (Georgia Institute of Technology), Bruce A. Schumm (University of Chicago), Dan Styer (Oberlin College), Daniel Marlow (Princeton University), Jeffrey Sundquist (Palm Beach Community College–South), Cyrus Taylor (Case Western Reserve University), and Fulin Zuo (University of Miami), for their reviews of the problems and their solutions.

Jerome Licini (Lehigh University), Michael Crivello (San Diego Mesa College), Paul Quinn (University of Kansas), and Daniel Lucas (University of Wisconsin–Madison) error-checked the solutions. Without their thorough and critical work, many errors would have remained to be discovered by the users of this solutions manual. Their assistance is greatly appreciated. In spite of their best efforts, there may still be errors in some of the solutions, and for those I (dm) assume full responsibility. Should you find errors or think of alternative solutions that you would like to call to my attention, I would appreciate it if you would communicate them to me by sending them to asktipler@whfreeman.com.

It was a pleasure to work with Brian Donnellan, Media and Supplements Editor for Physics, who guided us through the creation of this solution manual. Our thanks to Amanda McCorquodale and Eileen McGinnis for organizing the reviewing and error-checking process.

September 2003

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David Mills, Professor Emeritus, College of the Redwoods, retired in May of 2000 after a teaching career of 42 years. He earned his bachelor's degree at Humboldt State College, his master's degree at California State University–Hayward, and his doctoral degree at the University of Northern Colorado. His teaching career included experience with the Physical Science Study Committee materials, the Harvard Project curriculum, the Personalized System of Instruction, Microcomputer-Based Laboratory instruction, and the interactive-engagement movement in physics education. A 1996 NSF.ILI grant allowed him to transform instruction in physics at the College of the Redwoods from a traditional lecture-laboratory delivery system to one that was microcomputer based, eliminate the distinction between lecture and laboratory, and utilize interactive-engagement teaching and learning strategies. He authored the Test Bank to accompany *Physics for Scientists and Engineers, 3e* and *4e*. He now lives in Henderson, NV and is an Adjunct Professor at the Community College of Southern Nevada.

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Chapter 21
The Electric Field 1: Discrete Charge Distributions

Conceptual Problems

***1** •• Discuss the similarities and differences in the properties of electric charge and gravitational mass.

Similarities:

The force between charges and masses varies as $1/r^2$.

The force is directly proportional to the product of the charges or masses.

Differences:

There are positive and negative charges but only positive masses.

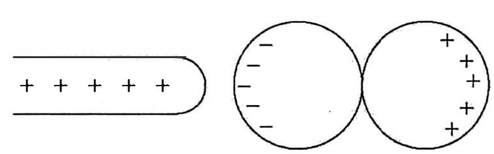
Like charges repel; like masses attract.

The gravitational constant G is many orders of magnitude smaller than the Coulomb constant k.

***5** •• Two uncharged conducting spheres with their conducting surfaces in contact are supported on a large wooden table by insulated stands. A positively charged rod is brought up close to the surface of one of the spheres on the side opposite its point of contact with the other sphere. (*a*) Describe the induced charges on the two conducting spheres, and sketch the charge distributions on them. (*b*) The two spheres are separated far apart and the charged rod is removed. Sketch the charge distributions on the separated spheres.

Determine the Concept Because the spheres are conductors, there are free electrons on them that will reposition themselves when the positively charged rod is brought nearby.

(*a*) On the sphere near the positively charged rod, the induced charge is negative and near the rod. On the other sphere, the net charge is positive and on the side far from the rod. This is shown in the diagram.



(*b*) When the spheres are separated and far apart and the rod has been removed, the induced charges are distributed uniformly over each sphere. The charge distributions are shown in the

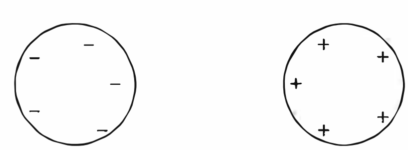


diagram.

***7** • A positive charge that is free to move but is at rest in an electric field $\vec{E}$ will
(a) accelerate in the direction perpendicular to $\vec{E}$.
(b) remain at rest.
(c) accelerate in the direction opposite to $\vec{E}$.
(d) accelerate in the same direction as $\vec{E}$.
(e) do none of the above.

Determine the Concept The acceleration of the positive charge is given by $\vec{a} = \frac{\vec{F}}{m} = \frac{q_0}{m}\vec{E}$. Because q_0 and m are both positive, the acceleration is in the same direction as the electric field. $\boxed{(d) \text{ is correct.}}$

***8** • If four charges are placed at the corners of a square as shown in Figure 21-33, the field $\vec{E}$ is zero at

(a) all points along the sides of the square midway between two charges.
(b) the midpoint of the square.
(c) midway between the top two charges and midway between the bottom two charges.
(d) none of the above.

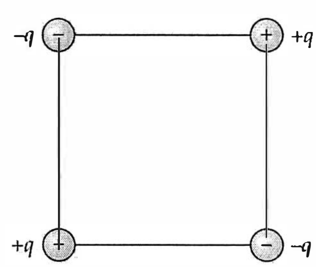
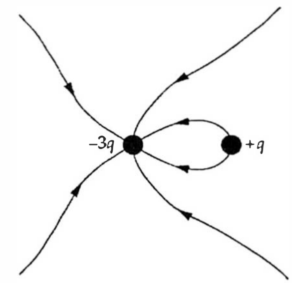


Figure 21-33 Problem 8

Determine the Concept $\vec{E}$ is zero wherever the net force acting on a test charge is zero. At the center of the square the two positive charges alone would produce a net electric field of zero, and the two negative charges alone would also produce a net electric field of zero. Thus, the net force acting on a test charge at the midpoint of the square will be zero. $\boxed{(b) \text{ is correct.}}$

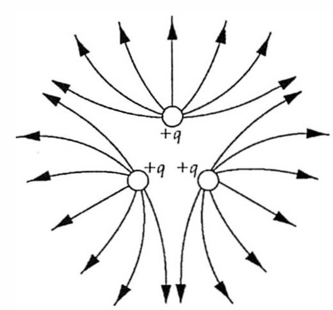
***11** • Two charges $+q$ and $-3q$ are separated by a small distance. Draw the electric field lines for this system.

Determine the Concept We can use the rules for drawing electric field lines to draw the electric field lines for this system. In the field-line sketch to the right we've assigned 2 field lines to each charge q.



***12** • Three equal positive point charges are situated at the corners of an equilateral triangle. Sketch the electric field lines in the plane of the triangle.

Determine the Concept We can use the rules for drawing electric field lines to draw the electric field lines for this system. In the field-line sketch to the right we've assigned 7 field lines to each charge q.



***14** • The electric field lines around an electrical dipole are best represented by which, if any, of the diagrams in Figure 21-34?

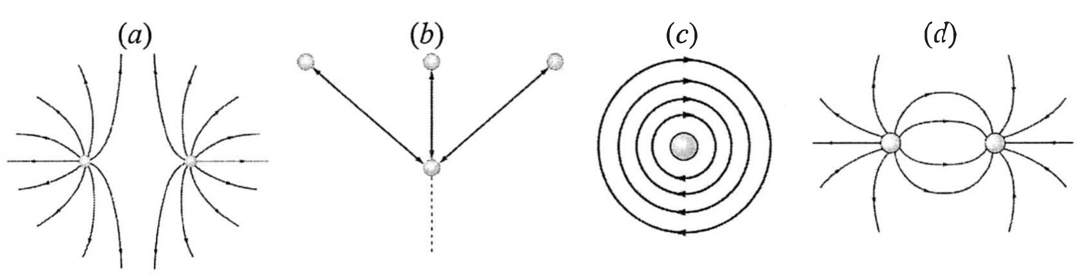


Figure 21-34 Problem 14

Determine the Concept Electric field lines around an electric dipole originate at the positive charge and terminate at the negative charge. Only the lines shown in (*d*) satisfy this requirement. $\boxed{(d)\text{ is correct.}}$

***15** •• A molecule with electric dipole moment $\vec{p}$ is oriented so that $\vec{p}$ makes an angle θ with a uniform electric field $\vec{E}$ that is in the direction of increasing x. The dipole is free to move in response to the force from the field. Describe the motion of the dipole. Suppose the electric field is nonuniform and is larger in the x direction. How will the motion be changed?

Determine the Concept Because $\theta \neq 0$, a dipole in a uniform electric field will experience a restoring torque whose magnitude is $pE_x \sin\theta$. Hence it will oscillate about its equilibrium orientation, $\theta = 0$. If $\theta << 1$, $\sin\theta \approx \theta$, and the motion will be simple harmonic motion. Because the field is nonuniform and is larger in the x direction, the force acting on the positive charge of the dipole (in the direction of increasing x) will be greater than the force acting on the negative charge of the dipole (in the direction of decreasing x) and thus there will be a net electric force on the dipole in the direction of increasing x. Hence, the dipole will accelerate in the x direction as it oscillates about $\theta = 0$.

***18** •• A metal ball is positively charged. Is it possible for it to attract another positively charged ball? Explain.

Determine the Concept Yes. A positively charged ball will induce a dipole on the metal ball, and if the two are in close proximity, the net force can be attractive.

***19** •• A simple demonstration of electrostatic attraction can be done simply by tying a small ball of tinfoil on a hanging string, and bringing a charged wand near it. Initially, the ball will be attracted to the wand, but once they touch, the ball will be repelled violently from it. Explain this behavior.

Determine the Concept Assume that the wand has a negative charge. When the charged wand is brought near the tinfoil, the side nearer the wand becomes positively charged by induction, and so it swings toward the wand. When it touches the wand, some of the negative charge is transferred to the foil, which, as a result, acquires a net negative charge and is now repelled by the wand.

Estimation and Approximation

***23** •• A popular classroom demonstration consists of rubbing a "magic wand" made of plastic with fur to charge it, and then placing it near an empty soda can on its side (Figure 21-35). The can will roll toward the wand, as it acquires a charge on the side nearest the wand by induction. Typically, if the wand is held about 10 cm away from the can, the can will have an initial acceleration of about 1 m/s^2. If the mass of the can is 0.018 kg, estimate the charge on the rod.

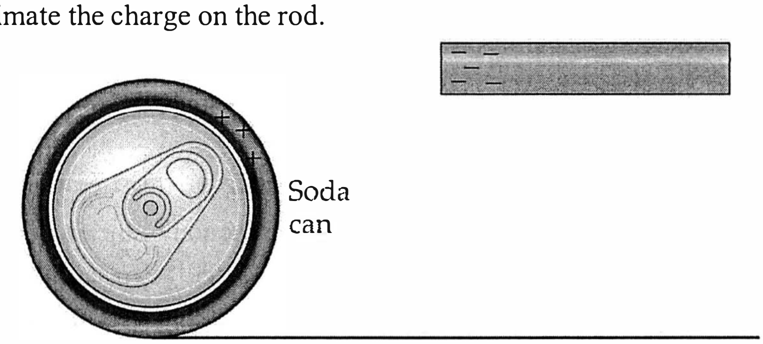


Figure 21-35 Problem 23

Picture the Problem We can use Coulomb's law to express the charge on the rod in terms of the force exerted on it by the soda can and its distance from the can. We can apply Newton's 2nd law in rotational form to the can to relate its acceleration to the electric force exerted on it by the rod. Combining these equations will yield an expression for Q as a function of the mass of the can, its distance from the rod, and its acceleration.

Use Coulomb's law to relate the force on the rod to its charge Q and distance r from the soda can:

$$F=\frac{kQ^2}{r^2}$$

Solve for Q to obtain:

$$Q=\sqrt{\frac{r^2F}{k}} \qquad (1)$$

Apply $\sum\tau_{\text{center of mass}}=I\alpha$ to the can:

$$FR=I\alpha$$

Because the can rolls without slipping, we know that its linear acceleration a and angular acceleration α are related according to:

$$\alpha=\frac{a}{R}$$

where R is the radius of the soda can.

Because the empty can is a hollow cylinder:

$$I=MR^2$$

where M is the mass of the can.

Substitute for I and α and solve for F to obtain:

$$F=\frac{MR^2a}{R^2}=Ma$$

Substitute for F in equation (1):

$$Q=\sqrt{\frac{r^2Ma}{k}}$$

Substitute numerical values and evaluate Q:

$$Q=\sqrt{\frac{(0.1\,\text{m})^2(0.018\,\text{kg})(1\,\text{m/s}^2)}{8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2}}=\boxed{141\,\text{nC}}$$

Electric Charge

***27** • How many coulombs of positive charge are there in 1 kg of carbon? Twelve grams of carbon contain Avogadro's number of atoms, with each atom having six protons and six electrons.

Picture the Problem We can find the number of coulombs of positive charge there are in 1 kg of carbon from $Q=6n_\text{C}e$, where n_C is the number of atoms in 1 kg of carbon and the factor of 6 is present to account for the presence of 6 protons in each atom. We can find the number of atoms in 1kg of carbon by setting up a proportion relating Avogadro's number, the mass of carbon, and the molecular mass of carbon to n_C.

Express the positive charge in terms of the electronic charge, the number of protons per atom, and the number of atoms in 1 kg of carbon:

$$Q = 6n_C e$$

Using a proportion, relate the number of atoms in 1 kg of carbon n_C, to Avogadro's number and the molecular mass M of carbon:

$$\frac{n_C}{N_A} = \frac{m_C}{M} \Rightarrow n_C = \frac{N_A m_C}{M}$$

Substitute to obtain:

$$Q = \frac{6N_A m_C e}{M}$$

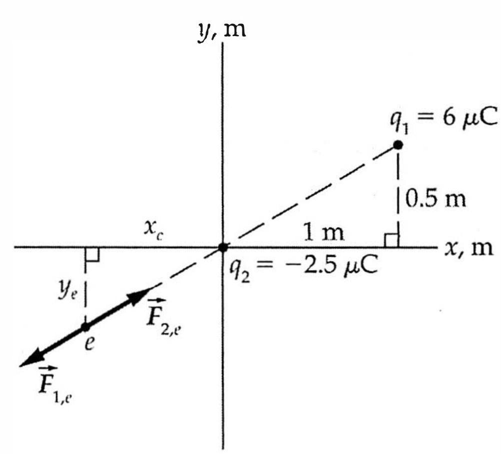
Substitute numerical values and evaluate Q:

$$Q = \frac{6(6.02\times10^{23}\ \text{atoms/mol})(1\,\text{kg})(1.6\times10^{-19}\ \text{C})}{0.012\,\text{kg/mol}} = \boxed{4.82\times10^{7}\ \text{C}}$$

Coulomb's Law

***32 ••** A point charge of −2.5 μC is located at the origin. A second point charge of 6 μC is at x = 1 m, y = 0.5 m. Find the x and y coordinates of the position at which an electron would be in equilibrium.

Picture the Problem The positions of the charges are shown in the diagram. It is apparent that the electron must be located along the line joining the two charges. Moreover, because it is negatively charged, it must be closer to the −2.5 μC than to the 6.0 μC charge, as is indicated in the figure. We can find the x and y coordinates of the electron's position by equating the two electrostatic forces acting on it and solving for its distance from the origin.



We can use similar triangles to express this radial distance in terms of the x and y coordinates of the electron.

Express the condition that must be satisfied if the electron is to be in equilibrium:

$$F_{1,e} = F_{2,e}$$

Express the magnitude of the force that q_1 exerts on the electron:

$$F_{1,e} = \frac{kq_1e}{\left(r+\sqrt{1.25\,\text{m}}\right)^2}$$

Express the magnitude of the force that q_2 exerts on the electron:

$$F_{2,e} = \frac{k|q_2|e}{r^2}$$

Substitute and simplify to obtain:

$$\frac{q_1}{\left(r+\sqrt{1.25\,\text{m}}\right)^2} = \frac{|q_2|}{r^2}$$

Substitute for q_1 and q_2 and simplify:

$$\left(-1.4\,\text{m}^{-2}\right)r^2 + \left(2.2361\,\text{m}^{-1}\right)r + 1.25\,\text{m} = 0$$

Solve for r to obtain:

$r = 2.036\,\text{m}$

and

$r = -0.4386\,\text{m}$

Because $r < 0$ is unphysical, we'll consider only the positive root.

Use the similar triangles in the diagram to establish the proportion involving the y coordinate of the electron:

$$\frac{y_e}{0.5\,\text{m}} = \frac{2.036\,\text{m}}{1.12\,\text{m}}$$

Solve for y_e:

$$y_e = 0.909\,\text{m}$$

Use the similar triangles in the diagram to establish the proportion involving the x coordinate of the electron:

$$\frac{x_e}{1\,\text{m}} = \frac{2.036\,\text{m}}{1.12\,\text{m}}$$

Solve for x_e:

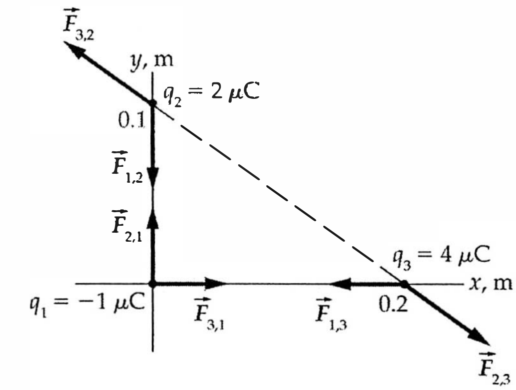
$$x_e = 1.82\,\text{m}$$

The coordinates of the electron's position are:

$$(x_e, y_e) = \boxed{(-1.82\,\text{m}, -0.909\,\text{m})}$$

***33** •• A charge of $-1.0\ \mu\text{C}$ is located at the origin; a second charge of $2.0\ \mu\text{C}$ is located at $x = 0$, $y = 0.1$ m; and a third charge of $4.0\ \mu\text{C}$ is located at $x = 0.2$ m, $y = 0$. Find the forces that act on each of the three charges.

Picture the Problem Let q_1 represent the charge at the origin, q_2 the charge at (0, 0.1 m), and q_3 the charge at (0.2 m, 0). The diagram shows the forces acting on each of the charges. Note the action-and-reaction pairs. We can apply Coulomb's law and the principle of superposition of forces to find the net force acting on each of the charges.



Express the net force acting on q_1:

$$\vec{F}_1 = \vec{F}_{2,1} + \vec{F}_{3,1}$$

Express the force that q_2 exerts on q_1:

$$\vec{F}_{2,1} = \frac{kq_2q_1}{r_{2,1}^2}\hat{r}_{2,1} = \frac{kq_2q_1}{r_{2,1}^2}\frac{\vec{r}_{2,1}}{r_{2,1}} = \frac{kq_2q_1}{r_{2,1}^3}\vec{r}_{2,1}$$

Substitute numerical values and evaluate $\vec{F}_{2,1}$:

$$\vec{F}_{2,1} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2\ \mu\text{C})\frac{(-1\ \mu\text{C})}{(0.1\,\text{m})^3}(-0.1\,\text{m})\hat{j} = (1.80\,\text{N})\hat{j}$$

Express the force that q_3 exerts on q_1:

$$\vec{F}_{3,1} = \frac{kq_3q_1}{r_{3,1}^3}\vec{r}_{3,1}$$

Substitute numerical values and evaluate $\vec{F}_{3,1}$:

$$\vec{F}_{3,1} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(4\ \mu\text{C})\frac{(-1\ \mu\text{C})}{(0.2\,\text{m})^3}(-0.2\,\text{m})\hat{i} = (0.899\,\text{N})\hat{i}$$

Substitute to find $\vec{F}_1$:

$$\vec{F}_1 = \boxed{(0.899\,\text{N})\hat{i} + (1.80\,\text{N})\hat{j}}$$

Express the net force acting on q_2:

$$\begin{aligned}\vec{F}_2 &= \vec{F}_{3,2} + \vec{F}_{1,2}\\ &= \vec{F}_{3,2} - \vec{F}_{2,1}\\ &= \vec{F}_{3,2} - (1.80\,\text{N})\hat{j}\end{aligned}$$

because $\vec{F}_{1,2}$ and $\vec{F}_{2,1}$ are action-and-reaction forces.

Express the force that q_3 exerts on q_2:

$$\vec{F}_{3,2} = \frac{kq_3q_2}{r_{3,2}^3}\vec{r}_{3,2} = \frac{kq_3q_2}{r_{3,2}^3}\left[(-0.2\,\text{m})\hat{i} + (0.1\,\text{m})\hat{j}\right]$$

Substitute numerical values and evaluate $\vec{F}_{3,2}$:

$$\vec{F}_{3,2} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(4\,\mu\text{C})\frac{(2\,\mu\text{C})}{(0.224\,\text{m})^3}\left[(-0.2\,\text{m})\hat{i} + (0.1\,\text{m})\hat{j}\right]$$
$$= (-1.28\,\text{N})\hat{i} + (0.640\,\text{N})\hat{j}$$

Find the net force acting on q_2:

$$\vec{F}_2 = \vec{F}_{3,2} - (1.80\,\text{N})\hat{j} = (-1.28\,\text{N})\hat{i} + (0.640\,\text{N})\hat{j} - (1.80\,\text{N})\hat{j}$$
$$= \boxed{(-1.28\,\text{N})\hat{i} - (1.16\,\text{N})\hat{j}}$$

Noting that $\vec{F}_{1,3}$ and $\vec{F}_{3,1}$ are an action-and-reaction pair, as are $\vec{F}_{2,3}$ and $\vec{F}_{3,2}$, express the net force acting on q_3:

$$\vec{F}_3 = \vec{F}_{1,3} + \vec{F}_{2,3} = -\vec{F}_{3,1} - \vec{F}_{3,2} = -(0.899\,\text{N})\hat{i} - \left[(-1.28\,\text{N})\hat{i} + (0.640\,\text{N})\hat{j}\right]$$
$$= \boxed{(0.381\,\text{N})\hat{i} - (0.640\,\text{N})\hat{j}}$$

The Electric Field

***37** • A charge of 4.0 μC is at the origin. What is the magnitude and direction of the electric field on the x axis at (a) $x = 6$ m, and (b) $x = -10$ m? (c) Sketch the function E_x versus x for both positive and negative values of x. (Remember that E_x is negative when $\vec{E}$ points in the negative x direction.)

Picture the Problem Let q represent the charge at the origin and use Coulomb's law for $\vec{E}$ due to a point charge to find the electric field at $x = 6$ m and -10 m.

(a) Express the electric field at a point P located a distance x from a charge q:

$$\vec{E}(x) = \frac{kq}{x^2}\hat{r}_{\text{P},0}$$

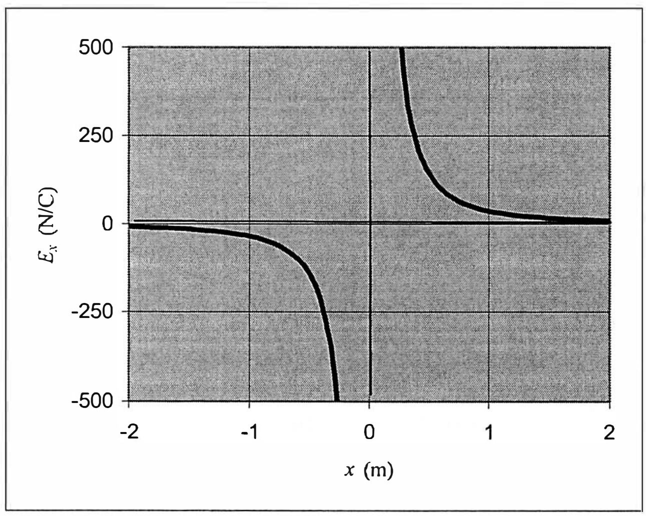
Evaluate this expression for $x = 6$ m:

$$\vec{E}(6\,\text{m}) = \frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(4\,\mu\text{C})}{(6\,\text{m})^2}\hat{i}$$
$$= \boxed{(999\ \text{N/C})\hat{i}}$$

(*b*) Evaluate $\vec{E}$ at $x = -10$ m:

$$\vec{E}(-10\,\text{m}) = \frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(4\,\mu\text{C})}{(10\,\text{m})^2}(-\hat{i}) = \boxed{(-360\ \text{N/C})\hat{i}}$$

(*c*) The following graph was plotted using a spreadsheet program:



***38** • Two charges, each +4 μC, are on the x axis, one at the origin and the other at $x = 8$ m. Find the electric field on the x axis at (*a*) $x = -2$ m, (*b*) $x = 2$ m, (*c*) $x = 6$ m, and (*d*) $x = 10$ m. (*e*) At what point on the x axis is the electric field zero? (*f*) Sketch E_x versus x.

Picture the Problem Let q represent the charges of +4 μC and use Coulomb's law for $\vec{E}$ due to a point charge and the principle of superposition for fields to find the electric field at the locations specified.

Noting that $q_1 = q_2$, use Coulomb's law and the principle of superposition to express the electric field due to the given charges at a point P a distance x from the origin:

$$\vec{E}(x)=\vec{E}_{q_1}(x)+\vec{E}_{q_2}(x)=\frac{kq_1}{x^2}\hat{r}_{q_1,\mathrm{P}}+\frac{kq_2}{(8\,\mathrm{m}-x)^2}\hat{r}_{q_2,\mathrm{P}}=kq_1\left(\frac{1}{x^2}\hat{r}_{q_1,\mathrm{P}}+\frac{1}{(8\,\mathrm{m}-x)^2}\hat{r}_{q_2,\mathrm{P}}\right)$$

$$=(36\,\mathrm{kN\cdot m^2/C})\left(\frac{1}{x^2}\hat{r}_{q_1,\mathrm{P}}+\frac{1}{(8\,\mathrm{m}-x)^2}\hat{r}_{q_2,\mathrm{P}}\right)$$

(*a*) Apply this equation to the point at $x = -2$ m:

$$\vec{\mathbf{E}}(-2\,\mathrm{m})=(36\,\mathrm{kN\cdot m^2/C})\left[\frac{1}{(2\,\mathrm{m})^2}(-\hat{\mathbf{i}})+\frac{1}{(10\,\mathrm{m})^2}(-\hat{\mathbf{i}})\right]=\boxed{(-9.36\,\mathrm{kN/C})\hat{\mathbf{i}}}$$

(*b*) Evaluate $\vec{E}$ at $x = 2$ m:

$$\vec{E}(2\,\mathrm{m})=(36\,\mathrm{kN\cdot m^2/C})\left[\frac{1}{(2\,\mathrm{m})^2}(\hat{i})+\frac{1}{(6\,\mathrm{m})^2}(-\hat{i})\right]=\boxed{(8.00\,\mathrm{kN/C})\hat{i}}$$

(*c*) Evaluate $\vec{E}$ at $x = 6$ m:

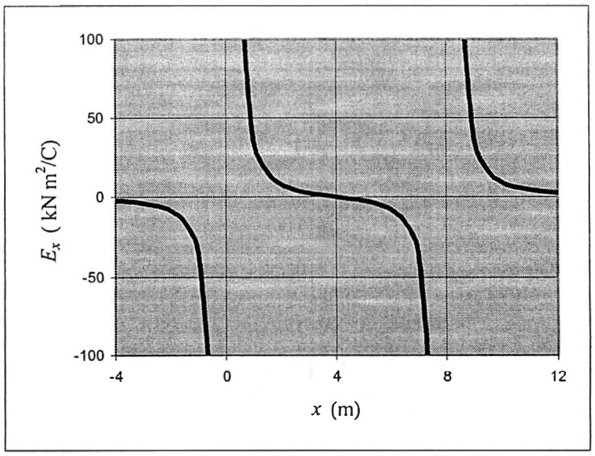
$$\vec{E}(6\,\mathrm{m})=(36\,\mathrm{kN\cdot m^2/C})\left[\frac{1}{(6\,\mathrm{m})^2}(\hat{i})+\frac{1}{(2\,\mathrm{m})^2}(-\hat{i})\right]=\boxed{(-8.00\,\mathrm{kN/C})\hat{i}}$$

(*d*) Evaluate $\vec{E}$ at $x = 10$ m:

$$\vec{E}(10\,\mathrm{m})=(36\,\mathrm{kN\cdot m^2/C})\left[\frac{1}{(10\,\mathrm{m})^2}(\hat{i})+\frac{1}{(2\,\mathrm{m})^2}(\hat{i})\right]=\boxed{(9.35\,\mathrm{kN/C})\hat{i}}$$

(*e*) From symmetry considerations: $E(4\,\mathrm{m})=\boxed{0}$

(*f*) The following graph was plotted using a spreadsheet program:



***42 ••** A point charge of +5.0 μC is located at $x = -3.0$ cm, and a second point charge of -8.0 μC is located at $x = +4.0$ cm. Where should a third charge of +6.0 μC be placed so that the electric field at $x = 0$ is zero?

Picture the Problem If the electric field at $x = 0$ is zero, both its x and y components must be zero. The only way this condition can be satisfied with the point charges of +5.0 μC and -8.0 μC are on the x axis is if the point charge of +6.0 μC is also on the x axis. Let the subscripts 5, -8, and 6 identify the point charges and their fields. We can use Coulomb's law for $\vec{E}$ due to a point charge and the principle of superposition for fields to determine where the +6.0 μC charge should be located so that the electric field at $x = 0$ is zero.

Express the electric field at $x = 0$ in terms of the fields due to the charges of +5.0 μC, -8.0 μC, and +6.0 μC:

$$\vec{E}(0) = \vec{E}_{5\,\mu\text{C}} + \vec{E}_{-8\,\mu\text{C}} + \vec{E}_{6\,\mu\text{C}} = 0$$

Substitute for each of the fields to obtain:

$$\frac{kq_5}{r_5^2}\hat{\boldsymbol{r}}_5 + \frac{kq_6}{r_6^2}\hat{\boldsymbol{r}}_6 + \frac{kq_{-8}}{r_{-8}^2}\hat{\boldsymbol{r}}_{-8} = 0$$

or

$$\frac{kq_5}{r_5^2}\hat{\boldsymbol{i}} + \frac{kq_6}{r_6^2}\left(-\hat{\boldsymbol{i}}\right) + \frac{kq_{-8}}{r_{-8}^2}\left(-\hat{\boldsymbol{i}}\right) = 0$$

Divide out the unit vector $\hat{\boldsymbol{i}}$ to obtain:

$$\frac{q_5}{r_5^2} - \frac{q_6}{r_6^2} - \frac{q_{-8}}{r_{-8}^2} = 0$$

Substitute numerical values to obtain:

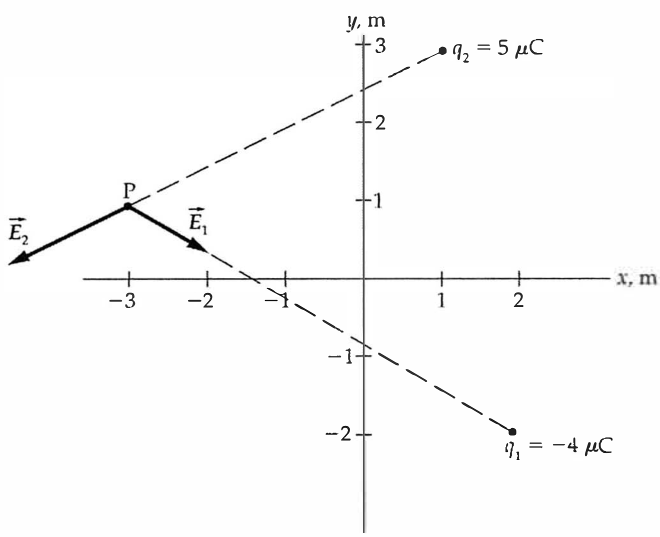
$$\frac{5}{(3\,\text{cm})^2} - \frac{6}{r_6^2} - \frac{-8}{(4\,\text{cm})^2} = 0$$

Solve for r_6:

$$r_6 = \boxed{2.38\,\text{cm}}$$

*45 •• A 5-μC point charge is located at $x = 1$ m, $y = 3$ m; and a -4-μC point charge is located at $x = 2$ m, $y = -2$ m. (*a*) Find the magnitude and direction of the electric field at $x = -3$ m, $y = 1$ m. (*b*) Find the magnitude and direction of the force on a proton at $x = -3$ m, $y = 1$ m.

Picture the Problem The diagram shows the electric field vectors at the point of interest P due to the two charges. We can use Coulomb's law for $\vec{E}$ due to point charges and the superposition principle for electric fields to find $\vec{E}_P$. We can apply $\vec{F} = q\vec{E}$ to find the force on a proton at (−3 m, 1 m).



(*a*) Express the electric field at (−3 m, 1 m) due to the charges q_1 and q_2:

$$\vec{E}_P = \vec{E}_1 + \vec{E}_2$$

Evaluate $\vec{E}_1$:

$$\vec{E}_1 = \frac{kq_1}{r_{1,P}^2}\hat{r}_{1,P} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-4\ \mu\text{C})}{(5\ \text{m})^2+(3\ \text{m})^2}\left(\frac{(-5\ \text{m})\hat{i}+(3\ \text{m})\hat{j}}{\sqrt{(5\ \text{m})^2+(3\ \text{m})^2}}\right)$$
$$= (-1.06\ \text{kN/C})(-0.857\hat{i}+0.514\hat{j}) = (0.908\ \text{kN/C})\hat{i}+(-0.544\ \text{kN/C})\hat{j}$$

Evaluate $\vec{E}_2$:

$$\vec{E}_2 = \frac{kq_2}{r_{2,P}^2}\hat{r}_{2,P} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(5\ \mu\text{C})}{(4\ \text{m})^2+(2\ \text{m})^2}\left(\frac{(-4\ \text{m})\hat{i}+(-2\ \text{m})\hat{j}}{\sqrt{(4\ \text{m})^2+(2\ \text{m})^2}}\right)$$
$$= (2.25\ \text{kN/C})(-0.894\hat{i}-0.447\hat{j}) = (-2.01\ \text{kN/C})\hat{i}+(-1.01\ \text{kN/C})\hat{j}$$

Substitute and simplify to find $\vec{E}_P$:

$$\vec{E}_P = (0.908\,\text{kN/C})\hat{i} + (-0.544\,\text{kN/C})\hat{j} + (-2.01\,\text{kN/C})\hat{i} + (-1.01\,\text{kN/C})\hat{j}$$
$$= (-1.10\,\text{kN/C})\hat{i} + (-1.55\,\text{kN/C})\hat{j}$$

The magnitude of $\vec{E}_P$ is:

$$E_P = \sqrt{(1.10\,\text{kN/C})^2 + (1.55\,\text{kN/C})^2} = \boxed{1.90\,\text{kN/C}}$$

The direction of $\vec{E}_P$ is:

$$\theta_E = \tan^{-1}\left(\frac{-1.55\,\text{kN/C}}{-1.10\,\text{kN/C}}\right) = \boxed{235°}$$

Note that the angle returned by your calculator for $\tan^{-1}\left(\frac{-1.55\,\text{kN/C}}{-1.10\,\text{kN/C}}\right)$ is the reference angle and must be increased by 180° to yield θ_E.

(*b*) Express and evaluate the force on a proton at point P:

$$\vec{F} = q\vec{E}_P = (1.6\times10^{-19}\,\text{C})\left[(-1.10\,\text{kN/C})\hat{i} + (-1.55\,\text{kN/C})\hat{j}\right]$$
$$= (-1.76\times10^{-16}\,\text{N})\hat{i} + (-2.48\times10^{-16}\,\text{N})\hat{j}$$

The magnitude of $\vec{F}$ is:

$$F = \sqrt{(-1.76\times10^{-16}\text{N})^2 + (-2.48\times10^{-16}\text{N})^2} = \boxed{3.04\times10^{-16}\,\text{N}}$$

The direction of $\vec{F}$ is:

$$\theta_F = \tan^{-1}\left(\frac{-2.48\times10^{-16}\,\text{N}}{-1.76\times10^{-16}\,\text{N}}\right) = \boxed{235°}$$

where, as noted above, the angle returned by your calculator for $\tan^{-1}\left(\frac{-2.48\times10^{-16}\,\text{N}}{-1.76\times10^{-16}\,\text{N}}\right)$ is the reference angle and must be increased by 180° to yield θ_E.

***48** ••• Two positive point charges $+q$ are on the y axis at $y = +a$ and $y = -a$ as in Problem 44. A bead of mass m carrying a negative charge $-q$ slides without friction along a thread that runs along the x axis. (a) Show that for small displacements of $x << a$, the bead experiences a restoring force that is proportional to x and therefore undergoes simple harmonic motion. (b) Find the period of the motion.

Picture the Problem In Problem 44 it is shown that the electric field on the x axis, due to equal positive charges located at (0, a) and (0,–a), is given by $E_x = 2kqx\left(x^2 + a^2\right)^{-3/2}$. We can use $T = 2\pi\sqrt{m/k'}$ to express the period of the motion in terms of the restoring constant k'.

(a) Express the force acting on the on the bead when its displacement from the origin is x:

$$F_x = -qE_x = -\frac{2kq^2x}{\left(x^2 + a^2\right)^{3/2}}$$

Factor a^2 from the denominator to obtain:

$$F_x = -\frac{2kq^2x}{a^2\left(\frac{x^2}{a^2} + 1\right)^{3/2}}$$

For $x << a$:

$$F_x = \boxed{-\frac{2kq^2}{a^3}x}$$

i.e., the bead experiences a linear restoring force.

(b) Express the period of a simple harmonic oscillator:

$$T = 2\pi\sqrt{\frac{m}{k'}}$$

Obtain k' from our result in part (a):

$$k' = \frac{2kq^2}{a^3}$$

Substitute to obtain:

$$T = 2\pi\sqrt{\frac{m}{\frac{2kq^2}{a^3}}} = \boxed{2\pi\sqrt{\frac{ma^3}{2kq^2}}}$$

Motion of Point Charges in Electric Fields

***50** • (a) Compute e/m for a proton, and find its acceleration in a uniform electric field with a magnitude of 100 N/C. (b) Find the time it takes for a proton initially at rest in such a field to reach a speed of $0.01c$ (where c is the speed of light).

Picture the Problem We can use Newton's 2nd law of motion to find the acceleration of the proton in the uniform electric field and constant-acceleration equations to find the time required for it to reach a speed of $0.01c$ and the distance it travels while acquiring this speed.

(*a*) Use data found at the back of your text to compute *e*/*m* for an electron:

$$\frac{e}{m_p}=\frac{1.6\times10^{-19}\ \text{C}}{1.67\times10^{-27}\ \text{kg}}=\boxed{9.58\times10^{7}\ \text{C/kg}}$$

Apply Newton's 2nd law to relate the acceleration of the electron to the electric field:

$$a=\frac{F_{\text{net}}}{m_p}=\frac{eE}{m_p}$$

Substitute numerical values and evaluate *a*:

$$a=\frac{(1.6\times10^{-19}\ \text{C})(100\ \text{N/C})}{1.67\times10^{-27}\ \text{kg}}=\boxed{9.58\times10^{9}\ \text{m/s}^2}$$

$\boxed{\text{The direction of the acceleration of a proton is in the direction of the electric field.}}$

(*b*) Using the definition of acceleration, relate the time required for an electron to reach $0.01c$ to its acceleration:

$$\Delta t=\frac{v}{a}=\frac{0.01c}{a}$$

Substitute numerical values and evaluate Δt:

$$\Delta t=\frac{0.01(3\times10^{8}\ \text{m/s})}{9.58\times10^{9}\ \text{m/s}^2}=\boxed{313\ \mu\text{s}}$$

***54** •• A particle leaves the origin with a speed of 3×10^{6} m/s at 35° to the *x* axis. It moves in a constant electric field $\vec{E}=E_y\hat{j}$. Find E_y such that the particle will cross the *x* axis at $x = 1.5$ cm if the particle is (*a*) an electron, and (*b*) a proton.

Picture the Problem We can use constant-acceleration equations to express the *x* and *y* coordinates of the particle in terms of the parameter *t* and Newton's 2nd law to express the constant acceleration in terms of the electric field. Eliminating the parameter will yield an equation for *y* as a function of *x*, *q*, and *m* that we can solve for E_y.

Express the *x* and *y* coordinates of the particle as functions of time:

$$x=(v\cos\theta)t$$

and

$$y = (v\sin\theta)t - \tfrac{1}{2}a_y t^2$$

Apply Newton's 2nd law to relate the acceleration of the particle to the net force acting on it:

$$a_y = \frac{F_{\text{net, y}}}{m} = \frac{qE_y}{m}$$

Substitute in the y-coordinate equation to obtain:

$$y = (v\sin\theta)t - \frac{qE_y}{2m}t^2$$

Eliminate the parameter t between the two equations to obtain:

$$y = (\tan\theta)x - \frac{qE_y}{2mv^2\cos^2\theta}x^2$$

Set $y = 0$ and solve for E_y:

$$E_y = \frac{mv^2\sin 2\theta}{qx}$$

Substitute the non-particle specific data to obtain:

$$E_y = \frac{m(3\times10^6\ \text{m/s})^2\sin 70°}{q(0.015\,\text{m})} = (5.64\times10^{14}\ \text{m/s}^2)\frac{m}{q}$$

(*a*) Substitute for the mass and charge of an electron and evaluate E_y:

$$E_y = (5.64\times10^{14}\ \text{m/s}^2)\frac{9.11\times10^{-31}\ \text{kg}}{1.6\times10^{-19}\text{C}} = \boxed{3.21\,\text{kN/C}}$$

(*b*) Substitute for the mass and charge of a proton and evaluate E_y:

$$E_y = (5.64\times10^{14}\ \text{m/s}^2)\frac{1.67\times10^{-27}\ \text{kg}}{1.6\times10^{-19}\text{C}} = \boxed{5.89\,\text{MN/C}}$$

***58** • A dipole of moment 0.5 e·nm is placed in a uniform electric field with a magnitude of 4.0×10^4 N/C. What is the magnitude of the torque on the dipole when (*a*) the dipole is parallel to the electric field, (*b*) the dipole is perpendicular to the electric field, and (*c*) the dipole makes an angle of 30° with the electric field? (*d*) Find the potential energy of the dipole in the electric field for each case.

Picture the Problem The torque on an electric dipole in an electric field is given by $\vec{\tau} = \vec{p}\times\vec{E}$ and the potential energy of the dipole by $U = -\vec{p}\cdot\vec{E}$.

Using its definition, express the torque

$$\vec{\tau} = \vec{p}\times\vec{E}$$

on a dipole moment in a uniform electric field:

and
$\tau = pE\sin\theta$
where θ is the angle between the electric dipole moment and the electric field.

(*a*) Evaluate τ for $\theta = 0°$:

$$\tau = pE\sin 0° = \boxed{0}$$

(*b*) Evaluate τ for $\theta = 90°$:

$$\tau = (0.5\,e\cdot\text{nm})(4.0\times10^4\ \text{N/C})\sin 90° = \boxed{3.20\times10^{-24}\ \text{N}\cdot\text{m}}$$

(*c*) Evaluate τ for $\theta = 30°$:

$$\tau = (0.5\,e\cdot\text{nm})(4.0\times10^4\ \text{N/C})\sin 30° = \boxed{1.60\times10^{-24}\ \text{N}\cdot\text{m}}$$

(*d*) Using its definition, express the potential energy of a dipole in an electric field:

$$U = -\vec{p}\cdot\vec{E} = -pE\cos\theta$$

Evaluate U for $\theta = 0°$:

$$U = -(0.5\,e\cdot\text{nm})(4.0\times10^4\ \text{N/C})\cos 0° = \boxed{-3.20\times10^{-24}\ \text{J}}$$

Evaluate U for $\theta = 90°$:

$$U = -(0.5\,e\cdot\text{nm})(4.0\times10^4\ \text{N/C})\cos 90° = \boxed{0}$$

Evaluate U for $\theta = 30°$:

$$U = -(0.5\,e\cdot\text{nm})(4.0\times10^4\ \text{N/C})\cos 30° = \boxed{-2.77\times10^{-24}\ \text{J}}$$

***59 ••** For a dipole oriented along the x axis, the electric field falls off as $1/x^3$ in the x direction and $1/y^3$ in the y direction. Use dimensional analysis to prove that, in any direction, the field far from the dipole falls off as $1/r^3$.

Picture the Problem We can combine the dimension of an electric field with the dimension of an electric dipole moment to prove that, in any direction, the dimension of the far field is proportional to $1/[L]^3$ and, hence, the electric field far from the dipole falls off as $1/r^3$.

Express the dimension of an electric field:

$$[E] = \frac{[kQ]}{[L]^2}$$