SOLUTIONS MANUAL

POWER SYSTEMS ANALYSIS

SECOND EDITION

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VIJAY VITTAL

PART I. SOLUTIONS TO PROBLEM SETS
PART II. DISCUSSION OF SOLUTIONS TO DESIGN EXERCISES

PRENTICE HALL, Upper Saddle River, New Jersey 07458

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Manufacturing Buyer: Pat Brown

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

ISBN 0-13-022561-4

Prentice-Hall International (UK) Limited, London
Prentice-Hall of Australia Pty. Limited, Sydney
Prentice-Hall Canada, Inc., Toronto
Prentice-Hall Hispanoamericana, S.A., Mexico
Prentice-Hall of India Private Limited, New Delhi
Prentice-Hall (Singapore) Pte. Ltd.
Prentice-Hall of Japan, Inc., Tokyo
Editora Prentice-Hall do Brazil, Ltda., Rio de Janeiro

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PART I. SOLUTIONS TO PROBLEM SETS

2.1 $v = \sqrt{2} \times 120 \cos(\omega t + 30^\circ) \Rightarrow V = 120\angle 30^\circ$ V

$i = \sqrt{2} \times 10 \cos(\omega t - 30^\circ) \Rightarrow I = 10 \angle -30^\circ$ A

(a) $p(t) = |V||I| [\cos\phi + \cos(2\omega t + \angle V + \angle I)]$

$= 600 + 1200 \cos 2\omega t$ W

$S = VI^* = 1200\angle 60^\circ = P + jQ \Rightarrow P = 600\,W, \; Q = 1039$ VAr

(b) $Z = V/I = 12\angle 60^\circ = 6 + j10.39 = R + jX \Rightarrow R = 6, \; X = 10.39$

2.2 (a) Using (2.3) we find $p_{max} = 1707 = |V||I|(\cos\phi + 1)$ and $p_{min} = -293 = |V||I|(\cos\phi - 1)$. Then, since $|V| = 100$, we get $|I| = 10$ and $\cos\phi = \pm 45^\circ$. Pick $\phi = 45^\circ \Rightarrow Z = 10\angle 45^\circ = 7.07 + j7.07 = R + jX \Rightarrow R = 7.07, \; X = 7.07$

(b) $S = VI^* = Z|I|^2 = (7.07 + j7.07)\,10^2 \Rightarrow P = 707, \; Q = 707$

(c) For simplicity assume $i(t) = \sqrt{2}\,|I| \cos\omega t$. Then

$p_L(t) = v_L(t)\,i(t) = L\dfrac{di}{dt}\,i = -2\omega L|I|^2 \cos\omega t \sin\omega t = -\omega L|I|^2 \sin 2\omega t$

$p_{L\,max} = \omega L|I|^2 = 707 = Q$. Thus $p_{L\,max} = Q$. The same!

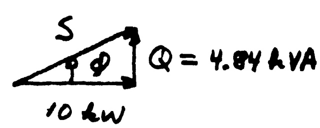
2.3 0.7 PF lagging $\Rightarrow \phi = 45.57^\circ$, $Q = 5.10$ MVAr

0.9 PF lagging $\Rightarrow \phi = 25.84^\circ$, $Q = 2.42$ MVAr. Capacitor must supply $5.10 - 2.42 = 2.68$ MVAr.

2.4 0.707 PF lagging $\Rightarrow S_{3\phi} = 200 + j200$ kVA. Cap supplies 50 kVAr. Resultant $S_{3\phi} = 200 + j150$ kVA $\Rightarrow$ PF $= 0.80$.

$|S| = \dfrac{|S_{3\phi}|}{3} = \dfrac{250 \times 10^3}{3} = |V||I| = \dfrac{440}{\sqrt{3}}|I| \Rightarrow |I| = 328$ A

2.5 0.9 PF lagging $\Rightarrow \phi = 25.84^\circ$



(a) $S = 10 + j\,4.84$ kVA

(b) $10 \times 10^3 = 416 \times |I| \times 0.9 \Rightarrow |I| = 26.71$ A

(c) Using (2.3), (or first principles) we get

$p(t) = 10 \times 10^3 + 11.11 \times 10^3 \cos(2\omega t + 25.84^\circ)$

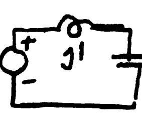
Note: the average value of $p(t)$ is 10 kW

2.6 Because system is balanced $V_{ab} = 208\angle 120°$, $V_{bc} = 208\angle 0°$. Using (2.17) or Fig 2.11, $V_{an} = 120\angle 90° \Rightarrow V_{bn} = 120\angle -30°$, $V_{cn} = 120\angle -150°$. Using per phase analysis, $I_a = 12\angle 105° \Rightarrow$ $I_b = 12\angle -15°$, $I_c = 12\angle -135°$.

2.7 $S = VI^* = V(YV)^* = Y^*|V|^2 = Y_C^* + Y_L^* + Y_R^*$
$= -j5 + j10 + 0.1 = 0.1 + j5$

2.8 (a) Using loop or nodal analysis we find, after much work,
$I_a = 0.9123\angle -90.351°$, $I_b = 0.9123\angle -209.65°$, $I_c = 0.9929\angle 30°$.
(b) Using per phase analysis $I_a = 1\angle 0° / j1.1 = 0.9091\angle -90°$,
then, $I_b = 0.9091\angle -210°$, $I_c = 0.9091\angle 30°$.

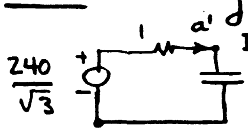
2.9 Proceeding by analogy with 3ϕ, we note
$E_{ab} = E_{an} - E_{bn} = E_{an}(1 - e^{-j\pi/2}) = \sqrt{2}\, E_{an}\, e^{j\pi/4}$.
Thus $E_{an} = \frac{1}{\sqrt{2}} E_{ab}\, e^{-j\pi/4}$, and $E_{an}, E_{bn}, E_{cn}, E_{dn}$ form a pos. seq. set of 4ϕ voltages. Doing per phase (phase a) analysis we have

$\frac{1}{\sqrt{2}}\angle 45°$ 

$-j0.5 \Rightarrow I_a = \dfrac{\frac{1}{\sqrt{2}}\angle -45°}{j0.5} = \sqrt{2}\angle -135°$

Then $I_b = \sqrt{2}\angle -225°$, $I_c = \sqrt{2}\angle -315°$, $I_d = \sqrt{2}\angle -405°$

2.10 Using per phase circuit we find $I_a = 103.8\angle 41.5°$.
Then $|I_b| = 103.8$ A



$Z_\lambda = \frac{1}{3}\frac{1}{j\omega C} = -j0.884$

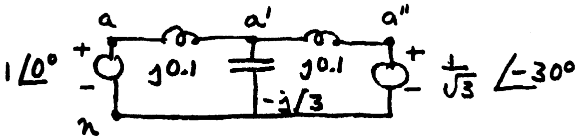
$V_{a'n} = Z_\lambda I_a = 91.76\angle -48.5° \Rightarrow |V_{a'b'}| = \sqrt{3}\cdot 91.76 = 158.9$ V

$S_{load} = V_{a'n} I_a^* = 9524\angle -90°$

$S_{load}^{3\phi} = 3\, S_{load} = 28574\angle -90°$ W

2.11 Assume pos. seq. operation. $V_{a''b''} = V_{a''n} - V_{b''n} = \sqrt{3}\, V_{a''n}\, e^{j\pi/6} \Rightarrow V_{a''n} = \frac{1}{\sqrt{3}} V_{a''b''} e^{-j\pi/6} = \frac{1}{\sqrt{3}} \angle{-30^\circ}$

Per Phase Ckt



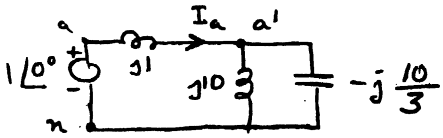
Using superposition & voltage divider law we get

$V_{a'n} = 0.899 \angle{-10.89^\circ} \Rightarrow V_{b'n} = 0.899 \angle{-130.89^\circ}$ and

$V_{c'n} = 0.899 \angle{-250.89^\circ}$. Then $V_{a'b'} = 1.557 \angle{19.11^\circ}$

2.12 Assume pos. seq..

Per Phase Cht



Combining parallel elements we have $Z_{\parallel} = -j5$. $I_a = 0.25 \angle{90^\circ}$

$V_{a'n} = -j5 \cdot j0.25 = 1.25 \angle{0^\circ}$

$V_{a'b'} = 2.165 \angle{30^\circ} \Rightarrow I_{cap} = .2165\angle{120^\circ}$

$S_{load}^{3\phi} = 3\, V_{a'n}\, I_a^* = 0.3125 \angle{-90^\circ}$

2.13

(a) $V_{bc} = 208\angle{-120^\circ}$, $V_{ca} = 208 \angle{120^\circ}$

$V_{an} = \frac{208}{\sqrt{3}} \angle{-30^\circ} \Rightarrow I_a = 1.20 \angle{-90^\circ}$

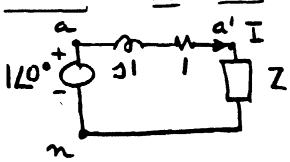
Then $I_b = 1.20\angle{-210^\circ}$, $I_c = 1.20 \angle{-330^\circ}$

(b) $V_{bc} = 208 \angle{120^\circ}$, $V_{ca} = 208 \angle{-120^\circ}$

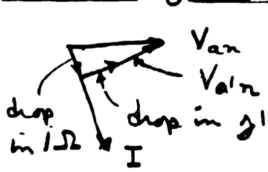
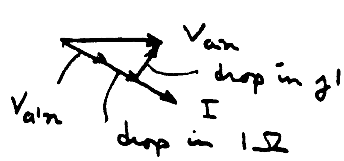
$V_{an} = \frac{208}{\sqrt{3}} \angle{30^\circ} \Rightarrow I_a = 1.20 \angle{-30^\circ}$

$I_b = 1.20 \angle{90^\circ}$, $I_c = 1.20 \angle{-150^\circ}$

2.14 Per Phase Cht.



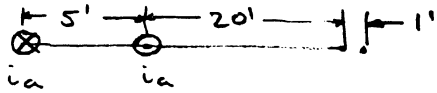
Problem reduces to picking Z so that $|V_{a'n}| > |V_{an}|$. It helps to draw some phasor diagrams.

I. $Z = j\omega L$	II $Z = R$	III $Z = -j\frac{1}{\omega C}$
		For example $Z = -j2$ $I = \frac{1}{\sqrt{2}} \angle{45^\circ}$ $V_{a'n} = \sqrt{2} \angle{-45^\circ}$
Clearly $\lvert V_{a'n}\rvert < \lvert V_{an}\rvert$	Clearly $\lvert V_{a'n}\rvert < \lvert V_{an}\rvert$	$\lvert V_{a'n}\rvert > \lvert V_{an}\rvert$ This is O.K.

2.15 Since $E_a+E_b+E_c=0$, neutrals are at same potential. $Z = E_a/I_a = \sqrt{2}\angle 55°$. For each Z, $S = VI^* = |V|^2/Z^*$. thus

$$S^{3\phi} = \frac{(\sqrt{2})^2+1^2+1^2}{\sqrt{2}\angle -55°} = 2\sqrt{2}\angle 55°.$$

3.1



$\frac{\mu_0}{2\pi} = 2\times 10^{-7}$, $|I_a| = 100$

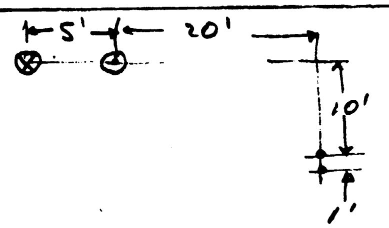
(Upward) flux linkages of telephone wires

$$\lambda = 2\times10^{-7}\left[i_a \ln\frac{26}{25} - i_a \ln\frac{21}{20}\right] = -.00957\times2\times10^{-7}\, i_a$$

$$v_{tel} = \frac{d\lambda}{dt} \Rightarrow V_{tel} = j\omega\times(-0.00957\times2\times10^{-7})\times 100 \quad V/\text{meter}$$

Since 1 mile = 1.609 km, $|V_{tel}| = 0.116$ V/mile

3.2

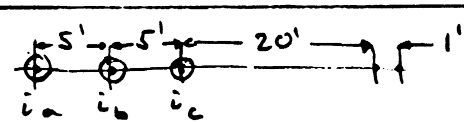


$$\lambda = 2\times10^{-7}\, i_a\left[\ln\frac{\sqrt{25^2+11^2}}{\sqrt{25^2+10^2}} - \ln\frac{\sqrt{20^2+11^2}}{\sqrt{20^2+10^2}}\right]$$

$$= -0.066294\times2\times10^{-7}\, i_a$$

$$\Rightarrow |V_{tel}| = 0.076 \text{ V/mile}$$

3.3



$$\lambda = 2\times10^{-7}\left[i_a\ln\frac{31}{30} + i_b\ln\frac{26}{25} + i_c\ln\frac{21}{20}\right]$$

$$= 2\times10^{-7}\left[0.0328\, i_a + 0.0392\, i_b + 0.0488\, i_c\right]$$

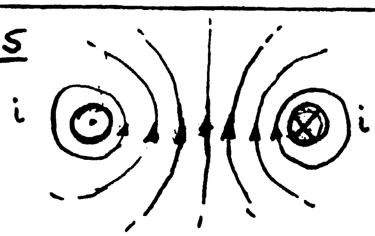
$$V_{tel} = j\omega\times2\times10^{-7}\left[0.0328\, I_a + 0.0392\, I_b + 0.0488\, I_c\right]$$

$$|V_{tel}| = 377\times2\times10^{-7}\times100\times\left|0.0328+0.0392\angle -120° + 0.0488\angle 120°\right|$$

$$= 1048\times10^{-7} \text{ V/meter} = 0.1692 \text{ V/mile}$$

3.4 The telephone wires are transposed every 1000 ft. Cancellation occurs in all but 720' of line $\Rightarrow$ 0.0158 V.

3.5



The flux linkages due to the current in the left conductor is ≈

$$\lambda_1 = \frac{\mu_0}{2\pi} i \left[\frac{\mu_r}{4} + \ln\frac{D}{r}\right] = i\frac{\mu_0}{2\pi}\ln\frac{D}{r'}$$

Here we are neglecting partial flux linkages of the far (right) conductor. The current in the right conductor contributes an equal number of flux linkages. Thus (at least approximately),

$$l = 2\lambda_1/i = \frac{\mu_0}{\pi}\ln\frac{D}{r'} = 4\times10^{-7}\ln\frac{D}{r'} \quad \text{H/meter}$$

3.6 If each conductor is hollow then there are no partial flux linkages and in (3.13) the term involving μ_r is absent. Then $r' = r$ and

$$l = \frac{\mu_0}{2\pi}\ln\frac{D}{r} \qquad \text{H/m}$$

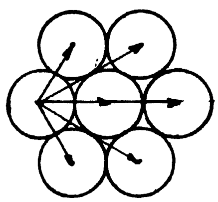
3.7 The hint is misleading. A better hint for combining the 7 (unequal) inductances $l_1, l_2, \cdots, l_7$ would be to use the average inductance i.e. let $l_a = \frac{l_{av}}{7} = \frac{l_1 + l_2 + \cdots + l_7}{7^2}$ Returning to the problem, paralleling the steps which lead to (3.27) we have

$$\lambda_1 = \frac{\mu_0}{2\pi}\left[\frac{i_a}{7}\left(\ln\frac{1}{d_{11}} + \ln\frac{1}{d_{12}} + \cdots + \ln\frac{1}{d_{17}}\right)\right.$$
$$+ \frac{i_b}{7}\left(\ln\frac{1}{d_{18}} + \ln\frac{1}{d_{19}} + \cdots + \ln\frac{1}{d_{1,21}}\right)$$
$$\left. + \frac{i_c}{7}\left(\ln\frac{1}{d_{1,15}} + \ln\frac{1}{d_{1,16}} + \cdots + \ln\frac{1}{d_{1,21}}\right)\right]$$
$$\approx \frac{\mu_0}{2\pi} i_a \ln\frac{D}{(d_{11}\cdots d_{17})^{1/7}} \Rightarrow l_1 = 7\times\frac{\mu_0}{2\pi}\ln\frac{D}{(d_{11}\cdots d_{17})^{1/7}}$$

$$l_a = \frac{l_{av}}{7} = \frac{l_1 + \cdots + l_7}{7^2} = \frac{\mu_0}{2\pi} \ln \frac{D}{R_s}$$

where $R_s \triangleq \left[(d_{11} \cdots d_{17})^{1/7} \cdots (d_{71} \cdots d_{77})^{1/7} \right]^{1/7}$

3.8

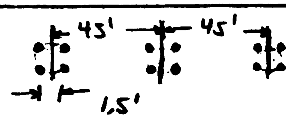


Six of the product terms are the same; the relevant distances are shown on the left. The different geometry is for the center wire to the outside wires. Let $d = 0.0876$ in. be the wire diameter. Noting that $d_{ii} = 0.7788 \cdot \frac{d}{2}$ we have

$$R_s = \left[\left(\left(0.7788 \tfrac{d}{2}\right) \cdot d^3 \cdot 2d \cdot (\sqrt{3}d)^2 \right)^6 \left(0.7788 \tfrac{d}{2} \cdot d^6 \right) \right]^{1/7^2}$$

$= 1.088\, d = 0.0943$ in. $= 0.00786$ ft.. very close!

3.9



$D_m = (45' \times 45' \times 90')^{1/3} = 56.70$ ft

$R_b = (0.0479 \times 1.5^2 \times \sqrt{2} \times 1.5)^{1/4} = .6915'$

$$l = 2 \times 10^{-7} \ln \frac{D_m}{R_b} = 8.81 \times 10^{-7} \ H/m$$

3.10 Using $r' = 0.7788 \times \frac{1.424}{2} \times \frac{1}{12} = 0.0462'$

(instead of $GMR = 0.0479$) we get $R_b = 0.6853'$.

$l = 2 \times 10^{-7} \ln \frac{D_m}{R_b} = 8.83\ H/m$. $\approx 0.23\%$ error.

3.11 $l = 9.47 \times 10^{-7}\ H/m$

$X_L = 1609 \times 377 \times 9.47 \times 10^{-7} = 0.574\ \Omega s/\text{mile}$

3.12 $D_m = (26' \times 26' \times 52')^{1/3}$, $R_b = (0.0386' \times 1.5)^{1/2} = 0.2406'$

$l = 2 \times 10^{-7} \ln \frac{D_m}{R_b} = 9.83 \times 10^{-7}\ H/m$

3.13 Using $r' = 0.7788 \times \frac{1.165}{2} \times \frac{1}{12} = 0.0378'$ we get $R_b = 0.2381'$

$$l = 2 \times 10^{-7} \ln \frac{D_m}{R_b} = 9.85 \ \text{H/m} . \approx 0.20\% \text{ error.}$$

3.14 $X_L = 1609 \times 377 \times 9.83 \times 10^{-7} = 0.596\,\Omega$ / mile

---

3.15 $D_m = (45' \times 45' \times 90')^{1/3} = 56.70'$

$$r = \frac{1.424''}{2} \times \frac{1}{12} = 0.0593', \quad R_b^c = (0.0593 \times 1.5^2 \times \sqrt{2}\, 1.5)^{1/4} = .07295'$$

$$c = \frac{2\pi \cdot 8.854 \cdot 10^{-12}}{\ln \frac{D_m}{R_b^c}} = 12.78 \times 10^{-12} \ \text{F/m (to neutral)}$$

3.16 $B_c = 1609 \times 377 \times 12.78 \times 10^{-12} = 7.75 \times 10^{-6}$ ℧/mile

$|X_c| = 1/B_c = 0.129$ MΩ/mile

3.17 $D_m = (26' \times 26' \times 52')^{1/3} = 32.76'$

$$r = \frac{1.165''}{2} \times \frac{1}{12} = 0.0485'$$

$$R_b^c = (0.0485' \times 1.5')^{1/2} = 0.2698'$$

$$c = 11.59 \times 10^{-12} \ \text{F/m (to neutral)}$$

3.18 $B_c = 1609 \times 377 \times 11.59 \times 10^{-12} = 7.03 \times 10^{6}$ ℧/mi.

$|X_c| = 1/B_c = 0.142$ MΩ/mile

3.19 $lc = 8.81 \times 10^{-7} \times 12.78 \times 10^{-12} = 11.259 \times 10^{-18}$

$\mu_0 \epsilon_0 = 4\pi \times 10^{-7} \times 8.854 \times 10^{-12} = 11.126 \times 10^{-18}$

3.20 $\ell c = 9.83 \times 10^{-7} \times 11.59 \times 10^{-12} = 11.393 \times 10^{-18}$

$\mu_0 \epsilon_0 = 111.126 \times 10^{-18}$

Note: $\mu_0 \epsilon_0$ is a universal constant.

Velocity of light in vacuum "c" $= \frac{1}{\sqrt{\mu_0 \epsilon_0}} \approx 2.998 \times 10^8$ m/sec.

3.21

$r_a = r_b = r_c = 0.145\ \Omega/\text{mi}$, from Table A8.1, for Grosbeak, at 25°C.

$GMR_a = GMR_b = GMR_c = 0.0355$ ft. (From Table A8.1).

Assume $\rho = 100\ \Omega$-m (as in Example 3.6), $f = 60$ Hz, then

$$D_e = 2160\sqrt{\rho/f} = 2160\sqrt{100/60} = 2790 \text{ ft.}$$

At 60 Hz, $r_d = 9.869 \times 10^{-7} \times f \quad \Omega/\text{m}$

Using the conversion factor: 1 mile = 1.609 km we get

$r_d = 0.09528\ \Omega/\text{mi}$

Then $z_{aa} = z_{bb} = z_{cc} = r_a + r_d + j\omega \times 2 \times 10^{-7}\, \ell n \frac{D_e}{GMR_i}\ \Omega/\text{m}$

$$= (0.145 + 0.09528) + j(377 \times 2 \times 10^{-7})\ell n \frac{2790}{0.0355} \times 1.609 \times 10^3 = 0.2403 + j1.3675\ \Omega/\text{mi}$$

$d_{ab} = \sqrt{4^2 + 5.5^2} = 6.807$ ft, $d_{ca} = 4$ ft, $d_{bc} = 5.5$ ft.

$$z_{ab} = r_d + j\omega \times 2 \times 10^{-7}\, \ell n \frac{D_e}{d_{ab}}\ \Omega/\text{m}$$

$$= 0.09528 + j(377 \times 2 \times 10^{-7})\ell n \frac{2790}{6.8007} \times 1.609 \times 10^3 = 0.09528 + j0.7299\ \Omega/\text{mi}$$

Similarly using $d_{bc} = 5.5$ ft. and $d_{ca} = 4$ ft., we get

$z_{bc} = 0.09528 + j0.7557\ \Omega/\text{mi}$, $z_{ca} = 0.09528 + j0.7943\ \Omega/\text{mi}$

For 30 miles of line we multiply the above values by 30 to write, in matrix notation

$$Z_{abc} = \begin{bmatrix} (7.209 + j41.025) & (2.858 + j21.8970) & (2.858 + j23.8290) \\ (2.858 + j21.8970) & (7.209 + j41.025) & (2.858 + j22.6710) \\ (2.858 + j23.8290) & (2.858 + j22.6710) & (7.209 + j41.025) \end{bmatrix} \Omega$$

3.22

$r_a = r_b = r_c = 0.306\ \Omega/\text{mi}$ (From Table A8.1, for Ostrich, at 25°).

$GMR_a = GMR_b = GMR_c = 0.0230$ ft. (From Table A8.1).

Assume $\rho = 100\ \Omega$-m as in Example 3.6, $f = 60$ Hz, then

$$D_e = 2160\sqrt{\rho/f} \text{ ft} = 2160\sqrt{100/60} = 2790 \text{ ft.}$$

At 60 Hz, $r_d = 9.869 \times 10^{-7} \times f \quad \Omega / \mathrm{m}$

Using the conversion factor: 1 mile = 1.609 km we get

$r_d = 0.09528 \ \Omega / \mathrm{mi}$

Then $z_{aa} = z_{bb} = z_{cc} = r_a + r_d + j\omega \times 2 \times 10^{-7} \ell n \dfrac{D_e}{GMR_i} \quad \Omega / \mathrm{m}$

$$= (0.306 + 0.09528) + j\left(377 \times 2 \times 10^{-7}\right) \ell n \frac{2790}{0.0230} \times 1.609 \times 10^3 \ \Omega/\mathrm{mi}$$

$d_{ab} = d_{bc} = \sqrt{4^2 + 1.5^2} = 4.2720 \text{ ft}$, $d_{ca} = 8.0$ ft.

$$z_{ab} = r_d + j\omega \times 2 \times 10^{-7} \ell n \frac{D_e}{d_{ab}} \ \Omega/\mathrm{m}$$

$$= 0.09528 + j\left(377 \times 2 \times 10^{-7}\right) \ell n \frac{2790}{4.2720} \times 1.609 \times 10^3 = 0.09528 + j0.7864 \ \Omega/\mathrm{mi}$$

$z_{bc} = z_{ab} = 0.09528 + j0.7864 \ \Omega/\mathrm{mi}.$

Similarly using d_{ca} = 8.0 ft. we get

$z_{ca} = 0.09528 + j0.7102 \ \Omega/\mathrm{mi}.$

For 40 miles of line we multiply the above values by 40 to write, in matrix notation

$$Z_{abc} = \begin{bmatrix} (16.0512 + j56.804) & (3.8112 + j31.456) & (3.8112 + j28.408) \\ (3.8112 + j31.456) & (16.0512 + j56.804) & (3.8112 + j31.456) \\ (3.8112 + j28.408) & (3.8112 + j31.456) & (16.0512 + j56.804) \end{bmatrix} \Omega$$

4.1 $z = 0.17 + j0.79 = 0.8081 \angle 77.86° \ \Omega/\text{mi}$

$y = j5.4 \times 10^{-6} = 5.4 \times 10^{-6} \angle 90° \ \mho/\text{mi}$

$Z_c = \sqrt{\dfrac{z}{y}} = 386.8 \angle -6.07°$

$\gamma = 2.09 \times 10^{-3} \angle 83.9° = \underbrace{0.221 \times 10^{-3}}_{\alpha} + j\underbrace{2.08 \times 10^{-3}}_{\beta} = \alpha + j\beta$

4.2 $\left.\begin{array}{l} z = 0.02 + j0.54 = 0.54 \angle 87.88° \\ y = 7.8 \times 10^{-6} \angle 90° \end{array}\right\} \Rightarrow Z_c = 263.2 \angle -1.06°$

$\gamma = 2.05 \times 10^{-3} \angle 88.9° = 0.038 \times 10^{-3} + j\ 2.05 \times 10^{-3} = \alpha + j\beta.$

Note: Z_c is very different; so is α. But β is $\approx$ the same.

4.3 $V_1 = V_2 \cosh \gamma l + Z_c I_2 \sinh \gamma l$

$I_1 = I_2 \cosh \gamma l + \dfrac{V_2}{Z_c} \sinh \gamma l$

$Z_{oc} = \left.\dfrac{V_1}{I_1}\right|_{I_2=0} = Z_c \coth \gamma l = 800 \angle -89°$

$Z_{sc} = \left.\dfrac{V_1}{I_1}\right|_{V_2=0} = Z_c \tanh \gamma l = 200 \angle 77°$

$Z_{oc} Z_{sc} = Z_c^2 = 16 \times 10^4 \angle -12° \Rightarrow Z_c = 400 \angle -6°$

$\dfrac{Z_{sc}}{Z_{oc}} = (\tanh \gamma l)^2 = 0.25 \angle 166° \Rightarrow \tanh \gamma l = 0.5 \angle 83°$

$\tanh \gamma l = \dfrac{e^{\gamma l} - e^{-\gamma l}}{e^{\gamma l} + e^{-\gamma l}} = \dfrac{e^{2\gamma l} - 1}{e^{2\gamma l} + 1} = \dfrac{x-1}{x+1} = 0.5 \angle 83° = y$

Solve for $x = \dfrac{1+y}{1-y} = 1.103 \angle 52.92° = 1.103 \angle 0.9236 \text{ rad}$

$= e^{2\gamma l} = e^{2\alpha l} e^{j2\beta l}$

$\Rightarrow \alpha l = 0.0490, \quad \beta l = 0.4618$

$\gamma l = \alpha l + j\beta l = 0.0490 + j0.4618 = 0.4643 \angle 83.9°$

4.4 Agree. From (4.10), Z_c & γl specify the general relation between (V_1, I_1) & (V_2, I_2). With Z_{oc} & Z_{sc} we can calculate Z_c & γl (as in problem 4.3).

4.5
$$Z = z\,l = 20 + j\,80 = 82.46 \angle 75.96^\circ$$
$$Y = y\,l = j\,5\times10^{-4} = 5\times10^{-4} \angle 90^\circ$$
$$Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{Z}{Y}} = 406.1 \angle -7.02^\circ, \quad \gamma l = \sqrt{ZY} = .203 \angle 82.98^\circ$$
$$= \underbrace{0.0248}_{\alpha l} + j\,\underbrace{0.2015}_{\beta l}$$
$$e^{\gamma l} = e^{\alpha l} \angle \overset{\text{rad.}}{\beta l} = 1.025 \angle 11.55^\circ$$
$$e^{-\gamma l} = 0.975 \angle -11.55^\circ$$
$$\Rightarrow \cosh \gamma l = 0.9801 \angle 0.290^\circ$$
$$\sinh \gamma l = 0.2017 \angle 83.08^\circ$$

4.6
$$\eta = \frac{-P_{21}}{P_{12}} = e^{-2\alpha l}. \quad 2\alpha l = 2\times 0.0248 \Rightarrow \eta = 0.952$$

4.7
$$\frac{V_1}{I_1} = Z_c\,\frac{Z_L \cosh \gamma l + Z_c \sinh \gamma l}{Z_c \cosh \gamma l + Z_L \sinh \gamma l}$$

$\gamma = \sqrt{zy}$. With $r, l, g, c > 0$, $0 < \angle\gamma < 90^\circ \Rightarrow \alpha > 0$

Then as $\gamma l \to \infty$, $e^{-\gamma l} = e^{-\alpha l}\, e^{-j\beta l} \to 0$

$$\Rightarrow \frac{\sinh \gamma l}{\cosh \gamma l} \to 1 \Rightarrow \frac{V_1}{I_1} \to Z_c$$

4.8 $P_{load} = \frac{15\,MW}{3} = 5\,MW$. $|V_2| = \frac{132\,kV}{\sqrt{3}} = 76.21\,kV$. Pick $\angle V_2 = 0$

$I_2 = 5\times10^6 / 76.21\times10^3 = 65.6\,A$ (P.F. = 1 !).

$$V_1 = V_2 \cosh \gamma l + Z_c I_2 \sinh \gamma l$$
$$I_1 = I_2 \cosh \gamma l + \frac{V_2}{Z_c} \sinh \gamma l$$

From problem 4.1, $\gamma = 0.221 \times 10^{-3} + j\, 2.08 \times 10^{-3}$ /mi

$\gamma l = 0.0331 + j\,0.3116 \Rightarrow \begin{cases} \cosh \gamma l = 0.9524 \angle 0.6113^\circ \\ \sinh \gamma l = 0.3083 \angle 84.12^\circ \end{cases}$

From prob. 4.1, $Z_c = 386.8 \angle -6.07^\circ$. then

$V_1 = 74.675 \angle 6.48^\circ$ kV. $I_1 = 87.46 \angle 44.60^\circ$ A.

$\theta_{12} = 6.48^\circ - 0^\circ = 6.48^\circ$. $P_{12} = Re\, V_1 I_1^* = 5.138$ MW.

$$\eta = \frac{-P_{21}}{P_{12}} = \frac{5}{5.138} = 0.973$$

4.9 $Z' = Z_c \sinh \gamma l = 386.8 \angle -6.07^\circ \times 0.3083 \angle 84.12^\circ$
$= 119.25 \angle 78.05^\circ$

$$\frac{Y'}{2} = \frac{\cosh \gamma l - 1}{Z_c \sinh \gamma l} = \frac{0.9524 \angle 0.6113 - 1}{119.25 \angle 78.05^\circ} = 409 \times 10^{-6} \angle 89.91^\circ$$

4.10 $|I_2| = \dfrac{5 \times 10^6}{0.9 \times 76.21 \times 10^3} = 72.90$. $I_2 = 72.90 \angle -25.84^\circ$ A

$V_1 = 78.28 \angle 5.60^\circ$ kV, $I_1 = 69.92 \angle 26.45^\circ$ A,

$\theta_{12} = 5.60^\circ$, $P_{12} = Re\, V_1 I_1^* = 5.115$ MW, $\eta = \dfrac{5}{5.115} = .978$

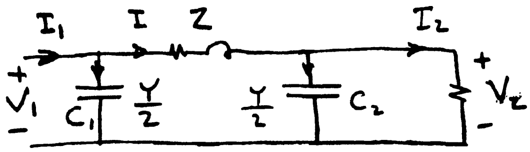
4.11 $|V_1| = 74.675$ kV. Let $\angle V_1 = 0$. $V_2 = 78.407 \angle -0.61^\circ$ kV

$$I_1 = \frac{V_2}{Z_c} \sinh \gamma l = 62.49 \angle 89.58^\circ$$

4.12 $Z = z l = 121.22 \angle 77.86^\circ$

$$\frac{Y}{2} = \frac{y l}{2} = 405 \times 10^{-6} \angle 90^\circ$$

(Pretty close to $Z' = 119.25 \angle 78.05^\circ$, $\dfrac{Y'}{2} = 409 \times 10^{-6} \angle 89.91^\circ$ the values found in problem 4.9).



$V_2 = 76.21 \angle 0^\circ$ kV

$I_{c2} = \frac{Y}{2} V_2 = 30.865 \angle 90^\circ$ A

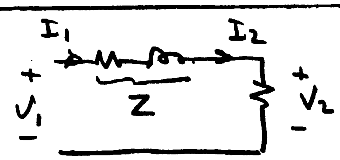
$I_2 = 65.6 \angle 0^\circ$ A. $\quad I = I_{c2} + I_2 = 72.50 \angle 25.20^\circ$ A

$V_1 = V_2 + ZI = 74.72 \angle 6.58^\circ$ kV.

$I_1 = I + I_{c1} = 87.02 \angle 44.44^\circ$ A

$\theta_{12} = 6.58^\circ$, $P_{12} = \text{Re}\, V_1 I_1^* = 5.134$ MW, $\eta = \frac{5}{5.134} = 0.974$

4.13



$I_2 = I_1 = 65.6$ A

$V_2 = 76.21$ kV

$V_1 = V_2 + Z I_2 = 78.27 \angle 5.70^\circ$ kV

$\theta_{12} = 5.70^\circ$, $P_{12} = \text{Re}\, V_1 I_1^* = 5.109$ MW, $\eta = \frac{5}{5.109} = 0.979$

4.14 $\quad x = \omega l = 377 \times 2 \times 10^{-3} = 0.754\ \Omega$ / mi

$z = r + j\omega l = 0.1 + j0.754 = 0.761 \angle 82.45^\circ\ \Omega$ / mi

$y = j\omega c = j377 \times 0.01 \times 10^{-6} = 3.77 \times 10^{-6} \angle 90^\circ\ \mho$ / mi

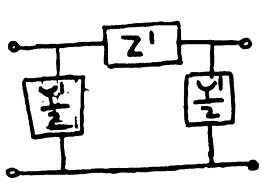
$Z = 152.2 \angle 82.45^\circ$, $Y = 754 \times 10^{-6} \angle 90^\circ$

$\gamma l = \sqrt{ZY} = 0.3388 \angle 86.23^\circ = 0.0223 + j\,0.3381$

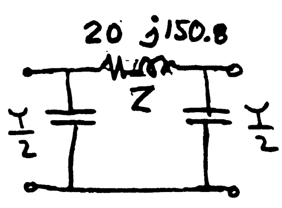
$Z_c = \sqrt{\frac{Z}{Y}} = 449.3 \angle -3.78^\circ$, $e^{\gamma l} = e^{\alpha l} \angle \beta l = 1.023 \angle 19.37^\circ$

$\cosh \gamma l = 0.944 \angle 0.453^\circ$, $\sinh \gamma l = 0.333 \angle 86.34^\circ$

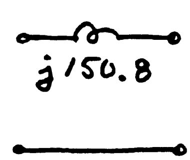
$Z' = Z_c \sinh \gamma l = 149.6 \angle 82.56^\circ$, $\frac{Y'}{2} = \frac{\cosh \gamma l - 1}{Z'} = 378 \times 10^{-6} \angle 89.85^\circ$



(a)



(b)



(c)

4.15 $|V_2| = 750/\sqrt{3} = 433\,kV$. Let $\angle V_2 = 0°$.

$P_{Load} = 100\,MW/3 = 0.95\,|V_2||I_2| \Rightarrow |I_2| = 81.03A$

$I_2 = 81.03 \angle{-18.19°}\ A$. (Note: $|I_2|$ is very small!).

From prob. 4.2, $Z_c = 2632 \angle{-1.06°}$, $\gamma l = \alpha l + j\beta l =$

$0.0152 + j\,0.820$. $e^{\gamma l} = 1.015 \angle{46.98°}$

$\cosh\gamma l = 0.6823 \angle{0.924°}$, $\sinh\gamma l = 0.7312 \angle{89.20°}$.

$V_1 = V_2 \cosh\gamma l + Z_c I_2 \sinh\gamma l = 301.3 \angle{3.69°}\ kV$.

$I_1 = I_2 \cosh\gamma l + \dfrac{V_2}{Z_c} \sinh\gamma l = 1188.5 \angle{85.57°}\ A$.

$P_{12} = Re\, V_1 I_1^* = 50.58\,MW$, $\eta = \dfrac{33.3}{50.58} = 0.658$

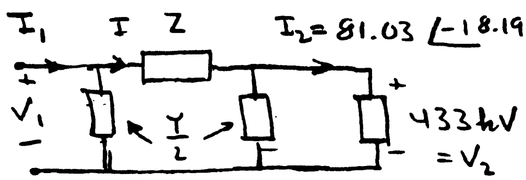
4.16 $|V_1| = 301.3\,kV$. Let $\angle V_1 = 0$.

$V_2 = \dfrac{V_1}{\cosh\gamma l} = 441.6 \angle{-0.924°}\ kV$

$I_1 = \dfrac{V_2}{Z_c} \sinh\gamma l = 1226.8 \angle{89.34°}\ A$

$S_{12}^{3\phi} = 3 S_{12} = 3 V_1 I_1^* = 1108 \angle{-89.34°}\ MVA$

4.17 $\dfrac{Y}{2} = \dfrac{yl}{2} = 1.56 \times 10^{-3} \angle{90°}$, $Z = 216 \angle{87.88°}$



$I = I_2 + \dfrac{Y}{2} V_2$

$= 654.7 \angle{83.25°}$

$V_1 = V_2 + Z I = 294.1 \angle{4.25°}\ kV$

$I_1 = I + \dfrac{Y}{2} V_1 = 1108.5 \angle{87.78°}$

$P_{12} = Re\, V_1 I_1^* = 36.73\,MW$

$\eta = \dfrac{33}{36.73} = 0.898$

4.18 $A = D = \cosh \gamma l = 0.9524 \angle 0.6113°$

$B = Z_c \sinh \gamma l = 119.25 \angle 78.05°$

$C = \frac{1}{Z_c} \sinh \gamma l = 0.797 \times 10^{-3} \angle 90.19°$

4.19
$$T_1 T_2 = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A^2+BC & AB+BD \\ AC+DC & BC+D^2 \end{bmatrix}$$
$$= \begin{bmatrix} A^2+BC & 2AB \\ 2AC & A^2+BC \end{bmatrix} = \begin{bmatrix} .8148 \angle 2.72° & 309.4 \angle 78.7 \\ 1.52 \angle 90.8° & .8148 \angle 2.72° \end{bmatrix}$$

4.20 In general, $S = VI^* = V(YV)^* = Y^*|V|^2 = \frac{|V|^2}{Z^*}$.
For π-equivalent circuit the sum of S_i is
$$S_{12} + S_{21} = \frac{Y^*}{2}\left(|V_1|^2 + |V_2|^2\right) + \frac{1}{Z^*}|V_1 - V_2|^2.$$ Alternatively
$$S_{12} + S_{21} = \left(\frac{1}{Z^*} + \frac{Y^*}{2}\right)\left(|V_1|^2 + |V_2|^2\right) - \frac{1}{Z^*}\left(V_1 V_2^* + V_2 V_1^*\right).$$

4.21 Using (4.33), $P_{12} = P_{SIL} \dfrac{\sin \theta_{12}}{\sin \beta l}$ where βl is in radians.

$\beta = 0.002 \Rightarrow \beta l = 0.6$ radians $= 34.38°$. $\sin \beta l = 0.565$.
$\theta_{12} = 45°$, $\sin 45° = 0.707$. Thus for a 138 kV line with $P_{SIL} = 50$ MVA, $P_{12} = 50\,\text{MVA} \times \dfrac{0.707}{0.565} = 62.6$ MVA is too small. The 345 & 765 kV lines are O.K.

4.22
$$\gamma = (yz)^{1/2} = [j\omega c(r + j\omega l)]^{1/2} = \left[(j\omega)^2 lc\left(1 + \frac{r}{j\omega l}\right)\right]^{1/2}$$
$$\left(1 + \frac{r}{2j\omega l}\right)^2 = 1 + \frac{r}{j\omega l} - \frac{r^2}{4\omega^2 l^2} \approx 1 + \frac{r}{j\omega l} \quad \text{if } r \ll \omega l$$

Prob 4.22 should read $r \ll \omega l$. Then
$$\gamma = j\omega\sqrt{lc}\left[1 + \frac{r}{2j\omega l}\right] = \frac{r}{2}\sqrt{\frac{c}{l}} + j\omega\sqrt{lc} \Rightarrow \beta = \omega\sqrt{lc}$$