Chapter 2

Limits and Derivatives

2.1 Introduction to Derivatives

2.1.1. With $\Delta t = 1.0$, $\Delta f = f(2.0) - f(1.0) = 3.0$, so $\frac{\Delta f}{\Delta t} = 3.0$. With $\Delta t = 0.5$, $\Delta f = f(1.5) - f(1.0) = 1.5$, so $\frac{\Delta f}{\Delta t} = 3.0$. With $\Delta t = 0.1$, $\Delta f = f(1.1) - f(1.0) = 0.3$, so $\frac{\Delta f}{\Delta t} = 3.0$. With $\Delta t = 0.01$, $\Delta f = f(1.01) - f(1.0) = 0.03$, so $\frac{\Delta f}{\Delta t} = 3.0$.

2.1.2. With $\Delta t = 1.0$, $\Delta g = g(1.0) - g(0.0) = -3.0$, so $\frac{\Delta g}{\Delta t} = -3.0$. With $\Delta t = 0.5$, $\Delta g = g(0.5) - g(0.0) = -1.5$, so $\frac{\Delta g}{\Delta t} = -3.0$. With $\Delta t = 0.1$, $\Delta g = g(0.1) - g(0.0) = -0.3$, so $\frac{\Delta g}{\Delta t} = -3.0$. With $\Delta t = 0.01$, $\Delta g = g(0.01) - g(0.0) = -0.03$, so $\frac{\Delta g}{\Delta t} = -3.0$.

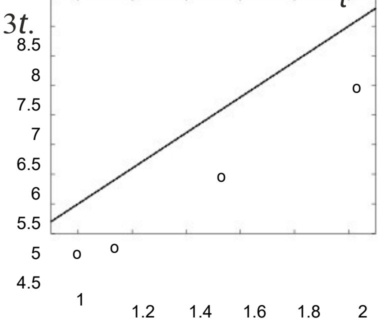
2.1.3. With $\Delta t = 1.0$, $\Delta h = h(2.0) - h(1.0) = 6.0$, so $\frac{\Delta h}{\Delta t} = 6.0$. With $\Delta t = 0.5$, $\Delta h = h(1.5) - h(1.0) = 2.5$, so $\frac{\Delta h}{\Delta t} = 5.0$. With $\Delta t = 0.1$, $\Delta h = h(1.1) - h(1.0) = 0.42$, so $\frac{\Delta h}{\Delta t} = 4.2$. With $\Delta t = 0.01$, $\Delta h = h(1.01) - h(1.0) = 0.0402$, so $\frac{\Delta h}{\Delta t} = 4.02$.

2.1.4. With $\Delta t = 1.0$, $\Delta h = h(1.0) - h(0.0) = 1.0$, so $\frac{\Delta h}{\Delta t} = 1.0$. With $\Delta t = 0.5$, $\Delta h = h(0.5) - h(0.0) = 0.25$, so $\frac{\Delta h}{\Delta t} = 0.5$. With $\Delta t = 0.1$, $\Delta h = h(0.1) - h(0.0) = 0.01$, so $\frac{\Delta h}{\Delta t} = 0.1$. With $\Delta t = 0.01$, $\Delta h = h(0.01) - h(0.0) = 0.0001$, so $\frac{\Delta h}{\Delta t} = 0.01$.

2.1.5. With $\Delta t = 1.0$, $\Delta G = G(1.0) - G(0.0) = 6.389$, so $\frac{\Delta G}{\Delta t} = 6.389$. With $\Delta t = 0.5$, $\Delta G = G(0.5) - G(0.0) = 1.718$, so $\frac{\Delta G}{\Delta t} = 3.436$. With $\Delta t = 0.1$, $\Delta G = G(0.1) - G(0.0) = 0.221$, so $\frac{\Delta G}{\Delta t} = 2.21$. With $\Delta t = 0.01$, $\Delta G = G(0.01) - G(0.0) = 0.0202$, so $\frac{\Delta G}{\Delta t} = 2.02$.

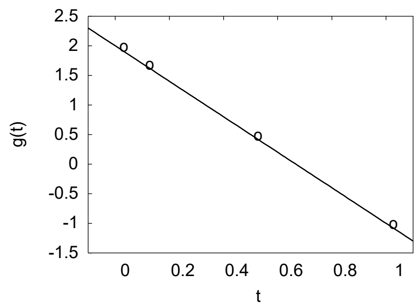
2.1.6. With $\Delta t = 1.0$, $\Delta G = G(1.0) - G(0.0) = -0.632$, so $\frac{\Delta G}{\Delta t} = -0.632$. With $\Delta t = 0.5$, $\Delta G = G(0.5) - G(0.0) = -0.393$, so $\frac{\Delta G}{\Delta t} = -0.787$. With $\Delta t = 0.1$, $\Delta G = G(0.1) - G(0.0) = -0.095$, so $\frac{\Delta G}{\Delta t} = -0.95$. With $\Delta t = 0.01$, $\Delta G = G(0.01) - G(0.0) = -0.00995$, so $\frac{\Delta G}{\Delta t} = -0.995$.

2.1.7. Each secant line is $f_s(t) = 2 + 3t$.

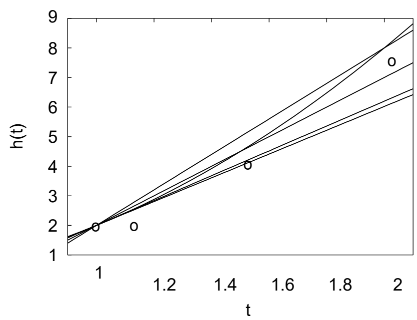


f(t)

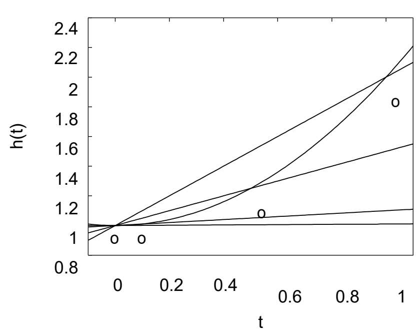
2.1.8. Each secant line is $g_s(t) = 2 - 3t$.



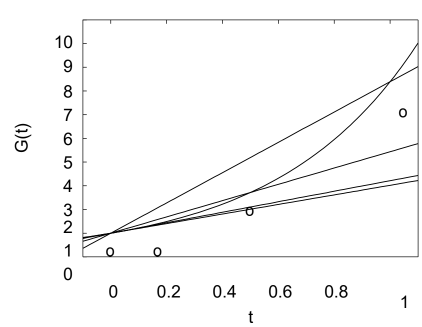
2.1.9. The coordinates of the base point are $(1, 2)$, so the secant lines are: with $\Delta t = 1.0$, $h_s(t) = 2+6(t-1)$, with $\Delta t = 0.5$, $h_s(t) = 2+5(t-1)$, with $\Delta t = 0.1$, $h_s(t) = 2+4.2(t-1)$, with $\Delta t = 0.01$, $h_s(t) = 2 + 4.02(t - 1)$.



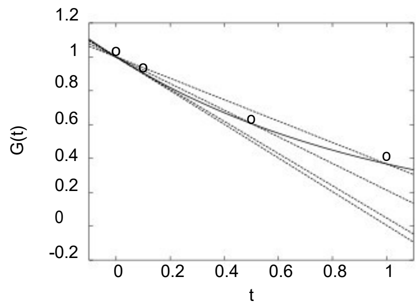
2.1.10. The coordinates of the base point are $(0, 1)$, so the secant lines are: with $\Delta t = 1.0$, $h_s(t) = 1+t$, with $\Delta t = 0.5$, $h_s(t) = 1 + 0.5t$, with $\Delta t = 0.1$, $h_s(t) = 1 + 0.1t$, with $\Delta t = 0.01$, $h_s(t) = 1 + 0.01t$.



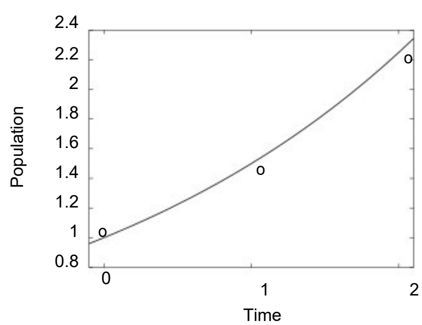
2.1.11. The coordinates of the base point are $(0, 1)$, so the secant lines are: with $\Delta t = 1.0$, $G_s(t) = 1 + 6.389t$, with $\Delta t = 0.5$, $G_s(t) = 1 + 3.436t$, with $\Delta t = 0.1$, $G_s(t) = 1 + 2.21t$, with $\Delta t = 0.01$, $G_s(t) = 1 + 2.02t$.



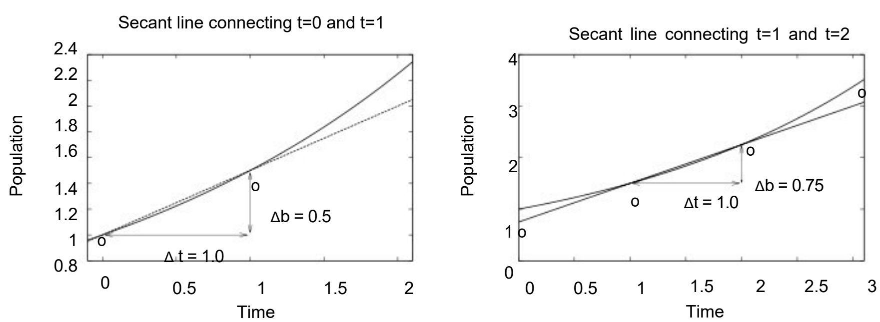
2.1.12. The coordinates of the base point are (0, 1), so the secant lines are: with $\Delta t = 1.0$, $G_s(t) = 1 - 0.632t$, with $\Delta t = 0.5$, $G_s(t) = 1 - 0.787t$, with $\Delta t = 0.1$, $G_s(t) = 1 - 0.95t$, with $\Delta t = 0.01$, $G_s(t) = 1 - 0.995t$.



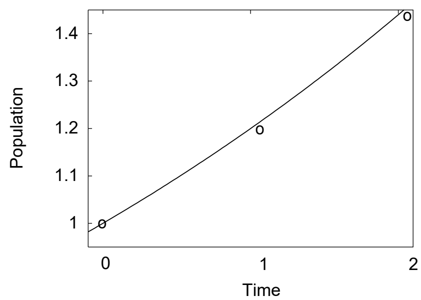
2.1.13. The slope is 3, so the tangent line i $f(t) = 2 + 3t$.
2.1.14. The slope is -3, so the tangent line is $\hat{g}(t) = 2 - 3t$.
2.1.15. It looks like the slopes are getting close to 4.0, so the tangent line is $\hat{h}(t) = 2 + 4(t - 1)$.
2.1.16. It looks like the slopes are getting close to 0.0, so the tangent line is $\hat{h}(t) = 1$.
2.1.17. It looks like the slopes are getting close to 2.0, so the tangent line is $\hat{G}(t) = 1 + 2t$.
2.1.18. It looks like the slopes are getting close to -1.0, so the tangent line is $\hat{G}(t) = 1 - t$.
2.1.19. The derivative of $g(t)$, the slope of the tangent line.
2.1.20. $g'(t)$, $\frac{dg}{dt}$, $\lim_{\Delta t \to 0} \frac{\quad}{\Delta t}$
2.1.21.



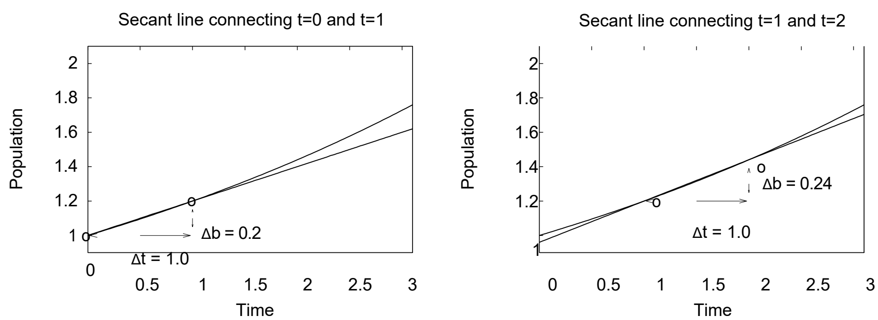
a. $b(0) = 1.0$, $b(1.0) = 1.5$, $b(2.0) = 2.25$.
b. $\Delta b = 1.5 - 1.0 = 0.5$, so $\Delta b/\Delta t = 0.5$.
c. $\Delta b = 2.25 - 1.5 = 0.75$, so $\Delta b/\Delta t = 0.75$.



2.1.22.

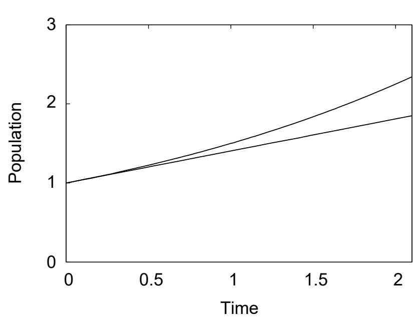


a. $b(0) = 1.0$, $b(1.0) = 1.2$, $b(2.0) = 1.44$.
b. $\Delta b = 1.2 - 1.0 = 0.2$, so $\Delta b/\Delta t = 0.2$.
c. $\Delta b = 1.44 - 1.2 = 0.24$, so $\Delta b/\Delta t = 0.24$.



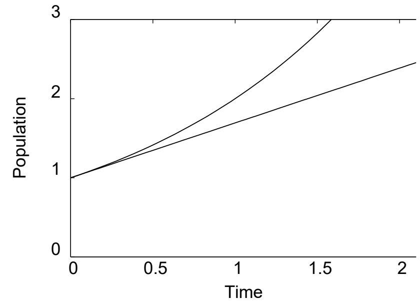
2.1.23.

a. $\Delta b = 1.5^{1.0} - 1.0 = 0.5$, and $\Delta b/\Delta t = 0.5$.
b. $\Delta b = 1.5^{0.1} - 1.0 = 0.0413$, and $\Delta b/\Delta t = 0.414$.
c. $\Delta b = 1.5^{0.01} - 1.0 = 0.00406$, and $\Delta b/\Delta t = 0.406$.
d. $\Delta b = 1.5^{0.001} - 1.0 = 0.000405$, and $\Delta b/\Delta t = 0.405$.
e. The limit looks like 0.405.

f. 

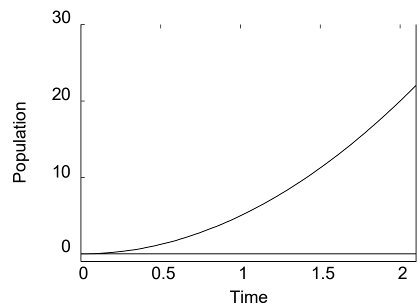
2.1.24.

a. The slope is $(2.0^{1.0} - 1.0)/1.0 = 1.0$.
b. The slope is $(2.0^{0.1} - 1.0)/0.1 = 0.718$.
c. The slope is $(2.0^{0.01} - 1.0)/0.01 = 0.696$.
d. The slope is $(2.0^{0.001} - 1.0)/0.001 = 0.693$.
e. It looks like the slope of the tangent is 0.693.
f.



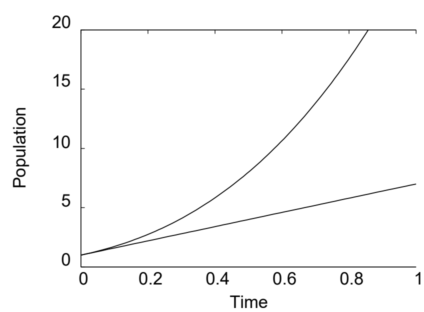
2.1.25.

a. The slope is $(5 \cdot 1.0^2 - 0.0)/1.0 = 5.0$.
b. The slope is $(5 \cdot 0.1^2 - 0.0)/0.1 = 0.5$.
c. The slope is $(5 \cdot 0.01^2 - 0.0)/0.01 = 0.05$.
d. The slope is $(5 \cdot 0.001^2 - 0.0)/0.001 = 0.005$.
e. The slope gets close to 0.
f.

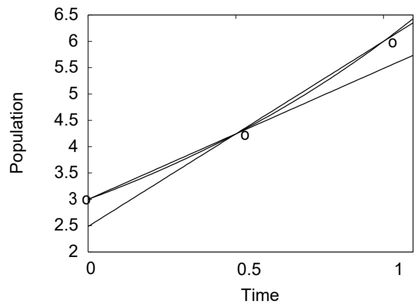


2.1.26.

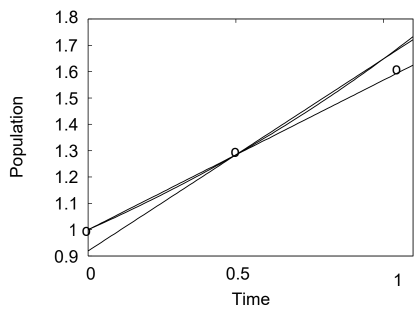
a. The slope is $((1 + 2 \cdot 1.0^3) - (1.0 + 2.0 \cdot 0.0^3))/1.0 = 26.0$.
b. The slope is $((1 + 2 \cdot 0.1^3) - (1.0 + 2.0 \cdot 0.0^3))/0.1 = 7.28$.
c. The slope is $((1 + 2 \cdot 0.01^3) - (1.0 + 2.0 \cdot 0.0^3))/0.01 = 6.1208$.
d. The slope is $((1 + 2 \cdot 0.001^3) - (1.0 + 2.0 \cdot 0.0^3))/0.001 = 6.012$.
e. The slope seems to be approaching 6.0.
f.



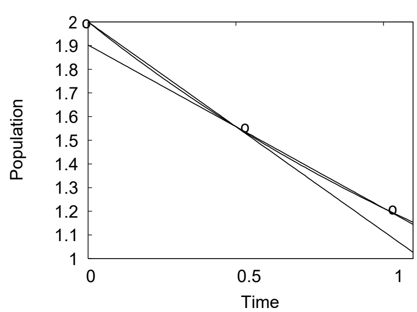
2.1.27. During the first hour, 3.0 bacteria/h. During the first half hour, 2.485 bacteria/h. During the second half hour, 3.515 bacteria/h. The population changes faster during the second half hour.



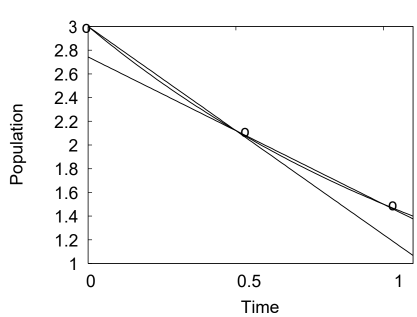
2.1.28. During the first hour, 0.648 bacteria/h. During the first half hour, 0.568 bacteria/h. During the second half hour, 0.729 bacteria/h. The population changes faster during the second half hour.



2.1.29. During the first hour, -0.79 bacteria/h. During the first half hour, -0.88 bacteria/h. During the second half hour, -0,69 bacteria/h. The population changes faster during the first half hour.



2.1.30. During the first hour, -1.5 bacteria/h. During the first half hour, -1.757 bacteria/h. During the second half hour, -1.243 bacteria/h. The population changes faster during the first half hour.



2.1.31.