Chapter 1
Graphs

Section 1.1

1. 0

2. $|5-(-3)| = |8| = 8$

3. $\sqrt{3^2+4^2} = \sqrt{25} = 5$

4. $11^2 + 60^2 = 121 + 3600 = 3721 = 61^2$
Since the sum of the squares of two of the sides of the triangle equals the square of the third side, the triangle is a right triangle.

5. $\frac{1}{2}bh$

6. true

7. x-coordinate or abscissa; y-coordinate or ordinate

8. quadrants

9. midpoint

10. False; the distance between two points is never negative.

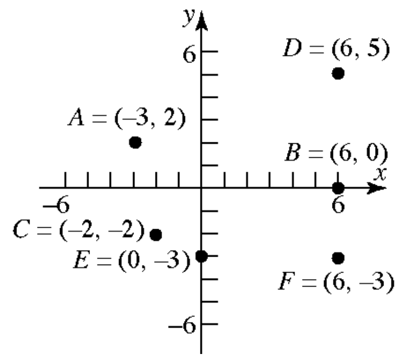
11. False; points that lie in Quadrant IV will have a positive x-coordinate and a negative y-coordinate. The point $(-1,4)$ lies in Quadrant II.

12. True; $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

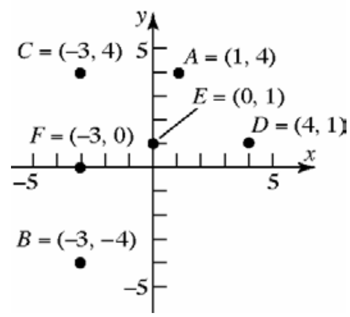
13. b

14. a

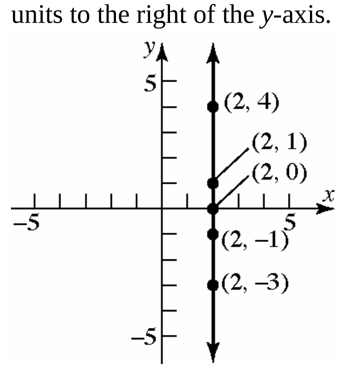
15. (a) Quadrant II
(b) x-axis
(c) Quadrant III
(d) Quadrant I
(e) y-axis
(f) Quadrant IV



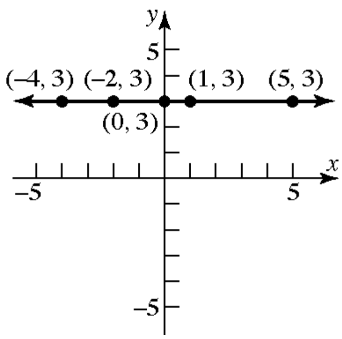
16. (a) Quadrant I
(b) Quadrant III
(c) Quadrant II
(d) Quadrant I
(e) y-axis
(f) x-axis



17. The points will be on a vertical line that is two units to the right of the y-axis.



18. The points will be on a horizontal line that is three units above the x-axis.



19. $d(P_1,P_2)=\sqrt{(2-0)^2+(1-0)^2}$
$=\sqrt{2^2+1^2}=\sqrt{4+1}=\sqrt{5}$

20. $d(P_1,P_2)=\sqrt{(-2-0)^2+(1-0)^2}$
$=\sqrt{(-2)^2+1^2}=\sqrt{4+1}=\sqrt{5}$

21. $d(P_1,P_2)=\sqrt{\left(2-(-1)\right)^2+(2-1)^2}$
$=\sqrt{3^2+1^2}=\sqrt{9+1}=\sqrt{10}$

22. $d(P_1,P_2)=\sqrt{(-2-1)^2+(2-1)^2}$
$=\sqrt{(-3)^2+1^2}=\sqrt{9+1}=\sqrt{10}$

23. $d(P_1,P_2)=\sqrt{(5-3)^2+\left(4-(-4)\right)^2}$
$=\sqrt{2^2+(8)^2}=\sqrt{4+64}=\sqrt{68}=2\sqrt{17}$

24. $d(P_1,P_2)=\sqrt{\left(2-(-1)\right)^2+(4-0)^2}$
$=\sqrt{(3)^2+4^2}=\sqrt{9+16}=\sqrt{25}=5$

25. $d(P_1,P_2)=\sqrt{(4-2)^2+\left(2-(-3)\right)^2}$
$=\sqrt{2^2+5^2}=\sqrt{4+25}=\sqrt{29}$

26. $d(P_1,P_2)=\sqrt{\left(6-(-3)\right)^2+(0-2)^2}$
$=\sqrt{9^2+(-2)^2}=\sqrt{81+4}=\sqrt{85}$

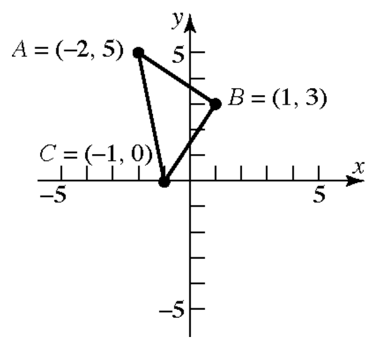
27. $d(P_1,P_2)=\sqrt{\left(6-(-4)\right)^2+\left(2-(-3)\right)^2}$
$=\sqrt{10^2+5^2}=\sqrt{100+25}$
$=\sqrt{125}=5\sqrt{5}$

28. $d(P_1,P_2)=\sqrt{(6-4)^2+\left(4-(-3)\right)^2}$
$=\sqrt{2^2+7^2}=\sqrt{4+49}=\sqrt{53}$

29. $d(P_1,P_2)=\sqrt{(0-a)^2+(0-a)^2}$
$=\sqrt{(-a)^2+(-a)^2}$
$=\sqrt{a^2+a^2}=\sqrt{2a^2}=|\,a\,|\sqrt{2}$

30. $d(P_1,P_2)=\sqrt{(0-a)^2+(0-b)^2}$
$=\sqrt{(-a)^2+(-b)^2}=\sqrt{a^2+b^2}$

31. $A=(-2,5),\ \ B=(1,3),\ \ C=(-1,0)$
$d(A,B)=\sqrt{\left(1-(-2)\right)^2+(3-5)^2}$
$=\sqrt{3^2+(-2)^2}=\sqrt{9+4}=\sqrt{13}$
$d(B,C)=\sqrt{\left(-1-1\right)^2+(0-3)^2}$
$=\sqrt{(-2)^2+(-3)^2}=\sqrt{4+9}=\sqrt{13}$
$d(A,C)=\sqrt{\left(-1-(-2)\right)^2+(0-5)^2}$
$=\sqrt{1^2+(-5)^2}=\sqrt{1+25}=\sqrt{26}$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

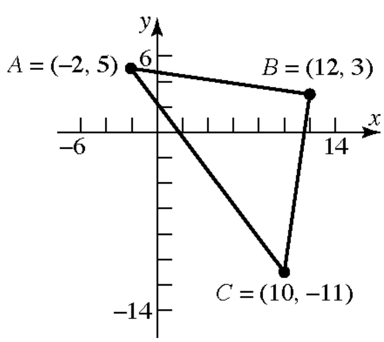
$$\left[d(A,B)\right]^2+\left[d(B,C)\right]^2=\left[d(A,C)\right]^2$$
$$\left(\sqrt{13}\right)^2+\left(\sqrt{13}\right)^2=\left(\sqrt{26}\right)^2$$
$$13+13=26$$
$$26=26$$

The area of a triangle is $A=\frac{1}{2}\cdot bh$. In this problem,

$$A = \frac{1}{2}\cdot\left[d(A,B)\right]\cdot\left[d(B,C)\right]$$
$$= \frac{1}{2}\cdot\sqrt{13}\cdot\sqrt{13} = \frac{1}{2}\cdot 13$$
$$= \frac{13}{2} \text{ square units}$$

32. $A = (-2, 5),\ B = (12, 3),\ C = (10, -11)$

$$d(A,B) = \sqrt{\left(12-(-2)\right)^2 + (3-5)^2}$$
$$= \sqrt{14^2 + (-2)^2}$$
$$= \sqrt{196+4} = \sqrt{200}$$
$$= 10\sqrt{2}$$
$$d(B,C) = \sqrt{\left(10-12\right)^2 + (-11-3)^2}$$
$$= \sqrt{(-2)^2 + (-14)^2}$$
$$= \sqrt{4+196} = \sqrt{200}$$
$$= 10\sqrt{2}$$
$$d(A,C) = \sqrt{\left(10-(-2)\right)^2 + (-11-5)^2}$$
$$= \sqrt{12^2 + (-16)^2}$$
$$= \sqrt{144+256} = \sqrt{400}$$
$$= 20$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

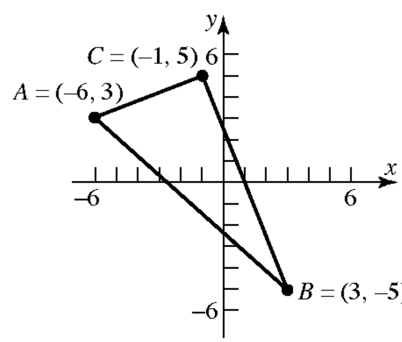
$$\left[d(A,B)\right]^2 + \left[d(B,C)\right]^2 = \left[d(A,C)\right]^2$$
$$\left(10\sqrt{2}\right)^2 + \left(10\sqrt{2}\right)^2 = \left(20\right)^2$$
$$200 + 200 = 400$$
$$400 = 400$$

The area of a triangle is $A = \frac{1}{2}bh$. In this problem,

$$A = \frac{1}{2}\cdot\left[d(A,B)\right]\cdot\left[d(B,C)\right]$$
$$= \frac{1}{2}\cdot 10\sqrt{2}\cdot 10\sqrt{2}$$
$$= \frac{1}{2}\cdot 100\cdot 2 = 100 \text{ square units}$$

33. $A = (-6, 3),\ B = (3, -5),\ C = (-1, 5)$

$$d(A,B) = \sqrt{\left(3-(-6)\right)^2 + (-5-3)^2}$$
$$= \sqrt{9^2 + (-8)^2} = \sqrt{81+64}$$
$$= \sqrt{145}$$
$$d(B,C) = \sqrt{\left(-1-3\right)^2 + (5-(-5))^2}$$
$$= \sqrt{(-4)^2 + 10^2} = \sqrt{16+100}$$
$$= \sqrt{116} = 2\sqrt{29}$$
$$d(A,C) = \sqrt{\left(-1-(-6)\right)^2 + (5-3)^2}$$
$$= \sqrt{5^2 + 2^2} = \sqrt{25+4}$$
$$= \sqrt{29}$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

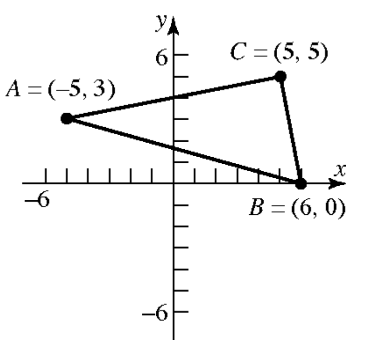
$$\left[d(A,C)\right]^2 + \left[d(B,C)\right]^2 = \left[d(A,B)\right]^2$$
$$\left(\sqrt{29}\right)^2 + \left(2\sqrt{29}\right)^2 = \left(\sqrt{145}\right)^2$$
$$29 + 4\cdot 29 = 145$$
$$29 + 116 = 145$$
$$145 = 145$$

The area of a triangle is $A = \frac{1}{2}bh$. In this problem,

$$A=\frac{1}{2}\cdot\left[d(A,C)\right]\cdot\left[d(B,C)\right]$$
$$=\frac{1}{2}\cdot\sqrt{29}\cdot 2\sqrt{29}$$
$$=\frac{1}{2}\cdot 2\cdot 29$$
$$= 29 \text{ square units}$$

34. $A=(-5,3),\ \ B=(6,0),\ \ C=(5,5)$

$$d(A,B)=\sqrt{\left(6-(-5)\right)^2+(0-3)^2}$$
$$=\sqrt{11^2+(-3)^2}=\sqrt{121+9}$$
$$=\sqrt{130}$$
$$d(B,C)=\sqrt{\left(5-6\right)^2+(5-0)^2}$$
$$=\sqrt{(-1)^2+5^2}=\sqrt{1+25}$$
$$=\sqrt{26}$$
$$d(A,C)=\sqrt{\left(5-(-5)\right)^2+(5-3)^2}$$
$$=\sqrt{10^2+2^2}=\sqrt{100+4}$$
$$=\sqrt{104}$$
$$=2\sqrt{26}$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

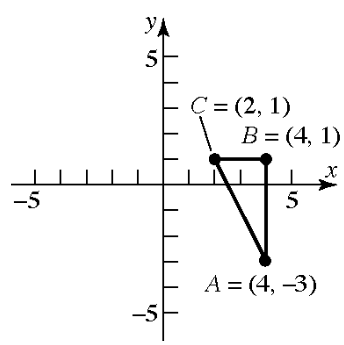
$$\left[d(A,C)\right]^2+\left[d(B,C)\right]^2=\left[d(A,B)\right]^2$$
$$\left(\sqrt{104}\right)^2+\left(\sqrt{26}\right)^2=\left(\sqrt{130}\right)^2$$
$$104+26=130$$
$$130=130$$

The area of a triangle is $A=\frac{1}{2}bh$. In this problem,

$$A=\frac{1}{2}\cdot\left[d(A,C)\right]\cdot\left[d(B,C)\right]$$
$$=\frac{1}{2}\cdot\sqrt{104}\cdot\sqrt{26}$$
$$=\frac{1}{2}\cdot 2\sqrt{26}\cdot\sqrt{26}$$
$$=\frac{1}{2}\cdot 2\cdot 26$$
$$= 26 \text{ square units}$$

35. $A=(4,-3),\ \ B=(4,1),\ \ C=(2,1)$

$$d(A,B)=\sqrt{(4-4)^2+\left(1-(-3)\right)^2}$$
$$=\sqrt{0^2+4^2}$$
$$=\sqrt{0+16}$$
$$=\sqrt{16}$$
$$=4$$
$$d(B,C)=\sqrt{\left(2-4\right)^2+\left(1-1\right)^2}$$
$$=\sqrt{(-2)^2+0^2}=\sqrt{4+0}$$
$$=\sqrt{4}$$
$$=2$$
$$d(A,C)=\sqrt{(2-4)^2+\left(1-(-3)\right)^2}$$
$$=\sqrt{(-2)^2+4^2}=\sqrt{4+16}$$
$$=\sqrt{20}$$
$$=2\sqrt{5}$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

$$\left[d(A,B)\right]^2+\left[d(B,C)\right]^2=\left[d(A,C)\right]^2$$
$$4^2+2^2=\left(2\sqrt{5}\right)^2$$
$$16+4=20$$
$$20=20$$

The area of a triangle is $A=\frac{1}{2}bh$. In this problem,

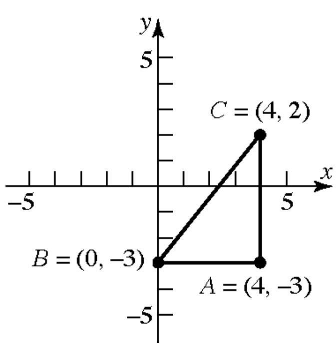
$$\begin{aligned}A&=\frac{1}{2}\cdot\left[d(A,B)\right]\cdot\left[d(B,C)\right]\\&=\frac{1}{2}\cdot 4\cdot 2\\&=4\text{ square units}\end{aligned}$$

36. $A=(4,-3),\ \ B=(0,-3),\ \ C=(4,2)$

$$\begin{aligned}d(A,B)&=\sqrt{(0-4)^2+\left(-3-(-3)\right)^2}\\&=\sqrt{(-4)^2+0^2}=\sqrt{16+0}\\&=\sqrt{16}\\&=4\end{aligned}$$

$$\begin{aligned}d(B,C)&=\sqrt{\left(4-0\right)^2+\left(2-(-3)\right)^2}\\&=\sqrt{4^2+5^2}=\sqrt{16+25}\\&=\sqrt{41}\end{aligned}$$

$$\begin{aligned}d(A,C)&=\sqrt{(4-4)^2+\left(2-(-3)\right)^2}\\&=\sqrt{0^2+5^2}=\sqrt{0+25}\\&=\sqrt{25}\\&=5\end{aligned}$$



Verifying that Δ ABC is a right triangle by the Pythagorean Theorem:

$$\left[d(A,B)\right]^2+\left[d(A,C)\right]^2=\left[d(B,C)\right]^2$$
$$4^2+5^2=\left(\sqrt{41}\right)^2$$
$$16+25=41$$
$$41=41$$

The area of a triangle is $A=\frac{1}{2}bh$. In this problem,

$$\begin{aligned}A&=\frac{1}{2}\cdot\left[d(A,B)\right]\cdot\left[d(A,C)\right]\\&=\frac{1}{2}\cdot 4\cdot 5\\&=10\text{ square units}\end{aligned}$$

37. The coordinates of the midpoint are:

$$\begin{aligned}(x,y)&=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)\\&=\left(\frac{3+5}{2},\frac{-4+4}{2}\right)\\&=\left(\frac{8}{2},\frac{0}{2}\right)\\&=(4,0)\end{aligned}$$

38. The coordinates of the midpoint are:

$$\begin{aligned}(x,y)&=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)\\&=\left(\frac{-2+2}{2},\frac{0+4}{2}\right)\\&=\left(\frac{0}{2},\frac{4}{2}\right)\\&=\left(0,2\right)\end{aligned}$$

39. The coordinates of the midpoint are:

$$\begin{aligned}(x,y)&=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)\\&=\left(\frac{2+4}{2},\frac{-3+2}{2}\right)\\&=\left(\frac{6}{2},\frac{-1}{2}\right)\\&=\left(3,-\frac{1}{2}\right)\end{aligned}$$

40. The coordinates of the midpoint are:

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$
$$=\left(\frac{-3+6}{2},\frac{2+0}{2}\right)$$
$$=\left(\frac{3}{2},\frac{2}{2}\right)$$
$$=\left(\frac{3}{2},1\right)$$

41. The coordinates of the midpoint are:

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$
$$=\left(\frac{-4+2}{2},\frac{-3+2}{2}\right)$$
$$=\left(\frac{-2}{2},\frac{-1}{2}\right)$$
$$=\left(-1,-\frac{1}{2}\right)$$

42. The coordinates of the midpoint are:

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$
$$=\left(\frac{4+6}{2},\frac{-3+1}{2}\right)$$
$$=\left(\frac{10}{2},\frac{-2}{2}\right)$$
$$=(5,-1)$$

43. The coordinates of the midpoint are:

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$
$$=\left(\frac{a+0}{2},\frac{a+0}{2}\right)$$
$$=\left(\frac{a}{2},\frac{a}{2}\right)$$

44. The coordinates of the midpoint are:

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$
$$=\left(\frac{a+0}{2},\frac{b+0}{2}\right)$$
$$=\left(\frac{a}{2},\frac{b}{2}\right)$$

45. The x coordinate would be $2+3=5$ and the y coordinate would be $5-2=3$. Thus the new point would be $(5,3)$.

46. The new x coordinate would be $-1-2=-3$ and the new y coordinate would be $6+4=10$. Thus the new point would be $(-3,10)$

47. **a.** If we use a right triangle to solve the problem, we know the hypotenuse is 13 units in length. One of the legs of the triangle will be 2+3=5. Thus the other leg will be:

$$5^2+b^2=13^2$$
$$25+b^2=169$$
$$b^2=144$$
$$b=12$$

Thus the coordinates will have an y value of $-1-12=-13$ and $-1+12=11$. So the points are $(3,11)$ and $(3,-13)$.

b. Consider points of the form $(3,y)$ that are a distance of 13 units from the point $(-2,-1)$.

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{(3-(-2))^2+(-1-y)^2}$$
$$=\sqrt{(5)^2+(-1-y)^2}$$
$$=\sqrt{25+1+2y+y^2}$$
$$=\sqrt{y^2+2y+26}$$
$$13=\sqrt{y^2+2y+26}$$
$$13^2=\left(\sqrt{y^2+2y+26}\right)^2$$
$$169=y^2+2y+26$$
$$0=y^2+2y-143$$
$$0=(y-11)(y+13)$$
$$y-11=0 \quad\text{or}\quad y+13=0$$
$$y=11 \qquad y=-13$$

Thus, the points $(3,11)$ and $(3,-13)$ are a distance of 13 units from the point $(-2,-1)$.

48. **a.** If we use a right triangle to solve the problem, we know the hypotenuse is 17 units in length. One of the legs of the triangle will be 2+6=8. Thus the other leg will be:

$$8^2 + b^2 = 17^2$$
$$64 + b^2 = 289$$
$$b^2 = 225$$
$$b = 15$$

Thus the coordinates will have an x value of $1-15=-14$ and $1+15=16$. So the points are $(-14,-6)$ and $(16,-6)$.

b. Consider points of the form $(x,-6)$ that are a distance of 17 units from the point $(1,2)$.

$$\begin{aligned} d &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(1-x)^2+(2-(-6))^2} \\ &= \sqrt{x^2-2x+1+(8)^2} \\ &= \sqrt{x^2-2x+1+64} \\ &= \sqrt{x^2-2x+65} \end{aligned}$$

$$17 = \sqrt{x^2-2x+65}$$
$$17^2 = \left(\sqrt{x^2-2x+65}\right)^2$$
$$289 = x^2-2x+65$$
$$0 = x^2-2x-224$$
$$0 = (x+14)(x-16)$$
$$x+14=0 \quad \text{or} \quad x-16=0$$
$$x=-14 \qquad\qquad x=16$$

Thus, the points $(-14,-6)$ and $(16,-6)$ are a distance of 13 units from the point $(1,2)$.

49. Points on the *x*-axis have a *y*-coordinate of 0. Thus, we consider points of the form $(x,0)$ that are a distance of 6 units from the point $(4,-3)$.

$$\begin{aligned} d &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(4-x)^2+(-3-0)^2} \\ &= \sqrt{16-8x+x^2+(-3)^2} \\ &= \sqrt{16-8x+x^2+9} \\ &= \sqrt{x^2-8x+25} \end{aligned}$$

$$6 = \sqrt{x^2-8x+25}$$
$$6^2 = \left(\sqrt{x^2-8x+25}\right)^2$$
$$36 = x^2-8x+25$$
$$0 = x^2-8x-11$$

$$\begin{aligned} x &= \frac{-(-8)\pm\sqrt{(-8)^2-4(1)(-11)}}{2(1)} \\ &= \frac{8\pm\sqrt{64+44}}{2} = \frac{8\pm\sqrt{108}}{2} \\ &= \frac{8\pm6\sqrt{3}}{2} = 4\pm3\sqrt{3} \end{aligned}$$

$$x = 4+3\sqrt{3} \quad \text{or} \quad x = 4-3\sqrt{3}$$

Thus, the points $\left(4+3\sqrt{3},0\right)$ and $\left(4-3\sqrt{3},0\right)$ are on the *x*-axis and a distance of 6 units from the point $(4,-3)$.

50. Points on the *y*-axis have an *x*-coordinate of 0. Thus, we consider points of the form $(0,y)$ that are a distance of 6 units from the point $(4,-3)$.

$$\begin{aligned} d &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(4-0)^2+(-3-y)^2} \\ &= \sqrt{4^2+9+6y+y^2} \\ &= \sqrt{16+9+6y+y^2} \\ &= \sqrt{y^2+6y+25} \end{aligned}$$

$$6 = \sqrt{y^2+6y+25}$$
$$6^2 = \left(\sqrt{y^2+6y+25}\right)^2$$
$$36 = y^2+6y+25$$
$$0 = y^2+6y-11$$

$$\begin{aligned} y &= \frac{(-6)\pm\sqrt{(6)^2-4(1)(-11)}}{2(1)} \\ &= \frac{-6\pm\sqrt{36+44}}{2} = \frac{-6\pm\sqrt{80}}{2} \\ &= \frac{-6\pm4\sqrt{5}}{2} = -3\pm2\sqrt{5} \end{aligned}$$

$$y = -3+2\sqrt{5} \quad \text{or} \quad y = -3-2\sqrt{5}$$

Thus, the points $\left(0,-3+2\sqrt{5}\right)$ and $\left(0,-3-2\sqrt{5}\right)$

are on the y-axis and a distance of 6 units from the point $(4,-3)$.

51. **a.** To shift 3 units left and 4 units down, we subtract 3 from the x-coordinate and subtract 4 from the y-coordinate.
$(2-3,5-4)=(-1,1)$

b. To shift left 2 units and up 8 units, we subtract 2 from the x-coordinate and add 8 to the y-coordinate.
$(2-2,5+8)=(0,13)$

52. Let the coordinates of point B be (x,y). Using the midpoint formula, we can write
$(2,3)=\left(\dfrac{-1+x}{2},\dfrac{8+y}{2}\right)$.
This leads to two equations we can solve.

$$\begin{aligned}\frac{-1+x}{2}&=2 & \frac{8+y}{2}&=3\\ -1+x&=4 & 8+y&=6\\ x&=5 & y&=-2\end{aligned}$$

Point B has coordinates $(5,-2)$.

53. $M=(x,y)=\left(\dfrac{x_1+x_2}{2},\dfrac{y_1+y_2}{2}\right)$.

$P_1=(x_1,y_1)=(-3,6)$ and $(x,y)=(-1,4)$, so

$$\begin{aligned}x&=\frac{x_1+x_2}{2} \quad\text{and} & y&=\frac{y_1+y_2}{2}\\ -1&=\frac{-3+x_2}{2} & 4&=\frac{6+y_2}{2}\\ -2&=-3+x_2 & 8&=6+y_2\\ 1&=x_2 & 2&=y_2\end{aligned}$$

Thus, $P_2=(1,2)$.

54. $M=(x,y)=\left(\dfrac{x_1+x_2}{2},\dfrac{y_1+y_2}{2}\right)$.

$P_2=(x_2,y_2)=(7,-2)$ and $(x,y)=(5,-4)$, so

$$\begin{aligned}x&=\frac{x_1+x_2}{2} \quad\text{and} & y&=\frac{y_1+y_2}{2}\\ 5&=\frac{x_1+7}{2} & -4&=\frac{y_1+(-2)}{2}\\ 10&=x_1+7 & -8&=y_1+(-2)\\ 3&=x_1 & -6&=y_1\end{aligned}$$

Thus, $P_1=(3,-6)$.

55. The midpoint of AB is: $D=\left(\dfrac{0+6}{2},\dfrac{0+0}{2}\right)=(3,0)$

The midpoint of AC is: $E=\left(\dfrac{0+4}{2},\dfrac{0+4}{2}\right)=(2,2)$

The midpoint of BC is: $F=\left(\dfrac{6+4}{2},\dfrac{0+4}{2}\right)=(5,2)$

$$\begin{aligned}d(C,D)&=\sqrt{(0-4)^2+(3-4)^2}\\&=\sqrt{(-4)^2+(-1)^2}=\sqrt{16+1}=\sqrt{17}\\ d(B,E)&=\sqrt{(2-6)^2+(2-0)^2}\\&=\sqrt{(-4)^2+2^2}=\sqrt{16+4}\\&=\sqrt{20}=2\sqrt{5}\\ d(A,F)&=\sqrt{(2-0)^2+(5-0)^2}\\&=\sqrt{2^2+5^2}=\sqrt{4+25}\\&=\sqrt{29}\end{aligned}$$

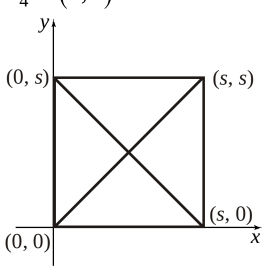
56. Let $P_1=(0,0)$, $P_2=(0,4)$, $P=(x,y)$

$$\begin{aligned}d(P_1,P_2)&=\sqrt{(0-0)^2+(4-0)^2}\\&=\sqrt{16}=4\\ d(P_1,P)&=\sqrt{(x-0)^2+(y-0)^2}\\&=\sqrt{x^2+y^2}=4\\ &\to\ x^2+y^2=16\\ d(P_2,P)&=\sqrt{(x-0)^2+(y-4)^2}\\&=\sqrt{x^2+(y-4)^2}=4\\ &\to\ x^2+(y-4)^2=16\end{aligned}$$

Therefore,

$$y^2 = (y-4)^2$$
$$y^2 = y^2 - 8y + 16$$
$$8y = 16$$
$$y = 2$$
which gives
$$x^2 + 2^2 = 16$$
$$x^2 = 12$$
$$x = \pm 2\sqrt{3}$$
Two triangles are possible. The third vertex is $\left(-2\sqrt{3}, 2\right)$ or $\left(2\sqrt{3}, 2\right)$.

57. Let $P_1 = (0,0)$, $P_2 = (0,s)$, $P_3 = (s,0)$, and $P_4 = (s,s)$.



The points P_1 and P_4 are endpoints of one diagonal and the points P_2 and P_3 are the endpoints of the other diagonal.
$$M_{1,4} = \left(\frac{0+s}{2}, \frac{0+s}{2}\right) = \left(\frac{s}{2}, \frac{s}{2}\right)$$
$$M_{2,3} = \left(\frac{0+s}{2}, \frac{s+0}{2}\right) = \left(\frac{s}{2}, \frac{s}{2}\right)$$
The midpoints of the diagonals are the same. Therefore, the diagonals of a square intersect at their midpoints.

58. Let $P_1 = (0,0)$, $P_2 = (a,0)$, and $P_3 = \left(\frac{a}{2}, \frac{\sqrt{3}\,a}{2}\right)$. To show that these vertices form an equilateral triangle, we need to show that the distance between any pair of points is the same constant value.
$$d(P_1, P_2) = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$
$$= \sqrt{(a-0)^2 + (0-0)^2} = \sqrt{a^2} = |a|$$
$$d(P_2, P_3) = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$
$$= \sqrt{\left(\frac{a}{2} - a\right)^2 + \left(\frac{\sqrt{3}\,a}{2} - 0\right)^2}$$
$$= \sqrt{\frac{a^2}{4} + \frac{3a^2}{4}} = \sqrt{\frac{4a^2}{4}} = \sqrt{a^2} = |a|$$
$$d(P_1, P_3) = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$
$$= \sqrt{\left(\frac{a}{2} - 0\right)^2 + \left(\frac{\sqrt{3}\,a}{2} - 0\right)^2}$$
$$= \sqrt{\frac{a^2}{4} + \frac{3a^2}{4}} = \sqrt{\frac{4a^2}{4}} = \sqrt{a^2} = |a|$$
Since all three distances have the same constant value, the triangle is an equilateral triangle.
Now find the midpoints:
$$P_4 = M_{P_1P_2} = \left(\frac{0+a}{2}, \frac{0+0}{2}\right) = \left(\frac{a}{2}, 0\right)$$
$$P_5 = M_{P_2P_3} = \left(\frac{a + \frac{a}{2}}{2}, \frac{0 + \frac{\sqrt{3}\,a}{2}}{2}\right) = \left(\frac{3a}{4}, \frac{\sqrt{3}\,a}{4}\right)$$
$$P_6 = M_{P_1P_3} = \left(\frac{0 + \frac{a}{2}}{2}, \frac{0 + \frac{\sqrt{3}a}{2}}{2}\right) = \left(\frac{a}{4}, \frac{\sqrt{3}\,a}{4}\right)$$

$$d(P_4, P_5) = \sqrt{\left(\frac{3a}{4} - \frac{a}{2}\right)^2 + \left(\frac{\sqrt{3}\,a}{4} - 0\right)^2}$$
$$= \sqrt{\left(\frac{a}{4}\right)^2 + \left(\frac{\sqrt{3}\,a}{4}\right)^2}$$
$$= \sqrt{\frac{a^2}{16} + \frac{3a^2}{16}} = \frac{|a|}{2}$$

$$d\left(P_4, P_6\right) = \sqrt{\left(\frac{a}{4} - \frac{a}{2}\right)^2 + \left(\frac{\sqrt{3}\,a}{4} - 0\right)^2}$$
$$= \sqrt{\left(-\frac{a}{4}\right)^2 + \left(\frac{\sqrt{3}\,a}{4}\right)^2}$$
$$= \sqrt{\frac{a^2}{16} + \frac{3a^2}{16}} = \frac{|a|}{2}$$
$$d\left(P_5, P_6\right) = \sqrt{\left(\frac{3a}{4} - \frac{a}{4}\right)^2 + \left(\frac{\sqrt{3}\,a}{4} - \frac{\sqrt{3}\,a}{4}\right)^2}$$
$$= \sqrt{\left(\frac{a}{2}\right)^2 + 0^2}$$
$$= \sqrt{\frac{a^2}{4}} = \frac{|a|}{2}$$

Since the sides are the same length, the triangle is equilateral.

59. $d(P_1, P_2) = \sqrt{(-4-2)^2 + (1-1)^2}$
$$= \sqrt{(-6)^2 + 0^2}$$
$$= \sqrt{36}$$
$$= 6$$
$$d(P_2, P_3) = \sqrt{\left(-4-(-4)\right)^2 + (-3-1)^2}$$
$$= \sqrt{0^2 + (-4)^2}$$
$$= \sqrt{16}$$
$$= 4$$
$$d(P_1, P_3) = \sqrt{(-4-2)^2 + (-3-1)^2}$$
$$= \sqrt{(-6)^2 + (-4)^2}$$
$$= \sqrt{36+16}$$
$$= \sqrt{52}$$
$$= 2\sqrt{13}$$

Since $\left[d(P_1, P_2)\right]^2 + \left[d(P_2, P_3)\right]^2 = \left[d(P_1, P_3)\right]^2$, the triangle is a right triangle.

60. $d(P_1, P_2) = \sqrt{\left(6-(-1)\right)^2 + (2-4)^2}$
$$= \sqrt{7^2 + (-2)^2}$$
$$= \sqrt{49+4}$$
$$= \sqrt{53}$$
$$d(P_2, P_3) = \sqrt{\left(4-6\right)^2 + (-5-2)^2}$$
$$= \sqrt{(-2)^2 + (-7)^2}$$
$$= \sqrt{4+49}$$
$$= \sqrt{53}$$
$$d(P_1, P_3) = \sqrt{\left(4-(-1)\right)^2 + (-5-4)^2}$$
$$= \sqrt{5^2 + (-9)^2}$$
$$= \sqrt{25+81}$$
$$= \sqrt{106}$$

Since $\left[d(P_1, P_2)\right]^2 + \left[d(P_2, P_3)\right]^2 = \left[d(P_1, P_3)\right]^2$, the triangle is a right triangle.

Since $d\left(P_1, P_2\right) = d\left(P_2, P_3\right)$, the triangle is isosceles.

Therefore, the triangle is an isosceles right triangle.

61. $d(P_1, P_2) = \sqrt{\left(0-(-2)\right)^2 + \left(7-(-1)\right)^2}$
$$= \sqrt{2^2 + 8^2} = \sqrt{4+64} = \sqrt{68}$$
$$= 2\sqrt{17}$$
$$d(P_2, P_3) = \sqrt{\left(3-0\right)^2 + (2-7)^2}$$
$$= \sqrt{3^2 + (-5)^2} = \sqrt{9+25}$$
$$= \sqrt{34}$$
$$d(P_1, P_3) = \sqrt{\left(3-(-2)\right)^2 + \left(2-(-1)\right)^2}$$
$$= \sqrt{5^2 + 3^2} = \sqrt{25+9}$$
$$= \sqrt{34}$$

Since $d(P_2, P_3) = d(P_1, P_3)$, the triangle is isosceles.

Since $\left[d(P_1, P_3)\right]^2 + \left[d(P_2, P_3)\right]^2 = \left[d(P_1, P_2)\right]^2$, the triangle is also a right triangle.

Therefore, the triangle is an isosceles right triangle.

62. $d(P_1,P_2)=\sqrt{(-4-7)^2+(0-2)^2}$
$=\sqrt{(-11)^2+(-2)^2}$
$=\sqrt{121+4}=\sqrt{125}$
$=5\sqrt{5}$

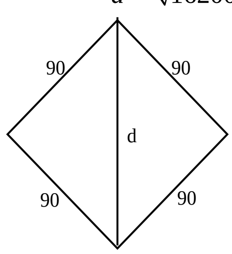
$d(P_2,P_3)=\sqrt{(4-(-4))^2+(6-0)^2}$
$=\sqrt{8^2+6^2}=\sqrt{64+36}$
$=\sqrt{100}$
$=10$

$d(P_1,P_3)=\sqrt{(4-7)^2+(6-2)^2}$
$=\sqrt{(-3)^2+4^2}=\sqrt{9+16}$
$=\sqrt{25}$
$=5$

Since $\left[d(P_1,P_3)\right]^2+\left[d(P_2,P_3)\right]^2=\left[d(P_1,P_2)\right]^2$, the triangle is a right triangle.

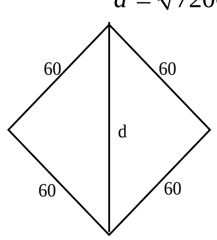
63. Using the Pythagorean Theorem:

$$90^2+90^2=d^2$$
$$8100+8100=d^2$$
$$16200=d^2$$
$$d=\sqrt{16200}=90\sqrt{2}\approx 127.28 \text{ feet}$$

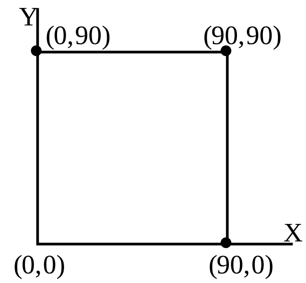


64. Using the Pythagorean Theorem:

$$60^2+60^2=d^2$$
$$3600+3600=d^2 \rightarrow 7200=d^2$$
$$d=\sqrt{7200}=60\sqrt{2}\approx 84.85 \text{ feet}$$



65. a. First: (90, 0), Second: (90, 90),
Third: (0, 90)



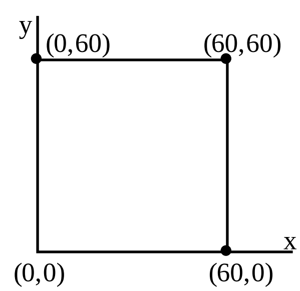
b. Using the distance formula:

$$d=\sqrt{(310-90)^2+(15-90)^2}$$
$$=\sqrt{220^2+(-75)^2}=\sqrt{54025}$$
$$=5\sqrt{2161}\approx 232.43 \text{ feet}$$

c. Using the distance formula:

$$d=\sqrt{(300-0)^2+(300-90)^2}$$
$$=\sqrt{300^2+210^2}=\sqrt{134100}$$
$$=30\sqrt{149}\approx 366.20 \text{ feet}$$

66. a. First: (60, 0), Second: (60, 60)
Third: (0, 60)



b. Using the distance formula:

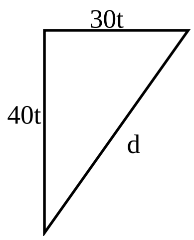
$$d=\sqrt{(180-60)^2+(20-60)^2}$$
$$=\sqrt{120^2+(-40)^2}=\sqrt{16000}$$
$$=40\sqrt{10}\approx 126.49 \text{ feet}$$

c. Using the distance formula:

$$d=\sqrt{(220-0)^2+(220-60)^2}$$
$$=\sqrt{220^2+160^2}=\sqrt{74000}$$
$$=20\sqrt{185}\approx 272.03 \text{ feet}$$

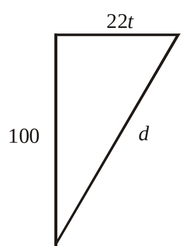
67. The Focus heading east moves a distance $30t$ after t hours. The truck heading south moves a distance $40t$ after t hours. Their distance apart after t hours is:

$$
\begin{aligned}
d &= \sqrt{(30t)^2 + (40t)^2} \\
&= \sqrt{900t^2 + 1600t^2} \\
&= \sqrt{2500t^2} \\
&= 50t \text{ miles}
\end{aligned}
$$



68. $\dfrac{15 \text{ miles}}{1 \text{ hr}} \cdot \dfrac{5280 \text{ ft}}{1 \text{ mile}} \cdot \dfrac{1 \text{ hr}}{3600 \text{ sec}} = 22 \text{ ft/sec}$

$$
\begin{aligned}
d &= \sqrt{100^2 + \left(22t\right)^2} \\
&= \sqrt{10000 + 484t^2} \text{ feet}
\end{aligned}
$$



69. a. The shortest side is between $P_1 = (2.6, 1.5)$ and $P_2 = (2.7, 1.7)$. The estimate for the desired intersection point is:

$$
\begin{aligned}
\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) &= \left(\frac{2.6 + 2.7}{2}, \frac{1.5 + 1.7}{2}\right) \\
&= \left(\frac{5.3}{2}, \frac{3.2}{2}\right) \\
&= \left(2.65, 1.6\right)
\end{aligned}
$$

b. Using the distance formula:

$$
\begin{aligned}
d &= \sqrt{(2.65 - 1.4)^2 + (1.6 - 1.3)^2} \\
&= \sqrt{(1.25)^2 + (0.3)^2} \\
&= \sqrt{1.5625 + 0.09} \\
&= \sqrt{1.6525} \\
&\approx 1.285 \text{ units}
\end{aligned}
$$

70. Let $P_1 = (2007, 345)$ and $P_2 = (2013, 466)$. The midpoint is:

$$
\begin{aligned}
\left(x, y\right) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \\
&= \left(\frac{2007 + 2013}{2}, \frac{345 + 466}{2}\right) \\
&= \left(\frac{4020}{2}, \frac{811}{2}\right) \\
&= \left(2010, 405.5\right)
\end{aligned}
$$

The estimate for 2010 is \$405.5 billion. The estimate net sales of Wal-Mart Stores, Inc. in 2010 is \$0.5 billion off from the reported value of \$405 billion.

71. For 2003 we have the ordered pair $\left(2003, 18660\right)$ and for 2013 we have the ordered pair $\left(2013, 23624\right)$. The midpoint is

$$
\begin{aligned}
\left(\text{year}, \$\right) &= \left(\frac{2003 + 2013}{2}, \frac{18660 + 23624}{2}\right) \\
&= \left(\frac{4016}{2}, \frac{42284}{2}\right) \\
&= \left(2008, 21142\right)
\end{aligned}
$$

Using the midpoint, we estimate the poverty level in 2008 to be \$21,142. This is lower than the actual value.

72. Answers will vary.

Section 1.2

1.
$$
\begin{aligned}
2\left(x + 3\right) - 1 &= -7 \\
2\left(x + 3\right) &= -6 \\
x + 3 &= -3 \\
x &= -6
\end{aligned}
$$

The solution set is $\{-6\}$.

2.
$$
\begin{aligned}
x^2 - 9 &= 0 \\
x^2 &= 9 \\
x &= \pm\sqrt{9} = \pm 3
\end{aligned}
$$

The solution set is $\{-3, 3\}$.

3. intercepts

4. $y = 0$

5. y-axis

6. 4

7. $(-3,4)$

8. True

9. False; the y-coordinate of a point at which the graph crosses or touches the x-axis is always 0. The x-coordinate of such a point is an x-intercept.

10. False; a graph can be symmetric with respect to both coordinate axes (in such cases it will also be symmetric with respect to the origin).
For example: $x^2 + y^2 = 1$

11. d

12. c

13. $y = x^4 - \sqrt{x}$

$0 = 0^4 - \sqrt{0}$ $1 = 1^4 - \sqrt{1}$ $4 = (2)^4 - \sqrt{2}$

$0 = 0$ $1 \neq 0$ $4 \neq 16 - \sqrt{2}$

The point (0, 0) is on the graph of the equation.

14. $y = x^3 - 2\sqrt{x}$

$0 = 0^3 - 2\sqrt{0}$ $1 = 1^3 - 2\sqrt{1}$ $-1 = 1^3 - 2\sqrt{1}$

$0 = 0$ $1 \neq -1$ $-1 = -1$

The points (0, 0) and (1, –1) are on the graph of the equation.

15. $y^2 = x^2 + 9$

$3^2 = 0^2 + 9$ $0^2 = 3^2 + 9$ $0^2 = (-3)^2 + 9$

$9 = 9$ $0 \neq 18$ $0 \neq 18$

The point (0, 3) is on the graph of the equation.

16. $y^3 = x + 1$

$2^3 = 1 + 1$ $1^3 = 0 + 1$ $0^3 = -1 + 1$

$8 \neq 2$ $1 = 1$ $0 = 0$

The points (0, 1) and (–1, 0) are on the graph of the equation.

17. $x^2 + y^2 = 4$

$0^2 + 2^2 = 4$ $(-2)^2 + 2^2 = 4$ $\left(\sqrt{2}\right)^2 + \left(\sqrt{2}\right)^2 = 4$

$4 = 4$ $8 \neq 4$ $4 = 4$

(0, 2) and $\left(\sqrt{2}, \sqrt{2}\right)$ are on the graph of the equation.

18. $x^2 + 4y^2 = 4$

$0^2 + 4 \cdot 1^2 = 4$ $2^2 + 4 \cdot 0^2 = 4$ $2^2 + 4\left(\frac{1}{2}\right)^2 = 4$

$4 = 4$ $4 = 4$ $5 \neq 4$

The points (0, 1) and (2, 0) are on the graph of the equation.

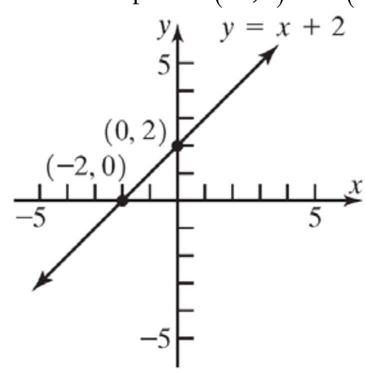
19. $y = x + 2$

x-intercept: y-intercept:

$0 = x + 2$ $y = 0 + 2$

$-2 = x$ $y = 2$

The intercepts are $(-2,0)$ and $(0,2)$.



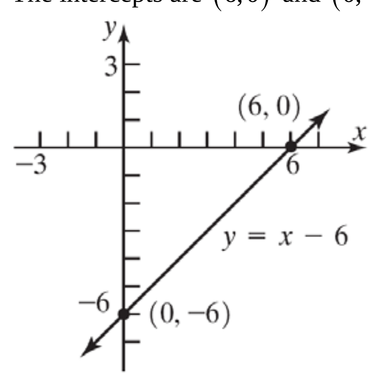
20. $y = x - 6$

x-intercept: y-intercept:

$0 = x - 6$ $y = 0 - 6$

$6 = x$ $y = -6$

The intercepts are $(6,0)$ and $(0,-6)$.



21. $y = 2x + 8$

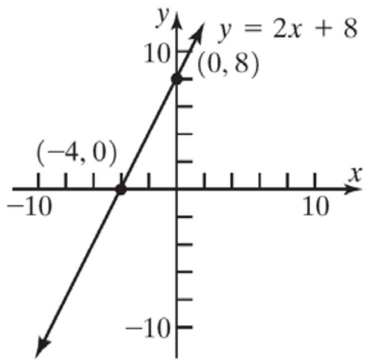
x-intercept: y-intercept:

$0 = 2x + 8$ $y = 2(0) + 8$

$2x = -8$ $y = 8$

$x = -4$

The intercepts are $(-4, 0)$ and $(0, 8)$.

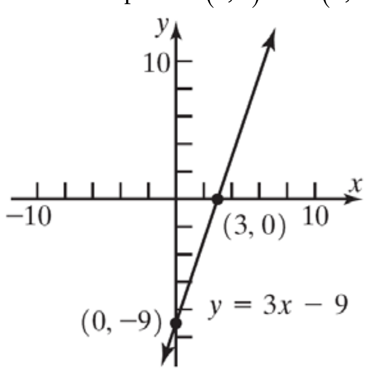


22. $y = 3x - 9$

x-intercept:
$0 = 3x - 9$
$3x = 9$
$x = 3$

y-intercept:
$y = 3(0) - 9$
$y = -9$

The intercepts are $(3, 0)$ and $(0, -9)$.

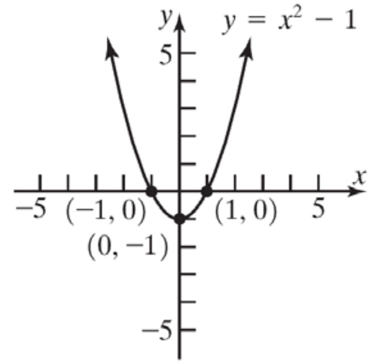


23. $y = x^2 - 1$

x-intercepts:
$0 = x^2 - 1$
$x^2 = 1$
$x = \pm 1$

y-intercept:
$y = 0^2 - 1$
$y = -1$

The intercepts are $(-1, 0)$, $(1, 0)$, and $(0, -1)$.

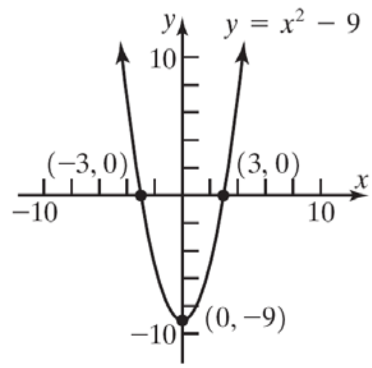


24. $y = x^2 - 9$

x-intercepts:
$0 = x^2 - 9$
$x^2 = 9$
$x = \pm 3$

y-intercept:
$y = 0^2 - 9$
$y = -9$

The intercepts are $(-3, 0)$, $(3, 0)$, and $(0, -9)$.

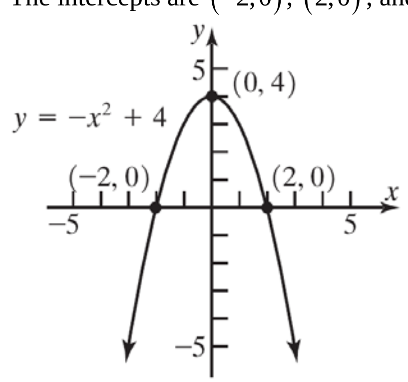


25. $y = -x^2 + 4$

x-intercepts:
$0 = -x^2 + 4$
$x^2 = 4$
$x = \pm 2$

y-intercepts:
$y = -(0)^2 + 4$
$y = 4$

The intercepts are $(-2, 0)$, $(2, 0)$, and $(0, 4)$.

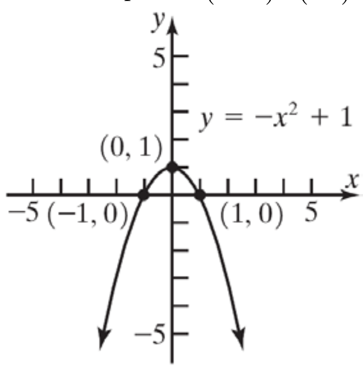


26. $y = -x^2 + 1$

x-intercepts: $0 = -x^2 + 1$; $x^2 = 1$; $x = \pm 1$

y-intercept: $y = -(0)^2 + 1$; $y = 1$

The intercepts are $(-1,0)$, $(1,0)$, and $(0,1)$.

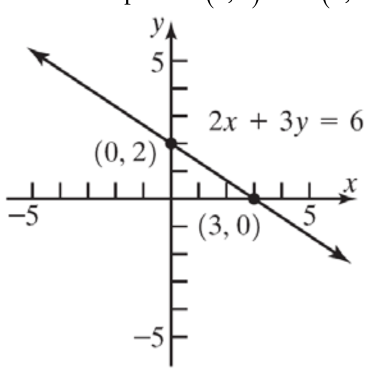


27. $2x + 3y = 6$

x-intercepts: $2x + 3(0) = 6$; $2x = 6$; $x = 3$

y-intercept: $2(0) + 3y = 6$; $3y = 6$; $y = 2$

The intercepts are $(3,0)$ and $(0,2)$.

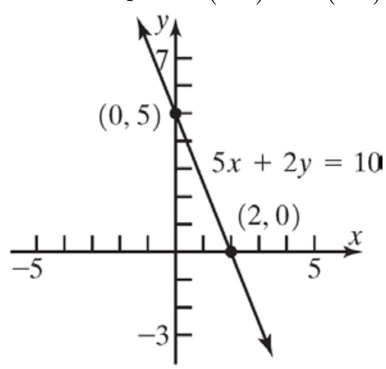


28. $5x + 2y = 10$

x-intercepts: $5x + 2(0) = 10$; $5x = 10$; $x = 2$

y-intercept: $5(0) + 2y = 10$; $2y = 10$; $y = 5$

The intercepts are $(2,0)$ and $(0,5)$.

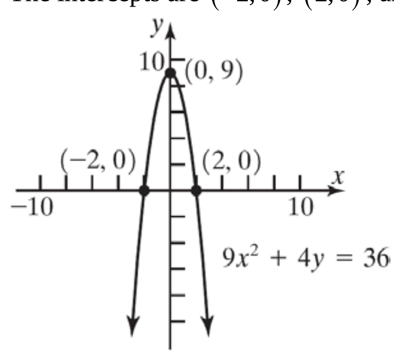


29. $9x^2 + 4y = 36$

x-intercepts: $9x^2 + 4(0) = 36$; $9x^2 = 36$; $x^2 = 4$; $x = \pm 2$

y-intercept: $9(0)^2 + 4y = 36$; $4y = 36$; $y = 9$

The intercepts are $(-2,0)$, $(2,0)$, and $(0,9)$.

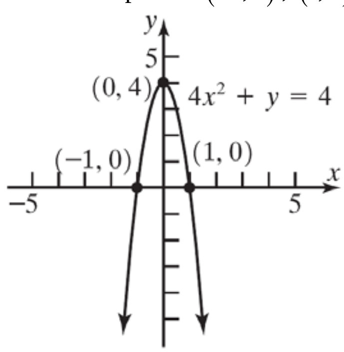


30. $4x^2 + y = 4$

x-intercepts: $4x^2 + 0 = 4$; $4x^2 = 4$; $x^2 = 1$; $x = \pm 1$

y-intercept: $4(0)^2 + y = 4$; $y = 4$

The intercepts are $(-1,0)$, $(1,0)$, and $(0,4)$.



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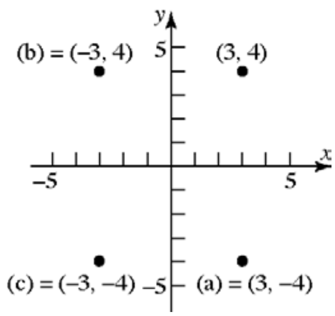

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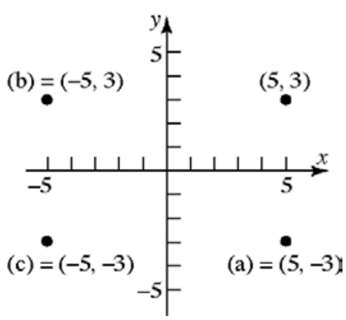

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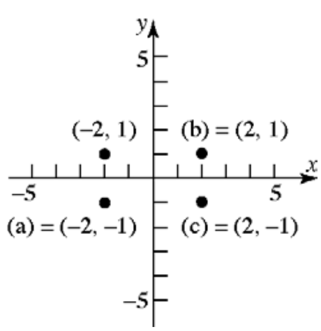

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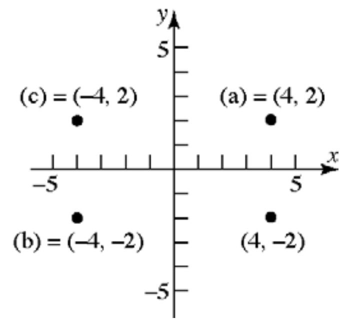

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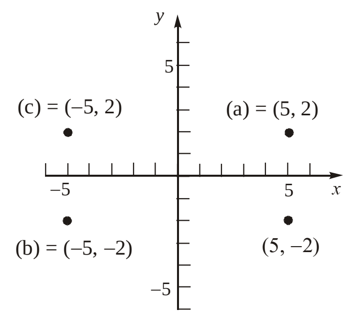

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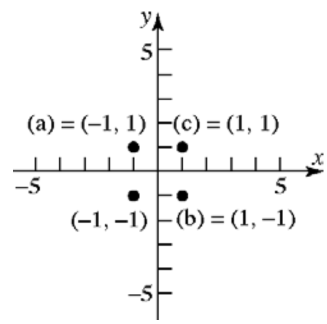

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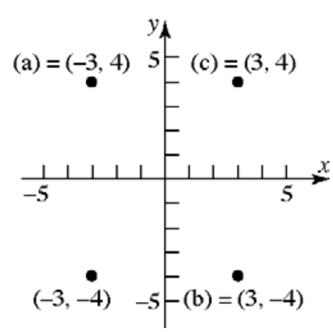

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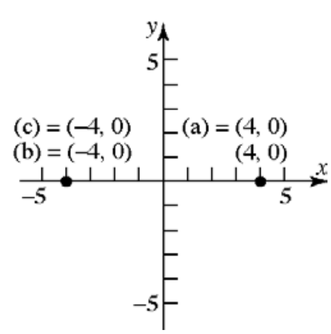

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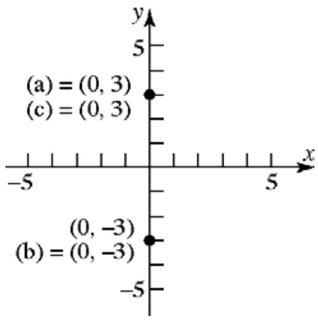

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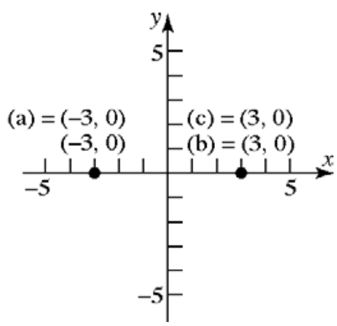


41. **a.** Intercepts: $(-1,0)$ and $(1,0)$

b. Symmetric with respect to the *x*-axis, *y*-axis, and the origin.

42. **a.** Intercepts: $(0,1)$

b. Not symmetric to the *x*-axis, the *y*-axis, nor the origin

43. **a.** Intercepts: $\left(-\frac{\pi}{2},0\right)$, $(0,1)$, and $\left(\frac{\pi}{2},0\right)$

b. Symmetric with respect to the *y*-axis.

44. **a.** Intercepts: $(-2,0)$, $(0,-3)$, and $(2,0)$

b. Symmetric with respect to the *y*-axis.

45. **a.** Intercepts: $(0,0)$

b. Symmetric with respect to the *x*-axis.

46. **a.** Intercepts: $(-2,0)$, $(0,2)$, $(0,-2)$, and $(2,0)$

b. Symmetric with respect to the *x*-axis, *y*-axis, and the origin.

47. **a.** Intercepts: $(-2,0)$, $(0,0)$, and $(2,0)$

b. Symmetric with respect to the origin.

48. **a.** Intercepts: $(-4,0)$, $(0,0)$, and $(4,0)$

b. Symmetric with respect to the origin.

49. **a.** x-intercept: $[-2,1]$, y-intercept 0

b. Not symmetric to x-axis, y-axis, or origin.

50. **a.** x-intercept: $[-1,2]$, y-intercept 0

b. Not symmetric to x-axis, y-axis, or origin.

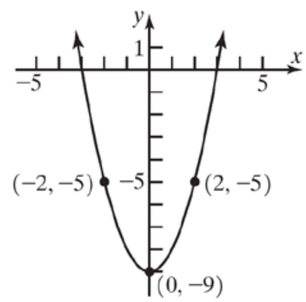
51. **a.** Intercepts: none

b. Symmetric with respect to the origin.

52. **a.** Intercepts: none

b. Symmetric with respect to the *x*-axis.

53.



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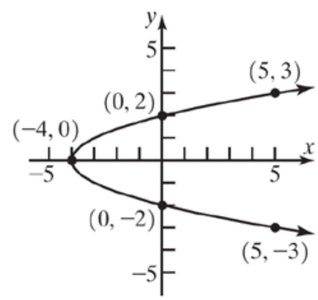

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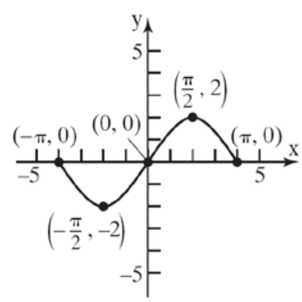

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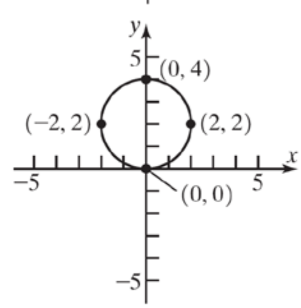


57. $y^2 = x + 4$

x-intercepts:
$0^2 = x + 4$
$-4 = x$

y-intercepts:
$y^2 = 0 + 4$
$y^2 = 4$
$y = \pm 2$

The intercepts are $(-4, 0)$, $(0, -2)$ and $(0, 2)$.

Test x-axis symmetry: Let $y = -y$
$(-y)^2 = x + 4$
$y^2 = x + 4$ same

Test y-axis symmetry: Let $x = -x$
$y^2 = -x + 4$ different

Test origin symmetry: Let $x = -x$ and $y = -y$.
$(-y)^2 = -x + 4$
$y^2 = -x + 4$ different

Therefore, the graph will have x-axis symmetry.

58. $y^2 = x + 9$

x-intercepts:
$(0)^2 = -x + 9$
$0 = -x + 9$
$x = 9$

y-intercepts:
$y^2 = 0 + 9$
$y^2 = 9$
$y = \pm 3$

The intercepts are $(-9, 0)$, $(0, -3)$ and $(0, 3)$.

Test x-axis symmetry: Let $y = -y$
$(-y)^2 = x + 9$
$y^2 = x + 9$ same

Test y-axis symmetry: Let $x = -x$
$y^2 = -x + 9$ different

Test origin symmetry: Let $x = -x$ and $y = -y$.
$(-y)^2 = -x + 9$
$y^2 = -x + 9$ different

Therefore, the graph will have x-axis symmetry.

59. $y = \sqrt[3]{x}$

x-intercepts:
$0 = \sqrt[3]{x}$
$0 = x$

y-intercepts:
$y = \sqrt[3]{0} = 0$

The only intercept is $(0, 0)$.

Test x-axis symmetry: Let $y = -y$
$-y = \sqrt[3]{x}$ different

Test y-axis symmetry: Let $x = -x$
$y = \sqrt[3]{-x} = -\sqrt[3]{x}$ different

Test origin symmetry: Let $x = -x$ and $y = -y$
$-y = \sqrt[3]{-x} = -\sqrt[3]{x}$
$y = \sqrt[3]{x}$ same

Therefore, the graph will have origin symmetry.

60. $y = \sqrt[5]{x}$

x-intercepts:
$0 = \sqrt[3]{x}$
$0 = x$

y-intercepts:
$y = \sqrt[5]{0} = 0$

The only intercept is $(0, 0)$.

Test x-axis symmetry: Let $y = -y$
$-y = \sqrt[5]{x}$ different

Test y-axis symmetry: Let $x = -x$
$y = \sqrt[5]{-x} = -\sqrt[5]{x}$ different

Test origin symmetry: Let $x = -x$ and $y = -y$
$-y = \sqrt[5]{-x} = -\sqrt[5]{x}$
$y = \sqrt[5]{x}$ same

Therefore, the graph will have origin symmetry.

61. $x^2 + y - 9 = 0$

x-intercepts:
$x^2 - 9 = 0$
$x^2 = 9$
$x = \pm 3$

y-intercepts:
$0^2 + y - 9 = 0$
$y = 9$

The intercepts are $(-3, 0)$, $(3, 0)$, and $(0, 9)$.

Test x-axis symmetry: Let $y = -y$
$x^2 - y - 9 = 0$ different

Test y-axis symmetry: Let $x = -x$
$(-x)^2 + y - 9 = 0$
$x^2 + y - 9 = 0$ same

Test origin symmetry: Let $x = -x$ and $y = -y$
$(-x)^2 - y - 9 = 0$
$x^2 - y - 9 = 0$ different

Therefore, the graph will have y-axis symmetry.

62. $x^2 - y - 4 = 0$

x-intercepts:

y-intercept:

$x^2 - 0 - 4 = 0$ $\quad$ $0^2 - y - 4 = 0$

$x^2 = 4$ $\quad$ $-y = 4$

$x = \pm 2$ $\quad$ $y = -4$

The intercepts are $(-2,0)$, $(2,0)$, and $(0,-4)$.

Test x-axis symmetry: Let $y = -y$

$x^2 - (-y) - 4 = 0$

$x^2 + y - 4 = 0$ different

Test y-axis symmetry: Let $x = -x$

$(-x)^2 - y - 4 = 0$

$x^2 - y - 4 = 0$ same

Test origin symmetry: Let $x = -x$ and $y = -y$

$(-x)^2 - (-y) - 4 = 0$

$x^2 + y - 4 = 0$ different

Therefore, the graph will have y-axis symmetry.

63. $9x^2 + 4y^2 = 36$

x-intercepts: $\quad$ y-intercepts:

$9x^2 + 4(0)^2 = 36$ $\quad$ $9(0)^2 + 4y^2 = 36$

$9x^2 = 36$ $\quad$ $4y^2 = 36$

$x^2 = 4$ $\quad$ $y^2 = 9$

$x = \pm 2$ $\quad$ $y = \pm 3$

The intercepts are $(-2,0)$, $(2,0)$, $(0,-3)$, and $(0,3)$.

Test x-axis symmetry: Let $y = -y$

$9x^2 + 4(-y)^2 = 36$

$9x^2 + 4y^2 = 36$ same

Test y-axis symmetry: Let $x = -x$

$9(-x)^2 + 4y^2 = 36$

$9x^2 + 4y^2 = 36$ same

Test origin symmetry: Let $x = -x$ and $y = -y$

$9(-x)^2 + 4(-y)^2 = 36$

$9x^2 + 4y^2 = 36$ same

Therefore, the graph will have x-axis, y-axis, and origin symmetry.

64. $4x^2 + y^2 = 4$

x-intercepts: $\quad$ y-intercepts:

$4x^2 + 0^2 = 4$ $\quad$ $4(0)^2 + y^2 = 4$

$4x^2 = 4$ $\quad$ $y^2 = 4$

$x^2 = 1$ $\quad$ $y = \pm 2$

$x = \pm 1$

The intercepts are $(-1,0)$, $(1,0)$, $(0,-2)$, and $(0,2)$.

Test x-axis symmetry: Let $y = -y$

$4x^2 + (-y)^2 = 4$

$4x^2 + y^2 = 4$ same

Test y-axis symmetry: Let $x = -x$

$4(-x)^2 + y^2 = 4$

$4x^2 + y^2 = 4$ same

Test origin symmetry: Let $x = -x$ and $y = -y$

$4(-x)^2 + (-y)^2 = 4$

$4x^2 + y^2 = 4$ same

Therefore, the graph will have x-axis, y-axis, and origin symmetry.

65. $y = x^3 - 27$

x-intercepts: $\quad$ y-intercepts:

$0 = x^3 - 27$ $\quad$ $y = 0^3 - 27$

$x^3 = 27$ $\quad$ $y = -27$

$x = 3$

The intercepts are $(3,0)$ and $(0,-27)$.

Test x-axis symmetry: Let $y = -y$

$-y = x^3 - 27$ different

Test y-axis symmetry: Let $x = -x$

$y = (-x)^3 - 27$

$y = -x^3 - 27$ different

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = (-x)^3 - 27$

$y = x^3 + 27$ different

Therefore, the graph has none of the indicated symmetries.

66. $y = x^4 - 1$

x-intercepts: $\quad$ y-intercepts:

$0 = x^4 - 1$ $\quad$ $y = 0^4 - 1$

$x^4 = 1$ $\quad$ $y = -1$

$x = \pm 1$

The intercepts are $(-1,0)$, $(1,0)$, and $(0,-1)$.

Test x-axis symmetry: Let $y = -y$

$-y = x^4 - 1$ different

Test y-axis symmetry: Let $x = -x$

$y = (-x)^4 - 1$

$y = x^4 - 1$ same

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = (-x)^4 - 1$

$-y = x^4 - 1$ different

Therefore, the graph will have y-axis symmetry.

67. $y = x^2 - 3x - 4$

x-intercepts:

$0 = x^2 - 3x - 4$

$0 = (x-4)(x+1)$

$x = 4$ or $x = -1$

y-intercepts:

$y = 0^2 - 3(0) - 4$

$y = -4$

The intercepts are $(4,0)$, $(-1,0)$, and $(0,-4)$.

Test x-axis symmetry: Let $y = -y$

$-y = x^2 - 3x - 4$ different

Test y-axis symmetry: Let $x = -x$

$y = (-x)^2 - 3(-x) - 4$

$y = x^2 + 3x - 4$ different

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = (-x)^2 - 3(-x) - 4$

$-y = x^2 + 3x - 4$ different

Therefore, the graph has none of the indicated symmetries.

68. $y = x^2 + 4$

x-intercepts:

$0 = x^2 + 4$

$x^2 = -4$

no real solution

y-intercepts:

$y = 0^2 + 4$

$y = 4$

The only intercept is $(0,4)$.

Test x-axis symmetry: Let $y = -y$

$-y = x^2 + 4$ different

Test y-axis symmetry: Let $x = -x$

$y = (-x)^2 + 4$

$y = x^2 + 4$ same

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = (-x)^2 + 4$

$-y = x^2 + 4$ different

Therefore, the graph will have y-axis symmetry.

69. $y = \dfrac{3x}{x^2+9}$

x-intercepts:

$0 = \dfrac{3x}{x^2+9}$

$3x = 0$

$x = 0$

y-intercepts:

$y = \dfrac{3(0)}{0^2+9} = \dfrac{0}{9} = 0$

The only intercept is $(0,0)$.

Test x-axis symmetry: Let $y = -y$

$-y = \dfrac{3x}{x^2+9}$ different

Test y-axis symmetry: Let $x = -x$

$y = \dfrac{3(-x)}{(-x)^2+9}$

$y = -\dfrac{3x}{x^2+9}$ different

Test origin symmetry: Let $x = -x$ and $y = -y$

$-y = \dfrac{3(-x)}{(-x)^2+9}$

$-y = -\dfrac{3x}{x^2+9}$

$y = \dfrac{3x}{x^2+9}$ same

Therefore, the graph has origin symmetry.

70. $y = \dfrac{x^2-4}{2x}$

x-intercepts:

$0 = \dfrac{x^2-4}{2x}$

$x^2 - 4 = 0$

$x^2 = 4$

$x = \pm 2$

y-intercepts:

$y = \dfrac{0^2-4}{2(0)} = \dfrac{-4}{0}$

undefined

The intercepts are $(-2,0)$ and $(2,0)$.

Test x-axis symmetry: Let $y = -y$

$-y = \dfrac{x^2-4}{2x}$ different

Test y-axis symmetry: Let $x = -x$

$$y = \frac{(-x)^2 - 4}{2(-x)}$$

$$y = -\frac{x^2 - 4}{2x} \quad \text{different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$-y = \frac{(-x)^2 - 4}{2(-x)}$$

$$-y = \frac{x^2 - 4}{-2x}$$

$$y = \frac{x^2 - 4}{2x} \quad \text{same}$$

Therefore, the graph has origin symmetry.

71. $y = \dfrac{-x^3}{x^2 - 9}$

x-intercepts:

$$0 = \frac{-x^3}{x^2 - 9}$$

$$-x^3 = 0$$

$$x = 0$$

y-intercepts:

$$y = \frac{-0^3}{0^2 - 9} = \frac{0}{-9} = 0$$

The only intercept is $(0, 0)$.

Test x-axis symmetry: Let $y = -y$

$$-y = \frac{-x^3}{x^2 - 9}$$

$$y = \frac{x^3}{x^2 - 9} \quad \text{different}$$

Test y-axis symmetry: Let $x = -x$

$$y = \frac{-(-x)^3}{(-x)^2 - 9}$$

$$y = \frac{x^3}{x^2 - 9} \quad \text{different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

$$-y = \frac{-(-x)^3}{(-x)^2 - 9}$$

$$-y = \frac{x^3}{x^2 - 9}$$

$$y = \frac{-x^3}{x^2 - 9} \quad \text{same}$$

Therefore, the graph has origin symmetry.

72. $y = \dfrac{x^4 + 1}{2x^5}$

x-intercepts:

$$0 = \frac{x^4 + 1}{2x^5}$$

$$x^4 = -1$$

no real solution

y-intercepts:

$$y = \frac{0^4 + 1}{2(0)^5} = \frac{1}{0}$$

undefined

There are no intercepts for the graph of this equation.

Test x-axis symmetry: Let $y = -y$

$$-y = \frac{x^4 + 1}{2x^5} \quad \text{different}$$

Test y-axis symmetry: Let $x = -x$

$$y = \frac{(-x)^4 + 1}{2(-x)^5}$$

$$y = \frac{x^4 + 1}{-2x^5} \quad \text{different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$

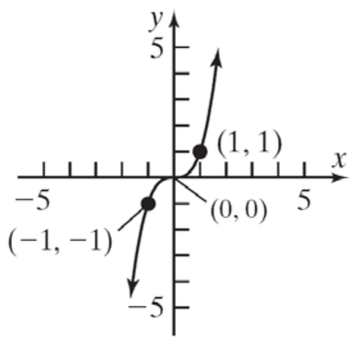
$$-y = \frac{(-x)^4 + 1}{2(-x)^5}$$

$$-y = \frac{x^4 + 1}{-2x^5}$$

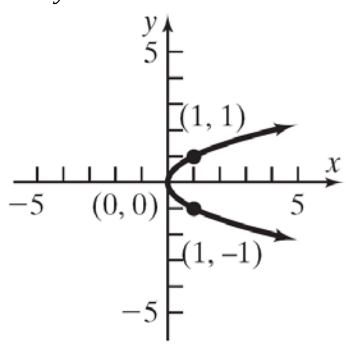
$$y = \frac{x^4 + 1}{2x^5} \quad \text{same}$$

Therefore, the graph has origin symmetry.

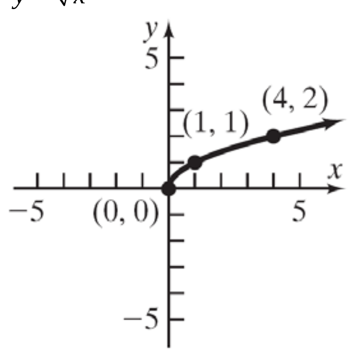
73. $y = x^3$



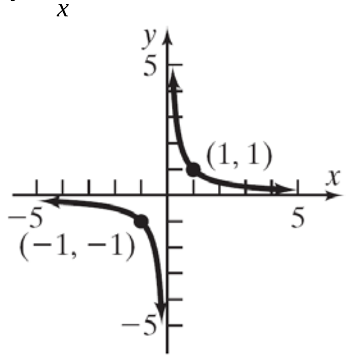
74. $x = y^2$



75. $y = \sqrt{x}$



76. $y = \frac{1}{x}$



77. If the point $(a,4)$ is on the graph of $y = x^2 + 3x$, then we have

$$4 = a^2 + 3a$$
$$0 = a^2 + 3a - 4$$
$$0 = (a+4)(a-1)$$
$$a + 4 = 0 \quad \text{or} \quad a - 1 = 0$$
$$a = -4 \qquad a = 1$$

Thus, $a = -4$ or $a = 1$.

78. If the point $(a,-5)$ is on the graph of $y = x^2 + 6x$, then we have

$$-5 = a^2 + 6a$$
$$0 = a^2 + 6a + 5$$
$$0 = (a+5)(a+1)$$
$$a + 5 = 0 \quad \text{or} \quad a + 1 = 0$$
$$a = -5 \qquad a = -1$$

Thus, $a = -5$ or $a = -1$.

79. For a graph with origin symmetry, if the point (a,b) is on the graph, then so is the point $(-a,-b)$. Since the point $(1,2)$ is on the graph of an equation with origin symmetry, the point $(-1,-2)$ must also be on the graph.

80. For a graph with *y*-axis symmetry, if the point (a,b) is on the graph, then so is the point $(-a,b)$. Since 6 is an *x*-intercept in this case, the point $(6,0)$ is on the graph of the equation. Due to the *y*-axis symmetry, the point $(-6,0)$ must also be on the graph. Therefore, -6 is another *x*-intercept.

81. For a graph with origin symmetry, if the point (a,b) is on the graph, then so is the point $(-a,-b)$. Since -4 is an *x*-intercept in this case, the point $(-4,0)$ is on the graph of the equation. Due to the origin symmetry, the point $(4,0)$ must also be on the graph. Therefore, 4 is another *x*-intercept.

82. For a graph with *x*-axis symmetry, if the point (a,b) is on the graph, then so is the point

$(a,-b)$. Since 2 is a y-intercept in this case, the point $(0,2)$ is on the graph of the equation. Due to the x-axis symmetry, the point $(0,-2)$ must also be on the graph. Therefore, -2 is another y-intercept.

83. a. $\left(x^2+y^2-x\right)^2 = x^2+y^2$

x-intercepts:

$$\left(x^2+(0)^2-x\right)^2 = x^2+(0)^2$$
$$\left(x^2-x\right)^2 = x^2$$
$$x^4-2x^3+x^2 = x^2$$
$$x^4-2x^3 = 0$$
$$x^3(x-2) = 0$$
$$x^3 = 0 \quad \text{or} \quad x-2=0$$
$$x = 0 \qquad\qquad x = 2$$

y-intercepts:

$$\left((0)^2+y^2-0\right)^2 = (0)^2+y^2$$
$$\left(y^2\right)^2 = y^2$$
$$y^4 = y^2$$
$$y^4-y^2 = 0$$
$$y^2\left(y^2-1\right) = 0$$
$$y^2 = 0 \quad \text{or} \quad y^2-1=0$$
$$y = 0 \qquad\qquad y^2 = 1$$
$$y = \pm 1$$

The intercepts are $(0,0)$, $(2,0)$, $(0,-1)$, and $(0,1)$.

b. <u>Test x-axis symmetry:</u> Let $y=-y$

$$\left(x^2+(-y)^2-x\right)^2 = x^2+(-y)^2$$
$$\left(x^2+y^2-x\right)^2 = x^2+y^2 \quad \text{same}$$

<u>Test y-axis symmetry:</u> Let $x=-x$

$$\left((-x)^2+y^2-(-x)\right)^2 = (-x)^2+y^2$$
$$\left(x^2+y^2+x\right)^2 = x^2+y^2 \quad \text{different}$$

<u>Test origin symmetry</u>: Let $x=-x$ and $y=-y$

$$\left((-x)^2+(-y)^2-(-x)\right)^2 = (-x)^2+(-y)^2$$
$$\left(x^2+y^2+x\right)^2 = x^2+y^2 \quad \text{different}$$

Thus, the graph will have x-axis symmetry.

84. a. $16y^2 = 120x-225$

x-intercepts:

$$16y^2 = 120(0)-225$$
$$16y^2 = -225$$
$$y^2 = -\frac{225}{16}$$
no real solution

y-intercepts:

$$16(0)^2 = 120x-225$$
$$0 = 120x-225$$
$$-120x = -225$$
$$x = \frac{-225}{-120} = \frac{15}{8}$$

The only intercept is $\left(\frac{15}{8},0\right)$.

b. <u>Test x-axis symmetry:</u> Let $y=-y$

$$16(-y)^2 = 120x-225$$
$$16y^2 = 120x-225 \quad \text{same}$$

<u>Test y-axis symmetry:</u> Let $x=-x$

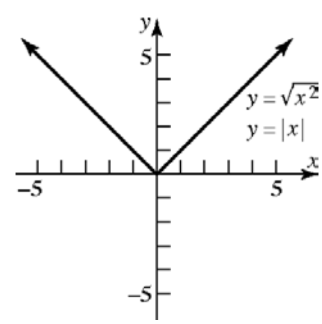
$$16y^2 = 120(-x)-225$$
$$16y^2 = -120x-225 \quad \text{different}$$

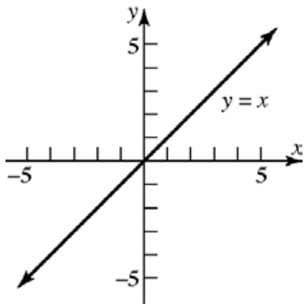
<u>Test origin symmetry</u>: Let $x=-x$ and $y=-y$

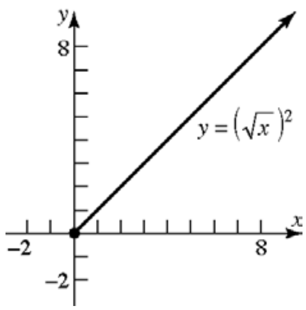
$$16(-y)^2 = 120(-x)-225$$
$$16y^2 = -120x-225 \quad \text{different}$$

Thus, the graph will have x-axis symmetry.

85. a.







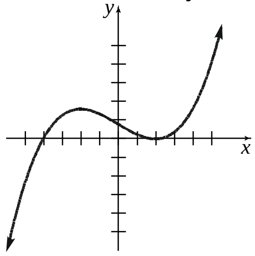
b. Since $\sqrt{x^2}=|x|$ for all x, the graphs of $y=\sqrt{x^2}$ and $y=|x|$ are the same.

c. For $y=\left(\sqrt{x}\right)^2$, the domain of the variable x is $x \ge 0$; for $y=x$, the domain of the variable x is all real numbers. Thus, $\left(\sqrt{x}\right)^2=x$ only for $x \ge 0$.

d. For $y=\sqrt{x^2}$, the range of the variable y is $y \ge 0$; for $y=x$, the range of the variable y is all real numbers. Also, $\sqrt{x^2}=x$ only if $x \ge 0$. Otherwise, $\sqrt{x^2}=-x$.

86. Answers will vary. A complete graph presents enough of the graph to the viewer so they can "see" the rest of the graph as an obvious continuation of what is shown.

87. Answers will vary. One example:



88. Answers will vary

89. Answers will vary

90. Answers will vary.
Case 1: Graph has *x*-axis and *y*-axis symmetry, show origin symmetry.
(x,y) on graph $\rightarrow (x,-y)$ on graph
(from *x*-axis symmetry)
$(x,-y)$ on graph $\rightarrow (-x,-y)$ on graph
(from *y*-axis symmetry)
Since the point $(-x,-y)$ is also on the graph, the graph has origin symmetry.

Case 2: Graph has *x*-axis and origin symmetry, show *y*-axis symmetry.
(x,y) on graph $\rightarrow (x,-y)$ on graph
(from *x*-axis symmetry)
$(x,-y)$ on graph $\rightarrow (-x,y)$ on graph
(from origin symmetry)
Since the point $(-x,y)$ is also on the graph, the graph has *y*-axis symmetry.

Case 3: Graph has *y*-axis and origin symmetry, show *x*-axis symmetry.
(x,y) on graph $\rightarrow (-x,y)$ on graph
(from *y*-axis symmetry)
$(-x,y)$ on graph $\rightarrow (x,-y)$ on graph
(from origin symmetry)
Since the point $(x,-y)$ is also on the graph, the graph has *x*-axis symmetry.

91. Answers may vary. The graph must contain the points $(-2,5)$, $(-1,3)$, and $(0,2)$. For the graph to be symmetric about the *y*-axis, the graph must also contain the points $(2,5)$ and $(1,3)$ (note that (0, 2) is on the *y*-axis).

For the graph to also be symmetric with respect to the *x*-axis, the graph must also contain the points $(-2,-5)$, $(-1,-3)$, $(0,-2)$, $(2,-5)$, and $(1,-3)$. Recall that a graph with two of the symmetries (x-axis, y-axis, origin) will necessarily have the third. Therefore, if the original graph with y-axis symmetry also has x-axis symmetry, then it will also have origin symmetry.

Section 1.3

1. undefined; 0

2. 3; 2

x-intercept: $2x+3(0)=6$

$$2x=6$$
$$x=3$$

y-intercept: $2(0)+3y=6$

$$3y=6$$
$$y=2$$

3. True

4. False; the slope is $\frac{3}{2}$.

$$2y=3x+5$$
$$y=\frac{3}{2}x+\frac{5}{2}$$

5. True; $2(1)+(2)\stackrel{?}{=}4$

$$2+2\stackrel{?}{=}4$$
$$4=4 \text{ True}$$

6. $m_1=m_2$; y-intercepts; $m_1 \cdot m_2=-1$

7. 2

8. $-\frac{1}{2}$

9. False; perpendicular lines have slopes that are opposite-reciprocals of each other.

10. d

11. c

12. b

13. a. Slope $=\frac{1-0}{2-0}=\frac{1}{2}$

b. If x increases by 2 units, y will increase by 1 unit.

14. a. Slope $=\frac{1-0}{-2-0}=-\frac{1}{2}$

b. If x increases by 2 units, y will decrease by 1 unit.

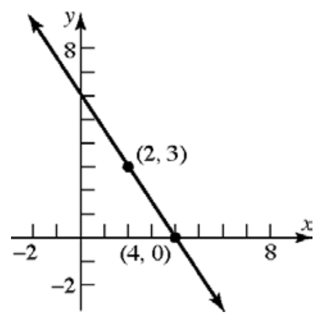
15. a. Slope $=\frac{1-2}{1-(-2)}=-\frac{1}{3}$

b. If x increases by 3 units, y will decrease by 1 unit.

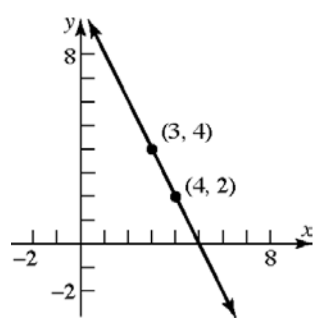
16. a. Slope $=\frac{2-1}{2-(-1)}=\frac{1}{3}$

b. If x increases by 3 units, y will increase by 1 unit.

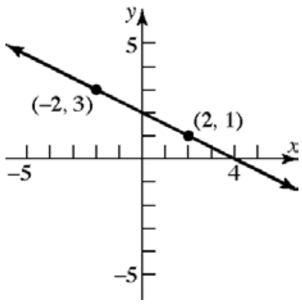
17. Slope $=\frac{y_2-y_1}{x_2-x_1}=\frac{0-3}{4-2}=-\frac{3}{2}$



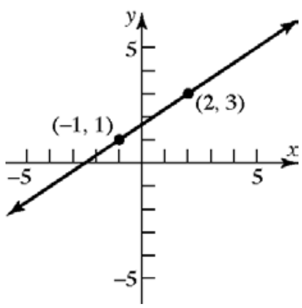
18. Slope $=\frac{y_2-y_1}{x_2-x_1}=\frac{4-2}{3-4}=\frac{2}{-1}=-2$



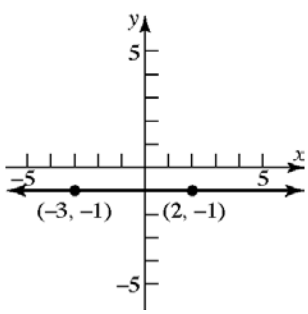
19. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1-3}{2-(-2)} = \dfrac{-2}{4} = -\dfrac{1}{2}$



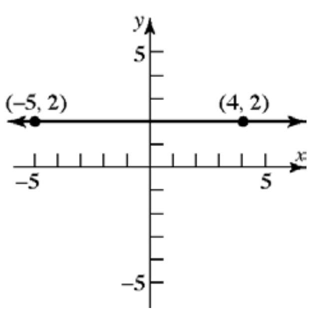
20. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3-1}{2-(-1)} = \dfrac{2}{3}$



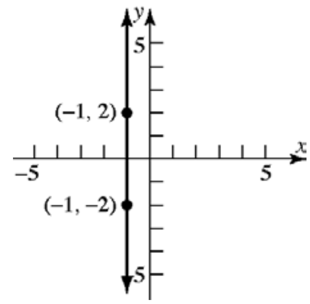
21. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-1-(-1)}{2-(-3)} = \dfrac{0}{5} = 0$



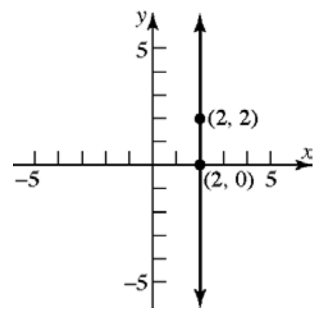
22. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2-2}{-5-4} = \dfrac{0}{-9} = 0$



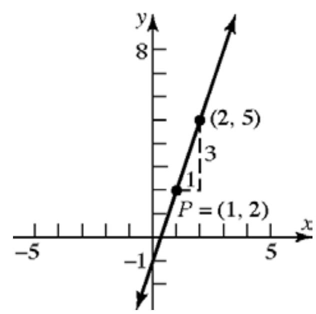
23. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-2-2}{-1-(-1)} = \dfrac{-4}{0}$ undefined.



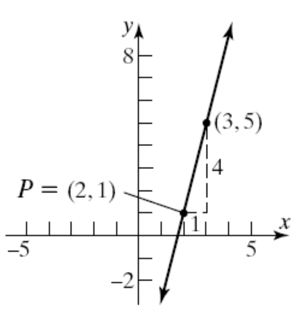
24. Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2-0}{2-2} = \dfrac{2}{0}$ undefined.



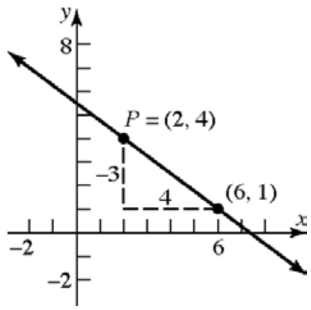
25. $P = (1,2); m = 3$ **;** $y - 2 = 3(x - 1)$



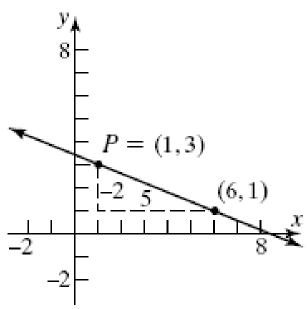
26. $P = (2,1); m = 4$ **;** $y - 1 = 4(x - 2)$



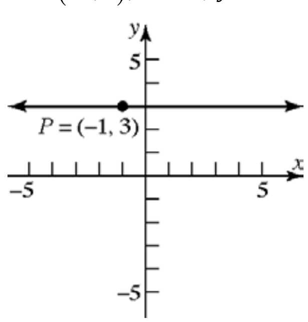
27. $P=(2,4); m=-\frac{3}{4}$**;** $y-4=-\frac{3}{4}(x-2)$



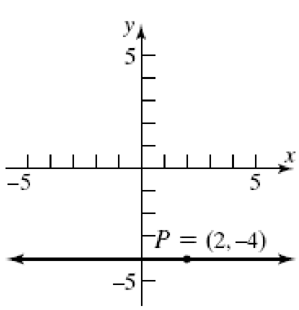
28. $P=(1,3); m=-\frac{2}{5}$**;** $y-3=-\frac{2}{5}(x-1)$



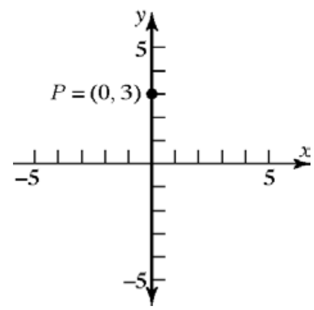
29. $P=(-1,3); m=0$**;** $y-3=0$



30. $P=(2,-4); m=0$**;** $y=-4$

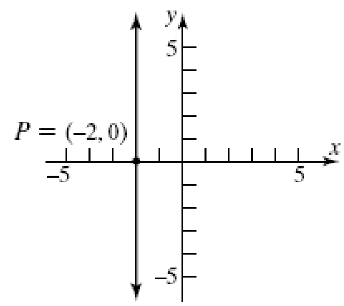


31. $P=(0,3)$; slope undefined **;** $x=0$



(note: the line is the *y*-axis)

32. $P=(-2,0)$; slope undefined $x=-2$



33. Slope $=4=\frac{4}{1}$; point: $(1,2)$

If *x* increases by 1 unit, then *y* increases by 4 units.

Answers will vary. Three possible points are:

$x=1+1=2$ and $y=2+4=6$

$(2,6)$

$x=2+1=3$ and $y=6+4=10$

$(3,10)$

$x=3+1=4$ and $y=10+4=14$

$(4,14)$

34. Slope $=2=\frac{2}{1}$; point: $(-2,3)$

If *x* increases by 1 unit, then *y* increases by 2 units.

Answers will vary. Three possible points are:

$x=-2+1=-1$ and $y=3+2=5$

$(-1,5)$

$x=-1+1=0$ and $y=5+2=7$

$(0,7)$

$x=0+1=1$ and $y=7+2=9$

$(1,9)$

35. Slope $= -\frac{3}{2} = \frac{-3}{2}$; point: $(2,-4)$

If x increases by 2 units, then y decreases by 3 units.

Answers will vary. Three possible points are:

$x = 2+2 = 4$ and $y = -4-3 = -7$

$(4,-7)$

$x = 4+2 = 6$ and $y = -7-3 = -10$

$(6,-10)$

$x = 6+2 = 8$ and $y = -10-3 = -13$

$(8,-13)$

36. Slope $= \frac{4}{3}$; point: $(-3,2)$

If x increases by 3 units, then y increases by 4 units.

Answers will vary. Three possible points are:

$x = -3+3 = 0$ and $y = 2+4 = 6$

$(0,6)$

$x = 0+3 = 3$ and $y = 6+4 = 10$

$(3,10)$

$x = 3+3 = 6$ and $y = 10+4 = 14$

$(6,14)$

37. Slope $= -2 = \frac{-2}{1}$; point: $(-2,-3)$

If x increases by 1 unit, then y decreases by 2 units.

Answers will vary. Three possible points are:

$x = -2+1 = -1$ and $y = -3-2 = -5$

$(-1,-5)$

$x = -1+1 = 0$ and $y = -5-2 = -7$

$(0,-7)$

$x = 0+1 = 1$ and $y = -7-2 = -9$

$(1,-9)$

38. Slope $= -1 = \frac{-1}{1}$; point: $(4,1)$

If x increases by 1 unit, then y decreases by 1 unit.

Answers will vary. Three possible points are:

$x = 4+1 = 5$ and $y = 1-1 = 0$

$(5,0)$

$x = 5+1 = 6$ and $y = 0-1 = -1$

$(6,-1)$

$x = 6+1 = 7$ and $y = -1-1 = -2$

$(7,-2)$

39. (0, 0) and (2, 1) are points on the line.

Slope $= \frac{1-0}{2-0} = \frac{1}{2}$

y-intercept is 0; using $y = mx+b$:

$$y = \frac{1}{2}x+0$$
$$2y = x$$
$$0 = x-2y$$
$$x-2y = 0 \text{ or } y = \frac{1}{2}x$$

40. (0, 0) and (–2, 1) are points on the line.

Slope $= \frac{1-0}{-2-0} = \frac{1}{-2} = -\frac{1}{2}$

y-intercept is 0; using $y = mx+b$:

$$y = -\frac{1}{2}x+0$$
$$2y = -x$$
$$x+2y = 0$$
$$x+2y = 0 \text{ or } y = -\frac{1}{2}x$$

41. (–1, 3) and (1, 1) are points on the line.

Slope $= \frac{1-3}{1-(-1)} = \frac{-2}{2} = -1$

Using $y - y_1 = m(x - x_1)$

$$y-1 = -1(x-1)$$
$$y-1 = -x+1$$
$$y = -x+2$$
$$x+y = 2 \text{ or } y = -x+2$$

42. (–1, 1) and (2, 2) are points on the line.

Slope $= \dfrac{2-1}{2-(-1)} = \dfrac{1}{3}$

Using $y - y_1 = m(x - x_1)$

$$y-1=\frac{1}{3}\left(x-(-1)\right)$$
$$y-1=\frac{1}{3}(x+1)$$
$$y-1=\frac{1}{3}x+\frac{1}{3}$$
$$y=\frac{1}{3}x+\frac{4}{3}$$
$$x-3y=-4 \text{ or } y=\frac{1}{3}x+\frac{4}{3}$$

43. $y-y_1=m(x-x_1),\ m=2$

$$y-3=2(x-3)$$
$$y-3=2x-6$$
$$y=2x-3$$
$$2x-y=3 \text{ or } y=2x-3$$

44. $y-y_1=m(x-x_1),\ m=-1$

$$y-2=-1(x-1)$$
$$y-2=-x+1$$
$$y=-x+3$$
$$x+y=3 \text{ or } y=-x+3$$

45. $y-y_1=m(x-x_1),\ m=-\dfrac{1}{2}$

$$y-2=-\frac{1}{2}(x-1)$$
$$y-2=-\frac{1}{2}x+\frac{1}{2}$$
$$y=-\frac{1}{2}x+\frac{5}{2}$$
$$x+2y=5 \text{ or } y=-\frac{1}{2}x+\frac{5}{2}$$

46. $y-y_1=m(x-x_1),\ m=1$

$$y-1=1(x-(-1))$$
$$y-1=x+1$$
$$y=x+2$$
$$x-y=-2 \text{ or } y=x+2$$

47. Slope = 3; containing (–2, 3)

$$y-y_1=m(x-x_1)$$
$$y-3=3(x-(-2))$$
$$y-3=3x+6$$
$$y=3x+9$$
$$3x-y=-9 \text{ or } y=3x+9$$

48. Slope = 2; containing the point (4, –3)

$$y-y_1=m(x-x_1)$$
$$y-(-3)=2(x-4)$$
$$y+3=2x-8$$
$$y=2x-11$$
$$2x-y=11 \text{ or } y=2x-11$$

49. Slope $=-\dfrac{2}{3}$; containing (1, –1)

$$y-y_1=m(x-x_1)$$
$$y-(-1)=-\frac{2}{3}(x-1)$$
$$y+1=-\frac{2}{3}x+\frac{2}{3}$$
$$y=-\frac{2}{3}x-\frac{1}{3}$$
$$2x+3y=-1 \text{ or } y=-\frac{2}{3}x-\frac{1}{3}$$

50. Slope $=\dfrac{1}{2}$; containing the point (3, 1)

$$y-y_1=m(x-x_1)$$
$$y-1=\frac{1}{2}(x-3)$$
$$y-1=\frac{1}{2}x-\frac{3}{2}$$
$$y=\frac{1}{2}x-\frac{1}{2}$$
$$x-2y=1 \text{ or } y=\frac{1}{2}x-\frac{1}{2}$$

51. Containing (1, 3) and (–1, 2)

$$m=\frac{2-3}{-1-1}=\frac{-1}{-2}=\frac{1}{2}$$

$$y-y_1=m(x-x_1)$$

$$y-3=\frac{1}{2}(x-1)$$

$$y-3=\frac{1}{2}x-\frac{1}{2}$$

$$y=\frac{1}{2}x+\frac{5}{2}$$

$$x-2y=-5 \text{ or } y=\frac{1}{2}x+\frac{5}{2}$$

52. Containing the points (–3, 4) and (2, 5)

$$m=\frac{5-4}{2-(-3)}=\frac{1}{5}$$

$$y-y_1=m(x-x_1)$$

$$y-5=\frac{1}{5}(x-2)$$

$$y-5=\frac{1}{5}x-\frac{2}{5}$$

$$y=\frac{1}{5}x+\frac{23}{5}$$

$$x-5y=-23 \text{ or } y=\frac{1}{5}x+\frac{23}{5}$$

53. Slope = –3; y-intercept =3

$$y=mx+b$$

$$y=-3x+3$$

$$3x+y=3 \text{ or } y=-3x+3$$

54. Slope = –2; y-intercept = –2

$$y=mx+b$$

$$y=-2x+(-2)$$

$$2x+y=-2 \text{ or } y=-2x-2$$

55. x-intercept = 2; y-intercept = –1
Points are (2,0) and (0,–1)

$$m=\frac{-1-0}{0-2}=\frac{-1}{-2}=\frac{1}{2}$$

$$y=mx+b$$

$$y=\frac{1}{2}x-1$$

$$x-2y=2 \text{ or } y=\frac{1}{2}x-1$$

56. x-intercept = –4; y-intercept = 4
Points are (–4, 0) and (0, 4)

$$m=\frac{4-0}{0-(-4)}=\frac{4}{4}=1$$

$$y=mx+b$$

$$y=1x+4$$

$$y=x+4$$

$$x-y=-4 \text{ or } y=x+4$$

57. Slope undefined; containing the point (2, 4)
This is a vertical line.
$x=2$ No slope-intercept form.

58. Slope undefined; containing the point (3, 8)
This is a vertical line.
$x=3$ No slope-intercept form.

59. Horizontal lines have slope $m=0$ and take the form $y=b$. Therefore, the horizontal line passing through the point $(-3,2)$ is $y=2$.

60. Vertical lines have an undefined slope and take the form $x=a$. Therefore, the vertical line passing through the point $(4,-5)$ is $x=4$.

61. Parallel to $y=2x$; Slope = 2
Containing (–1, 2)

$$y-y_1=m(x-x_1)$$

$$y-2=2(x-(-1))$$

$$y-2=2x+2 \rightarrow y=2x+4$$

$$2x-y=-4 \text{ or } y=2x+4$$

62. Parallel to $y=-3x$; Slope = –3; Containing the point (–1, 2)

$$y-y_1=m(x-x_1)$$

$$y-2=-3(x-(-1))$$

$$y-2=-3x-3 \rightarrow y=-3x-1$$

$$3x+y=-1 \text{ or } y=-3x-1$$

63. Parallel to $2x - y = -2$; Slope = 2
Containing the point (0, 0)

$$y - y_1 = m(x - x_1)$$
$$y - 0 = 2(x - 0)$$
$$y = 2x$$
$$2x - y = 0 \text{ or } y = 2x$$

64. Parallel to $x - 2y = -5$;

Slope $= \frac{1}{2}$; Containing the point $(0,0)$

$$y - y_1 = m(x - x_1)$$
$$y - 0 = \frac{1}{2}(x - 0) \to y = \frac{1}{2}x$$
$$x - 2y = 0 \text{ or } y = \frac{1}{2}x$$

65. Parallel to $x = 5$; Containing (4,2)
This is a vertical line.
$x = 4$ No slope-intercept form.

66. Parallel to $y = 5$; Containing the point (4, 2)
This is a horizontal line. Slope = 0
$y = 2$

67. Perpendicular to $y = \frac{1}{2}x + 4$; Containing (1, –2)
Slope of perpendicular = –2

$$y - y_1 = m(x - x_1)$$
$$y - (-2) = -2(x - 1)$$
$$y + 2 = -2x + 2 \to y = -2x$$
$$2x + y = 0 \text{ or } y = -2x$$

68. Perpendicular to $y = 2x - 3$; Containing the point (1, –2)

Slope of perpendicular $= -\frac{1}{2}$

$$y - y_1 = m(x - x_1)$$
$$y - (-2) = -\frac{1}{2}(x - 1)$$
$$y + 2 = -\frac{1}{2}x + \frac{1}{2} \to y = -\frac{1}{2}x - \frac{3}{2}$$
$$x + 2y = -3 \text{ or } y = -\frac{1}{2}x - \frac{3}{2}$$

69. Perpendicular to $2x + y = 2$; Containing the point (–3, 0)

Slope of perpendicular $= \frac{1}{2}$

$$y - y_1 = m(x - x_1)$$
$$y - 0 = \frac{1}{2}(x - (-3)) \to y = \frac{1}{2}x + \frac{3}{2}$$
$$x - 2y = -3 \text{ or } y = \frac{1}{2}x + \frac{3}{2}$$

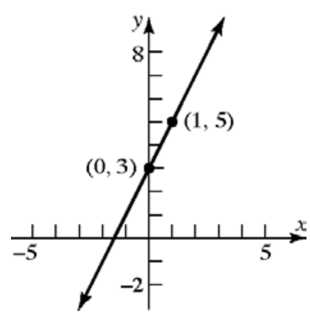
70. Perpendicular to $x - 2y = -5$; Containing the point (0, 4)
Slope of perpendicular = –2

$$y = mx + b$$
$$y = -2x + 4$$
$$2x + y = 4 \text{ or } y = -2x + 4$$

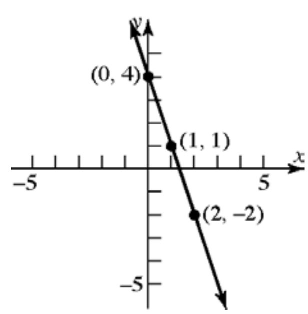
71. Perpendicular to $x = 8$; Containing (3, 4)
Slope of perpendicular = 0 (horizontal line)
$y = 4$

72. Perpendicular to $y = 8$;
Containing the point (3, 4)
Slope of perpendicular is undefined (vertical line). $x = 3$ No slope-intercept form.

73. $y = 2x + 3$; Slope = 2; y-intercept = 3

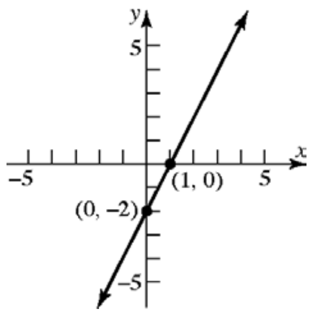


74. $y = -3x + 4$; Slope = –3; y-intercept = 4



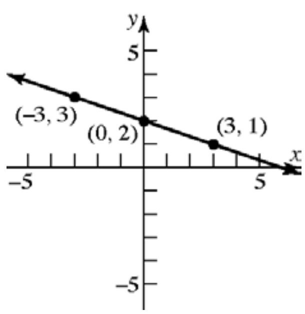
75. $\frac{1}{2}y = x - 1$; $y = 2x - 2$

Slope = 2; y-intercept = –2

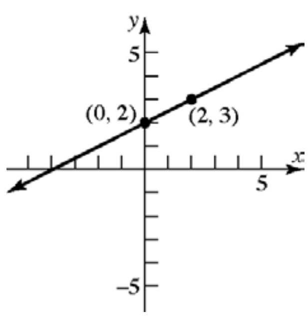


76. $\frac{1}{3}x + y = 2$; $y = -\frac{1}{3}x + 2$

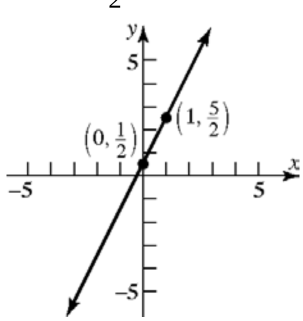
Slope = $-\frac{1}{3}$; y-intercept = 2



77. $y = \frac{1}{2}x + 2$; Slope = $\frac{1}{2}$; y-intercept = 2

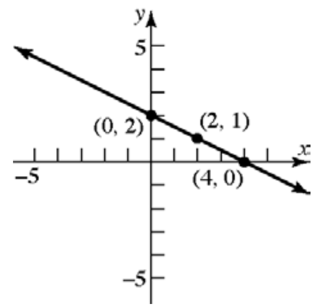


78. $y = 2x + \frac{1}{2}$; Slope = 2; y-intercept = $\frac{1}{2}$



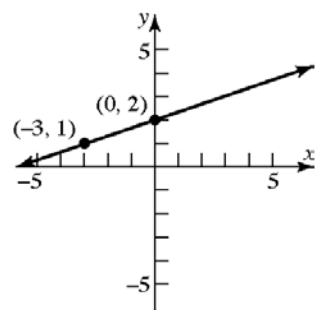
79. $x + 2y = 4$; $2y = -x + 4 \rightarrow y = -\frac{1}{2}x + 2$

Slope = $-\frac{1}{2}$; y-intercept = 2



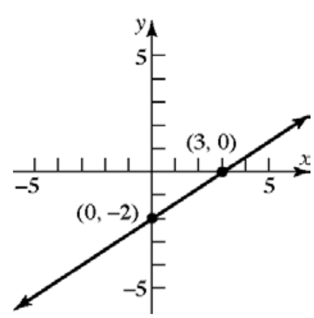
80. $-x + 3y = 6$; $3y = x + 6 \rightarrow y = \frac{1}{3}x + 2$

Slope = $\frac{1}{3}$; y-intercept = 2



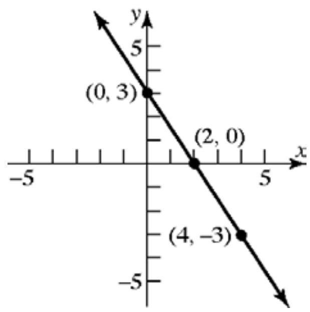
81. $2x - 3y = 6$; $-3y = -2x + 6 \rightarrow y = \frac{2}{3}x - 2$

Slope = $\frac{2}{3}$; y-intercept = –2

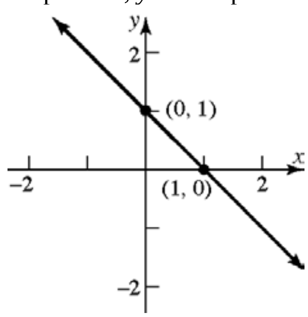


82. $3x+2y=6$; $2y=-3x+6 \to y=-\dfrac{3}{2}x+3$

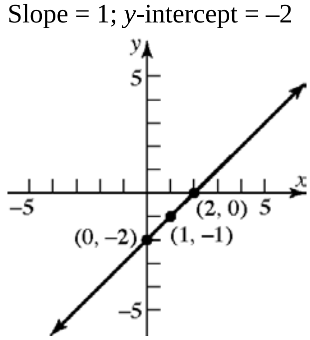
Slope $=-\dfrac{3}{2}$; y-intercept = 3



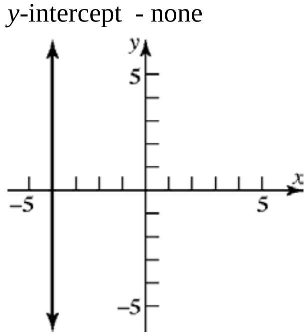
83. $x+y=1$; $y=-x+1$
Slope = –1; y-intercept = 1



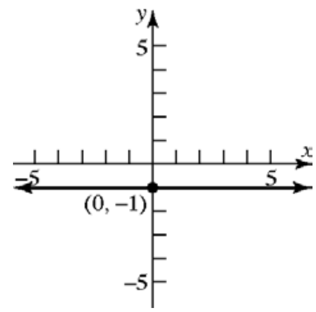
84. $x-y=2$; $y=x-2$
Slope = 1; y-intercept = –2



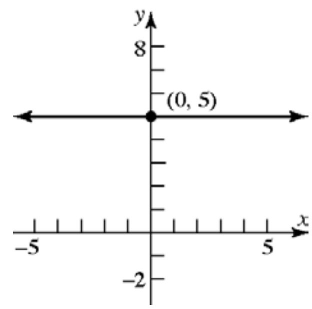
85. $x=-4$; Slope is undefined
y-intercept - none



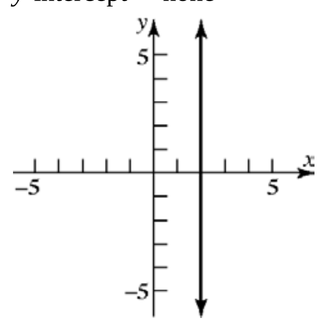
86. $y=-1$; Slope = 0; y-intercept = –1



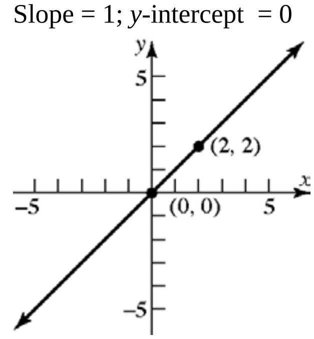
87. $y=5$; Slope = 0; y-intercept = 5



88. $x=2$; Slope is undefined
y-intercept - none

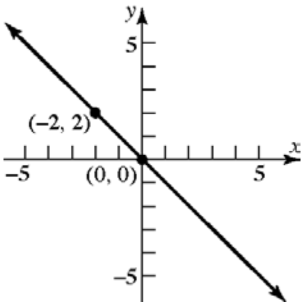


89. $y-x=0$; $y=x$
Slope = 1; y-intercept = 0



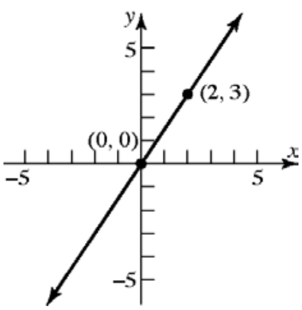
90. $x+y=0$; $y=-x$

Slope = –1; y-intercept = 0



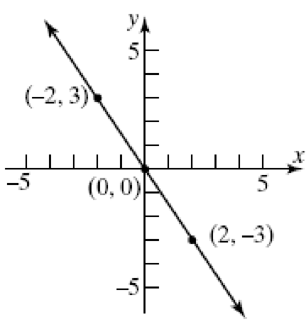
91. $2y-3x=0$; $2y=3x \to y=\frac{3}{2}x$

Slope $=\frac{3}{2}$; y-intercept = 0



92. $3x+2y=0$; $2y=-3x \to y=-\frac{3}{2}x$

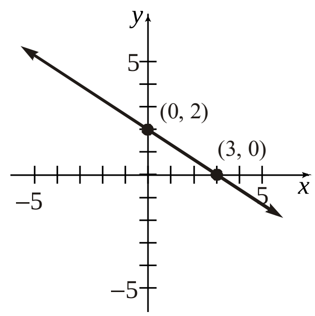
Slope $=-\frac{3}{2}$; y-intercept = 0



93. a. x-intercept:
$$\begin{aligned} 2x+3(0)&=6\\ 2x&=6\\ x&=3 \end{aligned}$$
The point $(3,0)$ is on the graph.

y-intercept:
$$\begin{aligned} 2(0)+3y&=6\\ 3y&=6\\ y&=2 \end{aligned}$$
The point $(0,2)$ is on the graph.

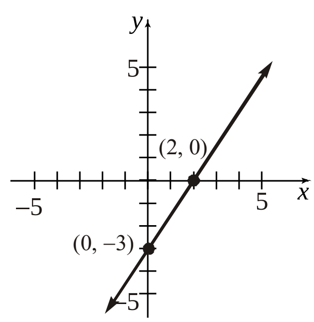
b.



94. a. x-intercept:
$$\begin{aligned} 3x-2(0)&=6\\ 3x&=6\\ x&=2 \end{aligned}$$
The point $(2,0)$ is on the graph.

y-intercept:
$$\begin{aligned} 3(0)-2y&=6\\ -2y&=6\\ y&=-3 \end{aligned}$$
The point $(0,-3)$ is on the graph.

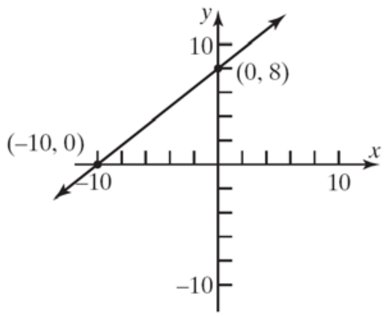
b.



95. **a.** x-intercept: $-4x+5(0)=40$
$$-4x=40$$
$$x=-10$$
The point $(-10,0)$ is on the graph.

y-intercept: $-4(0)+5y=40$
$$5y=40$$
$$y=8$$
The point $(0,8)$ is on the graph.

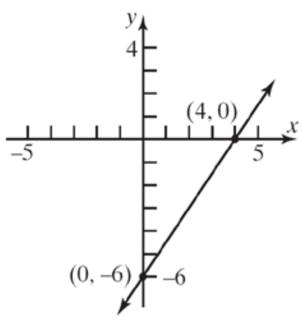
b.



96. **a.** x-intercept: $6x-4(0)=24$
$$6x=24$$
$$x=4$$
The point $(4,0)$ is on the graph.

y-intercept: $6(0)-4y=24$
$$-4y=24$$
$$y=-6$$
The point $(0,-6)$ is on the graph.

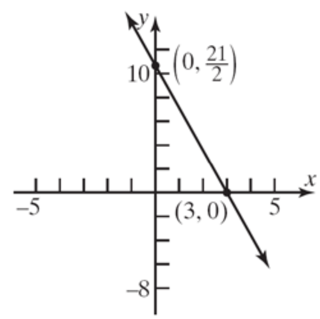
b.



97. **a.** x-intercept: $7x+2(0)=21$
$$7x=21$$
$$x=3$$
The point $(3,0)$ is on the graph.

y-intercept: $7(0)+2y=21$
$$2y=21$$
$$y=\frac{21}{2}$$
The point $\left(0,\frac{21}{2}\right)$ is on the graph.

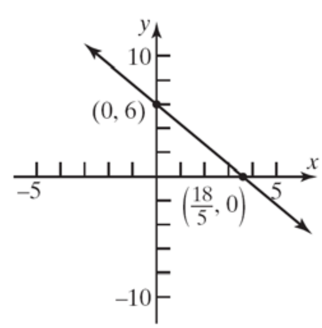
b.



98. **a.** x-intercept: $5x+3(0)=18$
$$5x=18$$
$$x=\frac{18}{5}$$
The point $\left(\frac{18}{5},0\right)$ is on the graph.

y-intercept: $5(0)+3y=18$
$$3y=18$$
$$y=6$$
The point $(0,6)$ is on the graph.

b.



99. a. x-intercept: $\frac{1}{2}x+\frac{1}{3}(0)=1$

$$\frac{1}{2}x=1$$
$$x=2$$

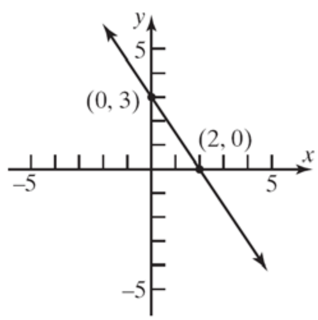
The point $(2,0)$ is on the graph.

y-intercept: $\frac{1}{2}(0)+\frac{1}{3}y=1$

$$\frac{1}{3}y=1$$
$$y=3$$

The point $(0,3)$ is on the graph.

b.



100. a. x-intercept: $x-\frac{2}{3}(0)=4$

$$x=4$$

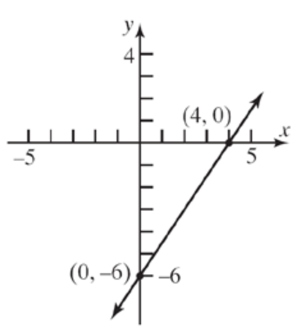
The point $(4,0)$ is on the graph.

y-intercept: $(0)-\frac{2}{3}y=4$

$$-\frac{2}{3}y=4$$
$$y=-6$$

The point $(0,-6)$ is on the graph.

b.



101. a. x-intercept: $0.2x-0.5(0)=1$

$$0.2x=1$$
$$x=5$$

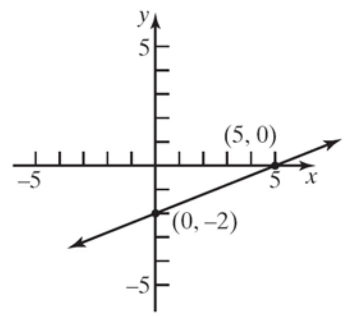
The point $(5,0)$ is on the graph.

y-intercept: $0.2(0)-0.5y=1$

$$-0.5y=1$$
$$y=-2$$

The point $(0,-2)$ is on the graph.

b.



102. a. x-intercept: $-0.3x+0.4(0)=1.2$

$$-0.3x=1.2$$
$$x=-4$$

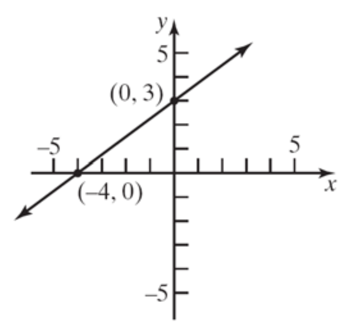
The point $(-4,0)$ is on the graph.

y-intercept: $-0.3(0)+0.4y=1.2$

$$0.4y=1.2$$
$$y=3$$

The point $(0,3)$ is on the graph.

b.



103. The equation of the x-axis is $y=0$. (The slope is 0 and the y-intercept is 0.)

104. The equation of the y-axis is $x=0$. (The slope is undefined.)

105. The slopes are the same but the y-intercepts are different. Therefore, the two lines are parallel.

106. The slopes are opposite-reciprocals. That is, their product is -1. Therefore, the lines are perpendicular.

107. The slopes are different and their product does not equal -1. Therefore, the lines are neither parallel nor perpendicular.

108. The slopes are different and their product does not equal -1 (in fact, the signs are the same so the product is positive). Therefore, the lines are neither parallel nor perpendicular.

109. Intercepts: $(0,2)$ and $(-2,0)$. Thus, slope = 1.
$y = x + 2$ or $x - y = -2$

110. Intercepts: $(0,1)$ and $(1,0)$. Thus, slope = –1.
$y = -x + 1$ or $x + y = 1$

111. Intercepts: $(3,0)$ and $(0,1)$. Thus, slope = $-\frac{1}{3}$.

$y = -\frac{1}{3}x + 1$ or $x + 3y = 3$

112. Intercepts: $(0,-1)$ and $(-2,0)$. Thus,

slope = $-\frac{1}{2}$.

$y = -\frac{1}{2}x - 1$ or $x + 2y = -2$

113. $P_1 = (-2,5)$, $P_2 = (1,3)$: $m_1 = \frac{5-3}{-2-1} = \frac{2}{-3} = -\frac{2}{3}$

$P_2 = (1,3)$, $P_3 = (-1,0)$: $m_2 = \frac{3-0}{1-(-1)} = \frac{3}{2}$

Since $m_1 \cdot m_2 = -1$, the line segments $\overline{P_1P_2}$ and $\overline{P_2P_3}$ are perpendicular. Thus, the points P_1, P_2, and P_3 are vertices of a right triangle.

114. $P_1 = (1,-1)$, $P_2 = (4,1)$, $P_3 = (2,2)$, $P_4 = (5,4)$

$m_{12} = \frac{1-(-1)}{4-1} = \frac{2}{3}$; $m_{24} = \frac{4-1}{5-4} = 3$;

$m_{34} = \frac{4-2}{5-2} = \frac{2}{3}$; $m_{13} = \frac{2-(-1)}{2-1} = 3$

Each pair of opposite sides are parallel (same slope) and adjacent sides are not perpendicular. Therefore, the vertices are for a parallelogram.

115. $P_1 = (-1,0)$, $P_2 = (2,3)$, $P_3 = (1,-2)$, $P_4 = (4,1)$

$m_{12} = \frac{3-0}{2-(-1)} = \frac{3}{3} = 1$; $m_{24} = \frac{1-3}{4-2} = -1$;

$m_{34} = \frac{1-(-2)}{4-1} = \frac{3}{3} = 1$; $m_{13} = \frac{-2-0}{1-(-1)} = -1$

Opposite sides are parallel (same slope) and adjacent sides are perpendicular (product of slopes is -1). Therefore, the vertices are for a rectangle.

116. $P_1 = (0,0)$, $P_2 = (1,3)$, $P_3 = (4,2)$, $P_4 = (3,-1)$

$m_{12} = \frac{3-0}{1-0} = 3$; $m_{23} = \frac{2-3}{4-1} = -\frac{1}{3}$;

$m_{34} = \frac{-1-2}{3-4} = 3$; $m_{14} = \frac{-1-0}{3-0} = -\frac{1}{3}$

$d_{12} = \sqrt{(1-0)^2 + (3-0)^2} = \sqrt{1+9} = \sqrt{10}$

$d_{23} = \sqrt{(4-1)^2 + (2-3)^2} = \sqrt{9+1} = \sqrt{10}$

$d_{34} = \sqrt{(3-4)^2 + (-1-2)^2} = \sqrt{1+9} = \sqrt{10}$

$d_{14} = \sqrt{(3-0)^2 + (-1-0)^2} = \sqrt{9+1} = \sqrt{10}$

Opposite sides are parallel (same slope) and adjacent sides are perpendicular (product of slopes is -1). In addition, the length of all four sides is the same. Therefore, the vertices are for a square.

117. Let x = number of miles driven, and let C = cost in dollars.
Total cost = (cost per mile)(number of miles) + fixed cost
$C = 0.09x + 33$
When x = 175, $C = (0.09)(175) + 33 = \$48.75$.
When x = 403, $C = (0.09)(403) + 33 = \$69.27$.

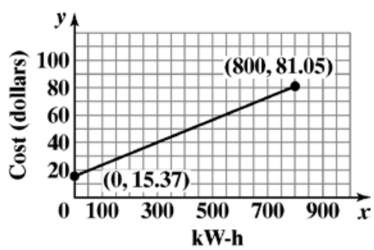
118. Let x = number of pairs of jeans manufactured, and let C = cost in dollars.
Total cost = (cost per pair)(number of pairs) + fixed cost
$C = 8x + 500$
When x = 400, $C = (8)(400) + 500 = \$3700$.
When x = 740, $C = (8)(740) + 500 = \$6420$.

119. Let x = number of miles driven annually, and let C = cost in dollars.
Total cost = (approx cost per mile)(number of miles) + fixed cost
$C = 0.16x + 4436$

120. Let x = profit in dollars, and let S = salary in dollars.
Weekly salary = (% share of profit)(profit) + weekly pay
$S = 0.05x + 375$

121. **a.** $C = 0.0821x + 15.37$; $0 \le x \le 800$

b.



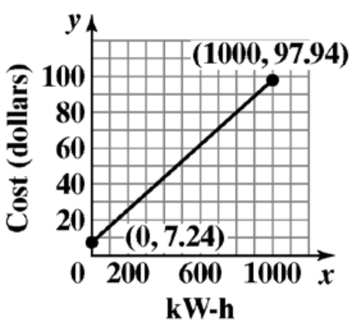
c. For 200 kWh,
$C = 0.0821(200) + 15.37 = \$31.79$

d. For 500 kWh,
$C = 0.0821(500) + 15.37 = \$56.42$

e. For each usage increase of 1 kWh, the monthly charge increases by \$0.0821 (that is, 8.21 cents).

122. **a.** $C = 0.0907x + 7.24$; $0 \le x \le 1000$

b.



c. For 200 kWh,
$C = 0.0907(200) + 7.24 = \$25.38$

d. For 500 kWh,
$C = 0.0907(500) + 7.24 = \$52.59$

e. For each usage increase of 1 kWh, the monthly charge increases by \$0.0907 (that is, 9.07 cents).

123. $(°C, °F) = (0, 32)$; $(°C, °F) = (100, 212)$

$$\text{slope} = \frac{212-32}{100-0} = \frac{180}{100} = \frac{9}{5}$$

$$°F - 32 = \frac{9}{5}(°C - 0)$$

$$°F - 32 = \frac{9}{5}(°C)$$

$$°C = \frac{5}{9}(°F - 32)$$

If $°F = 78$, then

$$°C = \frac{5}{9}(78-32) = \frac{5}{9}(46)$$

$$°C \approx 25.6°C$$

124. **a.** $K = °C + 273$

b. $$°C = \frac{5}{9}(°F - 32)$$

$$K = \frac{5}{9}(°F - 32) + 273$$

$$K = \frac{5}{9}°F - \frac{160}{9} + 273$$

$$K = \frac{5}{9}°F + \frac{2297}{9}$$

125. **a.** The y-intercept is (0, 45), so b = 45. Since the ramp drops 3 inches for every 29 inch run, the slope is $m = \frac{-3}{29} = \frac{-3}{29}$. Thus, the equation is $y = -\frac{3}{29}x + 45$.

b. Let $y = 0$.

$$0 = \frac{-3}{29}x + 45$$

$$\frac{3}{29}x = 45$$

$$\frac{29}{3}\left(\frac{3}{29}x\right) = \frac{29}{3}(45)$$

$$x = 435$$

The x-intercept is (435, 0). This means that the ramp meets the floor 435 inches (or 36.25 feet) from the base of the platform.

c. No. From part (b), the run is 36.25 feet which exceeds the required maximum of 30 feet.

d. First, design requirements state that the maximum slope is a drop of 1 inch for each 8 inches of run. This means $|1| \le \frac{1}{8}$.

Second, the run is restricted to no more than 30 feet = 360 inches. For a rise of 45 inches, this means the minimum slope is $\frac{45}{360} = \frac{1}{8}$. That is, $|m| \ge \frac{1}{8}$. Thus, the only possible slope that can be used to obtain the 45-inch rise and still meet design requirements is $m = -\frac{1}{8}$. In words, for every 8 inches of run, the ramp must drop *exactly* 1 inch.

126. a. The year 2000 corresponds to $x = 0$, and the year 2012 corresponds to $x = 12$. Therefore, the points (0, 20.6) and (12, 9.3) are on the line. Thus,
$m = \frac{9.3 - 20.6}{12 - 0} = -\frac{11.3}{12} = -0.942$. The y-intercept is 20.6, so $b = 20.6$ and the equation is $y = -0.942x + 20.6$

b. x-intercept:

$$0 = -0.942x + 20.6$$
$$0.942x = 20.6$$
$$x = 21.9$$

y-intercept: $y = -0.942(0) + 20.6 = 20.6$

The intercepts are (21.9, 0) and (0, 20.6).

c. The y-intercept represents the percentage of twelfth graders in 2000 who had reported daily use of cigarettes. The x-intercept represents the number of years after 2000 when 0% of twelfth graders will have reported daily use of cigarettes.

d. The year 2025 corresponds to $x = 25$.
$y = -0.942(25) + 20.6 = -2.95$
This prediction is not reasonable.

127. a. Let x = number of boxes sold, and A = money, in dollar spent on advertising. We have the points
$(x_1, A_1) = (100,000, 10,000)$;
$(x_2, A_2) = (200,000, 50,000)$

$$\text{slope} = \frac{50,000 - 10,000}{200,000 - 100,000} = \frac{40,000}{100,000} = \frac{2}{5}$$

$$A - 10,000 = \frac{2}{5}(x - 100,000)$$
$$A - 10,000 = \frac{2}{5}x - 40,000$$
$$A = \frac{2}{5}x - 30,000$$

b. If x = 700,000, then
$A = \frac{2}{5}(700,000) - 30,000 = \$250,000$

c. Each additional box sold requires an additional $0.40 in advertising.

128. Find the slope of the line containing (a,b) and (b,a):

$$\text{slope} = \frac{a-b}{b-a} = -1$$

The slope of the line $y = x$ is 1.

Since $-1 \cdot 1 = -1$, the line containing the points (a,b) and (b,a) is perpendicular to the line $y = x$.

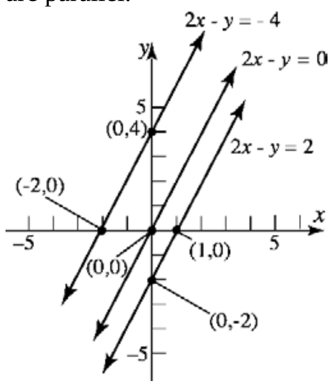
The midpoint of (a,b) and (b,a) is
$M = \left(\frac{a+b}{2}, \frac{b+a}{2}\right)$.

Since the coordinates are the same, the midpoint lies on the line $y = x$.

Note: $\frac{a+b}{2} = \frac{b+a}{2}$

129. $2x - y = C$
Graph the lines:
$2x - y = -4$
$2x - y = 0$
$2x - y = 2$
All the lines have the same slope, 2. The lines

are parallel.



130. Refer to Figure 47.

length of $\overline{OA} = d(O,A) = \sqrt{1+m_1^2}$

length of $\overline{OB} = d(O,B) = \sqrt{1+m_2^2}$

length of $\overline{AB} = d(A,B) = m_1 - m_2$

Now consider the equation

$$\left(\sqrt{1+m_1^2}\right)^2 + \left(\sqrt{1+m_2^2}\right)^2 = (m_1 - m_2)^2$$

If this equation is valid, then ΔAOB is a right triangle with right angle at vertex *O*.

$$\left(\sqrt{1+m_1^2}\right)^2 + \left(\sqrt{1+m_2^2}\right)^2 = (m_1 - m_2)^2$$

$$1+m_1^2+1+m_2^2 = m_1^2 - 2m_1m_2 + m_2^2$$

$$2+m_1^2+m_2^2 = m_1^2 - 2m_1m_2 + m_2^2$$

But we are assuming that $m_1m_2 = -1$, so we have

$$2+m_1^2+m_2^2 = m_1^2 - 2(-1) + m_2^2$$

$$2+m_1^2+m_2^2 = m_1^2 + 2 + m_2^2$$

$$0 = 0$$

Therefore, by the converse of the Pythagorean Theorem, ΔAOB is a right triangle with right angle at vertex *O*. Thus Line 1 is perpendicular to Line 2.

131. (b), (c), (e) and (g)
The line has positive slope and positive *y*-intercept.

132. (a), (c), and (g)
The line has negative slope and positive *y*-intercept.

133. (c)
The equation $x - y = -2$ has slope 1 and *y*-intercept $(0, 2)$. The equation $x - y = 1$ has slope 1 and *y*-intercept $(0, -1)$. Thus, the lines are parallel with positive slopes. One line has a positive *y*-intercept and the other with a negative *y*-intercept.

134. (d)
The equation $y - 2x = 2$ has slope 2 and *y*-intercept $(0, 2)$. The equation $x + 2y = -1$ has slope $-\frac{1}{2}$ and *y*-intercept $\left(0, -\frac{1}{2}\right)$. The lines are perpendicular since $2\left(-\frac{1}{2}\right) = -1$. One line has a positive *y*-intercept and the other with a negative *y*-intercept.

135 – 137. Answers will vary.

138. No, the equation of a vertical line cannot be written in slope-intercept form because the slope is undefined.

139. No, a line does not need to have both an *x*-intercept and a *y*-intercept. Vertical and horizontal lines have only one intercept (unless they are a coordinate axis). Every line must have at least one intercept.

140. Two lines with equal slopes and equal *y*-intercepts are coinciding lines (i.e. the same).

141. Two lines that have the same *x*-intercept and *y*-intercept (assuming the *x*-intercept is not 0) are the same line since a line is uniquely defined by two distinct points.

142. No. Two lines with the same slope and different *x*-intercepts are distinct parallel lines and have no points in common.
Assume Line 1 has equation $y = mx + b_1$ and Line 2 has equation $y = mx + b_2$,

Line 1 has *x*-intercept $-\frac{b_1}{m}$ and *y*-intercept b_1.

Line 2 has *x*-intercept $-\frac{b_2}{m}$ and *y*-intercept b_2.

Assume also that Line 1 and Line 2 have unequal *x*-intercepts.
If the lines have the same *y*-intercept, then $b_1 = b_2$.

$b_1 = b_2 \Rightarrow \frac{b_1}{m} = \frac{b_2}{m} \Rightarrow -\frac{b_1}{m} = -\frac{b_2}{m}$

But $-\frac{b_1}{m} = -\frac{b_2}{m} \Rightarrow$ Line 1 and Line 2 have the same x-intercept, which contradicts the original assumption that the lines have unequal x-intercepts. Therefore, Line 1 and Line 2 cannot have the same y-intercept.

143. Yes. Two distinct lines with the same y-intercept, but different slopes, can have the same x-intercept if the x-intercept is $x = 0$.

Assume Line 1 has equation $y = m_1 x + b$ and Line 2 has equation $y = m_2 x + b$,

Line 1 has x-intercept $-\frac{b}{m_1}$ and y-intercept b.

Line 2 has x-intercept $-\frac{b}{m_2}$ and y-intercept b.

Assume also that Line 1 and Line 2 have unequal slopes, that is $m_1 \neq m_2$.

If the lines have the same x-intercept, then

$-\frac{b}{m_1} = -\frac{b}{m_2}$.

$$-\frac{b}{m_1} = -\frac{b}{m_2}$$
$$-m_2 b = -m_1 b$$
$$-m_2 b + m_1 b = 0$$

But $-m_2 b + m_1 b = 0 \Rightarrow b(m_1 - m_2) = 0$

$\Rightarrow b = 0$

or $m_1 - m_2 = 0 \Rightarrow m_1 = m_2$

Since we are assuming that $m_1 \neq m_2$, the only way that the two lines can have the same x-intercept is if $b = 0$.

144. Answers will vary.

145. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-2}{1-(-3)} = \frac{-6}{4} = -\frac{3}{2}$

It appears that the student incorrectly found the slope by switching the direction of one of the subtractions.

Section 1.4

1. add; $\left(\frac{1}{2}\cdot 10\right)^2 = 25$

2. $(x-2)^2 = 9$

$$x - 2 = \pm\sqrt{9}$$
$$x - 2 = \pm 3$$
$$x = 2 \pm 3$$
$$x = 5 \quad \text{or} \quad x = -1$$

The solution set is $\{-1, 5\}$.

3. False. For example, $x^2 + y^2 + 2x + 2y + 8 = 0$ is not a circle. It has no real solutions.

4. radius

5. True; $r^2 = 9 \rightarrow r = 3$

6. False; the center of the circle $(x+3)^2 + (y-2)^2 = 13$ is $(-3, 2)$.

7. d

8. a

9. Center = (2, 1)

Radius = distance from (0,1) to (2,1)

$= \sqrt{(2-0)^2 + (1-1)^2} = \sqrt{4} = 2$

Equation: $(x-2)^2 + (y-1)^2 = 4$

10. Center = (1, 2)

Radius = distance from (1,0) to (1,2)

$= \sqrt{(1-1)^2 + (2-0)^2} = \sqrt{4} = 2$

Equation: $(x-1)^2 + (y-2)^2 = 4$

11. Center = midpoint of (1, 2) and (4, 2)

$= \left(\frac{1+4}{2}, \frac{2+2}{2}\right) = \left(\frac{5}{2}, 2\right)$

Radius = distance from $\left(\frac{5}{2}, 2\right)$ to (4,2)

$= \sqrt{\left(4 - \frac{5}{2}\right)^2 + (2-2)^2} = \sqrt{\frac{9}{4}} = \frac{3}{2}$

Equation: $\left(x - \frac{5}{2}\right)^2 + (y-2)^2 = \frac{9}{4}$

12. Center = midpoint of (0, 1) and (2, 3)

$= \left(\frac{0+2}{2}, \frac{1+3}{2}\right) = (1, 2)$

Radius = distance from $(1,2)$ to $(2,3)$

$$= \sqrt{(2-1)^2 + (3-2)^2} = \sqrt{2}$$

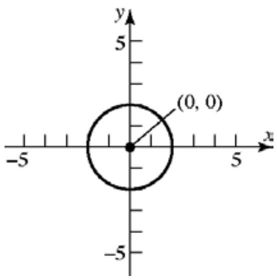
Equation: $(x-1)^2 + (y-2)^2 = 2$

13. $(x-h)^2 + (y-k)^2 = r^2$

$(x-0)^2 + (y-0)^2 = 2^2$

$x^2 + y^2 = 4$

General form: $x^2 + y^2 - 4 = 0$

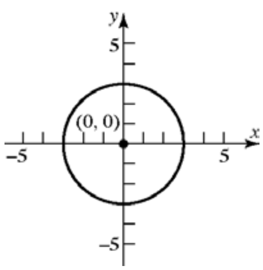


14. $(x-h)^2 + (y-k)^2 = r^2$

$(x-0)^2 + (y-0)^2 = 3^2$

$x^2 + y^2 = 9$

General form: $x^2 + y^2 - 9 = 0$



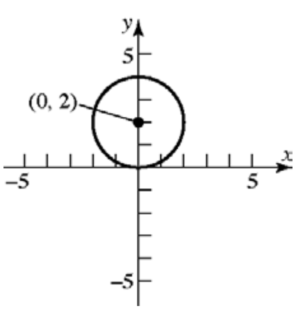
15. $(x-h)^2 + (y-k)^2 = r^2$

$(x-0)^2 + (y-2)^2 = 2^2$

$x^2 + (y-2)^2 = 4$

General form: $x^2 + y^2 - 4y + 4 = 4$

$x^2 + y^2 - 4y = 0$



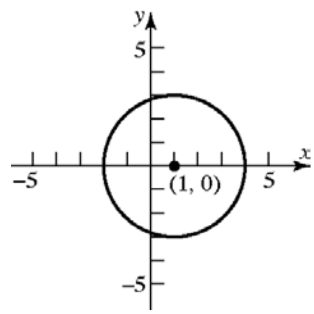
16. $(x-h)^2 + (y-k)^2 = r^2$

$(x-1)^2 + (y-0)^2 = 3^2$

$(x-1)^2 + y^2 = 9$

General form: $x^2 - 2x + 1 + y^2 = 9$

$x^2 + y^2 - 2x - 8 = 0$



17. $(x-h)^2 + (y-k)^2 = r^2$

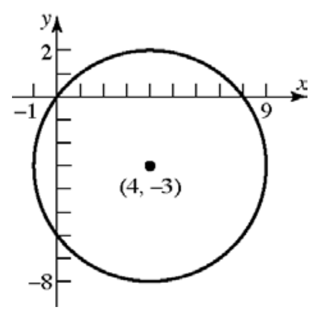
$(x-4)^2 + (y-(-3))^2 = 5^2$

$(x-4)^2 + (y+3)^2 = 25$

General form:

$x^2 - 8x + 16 + y^2 + 6y + 9 = 25$

$x^2 + y^2 - 8x + 6y = 0$



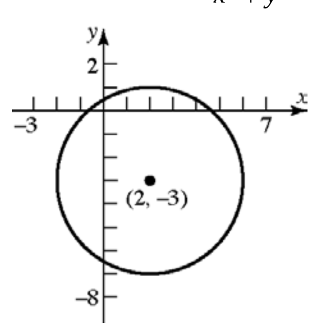
18. $(x-h)^2 + (y-k)^2 = r^2$

$(x-2)^2 + (y-(-3))^2 = 4^2$

$(x-2)^2 + (y+3)^2 = 16$

General form: $x^2 - 4x + 4 + y^2 + 6y + 9 = 16$

$x^2 + y^2 - 4x + 6y - 3 = 0$



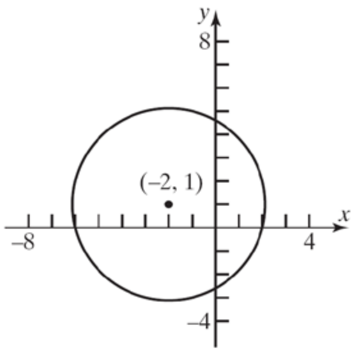
19. $(x-h)^2+(y-k)^2=r^2$

$(x-(-2))^2+(y-1)^2=4^2$

$(x+2)^2+(y-1)^2=16$

General form: $x^2+4x+4+y^2-2y+1=16$

$x^2+y^2+4x-2y-11=0$



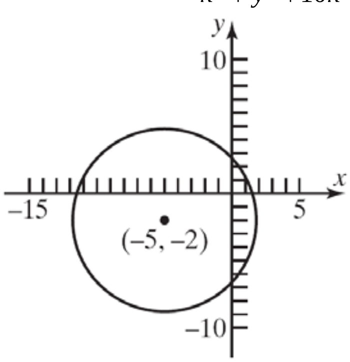
20. $(x-h)^2+(y-k)^2=r^2$

$(x-(-5))^2+(y-(-2))^2=7^2$

$(x+5)^2+(y+2)^2=49$

General form: $x^2+10x+25+y^2+4y+4=49$

$x^2+y^2+10x+4y-20=0$



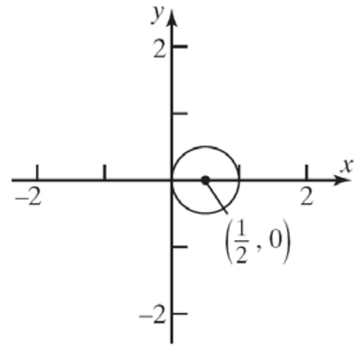
21. $(x-h)^2+(y-k)^2=r^2$

$\left(x-\frac{1}{2}\right)^2+(y-0)^2=\left(\frac{1}{2}\right)^2$

$\left(x-\frac{1}{2}\right)^2+y^2=\frac{1}{4}$

General form: $x^2-x+\frac{1}{4}+y^2=\frac{1}{4}$

$x^2+y^2-x=0$



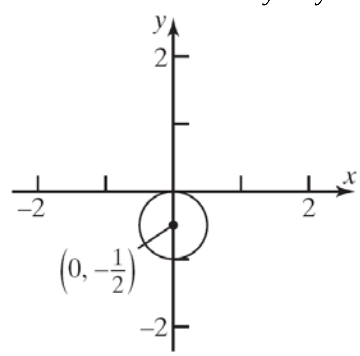
22. $(x-h)^2+(y-k)^2=r^2$

$(x-0)^2+\left(y-\left(-\frac{1}{2}\right)\right)^2=\left(\frac{1}{2}\right)^2$

$x^2+\left(y+\frac{1}{2}\right)^2=\frac{1}{4}$

General form: $x^2+y^2+y+\frac{1}{4}=\frac{1}{4}$

$x^2+y^2+y=0$

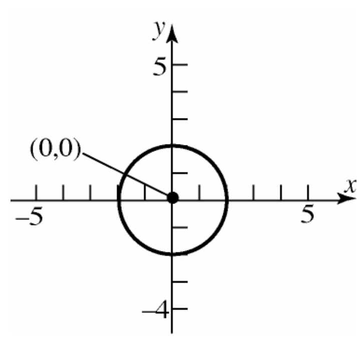


23. $x^2+y^2=4$

$x^2+y^2=2^2$

a. Center: $(0,0)$; Radius $=2$

b.



c. x-intercepts: $x^2 + (0)^2 = 4$

$$x^2 = 4$$
$$x = \pm\sqrt{4} = \pm 2$$

y-intercepts: $(0)^2 + y^2 = 4$

$$y^2 = 4$$
$$y = \pm\sqrt{4} = \pm 2$$

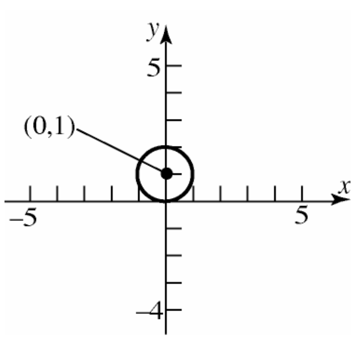
The intercepts are $(-2, 0), (2, 0), (0, -2),$ and $(0, 2)$.

24. $x^2 + (y-1)^2 = 1$

$$x^2 + (y-1)^2 = 1^2$$

a. Center:$(0, 1)$; Radius $= 1$

b.



c. x-intercepts: $x^2 + (0-1)^2 = 1$

$$x^2 + 1 = 1$$
$$x^2 = 0$$
$$x = \pm\sqrt{0} = 0$$

y-intercepts: $(0)^2 + (y-1)^2 = 1$

$$(y-1)^2 = 1$$
$$y - 1 = \pm\sqrt{1}$$
$$y - 1 = \pm 1$$
$$y = 1 \pm 1$$
$$y = 2 \quad \text{or} \quad y = 0$$

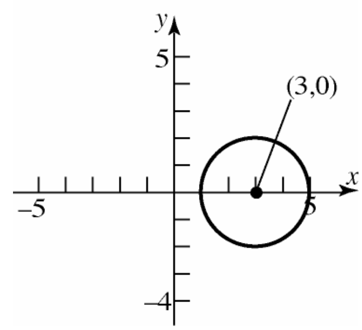
The intercepts are $(0, 0)$ and $(0, 2)$.

25. $2(x-3)^2 + 2y^2 = 8$

$$(x-3)^2 + y^2 = 4$$

a. Center: $(3, 0)$; Radius $= 2$

b.



c. x-intercepts: $(x-3)^2 + (0)^2 = 4$

$$(x-3)^2 = 4$$
$$x - 3 = \pm\sqrt{4}$$
$$x - 3 = \pm 2$$
$$x = 3 \pm 2$$
$$x = 5 \quad \text{or} \quad x = 1$$

y-intercepts: $(0-3)^2 + y^2 = 4$

$$(-3)^2 + y^2 = 4$$
$$9 + y^2 = 4$$
$$y^2 = -5$$

No real solution.

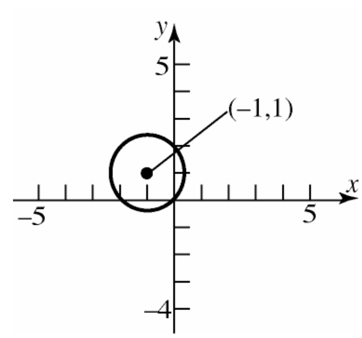
The intercepts are $(1, 0)$ and $(5, 0)$.

26. $3(x+1)^2 + 3(y-1)^2 = 6$

$$(x+1)^2 + (y-1)^2 = 2$$

a. Center: $(-1,1)$; Radius = $\sqrt{2}$

b.



c. x-intercepts:
$$\begin{aligned}(x+1)^2+(0-1)^2&=2\\(x+1)^2+(-1)^2&=2\\(x+1)^2+1&=2\\(x+1)^2&=1\\x+1&=\pm\sqrt{1}\\x+1&=\pm 1\\x&=-1\pm 1\\x=0 \quad &\text{or} \quad x=-2\end{aligned}$$

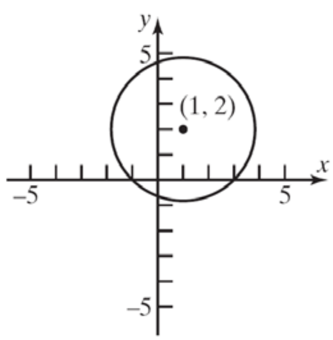
y-intercepts:
$$\begin{aligned}(0+1)^2+(y-1)^2&=2\\(1)^2+(y-1)^2&=2\\1+(y-1)^2&=2\\(y-1)^2&=1\\y-1&=\pm\sqrt{1}\\y-1&=\pm 1\\y&=1\pm 1\\y=2 \quad &\text{or} \quad y=0\end{aligned}$$

The intercepts are $(-2, 0)$, $(0, 0)$, and $(0, 2)$.

27.
$$\begin{aligned}x^2+y^2-2x-4y-4&=0\\x^2-2x+y^2-4y&=4\\(x^2-2x+1)+(y^2-4y+4)&=4+1+4\\(x-1)^2+(y-2)^2&=3^2\end{aligned}$$

a. Center: (1, 2); Radius = 3

b.



c. x-intercepts:
$$\begin{aligned}(x-1)^2+(0-2)^2&=3^2\\(x-1)^2+(-2)^2&=3^2\\(x-1)^2+4&=9\\(x-1)^2&=5\\x-1&=\pm\sqrt{5}\\x&=1\pm\sqrt{5}\end{aligned}$$

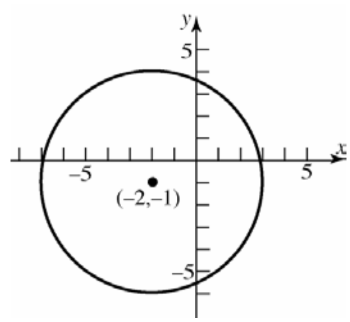
y-intercepts:
$$\begin{aligned}(0-1)^2+(y-2)^2&=3^2\\(-1)^2+(y-2)^2&=3^2\\1+(y-2)^2&=9\\(y-2)^2&=8\\y-2&=\pm\sqrt{8}\\y-2&=\pm 2\sqrt{2}\\y&=2\pm 2\sqrt{2}\end{aligned}$$

The intercepts are $\left(1-\sqrt{5}, 0\right)$, $\left(1+\sqrt{5}, 0\right)$, $\left(0, 2-2\sqrt{2}\right)$, and $\left(0, 2+2\sqrt{2}\right)$.

28.
$$\begin{aligned}x^2+y^2+4x+2y-20&=0\\x^2+4x+y^2+2y&=20\\(x^2+4x+4)+(y^2+2y+1)&=20+4+1\\(x+2)^2+(y+1)^2&=5^2\end{aligned}$$

a. Center: (–2,–1); Radius = 5

b.



c. x-intercepts:
$$\begin{aligned}(x+2)^2+(0+1)^2&=5^2\\(x+2)^2+1&=25\\(x+2)^2&=24\\x+2&=\pm\sqrt{24}\\x+2&=\pm 2\sqrt{6}\\x&=-2\pm 2\sqrt{6}\end{aligned}$$

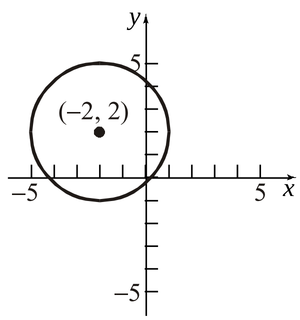
y-intercepts:
$$\begin{aligned}(0+2)^2+(y+1)^2&=5^2\\4+(y+1)^2&=25\\(y+1)^2&=21\\y+1&=\pm\sqrt{21}\\y&=-1\pm\sqrt{21}\end{aligned}$$

The intercepts are $\left(-2-2\sqrt{6}, 0\right)$, $\left(-2+2\sqrt{6}, 0\right)$, $\left(0, -1-\sqrt{21}\right)$, and $\left(0, -1+\sqrt{21}\right)$.

29.
$$x^2+y^2+4x-4y-1=0$$
$$x^2+4x+y^2-4y=1$$
$$(x^2+4x+4)+(y^2-4y+4)=1+4+4$$
$$(x+2)^2+(y-2)^2=3^2$$

a. Center: (–2, 2); Radius = 3

b.



c. x-intercepts: $(x+2)^2+(0-2)^2=3^2$
$$(x+2)^2+4=9$$
$$(x+2)^2=5$$
$$x+2=\pm\sqrt{5}$$
$$x=-2\pm\sqrt{5}$$

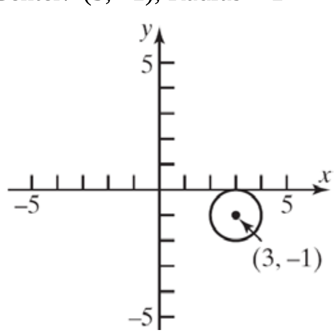
y-intercepts: $(0+2)^2+(y-2)^2=3^2$
$$4+(y-2)^2=9$$
$$(y-2)^2=5$$
$$y-2=\pm\sqrt{5}$$
$$y=2\pm\sqrt{5}$$

The intercepts are $\left(-2-\sqrt{5},\,0\right)$, $\left(-2+\sqrt{5},\,0\right)$, $\left(0,\,2-\sqrt{5}\right)$, and $\left(0,\,2+\sqrt{5}\right)$.

30.
$$x^2+y^2-6x+2y+9=0$$
$$x^2-6x+y^2+2y=-9$$
$$(x^2-6x+9)+(y^2+2y+1)=-9+9+1$$
$$(x-3)^2+(y+1)^2=1^2$$

a. Center: (3, –1); Radius = 1

b.



c. x-intercepts: $(x-3)^2+(0+1)^2=1^2$
$$(x-3)^2+1=1$$
$$\left(x-3\right)^2=0$$
$$x-3=0$$
$$x=3$$

y-intercepts: $(0-3)^2+(y+1)^2=1^2$
$$9+(y+1)^2=1$$
$$\left(y+1\right)^2=-8$$

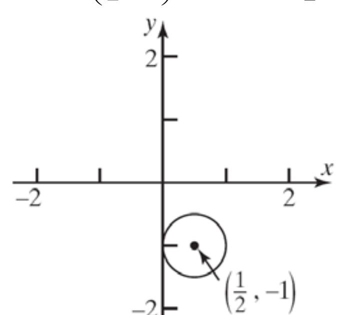
No real solution.

The intercept only intercept is $\left(3,0\right)$.

31.
$$x^2+y^2-x+2y+1=0$$
$$x^2-x+y^2+2y=-1$$
$$\left(x^2-x+\frac{1}{4}\right)+(y^2+2y+1)=-1+\frac{1}{4}+1$$
$$\left(x-\frac{1}{2}\right)^2+(y+1)^2=\left(\frac{1}{2}\right)^2$$

a. Center: $\left(\frac{1}{2},-1\right)$; Radius = $\frac{1}{2}$

b.



c. x-intercepts: $\left(x-\frac{1}{2}\right)^2+(0+1)^2=\left(\frac{1}{2}\right)^2$
$$\left(x-\frac{1}{2}\right)^2+1=\frac{1}{4}$$
$$\left(x-\frac{1}{2}\right)^2=-\frac{3}{4}$$

No real solutions

y-intercepts: $\left(0-\frac{1}{2}\right)^2+(y+1)^2=\left(\frac{1}{2}\right)^2$
$$\frac{1}{4}+\left(y+1\right)^2=\frac{1}{4}$$
$$\left(y+1\right)^2=0$$
$$y+1=0$$
$$y=-1$$

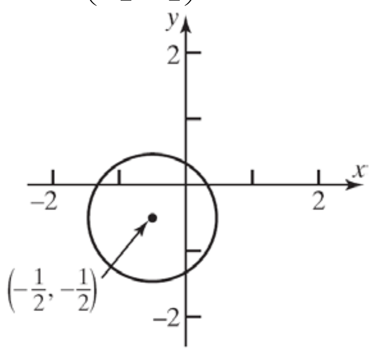
The only intercept is $\left(0,-1\right)$.

32.

$$x^2+y^2+x+y-\frac{1}{2}=0$$
$$x^2+x+y^2+y=\frac{1}{2}$$
$$\left(x^2+x+\frac{1}{4}\right)+\left(y^2+y+\frac{1}{4}\right)=\frac{1}{2}+\frac{1}{4}+\frac{1}{4}$$
$$\left(x+\frac{1}{2}\right)^2+\left(y+\frac{1}{2}\right)^2=1^2$$

a. Center: $\left(-\frac{1}{2},-\frac{1}{2}\right)$; Radius = 1

b.



c. x-intercepts: $\left(x+\frac{1}{2}\right)^2+\left(0+\frac{1}{2}\right)^2=1^2$

$$\left(x+\frac{1}{2}\right)^2+\frac{1}{4}=1$$
$$\left(x+\frac{1}{2}\right)^2=\frac{3}{4}$$
$$x+\frac{1}{2}=\pm\frac{\sqrt{3}}{2}$$
$$x=\frac{-1\pm\sqrt{3}}{2}$$

y-intercepts: $\left(0+\frac{1}{2}\right)^2+\left(y+\frac{1}{2}\right)^2=1^2$

$$\frac{1}{4}+\left(y+\frac{1}{2}\right)^2=1$$
$$\left(y+\frac{1}{2}\right)^2=\frac{3}{4}$$
$$y+\frac{1}{2}=\pm\frac{\sqrt{3}}{2}$$
$$y=\frac{-1\pm\sqrt{3}}{2}$$

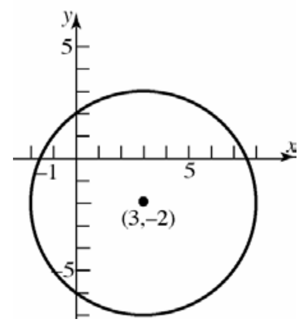
The intercepts are $\left(\frac{-1-\sqrt{3}}{2},0\right)$, $\left(\frac{-1+\sqrt{3}}{2},0\right)$, $\left(0,\frac{-1-\sqrt{3}}{2}\right)$, and $\left(0,\frac{-1+\sqrt{3}}{2}\right)$.

33.

$$2x^2+2y^2-12x+8y-24=0$$
$$x^2+y^2-6x+4y=12$$
$$x^2-6x+y^2+4y=12$$
$$(x^2-6x+9)+(y^2+4y+4)=12+9+4$$
$$(x-3)^2+(y+2)^2=5^2$$

a. Center: (3,–2); Radius = 5

b.



c. x-intercepts: $(x-3)^2+(0+2)^2=5^2$

$$(x-3)^2+4=25$$
$$(x-3)^2=21$$
$$x-3=\pm\sqrt{21}$$
$$x=3\pm\sqrt{21}$$

y-intercepts: $(0-3)^2+(y+2)^2=5^2$

$$9+(y+2)^2=25$$
$$(y+2)^2=16$$
$$y+2=\pm4$$
$$y=-2\pm4$$
$$y=2 \quad\text{or}\quad y=-6$$

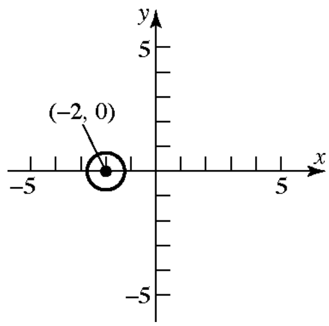
The intercepts are $(3-\sqrt{21},0)$, $(3+\sqrt{21},0)$, $(0,-6)$, and $(0,2)$.

34. a.

$$2x^2+2y^2+8x+7=0$$
$$2x^2+8x+2y^2=-7$$
$$x^2+4x+y^2=-\frac{7}{2}$$
$$(x^2+4x+4)+y^2=-\frac{7}{2}+4$$
$$(x+2)^2+y^2=\frac{1}{2}$$
$$(x+2)^2+y^2=\left(\frac{\sqrt{2}}{2}\right)^2$$

Center: (–2, 0); Radius = $\frac{\sqrt{2}}{2}$

b.



c. x-intercepts:
$$\begin{aligned}(x+2)^2+(0)^2&=\frac{1}{2}\\(x+2)^2&=\frac{1}{2}\\x+2&=\pm\sqrt{\frac{1}{2}}\\x+2&=\pm\frac{\sqrt{2}}{2}\\x&=-2\pm\frac{\sqrt{2}}{2}\end{aligned}$$

y-intercepts:
$$\begin{aligned}(0+2)^2+y^2&=\frac{1}{2}\\4+y^2&=\frac{1}{2}\\y^2&=-\frac{7}{2}\end{aligned}$$

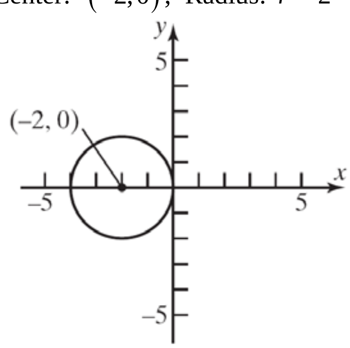
No real solutions.

The intercepts are $\left(-2-\frac{\sqrt{2}}{2},\,0\right)$ and $\left(-2+\frac{\sqrt{2}}{2},\,0\right)$.

35.
$$\begin{aligned}2x^2+8x+2y^2&=0\\x^2+4x+y^2&=0\\x^2+4x+4+y^2&=0+4\\(x+2)^2+y^2&=2^2\end{aligned}$$

a. Center: $(-2,0)$; Radius: $r=2$

b.



c. x-intercepts:
$$\begin{aligned}(x+2)^2+(0)^2&=2^2\\(x+2)^2&=4\\(x+2)^2&=\pm\sqrt{4}\\x+2&=\pm2\\x&=-2\pm2\\x=0\quad&\text{or}\quad x=-4\end{aligned}$$

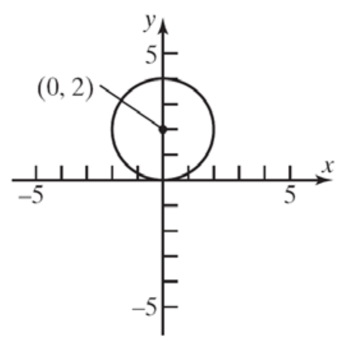
y-intercepts:
$$\begin{aligned}(0+2)^2+y^2&=2^2\\4+y^2&=4\\y^2&=0\\y&=0\end{aligned}$$

The intercepts are $(-4,0)$ and $(0,0)$.

36.
$$\begin{aligned}3x^2+3y^2-12y&=0\\x^2+y^2-4y&=0\\x^2+y^2-4y+4&=0+4\\x^2+(y-2)^2&=4\end{aligned}$$

a. Center: $(0,2)$; Radius: $r=2$

b.



c. x-intercepts:
$$\begin{aligned}x^2+(0-2)^2&=4\\x^2+4&=4\\x^2&=0\\x&=0\end{aligned}$$

y-intercepts:
$$\begin{aligned}0^2+(y-2)^2&=4\\(y-2)^2&=4\\y-2&=\pm\sqrt{4}\\y-2&=\pm2\\y&=2\pm2\\y=4\quad&\text{or}\quad y=0\end{aligned}$$

The intercepts are $(0,0)$ and $(0,4)$.

37. Center at (0, 0); containing point (–2, 3).
$r=\sqrt{\left(-2-0\right)^2+\left(3-0\right)^2}=\sqrt{4+9}=\sqrt{13}$

Equation: $(x-0)^2+(y-0)^2=\left(\sqrt{13}\right)^2$
$$x^2+y^2=13$$

38. Center at (1, 0); containing point (–3, 2).
$r=\sqrt{\left(-3-1\right)^2+\left(2-0\right)^2}=\sqrt{16+4}=\sqrt{20}=2\sqrt{5}$

Equation: $(x-1)^2+(y-0)^2=\left(\sqrt{20}\right)^2$
$$(x-1)^2+y^2=20$$

39. Center at (2, 3); tangent to the x-axis.
$r=3$
Equation: $(x-2)^2+(y-3)^2=3^2$
$$(x-2)^2+(y-3)^2=9$$

40. Center at (–3, 1); tangent to the y-axis.
$r=3$
Equation: $(x+3)^2+(y-1)^2=3^2$
$$(x+3)^2+(y-1)^2=9$$

41. Endpoints of a diameter are (1, 4) and (–3, 2).
The center is at the midpoint of that diameter:
Center: $\left(\dfrac{1+(-3)}{2},\dfrac{4+2}{2}\right)=\left(-1,3\right)$

Radius: $r=\sqrt{(1-(-1))^2+(4-3)^2}=\sqrt{4+1}=\sqrt{5}$

Equation: $(x-(-1))^2+(y-3)^2=\left(\sqrt{5}\right)^2$
$$(x+1)^2+(y-3)^2=5$$

42. Endpoints of a diameter are (4, 3) and (0, 1).
The center is at the midpoint of that diameter:
Center: $\left(\dfrac{4+0}{2},\dfrac{3+1}{2}\right)=\left(2,2\right)$

Radius: $r=\sqrt{(4-2)^2+(3-2)^2}=\sqrt{4+1}=\sqrt{5}$

Equation: $(x-2)^2+(y-2)^2=\left(\sqrt{5}\right)^2$
$$(x-2)^2+(y-2)^2=5$$

43. Center at (–1, 3); tangent to the line $y = 2$.
This means that the circle contains the point (–1, 2), so the radius is $r = 1$.
Equation: $(x+1)^2+(y-3)^2=(1)^2$
$$(x+1)^2+(y-3)^2=1$$

44. Center at (4, –2); tangent to the line $x = 1$.
This means that the circle contains the point (1, –2), so the radius is $r = 3$.
Equation: $(x-4)^2+(y+2)^2=(3)^2$
$$(x-4)^2+(y+2)^2=9$$

45. (c); Center: $\left(1,-2\right)$; Radius = 2

46. (d) ; Center: $\left(-3,3\right)$; Radius = 3

47. (b) ; Center: $\left(-1,2\right)$; Radius = 2

48. (a) ; Center: $\left(-3,3\right)$; Radius = 3

49. Let the upper right corner of the square be the point $\left(x,y\right)$. The circle and the square are both centered about the origin. Because of symmetry, we have $x=y$ at the upper right corner of the square. Therefore, we get
$$x^2+y^2=25$$
$$x^2+x^2=25$$
$$2x^2=25$$
$$x^2=\frac{25}{2}$$
$$x=\sqrt{\frac{25}{2}}=\frac{5\sqrt{2}}{2}$$
The length of one side of the square is $2x$. Thus, the area is
$$A=s^2=\left(2\cdot\frac{5\sqrt{2}}{2}\right)^2=\left(5\sqrt{2}\right)^2=50 \text{ sq units.}$$

50. The area of the shaded region is the area of the circle, less the area of the square. Let the upper-right corner of the square be the point $\left(x,y\right)$. The circle and the square are both centered about the origin. Because of symmetry, we have that $x=y$ at the upper-right corner of the square. Therefore, we get

$$x^2 + y^2 = 36$$
$$x^2 + x^2 = 36$$
$$2x^2 = 36$$
$$x^2 = 18$$
$$x = 3\sqrt{2}$$

The length of one side of the square is $2x$. Thus, the area of the square is $\left(2 \cdot 3\sqrt{2}\right)^2 = 72$ square units. From the equation of the circle, we have $r = 6$. The area of the circle is $\pi r^2 = \pi(6)^2 = 36\pi$ square units.

Therefore, the area of the shaded region is $A = 36\pi - 72$ square units.

51. The diameter of the Ferris wheel was 260 feet, so the radius was 130 feet. The maximum height was 270 feet, so the center was at a height of $270 - 130 = 140$ feet above the ground. Since the center of the wheel is on the y-axis, it is the point (0, 140). Thus, an equation for the wheel is:

$$(x-0)^2 + (y-140)^2 = (130)^2$$
$$x^2 + (y-140)^2 = 16900$$

52. The diameter of the wheel is 520 feet, so the radius is 260 feet. The maximum height is 550 feet, so the center of the wheel is at a height of $550 - 260 = 290$ feet above the ground. Since the center of the wheel is on the y-axis, it is the point (0, 290). Thus, an equation for the wheel is:

$$(x-0)^2 + (y-290)^2 = 260^2$$
$$x^2 + (y-290)^2 = 67,600$$

53. $x^2 + y^2 + 10x + 8y - 3559 = 0$

$$x^2 + 10x + y^2 + 8y - 3559 = 0$$
$$x^2 + 10x + 25 + y^2 + 8y + 16 = 3559 + 25 + 16$$
$$(x+5)^2 + (y+4)^2 = 3600$$

The circle representing Earth has center $(-5, -4)$ and radius = $\sqrt{3600} = 60$.

So the radius of the satellite's orbit is $60 + 0.7 = 60.7$ units.

The equation of the orbit is

$$(x+5)^2 + (y+4)^2 = (60.7)^2$$
$$x^2 + y^2 + 10x + 8y - 3643.49 = 0$$

54. **a.**

$$x^2 + (mx+b)^2 = r^2$$
$$x^2 + m^2x^2 + 2bmx + b^2 = r^2$$
$$(1+m^2)x^2 + 2bmx + b^2 - r^2 = 0$$

There is one solution if and only if the discriminant is zero.

$$(2bm)^2 - 4(1+m^2)(b^2 - r^2) = 0$$
$$4b^2m^2 - 4b^2 + 4r^2 - 4b^2m^2 + 4m^2r^2 = 0$$
$$-4b^2 + 4r^2 + 4m^2r^2 = 0$$
$$-b^2 + r^2 + m^2r^2 = 0$$
$$r^2(1+m^2) = b^2$$

b. Using the quadratic formula, the result from part (a), and knowing that the discriminant is zero, we get:

$$(1+m^2)x^2 + 2bmx + b^2 - r^2 = 0$$
$$x = \frac{-2bm}{2(1+m^2)} = \frac{-bm}{\left(\frac{b^2}{r^2}\right)} = \frac{-bmr^2}{b^2} = \frac{-mr^2}{b}$$
$$y = m\left(\frac{-mr^2}{b}\right) + b$$
$$= \frac{-m^2r^2}{b} + b = \frac{-m^2r^2 + b^2}{b} = \frac{r^2}{b}$$

c. The slope of the tangent line is m.

The slope of the line joining the point of tangency and the center is:

$$\frac{\left(\frac{r^2}{b} - 0\right)}{\left(\frac{-mr^2}{b} - 0\right)} = \frac{r^2}{b} \cdot \frac{b}{-mr^2} = -\frac{1}{m}$$

Therefore, the tangent line is perpendicular to the line containing the center of the circle and the point of tangency.

55. $x^2 + y^2 = 49$

Center: (0, 0)

Slope from center to $\left(1, 4\sqrt{3}\right)$ is

$$\frac{4\sqrt{3} - 0}{1 - 0} = \frac{4\sqrt{3}}{1} = 4\sqrt{3}.$$

Slope of the tangent line is $\frac{-1}{4\sqrt{3}} = \frac{-\sqrt{3}}{12}$.

Equation of the tangent line is:

$$y-4\sqrt{3}=\frac{-\sqrt{3x}}{12}(x-1)$$
$$y-4\sqrt{3}=\frac{-\sqrt{3x}}{12}+\frac{\sqrt{3}}{12}$$
$$12y-48\sqrt{3}=-\sqrt{3}\,x+\sqrt{3}$$
$$\sqrt{3}\,x+12y=49\sqrt{3}$$
$$\sqrt{3x}+12y-49\sqrt{3}=0$$

56. $x^2+y^2-4x+6y+4=0$

$$(x^2-4x+4)+(y^2+6y+9)=-4+4+9$$
$$(x-2)^2+(y+3)^2=9$$

Center: (2, –3)

Slope from center to $\left(3, 2\sqrt{2}-3\right)$ is

$$\frac{2\sqrt{2}-3-(-3)}{3-2}=\frac{2\sqrt{2}}{1}=2\sqrt{2}$$

Slope of the tangent line is: $\dfrac{-1}{2\sqrt{2}}=-\dfrac{\sqrt{2}}{4}$

Equation of the tangent line:

$$y-\left(2\sqrt{2}-3\right)=-\frac{\sqrt{2}}{4}(x-3)$$
$$y-2\sqrt{2}+3=-\frac{\sqrt{2}}{4}x+\frac{3\sqrt{2}}{4}$$
$$4y-8\sqrt{2}+12=-\sqrt{2}x+3\sqrt{2}$$
$$\sqrt{2}x+4y-11\sqrt{2}+12=0$$

57. Let (h, k) be the center of the circle.

$$x-2y+16=0$$
$$2y=x+16$$

The slope of the tangent line is $\dfrac{1}{2}$. The slope from (h, k) to (0, 8) is –2.

$$\frac{8-k}{0-h}=-2$$
$$8-k=2h$$

The other tangent line is $y=2x-1$, and it has slope 2. The slope from (h, k) to (3, 5) is $-\dfrac{1}{2}$

$$\frac{5-k}{3-h}=\frac{-1}{2}$$
$$10-2k=-3+h$$
$$h=13-2k$$

Solve the two equations in h and k:

$$8-k=2(13-2k)$$
$$8-k=26-4k$$
$$3k=18$$
$$k=6$$
$$h=13-2(6)=1$$

The center of the circle is (1, 6).

58. Find the centers of the two circles:

$$x^2+y^2-4x+6y+4=0$$
$$(x^2-4x+4)+(y^2+6y+9)=-4+4+9$$
$$(x-2)^2+(y+3)^2=9$$

Center: $(2,-3)$

$$x^2+y^2+6x+4y+9=0$$
$$(x^2+6x+9)+(y^2+4y+4)=-9+9+4$$
$$(x+3)^2+(y+2)^2=4$$

Center: $(-3,-2)$

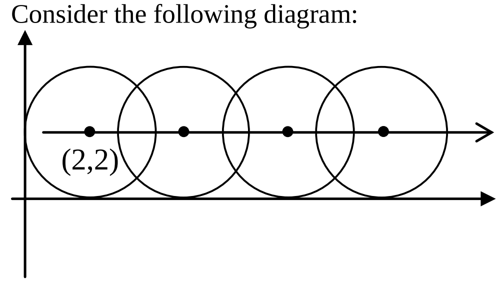
Find the slope of the line containing the centers:

$$m=\frac{-2-(-3)}{-3-2}=-\frac{1}{5}$$

Find the equation of the line containing the centers:

$$y+3=-\frac{1}{5}(x-2)$$
$$5y+15=-x+2$$
$$x+5y=-13$$
$$x+5y+13=0$$

59. Consider the following diagram:



Therefore, the path of the center of the circle has the equation $y=2$.

60. $C = 2\pi r$
$6\pi = 2\pi r$
$\dfrac{6\pi}{2\pi} = \dfrac{2\pi r}{2\pi}$
$3 = r$
The radius is 3 units long.

61. (b), (c), (e) and (g)
We need $h, k > 0$ and $(0,0)$ on the graph.

62. (b), (e) and (g)
We need $h < 0$, $k = 0$, and $|h| > r$.

63. Answers will vary.

64. The student has the correct radius, but the signs of the coordinates of the center are incorrect. The student needs to write the equation in the standard form $(x-h)^2 + (y-k)^2 = r^2$.
$(x+3)^2 + (y-2)^2 = 16$
$(x-(-3))^2 + (y-2)^2 = 4^2$
Thus, $(h,k) = (-3,2)$ and $r = 4$.

Chapter 1 Review Exercises

1. $P_1 = (0,0)$ and $P_2 = (4,2)$

a. $d(P_1,P_2) = \sqrt{(4-0)^2 + (2-0)^2}$
$= \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$

b. The coordinates of the midpoint are:
$(x,y) = \left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$
$= \left(\dfrac{0+4}{2}, \dfrac{0+2}{2}\right) = \left(\dfrac{4}{2}, \dfrac{2}{2}\right) = (2,1)$

c. slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{2-0}{4-0} = \dfrac{2}{4} = \dfrac{1}{2}$

d. For each run of 2, there is a rise of 1.

2. $P_1 = (1,-1)$ and $P_2 = (-2,3)$

a. $d(P_1,P_2) = \sqrt{(-2-1)^2 + (3-(-1))^2}$
$= \sqrt{9+16} = \sqrt{25} = 5$

b. The coordinates of the midpoint are:
$(x,y) = \left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$
$= \left(\dfrac{1+(-2)}{2}, \dfrac{-1+3}{2}\right)$
$= \left(\dfrac{-1}{2}, \dfrac{2}{2}\right) = \left(-\dfrac{1}{2}, 1\right)$

c. slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{3-(-1)}{-2-1} = \dfrac{4}{-3} = -\dfrac{4}{3}$

d. For each run of 3, there is a rise of -4.

3. $P_1 = (4,-4)$ and $P_2 = (4,8)$

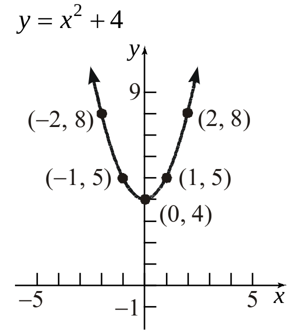
a. $d(P_1,P_2) = \sqrt{(4-4)^2 + (8-(-4))^2}$
$= \sqrt{0+144} = \sqrt{144} = 12$

b. The coordinates of the midpoint are:
$(x,y) = \left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$
$= \left(\dfrac{4+4}{2}, \dfrac{-4+8}{2}\right) = \left(\dfrac{8}{2}, \dfrac{4}{2}\right) = (4,2)$

c. slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{8-(-4)}{4-4} = \dfrac{12}{0}$, undefined

d. An undefined slope means the points lie on a vertical line. There is no change in *x*.

4. $y = x^2 + 4$



5. *x*-intercepts: $-4, 0, 2$; *y*-intercepts: $-2, 0, 2$
Intercepts: $(-4, 0)$, $(0, 0)$, $(2, 0)$, $(0, -2)$, $(0, 2)$

6. $3x^2 = 2y$

x-intercepts:
$3x^2 = 2(0)$
$2x = 0$
$x = 0$

y-intercepts:
$3(0)^2 = 2y$
$0 = 2y$
$0 = y$

The only intercept is $(0, 0)$.

<u>Test x-axis symmetry</u>: Let $y = -y$

$3x^2 = 2(-y)$
$3x^2 = -2y$ different

<u>Test y-axis symmetry</u>: Let $x = -x$

$3(-x)^2 = 2y$
$3x^2 = 2y$ same

<u>Test origin symmetry</u>: Let $x = -x$ and $y = -y$.

$3(-x)^2 = 2(-y)$
$3x^2 = -2y$ different

Thus the graph will have only y-axis symmetry.

7. $4x^2 + y^2 = 16$

x-intercepts:
$4x^2 + (0)^2 = 16$
$x^2 = 4$
$x = 2, -2$

y-intercepts:
$4(0)^2 + y^2 = 16$
$y^2 = 16$
$y = 4, -4$

The intercepts are $(2, 0)$, $(-2, 0)$, $(0, 4)$, and $(0, -4)$.

<u>Test x-axis symmetry</u>: Let $y = -y$

$4x^2 + (-y)^2 = 16$
$4x^2 + y^2 = 16$ same

<u>Test y-axis symmetry</u>: Let $x = -x$

$4(-x)^2 + y^2 = 16$
$4x^2 + y^2 = 16$ same

<u>Test origin symmetry</u>: Let $x = -x$ and $y = -y$.

$4(-x)^2 + (-y)^2 = 16$
$4x^2 + y^2 = 16$ same

Thus the graph will have x-axis, y-axis, and origin symmetry.

8. $y = x^4 + 2x^2 + 1$

x-intercepts:
$0 = x^4 + 2x^2 + 1$
$0 = (x^2 + 1)(x^2 + 1)$
$x^2 + 1 = 0$
$x^2 = -1$
no real solutions

y-intercepts:
$y = (0)^4 + 2(0)^2 + 1$
$= 1$

The only intercept is (0, 1).

<u>Test x-axis symmetry</u>: Let $y = -y$

$-y = x^4 + 2x^2 + 1$
$y = -x^4 - 2x^2 - 1$ different

<u>Test y-axis symmetry</u>: Let $x = -x$

$y = (-x)^4 + 2(-x)^2 + 1$
$y = x^4 + 2x^2 + 1$ same

<u>Test origin symmetry</u>: Let $x = -x$ and $y = -y$.

$-y = (-x)^4 + 2(-x)^2 + 1$
$-y = x^4 + 2x^2 + 1$
$y = -x^4 - 2x^2 - 1$ different

Therefore, the graph will have y-axis symmetry.

9. $y^5 = x^3 - 4x$

x-intercepts:
$(0)^5 = x^3 - 4x$
$0 = x(x-2)(x+2)$
$x = 0, 2, -2$

y-intercepts:
$y^5 = (0)^3 - 4(0)$
$y^5 = 0$
$y = 0$

$x = 0$, $x = -1$, $x = 1$

The intercepts are $(0, 0)$, $(2, 0)$, and $(-2, 0)$.

<u>Test x-axis symmetry</u>: Let $y = -y$

$(-y)^5 = x^3 - 4x$
$-y^5 = x^3 - 4x$ different

<u>Test y-axis symmetry</u>: Let $x = -x$

$y^5 = (-x)^3 - 4(-x)$
$y^5 = -x^3 + 4x$ different

<u>Test origin symmetry</u>: Let $x = -x$ and $y = -y$.

$(-y)^5 = (-x)^3 - 4(-x)$
$-y^5 = -x^3 + 4x$
$y^5 = x^3 - 4x$ same

Thus the graph will have only origin symmetry.

10. $x^2 + x + y^2 + 2y = 0$

x-intercepts: $x^2 + x + (0)^2 + 2(0) = 0$
$$x^2 + x = 0$$
$$x(x+1) = 0$$
$$x = 0, x = -1$$

y-intercepts: $(0)^2 + 0 + y^2 + 2y = 0$
$$y^2 + 2y = 0$$
$$y(y+2) = 0$$
$$y = 0, y = -2$$

The intercepts are $(-1, 0)$, $(0, 0)$, and $(0, -2)$.

Test x-axis symmetry: Let $y = -y$
$$x^2 + x + (-y)^2 + 2(-y) = 0$$
$$x^2 + x + y^2 - 2y = 0 \quad \text{different}$$

Test y-axis symmetry: Let $x = -x$
$$(-x)^2 + (-x) + y^2 + 2y = 0$$
$$x^2 - x + y^2 + 2y = 0 \quad \text{different}$$

Test origin symmetry: Let $x = -x$ and $y = -y$.
$$(-x)^2 + (-x) + (-y)^2 + 2(-y) = 0$$
$$x^2 - x + y^2 - 2y = 0 \quad \text{different}$$

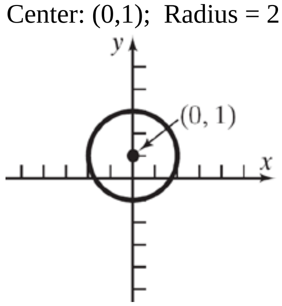
The graph has none of the indicated symmetries.

11. $(x-h)^2 + (y-k)^2 = r^2$
$$\left(x-(-3)\right)^2 + \left(y-4\right)^2 = 5^2$$
$$\left(x+3\right)^2 + \left(y-4\right)^2 = 25$$

12. $(x-h)^2 + (y-k)^2 = r^2$
$$\left(x-(-1)\right)^2 + \left(y-(-2)\right)^2 = 1^2$$
$$\left(x+1\right)^2 + \left(y+2\right)^2 = 1$$

13. $x^2 + \left(y-1\right)^2 = 4$
$$x^2 + \left(y-1\right)^2 = 2^2$$

Center: (0,1); Radius = 2



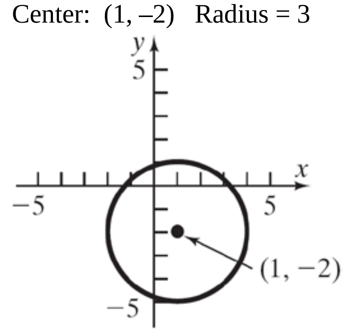
x-intercepts: $x^2 + \left(0-1\right)^2 = 4$
$$x^2 + 1 = 4$$
$$x^2 = 3$$
$$x = \pm\sqrt{3}$$

y-intercepts: $0^2 + \left(y-1\right)^2 = 4$
$$(y-1)^2 = 4$$
$$y - 1 = \pm 2$$
$$y = 1 \pm 2$$
$$y = 3 \text{ or } y = -1$$

The intercepts are $\left(-\sqrt{3}, 0\right)$, $\left(\sqrt{3}, 0\right)$, $(0, -1)$, and $(0, 3)$.

14. $x^2 + y^2 - 2x + 4y - 4 = 0$
$$x^2 - 2x + y^2 + 4y = 4$$
$$\left(x^2 - 2x + 1\right) + \left(y^2 + 4y + 4\right) = 4 + 1 + 4$$
$$\left(x-1\right)^2 + \left(y+2\right)^2 = 3^2$$

Center: (1, –2) Radius = 3



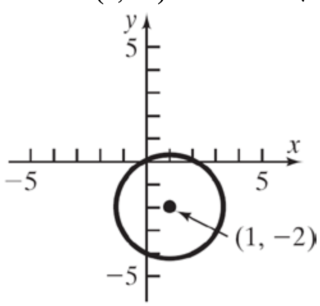
x-intercepts: $\left(x-1\right)^2 + \left(0+2\right)^2 = 3^2$
$$\left(x-1\right)^2 + 4 = 9$$
$$\left(x-1\right)^2 = 5$$
$$x - 1 = \pm\sqrt{5}$$
$$x = 1 \pm \sqrt{5}$$

y-intercepts: $\left(0-1\right)^2 + \left(y+2\right)^2 = 3^2$
$$1 + \left(y+2\right)^2 = 9$$
$$\left(y+2\right)^2 = 8$$
$$y + 2 = \pm\sqrt{8}$$
$$y + 2 = \pm 2\sqrt{2}$$
$$y = -2 \pm 2\sqrt{2}$$

The intercepts are $\left(1-\sqrt{5}, 0\right)$, $\left(1+\sqrt{5}, 0\right)$, $\left(0, -2-2\sqrt{2}\right)$, and $\left(0, -2+2\sqrt{2}\right)$.

15.
$$3x^2 + 3y^2 - 6x + 12y = 0$$
$$x^2 + y^2 - 2x + 4y = 0$$
$$x^2 - 2x + y^2 + 4y = 0$$
$$\left(x^2 - 2x + 1\right) + \left(y^2 + 4y + 4\right) = 1 + 4$$
$$(x-1)^2 + (y+2)^2 = \left(\sqrt{5}\right)^2$$

Center: (1, –2) Radius = $\sqrt{5}$



x-intercepts: $(x-1)^2 + (0+2)^2 = \left(\sqrt{5}\right)^2$
$$(x-1)^2 + 4 = 5$$
$$(x-1)^2 = 1$$
$$x - 1 = \pm 1$$
$$x = 1 \pm 1$$
$$x = 2 \quad \text{or} \quad x = 0$$

y-intercepts: $(0-1)^2 + (y+2)^2 = \left(\sqrt{5}\right)^2$
$$1 + (y+2)^2 = 5$$
$$(y+2)^2 = 4$$
$$y + 2 = \pm 2$$
$$y = -2 \pm 2$$
$$y = 0 \quad \text{or} \quad y = -4$$

The intercepts are $(0, 0)$, $(2, 0)$, and $(0, -4)$.

16. Slope = –2; containing (3,–1)
$$y - y_1 = m(x - x_1)$$
$$y - (-1) = -2(x - 3)$$
$$y + 1 = -2x + 6$$
$$y = -2x + 5 \quad \text{or} \quad 2x + y = 5$$

17. Horizontal; containing the point (1, –4)
Vertical lines have equations of the form $y = a$, where a is the y-intercept. Now, a vertical line containing the point (1, –4) must have a y-intercept of –4, so the equation of the line is $y = -4$. The line does not have a slope-intercept form.

18. y-intercept = –2; containing (5,–3)
Points are (5,–3) and (0,–2)
$$m = \frac{-2-(-3)}{0-5} = \frac{1}{-5} = -\frac{1}{5}$$
$$y = mx + b$$
$$y = -\frac{1}{5}x - 2 \quad \text{or} \quad x + 5y = -10$$

19. Containing the points (2,–3) and (4, 1)
$$m = \frac{1-(-3)}{(4-2)} = \frac{4}{2} = 2$$
$$y - y_1 = m(x - x_1)$$
$$y - (-3) = 2(x - 2)$$
$$y + 3 = 2x - 4$$
$$y - 2x + 7 = 0$$

20. Parallel to $2x - 3y = -4$
$$2x - 3y = -4$$
$$-3y = -2x - 4$$
$$\frac{-3y}{-3} = \frac{-2x-4}{-3}$$
$$y = \frac{2}{3}x + \frac{4}{3}$$

Slope = $\frac{2}{3}$; containing (–5,3)
$$y - y_1 = m(x - x_1)$$
$$y - 3 = \frac{2}{3}(x - (-5))$$
$$y - 3 = \frac{2}{3}(x + 5)$$
$$y - 3 = \frac{2}{3}x + \frac{10}{3}$$
$$y = \frac{2}{3}x + \frac{19}{3} \quad \text{or} \quad 2x - 3y = -19$$

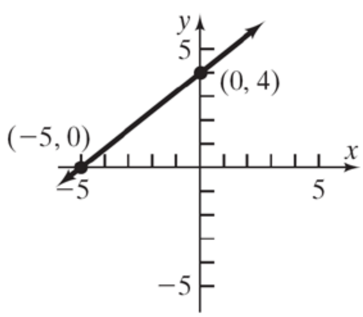
21. Perpendicular to $x - y = 3$
$$x - y = 3$$
$$y = x - 3$$
The slope of this line is 1, so the slope of a line perpendicular to this line would be –1.
Slope = –1; containing the point (-3,5)

$$y - y_1 = m(x - x_1)$$
$$y - 5 = (-1)(x - (-3))$$
$$x + y - 8 = 0$$

22. $4x - 5y = -20$
$$-5y = -4x - 20$$
$$y = \frac{4}{5}x + 4$$

slope $= \frac{4}{5}$; y-intercept $= 4$

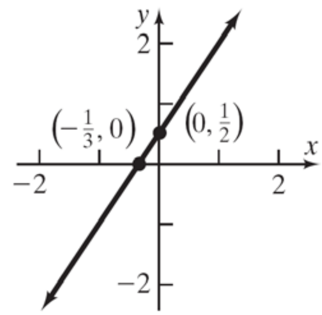
x-intercept: Let $y = 0$.
$$4x - 5(0) = -20$$
$$4x = -20$$
$$x = -5$$



23. $\frac{1}{2}x - \frac{1}{3}y = -\frac{1}{6}$
$$-\frac{1}{3}y = -\frac{1}{2}x - \frac{1}{6}$$
$$y = \frac{3}{2}x + \frac{1}{2}$$

slope $= \frac{3}{2}$; y-intercept $= \frac{1}{2}$

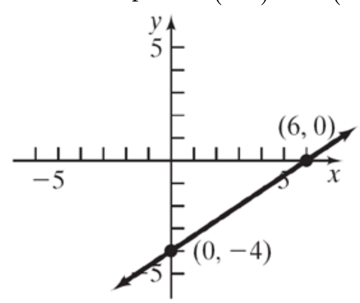
x-intercept: Let $y = 0$.
$$\frac{1}{2}x - \frac{1}{3}(0) = -\frac{1}{6}$$
$$\frac{1}{2}x = -\frac{1}{6}$$
$$x = -\frac{1}{3}$$



24. $2x - 3y = 12$

x-intercept:	y-intercept:
$2x - 3(0) = 12$	$2(0) - 3y = 12$
$2x = 12$	$-3y = 12$
$x = 6$	$y = -4$

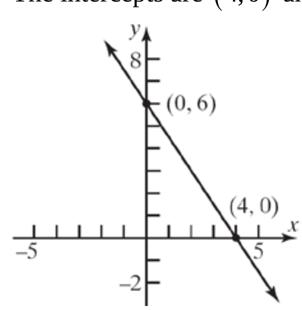
The intercepts are $(6, 0)$ and $(0, -4)$.



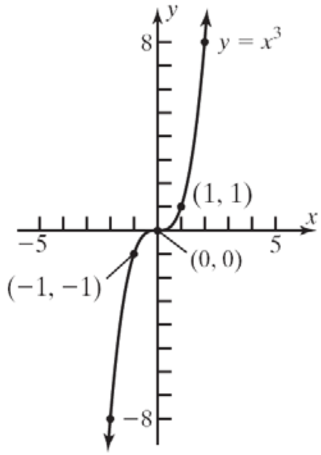
25. $\frac{1}{2}x + \frac{1}{3}y = 2$

x-intercept:	y-intercept:
$\frac{1}{2}x + \frac{1}{3}(0) = 2$	$\frac{1}{2}(0) + \frac{1}{3}y = 2$
$\frac{1}{2}x = 2$	$\frac{1}{3}y = 2$
$x = 4$	$y = 6$

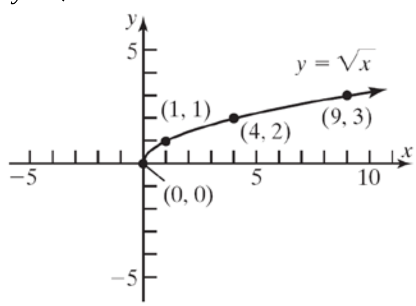
The intercepts are $(4, 0)$ and $(0, 6)$.



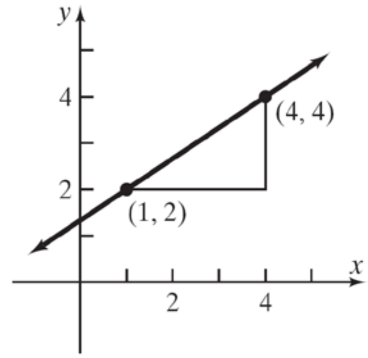
26. $y = x^3$



27. $y = \sqrt{x}$



28. slope = $\frac{2}{3}$, containing the point (1,2)



29. Find the distance between each pair of points.

$$d_{A,B} = \sqrt{(5-2)^2 + (6-2)^2} = \sqrt{9+16} = 5$$

$$d_{B,C} = \sqrt{(5-(-1))^2 + (6-(-2))^2} = \sqrt{36+64} = 10$$

$$d_{A,C} = \sqrt{(2-(-2))^2 + (2-(-1))^2} = \sqrt{16+9} = 5$$

Since *AB* = *BC*, triangle *ABC* is isosceles.

30. Given the points $A = (1, 2)$, $B = (1, 6)$, and $C = (5, 2)$.

a. Find the distance between each pair of points.

$$d(A,B) = \sqrt{(6-2)^2 + (1-1)^2} = \sqrt{16+0} = 4$$

$$d(B,C) = \sqrt{(6-2)^2 + (5-1)^2} = \sqrt{16+16} = 4\sqrt{2}$$

$$d(A,C) = \sqrt{(5-1)^2 + (2-2)^2} = \sqrt{16+0} = 4$$

$$\left[d(A,B)\right]^2 + \left[d(A,C)\right]^2 = \left[d(B,C)\right]^2$$

$$4^2 + 4^2 = \left(4\sqrt{2}\right)^2$$

$$16 + 16 = 32$$

$$32 = 32$$

The Pythagorean Theorem is satisfied, so this is a right triangle.

b. Find the slopes:

$$m_{AB} = \frac{6-2}{1-1} = \frac{4}{0} = \infty$$

$$m_{BC} = \frac{6-2}{5-1} = \frac{4}{4} = 1$$

$$m_{AC} = \frac{2-2}{5-1} = \frac{0}{4} = 0$$

Since *AB* is a line parallel to the *y*-axis and *AC* is a line parallel to the *x*-axis, the sides *AB* and *AC* are perpendicular and the triangle is a right triangle.

31. Endpoints of the diameter are (–5, 4) and (5, 10). The center is at the midpoint of the diameter:

Center: $\left(\frac{-5+5}{2}, \frac{4+10}{2}\right) = (0, 7)$

Radius: $r = \sqrt{(0-5)^2 + (7-10)^2}$

$= \sqrt{25+9} = \sqrt{34}$

Equation: $(x-0)^2 + (y-7)^2 = \left(\sqrt{34}\right)^2$

$x^2 + (y-7)^2 = 34$

32. slope of $\overline{AB} = \dfrac{7-3}{1-2} = -4$

slope of $\overline{AC} = \dfrac{3-(-1)}{2-3} = -4$

Therefore, the points lie on a line.

Chapter 1 Test

1. $d(P_1, P_2) = \sqrt{(5-(-1))^2 + (-1-3)^2}$

$= \sqrt{6^2 + (-4)^2}$

$= \sqrt{36+16}$

$= \sqrt{52} = 2\sqrt{13}$

2. The coordinates of the midpoint are:

$(x, y) = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

$= \left(\dfrac{-1+5}{2}, \dfrac{3+(-1)}{2}\right)$

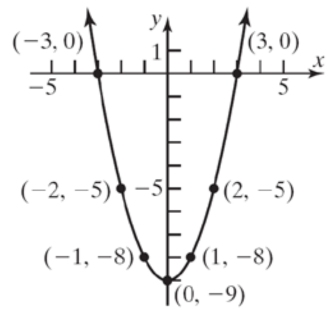
$= \left(\dfrac{4}{2}, \dfrac{2}{2}\right)$

$= (2, 1)$

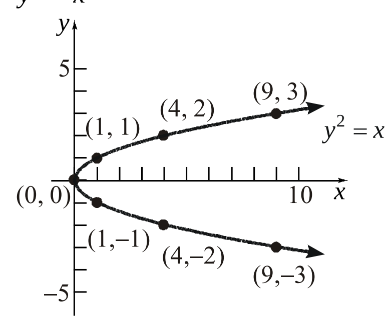
3. **a.** $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-1-3}{5-(-1)} = \dfrac{-4}{6} = -\dfrac{2}{3}$

b. If x increases by 3 units, y will decrease by 2 units.

4. $y = x^2 - 9$



5. $y^2 = x$



6. $x^2 + y = 9$

x-intercepts: $x^2 + 0 = 9$; $x^2 = 9$; $x = \pm 3$

y-intercept: $(0)^2 + y = 9$; $y = 9$

The intercepts are $(-3,0)$, $(3,0)$, and $(0,9)$.

Test x-axis symmetry: Let $y = -y$

$x^2 + (-y) = 9$

$x^2 - y = 9$ different

Test y-axis symmetry: Let $x = -x$

$(-x)^2 + y = 9$

$x^2 + y = 9$ same

Test origin symmetry: Let $x = -x$ and $y = -y$

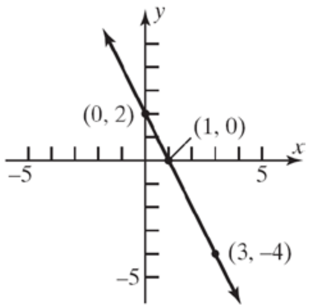
$(-x)^2 + (-y) = 9$

$x^2 - y = 9$ different

Therefore, the graph will have y-axis symmetry.

7. Slope = -2; containing $(3,-4)$

$$y-y_1=m(x-x_1)$$
$$y-(-4)=-2(x-3)$$
$$y+4=-2x+6$$
$$y=-2x+2$$



8. $(x-h)^2+(y-k)^2=r^2$

$$\left(x-4\right)^2+\left(y-(-3)\right)^2=5^2$$
$$\left(x-4\right)^2+\left(y+3\right)^2=25$$

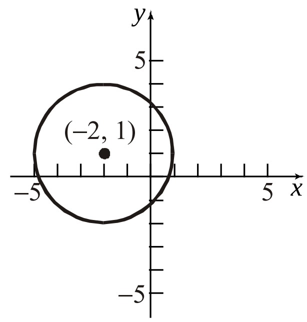
General form: $\left(x-4\right)^2+\left(y+3\right)^2=25$

$$x^2-8x+16+y^2+6y+9=25$$
$$x^2+y^2-8x+6y=0$$

9. $x^2+y^2+4x-2y-4=0$

$$x^2+4x+y^2-2y=4$$
$$(x^2+4x+4)+(y^2-2y+1)=4+4+1$$
$$(x+2)^2+(y-1)^2=3^2$$

Center: (–2, 1); Radius = 3



10. $2x+3y=6$

$$3y=-2x+6$$
$$y=-\frac{2}{3}x+2$$

<u>Parallel line</u>
Any line parallel to $2x+3y=6$ has slope $m=-\frac{2}{3}$. The line contains $(1,-1)$:

$$y-y_1=m(x-x_1)$$
$$y-(-1)=-\frac{2}{3}(x-1)$$
$$y+1=-\frac{2}{3}x+\frac{2}{3}$$
$$y=-\frac{2}{3}x-\frac{1}{3}$$

<u>Perpendicular line</u>
Any line perpendicular to $2x+3y=6$ has slope $m=\frac{3}{2}$. The line contains $(0,3)$:

$$y-y_1=m(x-x_1)$$
$$y-3=\frac{3}{2}(x-0)$$
$$y-3=\frac{3}{2}x$$
$$y=\frac{3}{2}x+3$$

Chapter 1 Project

Internet Based Project

Chapter 2

Functions and Their Graphs

Section 2.1

1. $(-1,3)$

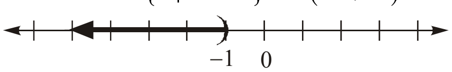
2. $3(-2)^2-5(-2)+\dfrac{1}{(-2)}=3(4)-5(-2)-\dfrac{1}{2}$

$$=12+10-\frac{1}{2}$$

$$=\frac{43}{2} \text{ or } 21\tfrac{1}{2} \text{ or } 21.5$$

3. We must not allow the denominator to be 0.
$x+4\neq 0 \Rightarrow x\neq -4$; Domain: $\{x \mid x\neq -4\}$.

4. $3-2x>5$
$-2x>2$
$x<-1$
Solution set: $\{x \mid x<-1\}$ or $(-\infty,-1)$

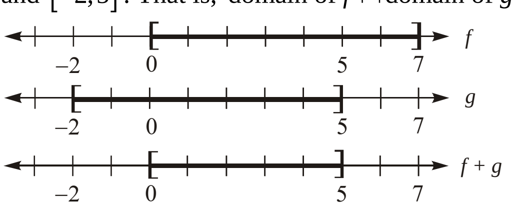


5. $\sqrt{5}+2$

6. radicals

7. independent; dependent

8. $[0,5]$
We need the intersection of the intervals $[0,7]$ and $[-2,5]$. That is, domain of $f \cap$ domain of g.



9. $\neq$; f; g

10. $(g-f)(x)$ or $g(x)-f(x)$

11. False; every function is a relation, but not every relation is a function. For example, the relation $x^2+y^2=1$ is not a function.

12. True

13. False; if the domain is not specified, we assume it is the largest set of real numbers for which the value of f is a real number.

14. False; the domain of $f(x)=\dfrac{x^2-4}{x}$ is $\{x \mid x\neq 0\}$.

15. a

16. c

17. d

18. a

19. Function
Domain: {Elvis, Colleen, Kaleigh, Marissa}
Range: {Jan. 8, Mar. 15, Sept. 17}

20. Not a function

21. Function
Domain: {Less than 9th grade, 9th-12th grade, High School Graduate, Some College, College Graduate}
Range: {\$18,120, \$23,251, \$36,055, \$45,810, \$67,165}

22. Not a function

23. Not a function

24. Function
Domain: {–2, –1, 3, 4}
Range: {3, 5, 7, 12}

25. Function
Domain: {0, 1, 2, 3}
Range: {–2, 3, 7}

26. Function
Domain: {1, 2, 3, 4}
Range: {3}

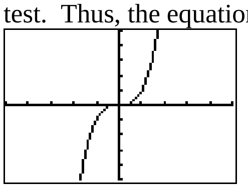
27. Not a function

28. Not a function

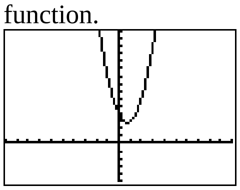
29. Function
Domain: {–2, –1, 0, 1}
Range: {3, 4, 16}

30. Function
Domain: {–2, –1, 0, 1}
Range: {0, 1, 4}

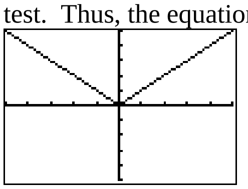
31. Graph $y = x^3$. The graph passes the vertical line test. Thus, the equation represents a function.



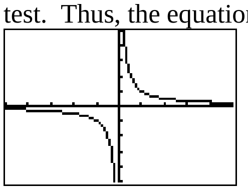
32. Graph $y = 2x^2 - 3x + 4$. The graph passes the vertical line test. Thus, the equation represents a function.



33. Graph $y = |x|$. The graph passes the vertical line test. Thus, the equation represents a function.



34. Graph $y = \frac{1}{x}$. The graph passes the vertical line test. Thus, the equation represents a function.



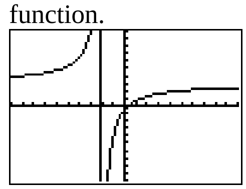
35. $y = \pm\sqrt{1-2x}$
For $x = 0$, $y = \pm 1$. Thus, (0, 1) and (0, –1) are on the graph. This is not a function, since a distinct *x*-value corresponds to two different *y*-values.

36. $y^2 = 4 - x^2$
Solve for y: $y = \pm\sqrt{4-x^2}$
For $x = 0$, $y = \pm 2$. Thus, (0, 2) and (0, –2) are on the graph. This is not a function, since a distinct *x*-value corresponds to two different *y*-values.

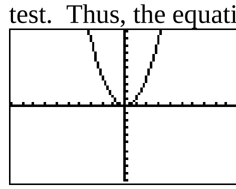
37. $x = y^2$
Solve for y: $y = \pm\sqrt{x}$
For $x = 1$, $y = \pm 1$. Thus, (1, 1) and (1, –1) are on the graph. This is not a function, since a distinct *x*-value corresponds to two different *y*-values.

38. $x + y^2 = 1$
Solve for y: $y = \pm\sqrt{1-x}$
For $x = 0$, $y = \pm 1$. Thus, (0, 1) and (0, –1) are on the graph. This is not a function, since a distinct *x*-value corresponds to two different *y*-values.

39. Graph $y = \frac{3x-1}{x+2}$. The graph passes the vertical line test. Thus, the equation represents a function.



40. Graph $y = x^2$. The graph passes the vertical line test. Thus, the equation represents a function.



41. $x^2 - 4y^2 = 1$
Solve for *y*:
$$\begin{aligned} x^2 - 4y^2 &= 1 \\ 4y^2 &= x^2 - 1 \\ y^2 &= \frac{x^2-1}{4} \\ y &= \frac{\pm\sqrt{x^2-1}}{2} \end{aligned}$$
For $x = \sqrt{2}$, $y = \pm\frac{1}{2}$. Thus, $\left(\sqrt{2}, \frac{1}{2}\right)$ and $\left(\sqrt{2}, -\frac{1}{2}\right)$ are on the graph. This is not a function, since a distinct *x*-value corresponds to two different *y*-values.

42. $2x^2+3y^2=1$

Solve for y: $2x^2+3y^2=1$

$$3y^2=1-2x^2$$
$$y^2=\frac{1-2x^2}{3}$$
$$y=\pm\sqrt{\frac{1-2x^2}{3}}$$

For $x=0$, $y=\pm\sqrt{\frac{1}{3}}$. Thus, $\left(0,\sqrt{\frac{1}{3}}\right)$ and $\left(0,-\sqrt{\frac{1}{3}}\right)$ are on the graph. This is not a function, since a distinct x-value corresponds to two different y-values.

43. $f(x)=3x^2+2x-4$

a. $f(0)=3(0)^2+2(0)-4=-4$

b. $f(1)=3(1)^2+2(1)-4=3+2-4=1$

c. $f(-1)=3(-1)^2+2(-1)-4=3-2-4=-3$

d. $f(-x)=3(-x)^2+2(-x)-4=3x^2-2x-4$

e. $-f(x)=-\left(3x^2+2x-4\right)=-3x^2-2x+4$

f. $f(x+1)=3(x+1)^2+2(x+1)-4$
$$=3\left(x^2+2x+1\right)+2x+2-4$$
$$=3x^2+6x+3+2x+2-4$$
$$=3x^2+8x+1$$

g. $f(2x)=3(2x)^2+2(2x)-4=12x^2+4x-4$

h. $f(x+h)=3(x+h)^2+2(x+h)-4$
$$=3\left(x^2+2xh+h^2\right)+2x+2h-4$$
$$=3x^2+6xh+3h^2+2x+2h-4$$

44. $f(x)=-2x^2+x-1$

a. $f(0)=-2(0)^2+0-1=-1$

b. $f(1)=-2(1)^2+1-1=-2$

c. $f(-1)=-2(-1)^2+(-1)-1=-4$

d. $f(-x)=-2(-x)^2+(-x)-1=-2x^2-x-1$

e. $-f(x)=-\left(-2x^2+x-1\right)=2x^2-x+1$

f. $f(x+1)=-2(x+1)^2+(x+1)-1$
$$=-2\left(x^2+2x+1\right)+x+1-1$$
$$=-2x^2-4x-2+x$$
$$=-2x^2-3x-2$$

g. $f(2x)=-2(2x)^2+(2x)-1=-8x^2+2x-1$

h. $f(x+h)=-2(x+h)^2+(x+h)-1$
$$=-2\left(x^2+2xh+h^2\right)+x+h-1$$
$$=-2x^2-4xh-2h^2+x+h-1$$

45. $f(x)=\dfrac{x^2-1}{x+4}$

a. $f(0)=\dfrac{0^2-1}{0+4}=\dfrac{-1}{4}=-\dfrac{1}{4}$

b. $f(1)=\dfrac{1^2-1}{1+4}=\dfrac{0}{5}=0$

c. $f(-1)=\dfrac{(-1)^2-1}{-1+4}=\dfrac{0}{3}=0$

d. $f(-x)=\dfrac{(-x)^2-1}{-x+4}=\dfrac{x^2-1}{-x+4}$

e. $-f(x)=-\left(\dfrac{x^2-1}{x+4}\right)=\dfrac{-x^2+1}{x+4}$

f. $f(x+1)=\dfrac{(x+1)^2-1}{(x+1)+4}$
$$=\frac{x^2+2x+1-1}{x+5}=\frac{x^2+2x}{x+5}$$

g. $f(2x)=\dfrac{(2x)^2-1}{2x+4}=\dfrac{4x^2-1}{2x+4}$

h. $f(x+h)=\dfrac{(x+h)^2-1}{(x+h)+4}=\dfrac{x^2+2xh+h^2-1}{x+h+4}$

46. $f(x)=\dfrac{x}{x^2+1}$

a. $f(0)=\dfrac{0}{0^2+1}=\dfrac{0}{1}=0$

b. $f(1)=\dfrac{1}{1^2+1}=\dfrac{1}{2}$

c. $f(-1)=\dfrac{-1}{(-1)^2+1}=\dfrac{-1}{1+1}=-\dfrac{1}{2}$

d. $f(-x)=\dfrac{-x}{(-x)^2+1}=\dfrac{-x}{x^2+1}$

e. $-f(x)=-\left(\dfrac{x}{x^2+1}\right)=\dfrac{-x}{x^2+1}$

f. $$f(x+1)=\frac{x+1}{(x+1)^2+1}=\frac{x+1}{x^2+2x+1+1}=\frac{x+1}{x^2+2x+2}$$

g. $f(2x)=\dfrac{2x}{(2x)^2+1}=\dfrac{2x}{4x^2+1}$

h. $f(x+h)=\dfrac{x+h}{(x+h)^2+1}=\dfrac{x+h}{x^2+2xh+h^2+1}$

47. $f(x)=\sqrt{x^2+x}$

a. $f(0)=\sqrt{0^2+0}=\sqrt{0}=0$

b. $f(1)=\sqrt{1^2+1}=\sqrt{2}$

c. $f(-1)=\sqrt{(-1)^2+(-1)}=\sqrt{1-1}=\sqrt{0}=0$

d. $f(-x)=\sqrt{(-x)^2+(-x)}=\sqrt{x^2-x}$

e. $-f(x)=-\left(\sqrt{x^2+x}\right)=-\sqrt{x^2+x}$

f. $$f(x+1)=\sqrt{(x+1)^2+(x+1)}=\sqrt{x^2+2x+1+x+1}=\sqrt{x^2+3x+2}$$

g. $f(2x)=\sqrt{(2x)^2+2x}=\sqrt{4x^2+2x}$

h. $$f(x+h)=\sqrt{(x+h)^2+(x+h)}=\sqrt{x^2+2xh+h^2+x+h}$$

48. $f(x)=|x|+4$

a. $f(0)=|0|+4=0+4=4$

b. $f(1)=|1|+4=1+4=5$

c. $f(-1)=|-1|+4=1+4=5$

d. $f(-x)=|-x|+4=|x|+4$

e. $-f(x)=-(|x|+4)=-|x|-4$

f. $f(x+1)=|x+1|+4$

g. $f(2x)=|2x|+4=2|x|+4$

h. $f(x+h)=|x+h|+4$

49. $f(x)=1-\dfrac{1}{(x+2)^2}$

a. $f(0)=1-\dfrac{1}{(0+2)^2}=1-\dfrac{1}{4}=\dfrac{3}{4}$

b. $f(1)=1-\dfrac{1}{(1+2)^2}=1-\dfrac{1}{9}=\dfrac{8}{9}$

c. $f(-1)=1-\dfrac{1}{(-1+2)^2}=1-\dfrac{1}{1}=0$

d. $f(-x)=1-\dfrac{1}{(-x+2)^2}$

e. $-f(x)=-\left(1-\dfrac{1}{(x+2)^2}\right)=\dfrac{1}{(x+2)^2}-1$

f. $f(x+1)=1-\dfrac{1}{(x+1+2)^2}=1-\dfrac{1}{(x+3)^2}$

g. $f(2x)=1-\dfrac{1}{(2x+2)^2}=1-\dfrac{1}{4(x+1)^2}$

h. $f(x+h)=1-\dfrac{1}{(x+h+2)^2}$

50. $f(x)=\dfrac{2x+1}{3x-5}$

a. $f(0)=\dfrac{2(0)+1}{3(0)-5}=\dfrac{0+1}{0-5}=-\dfrac{1}{5}$

b. $f(1)=\dfrac{2(1)+1}{3(1)-5}=\dfrac{2+1}{3-5}=\dfrac{3}{-2}=-\dfrac{3}{2}$

c. $f(-1)=\dfrac{2(-1)+1}{3(-1)-5}=\dfrac{-2+1}{-3-5}=\dfrac{-1}{-8}=\dfrac{1}{8}$

d. $f(-x)=\dfrac{2(-x)+1}{3(-x)-5}=\dfrac{-2x+1}{-3x-5}=\dfrac{2x-1}{3x+5}$

e. $-f(x)=-\left(\dfrac{2x+1}{3x-5}\right)=\dfrac{-2x-1}{3x-5}$

f. $f(x+1)=\dfrac{2(x+1)+1}{3(x+1)-5}=\dfrac{2x+2+1}{3x+3-5}=\dfrac{2x+3}{3x-2}$

g. $f(2x)=\dfrac{2(2x)+1}{3(2x)-5}=\dfrac{4x+1}{6x-5}$

h. $f(x+h)=\dfrac{2(x+h)+1}{3(x+h)-5}=\dfrac{2x+2h+1}{3x+3h-5}$

51. $f(x)=x^2+2$

Domain: $\{x \mid x \text{ is any real number}\}$

52. $f(x)=-5x+4$

Domain: $\{x \mid x \text{ is any real number}\}$

53. $f(x)=\dfrac{x^2}{x^2+1}$

Domain: $\{x \mid x \text{ is any real number}\}$

54. $f(x)=\dfrac{x}{x^2+1}$

Domain: $\{x \mid x \text{ is any real number}\}$

55. $g(x)=\dfrac{x}{x^2-16}$

$x^2-16\neq 0$

$x^2\neq 16 \Rightarrow x\neq \pm 4$

Domain: $\{x \mid x\neq -4,\ x\neq 4\}$

56. $h(x)=\dfrac{2x}{x^2-4}$

$x^2-4\neq 0$

$x^2\neq 4 \Rightarrow x\neq \pm 2$

Domain: $\{x \mid x\neq -2,\ x\neq 2\}$

57. $G(x)=\dfrac{x+4}{x^3-4x}$

$x^3-4x\neq 0$

$x(x^2-4)\neq 0$

$x\neq 0,\quad x^2\neq 4$

$x\neq 0,\quad x\neq \pm 2$

Domain: $\{x \mid x\neq -2,\ x\neq 0,\ x\neq 2\}$

58. $F(x)=\dfrac{x-2}{x^3+x}$

$x^3+x\neq 0$

$x(x^2+1)\neq 0$

$x\neq 0,\quad x^2\neq -1$

Domain: $\{x \mid x\neq 0\}$

59. $G(x)=\sqrt{1-x}$

$1-x\geq 0$

$-x\geq -1$

$x\leq 1$

Domain: $\{x \mid x\leq 1\}$

60. $h(x)=\sqrt{3x-12}$

$3x-12\geq 0$

$3x\geq 12$

$x\geq 4$

Domain: $\{x \mid x\geq 4\}$

61. $f(x)=\dfrac{4}{\sqrt{x-9}}$

$x-9>0$

$x>9$

Domain: $\{x \mid x>9\}$

62. $p(x)=\sqrt{\dfrac{2}{x-1}}=\dfrac{\sqrt{2}}{\sqrt{x-1}}$

$x-1>0$

$x>1$

Domain: $\{x \mid x>1\}$

63. $q(x)=\frac{-x}{\sqrt{-x-2}}$

$-x-2>0$

$-x>2$

$x<-2$

Domain: $\{x \mid x<-2\}$

64. $f(x)=\dfrac{x}{\sqrt{x-4}}$

$x-4>0$

$x>4$

Domain: $\{x \mid x>4\}$

65. $h(z)=\dfrac{\sqrt{z+3}}{z-2}$

$z+3\ge 0$

$z\ge -3$

Also $z-2\ne 0$

$z\ne 2$

Domain: $\{z \mid z\ge -3, z\ne 2\}$

66. $P(t)=\dfrac{\sqrt{t-4}}{3t-21}$

$t-4\ge 0$

$t\ge 4$

Also $3t-21\ne 0$

$3t-21\ne 0$

$3t\ne 21$

$t\ne 7$

Domain: $\{t \mid t\ge 4, t\ne 7\}$

67. $f(x)=3x+4 \qquad g(x)=2x-3$

a. $(f+g)(x)=3x+4+2x-3=5x+1$

Domain: $\{x \mid x \text{ is any real number}\}$.

b. $(f-g)(x)=(3x+4)-(2x-3)$

$=3x+4-2x+3$

$=x+7$

Domain: $\{x \mid x \text{ is any real number}\}$.

c. $(f\cdot g)(x)=(3x+4)(2x-3)$

$=6x^2-9x+8x-12$

$=6x^2-x-12$

Domain: $\{x \mid x \text{ is any real number}\}$.

d. $\left(\dfrac{f}{g}\right)(x)=\dfrac{3x+4}{2x-3}$

$2x-3\ne 0 \Rightarrow 2x\ne 3 \Rightarrow x\ne \dfrac{3}{2}$

Domain: $\left\{x \,\middle|\, x\ne \dfrac{3}{2}\right\}$.

e. $(f+g)(3)=5(3)+1=15+1=16$

f. $(f-g)(4)=4+7=11$

g. $(f\cdot g)(2)=6(2)^2-2-12=24-2-12=10$

h. $\left(\dfrac{f}{g}\right)(1)=\dfrac{3(1)+4}{2(1)-3}=\dfrac{3+4}{2-3}=\dfrac{7}{-1}=-7$

68. $f(x)=2x+1 \qquad g(x)=3x-2$

a. $(f+g)(x)=2x+1+3x-2=5x-1$

Domain: $\{x \mid x \text{ is any real number}\}$.

b. $(f-g)(x)=(2x+1)-(3x-2)$

$=2x+1-3x+2$

$=-x+3$

Domain: $\{x \mid x \text{ is any real number}\}$.

c. $(f\cdot g)(x)=(2x+1)(3x-2)$

$=6x^2-4x+3x-2$

$=6x^2-x-2$

Domain: $\{x \mid x \text{ is any real number}\}$.

d. $\left(\dfrac{f}{g}\right)(x)=\dfrac{2x+1}{3x-2}$

$3x-2\ne 0$

$3x\ne 2 \Rightarrow x\ne \dfrac{2}{3}$

Domain: $\left\{x \,\middle|\, x\ne \dfrac{2}{3}\right\}$.

e. $(f+g)(3)=5(3)-1=15-1=14$

f. $(f-g)(4)=-4+3=-1$

g. $(f \cdot g)(2) = 6(2)^2 - 2 - 2$
$= 6(4) - 2 - 2$
$= 24 - 2 - 2 = 20$

h. $\left(\dfrac{f}{g}\right)(1) = \dfrac{2(1)+1}{3(1)-2} = \dfrac{2+1}{3-2} = \dfrac{3}{1} = 3$

69. $f(x) = 2x^2 + 3 \qquad g(x) = 4x^3 + 1$

a. $(f+g)(x) = 2x^2 + 3 + 4x^3 + 1$
$= 4x^3 + 2x^2 + 4$
Domain: $\{x \mid x \text{ is any real number}\}$.

b. $(f-g)(x) = \left(2x^2+3\right) - \left(4x^3+1\right)$
$= 2x^2 + 3 - 4x^3 - 1$
$= -4x^3 + 2x^2 + 2$
Domain: $\{x \mid x \text{ is any real number}\}$.

c. $(f \cdot g)(x) = \left(2x^2+3\right)\left(4x^3+1\right)$
$= 8x^5 + 12x^3 + 2x^2 + 3$
Domain: $\{x \mid x \text{ is any real number}\}$.

d. $\left(\dfrac{f}{g}\right)(x) = \dfrac{2x^2+3}{4x^3+1}$
$4x^3 + 1 \neq 0$
$4x^3 \neq -1$
$x^3 \neq -\dfrac{1}{4} \Rightarrow x \neq \sqrt[3]{-\dfrac{1}{4}} = -\dfrac{\sqrt[3]{2}}{2}$
Domain: $\left\{x \,\middle|\, x \neq -\dfrac{\sqrt[3]{2}}{2}\right\}$.

e. $(f+g)(3) = 4(3)^3 + 2(3)^2 + 4$
$= 4(27) + 2(9) + 4$
$= 108 + 18 + 4 = 130$

f. $(f-g)(4) = -4(4)^3 + 2(4)^2 + 2$
$= -4(64) + 2(16) + 2$
$= -256 + 32 + 2 = -222$

g. $(f \cdot g)(2) = 8(2)^5 + 12(2)^3 + 2(2)^2 + 3$
$= 8(32) + 12(8) + 2(4) + 3$
$= 256 + 96 + 8 + 3 = 363$

h. $\left(\dfrac{f}{g}\right)(1) = \dfrac{2(1)^2+3}{4(1)^3+1} = \dfrac{2(1)+3}{4(1)+1} = \dfrac{2+3}{4+1} = \dfrac{5}{5} = 1$

70. $f(x) = x - 1 \qquad g(x) = 2x^2$

a. $(f+g)(x) = x - 1 + 2x^2 = 2x^2 + x - 1$
Domain: $\{x \mid x \text{ is any real number}\}$.

b. $(f-g)(x) = (x-1) - (2x^2)$
$= x - 1 - 2x^2$
$= -2x^2 + x - 1$
Domain: $\{x \mid x \text{ is any real number}\}$.

c. $(f \cdot g)(x) = (x-1)(2x^2) = 2x^3 - 2x^2$
Domain: $\{x \mid x \text{ is any real number}\}$.

d. $\left(\dfrac{f}{g}\right)(x) = \dfrac{x-1}{2x^2}$
Domain: $\{x \mid x \neq 0\}$.

e. $(f+g)(3) = 2(3)^2 + 3 - 1$
$= 2(9) + 3 - 1$
$= 18 + 3 - 1 = 20$

f. $(f-g)(4) = -2(4)^2 + 4 - 1$
$= -2(16) + 4 - 1$
$= -32 + 4 - 1 = -29$

g. $(f \cdot g)(2) = 2(2)^3 - 2(2)^2$
$= 2(8) - 2(4)$
$= 16 - 8 = 8$

h. $\left(\dfrac{f}{g}\right)(1) = \dfrac{1-1}{2(1)^2} = \dfrac{0}{2(1)} = \dfrac{0}{2} = 0$

71. $f(x) = |x| \qquad g(x) = x$

a. $(f+g)(x) = |x| + x$

Domain: $\{x \mid x \text{ is any real number}\}$.

b. $(f-g)(x) = |x| - x$

Domain: $\{x \mid x \text{ is any real number}\}$.

c. $(f \cdot g)(x) = |x| \cdot x = x|x|$

Domain: $\{x \mid x \text{ is any real number}\}$.

d. $\left(\frac{f}{g}\right)(x) = \frac{|x|}{x}$

Domain: $\{x \mid x \neq 0\}$.

e. $(f+g)(3) = |3| + 3 = 3 + 3 = 6$

f. $(f-g)(4) = |4| - 4 = 4 - 4 = 0$

g. $(f \cdot g)(2) = 2|2| = 2 \cdot 2 = 4$

h. $\left(\frac{f}{g}\right)(1) = \frac{|1|}{1} = \frac{1}{1} = 1$

72. $f(x) = \sqrt{x} \qquad g(x) = 3x - 5$

a. $(f+g)(x) = \sqrt{x} + 3x - 5$

Domain: $\{x \mid x \geq 0\}$.

b. $(f-g)(x) = \sqrt{x} - (3x-5) = \sqrt{x} - 3x + 5$

Domain: $\{x \mid x \geq 0\}$.

c. $(f \cdot g)(x) = \sqrt{x}(3x-5) = 3x\sqrt{x} - 5\sqrt{x}$

Domain: $\{x \mid x \geq 0\}$.

d. $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x}}{3x-5}$

$x \geq 0$ and $3x - 5 \neq 0$

$$3x \neq 5 \Rightarrow x \neq \frac{5}{3}$$

Domain: $\left\{x \,\middle|\, x \geq 0 \text{ and } x \neq \frac{5}{3}\right\}$.

e. $(f+g)(3) = \sqrt{3} + 3(3) - 5$

$= \sqrt{3} + 9 - 5 = \sqrt{3} + 4$

f. $(f-g)(4) = \sqrt{4} - 3(4) + 5$

$= 2 - 12 + 5 = -5$

g. $(f \cdot g)(2) = 3(2)\sqrt{2} - 5\sqrt{2}$

$= 6\sqrt{2} - 5\sqrt{2} = \sqrt{2}$

h. $\left(\frac{f}{g}\right)(1) = \frac{\sqrt{1}}{3(1)-5} = \frac{1}{3-5} = \frac{1}{-2} = -\frac{1}{2}$

73. $f(x) = \sqrt{x-1} \qquad g(x) = \sqrt{4-x}$

a. $(f+g)(x) = \sqrt{x-1} + \sqrt{4-x}$

$x - 1 \geq 0$ and $4 - x \geq 0$

$x \geq 1$ and $-x \geq -4$

$x \leq 4$

Domain: $\{x \mid 1 \leq x \leq 4\}$.

b. $(f-g)(x) = \sqrt{x-1} - \sqrt{4-x}$

$x - 1 \geq 0$ and $4 - x \geq 0$

$x \geq 1$ and $-x \geq -4$

$x \leq 4$

Domain: $\{x \mid 1 \leq x \leq 4\}$.

c. $(f \cdot g)(x) = \left(\sqrt{x-1}\right)\left(\sqrt{4-x}\right)$

$= \sqrt{-x^2 + 5x - 4}$

$x - 1 \geq 0$ and $4 - x \geq 0$

$x \geq 1$ and $-x \geq -4$

$x \leq 4$

Domain: $\{x \mid 1 \leq x \leq 4\}$.

d. $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x-1}}{\sqrt{4-x}} = \sqrt{\frac{x-1}{4-x}}$

$x - 1 \geq 0$ and $4 - x > 0$

$x \geq 1$ and $-x > -4$

$x < 4$

Domain: $\{x \mid 1 \leq x < 4\}$.

e. $(f+g)(3) = \sqrt{3-1} + \sqrt{4-3}$

$= \sqrt{2} + \sqrt{1} = \sqrt{2} + 1$

f. $(f-g)(4) = \sqrt{4-1} - \sqrt{4-4}$

$= \sqrt{3} - \sqrt{0} = \sqrt{3} - 0 = \sqrt{3}$

g. $(f \cdot g)(2) = \sqrt{-(2)^2 + 5(2) - 4}$

$= \sqrt{-4 + 10 - 4} = \sqrt{2}$

h. $\left(\frac{f}{g}\right)(1) = \sqrt{\frac{1-1}{4-1}} = \sqrt{\frac{0}{3}} = \sqrt{0} = 0$

74. $f(x) = 1 + \frac{1}{x} \qquad g(x) = \frac{1}{x}$

a. $(f+g)(x) = 1 + \frac{1}{x} + \frac{1}{x} = 1 + \frac{2}{x}$

Domain: $\{x \mid x \neq 0\}$.

b. $(f-g)(x) = 1 + \frac{1}{x} - \frac{1}{x} = 1$

Domain: $\{x \mid x \neq 0\}$.

c. $(f \cdot g)(x) = \left(1 + \frac{1}{x}\right)\frac{1}{x} = \frac{1}{x} + \frac{1}{x^2}$

Domain: $\{x \mid x \neq 0\}$.

d. $\left(\frac{f}{g}\right)(x) = \frac{1 + \frac{1}{x}}{\frac{1}{x}} = \frac{\frac{x+1}{x}}{\frac{1}{x}} = \frac{x+1}{x} \cdot \frac{x}{1} = x + 1$

Domain: $\{x \mid x \neq 0\}$.

e. $(f+g)(3) = 1 + \frac{2}{3} = \frac{5}{3}$

f. $(f-g)(4) = 1$

g. $(f \cdot g)(2) = \frac{1}{2} + \frac{1}{(2)^2} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

h. $\left(\frac{f}{g}\right)(1) = 1 + 1 = 2$

75. $f(x) = \sqrt{x+1} \qquad g(x) = \frac{2}{x}$

a. $(f+g)(x) = \sqrt{x+1} + \frac{2}{x}$

$x + 1 \geq 0 \quad$ and $\quad x \neq 0$

$x \geq -1$

Domain: $\{x \mid x \geq -1, \text{ and } x \neq 0\}$.

b. $(f-g)(x) = \sqrt{x+1} - \frac{2}{x}$

$x + 1 \geq 0 \quad$ and $\quad x \neq 0$

$x \geq -1$

Domain: $\{x \mid x \geq -1, \text{ and } x \neq 0\}$.

c. $(f \cdot g)(x) = \sqrt{x+1} \cdot \frac{2}{x} = \frac{2\sqrt{x+1}}{x}$

$x + 1 \geq 0 \quad$ and $\quad x \neq 0$

$x \geq -1$

Domain: $\{x \mid x \geq -1, \text{ and } x \neq 0\}$.

d. $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x+1}}{\frac{2}{x}} = \frac{x\sqrt{x+1}}{2}$

$x + 1 \geq 0 \quad$ and $\quad x \neq 0$

$x \geq -1$

Domain: $\{x \mid x \geq -1, \text{ and } x \neq 0\}$.

e. $(f+g)(3) = \sqrt{3+1} + \frac{2}{3} = \sqrt{4} + \frac{2}{3} = 2 + \frac{2}{3} = \frac{8}{3}$

f. $(f-g)(4) = \sqrt{4+1} - \frac{2}{4} = \sqrt{5} - \frac{1}{2}$

g. $(f \cdot g)(2) = \frac{2\sqrt{2+1}}{2} = \frac{2\sqrt{3}}{2} = \sqrt{3}$

h. $\left(\frac{f}{g}\right)(1) = \frac{1\sqrt{1+1}}{2} = \frac{\sqrt{2}}{2}$

76. $f(x) = \frac{2x+3}{3x-2} \qquad g(x) = \frac{4x}{3x-2}$

a. $(f+g)(x) = \frac{2x+3}{3x-2} + \frac{4x}{3x-2}$

$= \frac{2x+3+4x}{3x-2} = \frac{6x+3}{3x-2}$

$3x - 2 \neq 0$

$3x \neq 2 \Rightarrow x \neq \frac{2}{3}$

Domain: $\left\{x \,\middle|\, x \neq \frac{2}{3}\right\}$.

b. $(f-g)(x) = \frac{2x+3}{3x-2} - \frac{4x}{3x-2}$

$= \frac{2x+3-4x}{3x-2} = \frac{-2x+3}{3x-2}$

$3x - 2 \neq 0$

$3x \neq 2 \Rightarrow x \neq \frac{2}{3}$

Domain: $\left\{x \,\middle|\, x \neq \frac{2}{3}\right\}$.

c. $(f \cdot g)(x) = \left(\dfrac{2x+3}{3x-2}\right)\left(\dfrac{4x}{3x-2}\right) = \dfrac{8x^2+12x}{(3x-2)^2}$

$3x-2 \neq 0$

$3x \neq 2 \Rightarrow x \neq \dfrac{2}{3}$

Domain: $\left\{x \,\middle|\, x \neq \dfrac{2}{3}\right\}$.

d. $\left(\dfrac{f}{g}\right)(x) = \dfrac{\frac{2x+3}{3x-2}}{\frac{4x}{3x-2}} = \dfrac{2x+3}{3x-2} \cdot \dfrac{3x-2}{4x} = \dfrac{2x+3}{4x}$

$3x-2 \neq 0$ and $x \neq 0$

$3x \neq 2$

$x \neq \dfrac{2}{3}$

Domain: $\left\{x \,\middle|\, x \neq \dfrac{2}{3} \text{ and } x \neq 0\right\}$.

e. $(f+g)(3) = \dfrac{6(3)+3}{3(3)-2} = \dfrac{18+3}{9-2} = \dfrac{21}{7} = 3$

f. $(f-g)(4) = \dfrac{-2(4)+3}{3(4)-2} = \dfrac{-8+3}{12-2} = \dfrac{-5}{10} = -\dfrac{1}{2}$

g. $(f \cdot g)(2) = \dfrac{8(2)^2+12(2)}{\left(3(2)-2\right)^2}$

$= \dfrac{8(4)+24}{\left(6-2\right)^2} = \dfrac{32+24}{\left(4\right)^2} = \dfrac{56}{16} = \dfrac{7}{2}$

h. $\left(\dfrac{f}{g}\right)(1) = \dfrac{2(1)+3}{4(1)} = \dfrac{2+3}{4} = \dfrac{5}{4}$

77. $f(x) = 3x+1 \qquad (f+g)(x) = 6 - \dfrac{1}{2}x$

$6 - \dfrac{1}{2}x = 3x+1+g(x)$

$5 - \dfrac{7}{2}x = g(x)$

$g(x) = 5 - \dfrac{7}{2}x$

78. $f(x) = \dfrac{1}{x} \qquad \left(\dfrac{f}{g}\right)(x) = \dfrac{x+1}{x^2-x}$

$\dfrac{x+1}{x^2-x} = \dfrac{\frac{1}{x}}{g(x)}$

$g(x) = \dfrac{\frac{1}{x}}{\frac{x+1}{x^2-x}} = \dfrac{1}{x} \cdot \dfrac{x^2-x}{x+1}$

$= \dfrac{1}{x} \cdot \dfrac{x(x-1)}{x+1} = \dfrac{x-1}{x+1}$

79. $f(x) = 4x+3$

$\dfrac{f(x+h)-f(x)}{h} = \dfrac{4(x+h)+3-(4x+3)}{h}$

$= \dfrac{4x+4h+3-4x-3}{h}$

$= \dfrac{4h}{h} = 4$

80. $f(x) = -3x+1$

$\dfrac{f(x+h)-f(x)}{h} = \dfrac{-3(x+h)+1-(-3x+1)}{h}$

$= \dfrac{-3x-3h+1+3x-1}{h}$

$= \dfrac{-3h}{h} = -3$

81. $f(x) = 3x^2+2$

$\dfrac{f(x+h)-f(x)}{h}$

$= \dfrac{3(x+h)^2+2-(3x^2+2)}{h}$

$= \dfrac{3x^2+6xh+3h^2+2-3x^2-2}{h}$

$= \dfrac{6xh+3h^2}{h}$

$= 6x+3h$

82. $f(x) = x^2 - 4$

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - 4 - (x^2 - 4)}{h} = \frac{x^2 + 2xh + h^2 - 4 - x^2 + 4}{h} = \frac{2xh + h^2}{h} = 2x + h$$

83. $f(x) = 3x^2 - 2x + 6$

$$\frac{f(x+h) - f(x)}{h} = \frac{\left[3(x+h)^2 - 2(x+h) + 6\right] - \left[3x^2 - 2x + 6\right]}{h}$$
$$= \frac{3\left(x^2 + 2xh + h^2\right) - 2x - 2h + 6 - 3x^2 + 2x - 6}{h}$$
$$= \frac{3x^2 + 6xh + 3h^2 - 2h - 3x^2}{h} = \frac{6xh + 3h^2 - 2h}{h}$$
$$= 6x + 3h - 2$$

84. $f(x) = x^2 - x + 4$

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - (x+h) + 4 - (x^2 - x + 4)}{h}$$
$$= \frac{x^2 + 2xh + h^2 - x - h + 4 - x^2 + x - 4}{h}$$
$$= \frac{2xh + h^2 - h}{h} = 2x + h - 1$$

85. $f(x) = \dfrac{1}{x+3}$

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{x+h+3} - \frac{1}{x+3}}{h} = \frac{\frac{x+3-(x+3+h)}{(x+h+3)(x+3)}}{h}$$
$$= \left(\frac{x+3-x-3-h}{(x+h+3)(x+3)}\right)\left(\frac{1}{h}\right) = \left(\frac{-h}{(x+h+3)(x+3)}\right)\left(\frac{1}{h}\right)$$
$$= \frac{-1}{(x+h+3)(x+3)}$$

86. $f(x) = \dfrac{1}{x^2}$

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} = \frac{\frac{x^2 - (x+h)^2}{x^2(x+h)^2}}{h} = \frac{\frac{x - \left(x^2 + 2xh + h^2\right)}{x^2(x+h)^2}}{h}$$
$$= \left(\frac{1}{h}\right)\frac{-2xh - h^2}{x^2(x+h)^2} = \left(\frac{1}{h}\right)\frac{h(-2x-h)}{x^2(x+h)^2}$$
$$= \frac{-2x-h}{x^2(x+h)^2} = \frac{-(2x+h)}{x^2(x+h)^2}$$

87. $f(x)=\dfrac{5x}{x-4}$

$$\frac{f(x+h)-f(x)}{h}=\frac{\dfrac{5(x+h)}{x+h-4}-\dfrac{5x}{x-4}}{h}$$

$$=\frac{\dfrac{5(x+h)(x-4)-5x(x-4+h)}{(x+h-4)(x-4)}}{h}$$

$$=\frac{\dfrac{5x^2-20x+5hx-20h-5x^2+20x-5xh}{(x+h-4)(x-4)}}{h}$$

$$=\left(\frac{-20h}{(x+h-4)(x-4)}\right)\left(\frac{1}{h}\right)$$

$$=-\frac{20}{(x+h-4)(x-4)}$$

88. $f(x)=\dfrac{2x}{x+3}$

$$\frac{f(x+h)-f(x)}{h}=\frac{\dfrac{2(x+h)}{x+h+3}-\dfrac{2x}{x+3}}{h}$$

$$=\frac{\dfrac{2(x+h)(x+3)-2x(x+3+h)}{(x+h+3)(x+3)}}{h}$$

$$=\frac{\dfrac{2x^2+6x+2hx+6h-2x^2-6x-2xh}{(x+h+3)(x+3)}}{h}$$

$$=\left(\frac{6h}{(x+h+3)(x+3)}\right)\left(\frac{1}{h}\right)$$

$$=\frac{6}{(x+h+3)(x+3)}$$

89. $f(x)=\sqrt{x+1}$

$$\frac{f(x+h)-f(x)}{h}$$

$$=\frac{\sqrt{x+h+1}-\sqrt{x+1}}{h}$$

$$=\frac{\sqrt{x+h+1}-\sqrt{x+1}}{h}\cdot\frac{\sqrt{x+h+1}+\sqrt{x+1}}{\sqrt{x+h+1}+\sqrt{x+1}}$$

$$=\frac{x+h+1-(x+1)}{h\left(\sqrt{x+h+1}+\sqrt{x+1}\right)}=\frac{h}{h\left(\sqrt{x+h+1}+\sqrt{x+1}\right)}$$

$$=\frac{1}{\sqrt{x+h+1}+\sqrt{x+1}}$$

90. $f(x)=\sqrt{x-2}$

$$\frac{f(x+h)-f(x)}{h}$$

$$=\frac{\sqrt{x+h-2}-\sqrt{x-2}}{h}$$

$$=\frac{\sqrt{x+h-2}-\sqrt{x-2}}{h}\cdot\frac{\sqrt{x+h-2}+\sqrt{x-2}}{\sqrt{x+h-2}+\sqrt{x-2}}$$

$$=\frac{x+h-2-x+2}{h\left(\sqrt{x+h-2}+\sqrt{x-2}\right)}$$

$$=\frac{h}{h\left(\sqrt{x+h-2}+\sqrt{x-2}\right)}$$

$$=\frac{1}{\sqrt{x+h-2}+\sqrt{x-2}}$$

91.

$$31=x^2-3x+3$$
$$0=x^2-3x-28$$
$$0=(x+4)(x-7)$$
$$x+4=0 \quad \text{or} \quad x-7=0$$
$$x=-4 \quad \text{or} \quad x=7$$

The solution set is: $\{-4,7\}$

92.
$$-\frac{7}{16}=\frac{5}{6}x-\frac{3}{4}$$
$$-\frac{7}{16}+\frac{3}{4}=\frac{5}{6}x$$
$$\frac{5}{6}x=-\frac{7}{16}+\frac{12}{16}$$
$$\frac{5}{6}x=\frac{5}{16}$$
$$x=\frac{5}{16}\cdot\frac{6}{5}=\frac{3}{8}$$

The solution set is: $\left\{\frac{3}{8}\right\}$

93. $f(x)=2x^3+Ax^2+7x-5$ and $f(2)=5$
$$f(2)=2(2)^3+A(2)^2+7(2)-5$$
$$5=16+4A+14-5$$
$$5=4A+25$$
$$A=\frac{-20}{4}=-5$$

94. $f(x)=3x^2-Bx+4$ and $f(-1)=12$:
$$f(-1)=3(-1)^2-B(-1)+4$$
$$12=3+B+4$$
$$B=5$$

95. $f(x)=\frac{3x+8}{2x-A}$ and $f(0)=2$
$$f(0)=\frac{3(0)+8}{2(0)-A}$$
$$2=\frac{8}{-A}$$
$$-2A=8$$
$$A=-4$$

96. $f(x)=\frac{2x-B}{3x+4}$ and $f(2)=\frac{1}{2}$
$$f(2)=\frac{2(2)-B}{3(2)+4}$$
$$\frac{1}{2}=\frac{4-B}{10}$$
$$5=4-B$$
$$B=-1$$

97. Let L represents the length of the rectangle, then, $2L$ represents the width of the rectangle, since the width is twice the length the function for Perimeter is
$$\begin{aligned}p(L)&=2(L+2L)\\&=2(3L)\\&=6L\\p(L)&=6L\end{aligned}$$

98. Let x represent the length of one of the two equal sides. The function for the area is:
$$A(x)=\frac{1}{2}\cdot x\cdot x=\frac{1}{2}x^2$$

99. Let x represent the number of hours worked. The function for the gross salary is:
$G(x)=17x$

100. Let x represent the number of items sold. The function for the gross salary is:
$G(x)=10x+100$

101. **a.** P is the dependent variable; y is the independent variable

b.
$$\begin{aligned}p(40)&=0.028(40)^2-2.678(40)+263.590\\&=44.8-107.12+263.590\\&=201.270\end{aligned}$$
In 2005 there are 201.270 million people who are 40 years of age or older.

c.
$$\begin{aligned}P(0)&=0.028(0)^2-2.678(0)+263.590\\&=263.590\end{aligned}$$
In 2005 there are 263.590 million people.

102. **a.** N is the dependent variable; r is the independent variable

b.
$$\begin{aligned}N(3)&=-1.35(3)^2+15.45(3)-20.71\\&=-12.15+46.35-20.71\\&=13.49\end{aligned}$$
In 2012, there are 13.49 million housing units with 3 rooms.

103. **a.**
$$\begin{aligned}H(1.3)&=31-4.9(1.3)^2\\&=31-8.281\\&=22.719\text{ meters.}\end{aligned}$$

b. $H(x) = 14$

$$14 = 31 - 4.9x^2$$
$$-17 = -4.9x^2$$
$$x^2 \approx 3.46938$$
$$x \approx 1.86 \text{ seconds.}$$

c. $H(x) = 0$

$$0 = 31 - 4.9x^2$$
$$-31 = -4.9x^2$$
$$x^2 \approx 6.326553$$
$$x \approx 2.52 \text{ seconds}$$

104. a. $H(1) = 20 - 13(1)^2 = 20 - 13 = 7$ meters

$$H(1.1) = 20 - 13(1.1)^2 = 20 - 13(1.21)$$
$$= 20 - 15.73 = 4.27 \text{ meters}$$
$$H(1.2) = 20 - 13(1.2)^2 = 20 - 13(1.44)$$
$$= 20 - 18.72 = 1.28 \text{ meters}$$

b. $H(x) = 15$

$$15 = 20 - 13x^2$$
$$-5 = -13x^2$$
$$x^2 \approx 0.3846$$
$$x \approx 0.62 \text{ seconds}$$

$H(x) = 10$

$$10 = 20 - 13x^2$$
$$-10 = -13x^2$$
$$x^2 \approx 0.7692$$
$$x \approx 0.88 \text{ seconds}$$

$H(x) = 5$

$$5 = 20 - 13x^2$$
$$-15 = -13x^2$$
$$x^2 \approx 1.1538$$
$$x \approx 1.07 \text{ seconds}$$

c. $H(x) = 0$

$$0 = 20 - 13x^2$$
$$-20 = -13x^2$$
$$x^2 \approx 1.5385$$
$$x \approx 1.24 \text{ seconds}$$

105. $C(x) = 150 + \dfrac{x}{15} + \dfrac{36,000}{x}$

a. $C(550) = 150 + \dfrac{550}{15} + \dfrac{36000}{550}$

$$= 150 + 36.67 + 65.45$$
$$= \$252.12$$

b. $C(500) = 150 + \dfrac{500}{15} + \dfrac{36000}{500}$

$$= 150 + 33.33 + 72$$
$$= \$255.33$$

c. $C(650) = 150 + \dfrac{650}{15} + \dfrac{36000}{650}$

$$= 150 + 43.33 + 55.38$$
$$= \$248.72$$

d. $C(450) = 150 + \dfrac{450}{15} + \dfrac{36000}{450}$

$$= 150 + 30 + 80$$
$$= \$260$$

106. $A(x) = 4x\sqrt{1 - x^2}$

a. $A\left(\dfrac{1}{3}\right) = 4 \cdot \dfrac{1}{3}\sqrt{1 - \left(\dfrac{1}{3}\right)^2} = \dfrac{4}{3}\sqrt{\dfrac{8}{9}} = \dfrac{4}{3} \cdot \dfrac{2\sqrt{2}}{3}$

$$= \frac{8\sqrt{2}}{9} \approx 1.26 \text{ ft}^2$$

b. $A\left(\dfrac{1}{2}\right) = 4 \cdot \dfrac{1}{2}\sqrt{1 - \left(\dfrac{1}{2}\right)^2} = 2\sqrt{\dfrac{3}{4}} = 2 \cdot \dfrac{\sqrt{3}}{2}$

$$= \sqrt{3} \approx 1.73 \text{ ft}^2$$

c. $A\left(\dfrac{2}{3}\right) = 4 \cdot \dfrac{2}{3}\sqrt{1 - \left(\dfrac{2}{3}\right)^2} = \dfrac{8}{3}\sqrt{\dfrac{5}{9}} = \dfrac{8}{3} \cdot \dfrac{\sqrt{5}}{3}$

$$= \frac{8\sqrt{5}}{9} \approx 1.99 \text{ ft}^2$$

107. $R(x) = \left(\dfrac{L}{p}\right)(x) = \dfrac{L(x)}{p(x)}$

108. $T(x) = (V + P)(x) = V(x) + P(x)$

109. $H(x) = (p.I)(x) = p(x).I(x)$

110. $N(x) = (I - T)(x) = I(x) - T(x)$

111. **a.** $p(x) = R(x) - C(x)$
$= \left(-1.7x^2 + 320x\right) - \left(0.06x^3 - 3x^2 + 85x + 400\right)$
$= -1.7x^2 + 320x - 0.06x^3 - 3x^2 - 85x - 400$
$= -0.06x^3 + 1.3x^2 + 235x - 400$

b. $p(12) = -0.06(12)^3 + 1.3(12)^2 + 235(12) - 400$
$= -103.68 + 187.2 + 2,820 - 400$
$= \$2503.52$

c. When 12 hundred cell phones are sold, the profit is

112. **a.** $P(x) = R(x) - C(x)$
$= 30x - \left(0.1x^2 + 7x + 400\right)$
$= 30x - 0.1x^2 - 7x - 400$
$= -0.1x^2 + 23x - 400$

b. $P(30) = -0.1(30)^2 + 23(30) - 400$
$= -90 + 690 - 400$
$= \$200$

c. When 30 clocks are sold, the profit is \$200.

113. **a.** $R(V) = 2.2V$
$B(V) = 0.03v^2 + 0.3v - 13$
$D(V) = R(V) + B(V)$
$= 2.2V + 0.03V^2 + 0.3V - 13$
$= 0.03V^2 + 2.5V - 13$

b. $D(50) = 0.03(50)^2 + 2.5(5) - 13$
$= 75 + 125 - 13$
$= 187$

c. The car will need 187 feet to stop once the impediment is observed.

114. **a.** $h(x) = 2x$
$h(a+b) = 2(a+b) = 2a + 2b$
$= h(a) + h(b)$
$h(x) = 2x$ has the property.

b. $g(x) = x^2$
$g(a+b) = (a+b)^2 = a^2 + 2ab + b^2$
Since
$a^2 + 2ab + b^2 \neq a^2 + b^2 = g(a) + g(b)$,
$g(x) = x^2$ does not have the property.

c. $F(x) = 5x - 2$
$F(a+b) = 5(a+b) - 2 = 5a + 5b - 2$
Since
$5a + 5b - 2 \neq 5a - 2 + 5b - 2 = F(a) + F(b)$,
$F(x) = 5x - 2$ does not have the property.

d. $G(x) = \dfrac{1}{x}$
$G(a+b) = \dfrac{1}{a+b} \neq \dfrac{1}{a} + \dfrac{1}{b} = G(a) + G(b)$
$G(x) = \dfrac{1}{x}$ does not have the property.

115. No. The domain of *f* is $\{x \mid x \text{ is any real number}\}$, but the domain of *g* is $\{x \mid x \neq -1\}$.

116. Answers will vary.

117. $\dfrac{3x - x^3}{(your\ age)}$

118. $(x+12)^2 + y^2 = 16$
x-intercept (y=0):
$(x+12)^2 + 0^2 = 16$
$(x+12)^2 = 16$
$(x+12) = \pm 4$
$x = -12 \pm 4$
$x = -16, x = -8$
$(-16, 0), (-8, 0)$
y-intercept (x=0):
$(0+12)^2 + y^2 = 16$
$(12)^2 + y^2 = 16$
$y^2 = 16 - 144 = -128$
There are no real solutions so there are no y-intercepts.
Symmetry: $(x+12)^2 + (-y)^2 = 16$
$(x+12)^2 + y^2 = 16$
This shows x-axis symmetry.

119. $y = 3x^2 - 8\sqrt{x}$
$y = 3(-1)^2 - 8\sqrt{-1}$

There is no solution so (-1,-5) is NOT a solution.

$y = 3x^2 - 8\sqrt{x}$

$y = 3(4)^2 - 8\sqrt{4}$

$= 48 - 16 = 32$

So (4,32) is a solution.

$y = 3x^2 - 8\sqrt{x}$

$y = 3(9)^2 - 8\sqrt{9}$

$= 243 - 24 = 219 \neq 171$

So (9,171) is NOT a solution.

120. $P_1 = (3,-4), P_2 = (-6,0)$

The formula for midpoint is:

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$\left(\frac{3+(-6)}{2}, \frac{-4+0}{2}\right)$

$\left(\frac{-3}{2}, \frac{-4}{2}\right)$

$\left(-\frac{3}{2}, -2\right)$

121. $(h,k) = (4,-1)$ and $r = 3$

The general form of a circle is:

$(x-h)^2 + (y-k)^2 = r^2$

$(x-4)^2 + (y-(-1))^2 = (3)^2$

$(x-4)^2 + (y+1)^2 = 9$

Section 2.2

1. $x^2 + 4y^2 = 16$

x-intercepts:

$x^2 + 4(0)^2 = 16$

$x^2 = 16$

$x = \pm 4 \Rightarrow (-4,0),(4,0)$

y-intercepts:

$(0)^2 + 4y^2 = 16$

$4y^2 = 16$

$y^2 = 4$

$y = \pm 2 \Rightarrow (0,-2),(0,2)$

2. False; $x = 2y - 2$

$-2 = 2y - 2$

$0 = 2y$

$0 = y$

The point $(-2,0)$ is on the graph.

3. vertical

4. $f(5) = -3$

5. $f(x) = ax^2 + 4$

$a(-1)^2 + 4 = 2 \Rightarrow a = -2$

6. False. The graph must pass the vertical line test in order to be the graph of a function.

7. False; e.g. $y = \frac{1}{x}$.

8. True

9. c

10. a

11. **a.** $f(0) = 3$ since $(0,3)$ is on the graph.
$f(-6) = -3$ since $(-6,-3)$ is on the graph.

b. $f(6) = 0$ since $(6, 0)$ is on the graph.
$f(11) = 1$ since $(11,1)$ is on the graph.

c. $f(3)$ is positive since $f(3) \approx 3.7$.

d. $f(-4)$ is negative since $f(-4) \approx -1$.

e. $f(x) = 0$ when $x = -3$, $x = 6$, and $x = 10$.

f. $f(x) > 0$ when $-3 < x < 6$, and $10 < x \leq 11$.

g. The domain of f is $\{x | -6 \leq x \leq 11\}$ or $[-6, 11]$.

h. The range of f is $\{y | -3 \leq y \leq 4\}$ or $[-3, 4]$.

i. The x-intercepts are -3, 6, and 10.

j. The y-intercept is 3.

k. The line $y=\frac{1}{2}$ intersects the graph 3 times.

l. The line $x=5$ intersects the graph 1 time.

m. $f(x)=3$ when $x=0$ and $x=4$.

n. $f(x)=-2$ when $x=-5$ and $x=8$.

12. a. $f(0)=0$ since $(0,0)$ is on the graph.
$f(6)=0$ since $(6,0)$ is on the graph.

b. $f(2)=-2$ since $(2,-2)$ is on the graph.
$f(-2)=1$ since $(-2,1)$ is on the graph.

c. $f(3)$ is negative since $f(3)\approx -1$.

d. $f(-1)$ is positive since $f(-1)\approx 1.0$.

e. $f(x)=0$ when $x=0$, $x=4$, and $x=6$.

f. $f(x)<0$ when $0<x<4$.

g. The domain of f is $\{x \mid -4\le x\le 6\}$ or $[-4, 6]$.

h. The range of f is $\{y \mid -2\le y\le 3\}$ or $[-2, 3]$.

i. The x-intercepts are 0, 4, and 6.

j. The y-intercept is 0.

k. The line $y=-1$ intersects the graph 2 times.

l. The line $x=1$ intersects the graph 1 time.

m. $f(x)=3$ when $x=5$.

n. $f(x)=-2$ when $x=2$.

13. Function

a. Domain: $\{x \mid x \text{ is any real number}\}$;
Range: $\{y \mid y>0\}$

b. Intercepts: $(0,1)$

c. None

14. Not a function since vertical lines will intersect the graph in more than one point.

15. Function

a. Domain: $\{x \mid -\pi\le x\le \pi\}$;
Range: $\{y \mid -1\le y\le 1\}$

b. Intercepts: $\left(-\frac{\pi}{2},0\right)$, $\left(\frac{\pi}{2},0\right)$, $(0,1)$

c. Symmetry about y-axis.

16. Function

a. Domain: $\{x \mid -\pi\le x\le \pi\}$;
Range: $\{y \mid -1\le y\le 1\}$

b. Intercepts: $(-\pi, 0)$, $(\pi, 0)$, $(0, 0)$

c. Symmetry about the origin.

17. Not a function since vertical lines will intersect the graph in more than one point.

18. Not a function since vertical lines will intersect the graph in more than one point.

19. Function

a. Domain: $\{x \mid 0\le x<4\}$;
Range: $\{y \mid 0\le y<3\}$

b. Intercepts: (0, 0)

c. None

20. Function

a. Domain: $\{x \mid 0 < x < 3\}$;

Range: $\{y \mid y < 2\}$

b. Intercepts: (1, 0)

c. None

21. Function

a. Domain: $\{x \mid x \ge -3\}$;

Range: $\{y \mid y \ge 0\}$

b. Intercepts: (–3, 0), (2,0), (0,2)

c. None

22. Function

a. Domain: $\{x \mid x \text{ is any real number}\}$;

Range: $\{y \mid y \le 2\}$

b. Intercepts: (–3, 0), (3, 0), (0,2)

c. Symmetry about y-axis.

23. Function

a. Domain: $\{x \mid x \text{ is any real number}\}$;

Range: $\{y \mid y \le 5\}$

b. Intercepts: (–1, 0), (2,0), (0,4)

c. None

24. Function

a. Domain: $\{x \mid x \text{ is any real number}\}$;

Range: $\{y \mid y \ge -3\}$

b. Intercepts: (1, 0), (3,0), (0,9)

c. None

25. $f(x) = -3x^2 + 5x$

a. $f(-1) = -3(-1)^2 + 5(-1) = -8 \ne 2$

The point $(-1, 2)$ is not on the graph of f.

b. $f(-2) = -3(-2)^2 + 5(-2) = -22$

The point $(-2, -22)$ is on the graph of f.

c. Solve for x:

$-2 = -3x^2 + 5x \Rightarrow 3x^2 - 5x - 2 = 0$

$(3x+1)(x-2) = 0 \Rightarrow x = -\frac{1}{3}, x = 2$

(2, –2) and $\left(-\frac{1}{3}, -2\right)$ on the graph of f.

d. The domain of f is $\{x \mid x \text{ is any real number}\}$.

e. x-intercepts:

$f(x) = 0 \Rightarrow -3x^2 + 5x = 0$

$x(-3x+5) = 0 \Rightarrow x = 0, x = \frac{5}{3}$

$(0,0)$ and $\left(\frac{5}{3}, 0\right)$

f. y-intercept:

$f(0) = -3(0)^2 + 5(0) = 0 \Rightarrow (0,0)$

26. $f(x) = 2x^2 - x - 1$

a. $f(-1) = 2(-1)^2 - (-1) - 1 = 2$

The point $(-1, 2)$ is on the graph of f.

b. $f(-2) = 2(-2)^2 - (-2) - 1 = 9$

The point $(-2, 9)$ is on the graph of f.

c. Solve for x:

$-1 = 2x^2 - x - 1$

$0 = 2x^2 - x$

$0 = x(2x-1) \Rightarrow x = 0, x = \frac{1}{2}$

(0, –1) and $\left(\frac{1}{2}, -1\right)$ are on the graph of f.

d. The domain of f is $\{x \mid x \text{ is any real number}\}$.

e. x-intercepts:

$f(x) = 0 \Rightarrow 2x^2 - x - 1 = 0$

$(2x+1)(x-1) = 0 \Rightarrow x = -\frac{1}{2}, x = 1$

$\left(-\frac{1}{2}, 0\right)$ and $(1, 0)$

f. y-intercept:

$f(0) = 2(0)^2 - 0 - 1 = -1 \Rightarrow (0, -1)$

27. $f(x)=\dfrac{x+2}{x-6}$

a. $f(3)=\dfrac{3+2}{3-6}=-\dfrac{5}{3}\neq 14$

The point $(3,14)$ is not on the graph of *f*.

b. $f(4)=\dfrac{4+2}{4-6}=\dfrac{6}{-2}=-3$

The point $(4,-3)$ is on the graph of *f*.

c. Solve for x:

$$2=\frac{x+2}{x-6}$$
$$2x-12=x+2$$
$$x=14$$

(14, 2) is a point on the graph of f.

d. The domain of *f* is $\{x \mid x\neq 6\}$.

e. *x*-intercepts:

$f(x)=0\Rightarrow \dfrac{x+2}{x-6}=0$

$x+2=0\Rightarrow x=-2\Rightarrow(-2,0)$

f. *y*-intercept: $f(0)=\dfrac{0+2}{0-6}=-\dfrac{1}{3}\Rightarrow\left(0,-\dfrac{1}{3}\right)$

28. $f(x)=\dfrac{x^2+2}{x+4}$

a. $f(1)=\dfrac{1^2+2}{1+4}=\dfrac{3}{5}$

The point $\left(1,\dfrac{3}{5}\right)$ is on the graph of *f*.

b. $f(0)=\dfrac{0^2+2}{0+4}=\dfrac{2}{4}=\dfrac{1}{2}$

The point $\left(0,\dfrac{1}{2}\right)$ is on the graph of *f*.

c. Solve for x:

$$\frac{1}{2}=\frac{x^2+2}{x+4}\Rightarrow x+4=2x^2+4$$
$$0=2x^2-x$$

$x(2x-1)=0\Rightarrow x=0$ or $x=\dfrac{1}{2}$

$\left(0,\dfrac{1}{2}\right)$ and $\left(\dfrac{1}{2},\dfrac{1}{2}\right)$ are on the graph of f.

d. The domain of *f* is $\{x \mid x\neq -4\}$.

e. *x*-intercepts:

$f(x)=0\Rightarrow\dfrac{x^2+2}{x+4}=0\Rightarrow x^2+2=0$

This is impossible, so there are no x-intercepts.

f. *y*-intercept:

$f(0)=\dfrac{0^2+2}{0+4}=\dfrac{2}{4}=\dfrac{1}{2}\Rightarrow\left(0,\dfrac{1}{2}\right)$

29. $f(x)=\dfrac{2x}{x-2}$

a. $f\left(\dfrac{1}{2}\right)=\dfrac{2\left(\dfrac{1}{2}\right)}{\dfrac{1}{2}-2}=\dfrac{1}{-\dfrac{3}{2}}=-\dfrac{2}{3}$

The point $\left(\dfrac{1}{2},-\dfrac{2}{3}\right)$ is on the graph of *f*.

b. $f(4)=\dfrac{2(4)}{4-2}=\dfrac{8}{2}=4$

The point $(4,4)$ is on the graph of *f*.

c. Solve for x:

$1=\dfrac{2x}{x-2}\Rightarrow x-2=2x\Rightarrow -2=x$

(–2,1) is a point on the graph of f.

d. The domain of *f* is $\{x \mid x\neq 2\}$.

e. *x*-intercept:

$f(x)=0\Rightarrow\dfrac{2x}{x-2}=0\Rightarrow 2x=0$

$\Rightarrow x=0\Rightarrow(0,0)$

f. *y*-intercept: $f(0)=\dfrac{0}{0-2}=0\Rightarrow(0,0)$

30. $f(x)=\dfrac{2x^2}{x^4+1}$

a. $f(-1)=\dfrac{2(-1)^2}{(-1)^4+1}=\dfrac{2}{2}=1$

The point (–1,1) is on the graph of *f*.

b. $f(2)=\dfrac{2(2)^2}{(2)^4+1}=\dfrac{8}{17}$

The point $\left(2,\dfrac{8}{17}\right)$ is on the graph of *f*.

c. Solve for x:

$$1=\frac{2x^2}{x^4+1}$$
$$x^4+1=2x^2$$
$$x^4-2x^2+1=0$$
$$(x^2-1)^2=0$$
$$x^2-1=0 \Rightarrow x=\pm 1$$

(1,1) and (–1,1) are on the graph of f.

d. The domain of *f* is $\{x \mid x \text{ is any real number}\}$.

e. *x*-intercept:

$$f(x)=0 \Rightarrow \frac{2x^2}{x^4+1}=0$$
$$2x^2=0 \Rightarrow x=0 \Rightarrow (0,0)$$

f. *y*-intercept:

$$f(0)=\frac{2(0)^2}{0^4+1}=\frac{0}{0+1}=0 \Rightarrow (0,0)$$

31. a. $(f+g)(3)=f(3)+g(3)=5+0=5$

b. $(f+g)(5)=f(5)+g(5)=3-4=-1$

c. $(f-g)(7)=f(7)-g(7)=1-0=1$

d. $(g-f)(7)=g(7)-f(7)=0-1=-1$

e. $(f.g)(3)=f(3).g(3)=5.0=0$

f. $\left(\frac{f}{g}\right)(5)=\frac{f(5)}{g(5)}=\frac{3}{-4}=\frac{-3}{4}$

32. $h(x)=-\frac{136x^2}{v^2}+2.7x+3.5$

a. We want $h(15)=10$.

$$-\frac{136(15)^2}{v^2}+2.7(15)+3.5=10$$
$$-\frac{30,600}{v^2}=-34$$
$$v^2=900$$
$$v=30 \text{ ft/sec}$$

The ball needs to be thrown with an initial velocity of 30 feet per second.

b. $h(x)=-\frac{126x^2}{30^2}+2.7x+3.5$

which simplifies to

$$h(x)=-\frac{34}{225}x^2+2.7x+3.5$$

c. Using the velocity from part (b),

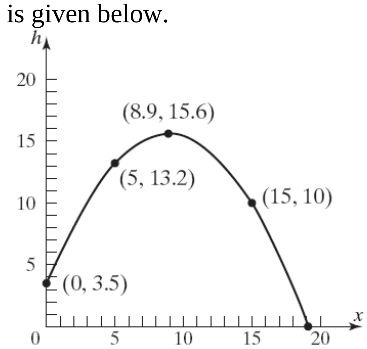
$$h(9)=-\frac{34}{225}(9)^2+2.7(9)+3.5=15.56 \text{ ft}$$

The ball will be 15.56 feet above the floor when it has traveled 9 feet in front of the foul line.

d. Select several values for *x* and use these to find the corresponding values for *h*. Use the results to form ordered pairs (x,h). Plot the points and connect with a smooth curve.

$$h(0)=-\frac{34}{225}(0)^2+2.7(0)+3.5=3.5 \text{ ft}$$
$$h(5)=-\frac{34}{225}(5)^2+2.7(5)+3.5\approx 13.2 \text{ ft}$$
$$h(15)=-\frac{24}{225}(15)^2+2.7(15)+3.5\approx 10 \text{ ft}$$

Thus, some points on the graph are $(0,3.5)$, $(5,13.2)$, and $(15,10)$. The complete graph is given below.



33. $h(x)=-\frac{44x^2}{v^2}+x+6$

a. $h(8)=-\frac{44(8)^2}{28^2}+(8)+6$

$$=-\frac{2816}{784}+14$$
$$\approx 10.4 \text{ feet}$$

b. $h(12) = -\dfrac{44(12)^2}{28^2} + (12) + 6$
$= -\dfrac{6336}{784} + 18$
≈ 9.9 feet

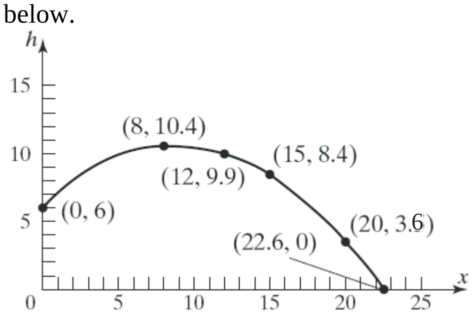
c. From part (a) we know the point $(8, 10.4)$ is on the graph and from part (b) we know the point $(12, 9.9)$ is on the graph. We could evaluate the function at several more values of x (e.g. $x = 0$, $x = 15$, and $x = 20$) to obtain additional points.

$h(0) = -\dfrac{44(0)^2}{28^2} + (0) + 6 = 6$

$h(15) = -\dfrac{44(15)^2}{28^2} + (15) + 6 \approx 8.4$

$h(20) = -\dfrac{44(20)^2}{28^2} + (20) + 6 \approx 3.6$

Some additional points are $(0, 6)$, $(15, 8.4)$ and $(20, 3.6)$. The complete graph is given below.



d. $h(15) = -\dfrac{44(15)^2}{28^2} + (15) + 6 \approx 8.4$ feet
No; when the ball is 15 feet in front of the foul line, it will be below the hoop. Therefore it cannot go through the hoop.

In order for the ball to pass through the hoop, we need to have $h(15) = 10$.

$10 = -\dfrac{44(15)^2}{v^2} + (15) + 6$

$-11 = -\dfrac{44(15)^2}{v^2}$

$v^2 = 4(225)$

$v^2 = 900$

$v = 30$ ft/sec

The ball must be shot with an initial velocity of 30 feet per second in order to go through the hoop.

34. $A(x) = 4x\sqrt{1 - x^2}$

a. Domain of $A(x) = 4x\sqrt{1 - x^2}$; we know that x must be greater than or equal to zero, since x represents a length. We also need $1 - x^2 \ge 0$, since this expression occurs under a square root. In fact, to avoid Area = 0, we require
$x > 0$ and $1 - x^2 > 0$.

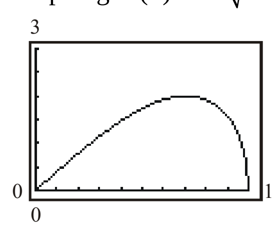
Solve: $1 - x^2 > 0$
$(1 + x)(1 - x) > 0$

Case1: $1 + x > 0$ and $1 - x > 0$
$x > -1$ and $x < 1$
(i.e. $-1 < x < 1$)

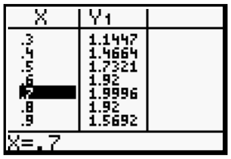
Case2: $1 + x < 0$ and $1 - x < 0$
$x < -1$ and $x > 1$
(which is impossible)

Therefore the domain of A is $\{x \mid 0 < x < 1\}$.

b. Graphing $A(x) = 4x\sqrt{1 - x^2}$



c. When $x = 0.7$ feet, the cross-sectional area is maximized at approximately 1.9996 square feet. Therefore, the length of the base of the beam should be 1.4 feet in order to maximize the cross-sectional area.



35. $h(x) = \dfrac{-32x^2}{130^2} + x$

a. $h(100) = \dfrac{-32(100)^2}{130^2} + 100$

$= \dfrac{-320{,}000}{16{,}900} + 100 \approx 81.07$ feet

b. $h(300) = \dfrac{-32(300)^2}{130^2} + 300$

$= \dfrac{-2{,}880{,}000}{16{,}900} + 300 \approx 129.59$ feet

c. $h(500) = \dfrac{-32(500)^2}{130^2} + 500$

$= \dfrac{-8{,}000{,}000}{16{,}900} + 500 \approx 26.63$ feet

d. Solving $h(x) = \dfrac{-32x^2}{130^2} + x = 0$

$$\frac{-32x^2}{130^2} + x = 0$$

$$x\left(\frac{-32x}{130^2} + 1\right) = 0$$

$$x = 0 \quad \text{or} \quad \frac{-32x}{130^2} + 1 = 0$$

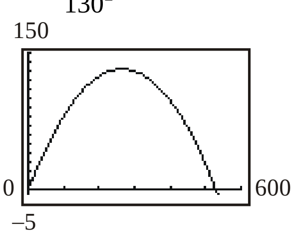
$$1 = \frac{32x}{130^2}$$

$$130^2 = 32x$$

$$x = \frac{130^2}{32} = 528.13 \text{ feet}$$

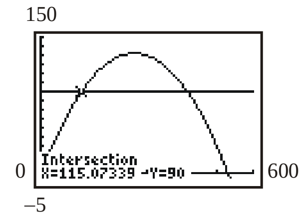
Therefore, the golf ball travels 528.13 feet.

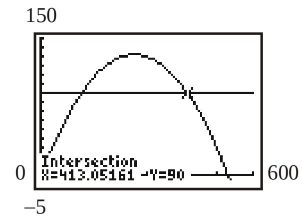
e. $y_1 = \dfrac{-32x^2}{130^2} + x$



f. Use INTERSECT on the graphs of

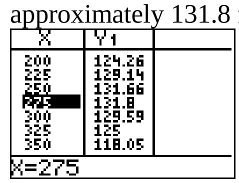
$y_1 = \dfrac{-32x^2}{130^2} + x$ and $y_2 = 90$.



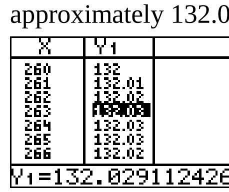


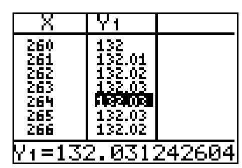
The ball reaches a height of 90 feet twice. The first time is when the ball has traveled approximately 115.07 feet, and the second time is when the ball has traveled about 413.05 feet.

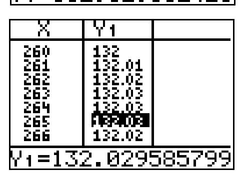
g. The ball travels approximately 275 feet before it reaches its maximum height of approximately 131.8 feet.



h. The ball travels approximately 264 feet before it reaches its maximum height of approximately 132.03 feet.







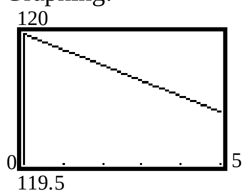
36. $W(h) = m\left(\dfrac{4000}{4000+h}\right)^2$

a. $h = 14110$ feet ≈ 2.67 miles;

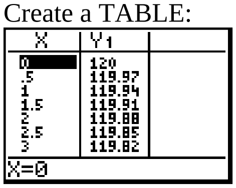
$W(2.67) = 120\left(\dfrac{4000}{4000+2.67}\right)^2 \approx 119.84$

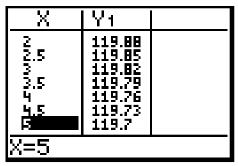
On Pike's Peak, Amy will weigh about 119.84 pounds.

b. Graphing:



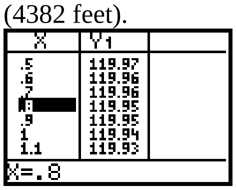
c. Create a TABLE:

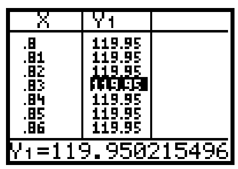




The weight *W* will vary from 120 pounds to about 119.7 pounds.

d. By refining the table, Amy will weigh 119.95 lbs at a height of about 0.83 miles (4382 feet).





e. Yes, 4382 feet is reasonable.

37. $C(x) = 100 + \dfrac{x}{10} + \dfrac{36000}{x}$

a. $C(480) = 100 + \dfrac{480}{10} + \dfrac{36000}{480}$

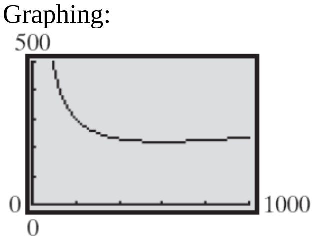
$= \$223$

$C(600) = 100 + \dfrac{600}{10} + \dfrac{36000}{600}$

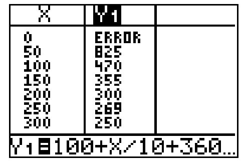
$= \$220$

b. $\{x \mid x > 0\}$

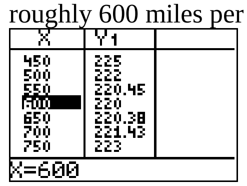
c. Graphing:



d. TblStart = 0; ΔTbl = 50



e. The cost per passenger is minimized to about \$220 when the ground speed is roughly 600 miles per hour.



38. **a.** $C(0) = 5000$

This represents the fixed overhead costs. That is, the company will incur costs of \$5000 per day even if no computers are manufactured.

b. $C(10) = 19{,}000$

It costs the company \$19,000 to produce 10 computers in a day.

c. $C(50) = 51{,}000$

It costs the company \$51,000 to produce 50 computers in a day.

d. The domain is $\{q \mid 0 \le q \le 100\}$. This indicates that production capacity is limited to 100 computers in a day.

e. The graph is curved down and rises slowly at first. As production increases, the graph becomes rises more quickly and changes to being curved up.

f. The inflection point is where the graph changes from being curved down to being curved up.

39. **a.** $C(0) = \$80$

It costs \$80 if you use no minutes.

b. $C(1000) = \$80$

It costs \$440, if you use 1000 minutes

c. $C(4000) = \$440$

It costs \$440 if you use 4000 minutes.

d. The domain is $\{m \, 0 \le m \le 0 \,|\, 14,400\}$. This indicates that there are at most 14,400 minutes in a month.

e. The graph is flat & increases at a constant rate.

40. Answers will vary. From a graph, the domain can be found by visually locating the x-values for which the graph is defined. The range can be found in a similar fashion by visually locating the *y*-values for which the function is defined.

If an equation is given, the domain can be found by locating any restricted values and removing them from the set of real numbers. The range can be found by using known properties of the graph of the equation, or estimated by means of a table of values.

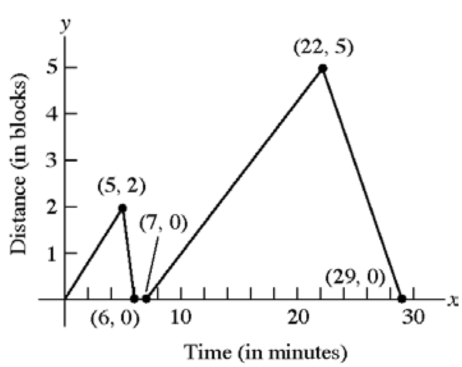
41. The graph of a function can have any number of *x*-intercepts. The graph of a function can have at most one *y*-intercept (otherwise the graph would fail the vertical line test).

42. Yes, the graph of a single point is the graph of a function since it would pass the vertical line test. The equation of such a function would be something like the following: $f(x) = 2$, where $x = 7$.

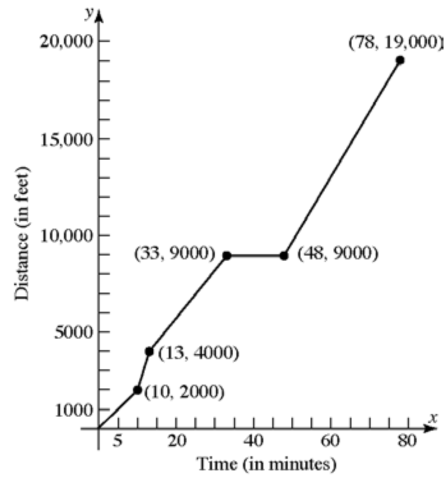
43. (a) III; (b) IV; (c) I; (d) V; (e) II

44. (a) II; (b) V; (c) IV; (d) III; I I

45.



46.



47. a. 2 hours elapsed; Kevin was between 0 and 3 miles from home.

b. 0.5 hours elapsed; Kevin was 3 miles from home.

c. 0.3 hours elapsed; Kevin was between 0 and 3 miles from home.

d. 0.2 hours elapsed; Kevin was at home.

e. 0.9 hours elapsed; Kevin was between 0 and 2.8 miles from home.

f. 0.3 hours elapsed; Kevin was 2.8 miles from home.

g. 1.1 hours elapsed; Kevin was between 0 and 2.8 miles from home.

h. The farthest distance Kevin is from home is 3 miles.

i. Kevin returned home 2 times.

48. a. Michael travels fastest between 7 and 7.4 minutes. That is, $(7, 7.4)$.

b. Michael's speed is zero between 4.2 and 6 minutes. That is, $(4.2, 6)$.

c. Between 0 and 2 minutes, Michael's speed increased from 0 to 30 miles/hour.

d. Between 4.2 and 6 minutes, Michael was stopped (i.e, his speed was 0 miles/hour).

e. Between 7 and 7.4 minutes, Michael was traveling at a steady rate of 50 miles/hour.

f. Michael's speed is constant between 2 and 4 minutes, between 4.2 and 6 minutes, between 7 and 7.4 minutes, and between 7.6 and 8 minutes. That is, on the intervals (2, 4), (4.2, 6), (7, 7.4), and (7.6, 8).

49. Answers (graphs) will vary. Points of the form (5, y) and of the form (x, 0) cannot be on the graph of the function.

50. The only such function is $f(x)=0$ because it is the only function for which $f(x)=-f(x)$. Any other such graph would fail the vertical line test.

51. Answers may vary.

52.
$$\begin{aligned} f(x-2) &= -(x-2)^2+(x-2)-3 \\ &= -(x^2-4x+4)+x-2-3 \\ &= -x^2+4x-4+x-5 \\ &= -x^2+5x-9 \end{aligned}$$

53.
$$\begin{aligned} d &= \sqrt{(1-3)^2+(0-(-6))^2} \\ &= \sqrt{(-2)^2+(-6)^2} \\ &= \sqrt{4+36}=\sqrt{40}=2\sqrt{10} \end{aligned}$$

54.
$$\begin{aligned} y-4 &= \frac{2}{3}(x-(-6)) \\ y-4 &= \frac{2}{3}x+4 \\ y &= \frac{2}{3}x+8 \end{aligned}$$

55. Since the function can be evaluated for any real number, the domain is: $(-\infty,\infty)$

Section 2.3

1. $2<x<5$

2. $\text{slope}=\dfrac{\Delta y}{\Delta x}=\dfrac{8-3}{3-(-2)}=\dfrac{5}{5}=1$

3. x-axis: $y\to -y$
$$\begin{aligned} (-y) &= 5x^2-1 \\ -y &= 5x^2-1 \\ y &= -5x^2+1 \quad \text{different} \end{aligned}$$

y-axis: $x\to -x$
$$\begin{aligned} y &= 5(-x)^2-1 \\ y &= 5x^2-1 \quad \text{same} \end{aligned}$$

origin: $x\to -x$ and $y\to -y$
$$\begin{aligned} (-y) &= 5(-x)^2-1 \\ -y &= 5x^2-1 \\ y &= -5x^2+1 \quad \text{different} \end{aligned}$$

The equation has symmetry with respect to the y-axis only.

4.
$$\begin{aligned} y-y_1 &= m(x-x_1) \\ y-(-2) &= 5(x-3) \\ y+2 &= 5(x-3) \end{aligned}$$

5. $y=x^2-9$

x-intercepts:

$0=x^2-9$

$x^2=9\to x=\pm 3$

y-intercept:

$y=(0)^2-9=-9$

The intercepts are $(-3,0)$, $(3,0)$, and $(0,-9)$.

6. increasing

7. even; odd

8. True

9. True

10. False; odd functions are symmetric with respect to the origin. Even functions are symmetric with respect to the y-axis.

11. c

12. d

13. Yes

14. No, it is increasing.

15. No

16. Yes

17. f is increasing on the intervals $(-8,-2)$, $(0,2)$, $(5,7)$.

18. f is decreasing on the intervals: $(-10,-8)$, $(-2,0)$, $(2,5)$.

19. Yes. The local maximum at $x = 2$ is 10.

20. No. There is a local minimum at $x = 5$; the local minimum is 0.

21. f has local maxima at $x = -2$ and $x = 2$. The local maxima are 6 and 10, respectively.

22. f has local minima at $x = -8, x = 0$ and $x = 5$. The local minima are –4, 0, and 0, respectively.

23. f has absolute maximum of 10 at x = 2.

24. f has absolute minimum of –4 at x = –8.

25. **a.** Intercepts: (–2, 0), (2, 0), and (0, 3).

b. Domain: $\{x \mid -4 \le x \le 4\}$ or $[-4, 4]$;

Range: $\{y \mid 0 \le y \le 3\}$ or $[0, 3]$.

c. Increasing: (–2, 0) and (2, 4);
Decreasing: (–4, –2) and (0, 2).

d. Since the graph is symmetric with respect to the y-axis, the function is <u>even</u>.

26. **a.** Intercepts: (–1, 0), (1, 0), and (0, 2).

b. Domain: $\{x \mid -3 \le x \le 3\}$ or $[-3, 3]$;

Range: $\{y \mid 0 \le y \le 3\}$ or $[0, 3]$.

c. Increasing: (–1, 0) and (1, 3);
Decreasing: (–3, –1) and (0, 1).

d. Since the graph is symmetric with respect to the y-axis, the function is <u>even</u>.

27. **a.** Intercepts: (1, 0).

b. Domain: $\{x \mid x > 0\}$ or $(0, \infty)$;

Range: $\{y \mid y \text{ is any real number}\}$.

c. Increasing: $(0, \infty)$; Decreasing: never.

d. Since the graph is not symmetric with respect to the y-axis or the origin, the function is <u>neither</u> even nor odd.

28. **a.** Intercepts: (0, 1).

b. Domain: $\{x \mid x \text{ is any real number}\}$;

Range: $\{y \mid y > 0\}$ or $(0, \infty)$.

c. Increasing: $(-\infty, \infty)$; Decreasing: never.

d. Since the graph is not symmetric with respect to the y-axis or the origin, the function is <u>neither</u> even nor odd.

29. **a.** Intercepts: $\left(-\frac{\pi}{2}, 0\right), \left(\frac{\pi}{2}, 0\right)$, and $(0, 1)$.

b. Domain: $\{x \mid -\pi \le x \le \pi\}$ or $[-\pi, \pi]$;

Range: $\{y \mid -1 \le y \le 1\}$ or $[-1, 1]$.

c. Increasing: $(-\pi, 0)$; Decreasing: $(0, \pi)$.

d. Since the graph is symmetric with respect to the y-axis, the function is <u>even</u>.

30. **a.** Intercepts: $(-\pi, 0), (\pi, 0)$, and $(0, 0)$.

b. Domain: $\{x \mid -\pi \le x \le \pi\}$ or $[-\pi, \pi]$;

Range: $\{y \mid -1 \le y \le 1\}$ or $[-1, 1]$.

c. Increasing: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$;

Decreasing: $\left(-\pi, -\frac{\pi}{2}\right)$ and $\left(\frac{\pi}{2}, \pi\right)$.

d. Since the graph is symmetric with respect to the origin, the function is <u>odd</u>.

31. **a.** Intercepts: $(-2.3, 0), (3, 0)$, and $(0, 1)$.

b. Domain: $\{x \mid -3 \le x \le 3\}$ or $[-3, 3]$;

Range: $\{y \mid -2 \le y \le 2\}$ or $[-2, 2]$.

c. Increasing: $(-3, -2)$ and $(0, 2)$;

Decreasing: $(2, 3)$; Constant: $(-2, 0)$.

d. Since the graph is not symmetric with respect to the y-axis or the origin, the function is <u>neither</u> even nor odd.

32. **a.** Intercepts: $\left(\frac{1}{3}, 0\right), \left(\frac{5}{2}, 0\right)$, and $\left(0, \frac{1}{2}\right)$.

b. Domain: $\{x \mid -3 \le x \le 3\}$ or $[-3, 3]$;

Range: $\{y \mid -1 \le y \le 2\}$ or $[-1, 2]$.

c. Increasing: $(2, 3)$; Decreasing: $(-1, 1)$;

Constant: $(-3, -1)$ and $(1, 2)$

d. Since the graph is not symmetric with respect to the y-axis or the origin, the function is <u>neither</u> even nor odd.

33. **a.** f has a local maximum of 2 at $x=0$.

b. f has a local minimum of 0 at both $x=-1$ and $x=1$.

34. **a.** f has a local maximum of 3 at $x=0$.

b. f has a local minimum of 0 at both $x=-2$ and $x=2$.

35. **a.** f has a local maximum of 1 at $x=0$.

b. f has a local minimum of –1 both at $x=-\pi$ and $x=\pi$.

36. **a.** f has a local maximum of 1 at $x=\frac{\pi}{2}$.

b. f has a local minimum of –1 at $x=-\frac{\pi}{2}$.

37. $f(x)=4x^3$

$f(-x)=4(-x)^3=-4x^3=-f(x)$

Therefore, f is odd.

38. $f(x)=2x^4-x^2$

$f(-x)=2(-x)^4-(-x)^2=2x^4-x^2=f(x)$

Therefore, f is even.

39. $h(x)=3x^3+5$

$h(-x)=3(-x)^3+5=-3x^3+5$

h is neither even nor odd.

40. $g(x)=-3x^2-5$

$g(-x)=-3(-x)^2-5=-3x^2-5=g(x)$

Therefore, g is even.

41. $G(x)=\sqrt{x}$

$G(-x)=\sqrt{-x}$

G is neither even nor odd.

42. $F(x)=\sqrt[3]{x}$

$F(-x)=\sqrt[3]{-x}=-\sqrt[3]{x}=-F(x)$

Therefore, F is odd.

43. $f(x)=\sqrt[3]{2x^2+1}$

$f(-x)=\sqrt[3]{2(-x)^2+1}=\sqrt[3]{2x^2+1}=f(x)$

Therefore, f is even.

44. $f(x)=x+|x|$

$f(-x)=-x+|-x|=-x+|x|$

f is neither even nor odd.

45. $h(x)=\dfrac{x}{x^2-1}$

$h(-x)=\dfrac{-x}{(-x)^2-1}=\dfrac{-x}{x^2-1}=-h(x)$

Therefore, h is odd.

46. $g(x)=\dfrac{1}{x^2}$

$g(-x)=\dfrac{1}{(-x)^2}=\dfrac{1}{x^2}=g(x)$

Therefore, g is even.

47. $F(x)=\dfrac{2x}{|x|}$

$F(-x)=\dfrac{2(-x)}{|-x|}=\dfrac{-2x}{|x|}=-F(x)$

Therefore, F is odd.

48. $h(x)=\dfrac{-x^3}{3x^2-9}$

$h(-x)=\dfrac{-(-x)^3}{3(-x)^2-9}=\dfrac{x^3}{3x^2-9}=-h(x)$

Therefore, h is odd.

49. f has an absolute maximum of 4 at $x=1$.
f has an absolute minimum of 1 at $x=5$.
f has an local maximum of 3 at $x=3$.
f has an local minimum of 2 at $x=2$.

50. f has an absolute maximum of 4 at $x=4$.
f has an absolute minimum of 0 at $x=5$.
f has an local maximum of 4 at $x=4$.
f has an local minimum of 1 at $x=1$.

51. f has an absolute minimum of 1 at $x = 0$.
f has no absolute maximum.
f has no local minimum.
f has no local maximum.

52. f has an absolute minimum of 1 at $x = 1$.
f has an absolute maximum of 4 at x = 3.
f has an local minimum of 1 at $x = 1$.
f has an local maximum of 4 at x = 3.

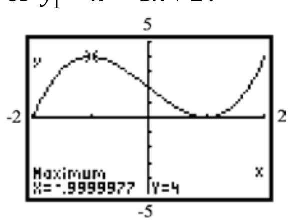
53. f has an absolute maximum of 4 at $x = 2$.
f has no absolute minimum.
f has an local maximum of 4 at $x = 2$.
f has an local minimum of 2 at $x = 0$.

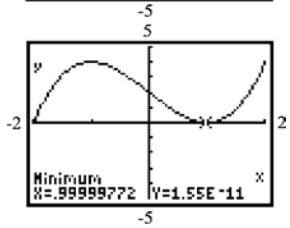
54. f has an absolute minimum of 0 at $x = 0$.
f has no absolute maximum.
f has an local minimum of 0 at $x = 0$.
f has an local minimum of 2 at $x = 3$.
f has an local maximum of 3 at $x = 2$.

55. f has no absolute maximum or minimum.
f has no local maximum or minimum.

56. f has no absolute maximum or minimum.
f has no local maximum or minimum.

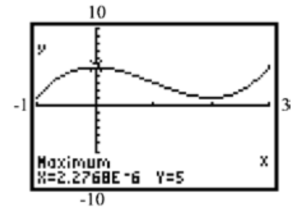
57. $f(x) = x^3 - 3x + 2$ on the interval $(-2, 2)$
Use MAXIMUM and MINIMUM on the graph of $y_1 = x^3 - 3x + 2$.

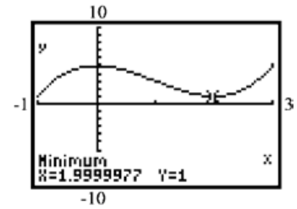




local maximum: $f(-1) = 4$
local minimum: $f(1) = 0$
f is increasing on: $(-2, -1)$ and $(1, 2)$;
f is decreasing on: $(-1, 1)$

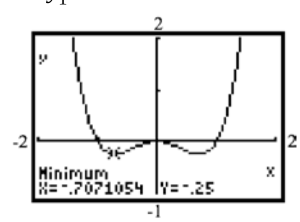
58. $f(x) = x^3 - 3x^2 + 5$ on the interval $(-1, 3)$
Use MAXIMUM and MINIMUM on the graph of $y_1 = x^3 - 3x^2 + 5$.

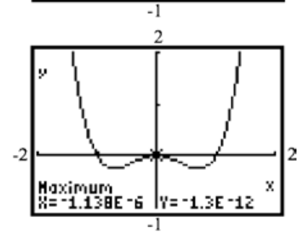


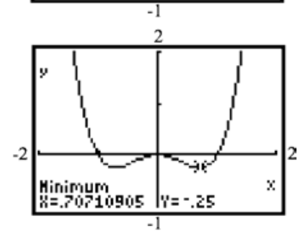


local maximum: $f(0) = 5$
local minimum: $f(2) = 1$
f is increasing on: $(-1, 0)$ and $(2, 3)$;
f is decreasing on: $(0, 2)$

59. $f(x) = x^4 - x^2$ on the interval $(-2, 2)$
Use MAXIMUM and MINIMUM on the graph of $y_1 = x^4 - x^2$.



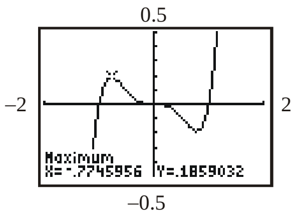


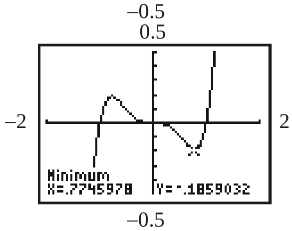


local maximum: $f(0) = 0$
local minimum:
$f(-0.71) = -0.25$; $f(0.71) = -0.25$

f is increasing on: $(-0.71, 0)$ and $(0.71, 2)$;
f is decreasing on: $(-2, -0.71)$ and $(0, 0.71)$

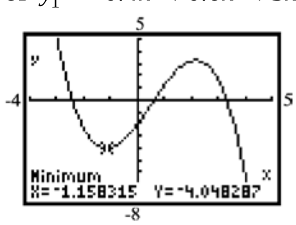
60. $f(x) = x^5 - x^3$ on the interval $(-2, 2)$
Use MAXIMUM and MINIMUM on the graph of $y_1 = x^5 - x^3$.

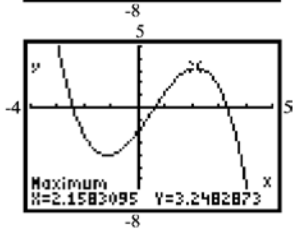




local maximum: $f(-0.77) = 0.19$
local minimum: $f(0.77) = -0.19$
f is increasing on: $(-2, -0.77)$ and $(0.77, 2)$;
f is decreasing on: $(-0.77, 0.77)$

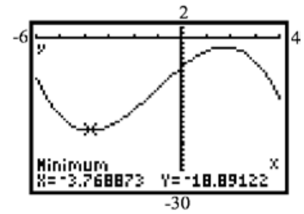
61. $f(x) = -0.4x^3 + 0.6x^2 + 3x - 2$ on the interval $(-4, 5)$
Use MAXIMUM and MINIMUM on the graph of $y_1 = -0.4x^3 + 0.6x^2 + 3x - 2$.

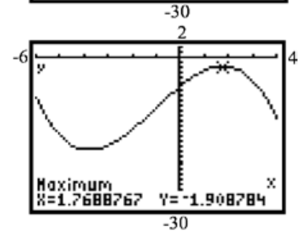




local maximum: $f(2.16) = 3.25$
local minimum: $f(-1.16) = -4.05$
f is increasing on: $(-1.16, 2.16)$;
f is decreasing on: $(-4, -1.16)$ and $(2.16, 5)$

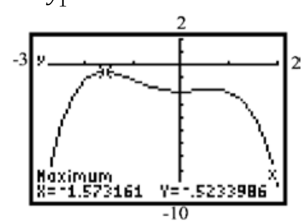
62. $f(x) = -0.2x^3 - 0.6x^2 + 4x - 6$ on the interval $(-6, 4)$
Use MAXIMUM and MINIMUM on the graph of $y_1 = -0.2x^3 - 0.6x^2 + 4x - 6$.

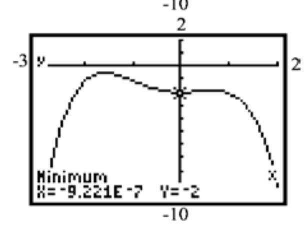


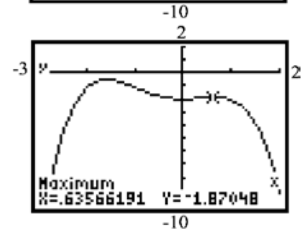


local maximum: $f(1.77) = -1.91$
local minimum: $f(-3.77) = -18.89$
f is increasing on: $(-3.77, 1.77)$;
f is decreasing on: $(-6, -3.77)$ and $(1.77, 4)$

63. $f(x) = -0.4x^4 - 0.5x^3 + 0.8x^2 - 2$ on the interval $(-3, 2)$
Use MAXIMUM and MINIMUM on the graph of $y_1 = -0.4x^4 - 0.5x^3 + 0.8x^2 - 2$.







local maxima: $f(-1.57) = -0.52$,
$f(0.64) = -1.87$

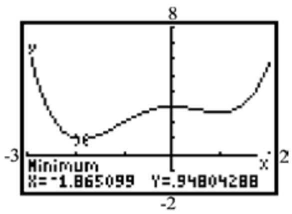
local minimum: $(0,-2)$ $f(0)=-2$

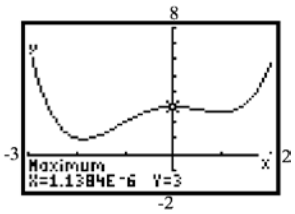
f is increasing on: $(-3,-1.57)$ and $(0,0.64)$;

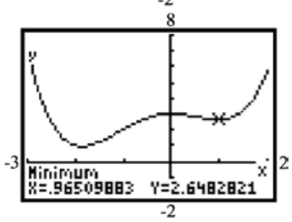
f is decreasing on: $(-1.57,0)$ and $(0.64,2)$

64. $f(x)=0.25x^4+0.3x^3-0.9x^2+3$ on the interval $(-3,2)$

Use MAXIMUM and MINIMUM on the graph of $y_1=0.25x^4+0.3x^3-0.9x^2+3$.







local maximum: $f(0)=3$

local minimum:

$f(-1.87)=0.95$, $f(0.97)=2.65$

f is increasing on: $(-1.87,0)$ and $(0.97,2)$;

f is decreasing on: $(-3,-1.87)$ and $(0,0.97)$

65. $f(x)=-2x^2+4$

a. Average rate of change of f from $x=0$ to $x=2$

$$\frac{f(2)-f(0)}{2-0}=\frac{\left(-2(2)^2+4\right)-\left(-2(0)^2+4\right)}{2}=\frac{(-4)-(4)}{2}=\frac{-8}{2}=-4$$

b. Average rate of change of f from $x = 1$ to $x = 3$:

$$\frac{f(3)-f(1)}{3-1}=\frac{\left(-2(3)^2+4\right)-\left(-2(1)^2+4\right)}{2}=\frac{(-14)-(2)}{2}=\frac{-16}{2}=-8$$

c. Average rate of change of f from $x = 1$ to $x = 4$:

$$\frac{f(4)-f(1)}{4-1}=\frac{\left(-2(4)^2+4\right)-\left(-2(1)^2+4\right)}{3}=\frac{(-28)-(2)}{3}=\frac{-30}{3}=-10$$

66. $f(x)=-x^3+1$

a. Average rate of change of f from $x = 0$ to $x = 2$:

$$\frac{f(2)-f(0)}{2-0}=\frac{\left(-(2)^3+1\right)-\left(-(0)^3+1\right)}{2}=\frac{-7-1}{2}=\frac{-8}{2}=-4$$

b. Average rate of change of f from $x = 1$ to $x = 3$:

$$\frac{f(3)-f(1)}{3-1}=\frac{\left(-(3)^3+1\right)-\left(-(1)^3+1\right)}{2}=\frac{-26-(0)}{2}=\frac{-26}{2}=-13$$

c. Average rate of change of f from $x = -1$ to $x = 1$:

$$\frac{f(1)-f(-1)}{1-(-1)}=\frac{\left(-(1)^3+1\right)-\left(-(-1)^3+1\right)}{2}=\frac{0-2}{2}=\frac{-2}{2}=-1$$

67. $g(x)=x^3-2x+1$

a. Average rate of change of g from $x=-3$ to $x=-2$:

$$\frac{g(-2)-g(-3)}{-2-(-3)}$$

$$=\frac{\left[(-2)^3-2(-2)+1\right]-\left[(-3)^3-2(-3)+1\right]}{1}$$

$$=\frac{(-3)-(-20)}{1}=\frac{17}{1}=17$$

b. Average rate of change of g from $x=-1$ to $x=1$:

$$\frac{g(1)-g(-1)}{1-(-1)}$$
$$=\frac{\left[(1)^3-2(1)+1\right]-\left[(-1)^3-2(-1)+1\right]}{2}$$
$$=\frac{(0)-(2)}{2}=\frac{-2}{2}=-1$$

c. Average rate of change of g from $x=1$ to $x=3$:

$$\frac{g(3)-g(1)}{3-1}$$
$$=\frac{\left[(3)^3-2(3)+1\right]-\left[(1)^3-2(1)+1\right]}{2}$$
$$=\frac{(22)-(0)}{2}=\frac{22}{2}=11$$

68. $h(x)=x^2-2x+3$

a. Average rate of change of h from $x=-1$ to $x=1$:

$$\frac{h(1)-h(-1)}{1-(-1)}$$
$$=\frac{\left[(1)^2-2(1)+3\right]-\left[(-1)^2-2(-1)+3\right]}{2}$$
$$=\frac{(2)-(6)}{2}=\frac{-4}{2}=-2$$

b. Average rate of change of h from $x=0$ to $x=2$:

$$\frac{h(2)-h(0)}{2-0}$$
$$=\frac{\left[(2)^2-2(2)+3\right]-\left[(0)^2-2(0)+3\right]}{2}$$
$$=\frac{(3)-(3)}{2}=\frac{0}{2}=0$$

c. Average rate of change of h from $x=2$ to $x=5$:

$$\frac{h(5)-h(2)}{5-2}$$
$$=\frac{\left[(5)^2-2(5)+3\right]-\left[(2)^2-2(2)+3\right]}{3}$$
$$=\frac{(18)-(3)}{3}=\frac{15}{3}=5$$

69. $f(x)=5x-2$

a. Average rate of change of f from 1 to 3:

$$\frac{\Delta y}{\Delta x}=\frac{f(3)-f(1)}{3-1}=\frac{13-3}{3-1}=\frac{10}{2}=5$$

Thus, the average rate of change of f from 1 to 3 is 5.

b. From (a), the slope of the secant line joining $(1, f(1))$ and $(3, f(3))$ is 5. We use the point-slope form to find the equation of the secant line:

$$y-y_1=m_{\text{sec}}(x-x_1)$$
$$y-3=5(x-1)$$
$$y-3=5x-5$$
$$y=5x-2$$

70. $f(x)=-4x+1$

a. Average rate of change of f from 2 to 5:

$$\frac{\Delta y}{\Delta x}=\frac{f(5)-f(2)}{5-2}=\frac{-19-(-7)}{5-2}$$
$$=\frac{-12}{3}=-4$$

Therefore, the average rate of change of f from 2 to 5 is -4.

b. From (a), the slope of the secant line joining $(2, f(2))$ and $(5, f(5))$ is -4. We use the point-slope form to find the equation of the secant line:

$$y-y_1=m_{\text{sec}}(x-x_1)$$
$$y-(-7)=-4(x-2)$$
$$y+7=-4x+8$$
$$y=-4x+1$$

71. $g(x) = x^2 - 2$

a. Average rate of change of g from -2 to 1:

$$\frac{\Delta y}{\Delta x} = \frac{g(1) - g(-2)}{1-(-2)} = \frac{-1-2}{1-(-2)} = \frac{-3}{3} = -1$$

Therefore, the average rate of change of g from -2 to 1 is -1.

b. From (a), the slope of the secant line joining $(-2, g(-2))$ and $(1, g(1))$ is -1. We use the point-slope form to find the equation of the secant line:

$$y - y_1 = m_{\text{sec}}(x - x_1)$$
$$y - 2 = -1(x - (-2))$$
$$y - 2 = -x - 2$$
$$y = -x$$

72. $g(x) = x^2 + 1$

a. Average rate of change of g from -1 to 2:

$$\frac{\Delta y}{\Delta x} = \frac{g(2) - g(-1)}{2-(-1)} = \frac{5-2}{2-(-1)} = \frac{3}{3} = 1$$

Therefore, the average rate of change of g from -1 to 2 is 1.

b. From (a), the slope of the secant line joining $(-1, g(-1))$ and $(2, g(2))$ is 1. We use the point-slope form to find the equation of the secant line:

$$y - y_1 = m_{\text{sec}}(x - x_1)$$
$$y - 2 = 1(x - (-1))$$
$$y - 2 = x + 1$$
$$y = x + 3$$

73. $h(x) = x^2 - 2x$

a. Average rate of change of h from 2 to 4:

$$\frac{\Delta y}{\Delta x} = \frac{h(4) - h(2)}{4-2} = \frac{8-0}{4-2} = \frac{8}{2} = 4$$

Therefore, the average rate of change of h from 2 to 4 is 4.

b. From (a), the slope of the secant line joining $(2, h(2))$ and $(4, h(4))$ is 4. We use the point-slope form to find the equation of the secant line:

$$y - y_1 = m_{\text{sec}}(x - x_1)$$
$$y - 0 = 4(x - 2)$$
$$y = 4x - 8$$

74. $h(x) = -2x^2 + x$

a. Average rate of change from 0 to 3:

$$\frac{\Delta y}{\Delta x} = \frac{h(3) - h(0)}{3-0} = \frac{-15-0}{3-0}$$
$$= \frac{-15}{3} = -5$$

Therefore, the average rate of change of h from 0 to 3 is -5.

b. From (a), the slope of the secant line joining $(0, h(0))$ and $(3, h(3))$ is -5. We use the point-slope form to find the equation of the secant line:

$$y - y_1 = m_{\text{sec}}(x - x_1)$$
$$y - 0 = -5(x - 0)$$
$$y = -5x$$

75. a. $g(x) = x^3 - 27x$

$$g(-x) = (-x)^3 - 27(-x)$$
$$= -x^3 + 27x$$
$$= -(x^3 - 27x)$$
$$= -g(x)$$

Since $g(-x) = -g(x)$, the function is odd.

b. Since $g(x)$ is odd then it is symmetric about the origin so there exist a local maximum at $x = -3$.

$$g(-3) = (-3)^3 - 27(-3) = -27 + 81 = 54$$

So there is a local maximum of 54 at $x = -3$.

76. $f(x) = -x^3 + 12x$

a. $f(-x) = -(-x)^3 + 12(-x)$

$$= x^3 - 12x$$
$$= -(-x^3 + 12x)$$
$$= -f(x)$$

Since $f(-x) = -f(x)$, the function is odd.

b. Since $f(x)$ is odd then it is symmetric about the origin so there exist a local maximum at $x = -3$.

$f(-2) = -(-2)^3 + 12(-2) = 8 - 24 = -16$

So there is a local maximum of -16 at $x = -2$.

77. $F(x) = -x^4 + 8x^2 + 8$

a. $F(-x) = -(-x)^4 + 8(-x)^2 + 8$
$= -x^4 + 8x + 8$
$= F(x)$

Since $F(-x) = F(x)$, the function is even.

b. Since the function is even, its graph has y-axis symmetry. The second local maximum is in quadrant II and is 24 and occurs at $x = -2$.

c. Because the graph has y-axis symmetry, the area under the graph between $x = 0$ and $x = 3$ bounded below by the x-axis is the same as the area under the graph between $x = -3$ and $x = 0$ bounded below the x-axis. Thus, the area is 47.4 square units.

78. $G(x) = -x^4 + 32x^2 + 144$

a. $G(-x) = -(-x)^4 + 32(-x)^2 + 144$
$= -x^4 + 32x^2 + 144$
$= G(x)$

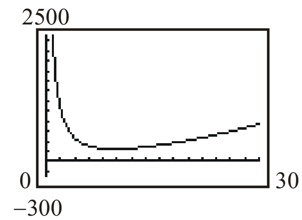
Since $G(-x) = G(x)$, the function is even.

b. Since the function is even, its graph has y-axis symmetry. The second local maximum is in quadrant II and is 400 and occurs at $x = -4$.

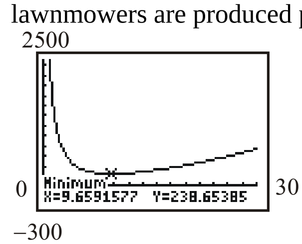
c. Because the graph has y-axis symmetry, the area under the graph between $x = 0$ and $x = 6$ bounded below by the x-axis is the same as the area under the graph between $x = -6$ and $x = 0$ bounded below the x-axis. Thus, the area is 1612.8 square units.

79. $\overline{C}(x) = 0.3x^2 + 21x - 251 + \frac{2500}{x}$

a. $y_1 = 0.3x^2 + 21x - 251 + \frac{2500}{x}$



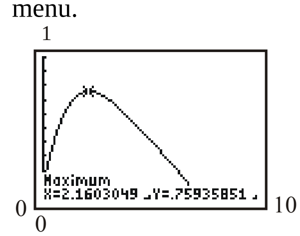
b. Use MINIMUM. Rounding to the nearest whole number, the average cost is minimized when approximately 10 lawnmowers are produced per hour.



c. The minimum average cost is approximately \$239 per mower.

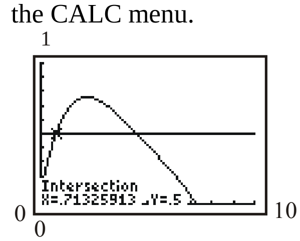
80. a. $C(t) = -.002t^4 + .039t^3 - .285t^2 + .766t + .085$

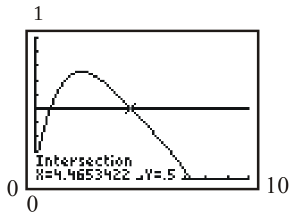
Graph the function on a graphing utility and use the Maximum option from the CALC menu.



The concentration will be highest after about 2.16 hours.

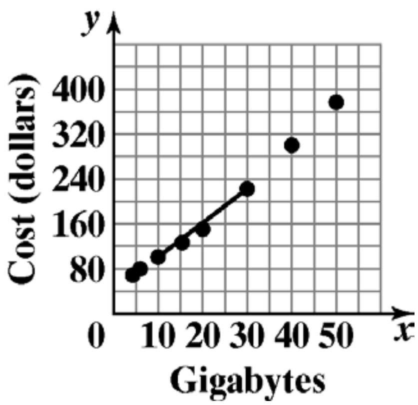
b. Enter the function in Y1 and 0.5 in Y2. Graph the two equations in the same window and use the Intersect option from the CALC menu.





After taking the medication, the woman can feed her child within the first 0.71 hours (about 42 minutes) or after 4.47 hours (about 4hours 28 minutes) have elapsed.

81. a. and b.



The slope represents the average rate of change of the cost of the plan from 10 to 30 gigabytes.

c. avg. rate of change $= \dfrac{C(10)-C(4)}{10-4}$

$= \dfrac{100-70}{6}$

$= \dfrac{30}{6}$

$=$ \$5 per gigabyte

On average, the cost per gigabyte is increasing at a rate of \$5 gram per gigabyte from 4 to 10 gigabytes.

d. avg. rate of change $= \dfrac{C(30)-C(10)}{30-10}$

$= \dfrac{225-100}{20}$

$= \dfrac{125}{20}$

$=$ \$6.25 per gigabyte

On average, the cost per gigabyte is increasing at a rate of \$6.25 gram per gigabyte from 10 to 30 gigabytes.

e. avg. rate of change $= \dfrac{C(50)-C(30)}{50-30}$

$= \dfrac{375-225}{20}$

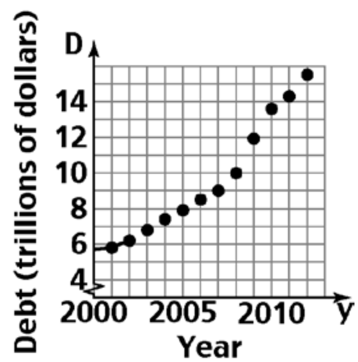
$= \dfrac{150}{20}$

$=$ \$7.50 per gigabyte

On average, the cost per gigabyte is increasing at a rate of \$7.50 gram per gigabyte from 30 to 50 gigabytes.

f. The average rate of change is increasing as the gigabyte use goes up. This indicates that the cost is increasing at an increasing rate.

82. a.



b. The slope represents the average rate of change of the debt from 2001 to 2006.

c. avg. rate of change $= \dfrac{P(2004)-P(2002)}{2004-2002}$

$= \dfrac{7379-6228}{2}$

$= \dfrac{1151}{2}$

$=$ \$ 575.5 billion/yr

d. avg. rate of change $= \dfrac{P(2008)-P(2006)}{2008-2006}$

$= \dfrac{10025-8507}{2}$

$= \dfrac{1518}{2}$

$=$ \$ 759 billion/yr

e. avg. rate of change $= \dfrac{P(2012)-P(2010)}{2012-2010}$
$= \dfrac{16066-13562}{2}$
$= \dfrac{2504}{2}$
$= \$\ 1252$ billion

f. The average rate of change is increasing as time passes.

83. a. avg. rate of change $= \dfrac{p(4.5)-p(0)}{4.5-0}$
$= \dfrac{0.15-0.06}{4.5-0}$
$= 0.020$ gram per hour

On average, the population is increasing at a rate of 0.036 gram per hour from 0 to 4.5 hours.

b. avg. rate of change $= \dfrac{p(8)-p(6.5)}{8-6.5}$
$= \dfrac{0.52-0.36}{8-6.5}$
$= \dfrac{0.16}{2.5}$
$= 0.107$ gram per hour

On average, the population is increasing at a rate of 0.107 gram per hour from 6.5 to 8 hours.

84. a. avg. rate of change $= \dfrac{P(2006)-P(2004)}{2006-2004}$
$= \dfrac{53.8-46.5}{2}$
$= \dfrac{7.3}{2}$
$= 3.65$ percentage points per year

On average, the percentage of returns that are e-filed is increasing at a rate of 3.65 percentage points per year from 2004 to 2006.

b. avg. rate of change $= \dfrac{P(2009)-P(2007)}{2009-2007}$
$= \dfrac{67.2-57.1}{2009-2007}$
$= \dfrac{10.1}{2}$
$= 5.05$ percentage points per year

On average, the percentage of returns that are e-filed is increasing at a rate of 5.05 percentage points per year from 2007 to 2009.

c.

avg. rate of change $= \dfrac{P(2012)-P(2010)}{2012-2010}$
$= \dfrac{82.7-69.8}{2012-2010}$
$= \dfrac{12.9}{2}$
$= 6.45$ percentage points per year

On average, the percentage of returns that are e-filed is increasing at a rate of 6.45 percentage points per year from 2010 to 2012.

d. The average rate of change is increasing as time passes. This indicates that the percentage of e-filers is increasing at an increasing rate.

85. $f(x) = x^2$

a. Average rate of change of *f* from $x=0$ to $x=1$:

$$\frac{f(1)-f(0)}{1-0} = \frac{1^2-0^2}{1} = \frac{1}{1} = 1$$

b. Average rate of change of *f* from $x=0$ to $x=0.5$:

$$\frac{f(0.5)-f(0)}{0.5-0} = \frac{(0.5)^2-0^2}{0.5} = \frac{0.25}{0.5} = 0.5$$

c. Average rate of change of *f* from $x=0$ to $x=0.1$:

$$\frac{f(0.1)-f(0)}{0.1-0} = \frac{(0.1)^2-0^2}{0.1} = \frac{0.01}{0.1} = 0.1$$

d. Average rate of change of *f* from $x = 0$ to $x = 0.01$:

$$\frac{f(0.01) - f(0)}{0.01 - 0} = \frac{(0.01)^2 - 0^2}{0.01}$$

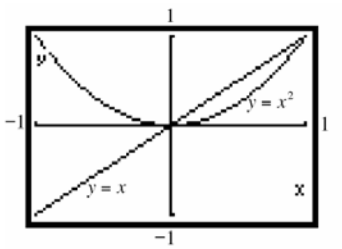
$$= \frac{0.0001}{0.01} = 0.01$$

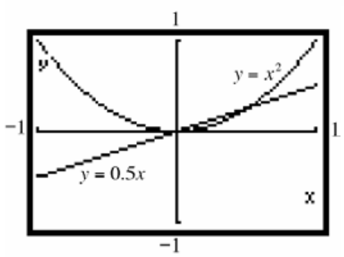
e. Average rate of change of *f* from $x = 0$ to $x = 0.001$:

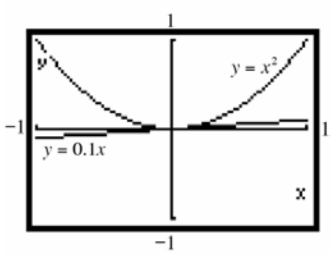
$$\frac{f(0.001) - f(0)}{0.001 - 0} = \frac{(0.001)^2 - 0^2}{0.001}$$

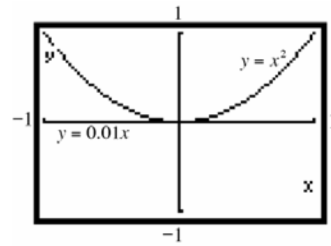
$$= \frac{0.000001}{0.001} = 0.001$$

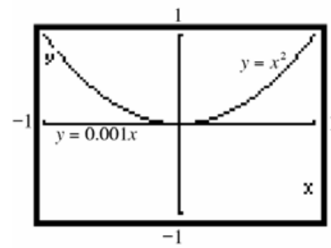
f. Graphing the secant lines:











g. The secant lines are beginning to look more and more like the tangent line to the graph of *f* at the point where $x = 0$.

h. The slopes of the secant lines are getting smaller and smaller. They seem to be approaching the number zero.

86. $f(x) = x^2$

a. Average rate of change of *f* from $x = 1$ to $x = 2$:

$$\frac{f(2) - f(1)}{2 - 1} = \frac{2^2 - 1^2}{1} = \frac{3}{1} = 3$$

b. Average rate of change of *f* from $x = 1$ to $x = 1.5$:

$$\frac{f(1.5) - f(1)}{1.5 - 1} = \frac{(1.5)^2 - 1^2}{0.5} = \frac{1.25}{0.5} = 2.5$$

c. Average rate of change of *f* from $x = 1$ to $x = 1.1$:

$$\frac{f(1.1) - f(1)}{1.1 - 1} = \frac{(1.1)^2 - 1^2}{0.1} = \frac{0.21}{0.1} = 2.1$$

d. Average rate of change of *f* from $x = 1$ to $x = 1.01$:

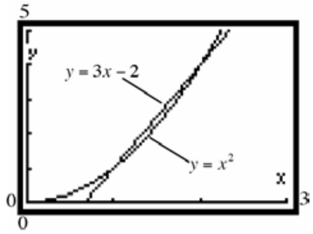
$$\frac{f(1.01) - f(1)}{1.01 - 1} = \frac{(1.01)^2 - 1^2}{0.01} = \frac{0.0201}{0.01} = 2.01$$

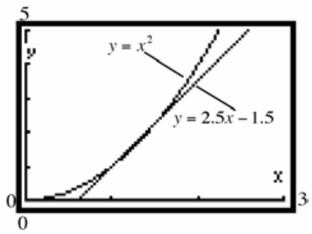
e. Average rate of change of *f* from $x = 1$ to $x = 1.001$:

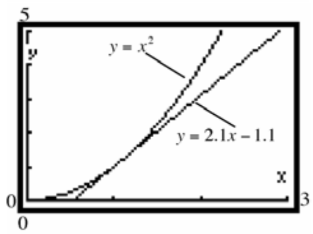
$$\frac{f(1.001) - f(1)}{1.001 - 1} = \frac{(1.001)^2 - 1^2}{0.001}$$

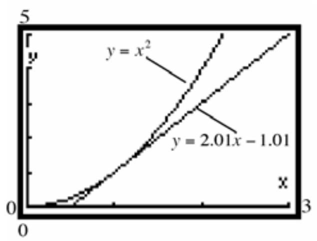
$$= \frac{0.002001}{0.001} = 2.001$$

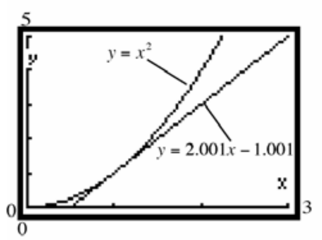
f. Graphing the secant lines:











g. The secant lines are beginning to look more and more like the tangent line to the graph of *f* at the point where $x = 1$.

h. The slopes of the secant lines are getting smaller and smaller. They seem to be approaching the number 2.

87. $f(x) = 2x + 5$

a. $m_{\text{sec}} = \dfrac{f(x+h) - f(x)}{h}$

$$= \frac{2(x+h) + 5 - 2x - 5}{h} = \frac{2h}{h} = 2$$

b. When $x = 1$:

$h = 0.5 \Rightarrow m_{\text{sec}} = 2$

$h = 0.1 \Rightarrow m_{\text{sec}} = 2$

$h = 0.01 \Rightarrow m_{\text{sec}} = 2$

as $h \to 0, \quad m_{\text{sec}} \to 2$

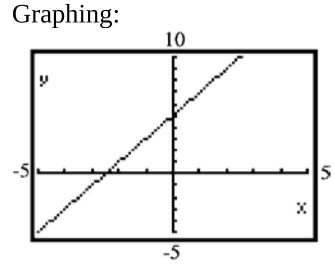
c. Using the point $\left(1, f(1)\right) = (1, 7)$ and slope, $m = 2$, we get the secant line:

$y - 7 = 2(x - 1)$

$y - 7 = 2x - 2$

$y = 2x + 5$

d. Graphing:



The graph and the secant line coincide.

88. $f(x) = -3x + 2$

a. $m_{\text{sec}} = \dfrac{f(x+h) - f(x)}{h}$

$$= \frac{-3(x+h) + 2 - (-3x + 2)}{h} = \frac{-3h}{h} = -3$$

b. When $x = 1$,

$h = 0.5 \Rightarrow m_{\text{sec}} = -3$

$h = 0.1 \Rightarrow m_{\text{sec}} = -3$

$h = 0.01 \Rightarrow m_{\text{sec}} = -3$

as $h \to 0, \quad m_{\text{sec}} \to -3$

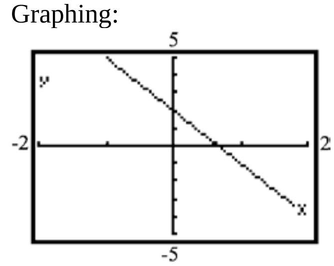
c. Using point $\left(1, f(1)\right) = (1, -1)$ and slope $= -3$, we get the secant line:

$y - (-1) = -3(x - 1)$

$y + 1 = -3x + 3$

$y = -3x + 2$

d. Graphing:



The graph and the secant line coincide.

89. $f(x)=x^2+2x$

a. $m_{\text{sec}}=\dfrac{f(x+h)-f(x)}{h}$

$=\dfrac{(x+h)^2+2(x+h)-(x^2+2x)}{h}$

$=\dfrac{x^2+2xh+h^2+2x+2h-x^2-2x}{h}$

$=\dfrac{2xh+h^2+2h}{h}$

$=2x+h+2$

b. When $x = 1$,

$h=0.5 \Rightarrow m_{\text{sec}}=2\cdot 1+0.5+2=4.5$

$h=0.1 \Rightarrow m_{\text{sec}}=2\cdot 1+0.1+2=4.1$

$h=0.01 \Rightarrow m_{\text{sec}}=2\cdot 1+0.01+2=4.01$

as $h\to 0$, $m_{\text{sec}}\to 2\cdot 1+0+2=4$

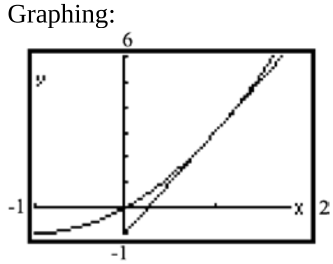
c. Using point $(1, f(1))=(1,3)$ and slope = 4.01, we get the secant line:

$y-3=4.01(x-1)$

$y-3=4.01x-4.01$

$y=4.01x-1.01$

d. Graphing:



90. $f(x)=2x^2+x$

a. $m_{\text{sec}}=\dfrac{f(x+h)-f(x)}{h}$

$=\dfrac{2(x+h)^2+(x+h)-(2x^2+x)}{h}$

$=\dfrac{2(x^2+2xh+h^2)+x+h-2x^2-x}{h}$

$=\dfrac{2x^2+4xh+2h^2+x+h-2x^2-x}{h}$

$=\dfrac{4xh+2h^2+h}{h}$

$=4x+2h+1$

b. When $x = 1$,

$h=0.5 \Rightarrow m_{\text{sec}}=4\cdot 1+2(0.5)+1=6$

$h=0.1 \Rightarrow m_{\text{sec}}=4\cdot 1+2(0.1)+1=5.2$

$h=0.01 \Rightarrow m_{\text{sec}}=4\cdot 1+2(0.01)+1=5.02$

as $h\to 0$, $m_{\text{sec}}\to 4\cdot 1+2(0)+1=5$

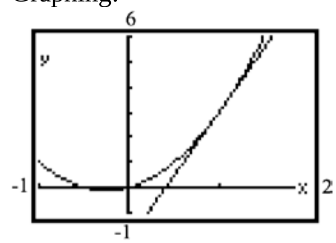
c. Using point $(1, f(1))=(1,3)$ and slope = 5.02, we get the secant line:

$y-3=5.02(x-1)$

$y-3=5.02x-5.02$

$y=5.02x-2.02$

d. Graphing:



91. $f(x)=2x^2-3x+1$

a. $m_{\text{sec}}=\dfrac{f(x+h)-f(x)}{h}$

$=\dfrac{2(x+h)^2-3(x+h)+1-(2x^2-3x+1)}{h}$

$=\dfrac{2(x^2+2xh+h^2)-3x-3h+1-2x^2+3x-1}{h}$

$=\dfrac{2x^2+4xh+2h^2-3x-3h+1-2x^2+3x-1}{h}$

$=\dfrac{4xh+2h^2-3h}{h}$

$=4x+2h-3$

b. When $x = 1$,

$h=0.5 \Rightarrow m_{\text{sec}}=4\cdot 1+2(0.5)-3=2$

$h=0.1 \Rightarrow m_{\text{sec}}=4\cdot 1+2(0.1)-3=1.2$

$h=0.01 \Rightarrow m_{\text{sec}}=4\cdot 1+2(0.01)-3=1.02$

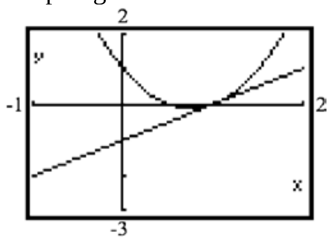
as $h\to 0$, $m_{\text{sec}}\to 4\cdot 1+2(0)-3=1$

c. Using point $(1, f(1))=(1,0)$ and slope = 1.02, we get the secant line:

$y-0=1.02(x-1)$

$y=1.02x-1.02$

d. Graphing:



92. $f(x) = -x^2 + 3x - 2$

a. $m_{\text{sec}} = \dfrac{f(x+h) - f(x)}{h}$

$= \dfrac{-(x+h)^2 + 3(x+h) - 2 - \left(-x^2 + 3x - 2\right)}{h}$

$= \dfrac{-(x^2 + 2xh + h^2) + 3x + 3h - 2 + x^2 - 3x + 2}{h}$

$= \dfrac{-x^2 - 2xh - h^2 + 3x + 3h - 2 + x^2 - 3x + 2}{h}$

$= \dfrac{-2xh - h^2 + 3h}{h}$

$= -2x - h + 3$

b. When $x = 1$,

$h = 0.5 \Rightarrow m_{\text{sec}} = -2 \cdot 1 - 0.5 + 3 = 0.5$

$h = 0.1 \Rightarrow m_{\text{sec}} = -2 \cdot 1 - 0.1 + 3 = 0.9$

$h = 0.01 \Rightarrow m_{\text{sec}} = -2 \cdot 1 - 0.01 + 3 = 0.99$

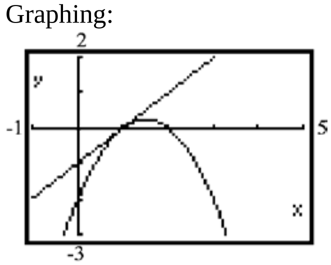
as $h \to 0$, $m_{\text{sec}} \to -2 \cdot 1 - 0 + 3 = 1$

c. Using point $\left(1, f(1)\right) = (1, 0)$ and slope = 0.99, we get the secant line:

$y - 0 = 0.99(x - 1)$

$y = 0.99x - 0.99$

d. Graphing:



93. $f(x) = \dfrac{1}{x}$

a. $m_{\text{sec}} = \dfrac{f(x+h) - f(x)}{h}$

$= \dfrac{\left(\dfrac{1}{x+h} - \dfrac{1}{x}\right)}{h} = \dfrac{\left(\dfrac{x - (x+h)}{(x+h)x}\right)}{h}$

$= \left(\dfrac{x - x - h}{(x+h)x}\right)\left(\dfrac{1}{h}\right) = \left(\dfrac{-h}{(x+h)x}\right)\left(\dfrac{1}{h}\right)$

$= -\dfrac{1}{(x+h)x}$

b. When $x = 1$,

$h = 0.5 \Rightarrow m_{\text{sec}} = -\dfrac{1}{(1+0.5)(1)}$

$= -\dfrac{1}{1.5} = -\dfrac{2}{3} \approx -0.667$

$h = 0.1 \Rightarrow m_{\text{sec}} = -\dfrac{1}{(1+0.1)(1)}$

$= -\dfrac{1}{1.1} = -\dfrac{10}{11} \approx -0.909$

$h = 0.01 \Rightarrow m_{\text{sec}} = -\dfrac{1}{(1+0.01)(1)}$

$= -\dfrac{1}{1.01} = -\dfrac{100}{101} \approx -0.990$

as $h \to 0$, $m_{\text{sec}} \to -\dfrac{1}{(1+0)(1)} = -\dfrac{1}{1} = -1$

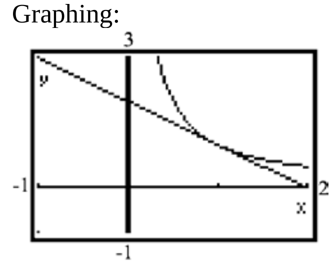
c. Using point $\left(1, f(1)\right) = (1, 1)$ and slope = $-\dfrac{100}{101}$, we get the secant line:

$y - 1 = -\dfrac{100}{101}(x - 1)$

$y - 1 = -\dfrac{100}{101}x + \dfrac{100}{101}$

$y = -\dfrac{100}{101}x + \dfrac{201}{101}$

d. Graphing:



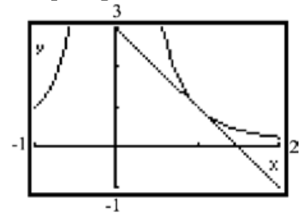
94. $f(x)=\dfrac{1}{x^2}$

a. $$m_{\text{sec}}=\frac{f(x+h)-f(x)}{h}$$
$$=\frac{\left(\dfrac{1}{(x+h)^2}-\dfrac{1}{x^2}\right)}{h}$$
$$=\frac{\left(\dfrac{x^2-(x+h)^2}{(x+h)^2x^2}\right)}{h}$$
$$=\left(\frac{x^2-\left(x^2+2xh+h^2\right)}{(x+h)^2x^2}\right)\left(\frac{1}{h}\right)$$
$$=\left(\frac{-2xh-h^2}{(x+h)^2x^2}\right)\left(\frac{1}{h}\right)$$
$$=\frac{-2x-h}{(x+h)^2x^2}=\frac{-2x-h}{\left(x^2+2xh+h^2\right)x^2}$$

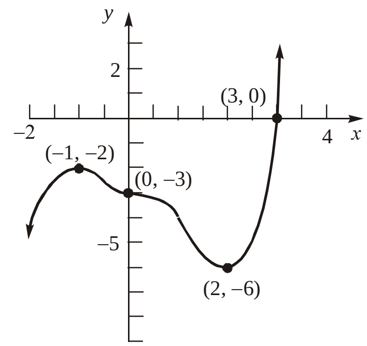
b. When $x = 1$,
$$h=0.5\Rightarrow m_{\text{sec}}=\frac{-2\cdot1-0.5}{(1+0.5)^2 1^2}=-\frac{10}{9}\approx-1.1111$$
$$h=0.1\Rightarrow m_{\text{sec}}=\frac{-2\cdot1-0.1}{(1+0.1)^2 1^2}=-\frac{210}{121}\approx-1.7355$$
$$h=0.01\Rightarrow m_{\text{sec}}=\frac{-2\cdot1-0.01}{(1+0.01)^2 1^2}$$
$$=-\frac{20{,}100}{10{,}201}\approx-1.9704$$
as $h\to0,\ m_{\text{sec}}\to\dfrac{-2\cdot1-0}{(1+0)^2 1^2}=-2$

c. Using point $(1, f(1))=(1,1)$ and slope = -1.9704, we get the secant line:
$$y-1=-1.9704(x-1)$$
$$y-1=-1.9704x+1.9704$$
$$y=-1.9704x+2.9704$$

d. Graphing:



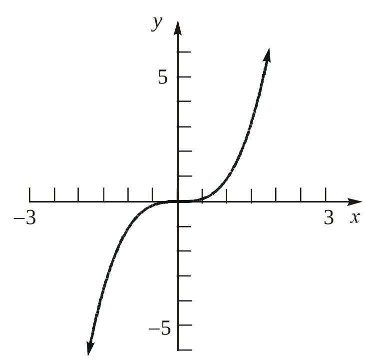
95. Answers will vary. One possibility follows:



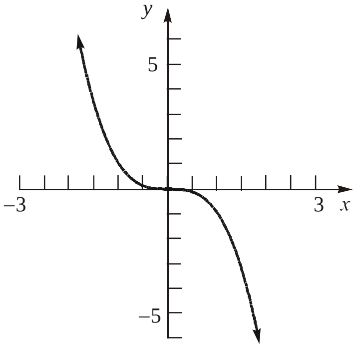
96. Answers will vary. See solution to Problem 89 for one possibility.

97. A function that is increasing on an interval can have at most one *x*-intercept on the interval. The graph of *f* could not "turn" and cross it again or it would start to decrease.

98. An increasing function is a function whose graph goes up as you read from left to right.

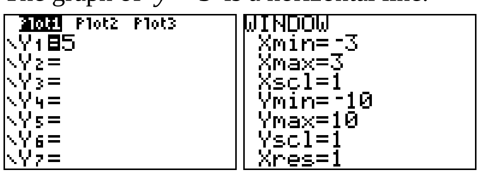


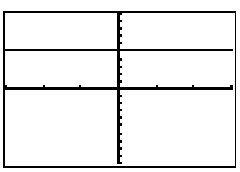
A decreasing function is a function whose graph goes down as you read from left to right.



99. To be an even function we need $f(-x) = f(x)$ and to be an odd function we need $f(-x) = -f(x)$. In order for a function be both even and odd, we would need $f(x) = -f(x)$. This is only possible if $f(x) = 0$.

100. The graph of $y = 5$ is a horizontal line.





The local maximum is $y = 5$ and it occurs at each *x*-value in the interval.

101. Not necessarily. It just means $f(5) > f(2)$. The function could have both increasing and decreasing intervals.

102. $$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{b - b}{x_2 - x_1} = 0$$
$$\frac{f(2) - f(-2)}{2 - (-2)} = \frac{0 - 0}{4} = 0$$

103. $$f(-3) = \frac{(-3)^2}{2(-3) + 5}$$
$$= \frac{9}{-6 + 5} = \frac{9}{-1} = -9$$
So the corresponding point is: $(-3, -9)$

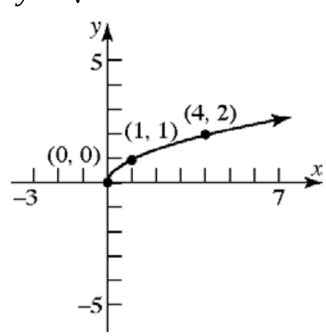
104. Let x be the number of miles driven. Then 0.80 represents the mileage charge. Let 40 be the fixed charge. Then the cost C to rent the truck is given by: $C(x) = 0.80x + 40$

105. The slope of the perpendicular line would be
$$-\frac{1}{m} = -\frac{1}{3/5} = -\frac{5}{3}$$
$$y - y_1 = m(x - x_1)$$
$$y - (-1) = -\frac{5}{3}(x - 3)$$
$$y + 1 = -\frac{5}{3}x + 5$$
$$y = -\frac{5}{3}x + 4$$

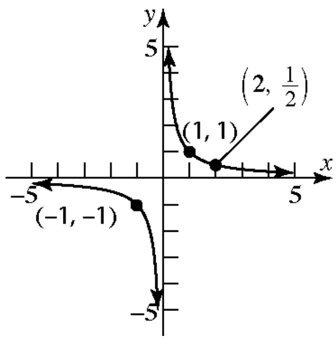
106. $$\frac{f(x+h) - f(x)}{h} = \frac{3(x+h)^2 - 5(x+h) - (3x^2 - 5x)}{h}$$
$$= \frac{3(x^2 + 2xh + h^2) - 5x - 5h - 3x^2 + 5x}{h}$$
$$= \frac{3x^2 + 6xh + 3h^2 - 5x - 5h - 3x^2 + 5x}{h}$$
$$= \frac{6xh + 3h^2 - 5h}{h} = \frac{h(6x + 3h - 5)}{h}$$
$$= 6x + 3h - 5$$

Section 2.4

1. $y = \sqrt{x}$



2. $y = \frac{1}{x}$



3. $y = x^3 - 8$

<u>*y*-intercept:</u>
Let $x = 0$, then $y = (0)^3 - 8 = -8$.

<u>*x*-intercept:</u>
Let $y = 0$, then $0 = x^3 - 8$

$$x^3 = 8$$
$$x = 2$$

The intercepts are $(0,-8)$ and $(2,0)$.

4. $(-\infty, 0)$

5. piecewise-defined

6. True

7. False; the cube root function is odd and increasing on the interval $(-\infty, \infty)$.

8. False; the domain and range of the reciprocal function are both the set of real numbers except for 0.

9. b

10. a

11. C

12. A

13. E

14. G

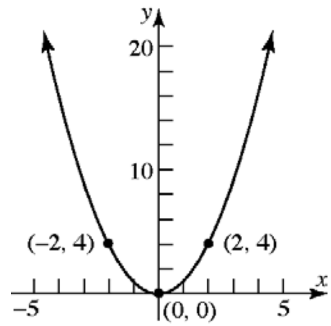
15. B

16. D

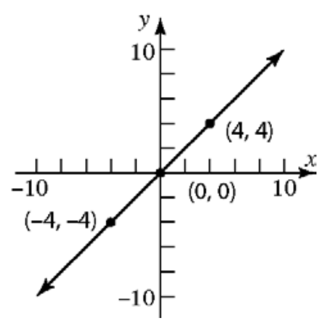
17. F

18. H

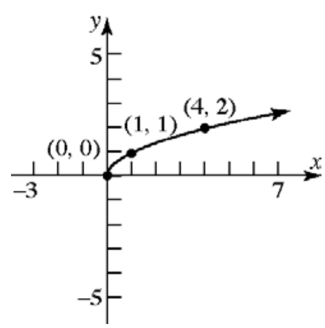
19. $f(x) = x^2$



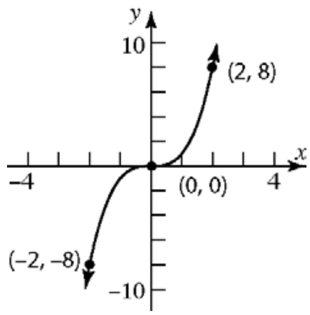
20. $f(x) = x$



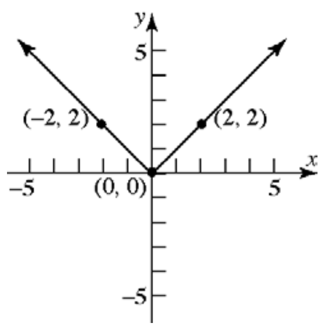
21. $f(x) = \sqrt{x}$



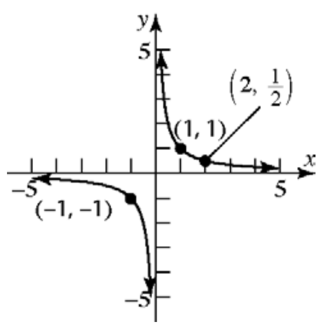
22. $f(x) = x^3$



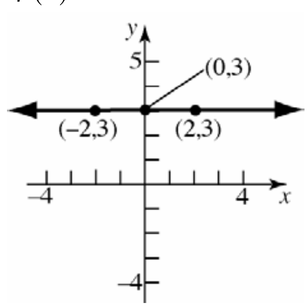
23. $f(x) = |x|$



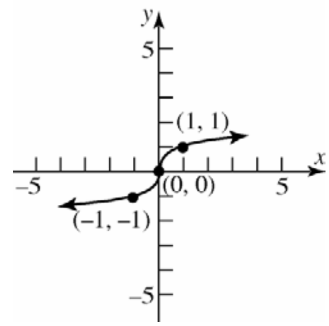
24. $f(x) = \dfrac{1}{x}$



25. $f(x) = 3$



26. $f(x) = \sqrt[3]{x}$



27. **a.** $f(-2) = (-2)^2 = 4$

b. $f(0) = 2$

c. $f(2) = 2(2) + 1 = 5$

28. **a.** $f(-2) = -3(-2) = 6$

b. $f(-1) = 0$

c. $f(0) = 2(0)^2 + 1 = 1$

29. **a.** $f(0) = 2(0) - 4 = -4$

b. $f(1) = 2(1) - 4 = -2$

c. $f(2) = 2(2) - 4 = 0$

d. $f(3) = (3)^3 - 2 = 25$

30. **a.** $f(-1) = (-1)^3 = -1$

b. $f(0) = (0)^3 = 0$

c. $f(1) = 3(1) + 2 = 5$

d. $f(3) = 3(3) + 2 = 11$

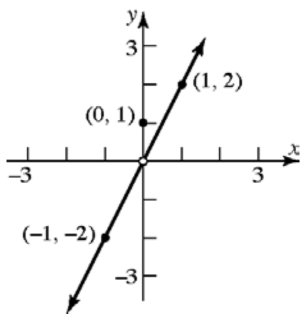
31. $f(x) = \begin{cases} 2x & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$

a. Domain: $\{x \mid x \text{ is any real number}\}$

b. x-intercept: none
y-intercept:
$f(0) = 1$

The only intercept is $(0,1)$.

c. Graph:



d. Range: $\{y \mid y \neq 0\}$; $(-\infty, 0) \cup (0, \infty)$

e. The graph is not continuous. There is a jump at $x = 0$.

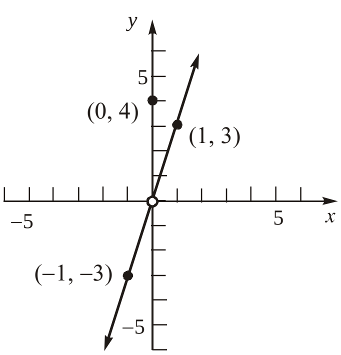
32. $f(x) = \begin{cases} 3x & \text{if } x \neq 0 \\ 4 & \text{if } x = 0 \end{cases}$

a. Domain: $\{x \mid x \text{ is any real number}\}$

b. x-intercept: none
y-intercept: $f(0) = 4$

The only intercept is $(0,4)$.

c. Graph:



d. Range: $\{y \mid y \neq 0\}$; $(-\infty, 0) \cup (0, \infty)$

e. The graph is not continuous. There is a jump at $x = 0$.

33. $f(x) = \begin{cases} x+3 & \text{if } x < -2 \\ -2x-3 & \text{if } x \geq -2 \end{cases}$

a. Domain: $\{x \mid x \text{ is any real number}\}$

b. $x+3=0$ $\qquad$ $-2x-3=0$

$x=-3$ $\qquad$ $-2x=3$

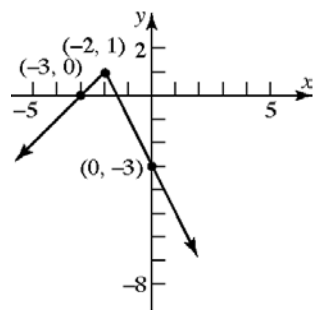
$x=-\dfrac{3}{2}$

x-intercepts: $-3, -\dfrac{3}{2}$

y-intercept: $f(0) = -2(0) - 3 = -3$

The intercepts are $(-3,0)$, $\left(-\dfrac{3}{2},0\right)$, and $(0,-3)$.

c. Graph:



d. Range: $\{y \mid y \leq 1\}$; $(-\infty, 1]$

e. The graph is continuous. There are no holes or gaps.

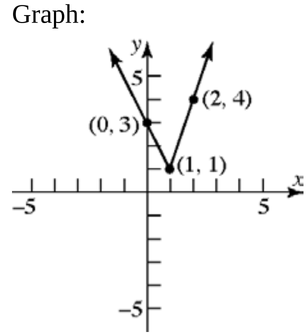
34. $f(x) = \begin{cases} -2x+3 & \text{if } x < 1 \\ 3x-2 & \text{if } x \geq 1 \end{cases}$

a. Domain: $\{x \mid x \text{ is any real number}\}$

b. x-intercept: none
y-intercept: $f(0) = -2(0) + 3 = 3$

The only intercept is $(0,3)$.

c. Graph:



d. Range: $\{y \mid y \geq 1\}$; $[1, \infty)$

e. The graph is continuous. There are no holes or gaps.

35. $f(x) = \begin{cases} 2x+5 & \text{if } -3 \leq x < 0 \\ -3 & \text{if } x = 0 \\ -5x & \text{if } x > 0 \end{cases}$

a. Domain: $\{x \mid x \geq -3\}$; $[-3, \infty)$

b.

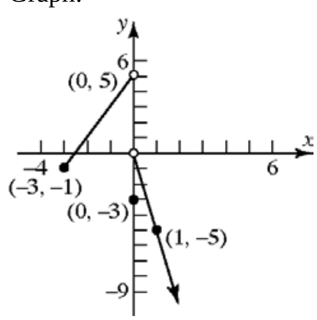
$$\begin{array}{ll} 2x+5=0 & -5x=0 \\ 2x=-5 & x=0 \\ x=-\dfrac{5}{2} & \text{(not in domain of piece)} \end{array}$$

x-intercept: $-\dfrac{5}{2}$

y-intercept: $f(0) = -3$

The intercepts are $\left(-\dfrac{5}{2}, 0\right)$ and $(0, -3)$.

c. Graph:



d. Range: $\{y \mid y < 5\}$; $(-\infty, 5)$

e. The graph is not continuous. There is a jump at $x = 0$.

36. $f(x) = \begin{cases} x+3 & \text{if } -2 \leq x < 1 \\ 5 & \text{if } x = 1 \\ -x+2 & \text{if } x > 1 \end{cases}$

a. Domain: $\{x \mid x \geq -2\}$; $[-2, \infty)$

b.

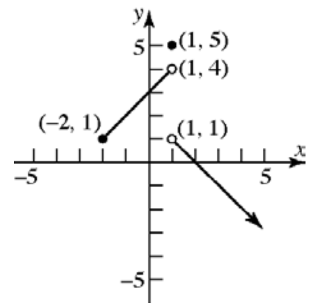
$$\begin{array}{ll} x+3=0 & -x+2=0 \\ x=-3 & -x=-2 \\ \text{(not in domain)} & x=2 \end{array}$$

x-intercept: 2

y-intercept: $f(0) = 0 + 3 = 3$

The intercepts are $(2, 0)$ and $(0, 3)$.

c. Graph:



d. Range: $\{y \mid y < 4, \ y = 5\}$; $(-\infty, 4) \cup \{5\}$

e. The graph is not continuous. There is a jump at $x = 1$.

37. $f(x) = \begin{cases} \dfrac{1}{x} & \text{if } x < 0 \\ \sqrt[3]{x} & \text{if } x \geq 0 \end{cases}$

a. Domain: $\{x \mid x \text{ is any real number}\}$

b.

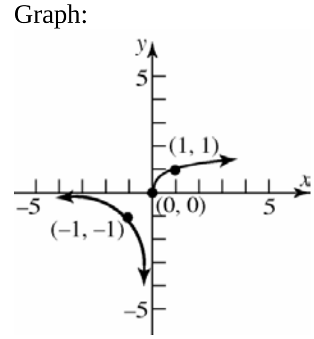
$$\begin{array}{ll} \dfrac{1}{x}=0 & \sqrt[3]{x}=0 \\ \text{(no solution)} & x=0 \end{array}$$

x-intercept: 0

y-intercept: $f(0) = \sqrt[3]{0} = 0$

The only intercept is $(0, 0)$.

c. Graph:



d. Range: $\{y \mid y \text{ is any real number}\}$

e. The graph is not continuous. There is a break at $x = 0$.

38. $f(x) = \begin{cases} 1+x & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$

a. Domain: $\{x \mid x \text{ is any real number}\}$

b. $1+x=0 \qquad x^2=0$

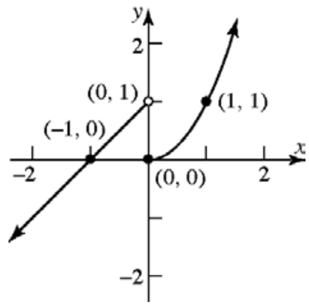
$x=-1 \qquad x=0$

x-intercepts: $-1, 0$

y-intercept: $f(0)=0^2=0$

The intercepts are $(-1,0)$ and $(0,0)$.

c. Graph:



d. Range: $\{y \mid y \text{ is any real number}\}$

e. The graph is not continuous. There is a jump at $x=0$.

39. $f(x)=\begin{cases} 2-x & \text{if } -3\le x<1 \\ \sqrt{x} & \text{if } x>1 \end{cases}$

a. Domain: $\{x \mid -3\le x<1 \text{ and } x>1\}$ or $\{x \mid x\ge -3, x\ne 1\}$; $[-3,1)\cup(1,\infty)$.

b. $2-x=0 \qquad \sqrt{x}=0$

$x=2 \qquad x=0$

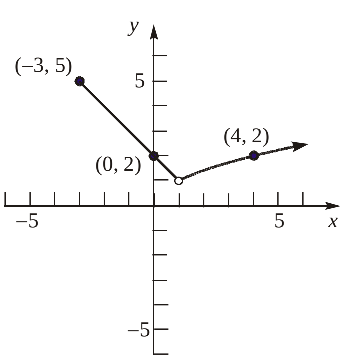
(not in domain of piece)

no x-intercepts

y-intercept: $f(0)=2-0=2$

The intercept is $(0,2)$.

c. Graph:



d. Range: $\{y \mid y>1\}$; $(1,\infty)$

e. The graph is not continuous. There is a hole at $x=1$.

40. $f(x)=\begin{cases} |x| & \text{if } -2\le x<0 \\ x^3 & \text{if } x>0 \end{cases}$

a. Domain: $\{x \mid -2\le x<0 \text{ and } x>0\}$ or $\{x \mid x\ge -2, x\ne 0\}$; $[-2,0)\cup(0,\infty)$.

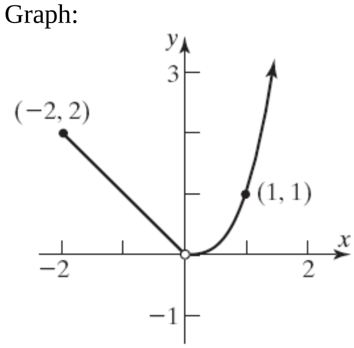
b. x-intercept: none

There are no x-intercepts since there are no values for x such that $f(x)=0$.

y-intercept:

There is no y-intercept since $x=0$ is not in the domain.

c. Graph:



d. Range: $\{y \mid y>0\}$; $(0,\infty)$

e. The graph is not continuous. There is a hole at $x=0$.

41. $f(x)=\text{int}(2x)$

a. Domain: $\{x \mid x \text{ is any real number}\}$

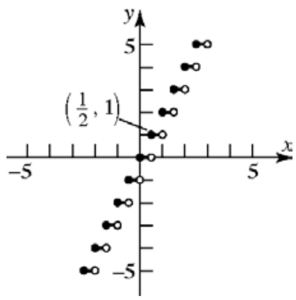
b. x-intercepts:

All values for x such that $0\le x<\frac{1}{2}$.

y-intercept: $f(0)=\text{int}(2(0))=\text{int}(0)=0$

The intercepts are all ordered pairs $(x, 0)$ when $0\le x<\frac{1}{2}$.

c. Graph:



d. Range: $\{y \mid y \text{ is an integer}\}$

e. The graph is not continuous. There is a jump at each $x = \dfrac{k}{2}$, where k is an integer.

42. $f(x) = 2\,\text{int}(x)$

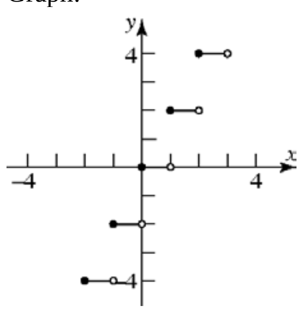
a. Domain: $\{x \mid x \text{ is any real number}\}$

b. x-intercepts:
All values for x such that $0 \le x < 1$.

y-intercept: $f(0) = 2\,\text{int}(0) = 0$

The intercepts are all ordered pairs $(x, 0)$ when $0 \le x < 1$.

c. Graph:



d. Range: $\{y \mid y \text{ is an even integer}\}$

e. The graph is not continuous. There is a jump at each integer value of x.

43. Answers may vary. One possibility follows:

$$f(x) = \begin{cases} x & \text{if } -1 \le x \le 0 \\ 1 & \text{if } 0 < x \le 2 \end{cases}$$

44. Answers may vary. One possibility follows:

$$f(x) = \begin{cases} -x & \text{if } -1 \le x \le 0 \\ \dfrac{1}{2}x & \text{if } 0 < x \le 2 \end{cases}$$

45. Answers may vary. One possibility follows:

$$f(x) = \begin{cases} 2x + 2 & \text{if } -1 \le x \le 0 \\ x & \text{if } x > 0 \end{cases}$$

46. Answers may vary. One possibility follows:

$$f(x) = \begin{cases} -x & \text{if } x \le 0 \\ -x + 2 & \text{if } 0 < x \le 2 \end{cases}$$

47. a. $f(1.2) = \text{int}\big(2(1.2)\big) = \text{int}(2.4) = 2$

b. $f(1.6) = \text{int}\big(2(1.6)\big) = \text{int}(3.2) = 3$

c. $f(-1.8) = \text{int}\big(2(-1.8)\big) = \text{int}(-3.6) = -4$

48. a. $f(1.2) = \text{int}\left(\dfrac{1.2}{2}\right) = \text{int}(0.6) = 0$

b. $f(1.6) = \text{int}\left(\dfrac{1.6}{2}\right) = \text{int}(0.8) = 0$

c. $f(-1.8) = \text{int}\left(\dfrac{-1.8}{2}\right) = \text{int}(-0.9) = -1$

49. $C = \begin{cases} 19.99 \text{ } if \text{ } 0 < x \le 250 \\ 0.25x - 42.51 \text{ if } x > 250 \end{cases}$

a. $C(100) = \$19.99$

b. $C(320) = 0.25(320) - 42.51$
$= \$37.49$

c. $C(251) = 0.25(251) - 42.51$
$= \$20.24$

50. $F(x) = \begin{cases} 3 & \text{if } 0 < x \le 3 \\ 5\,\text{int}(x+1) + 1 & \text{if } 3 < x < 9 \\ 50 & \text{if } 9 \le x \le 24 \end{cases}$

a. $F(2) = 3$
Parking for 2 hours costs \$3.

b. $F(7) = 5\,\text{int}(7+1) + 1 = 41$
Parking for 7 hours costs \$41.

c. $F(15) = 50$
Parking for 15 hours costs \$50.

d. $24 \text{ min} \cdot \dfrac{1 \text{ hr}}{60 \text{ min}} = 0.4 \text{ hr}$

$F(8.4) = 5\,\text{int}(8.4+1) + 1 = 5(9) + 1 = 46$

Parking for 8 hours and 24 minutes costs $46.

51. a. Charge for 20 therms:

$$C = 19.50 + 0.91686(20) + 0.3313(20)$$
$$= \$44.46$$

b. Charge for 150 therms:

$$C = 19.50 + 0.91686(30) + 0.3313(30)$$
$$+ 0.5757(120)$$
$$= \$126.03$$

c. For $0 \le x \le 30$:

$$C = 19.50 + 0.91686x + 0.3313x$$
$$= 1.24816x + 19.50$$

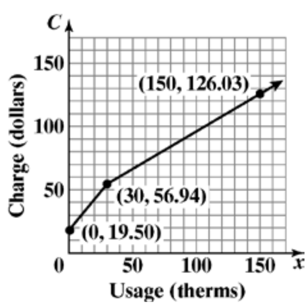
For $x > 30$:

$$C = 19.50 + 0.91686(30) + 0.5757(x-30)$$
$$+ 0.3313(30)$$
$$= 19.50 + 27.5058 + 0.5757x - 17.271$$
$$+ 9.939$$
$$= 0.5757x + 39.6738$$

The monthly charge function:

$$C = \begin{cases} 1.24816x + 19.50 & \text{for } 0 \le x \le 30 \\ 0.5757x + 39.6738 & \text{for } x > 30 \end{cases}$$

d. Graph:



52. a. Charge for 1000 therms:

$$C = 72.60 + 0.1201(150) + 0.0549(850)$$
$$+ 0.68(1000)$$
$$= \$817.28$$

b. Charge for 6000 therms:

$$C = 72.60 + 0.1201(150) + 0.0549(4850)$$
$$+0.0482(1000) + 0.68(6000)$$
$$= \$4485.08$$

c. For $0 \le x \le 150$:

$$C = 72.60 + 0.1201x + 0.68x$$
$$= 0.8001x + 72.60$$

For $150 < x \le 5000$:

$$C = 72.60 + 0.1201(150) + 0.0549(x-150)$$
$$+ 0.68x$$
$$= 72.60 + 18.015 + 0.0549x - 8.235$$
$$+ 0.68x$$
$$= 0.7349x + 82.38$$

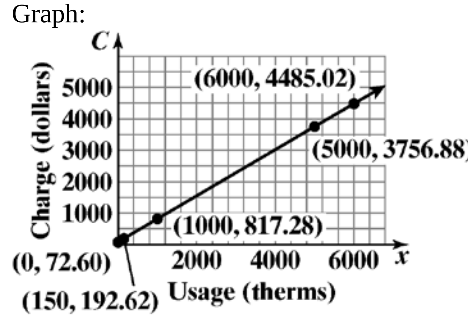
For $x > 5000$:

$$C = 72.60 + 0.1201(150) + 0.0549(4850)$$
$$+ 0.0482(x-5000) + 0.68x$$
$$= 72.60 + 18.015 + 266.265 + 0.0482x - 241$$
$$+ 0.68x$$
$$= 0.7282x + 115.88$$

The monthly charge function:

$$C(x) = \begin{cases} 0.8001x + 72.60 & \text{if} \quad 0 \le x \le 150 \\ 0.7349x + 82.38 & \text{if} \quad 150 < x \le 5000 \\ 0.7282x + 115.88 & \text{if} \quad x > 5000 \end{cases}$$

d. Graph:



53. For schedule X:

$$f(x) = \begin{cases} 0.11x \; if \; 0 < x \le 8000 \\ 880 + 0.17(x-8000) \; if \; 80000 < x < x30,600 \\ 4722 + 0.24(x-30,600) \; if \; 30,600 < x < 74,100 \\ 15162 + 0.32(x-74100) \; x > 74,100 \end{cases}$$

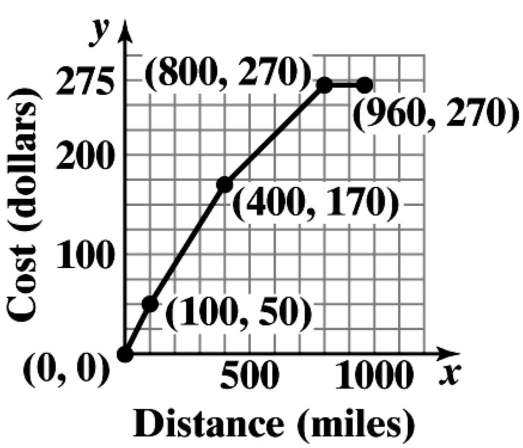
54. For Schedule Y − 1:

$$f(x)=\begin{cases} 0.10x & \text{if } 0 < x \le 18{,}150 \\ 1815.00+0.15(x-18{,}150) & \text{if } 18{,}150 < x \le 73{,}800 \\ 10{,}162.50+0.25(x-73{,}800) & \text{if } 73{,}800 < x \le 148{,}850 \\ 28{,}925.00+0.28(x-148{,}850) & \text{if } 148{,}850 < x \le 226{,}850 \\ 50{,}765.00+0.33(x-226{,}850) & \text{if } 226{,}850 < x \le 405{,}100 \\ 109{,}587.50+0.35(x-405{,}100) & \text{if } 405{,}100 < x \le 457{,}600 \\ 127{,}962.50+0.396(x-457{,}600) & \text{if } x > 457{,}600 \end{cases}$$

55. **a.** Let x represent the number of miles and C be the cost of transportation.

$$C(x)=\begin{cases} 0.50x & \text{if } 0 \le x \le 100 \\ 0.50(100)+0.40(x-100) & \text{if } 100 < x \le 400 \\ 0.50(100)+0.40(300)+0.25(x-400) & \text{if } 400 < x \le 800 \\ 0.50(100)+0.40(300)+0.25(400)+0(x-800) & \text{if } 800 < x \le 960 \end{cases}$$

$$C(x)=\begin{cases} 0.50x & \text{if } 0 \le x \le 100 \\ 10+0.40x & \text{if } 100 < x \le 400 \\ 70+0.25x & \text{if } 400 < x \le 800 \\ 270 & \text{if } 800 < x \le 960 \end{cases}$$

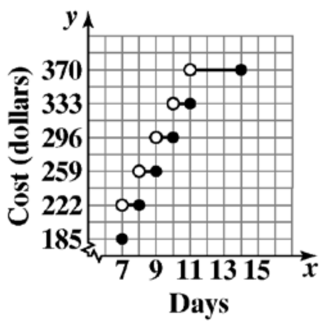


b. For hauls between 100 and 400 miles the cost is: $C(x)=10+0.40x$.

c. For hauls between 400 and 800 miles the cost is: $C(x)=70+0.25x$.

56. Let x = number of days car is used. The cost of renting is given by

$$C(x)=\begin{cases}185 & \text{if } x=7\\ 222 & \text{if } 7<x\le 8\\ 259 & \text{if } 8<x\le 9\\ 296 & \text{if } 9<x\le 10\\ 333 & \text{if } 10<x\le 11\\ 370 & \text{if } 11<x\le 14\end{cases}$$



57. a. Let s = the credit score of an individual who wishes to borrow \$250,000 with an 80% LTV ratio. The adverse market delivery charge is given by

$C(S)=12,250 \textit{If } s\le 659$

$8750 \textit{if } 660\le s\le 679$

$5250 \textit{if } 680\le s\le 699$

$3500 \textit{if } 700\le s\le 719$

$2625 \textit{if } 720\le s\le 739$

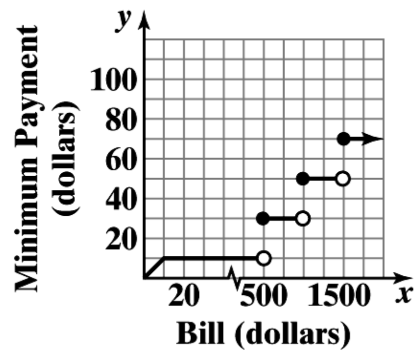
$875 \textit{if } s\ge 740$

b. 737 is between 720 and 739 so the charge would be \$2625.

c. 664 is between 660 and 679 so the charge would be \$8750.

58. Let x = the amount of the bill in dollars. The minimum payment due is given by

$$f(x)=\begin{cases}x & \text{if } 0\le x<10\\ 10 & \text{if } 10\le x<500\\ 30 & \text{if } 500\le x<1000\\ 50 & \text{if } 1000\le x<1500\\ 70 & \text{if } x\ge 1500\end{cases}$$



59. a. $W=10°C$

b. $W=33-\dfrac{(10.43+10\sqrt{15}-15)(33-10)}{22.03}\approx -3°C$

c. $w=33-1\cdot 5957(33-10)$

$\approx -4°C$

60. a. $W=-10°C$

b. $W=33-\dfrac{\left(10.45+10\sqrt{5}-5\right)\left(33-(-10)\right)}{22.04}$

$\approx -21°C$

c. $W=33-\dfrac{\left(10.45+10\sqrt{15}-15\right)\left(33-(-10)\right)}{22.04}$

$\approx -34°C$

d. $W=33-1.5958\left(33-(-10)\right)=-36°C$

61. Let x = the number of ounces and $C(x)$ = the postage due.

For $0<x\le 1$: $C(x)=\$0.98$

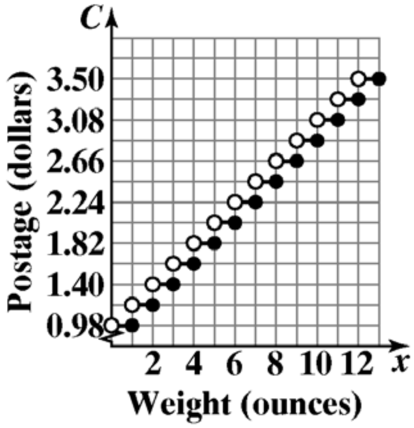
For $1<x\le 2$: $C(x)=0.98+0.21=\$1.19$

For $2<x\le 3$: $C(x)=0.98+2(0.21)=\$1.40$

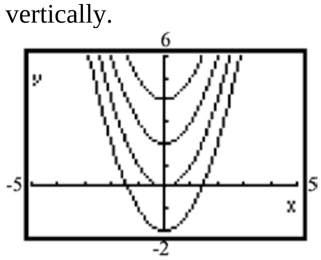
For $3<x\le 4$: $C(x)=0.98+3(0.21)=\$1.61$

$\vdots$

For $12 < x \le 13$: $C(x) = 0.98 + 12(0.21) = \$3.50$

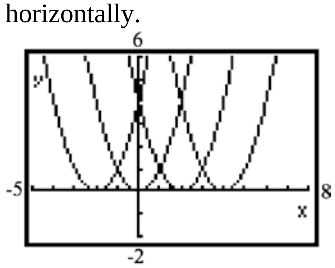


62. Each graph is that of $y = x^2$, but shifted vertically.



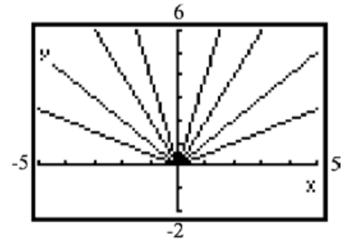
If $y = x^2 + k$, $k > 0$, the shift is up k units; if $y = x^2 - k$, $k > 0$, the shift is down k units. The graph of $y = x^2 - 4$ is the same as the graph of $y = x^2$, but shifted down 4 units. The graph of $y = x^2 + 5$ is the graph of $y = x^2$, but shifted up 5 units.

63. Each graph is that of $y = x^2$, but shifted horizontally.



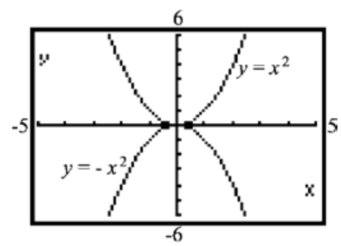
If $y = (x-k)^2$, $k > 0$, the shift is to the right k units; if $y = (x+k)^2$, $k > 0$, the shift is to the left k units. The graph of $y = (x+4)^2$ is the same as the graph of $y = x^2$, but shifted to the left 4 units. The graph of $y = (x-5)^2$ is the graph of $y = x^2$, but shifted to the right 5 units.

64. Each graph is that of $y = |x|$, but either compressed or stretched vertically.

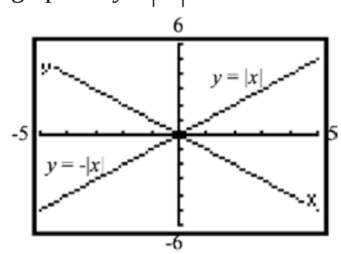


If $y = k|x|$ and $k > 1$, the graph is stretched vertically; if $y = k|x|$ and $0 < k < 1$, the graph is compressed vertically. The graph of $y = \frac{1}{4}|x|$ is the same as the graph of $y = |x|$, but compressed vertically. The graph of $y = 5|x|$ is the same as the graph of $y = |x|$, but stretched vertically.

65. The graph of $y = -x^2$ is the reflection of the graph of $y = x^2$ about the x-axis.

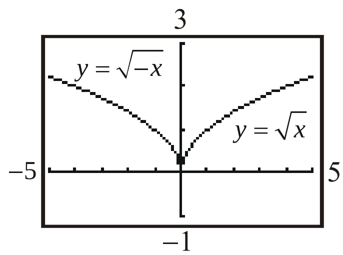


The graph of $y = -|x|$ is the reflection of the graph of $y = |x|$ about the x-axis.

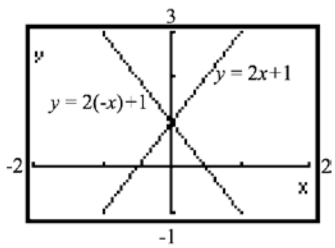


Multiplying a function by –1 causes the graph to be a reflection about the x-axis of the original function's graph.

66. The graph of $y=\sqrt{-x}$ is the reflection about the y-axis of the graph of $y=\sqrt{x}$.

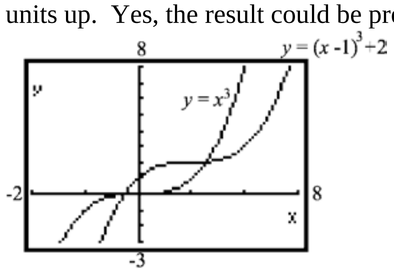


The same type of reflection occurs when graphing $y=2x+1$ and $y=2(-x)+1$.

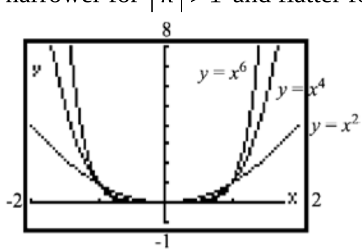


The graph of $y=f(-x)$ is the reflection about the y-axis of the graph of $y=f(x)$.

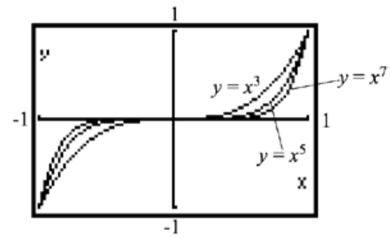
67. The graph of $y=(x-1)^3+2$ is a shifting of the graph of $y=x^3$ one unit to the right and two units up. Yes, the result could be predicted.



68. The graphs of $y=x^n$, n a positive even integer, **are all U-shaped and open upward. All go** through the points $(-1,1)$, $(0,0)$, and $(1,1)$. As n increases, the graph of the function is narrower for $|x|>1$ and flatter for $|x|<1$.



69. The graphs of $y=x^n$, n a positive odd integer, all have the same general shape. All go through the points $(-1,-1)$, $(0,0)$, and $(1,1)$. As n increases, the graph of the function increases at a greater rate for $|x|>1$ and is flatter around 0 for $|x|<1$.



70. $f(x)=\begin{cases}1 & \text{if } x \text{ is rational}\\ 0 & \text{if } x \text{ is irrational}\end{cases}$

Yes, it is a function.

Domain = $\{x \mid x \text{ is any real number}\}$ or $(-\infty,\infty)$

Range = {0, 1} or $\{y \mid y=0 \text{ or } y=1\}$

y-intercept: $x=0 \Rightarrow x$ is rational $\Rightarrow y=1$

So the y-intercept is $y=1$.

x-intercept: $y=0 \Rightarrow x$ is irrational

So the graph has infinitely many x-intercepts, namely, there is an x-intercept at each irrational value of x.

$f(-x)=1=f(x)$ when x is rational;

$f(-x)=0=f(x)$ when x is irrational.

Thus, f is even.

The graph of f consists of 2 infinite clusters of distinct points, extending horizontally in both directions. One cluster is located 1 unit above the x-axis, and the other is located along the x-axis.

71. For $0<x<1$, the graph of $y=x^r$, r rational and $r>0$, flattens down toward the x-axis as r gets bigger. For $x>1$, the graph of $y=x^r$ increases at a greater rate as r gets bigger.

72.
$$\begin{aligned}\sqrt{x^2-4}+7&=10\\ \sqrt{x^2-4}&=3\\ x^2-4&=9\\ x^2&=13\\ x&=\pm\sqrt{13}\end{aligned}$$

73.
$$x^2 + y^2 = 6y + 16$$
$$x^2 + y^2 - 6y = 16$$
$$x^2 + (y^2 - 6y + 9) = 16 + 9$$
$$x^2 + (y-3)^2 = 5^2$$

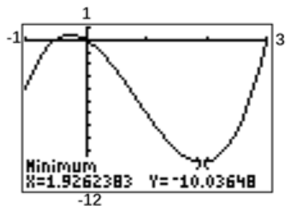
Center (h,k): (0, 3); Radius = 5

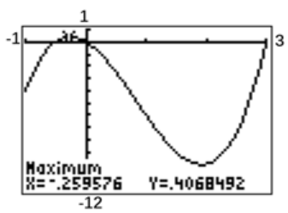
74.
$$3x - 4y = 12$$
$$-4y = -3x + 12$$
$$y = \frac{3}{4}x - 3$$

The lines would have equal slope so the slope would be $\frac{3}{4}$.

75. $f(x) = 2x^3 - 5x^2 - 3x$ on the interval $(-1, 3)$

Use MAXIMUM and MINIMUM on the graph of $y_1 = 2x^3 - 5x^2 - 3x$.





local maximum: $f(-0.26) \approx 0.41$

local minimum: $f(1.93) \approx -10.04$

Section 2.5

1. horizontal; right

2. y

3. False

4. True; the graph of $y = -f(x)$ is the reflection about the x-axis of the graph of $y = f(x)$.

5. d

6. a

7. E

8. B

9. D

10. H

11. A

12. I

13. C

14. L

15. J

16. F

17. K

18. G

19. $y = (x+4)^3$

20. $y = (x-4)^3$

21. $y = x^3 - 4$

22. $y = x^3 + 4$

23. $y = -x^3$

24. $y = (-x)^3 = -x^3$

25. $y = \left(\frac{1}{4}x\right)^3 = \frac{1}{64}x^3$

26. $y = 4x^3$

27. (1) $y = \sqrt{x} + 2$

(2) $y = -\left(\sqrt{x} + 2\right)$

(3) $y = -\left(\sqrt{-x} + 2\right) = -\sqrt{-x} - 2$

28. (1) $y = -\sqrt{x}$

(2) $y = -\sqrt{x-3}$

(3) $y = -\sqrt{x-3} - 2$

29. (1) $y=\sqrt{x}+2$

(2) $y=\sqrt{-x}+2$

(3) $y=\sqrt{-(x+3)}+2=\sqrt{-x-3}+2$

30. (1) $y=-\sqrt{x}$

(2) $y=-\sqrt{x}+2$

(3) $y=-\sqrt{x+3}+2$

31. (c); To go from $y=f(x)$ to $y=-f(x)$ we reflect about the *x*-axis. This means we change the sign of the *y*-coordinate for each point on the graph of $y=f(x)$. Thus, the point $(3, 6)$ would become $(3,-6)$.

32. (d); To go from $y=f(x)$ to $y=f(-x)$, we reflect each point on the graph of $y=f(x)$ about the *y*-axis. This means we change the sign of the *x*-coordinate for each point on the graph of $y=f(x)$. Thus, the point $(3,6)$ would become $(-3,6)$.

33. (c); To go from $y=f(x)$ to $y=2f(x)$, we stretch vertically by a factor of 2. Multiply the *y*-coordinate of each point on the graph of $y=f(x)$ by 2. Thus, the point $(1,3)$ would become $(1,6)$.

34. (c); To go from $y=f(x)$ to $y=f(2x)$, we compress horizontally by a factor of 2. Divide the *x*-coordinate of each point on the graph of $y=f(x)$ by 2. Thus, the point $(4,2)$ would become $(2,2)$.

35. **a.** The graph of $y=f(x+2)$ is the same as the graph of $y=f(x)$, but shifted 2 units to the left. Therefore, the *x*-intercepts are -7 and 1.

b. The graph of $y=f(x-2)$ is the same as the graph of $y=f(x)$, but shifted 2 units to the right. Therefore, the *x*-intercepts are -3 and 5.

c. The graph of $y=4f(x)$ is the same as the graph of $y=f(x)$, but stretched vertically by a factor of 4. Therefore, the *x*-intercepts are still -5 and 3 since the *y*-coordinate of each is 0.

d. The graph of $y=f(-x)$ is the same as the graph of $y=f(x)$, but reflected about the y-axis. Therefore, the *x*-intercepts are 5 and -3.

36. **a.** The graph of $y=f(x+4)$ is the same as the graph of $y=f(x)$, but shifted 4 units to the left. Therefore, the *x*-intercepts are -12 and -3.

b. The graph of $y=f(x-3)$ is the same as the graph of $y=f(x)$, but shifted 3 units to the right. Therefore, the *x*-intercepts are -5 and 4.

c. The graph of $y=2f(x)$ is the same as the graph of $y=f(x)$, but stretched vertically by a factor of 2. Therefore, the *x*-intercepts are still -8 and 1 since the *y*-coordinate of each is 0.

d. The graph of $y=f(-x)$ is the same as the graph of $y=f(x)$, but reflected about the *y*-axis. Therefore, the *x*-intercepts are 8 and -1.

37. **a.** The graph of $y=f(x+2)$ is the same as the graph of $y=f(x)$, but shifted 2 units to the left. Therefore, the graph of $f(x+2)$ is increasing on the interval $(-3,3)$.

b. The graph of $y=f(x-5)$ is the same as the graph of $y=f(x)$, but shifted 5 units to the right. Therefore, the graph of $f(x-5)$ is increasing on the interval $(4,10)$.

c. The graph of $y = -f(x)$ is the same as the graph of $y = f(x)$, but reflected about the x-axis. Therefore, we can say that the graph of $y = -f(x)$ must be *decreasing* on the interval $(-1, 5)$.

d. The graph of $y = f(-x)$ is the same as the graph of $y = f(x)$, but reflected about the y-axis. Therefore, we can say that the graph of $y = f(-x)$ must be *decreasing* on the interval $(-5, 1)$.

38. a. The graph of $y = f(x+2)$ is the same as the graph of $y = f(x)$, but shifted 2 units to the left. Therefore, the graph of $f(x+2)$ is decreasing on the interval $(-4, 5)$.

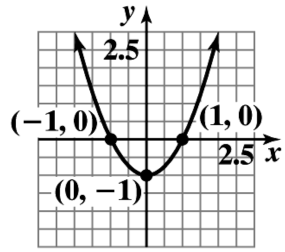
b. The graph of $y = f(x-5)$ is the same as the graph of $y = f(x)$, but shifted 5 units to the right. Therefore, the graph of $f(x-5)$ is decreasing on the interval $(3, 12)$.

c. The graph of $y = -f(x)$ is the same as the graph of $y = f(x)$, but reflected about the x-axis. Therefore, we can say that the graph of $y = -f(x)$ must be *increasing* on the interval $(-2, 7)$.

d. The graph of $y = f(-x)$ is the same as the graph of $y = f(x)$, but reflected about the y-axis. Therefore, we can say that the graph of $y = f(-x)$ must be *increasing* on the interval $(-7, 2)$.

39. $f(x) = x^2 - 1$

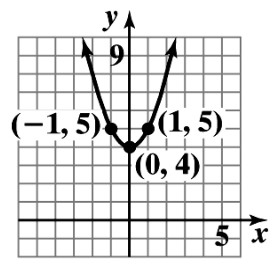
Using the graph of $y = x^2$, vertically shift downward 1 unit.



The domain is $(-\infty, \infty)$ and the range is $[-1, \infty)$.

40. $f(x) = x^2 + 4$

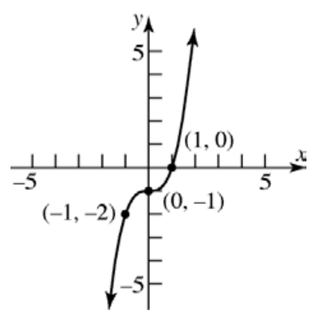
Using the graph of $y = x^2$, vertically shift upward 4 units.



The domain is $(-\infty, \infty)$ and the range is $[4, \infty)$.

41. $g(x) = x^3 - 1$

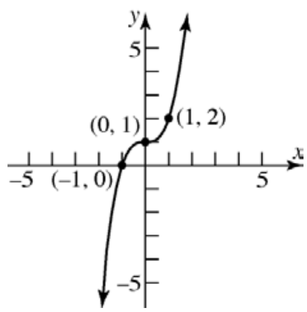
Using the graph of $y = x^3$, vertically shift downward 1 unit.



The domain is $(-\infty, \infty)$ and the range is $(-\infty, \infty)$.

42. $g(x) = x^3 + 1$

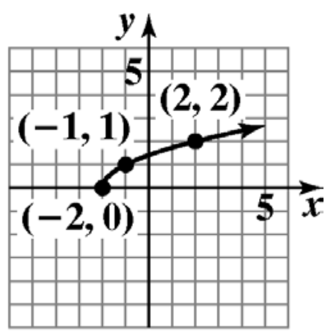
Using the graph of $y = x^3$, vertically shift upward 1 unit.



The domain is $(-\infty, \infty)$ and the range is $(-\infty, \infty)$.

43. $h(x) = \sqrt{x+2}$

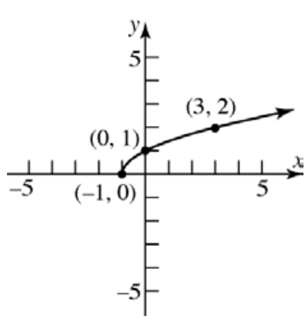
Using the graph of $y = \sqrt{x}$, horizontally shift to the left 2 units.



The domain is $[-2, \infty)$ and the range is $[0, \infty)$.

44. $h(x) = \sqrt{x+1}$

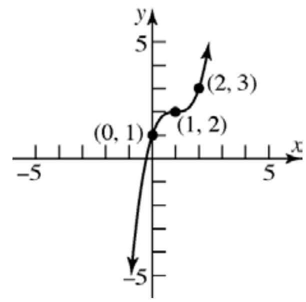
Using the graph of $y = \sqrt{x}$, horizontally shift to the left 1 unit.



The domain is $[-1, \infty)$ and the range is $[0, \infty)$.

45. $f(x) = (x-1)^3 + 2$

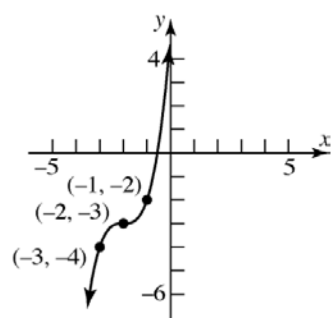
Using the graph of $y = x^3$, horizontally shift to the right 1 unit $\left[y = (x-1)^3 \right]$, then vertically shift up 2 units $\left[y = (x-1)^3 + 2 \right]$.



The domain is $(-\infty, \infty)$ and the range is $(-\infty, \infty)$.

46. $f(x) = (x+2)^3 - 3$

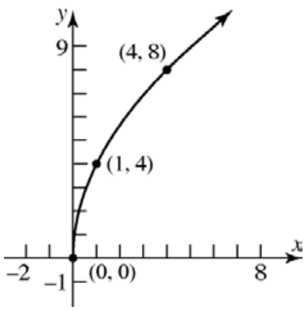
Using the graph of $y = x^3$, horizontally shift to the left 2 units $\left[y = (x+2)^3 \right]$, then vertically shift down 3 units $\left[y = (x+2)^3 - 3 \right]$.



The domain is $(-\infty, \infty)$ and the range is $(-\infty, \infty)$.

47. $g(x) = 4\sqrt{x}$

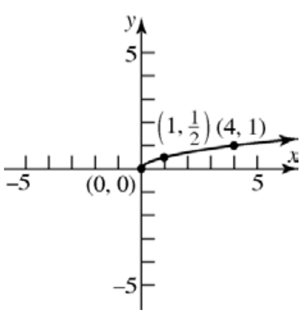
Using the graph of $y = \sqrt{x}$, vertically stretch by a factor of 4.



The domain is $\left[0, \infty\right)$ and the range is $\left[0, \infty\right)$.

48. $g(x) = \dfrac{1}{2}\sqrt{x}$

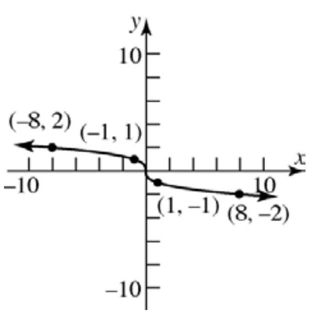
Using the graph of $y = \sqrt{x}$, vertically compress by a factor of $\dfrac{1}{2}$.



The domain is $\left[0, \infty\right)$ and the range is $\left[0, \infty\right)$.

49. $f(x) = -\sqrt[3]{x}$

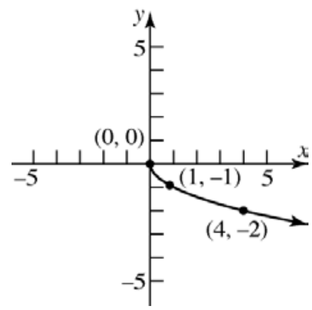
Using the graph of $y = \sqrt[3]{x}$, reflect the graph about the *x*-axis.



The domain is $\left(-\infty, \infty\right)$ and the range is $\left(-\infty, \infty\right)$.

50. $f(x) = -\sqrt{x}$

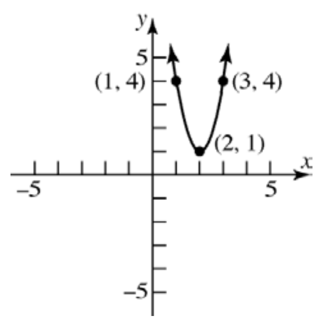
Using the graph of $y = \sqrt{x}$, reflect the graph about the *x*-axis.



The domain is $\left[0, \infty\right)$ and the range is $\left(-\infty, 0\right]$.

51. $f(x) = 3(x-2)^2 + 1$

Using the graph of $y = x^2$, horizontally shift to the right 2 units $\left[y = (x-2)^2\right]$, vertically stretch by a factor of 3 $\left[y = 3(x-2)^2\right]$, and then vertically shift upward 1 unit $\left[y = 3(x-2)^2 + 1\right]$.

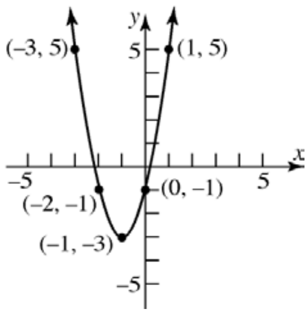


The domain is $\left(-\infty, \infty\right)$ and the range is $\left[1, \infty\right)$.

52. $f(x) = 2(x+1)^2 - 3$

Using the graph of $y = x^2$, horizontally shift to the left 1 unit $\left[y = (x+1)^2\right]$, vertically stretch by a factor of 2 $\left[y = 2(x+1)^2\right]$, and then vertically shift downward 3 units

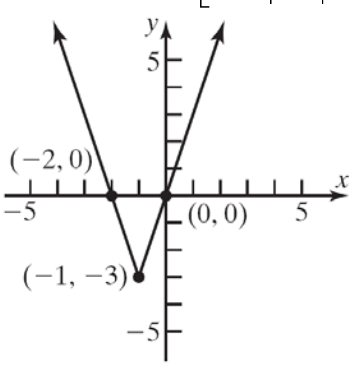
$\left[y = 2(x+1)^2 - 3\right]$.



The domain is $(-\infty, \infty)$ and the range is $[-3, \infty)$.

53. $g(x) = 3|x+1| - 3$

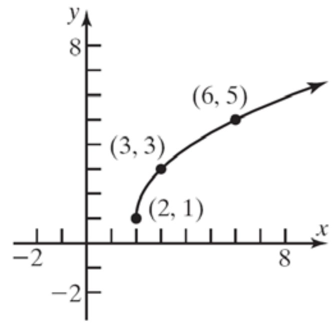
Using the graph of $y = |x|$, horizontally shift to the left 1 unit $\left[y = |x+1|\right]$, vertically stretch by a factor of 3 $\left[y = 3|x+1|\right]$, and vertically shift downward 3 units $\left[y = 3|x+1| - 3\right]$.



The domain is $(-\infty, \infty)$ and the range is $[-3, \infty)$.

54. $g(x) = 2\sqrt{x-2} + 1$

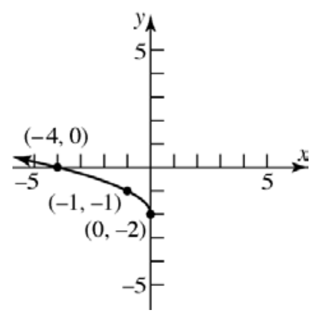
Using the graph of $y = \sqrt{x}$, horizontally shift to the right 2 units $\left[y = \sqrt{x-2}\right]$, vertically stretch by a factor of 2 $\left[y = 2\sqrt{x-2}\right]$, and vertically shift upward 1 unit $\left[y = 2\sqrt{x-2} + 1\right]$.



The domain is $[2, \infty)$ and the range is $[1, \infty)$.

55. $h(x) = \sqrt{-x} - 2$

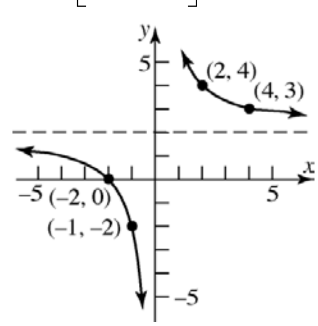
Using the graph of $y = \sqrt{x}$, reflect the graph about the *y*-axis $\left[y = \sqrt{-x}\right]$ and vertically shift downward 2 units $\left[y = \sqrt{-x} - 2\right]$.



The domain is $(-\infty, 0]$ and the range is $[-2, \infty)$.

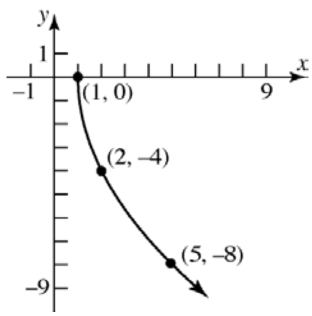
56. $h(x) = \frac{4}{x} + 2 = 4\left(\frac{1}{x}\right) + 2$

Stretch the graph of $y = \frac{1}{x}$ vertically by a factor of 4 $\left[y = 4 \cdot \frac{1}{x} = \frac{4}{x}\right]$ and vertically shift upward 2 units $\left[y = \frac{4}{x} + 2\right]$.



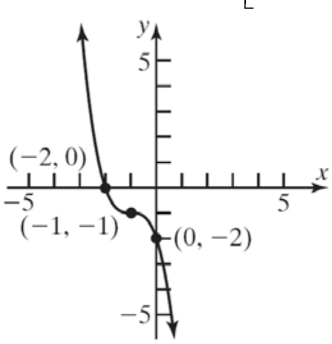
The domain is $(-\infty, 0) \cup (0, \infty)$ and the range is $(-\infty, 2) \cup (2, \infty)$.

57. $f(x) = -4\sqrt{x-1}$
Using the graph of $y = \sqrt{x}$, horizontally shift to the right 1 unit $\left[y = \sqrt{x-1}\right]$, reflect the graph about the x-axis $\left[y = -\sqrt{x-1}\right]$, and stretch vertically by a factor of 4 $\left[y = -4\sqrt{x-1}\right]$.



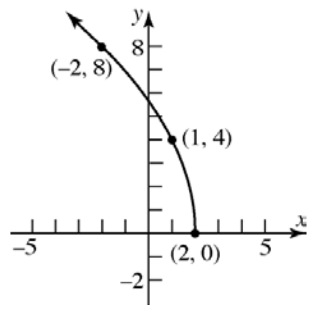
The domain is $[1, \infty)$ and the range is $(-\infty, 0]$.

58. $f(x) = -(x+1)^3 - 1$
Using the graph of $y = x^3$, horizontally shift to the left 1 unit $\left[y = (x+1)^3\right]$, reflect the graph about the x-axis $\left[y = -(x+1)^3\right]$, and vertically shift downward 1 unit $\left[y = -(x+1)^3 - 1\right]$.



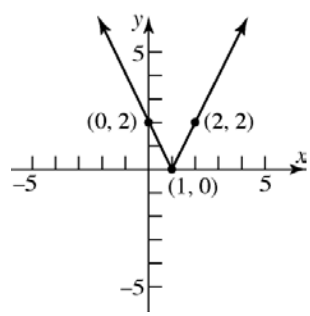
The domain is $(-\infty, \infty)$ and the range is $(-\infty, \infty)$.

59. $g(x) = 4\sqrt{2-x} = 4\sqrt{-(x-2)}$
Using the graph of $y = \sqrt{x}$, reflect the graph about the y-axis $\left[y = \sqrt{-x}\right]$, horizontally shift to the right 2 units $\left[y = \sqrt{-(x-2)}\right]$, and vertically stretch by a factor of 4 $\left[y = 4\sqrt{-(x-2)}\right]$.



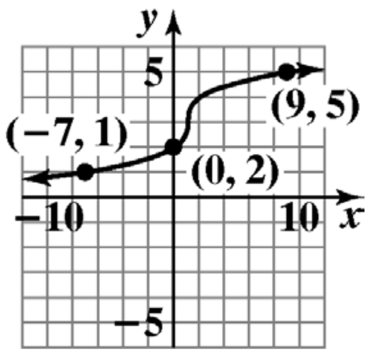
The domain is $(-\infty, 2]$ and the range is $[0, \infty)$.

60. $g(x) = 2|1-x| = 2|-(-1+x)| = 2|x-1|$
Using the graph of $y = |x|$, horizontally shift to the right 1 unit $\left[y = |x-1|\right]$, and vertically stretch by a factor or 2 $\left[y = 2|x-1|\right]$.



The domain is $(-\infty, \infty)$ and the range is $[0, \infty)$.

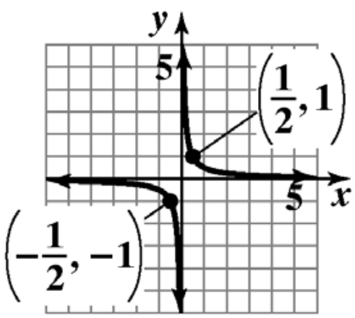
61. $f(x) = \sqrt[3]{x-1} + 3$
Using the graph of $f(x) = \sqrt[3]{x}$, horizontally shift to the right 1 unit $\left[y = \sqrt[3]{x-1}\right]$, then vertically shift up 3 units $\left[y = \sqrt[3]{x-1} + 3\right]$.



The domain is $(-\infty,\infty)$ and the range is $(-\infty,\infty)$.

62. $h(x)=\dfrac{1}{2x}$

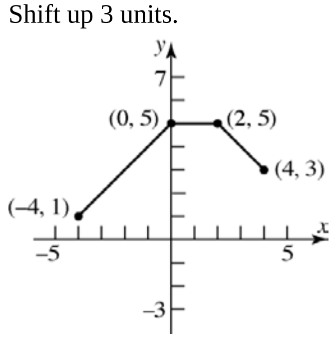
Using the graph of $y=\dfrac{1}{x}$, vertically compress by a factor of $\dfrac{1}{2}$.



The domain is $(-\infty,0)\cup(0,\infty)$ and the range is $(-\infty,0)\cup(0,\infty)$.

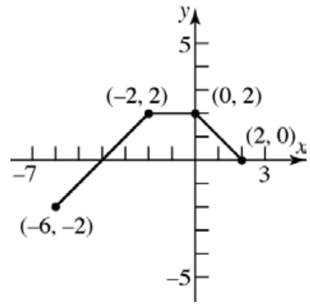
63. **a.** $F(x)=f(x)+3$

Shift up 3 units.



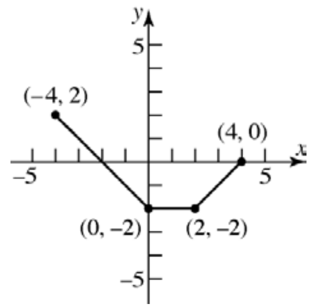
b. $G(x)=f(x+2)$

Shift left 2 units.



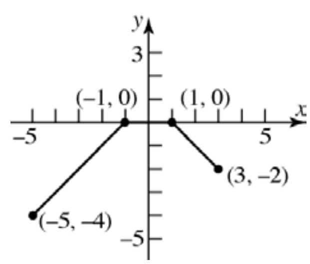
c. $P(x)=-f(x)$

Reflect about the x-axis.



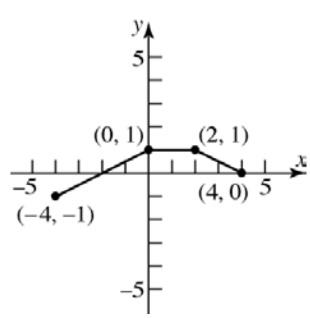
d. $H(x)=f(x+1)-2$

Shift left 1 unit and shift down 2 units.



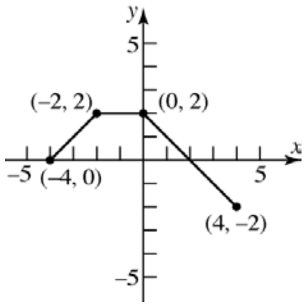
e. $Q(x)=\dfrac{1}{2}f(x)$

Compress vertically by a factor of $\dfrac{1}{2}$.



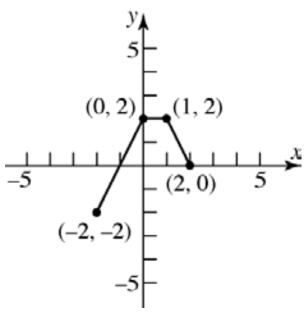
f. $g(x) = f(-x)$

Reflect about the y-axis.



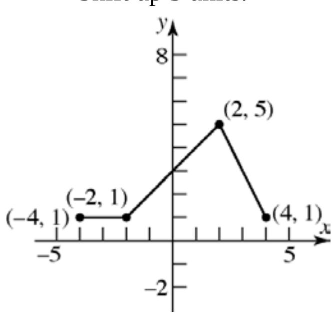
g. $h(x) = f(2x)$

Compress horizontally by a factor of $\frac{1}{2}$.



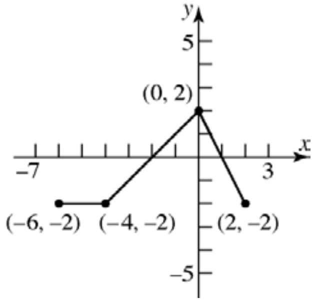
64. a. $F(x) = f(x) + 3$

Shift up 3 units.



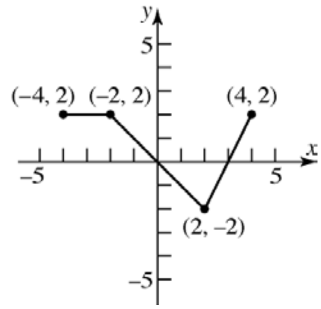
b. $G(x) = f(x+2)$

Shift left 2 units.



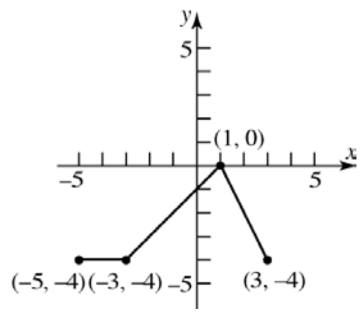
c. $P(x) = -f(x)$

Reflect about the x-axis.



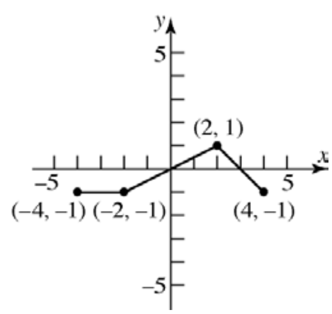
d. $H(x) = f(x+1) - 2$

Shift left 1 unit and shift down 2 units.



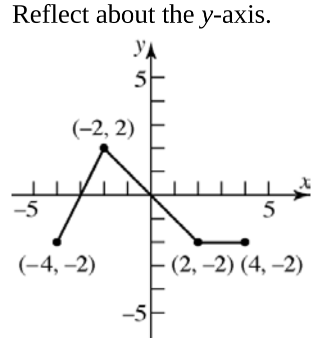
e. $Q(x) = \frac{1}{2} f(x)$

Compress vertically by a factor of $\frac{1}{2}$.



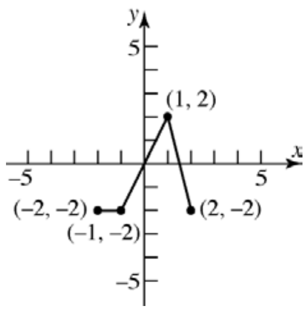
f. $g(x) = f(-x)$

Reflect about the y-axis.



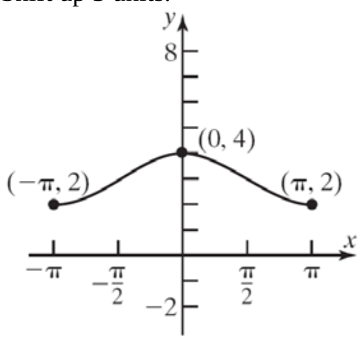
g. $h(x) = f(2x)$

Compress horizontally by a factor of $\frac{1}{2}$.



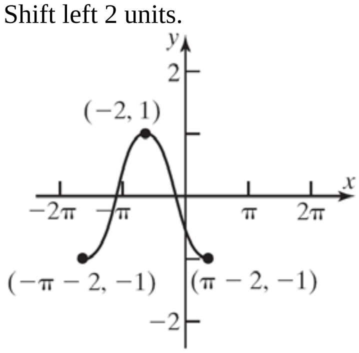
65. a. $F(x) = f(x) + 3$

Shift up 3 units.



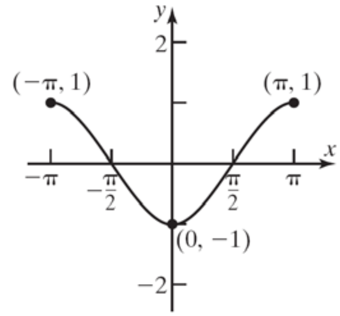
b. $G(x) = f(x + 2)$

Shift left 2 units.



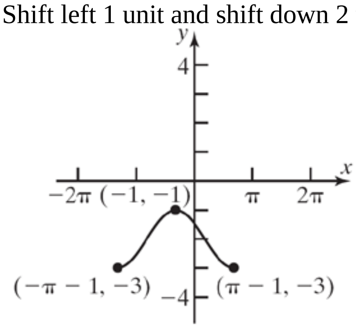
c. $P(x) = -f(x)$

Reflect about the x-axis.



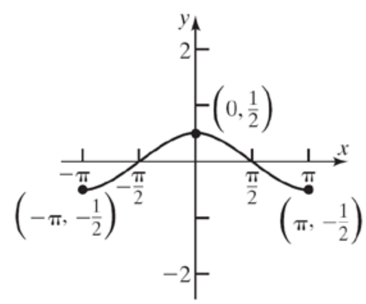
d. $H(x) = f(x + 1) - 2$

Shift left 1 unit and shift down 2 units.



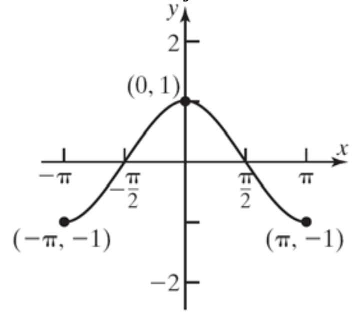
e. $Q(x) = \frac{1}{2} f(x)$

Compress vertically by a factor of $\frac{1}{2}$.



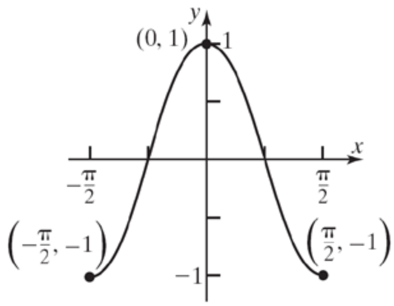
f. $g(x) = f(-x)$

Reflect about the y-axis.



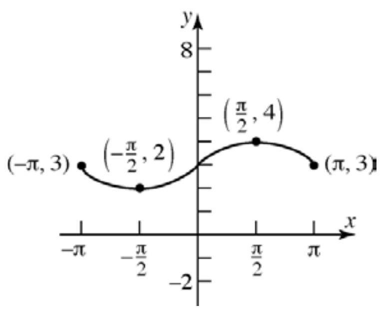
g. $h(x) = f(2x)$

Compress horizontally by a factor of $\frac{1}{2}$.



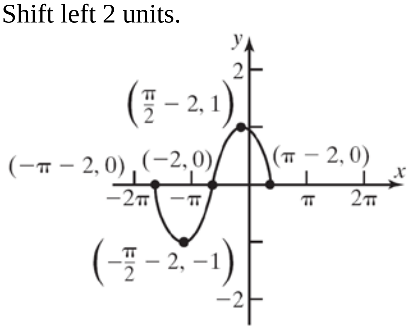
66. a. $F(x) = f(x) + 3$

Shift up 3 units.



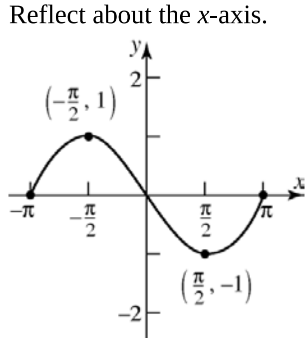
b. $G(x) = f(x+2)$

Shift left 2 units.



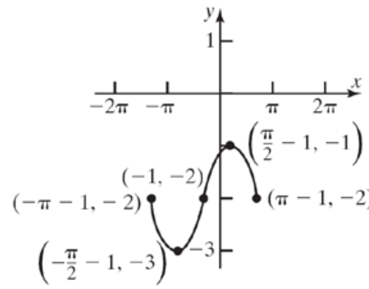
c. $P(x) = -f(x)$

Reflect about the x-axis.



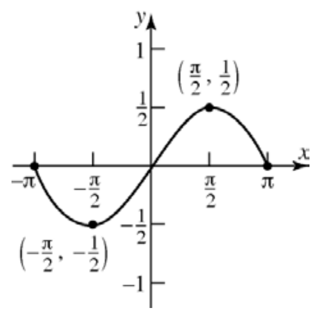
d. $H(x) = f(x+1) - 2$

Shift left 1 unit and shift down 2 units.



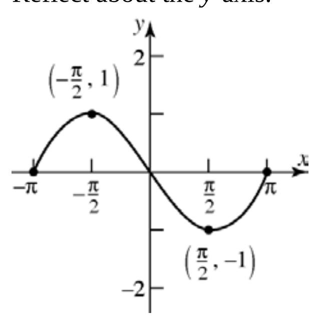
e. $Q(x) = \frac{1}{2} f(x)$

Compress vertically by a factor of $\frac{1}{2}$.



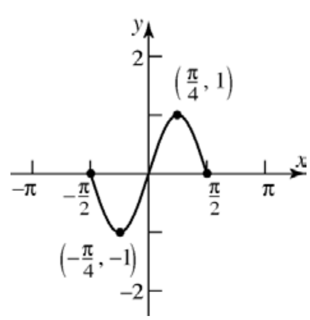
f. $g(x) = f(-x)$

Reflect about the y-axis.



g. $h(x) = f(2x)$

Compress horizontally by a factor of $\frac{1}{2}$.

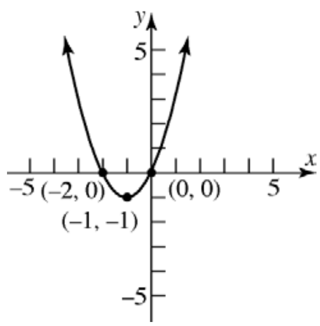


67. $f(x) = x^2 + 2x$

$f(x) = (x^2 + 2x + 1) - 1$

$f(x) = (x+1)^2 - 1$

Using $f(x) = x^2$, shift left 1 unit and shift down 1 unit.

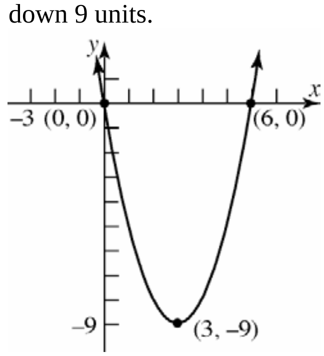


68. $f(x) = x^2 - 6x$

$f(x) = (x^2 - 6x + 9) - 9$

$f(x) = (x-3)^2 - 9$

Using $f(x) = x^2$, shift right 3 units and shift down 9 units.

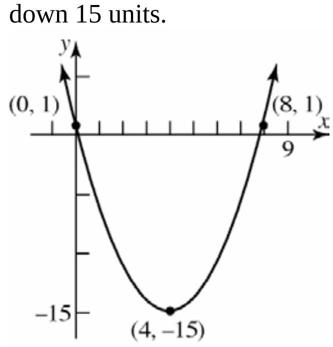


69. $f(x) = x^2 - 8x + 1$

$f(x) = \left(x^2 - 8x + 16\right) + 1 - 16$

$f(x) = \left(x-4\right)^2 - 15$

Using $f(x) = x^2$, shift right 4 units and shift down 15 units.

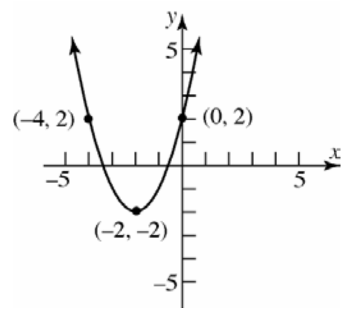


70. $f(x) = x^2 + 4x + 2$

$f(x) = \left(x^2 + 4x + 4\right) + 2 - 4$

$f(x) = \left(x+2\right)^2 - 2$

Using $f(x) = x^2$, shift left 2 units and shift down 2 units.



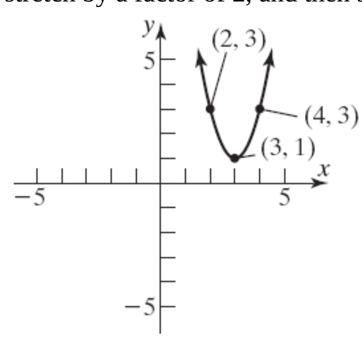
71. $f\left(x\right) = 2x^2 - 12x + 19$

$= 2\left(x^2 - 6x\right) + 19$

$= 2\left(x^2 - 6x + 9\right) + 19 - 18$

$= 2\left(x-3\right)^2 + 1$

Using $f\left(x\right) = x^2$, shift right 3 units, vertically stretch by a factor of 2, and then shift up 1 unit.



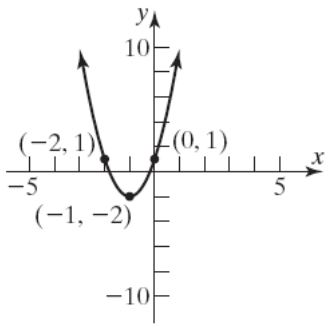
72. $f\left(x\right) = 3x^2 + 6x + 1$

$= 3\left(x^2 + 2x\right) + 1$

$= 3\left(x^2 + 2x + 1\right) + 1 - 3$

$= 3\left(x+1\right)^2 - 2$

Using $f\left(x\right) = x^2$, shift left 1 unit, vertically stretch by a factor of 3, and shift down 2 units.



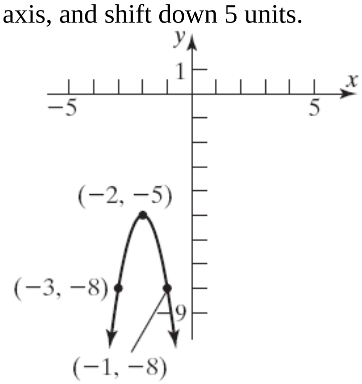
73. $f(x) = -3x^2 - 12x - 17$

$$= -3\left(x^2 + 4x\right) - 17$$

$$= -3\left(x^2 + 4x + 4\right) - 17 + 12$$

$$= -3(x+2)^2 - 5$$

Using $f(x) = x^2$, shift left 2 units, stretch vertically by a factor of 3, reflect about the *x*-axis, and shift down 5 units.



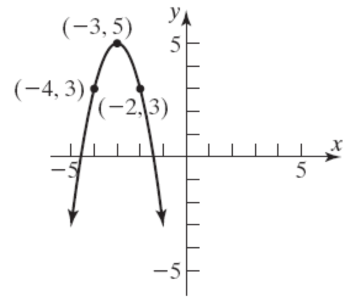
74. $f(x) = -2x^2 - 12x - 13$

$$= -2\left(x^2 + 6x\right) - 13$$

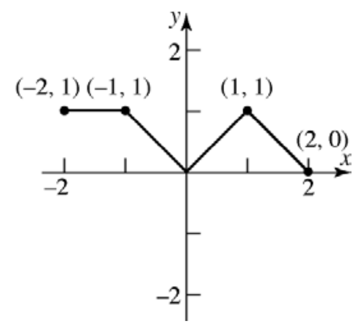
$$= -2\left(x^2 + 6x + 9\right) - 13 + 18$$

$$= -2(x+3)^2 + 5$$

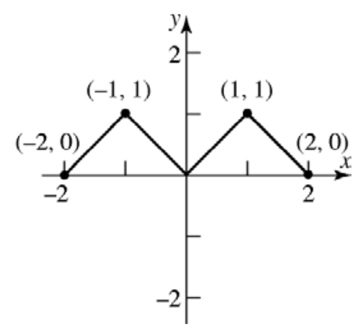
Using $f(x) = x^2$, shift left 3 units, stretch vertically by a factor of 2, reflect about the *x*-axis, and shift up 5 units.



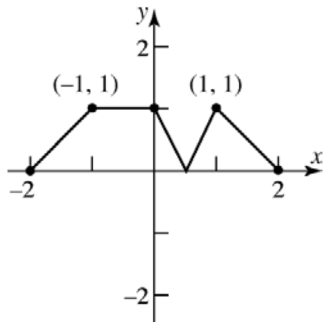
75. a. $y = |f(x)|$



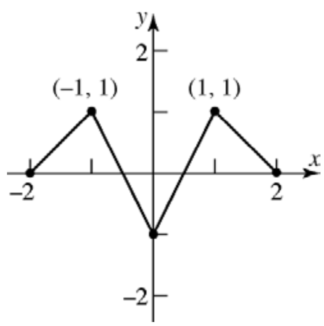
b. $y = f(|x|)$



76. a. To graph $y = |f(x)|$, the part of the graph for *f* that lies in quadrants III or IV is reflected about the *x*-axis.



b. To graph $y = f(|x|)$, the part of the graph for *f* that lies in quadrants II or III is replaced by the reflection of the part in quadrants I and IV reflected about the *y*-axis.



77. a. The graph of $y = f(x+3)-5$ is the graph of $y = f(x)$ but shifted left 3 units and down 5 units. Thus, the point $(1,3)$ becomes the point $(-2,-2)$.

b. The graph of $y = -2f(x-2)+1$ is the graph of $y = f(x)$ but shifted right 2 units, stretched vertically by a factor of 2, reflected about the *x*-axis, and shifted up 1 unit. Thus, the point $(1,3)$ becomes the point $(3,-5)$.

c. The graph of $y = f(2x+3)$ is the graph of $y = f(x)$ but shifted left 3 units and horizontally compressed by a factor of 2. Thus, the point $(1,3)$ becomes the point $(-1,3)$.

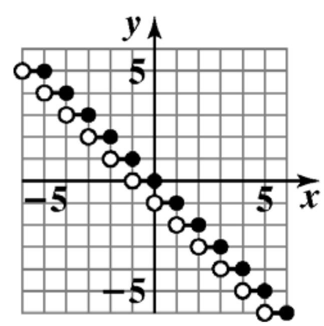
78. a. The graph of $y = g(x+1)-3$ is the graph of $y = g(x)$ but shifted left 1 unit and down 3 units. Thus, the point $(-3,5)$ becomes the point $(-4,2)$.

b. The graph of $y = -3g(x-4)+3$ is the graph of $y = g(x)$ but shifted right 4 units, stretched vertically by a factor of 3, reflected about the *x*-axis, and shifted up 3 units. Thus, the point $(-3,5)$ becomes the point $(1,-12)$.

c. The graph of $y = g(3x+9)$ is the graph of $y = f(x)$ but shifted left 9 units and horizontally compressed by a factor of 3. Thus, the point $(-3,5)$ becomes the point $(-4,5)$.

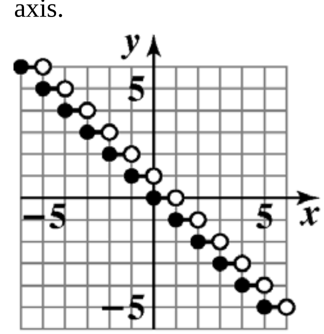
79. a. $f(x) = \text{int}(-x)$

Reflect the graph of $y = \text{int}(x)$ about the *y*-axis.



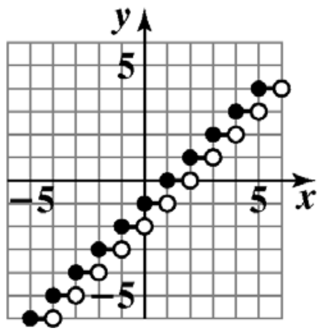
b. $g(x) = -\text{int}(x)$

Reflect the graph of $y = \text{int}(x)$ about the x-axis.



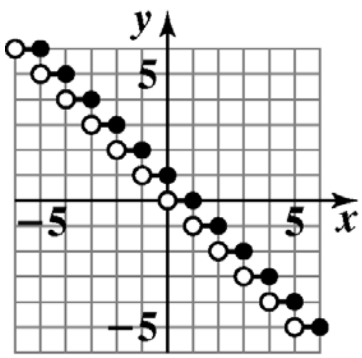
80. a. $f(x) = \text{int}(x-1)$

Shift the graph of $y = \text{int}(x)$ right 1 unit.



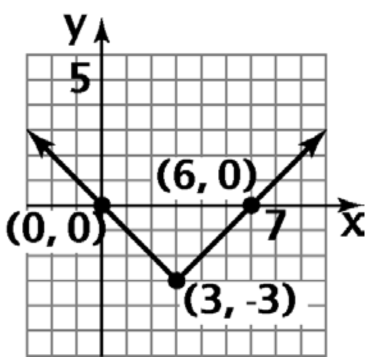
b. $g(x) = \text{int}(1-x) = \text{int}(-(x-1))$

Using the graph of $y = \text{int}(x)$, reflect the graph about the y-axis $\left[y = \text{int}(-x)\right]$, horizontally shift to the right 1 unit$\left[y = \text{int}(-(x-1))\right]$.



81. **a.** $f(x) = |x-3|-3$

Using the graph of $y = |x|$, horizontally shift to the right 3 units $\left[y = |x-3|\right]$ and vertically shift downward 3 units $\left[y = |x-3|-3\right]$.



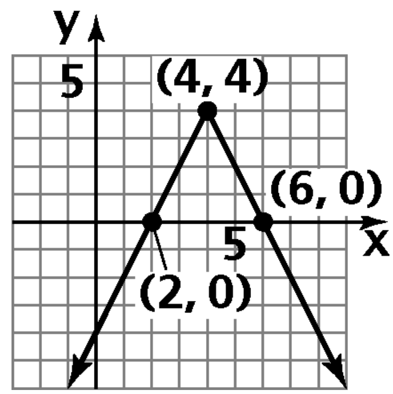
b. $A = \dfrac{1}{2}bh$

$= \dfrac{1}{2}(6)(3) = 9$

The area is 9 square units.

82. **a.** $f(x) = -2|x-4|+4$

Using the graph of $y = |x|$, horizontally shift to the right 4 units $\left[y = |x-4|\right]$, vertically stretch by a factor of 2 and flip on the x-axis $\left[y = -2|x-4|\right]$, and vertically shift upward 4 units $\left[y = -2|x-4|+4\right]$.



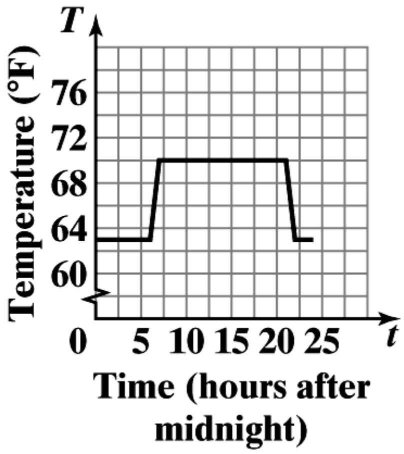
b. $A = \dfrac{1}{2}bh$

$= \dfrac{1}{2}(4)(4) = 8$

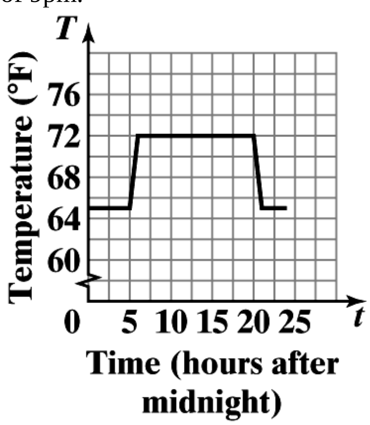
The area is 8 square units.

83. **a.** From the graph, the thermostat is set at $72°\text{F}$ during the daytime hours. The thermostat appears to be set at $65°\text{F}$ overnight.

b. To graph $y = T(t)-2$, the graph of $T(t)$ is shifted down 2 units. This change will lower the temperature in the house by 2 degrees.



c. To graph $y = T(t+1)$, the graph of $T(t)$ should be shifted left one unit. This change will cause the program to switch between the daytime temperature and overnight temperature one hour sooner. The home will begin warming up at 5am instead of 6am and will begin cooling down at 8pm instead of 9pm.



84. a. $R(0) = 28.6(0)^2 + 300(0) + 4843 = 4843$

The estimated worldwide music revenue for 2012 is \$4843 million.

$$R(3) = 28.6(3)^2 + 300(3) + 4843$$
$$= 6000.4$$

The estimated worldwide music revenue for 2015 is \$6000.4 million.

$$R(5) = 28.6(5)^2 + 300(5) + 4843$$
$$= 7058$$

The estimated worldwide music revenue for 2017 is \$7058 million.

b. $r(x) = R(x-2)$

$$= 28.6(x-2)^2 + 300(x-2) + 4843$$
$$= 28.6(x^2 - 4x + 4) + 300(x-2)$$
$$+ 4843$$
$$= 28.6x^2 - 114.4x + 114.4 + 300x$$
$$- 600 + 4843$$
$$= 28.6x^2 + 185.6x + 4357.4$$

c. The graph of $r(x)$ is the graph of $R(x)$ shifted 2 units to the left. Thus, $r(x)$ represents the estimated worldwide music revenue, x years after 2010.

$r(2) = 28.6(2)^2 + 185.6(2) + 4357.4 = 4843$

The estimated worldwide music revenue for 2012 is \$4843 million.

$$r(5) = 28.6(5)^2 + 185.6(5) + 4357.4$$
$$= 6000.4$$

The estimated worldwide music revenue for 2015 is \$6000.4 million.

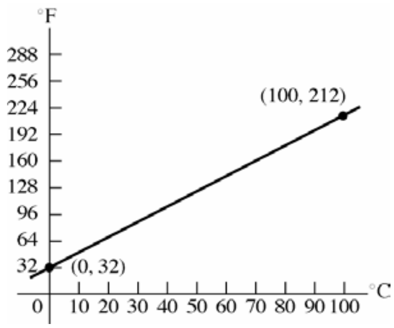
$$r(7) = 28.6(7)^2 + 185.6(7) + 4357.4$$
$$= 7058$$

The estimated worldwide music revenue for 2017 is \$7058 million.

d. In $r(x)$, x represents the number of years after 2010 (see the previous part).

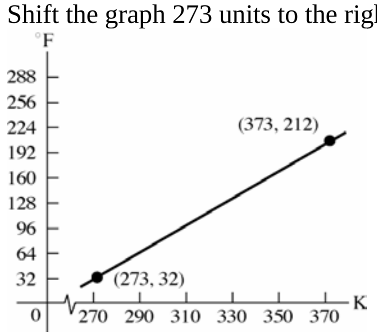
e. Answers will vary. One advantage might be that it is easier to determine what value should be substituted for x when using $r(x)$ instead of $R(x)$ to estimate worldwide music revenue.

85. $F = \frac{9}{5}C + 32$

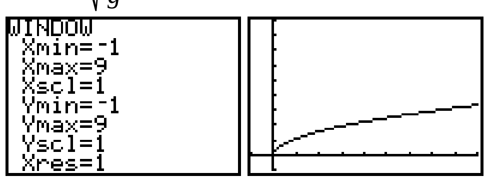


$F = \frac{9}{5}(K - 273) + 32$

Shift the graph 273 units to the right.

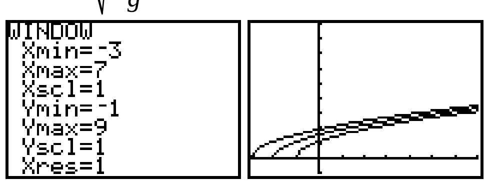


86. **a.** $T = 2\pi\sqrt{\frac{l}{g}}$



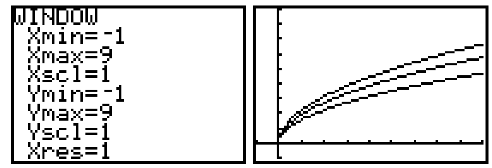
b. $T_1 = 2\pi\sqrt{\frac{l+1}{g}}$; $T_2 = 2\pi\sqrt{\frac{l+2}{g}}$;

$T_3 = 2\pi\sqrt{\frac{l+3}{g}}$



c. As the length of the pendulum increases, the period increases.

d. $T_1 = 2\pi\sqrt{\frac{2l}{g}}$; $T_2 = 2\pi\sqrt{\frac{3l}{g}}$; $T_3 = 2\pi\sqrt{\frac{4l}{g}}$



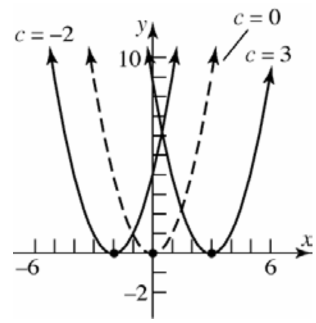
e. If the length of the pendulum is multiplied by k, the period is multiplied by $\sqrt{k}$.

87. $y = (x - c)^2$

If $c = 0$, $y = x^2$.

If $c = 3$, $y = (x-3)^2$; shift right 3 units.

If $c = -2$, $y = (x+2)^2$; shift left 2 units.

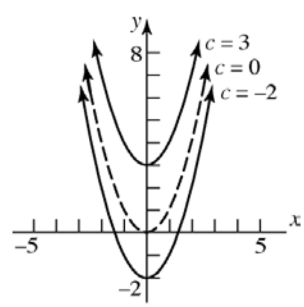


88. $y = x^2 + c$

If $c = 0$, $y = x^2$.

If $c = 3$, $y = x^2 + 3$; shift up 3 units.

If $c = -2$, $y = x^2 - 2$; shift down 2 units.



89. The graph of $y = 4f(x)$ is a vertical stretch of the graph of *f* by a factor of 4, while the graph of $y = f(4x)$ is a horizontal compression of the graph of *f* by a factor of $\frac{1}{4}$.

90. The graph of $y = f(x) - 2$ will shift the graph of $y = f(x)$ down by 2 units. The graph of $y = f(x-2)$ will shift the graph of $y = f(x)$ to the right by 2 units.

91. The graph of $y=\sqrt{-x}$ is the graph of $y=\sqrt{x}$ but reflected about the *y*-axis. Therefore, our region is simply rotated about the *y*-axis and does not change shape. Instead of the region being bounded on the right by $x=4$, it is bounded on the left by $x=-4$. Thus, the area of the second region would also be $\frac{16}{3}$ square units.

92. The range of $f(x)=x^2$ is $[0,\infty)$. The graph of $g(x)=f(x)+k$ is the graph of f shifted up k units if k > 0 and shifted down $|k|$ units if k < 0, so the range of g is $[k,\infty)$.

93. The domain of $g(x)=\sqrt{x}$ is $[0,\infty)$. The graph of $g(x-k)$ is the graph of g shifted k units to the right, so the domaine of g is $[k,\infty)$.

94.
$$\begin{aligned}3x-5y&=30\\ -5y&=-3x+30\\ y&=\frac{3}{5}x-6\end{aligned}$$

The slope is $\frac{3}{5}$ and the y-intercept is -6.

95.
$$\begin{aligned}f(-x)&=\frac{(-x)^2+2}{3(-x)}=\frac{x^2+2}{-3x}\\ &=-\frac{x^2+2}{3x}=-f(x)\end{aligned}$$

Since $f(-x)=-f(x)$ then $f(x)$ is odd.

96.
$$\begin{aligned}f(2)&=(2)^4-7(2)^2+3(2)+9\\ &=16-28+6+9=3\\ f(-2)&=(-2)^4-7(-2)^2+3(-2)+9\\ &=16-28-6+9=-9\end{aligned}$$
$$\begin{aligned}\frac{f(x_2)-f(x_1)}{x_2-x_1}&=\frac{3-(-9)}{2-(-2)}\\ &=\frac{12}{4}=3\end{aligned}$$

97. $y^2=x+4$

x-intercepts:
$$\begin{aligned}(0)^2&=x+4\\ 0&=x+4\\ x&=-4\end{aligned}$$

y-intercepts:
$$\begin{aligned}y^2&=0+4\\ y^2&=4\\ y&=\pm 2\end{aligned}$$

The intercepts are $(-4,0)$, $(0,-2)$ and $(0,2)$.

<u>Test *x*-axis symmetry</u>: Let $y=-y$
$$\begin{aligned}(-y)^2&=x+4\\ y^2&=x+4 \text{ same}\end{aligned}$$

<u>Test *y*-axis symmetry</u>: Let $x=-x$
$$y^2=-x+4 \text{ different}$$

<u>Test origin symmetry</u>: Let $x=-x$ and $y=-y$.
$$\begin{aligned}(-y)^2&=-x+4\\ y^2&=-x+4 \text{ different}\end{aligned}$$

Therefore, the graph will have *x*-axis symmetry.

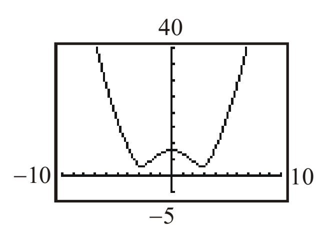
Section 2.6

1. a. The distance d from P to the origin is $d=\sqrt{x^2+y^2}$. Since P is a point on the graph of $y=x^2-8$, we have:
$$d(x)=\sqrt{x^2+(x^2-8)^2}=\sqrt{x^4-15x^2+64}$$

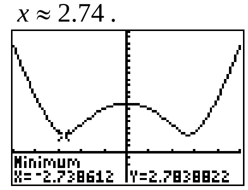
b. $d(0)=\sqrt{0^4-15(0)^2+64}=\sqrt{64}=8$

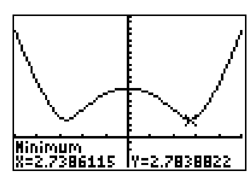
c.
$$\begin{aligned}d(1)&=\sqrt{(1)^4-15(1)^2+64}\\ &=\sqrt{1-15+64}=\sqrt{50}=5\sqrt{2}\approx 7.07\end{aligned}$$

d.



e. d is smallest when $x\approx -2.74$ or when $x\approx 2.74$.





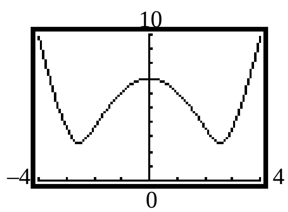
2. **a.** The distance d from P to $(0, -1)$ is $d = \sqrt{x^2 + (y+1)^2}$. Since P is a point on the graph of $y = x^2 - 8$, we have:

$$d(x) = \sqrt{x^2 + (x^2 - 8 + 1)^2}$$
$$= \sqrt{x^2 + \left(x^2 - 7\right)^2} = \sqrt{x^4 - 13x^2 + 49}$$

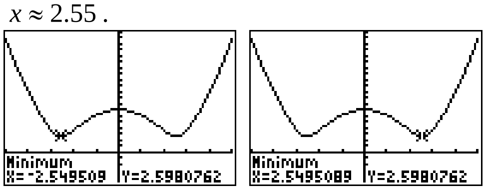
b. $d(0) = \sqrt{0^4 - 13(0)^2 + 49} = \sqrt{49} = 7$

c. $d(-1) = \sqrt{(-1)^4 - 13(-1)^2 + 49} = \sqrt{37} \approx 6.08$

d.



e. d is smallest when $x \approx -2.55$ or when $x \approx 2.55$.

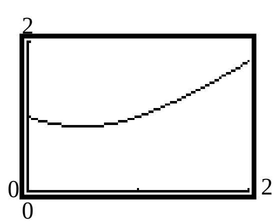


3. **a.** The distance d from P to the point $(1, 0)$ is $d = \sqrt{(x-1)^2 + y^2}$. Since P is a point on the graph of $y = \sqrt{x}$, we have:

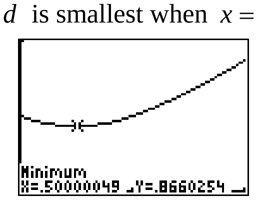
$$d(x) = \sqrt{(x-1)^2 + \left(\sqrt{x}\right)^2} = \sqrt{x^2 - x + 1}$$

where $x \ge 0$.

b.



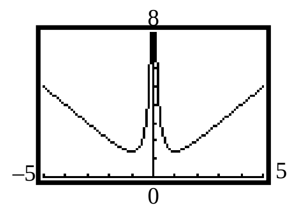
c. d is smallest when $x = \frac{1}{2}$.



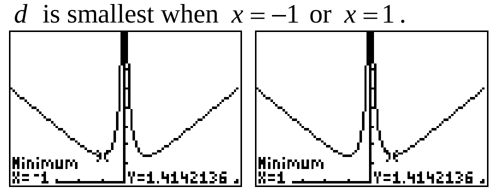
4. **a.** The distance d from P to the origin is $d = \sqrt{x^2 + y^2}$. Since P is a point on the graph of $y = \dfrac{1}{x}$, we have:

$$d(x) = \sqrt{x^2 + \left(\frac{1}{x}\right)^2} = \sqrt{x^2 + \frac{1}{x^2}} = \sqrt{\frac{x^4 + 1}{x^2}}$$
$$= \frac{\sqrt{x^2 + 1}}{|x|}$$

b.



c. d is smallest when $x = -1$ or $x = 1$.



5. By definition, a triangle has area $A = \dfrac{1}{2} b\,h$, b = base, h = height. From the figure, we know that $b = x$ and $h = y$. Expressing the area of the triangle as a function of x, we have:

$$A(x) = \frac{1}{2} xy = \frac{1}{2} x\left(x^4\right) = \frac{1}{2} x^5.$$

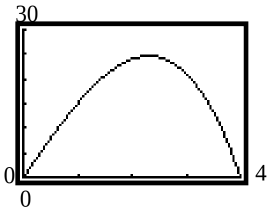
6. By definition, a triangle has area $A = \dfrac{1}{2} b\,h$, b=base, h = height. Because one vertex of the triangle is at the origin and the other is on the x-axis, we know that $b = x$ and $h = y$. Expressing the area of the triangle as a function of x, we have:

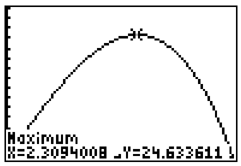
$$A(x) = \frac{1}{2} xy = \frac{1}{2} x\left(9 - x^2\right) = \frac{9}{2} x - \frac{1}{2} x^3.$$

7. **a.** $A(x) = xy = x\left(16 - x^2\right)$

b. Domain: $\left\{x \,\middle|\, 0 < x < 4\right\}$

c. The area is largest when $x \approx 2.31$.

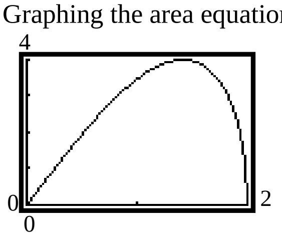


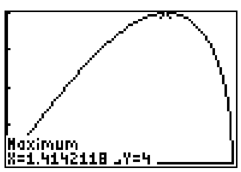


8. a. $A(x) = 2xy = 2x\sqrt{4-x^2}$

b. $p(x) = 2(2x) + 2(y) = 4x + 2\sqrt{4-x^2}$

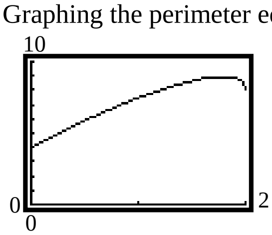
c. Graphing the area equation:

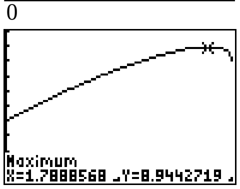




The area is largest when $x \approx 1.41$.

d. Graphing the perimeter equation:





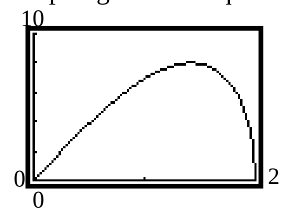
The perimeter is largest when $x \approx 1.79$.

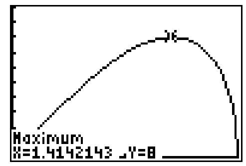
9. a. In Quadrant I, $x^2 + y^2 = 4 \rightarrow y = \sqrt{4-x^2}$

$A(x) = (2x)(2y) = 4x\sqrt{4-x^2}$

b. $p(x) = 2(2x) + 2(2y) = 4x + 4\sqrt{4-x^2}$

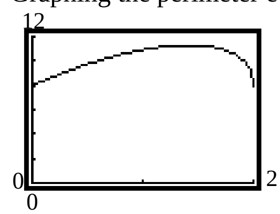
c. Graphing the area equation:

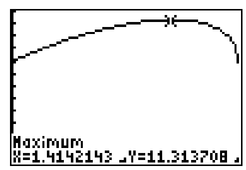




The area is largest when $x \approx 1.41$.

d. Graphing the perimeter equation:





The perimeter is largest when $x \approx 1.41$.

10. a. $A(r) = (2r)(2r) = 4r^2$

b. $p(r) = 4(2r) = 8r$

11. a. $C =$ circumference, $A =$ total area,

$r =$ radius, $x =$ side of square

$C = 2\pi r = 10 - 4x \quad \Rightarrow \quad r = \frac{5-2x}{\pi}$

Total Area $=$ area$_{\text{square}}$ + area$_{\text{circle}}$ $= x^2 + \pi r^2$

$$A(x) = x^2 + \pi\left(\frac{5-2x}{\pi}\right)^2 = x^2 + \frac{25 - 20x + 4x^2}{\pi}$$

b. Since the lengths must be positive, we have:

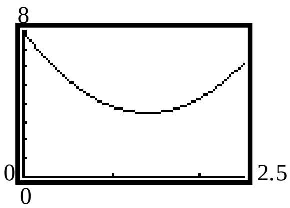
$10 - 4x > 0$ and $x > 0$

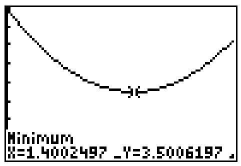
$-4x > -10$ and $x > 0$

$x < 2.5$ and $x > 0$

Domain: $\{x \mid 0 < x < 2.5\}$

c. The total area is smallest when $x \approx 1.40$ meters.





12. a. $C =$ circumference, $A =$ total area, $r =$ radius, $x =$ side of equilateral triangle

$$C = 2\pi r = 10 - 3x \Rightarrow r = \frac{10-3x}{2\pi}$$

The height of the equilateral triangle is $\frac{\sqrt{3}}{2}x$.

$$\begin{aligned}\text{Total Area} &= \text{area}_{\text{triangle}} + \text{area}_{\text{circle}} \\ &= \frac{1}{2}x\left(\frac{\sqrt{3}}{2}x\right) + \pi r^2\end{aligned}$$

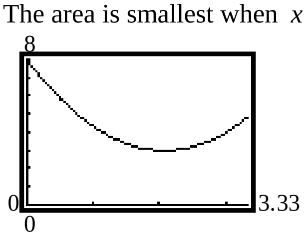
$$\begin{aligned}A(x) &= \frac{\sqrt{3}}{4}x^2 + \pi\left(\frac{10-3x}{2\pi}\right)^2 \\ &= \frac{\sqrt{3}}{4}x^2 + \frac{100-60x+9x^2}{4\pi}\end{aligned}$$

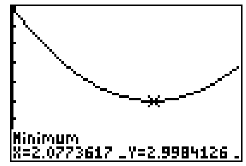
b. Since the lengths must be positive, we have:

$$\begin{aligned}10 - 3x > 0 &\quad \text{and } x > 0 \\ -3x > -10 &\quad \text{and } x > 0 \\ x < \frac{10}{3} &\quad \text{and } x > 0\end{aligned}$$

Domain: $\left\{x \,\middle|\, 0 < x < \frac{10}{3}\right\}$

c. The area is smallest when $x \approx 2.08$ meters.





13. a. Since the wire of length $6x$ is bent into a circle, the circumference is $6x$. Therefore, $c(x) = 6x$.

b. Since $C = 6x = 2\pi r$, $r = \frac{3x}{\pi}$.

$$A(x) = \pi r^2 = \pi\left(\frac{3x}{\pi}\right)^2 = \frac{9x^2}{\pi}.$$

14. a. Since the wire of length x is bent into a square, the perimeter is x. Therefore, $p(x) = x$.

b. Since $P = x = 4s$, $s = \frac{1}{4}x$, we have

$$A(x) = s^2 = \left(\frac{1}{4}x\right)^2 = \frac{1}{16}x^2.$$

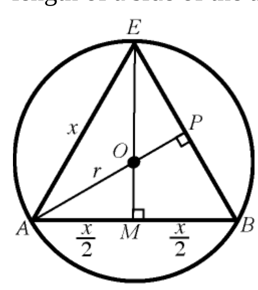
15. a. A = area, $3x =$ radius, diameter $= 6x$

$$A(x) = (6x)(3x) = 18x^2$$

b. P = perimeter

$$\begin{aligned}P(x) &= 2(6x) + 2(3x) \\ &= 18x\end{aligned}$$

16. $C =$ circumference, $r =$ radius; $x =$ length of a side of the triangle



Since ΔABC is equilateral, $EM = \frac{\sqrt{3}x}{2}$.

Therefore, $OM = \frac{\sqrt{3}x}{2} - OE = \frac{\sqrt{3}x}{2} - r$

In ΔOAM, $r^2=\left(\frac{x}{2}\right)^2+\left(\frac{\sqrt{3}x}{2}-r\right)^2$

$$r^2=\frac{x^2}{4}+\frac{3}{4}x^2-\sqrt{3}\,rx+r^2$$

$$\sqrt{3}\,rx=x^2$$

$$r=\frac{x}{\sqrt{3}}$$

Therefore, the circumference of the circle is

$$C(x)=2\pi r=2\pi\left(\frac{x}{\sqrt{3}}\right)=\frac{2\pi\sqrt{3}}{3}x$$

17. Area of the equilateral triangle

$$A=\frac{1}{2}x\cdot\frac{\sqrt{3}}{2}x=\frac{\sqrt{3}}{4}x^2$$

From problem 16, we have $r^2=\frac{x^2}{3}$.

Area inside the circle, but outside the triangle:

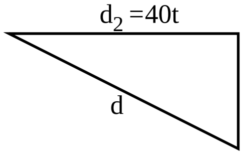
$$A(x)=\pi r^2-\frac{\sqrt{3}}{4}x^2$$

$$=\pi\frac{x^2}{3}-\frac{\sqrt{3}}{4}x^2=\left(\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right)x^2$$

18. $d^2=d_1{}^2+d_2{}^2$

$$d^2=(30t)^2+(40t)^2$$

$$d(t)=\sqrt{900t^2+1600t^2}=\sqrt{2500t^2}=50t$$



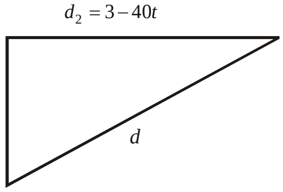
19. a. $d^2=d_1{}^2+d_2{}^2$

$$d^2=(2-30t)^2+(3-40t)^2$$

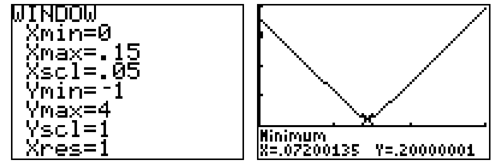
$$d(t)=\sqrt{(2-30t)^2+(3-40t)^2}$$

$$=\sqrt{4-120t+900t^2+9-240t+1600t^2}$$

$$=\sqrt{2500t^2-360t+13}$$



b. The distance is smallest at $t\approx 0.07$ hours.



20. r = radius of cylinder, h = height of cylinder, V = volume of cylinder

$$r^2+\left(\frac{h}{2}\right)^2=R^2\Rightarrow r^2+\frac{h^2}{4}=R^2\Rightarrow r^2=R^2-\frac{h^2}{4}$$

$$V=\pi r^2h$$

$$V(h)=\pi\left(R^2-\frac{h^2}{4}\right)h=\pi h\left(R^2-\frac{h^2}{4}\right)$$

21. r = radius of cylinder, h = height of cylinder, v = volume of cylinder

By similar triangles: $\frac{H}{R}=\frac{H-h}{r}$

$$\frac{H}{R}=\frac{H-h}{2x}$$

$$2Hx=RH-Rh$$

$$Rh=RH-2Hx$$

$$h=\frac{RH-2Hx}{R}$$

$$=\frac{H(R-2x)}{R}$$

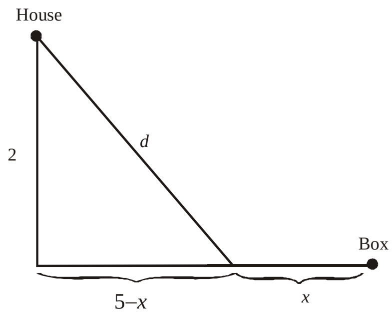
$$V=\pi r^2h$$

$$=\pi r^2\left(\frac{H(R-2x)}{R}\right)$$

$$=\pi(2x)^2\left(\frac{H(R-2x)}{R}\right)$$

$$=\frac{4H\pi(R-2x)x^2}{R}$$

22. a. The total cost of installing the cable along the road is $500x$. If cable is installed x miles along the road, there are $5-x$ miles between the road to the house and where the cable ends along the road.



$d = \sqrt{(5-x)^2 + 2^2}$

$= \sqrt{25 - 10x + x^2 + 4} = \sqrt{x^2 - 10x + 29}$

The total cost of installing the cable is:

$C(x) = 500x + 700\sqrt{x^2 - 10x + 29}$

Domain: $\{x \mid 0 \le x \le 5\}$

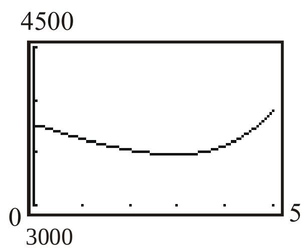
b. $C(1) = 500(1) + 700\sqrt{1^2 - 10(1) + 29}$

$= 500 + 700\sqrt{20} = \$3630.50$

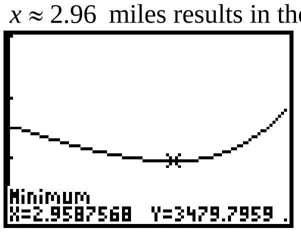
c. $C(3) = 500(3) + 700\sqrt{3^2 - 10(3) + 29}$

$= 1500 + 700\sqrt{8} = \$3479.90$

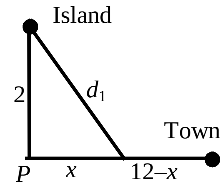
d.



e. Using MINIMUM, the graph indicates that $x \approx 2.96$ miles results in the least cost.



23. a. The time on the boat is given by $\frac{d_1}{3}$. The time on land is given by $\frac{12-x}{5}$.



$d_1 = \sqrt{x^2 + 2^2} = \sqrt{x^2 + 4}$

The total time for the trip is:

$T(x) = \frac{12-x}{5} + \frac{d_1}{3} = \frac{12-x}{5} + \frac{\sqrt{x^2+4}}{3}$

b. Domain: $\{x \mid 0 \le x \le 12\}$

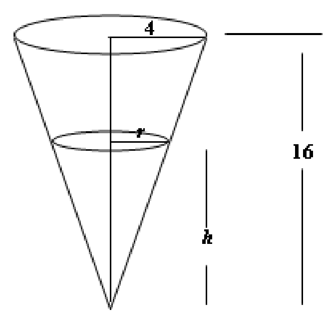
c. $T(4) = \frac{12-4}{5} + \frac{\sqrt{4^2+4}}{3}$

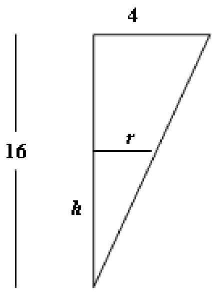
$= \frac{8}{5} + \frac{\sqrt{20}}{3} \approx 3.09$ hours

d. $T(8) = \frac{12-8}{5} + \frac{\sqrt{8^2+4}}{3}$

$= \frac{4}{5} + \frac{\sqrt{68}}{3} \approx 3.55$ hours

24. Consider the diagrams shown below.





There is a pair of similar triangles in the diagram. Since the smaller triangle is similar to the larger triangle, we have the proportion

$$\frac{r}{h}=\frac{4}{16}\Rightarrow\frac{r}{h}=\frac{1}{4}\Rightarrow r=\frac{1}{4}h$$

Substituting into the volume formula for the conical portion of water gives

$$V(h)=\frac{1}{3}\pi r^2 h=\frac{1}{3}\pi\left(\frac{1}{4}h\right)^2 h=\frac{\pi}{48}h^3.$$

25. a. length = $24-2x$; width = $24-2x$; height = x

$V(x)=x(24-2x)(24-2x)=x(24-2x)^2$

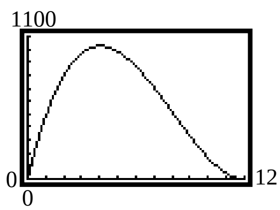
b. $V(3)=3(24-2(3))^2=3(18)^2$

$=3(324)=972\text{ in}^3.$

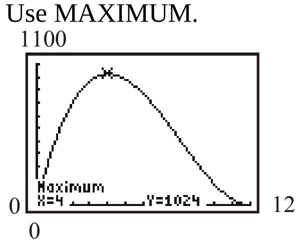
c. $V(10)=10(24-2(10))^2=10(4)^2$

$=10(16)=160\text{ in}^3.$

d. $y_1=x(24-2x)^2$



Use MAXIMUM.



The volume is largest when $x=4$ inches.

26. a. Let $A=$ amount of material, $x=$ length of the base, $h=$ height, and $V=$ volume.

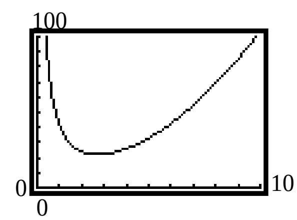
$V=x^2h=10\Rightarrow h=\dfrac{10}{x^2}$

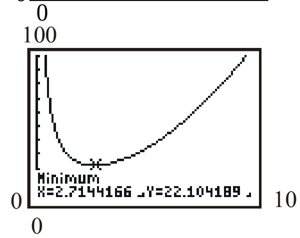
$$\text{Total Area } A=(\text{Area}_{\text{base}})+(4)(\text{Area}_{\text{side}})$$
$$=x^2+4xh$$
$$=x^2+4x\left(\frac{10}{x^2}\right)$$
$$=x^2+\frac{40}{x}$$
$$A(x)=x^2+\frac{40}{x}$$

b. $A(1)=1^2+\dfrac{40}{1}=1+40=41\text{ ft}^2$

c. $A(2)=2^2+\dfrac{40}{2}=4+20=24\text{ ft}^2$

d. $y_1=x^2+\dfrac{40}{x}$





The amount of material is least when $x=2.71$ ft.

27. The center would be the midpoint

$$(h,k)=\left(\frac{4+(-6)}{2},\frac{-5+3}{2}\right)$$
$$=\left(\frac{-2}{2},\frac{-2}{2}\right)=(-1,-1)$$

The distance from the midpoint to one of the point would be the radius.

$$r=\sqrt{(-1-4)^2+(-1-(-5))^2}=\sqrt{(-5)^2+(4)^2}$$
$$=\sqrt{25+16}=\sqrt{41}$$

28. In order for the 16-foot long Ford Fusion to pass the 50-foot truck, the Ford Fusion must travel the length of the truck and the length of itself in the time frame of 5 seconds. Thus the Fusion must travel an additional 66 feet in 5 seconds.

Convert this to miles-per-hour.

$5 \text{ sec} = \frac{5}{60} \text{ min} = \frac{5}{3600} \text{ hr} = \frac{1}{720} \text{ hr.}$

$66 \text{ ft} = \frac{66}{5280} \text{ mi}$

$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{\frac{66}{5280}}{\frac{1}{720}} = 9 \text{ mph}$

Since the truck is traveling 55 mph, the Fusion must travel 55 + 9 = 64 mph.

29. Start with $y = x^2$. To shift the graph left 4 units would change the function to $y = (x+4)^2$. To shift the graph down 2 units would change the function to $y = (x+4)^2 - 2$.

30. $(-\infty, -2) \cup (-2, 5]$

Chapter 2 Review Exercises

1. This relation represents a function.
Domain = {–1, 2, 4}; Range = {0, 3}.

2. This relation does not represent a function, since 4 is paired with two different values.

3. $f(x) = \frac{3x}{x^2 - 1}$

a. $f(3) = \frac{3(3)}{(3)^2 - 1} = \frac{9}{8}$

b. $f(-3) = \frac{3(-3)}{(-3)^2 - 1} = \frac{-9}{8}$

c. $f(-x) = \frac{3(-x)}{(-x)^2 - 1} = \frac{-3x}{x^2 - 1}$

d. $-f(x) = -\left(\frac{3x}{x^2 - 1}\right) = \frac{-3x}{x^2 - 1}$

e. $f(x-3) = \frac{3(x-3)}{(x-3)^2 - 1}$
$= \frac{3x - 3}{(x^2 - 6x + 9) - 1} = \frac{3x - 3}{x^2 - 6x + 8}$

f. $f(3x) = \frac{3(3x)}{(3x)^2 - 1} = \frac{9x}{9x^2 - 1}$

4. $f(x) = \sqrt{x^2 - 4}$

a. $f(3) = \sqrt{3^2 - 4} = \sqrt{5}$

b. $f(-3) = \sqrt{(-3)^2 - 4} = \sqrt{5}$

c. $f(-x) = \sqrt{(-x)^2 - 4} = \sqrt{x^2 - 4}$

d. $-f(x) = -\sqrt{x^2 - 4}$

e. $f(x-3) = \sqrt{(x-3)^2 - 4}$
$= \sqrt{(x^2 - 6x + 9) - 4}$
$= \sqrt{x^2 - 6x + 5}$

f. $f(3x) = \sqrt{(3x)^2 - 4} = \sqrt{9x^2 - 4}$

5. $f(x) = \frac{x^2 - 4}{x^2}$

a. $f(3) = \frac{3^2 - 4}{3^2} = \frac{5}{9}$

b. $f(-3) = \frac{(-3)^2 - 4}{(-3)^2} = \frac{5}{9}$

c. $f(-x) = \frac{(-x)^2 - 4}{(-x)^2} = \frac{x^2 - 4}{x^2}$

d. $-f(x) = -\left(\frac{x^2 - 4}{x^2}\right) = \frac{-x^2 + 4}{x^2}$

e. $f(x-3) = \frac{(x-3)^2 - 4}{(x-3)^2} = \frac{(x^2 - 6x + 9) - 4}{(x^2 - 6x + 9)}$
$= \frac{x^2 - 6x + 5}{x^2 - 6x + 9}$

f. $f(3x) = \frac{(3x)^2 - 4}{(3x)^2} = \frac{9x^2 - 4}{9x^2}$

6. $f(x)=\dfrac{x}{x^2-9}$

The denominator cannot be zero:

$x^2-9\neq 0$

$(x+3)(x-3)\neq 0$

$x\neq -3 \text{ or } 3$

Domain: $\{x \mid x\neq -3,\ x\neq 3\}$

7. $f(x)=\sqrt{7-2x}$

The radicand must be non-negative:

$7-2x\geq 0$

$x\leq \dfrac{7}{2}$

Domain: $\left\{x \,\middle|\, x\leq \dfrac{7}{2}\right\}$ or $\left(-\infty, \dfrac{7}{2}\right]$

8. $g(x)=\dfrac{|x|}{x}$

The denominator cannot be zero:

$x\neq 0$

Domain: $\{x \mid x\neq 0\}$

9. $f(x)=\dfrac{2x}{x^2+4x-5}$

The denominator cannot be zero:

$x^2+4x-5\neq 0$

$(x+4)(x-1)\neq 0$

$x\neq 1 \text{ or } -4$

Domain: $\{x \mid x\neq -4,\ x\neq 1\}$

10. $f(x)=\dfrac{\sqrt{x+1}}{x^2-4}$

The denominator cannot be zero:

$x^2-4\neq 0$

$(x+2)(x-2)\neq 0$

$x\neq -2 \text{ or } 2$

Also, the radicand must be non-negative:

$x+1\geq 0$

$x\geq -1$

Domain: $[-1,2)\cup(2,\infty)$

11. $g(x)=\dfrac{x}{\sqrt{3x+10}}$

The radicand must be non-negative and not zero:

$3x+10>0$

$x>-\dfrac{10}{3}$

Domain: $\left\{x \,\middle|\, x>-\dfrac{10}{3}\right\}$

12. $f(x)=2-x \quad g(x)=3x+1$

$$(f+g)(x)=f(x)+g(x)$$
$$=2-x+3x+1=2x+3$$

Domain: $\{x \mid x \text{ is any real number}\}$

$$(f-g)(x)=f(x)-g(x)$$
$$=2-x-(3x+1)$$
$$=2-x-3x-1$$
$$=-4x+1$$

Domain: $\{x \mid x \text{ is any real number}\}$

$$(f\cdot g)(x)=f(x)\cdot g(x)$$
$$=(2-x)(3x+1)$$
$$=6x+2-3x^2-x$$
$$=-3x^2+5x+2$$

Domain: $\{x \mid x \text{ is any real number}\}$

$$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}=\frac{2-x}{3x+1}$$

$3x+1\neq 0$

$3x\neq -1 \Rightarrow x\neq -\dfrac{1}{3}$

Domain: $\left\{x \,\middle|\, x\neq -\dfrac{1}{3}\right\}$

13. $f(x)=4x^2+3 \qquad g(x)=x-2$

$$(f+g)(x)=f(x)+g(x)$$
$$=4x^2+3+x-2$$
$$=4x^2+x+1$$

Domain: $\{x \mid x \text{ is any real number}\}$

$$(f\cdot g)(x)=f(x)\cdot g(x)$$
$$=(4x^2+3)(x-2)$$
$$=4x^3-8x^2+3x-6$$

Domain: $\{x \mid x \text{ is any real number}\}$

$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}=\frac{4x^2+3}{x-2}$
$x-2\neq 0 \Rightarrow x\neq 2$
Domain: $\{x \mid x\neq 2\}$

14. $f(x)=\frac{x+1}{x-1} \qquad g(x)=\frac{1}{x}$

$(f+g)(x)=f(x)+g(x)$
$=\frac{x+1}{x-1}+\frac{1}{x}=\frac{x(x+1)+1(x-1)}{x(x-1)}$
$=\frac{x^2+x+x-1}{x(x-1)}=\frac{x^2+2x-1}{x(x-1)}$
Domain: $\{x \mid x\neq 0, x\neq 1\}$

$(f-g)(x)=f(x)-g(x)$
$=\frac{x+1}{x-1}-\frac{1}{x}=\frac{x(x+1)-1(x-1)}{x(x-1)}$
$=\frac{x^2+x-x+1}{x(x-1)}=\frac{x^2+1}{x(x-1)}$
Domain: $\{x \mid x\neq 0, x\neq 1\}$

$(f\cdot g)(x)=f(x)\cdot g(x)=\left(\frac{x+1}{x-1}\right)\left(\frac{1}{x}\right)=\frac{x+1}{x(x-1)}$
Domain: $\{x \mid x\neq 0, x\neq 1\}$

$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}=\frac{\frac{x+1}{x-1}}{\frac{1}{x}}=\left(\frac{x+1}{x-1}\right)\left(\frac{x}{1}\right)=\frac{x(x+1)}{x-1}$
Domain: $\{x \mid x\neq 0, x\neq 1\}$

15. $f(x)=-x^2+7x+3$
$\frac{f(x+h)-f(x)}{h}$
$=\frac{-(x+h)^2+7(x+h)+3-(-x^2+7x+3)}{h}$
$=\frac{-x^2-h^2-2xh+7x+7h+3+x^2-7x-3}{h}$
$=\frac{-2xh-h^2+7h}{h}=-2x-h+7$

16. a. Domain: $\{x \mid -4\leq x\leq 3\}$; $[-4, 3]$
Range: $\{y \mid -3\leq y\leq 3\}$; $[-3, 3]$

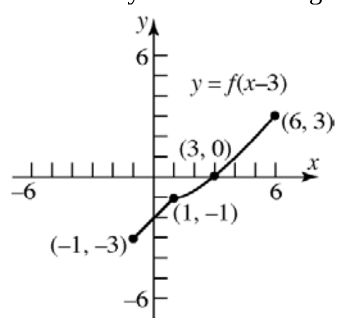
b. Intercept: $(0,0)$

c. $f(-2)=-1$

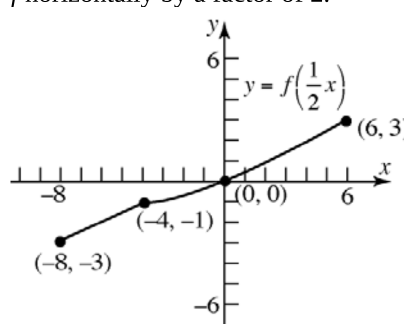
d. $f(x)=-3$ when $x=-4$

e. $f(x)>0$ when $0<x\leq 3$
$\{x \mid 0<x\leq 3\}$

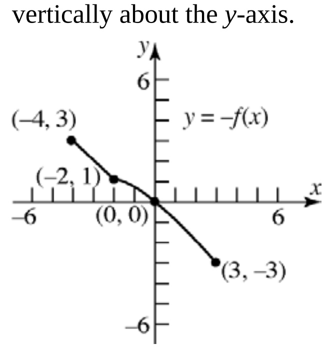
f. To graph $y=f(x-3)$, shift the graph of f horizontally 3 units to the right.



g. To graph $y=f\left(\frac{1}{2}x\right)$, stretch the graph of f horizontally by a factor of 2.



h. To graph $y=-f(x)$, reflect the graph of f vertically about the y-axis.



17. **a.** Domain: $(-\infty, 4]$
Range: $(-\infty, 3]$

b. Increasing: $(-\infty, -2)$ and $(2, 4)$;
Decreasing: $(-2, 2)$

c. Local minimum is -1 at $x = 2$;
Local maximum is 1 at $x = -2$

d. No absolute minimum;
Absolute maximum is 3 at $x = 4$

e. The graph has no symmetry.

f. The function is neither.

g. x-intercepts: $-3, 0, 3$;
y-intercept: 0

18. $f(x) = x^3 - 4x$

$f(-x) = (-x)^3 - 4(-x) = -x^3 + 4x$

$= -\left(x^3 - 4x\right) = -f(x)$

f is odd.

19. $g(x) = \dfrac{5 + 2x^2}{3 + x^6}$

$g(-x) = \dfrac{5 + 2(-x)^2}{3 + (-x)^6} = \dfrac{5 + 2x^2}{3 + x^6} = g(x)$

g is even.

20. $G(x) = 1 - x + x^3$

$G(-x) = 1 - (-x) + (-x)^3$

$= 1 + x - x^3 \neq -G(x)$ or $G(x)$

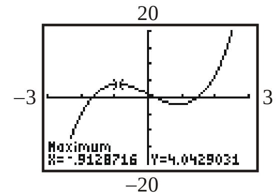
G is neither even nor odd.

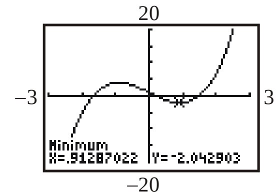
21. $f(x) = \dfrac{3x^3}{2 + x^2 + 2x^4}$

$f(-x) = \dfrac{3(-x)^3}{2 + (-x)^2 + 2(-x)^4} = \dfrac{-3x^3}{2 + x^2 + 2x^4} = -f(x)$

f is odd.

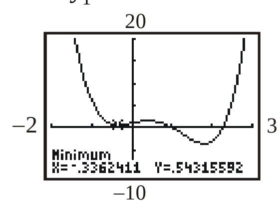
22. $f(x) = 2x^3 - 5x + 1$ on the interval $(-3, 3)$
Use MAXIMUM and MINIMUM on the graph of $y_1 = 2x^3 - 5x + 1$.

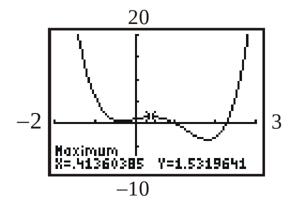


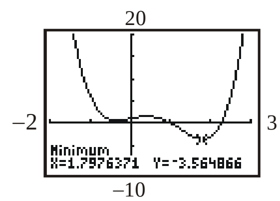


local maximum: 4.04 when $x \approx -0.91$
local minimum: -2.04 when $x = 0.91$
f is increasing on: $(-3, -0.91)$ and $(0.91, 3)$;
f is decreasing on: $(-0.91, 0.91)$.

23. $f(x) = 2x^4 - 5x^3 + 2x + 1$ on the interval $(-2, 3)$
Use MAXIMUM and MINIMUM on the graph of $y_1 = 2x^4 - 5x^3 + 2x + 1$.







local maximum: 1.53 when $x = 0.41$
local minima: 0.54 when $x = -0.34$, -3.56 when $x = 1.80$
f is increasing on: $(-0.34, 0.41)$ and $(1.80, 3)$;
f is decreasing on: $(-2, -0.34)$ and $(0.41, 1.80)$.

24. $f(x) = 8x^2 - x$

a. $\dfrac{f(2) - f(1)}{2 - 1} = \dfrac{8(2)^2 - 2 - [8(1)^2 - 1]}{1}$

$= 32 - 2 - (7) = 23$

b. $\dfrac{f(1) - f(0)}{1 - 0} = \dfrac{8(1)^2 - 1 - [8(0)^2 - 0]}{1}$

$= 8 - 1 - (0) = 7$

c. $\dfrac{f(4) - f(2)}{4 - 2} = \dfrac{8(4)^2 - 4 - [8(2)^2 - 2]}{2}$

$= \dfrac{128 - 4 - (30)}{2} = \dfrac{94}{2} = 47$

25. $f(x)=2-5x$

$$\frac{f(5)-f(3)}{5-3}=\frac{\left[2-5(5)\right]-\left[2-5(3)\right]}{5-3}$$
$$=\frac{(2-25)-(2-15)}{2}=-5$$

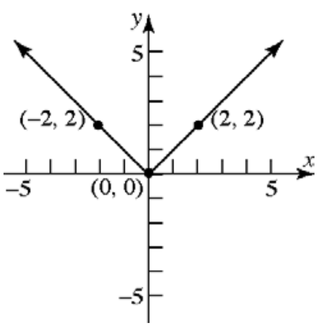
26. $f(x)=3x-4x^2$

$$\frac{f(5)-f(3)}{5-3}=\frac{\left[3(5)-4(5)^3\right]-\left[3(3)-4(3)^3\right]}{5-3}$$
$$=\frac{(15-500)-(9-108)}{2}=-193$$

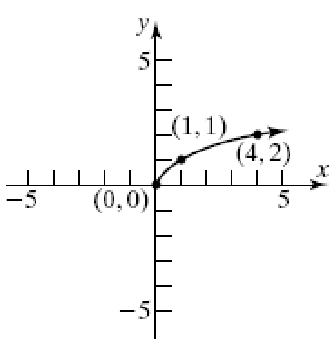
27. The graph does not pass the Vertical Line Test and is therefore not a function.

28. The graph passes the Vertical Line Test and is therefore a function.

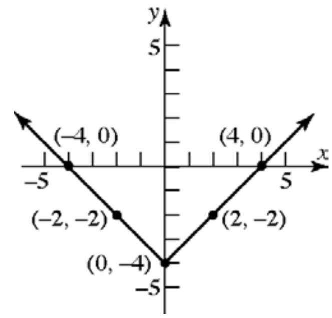
29. $f(x)=|x|$



30. $f(x)=\sqrt{x}$

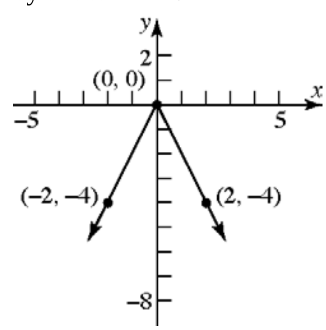


31. $F(x)=|x|-4$. Using the graph of $y=|x|$, vertically shift the graph downward 4 units.



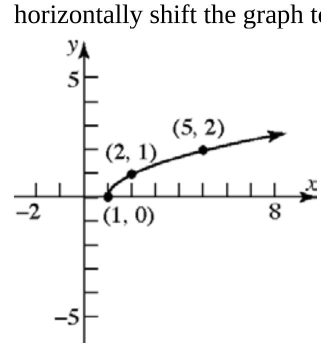
Intercepts: (–4,0), (4,0), (0,–4)
Domain: $\{x\,|\,x \text{ is any real number}\}$
Range: $\{y\,|\,y\geq -4\}$ or $[-4,\infty)$

32. $g(x)=-2|x|$. Reflect the graph of $y=|x|$ about the *x*-axis and vertically stretch the graph by a factor of 2.



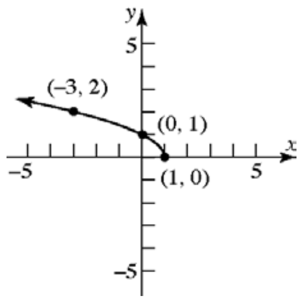
Intercepts: (0, 0)
Domain: $\{x\,|\,x \text{ is any real number}\}$
Range: $\{y\,|\,y\leq 0\}$ or $(-\infty,0]$

33. $h(x)=\sqrt{x-1}$. Using the graph of $y=\sqrt{x}$, horizontally shift the graph to the right 1 unit.



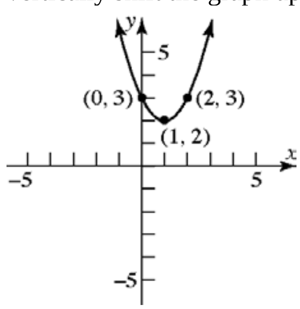
Intercept: (1, 0)
Domain: $\{x\,|\,x\geq 1\}$ or $[1,\infty)$
Range: $\{y\,|\,y\geq 0\}$ or $[0,\infty)$

34. $f(x)=\sqrt{1-x}=\sqrt{-(x-1)}$. Reflect the graph of $y=\sqrt{x}$ about the y-axis and horizontally shift the graph to the right 1 unit.



Intercepts: (1, 0), (0, 1)
Domain: $\{x \mid x \le 1\}$ or $(-\infty, 1]$
Range: $\{y \mid y \ge 0\}$ or $[0, \infty)$

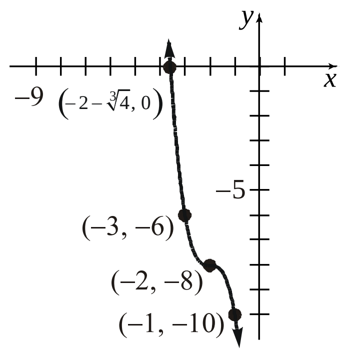
35. $h(x)=(x-1)^2+2$. Using the graph of $y=x^2$, horizontally shift the graph to the right 1 unit and vertically shift the graph up 2 units.



Intercepts: (0, 3)
Domain: $\{x \mid x \text{ is any real number}\}$
Range: $\{y \mid y \ge 2\}$ or $[2, \infty)$

36. $g(x)=-2(x+2)^3-8$

Using the graph of $y=x^3$, horizontally shift the graph to the left 2 units, vertically stretch the graph by a factor of 2, reflect about the x-axis, and vertically shift the graph down 8 units.



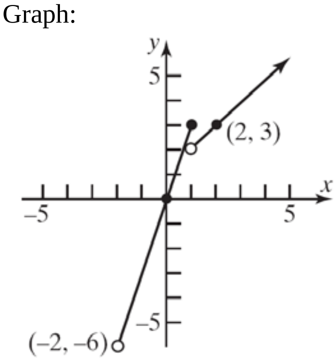
Intercepts: (0,–24), $\left(-2-\sqrt[3]{4},\ 0\right) \approx (-3.6,\ 0)$
Domain: $\{x \mid x \text{ is any real number}\}$
Range: $\{y \mid y \text{ is any real number}\}$

37. $f(x)=\begin{cases} 3x & \text{if } -2 < x \le 1 \\ x+1 & \text{if } x > 1 \end{cases}$

a. Domain: $\{\, x \mid x > -2 \,\}$ or $(-2, \infty)$

b. Intercept: $(0, 0)$

c. Graph:



d. Range: $\{\, y \mid y > -6 \,\}$ or $(-6, \infty)$

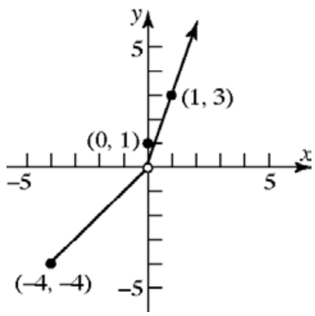
e. There is a jump in the graph at $x=1$. Therefore, the function is not continuous.

38. $f(x)=\begin{cases} x & \text{if } -4 \le x < 0 \\ 1 & \text{if } x = 0 \\ 3x & \text{if } x > 0 \end{cases}$

a. Domain: $\{x \mid x \ge -4\}$ or $[-4, \infty)$

b. Intercept: (0, 1)

c. Graph:



d. Range: $\{y \mid y \geq -4,\ y \neq 0\}$

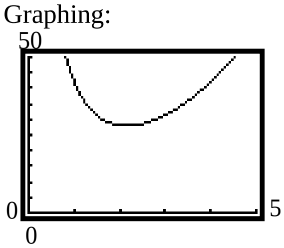
40. a. $x^2 h = 10 \quad \Rightarrow \quad h = \dfrac{10}{x^2}$

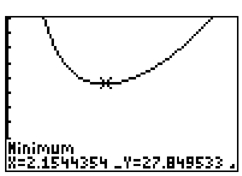
$$\begin{aligned} A(x) &= 2x^2 + 4xh \\ &= 2x^2 + 4x\left(\frac{10}{x^2}\right) \\ &= 2x^2 + \frac{40}{x} \end{aligned}$$

b. $A(1) = 2 \cdot 1^2 + \dfrac{40}{1} = 2 + 40 = 42\ \text{ft}^2$

c. $A(2) = 2 \cdot 2^2 + \dfrac{40}{2} = 8 + 20 = 28\ \text{ft}^2$

d. Graphing:





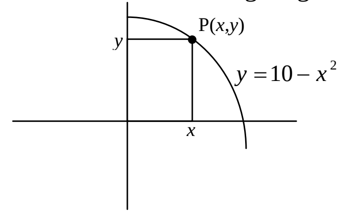
The area is smallest when $x \approx 2.15$ feet.

e. There is a jump at $x = 0$. Therefore, the function is not continuous.

39. $f(x) = \dfrac{Ax+5}{6x-2}$ and $f(1) = 4$

$$\begin{aligned} \frac{A(1)+5}{6(1)-2} &= 4 \\ \frac{A+5}{4} &= 4 \\ A+5 &= 16 \\ A &= 11 \end{aligned}$$

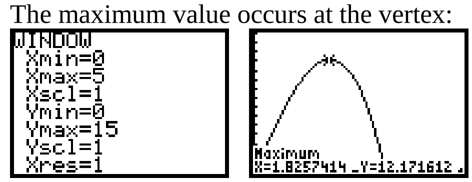
41. a. Consider the following diagram:



The area of the rectangle is $A = xy$. Thus, the area function for the rectangle is:

$A(x) = x(10 - x^2)$

b. The maximum value occurs at the vertex:



The maximum area is roughly:

$$\begin{aligned} A(1.83) &= -(1.83)^3 + 10(1.83) \\ &\approx 12.17 \text{ square units} \end{aligned}$$

Chapter 2 Test

1. a. $\{(2,5),(4,6),(6,7),(8,8)\}$

This relation is a function because there are no ordered pairs that have the same first element and different second elements.

Domain: $\{2,4,6,8\}$

Range: $\{5,6,7,8\}$

b. $\{(1,3),(4,-2),(-3,5),(1,7)\}$

This relation is not a function because there are two ordered pairs that have the same first element but different second elements.

c. This relation is not a function because the graph fails the vertical line test.

d. This relation is a function because it passes the vertical line test.
Domain: $\{x \mid x \text{ is any real number}\}$
Range: $\{y \mid y \geq 2\}$ or $[2, \infty)$

2. $f(x) = \sqrt{4-5x}$
The function tells us to take the square root of $4-5x$. Only nonnegative numbers have real square roots so we need $4-5x \geq 0$.

$$4-5x \geq 0$$
$$4-5x-4 \geq 0-4$$
$$-5x \geq -4$$
$$\frac{-5x}{-5} \leq \frac{-4}{-5}$$
$$x \leq \frac{4}{5}$$

Domain: $\left\{x \middle| x \leq \frac{4}{5}\right\}$ or $\left(-\infty, \frac{4}{5}\right]$

$$f(-1) = \sqrt{4-5(-1)} = \sqrt{4+5} = \sqrt{9} = 3$$

3. $g(x) = \dfrac{x+2}{|x+2|}$
The function tells us to divide $x+2$ by $|x+2|$. Division by 0 is undefined, so the denominator can never equal 0. This means that $x \neq -2$.
Domain: $\{x \mid x \neq -2\}$

$$g(-1) = \frac{(-1)+2}{|(-1)+2|} = \frac{1}{|1|} = 1$$

4. $h(x) = \dfrac{x-4}{x^2+5x-36}$
The function tells us to divide $x-4$ by $x^2+5x-36$. Since division by 0 is not defined, we need to exclude any values which make the denominator 0.

$$x^2+5x-36 = 0$$
$$(x+9)(x-4) = 0$$
$$x = -9 \text{ or } x = 4$$

Domain: $\{x \mid x \neq -9, x \neq 4\}$
(note: there is a common factor of $x-4$ but we must determine the domain prior to simplifying)

$$h(-1) = \frac{(-1)-4}{(-1)^2+5(-1)-36} = \frac{-5}{-40} = \frac{1}{8}$$

5. a. To find the domain, note that all the points on the graph will have an *x*-coordinate between -5 and 5, inclusive. To find the range, note that all the points on the graph will have a *y*-coordinate between -3 and 3, inclusive.
Domain: $\{x \mid -5 \leq x \leq 5\}$ or $[-5, 5]$
Range: $\{y \mid -3 \leq y \leq 3\}$ or $[-3, 3]$

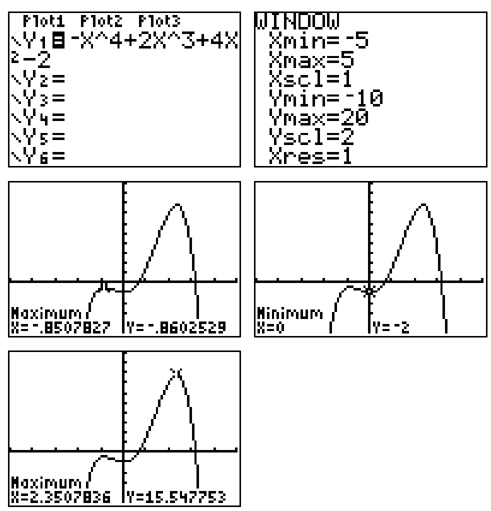
b. The intercepts are $(0,2)$, $(-2,0)$, and $(2,0)$.
x-intercepts: -2, 2
y-intercept: 2

c. $f(1)$ is the value of the function when $x = 1$. According to the graph, $f(1) = 3$.

d. Since $(-5,-3)$ and $(3,-3)$ are the only points on the graph for which $y = f(x) = -3$, we have $f(x) = -3$ when $x = -5$ and $x = 3$.

e. To solve $f(x) < 0$, we want to find *x*-values such that the graph is below the x-axis. The graph is below the x-axis for values in the domain that are less than -2 and greater than 2. Therefore, the solution set is $\{x \mid -5 \leq x < -2 \text{ or } 2 < x \leq 5\}$. In interval notation we would write the solution set as $[-5,-2) \cup (2,5]$.

6. $f(x) = -x^4 + 2x^3 + 4x^2 - 2$
We set Xmin = -5 and Xmax = 5. The standard Ymin and Ymax will not be good enough to see the whole picture so some adjustment must be made.



We see that the graph has a local maximum of -0.86 (rounded to two places) when $x=-0.85$ and another local maximum of 15.55 when $x=2.35$. There is a local minimum of -2 when $x=0$. Thus, we have

Local maxima: $f(-0.85)\approx -0.86$

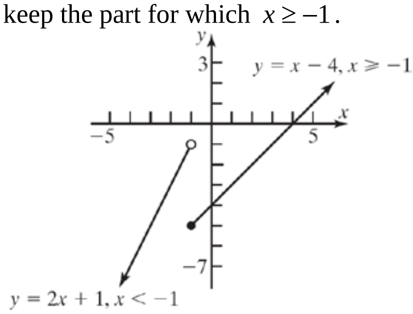
$f(2.35)\approx 15.55$

Local minima: $f(0)=-2$

The function is increasing on the intervals $(-5,-0.85)$ and $(0,2.35)$ and decreasing on the intervals $(-0.85,0)$ and $(2.35,5)$.

7. **a.** $f(x)=\begin{cases}2x+1 & x<-1\\ x-4 & x\ge -1\end{cases}$

To graph the function, we graph each "piece". First we graph the line $y=2x+1$ but only keep the part for which $x<-1$. Then we plot the line $y=x-4$ but only keep the part for which $x\ge -1$.



b. To find the intercepts, notice that the only piece that hits either axis is $y=x-4$.

$y=x-4$	$y=x-4$
$y=0-4$	$0=x-4$
$y=-4$	$4=x$

The intercepts are $(0,-4)$ and $(4,0)$.

c. To find $g(-5)$ we first note that $x=-5$ so we must use the first "piece" because $-5<-1$.

$g(-5)=2(-5)+1=-10+1=-9$

d. To find $g(2)$ we first note that $x=2$ so we must use the second "piece" because $2\ge -1$.

$g(2)=2-4=-2$

8. The average rate of change from 3 to 4 is given by

$$\frac{\Delta y}{\Delta x}=\frac{f(4)-f(3)}{4-3}$$
$$=\frac{\left(3(4)^2-2(4)+4\right)-\left(3(3)^2-2(3)+4\right)}{4-3}$$
$$=\frac{44-25}{4-3}=\frac{19}{1}=19$$

9. **a.** $(f-g)(x)=\left(2x^2+1\right)-\left(3x-2\right)$

$=2x^2+1-3x+2=2x^2-3x+3$

b. $(f\cdot g)(x)=\left(2x^2+1\right)\left(3x-2\right)$

$=6x^3-4x^2+3x-2$

c. $f(x+h)-f(x)$

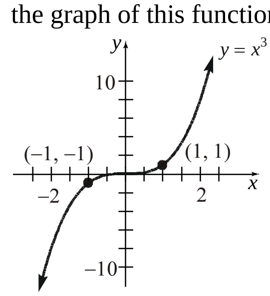
$=\left(2(x+h)^2+1\right)-\left(2x^2+1\right)$

$=\left(2\left(x^2+2xh+h^2\right)+1\right)-\left(2x^2+1\right)$

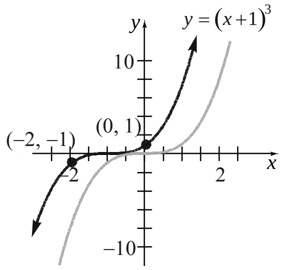
$=2x^2+4xh+2h^2+1-2x^2-1$

$=4xh+2h^2$

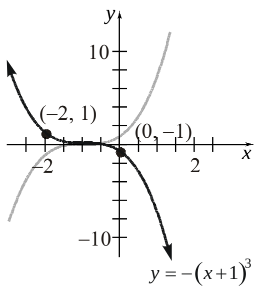
10. **a.** The basic function is $y=x^3$ so we start with the graph of this function.



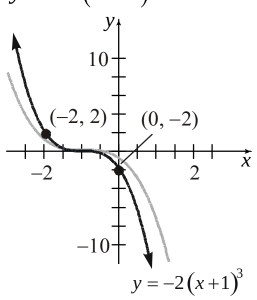
Next we shift this graph 1 unit to the left to obtain the graph of $y=(x+1)^3$.



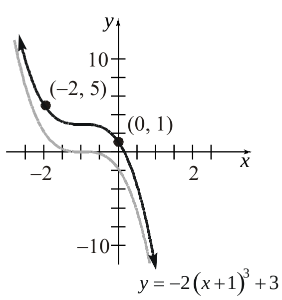
Next we reflect this graph about the x-axis to obtain the graph of $y=-(x+1)^3$.



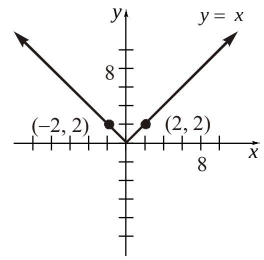
Next we stretch this graph vertically by a factor of 2 to obtain the graph of $y=-2(x+1)^3$.



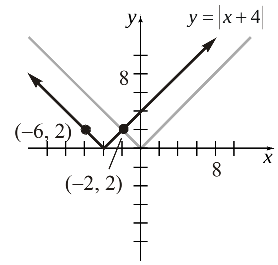
The last step is to shift this graph up 3 units to obtain the graph of $y=-2(x+1)^3+3$.



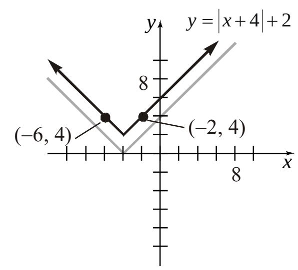
b. The basic function is $y=|x|$ so we start with the graph of this function.



Next we shift this graph 4 units to the left to obtain the graph of $y=|x+4|$.

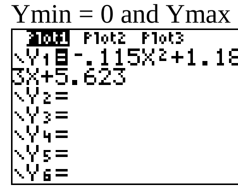


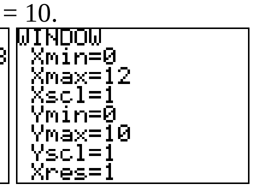
Next we shift this graph up 2 units to obtain the graph of $y=|x+4|+2$.

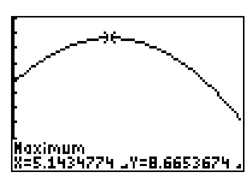


11. a. $r(x)=-0.115x^2+1.183x+5.623$

For the years 1992 to 2004, we have values of *x* between 0 and 12. Therefore, we can let Xmin = 0 and Xmax = 12. Since *r* is the interest rate as a percent, we can try letting Ymin = 0 and Ymax = 10.



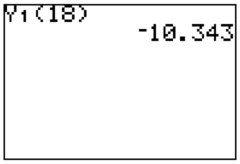




The highest rate during this period appears to be 8.67%, occurring in 1997 ($x\approx 5$).

b. For 2010, we have $x = 2010 - 1992 = 18$.

$$r(18) = -0.115(18)^2 + 1.183(18) + 5.623$$
$$= -10.343$$



The model predicts that the interest rate will be -10.343%. This is not a reasonable value since it implies that the bank would be paying interest to the borrower.

12. a. Let x = width of the rink in feet. Then the length of the rectangular portion is given by $2x - 20$. The radius of the semicircular portions is half the width, or $r = \frac{x}{2}$.

To find the volume, we first find the area of the volume. That is,

$$V(x) = \frac{1}{16}\left(2x^2 - 20x + \frac{\pi x^2}{4}\right)$$

$$V(x) = \frac{x^2}{8} - \frac{5x}{4} + \frac{\pi x^2}{64}$$

b. If the rink is 90 feet wide, then we have $x = 90$.

$$V(90) = \frac{90^2}{8} - \frac{5(90)}{4} + \frac{\pi(90)^2}{64} \approx 1297.61$$

The volume of ice is roughly 1297.61 ft^3.

Chapter 2 Cumulative Review

1.
$$3x - 8 = 10$$
$$3x - 8 + 8 = 10 + 8$$
$$3x = 18$$
$$\frac{3x}{3} = \frac{18}{3}$$
$$x = 6$$

The solution set is $\{6\}$.

the surface and multiply by the thickness of the ice. The two semicircles can be combined to form a complete circle, so the area is given by

$$A = l \cdot w + \pi r^2$$
$$= (2x - 20)(x) + \pi\left(\frac{x}{2}\right)^2$$
$$= 2x^2 - 20x + \frac{\pi x^2}{4}$$

We have expressed our measures in feet so we need to convert the thickness to feet as well.

$$0.75 \text{ in} \cdot \frac{1 \text{ ft}}{12 \text{ in}} = \frac{0.75}{12} \text{ ft} = \frac{1}{16} \text{ ft}$$

Now we multiply this by the area to obtain

2.
$$3x^2 - x = 0$$
$$x(3x - 1) = 0$$
$$x = 0 \quad \text{or} \quad 3x - 1 = 0$$
$$3x = 1$$
$$x = \frac{1}{3}$$

The solution set is $\left\{0, \frac{1}{3}\right\}$.

3.
$$x^2 - 8x - 9 = 0$$
$$(x - 9)(x + 1) = 0$$
$$x - 9 = 0 \quad \text{or} \quad x + 1 = 0$$
$$x = 9 \qquad x = -1$$

The solution set is $\{-1, 9\}$.

4.
$$6x^2 - 5x + 1 = 0$$
$$(3x - 1)(2x - 1) = 0$$
$$3x - 1 = 0 \quad \text{or} \quad 2x - 1 = 0$$
$$3x = 1 \qquad 2x = 1$$
$$x = \frac{1}{3} \qquad x = \frac{1}{2}$$

The solution set is $\left\{\frac{1}{3}, \frac{1}{2}\right\}$.

5. $|2x+3|=4$

$$\begin{aligned} 2x+3&=-4 \quad \text{or} \quad & 2x+3&=4 \\ 2x&=-7 & 2x&=1 \\ x&=-\frac{7}{2} & x&=\frac{1}{2} \end{aligned}$$

The solution set is $\left\{-\frac{7}{2}, \frac{1}{2}\right\}$.

6.
$$\begin{aligned} \sqrt{2x+3}&=2 \\ \left(\sqrt{2x+3}\right)^2&=2^2 \\ 2x+3&=4 \\ 2x&=1 \\ x&=\frac{1}{2} \end{aligned}$$

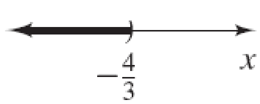
Check:
$$\begin{aligned} \sqrt{2\left(\frac{1}{2}\right)+3}&\stackrel{?}{=}2 \\ \sqrt{1+3}&\stackrel{?}{=}2 \\ \sqrt{4}&\stackrel{?}{=}2 \\ 2&=2 \quad \text{T} \end{aligned}$$

The solution set is $\left\{\frac{1}{2}\right\}$.

7.
$$\begin{aligned} 2-3x&>6 \\ -3x&>4 \\ x&<-\frac{4}{3} \end{aligned}$$

Solution set: $\left\{x \mid x<-\frac{4}{3}\right\}$

Interval notation: $\left(-\infty, -\frac{4}{3}\right)$



8. $|2x-5|<3$

$$\begin{aligned} -3<2x-5<3 \\ 2<2x<8 \\ 1<x<4 \end{aligned}$$

Solution set: $\{x \mid 1<x<4\}$

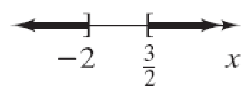
Interval notation: $(1,4)$

9. $|4x+1|\ge 7$

$$\begin{aligned} 4x+1&\le -7 \quad \text{or} \quad & 4x+1&\ge 7 \\ 4x&\le -8 & 4x&\ge 6 \\ x&\le -2 & x&\ge \frac{3}{2} \end{aligned}$$

Solution set: $\left\{x \mid x\le -2 \text{ or } x\ge \frac{3}{2}\right\}$

Interval notation: $(-\infty,-2]\cup\left[\frac{3}{2},\infty\right)$



10. a.
$$\begin{aligned} d&=\sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2} \\ &=\sqrt{\left(3-(-2)\right)^2+\left(-5-(-3)\right)^2} \\ &=\sqrt{(3+2)^2+(-5+3)^2} \\ &=\sqrt{5^2+(-2)^2}=\sqrt{25+4} \\ &=\sqrt{29} \end{aligned}$$

b.
$$\begin{aligned} M&=\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \\ &=\left(\frac{-2+3}{2}, \frac{-3+(-5)}{2}\right) \\ &=\left(\frac{1}{2}, -4\right) \end{aligned}$$

c. $m=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{-5-(-3)}{3-(-2)}=\dfrac{-2}{5}=-\dfrac{2}{5}$

11. $3x-2y=12$

x-intercept:
$$\begin{aligned} 3x-2(0)&=12 \\ 3x&=12 \\ x&=4 \end{aligned}$$

The point $(4,0)$ is on the graph.

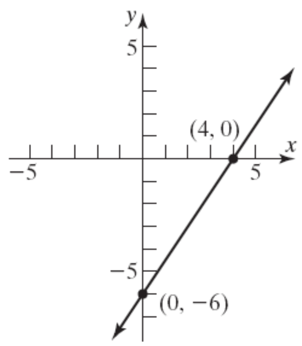
y-intercept:

$3(0) - 2y = 12$

$-2y = 12$

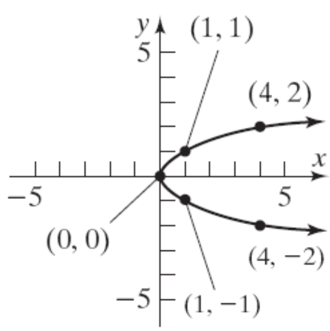
$y = -6$

The point $(0, -6)$ is on the graph.



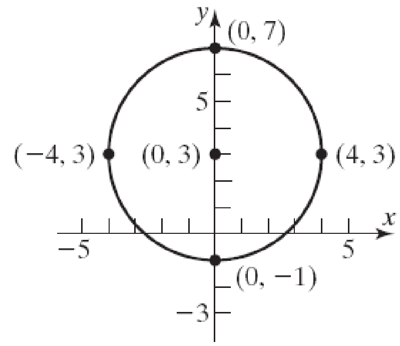
12. $x = y^2$

y	$x = y^2$	(x, y)
-2	$x = (-2)^2 = 4$	$(4, -2)$
-1	$x = (-1)^2 = 1$	$(1, -1)$
0	$x = 0^2 = 0$	$(0, 0)$
1	$x = 1^2 = 1$	$(1, 1)$
2	$x = 2^2 = 4$	$(4, 2)$



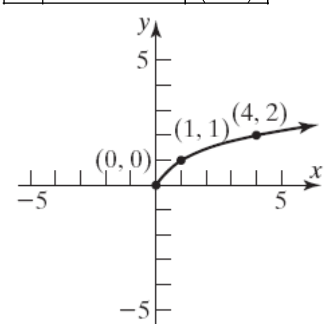
13. $x^2 + (y-3)^2 = 16$

This is the equation of a circle with radius $r = \sqrt{16} = 4$ and center at $(0, 3)$. Starting at the center we can obtain some points on the graph by moving 4 units up, down, left, and right. The corresponding points are $(0, 7)$, $(0, -1)$, $(-4, 3)$, and $(4, 3)$, respectively.



14. $y = \sqrt{x}$

x	$y = \sqrt{x}$	(x, y)
0	$y = \sqrt{0} = 0$	$(0, 0)$
1	$y = \sqrt{1} = 1$	$(1, 1)$
4	$y = \sqrt{4} = 2$	$(4, 2)$



15. $3x^2 - 4y = 12$

x-intercepts:

$3x^2 - 4(0) = 12$

$3x^2 = 12$

$x^2 = 4$

$x = \pm 2$

y-intercept:

$3(0)^2 - 4y = 12$

$-4y = 12$

$y = -3$

The intercepts are $(-2, 0)$, $(2, 0)$, and $(0, -3)$.

Check x-axis symmetry:

$3x^2 - 4(-y) = 12$

$3x^2 + 4y = 12$ different

Check y-axis symmetry:

$$3(-x)^2 - 4y = 12$$
$$3x^2 - 4y = 12 \quad \text{same}$$

Check origin symmetry:

$$3(-x)^2 - 4(-y) = 12$$
$$3x^2 + 4y = 12 \quad \text{different}$$

The graph of the equation has y-axis symmetry.

16. First we find the slope:

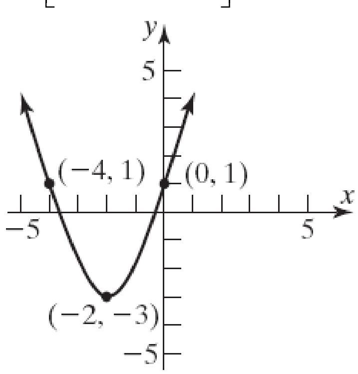
$$m = \frac{8-4}{6-(-2)} = \frac{4}{8} = \frac{1}{2}$$

Next we use the slope and the given point $(6,8)$ in the point-slope form of the equation of a line:

$$y - y_1 = m(x - x_1)$$
$$y - 8 = \frac{1}{2}(x-6)$$
$$y - 8 = \frac{1}{2}x - 3$$
$$y = \frac{1}{2}x + 5$$

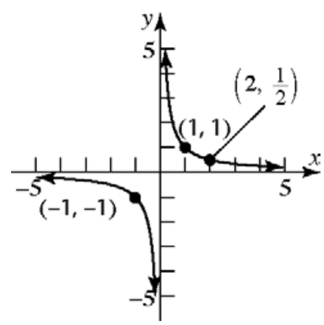
17. $f(x) = (x+2)^2 - 3$

Starting with the graph of $y = x^2$, shift the graph 2 units to the left $\left[y = (x+2)^2\right]$ and down 3 units $\left[y = (x+2)^2 - 3\right]$.



18. $f(x) = \dfrac{1}{x}$

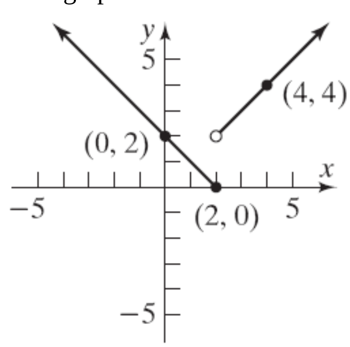
x	$y = \frac{1}{x}$	(x, y)
-1	$y = \frac{1}{-1} = -1$	$(-1,-1)$
1	$y = \frac{1}{1} = 1$	$(1,1)$
2	$y = \frac{1}{2}$	$\left(2, \frac{1}{2}\right)$



19. $f(x) = \begin{cases} 2 - x & \text{if } x \le 2 \\ |x| & \text{if } x > 2 \end{cases}$

Graph the line $y = 2 - x$ for $x \le 2$. Two points on the graph are $(0,2)$ and $(2,0)$.

Graph the line $y = x$ for $x > 2$. There is a hole in the graph at $x = 2$.



Chapter 2 Projects

Project I – Internet Based Project – Answers will vary

Project II

1. Silver: $C(x) = 20 + 0.16(x - 200) = 0.16x - 12$

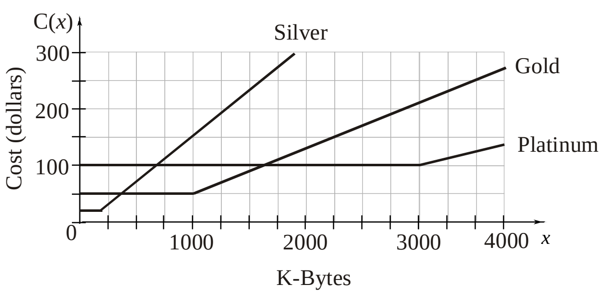
$$C(x) = \begin{cases} 20 & 0 \le x \le 200 \\ 0.16x - 12 & x > 200 \end{cases}$$

Gold: $C(x) = 50 + 0.08(x - 1000) = 0.08x - 30$

$$C(x) = \begin{cases} 50.00 & 0 \le x \le 1000 \\ 0.08x - 30 & x > 1000 \end{cases}$$

Platinum: $C(x) = 100 + 0.04(x - 3000)$
$= 0.04x - 20$

$$C(x) = \begin{cases} 100.00 & 0 \le x \le 3000 \\ 0.04x - 20 & x > 3000 \end{cases}$$



3. Let y = #K-bytes of service over the plan minimum.

Silver: $20 + 0.16y \le 50$
$0.16y \le 30$
$y \le 187.5$

Silver is the best up to $187.5 + 200 = 387.5$ K-bytes of service.

Gold: $50 + 0.08y \le 100$
$0.08y \le 50$
$y \le 625$

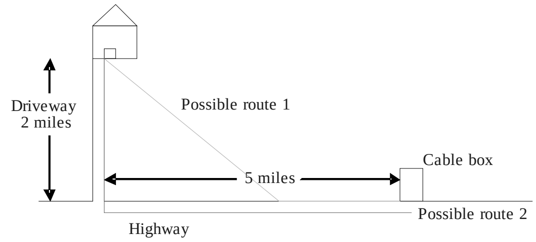
Gold is the best from 387.5 K-bytes to $625 + 1000 = 1625$ K-bytes of service.

Platinum: Platinum will be the best if more than 1625 K-bytes is needed.

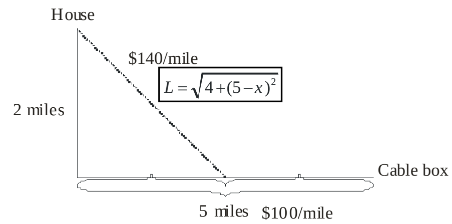
4. Answers will vary.

Project III

1.



2.



$C(x) = 100x + 140L$

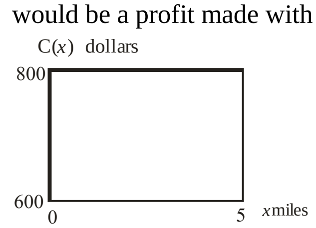
$C(x) = 100x + 140\sqrt{4 + (5 - x)^2}$

3.

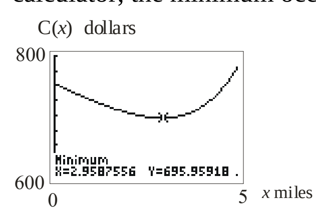
x	$C(x)$
0	$100(0) + 140\sqrt{4+25} \approx \$753.92$
1	$100(1) + 140\sqrt{4+16} \approx \$726.10$
2	$100(2) + 140\sqrt{4+9} \approx \$704.78$
3	$100(3) + 140\sqrt{4+4} \approx \$695.98$
4	$100(4) + 140\sqrt{4+1} \approx \$713.05$
5	$100(5) + 140\sqrt{4+0} = \$780.00$

The choice where the cable goes 3 miles down the road then cutting up to the house seems to yield the lowest cost.

4. Since all of the costs are less than \$800, there would be a profit made with any of the plans.



Using the MINIMUM function on a graphing calculator, the minimum occurs at $x \approx 2.96$.



The minimum cost occurs when the cable runs for 2.96 mile along the road.

6. $C(4.5) = 100(4.5) + 140\sqrt{4+(5-4.5)^2}$

$\approx \$738.62$

The cost for the Steven's cable would be \$738.62.

7. 5000(738.62) = \$3,693,100 State legislated
5000(695.96) = \$3,479,800 cheapest cost
It will cost the company \$213,300 more.

Project IV

1. $A = \pi r^2$

2. $r = 2.2t$

3. $r = 2.2(2) = 4.4 \text{ ft}$

$r = 2.2(2.5) = 5.5 \text{ ft}$

4. $A = \pi(4.4)^2 = 60.82 \text{ ft}^2$

$A = \pi(5.5)^2 = 95.03 \text{ ft}^2$

5. $A = \pi(2.2t)^2 = 4.84\pi t^2$

6. $A = 4.84\pi(2)^2 = 60.82 \text{ ft}^2$

$A = 4.84\pi(2.5)^2 = 95.03 \text{ ft}^2$

7. $\dfrac{A(2.5)-A(2)}{2.5-2} = \dfrac{95.03-60.82}{0.5} = 68.42 \text{ ft/hr}$

8. $\dfrac{A(3.5)-A(3)}{3.5-3} = \dfrac{186.27-136.85}{0.5} = 98.84 \text{ ft/hr}$

9. The average rate of change is increasing.

10. 150 yds = 450 ft
$r = 2.2t$

$t = \dfrac{450}{2.2} = 204.5 \text{ hours}$

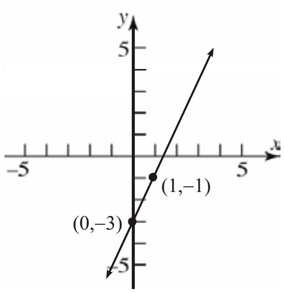
11. 6 miles = 31680 ft
Therefore, we need a radius of 15,840 ft.

$t = \dfrac{15{,}840}{2.2} = 7200 \text{ hours}$

Chapter 3
Linear and Quadratic Functions

Section 3.1

1. From the equation $y = 2x - 3$, we see that the *y*-intercept is -3. Thus, the point $(0,-3)$ is on the graph. We can obtain a second point by choosing a value for *x* and finding the corresponding value for *y*. Let $x = 1$, then $y = 2(1) - 3 = -1$. Thus, the point $(1,-1)$ is also on the graph. Plotting the two points and connecting with a line yields the graph below.



2. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3-5}{-1-2} = \dfrac{-2}{-3} = \dfrac{2}{3}$

3. $f(2) = 3(2)^2 - 2 = 10$

$f(4) = 3(4)^2 - 2 = 46$

$\dfrac{\Delta y}{\Delta x} = \dfrac{f(4) - f(2)}{4-2} = \dfrac{46-10}{4-2} = \dfrac{36}{2} = 18$

4. $60x - 900 = -15x + 2850$

$75x - 900 = 2850$

$75x = 3750$

$x = 50$

The solution set is {50}.

5. $f(-2) = (-2)^2 - 4 = 4 - 4 = 0$

6. True

7. slope; *y*-intercept

8. positive

9. True

10. False. The *y*-intercept is 8. The average rate of change is 2 (the slope).

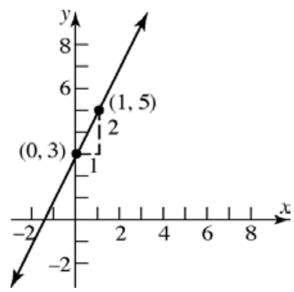
11. a

12. d

13. $f(x) = 2x + 3$

a. Slope = 2; *y*-intercept = 3

b. Plot the point (0, 3). Use the slope to find an additional point by moving 1 unit to the right and 2 units up.



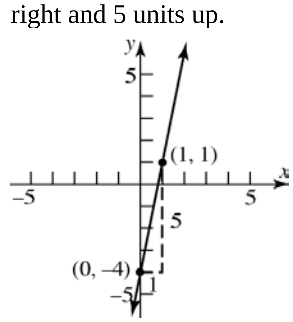
c. average rate of change = 2

d. increasing

14. $g(x) = 5x - 4$

a. Slope = 5; *y*-intercept = -4

b. Plot the point $(0,-4)$. Use the slope to find an additional point by moving 1 unit to the right and 5 units up.



c. average rate of change = 5

d. increasing