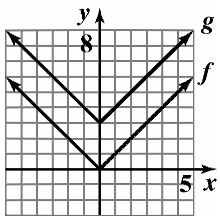
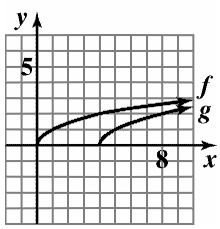
Section 1.6

Check Point Exercises

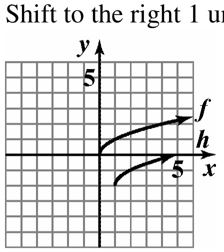
1. Shift up vertically 3 units.



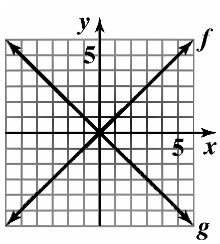
2. Shift to the right 4 units.



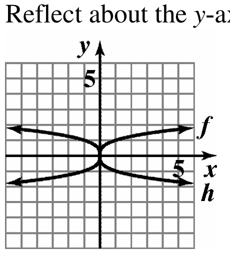
3. Shift to the right 1 unit and down 2 units.



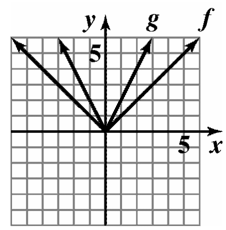
4. Reflect about the *x*-axis.



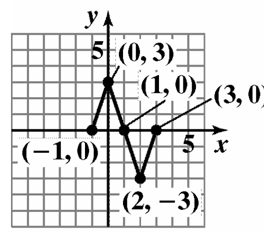
5. Reflect about the *y*-axis.



6. Vertically stretch the graph of $f(x)=|x|$.

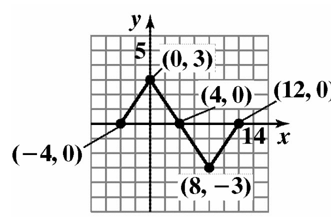


7. **a.** Horizontally shrink the graph of $y = f(x)$.



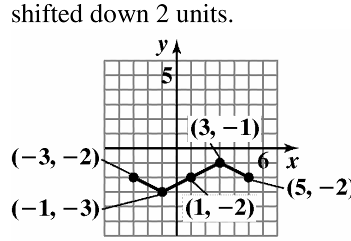
$g(x) = f(2x)$

b. Horizontally stretch the graph of $y = f(x)$.



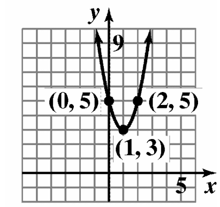
$h(x) = f\left(\frac{1}{2}x\right)$

8. The graph of $y = f(x)$ is shifted 1 unit left, shrunk by a factor of $\frac{1}{3}$, reflected about the x-axis, then shifted down 2 units.



$y = -\frac{1}{3}f(x+1) - 2$

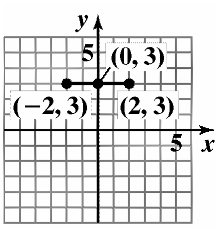
9. The graph of $f(x) = x^2$ is shifted 1 unit right, stretched by a factor of 2, then shifted up 3 units.



$g(x) = 2(x-1)^2 + 3$

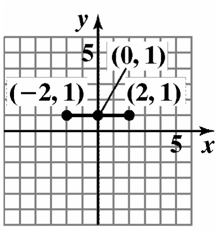
Exercise Set 1.6

1.



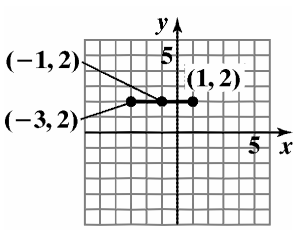
$g(x) = f(x) + 1$

2.



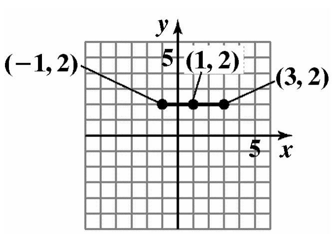
$g(x) = f(x) - 1$

3.



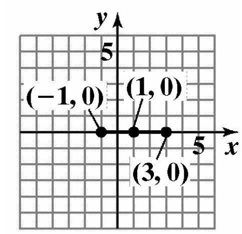
$g(x) = f(x+1)$

4.



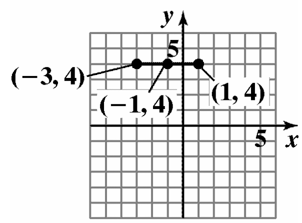
$g(x) = f(x-1)$

5.



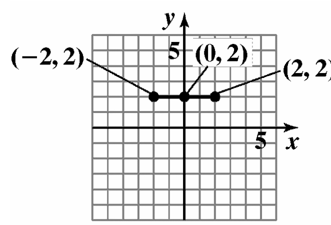
$g(x) = f(x-1) - 2$

6.



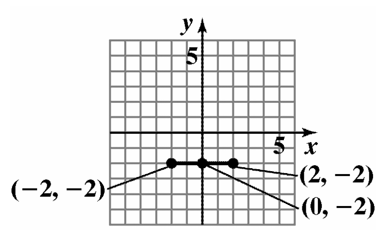
$g(x) = f(x+1) + 2$

7.



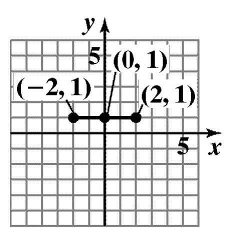
$g(x) = -f(x)$

8.



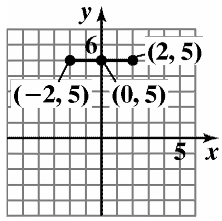
$g(x) = -f(x)$

9.



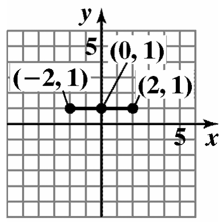
$g(x) = -f(x) + 3$

10.



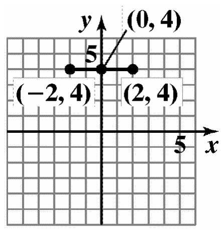
$g(x) = f(-x) + 3$

11.



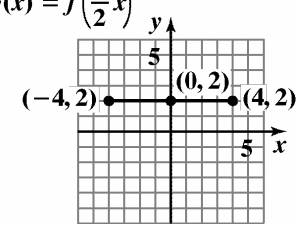
$g(x) = \frac{1}{2}f(x)$

12.

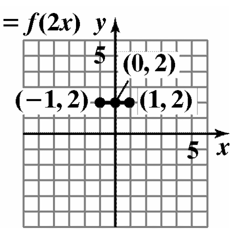


$g(x) = 2f(x)$

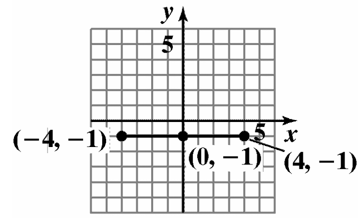
13. $g(x) = f\left(\frac{1}{2}x\right)$



14. $g(x) = f(2x)$

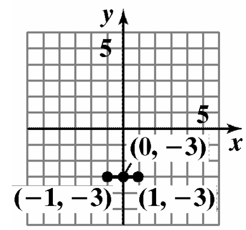


15.



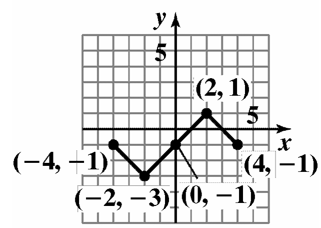
$g(x) = -f\left(\frac{1}{2}x\right) + 1$

16.



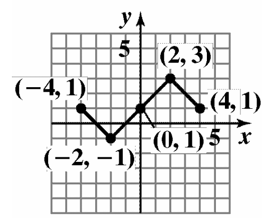
$g(x) = -f(2x) - 1$

17.



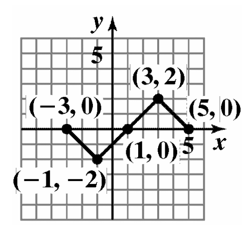
$g(x) = f(x) - 1$

18.



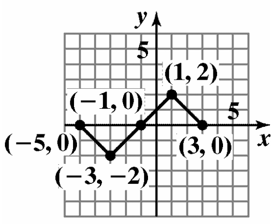
$g(x) = f(x) + 1$

19.



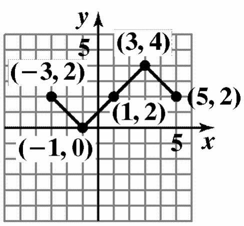
$g(x) = f(x - 1)$

20.



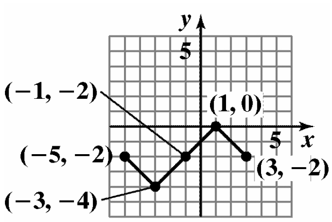
$g(x) = f(x + 1)$

21.



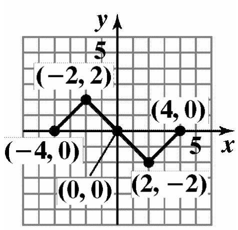
$g(x) = f(x - 1) + 2$

22.



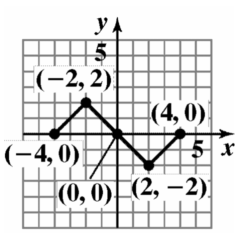
$g(x) = f(x + 1) - 2$

23.



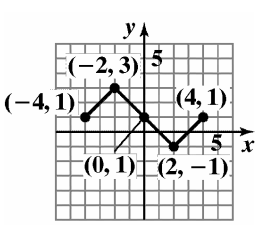
$g(x) = -f(x)$

24.



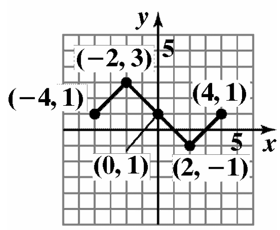
$g(x) = f(-x)$

25.



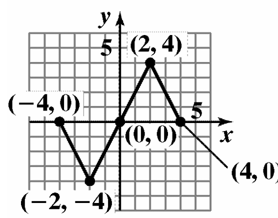
$g(x) = f(-x) + 1$

26.



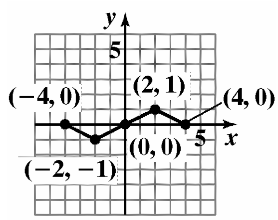
$g(x) = -f(x) + 1$

27.



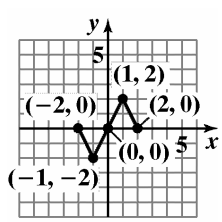
$g(x) = 2f(x)$

28.



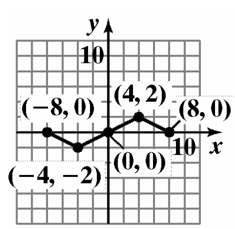
$g(x) = \frac{1}{2}f(x)$

29.



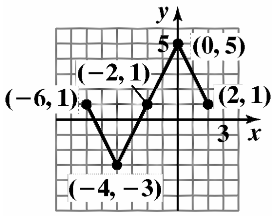
$g(x) = f(2x)$

30.



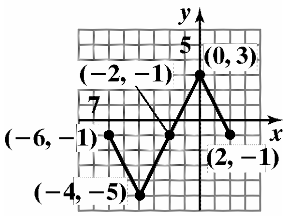
$g(x) = f\left(\frac{1}{2}x\right)$

31.



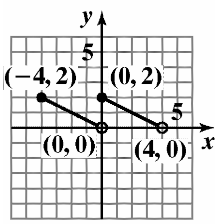
$g(x) = 2f(x + 2) + 1$

32.



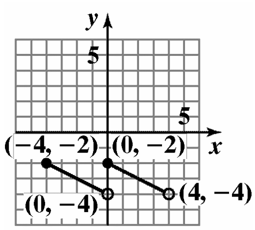
$g(x) = 2f(x + 2) - 1$

33.



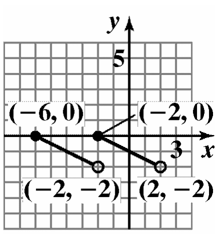
$g(x) = f(x) + 2$

34.



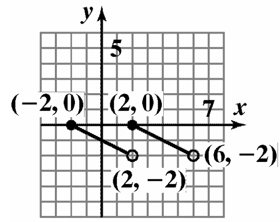
$g(x) = f(x) - 2$

35.



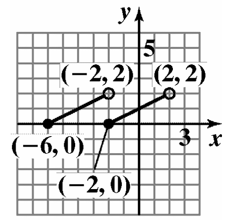
$g(x) = f(x + 2)$

36.



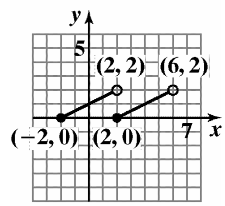
$g(x) = f(x - 2)$

37.



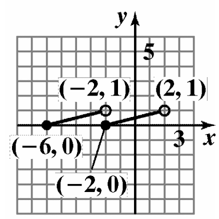
$g(x) = -f(x + 2)$

38.



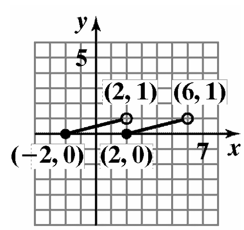
$g(x) = -f(x - 2)$

39.



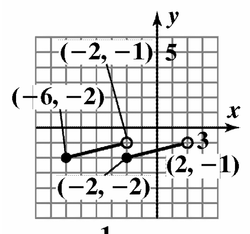
$g(x) = -\frac{1}{2}f(x + 2)$

40.



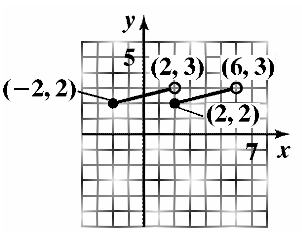
$g(x) = -\frac{1}{2}f(x - 2)$

41.



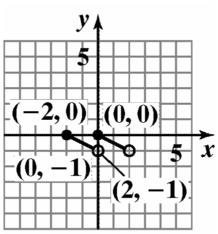
$g(x) = -\frac{1}{2}f(x+2) - 2$

42.



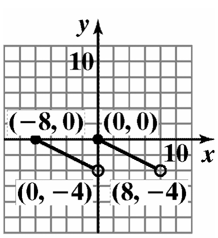
$g(x) = -\frac{1}{2}f(x-2) + 2$

43.



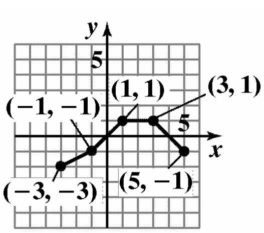
$g(x) = \frac{1}{2}f(2x)$

44.



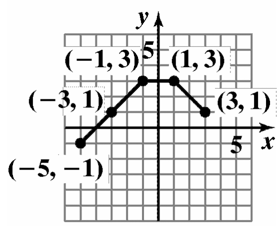
$g(x) = 2f\left(\frac{1}{2}x\right)$

45.



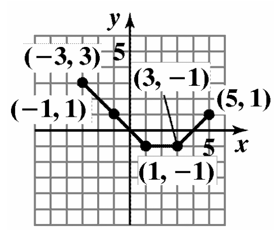
$g(x) = f(x-1) - 1$

46.



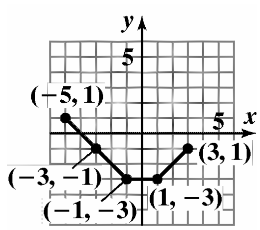
$g(x) = f(x+1) + 1$

47.



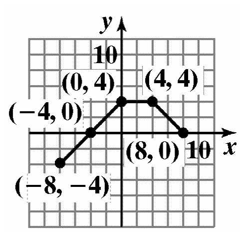
$g(x) = -f(x-1) + 1$

48.



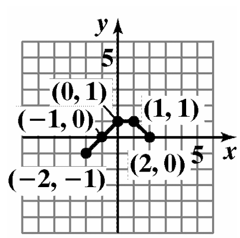
$g(x) = -f(x+1) - 1$

49.



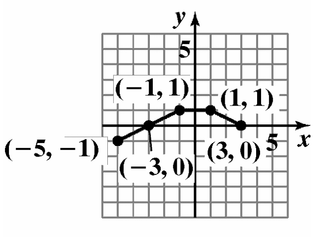
$g(x) = 2f\left(\frac{1}{2}x\right)$

50.



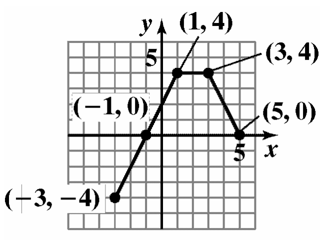
$g(x) = \frac{1}{2}f(2x)$

51.



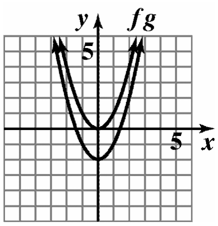
$g(x) = \frac{1}{2}f(x+1)$

52.

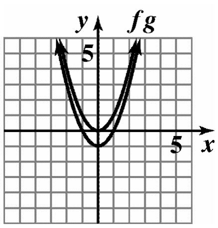


$g(x) = 2f(x-1)$

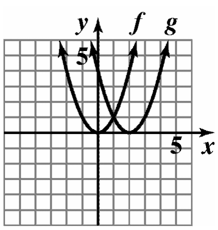
53.



54.



55.



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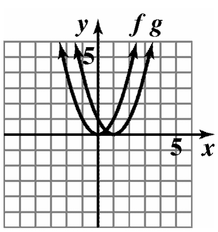

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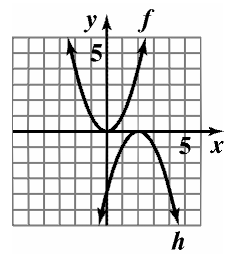

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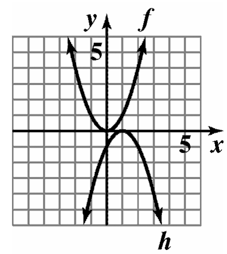

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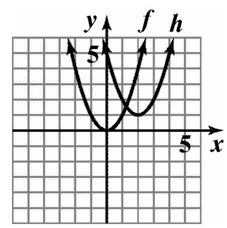

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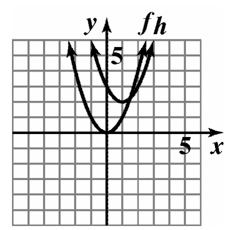

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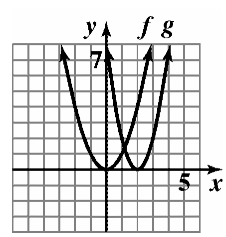

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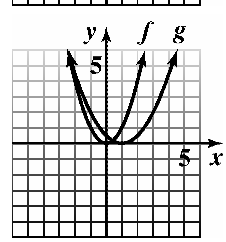

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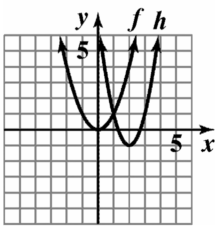

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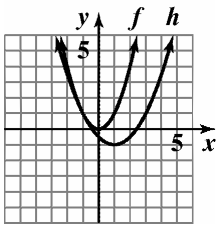

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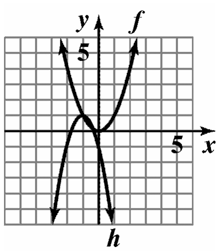

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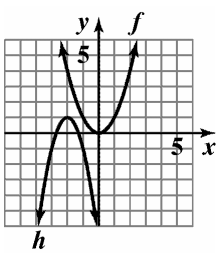

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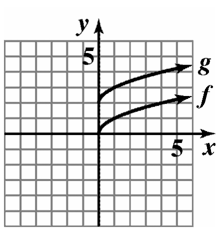

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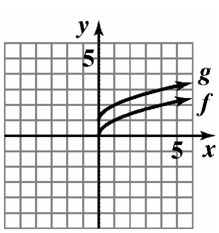

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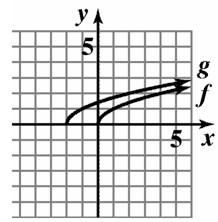

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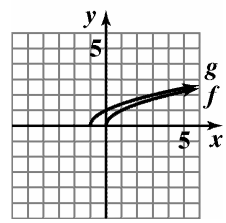

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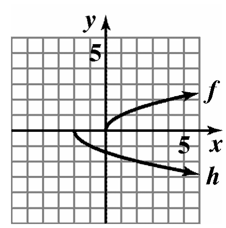

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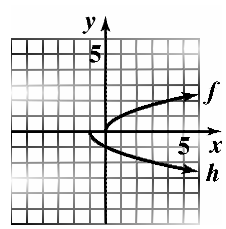

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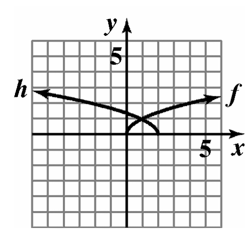

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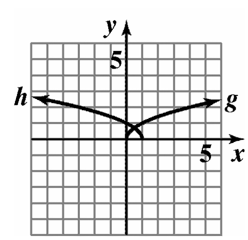

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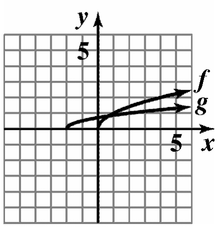

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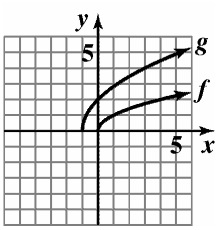

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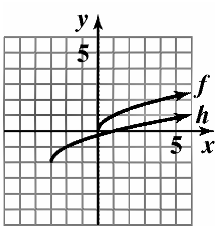

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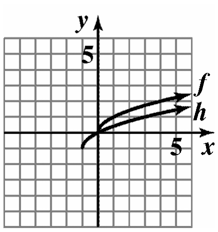

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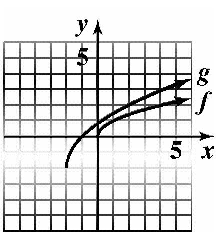

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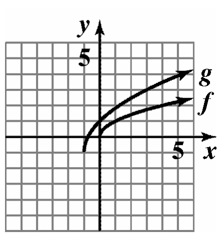

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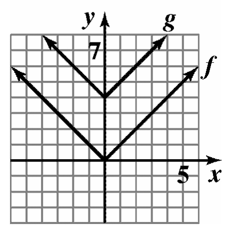

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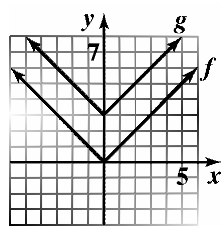

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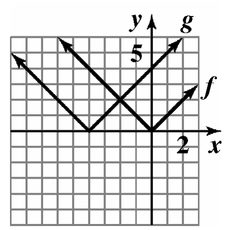

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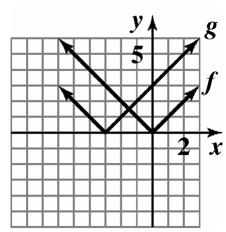

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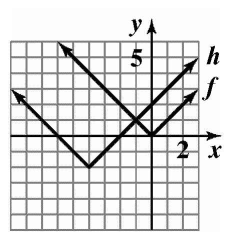

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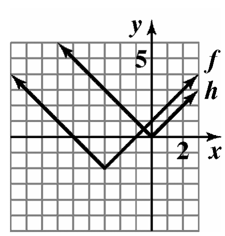

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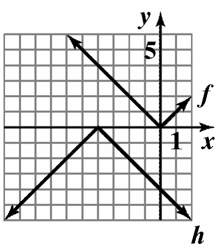

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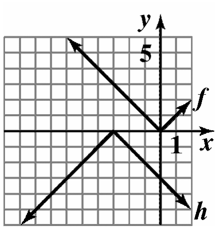

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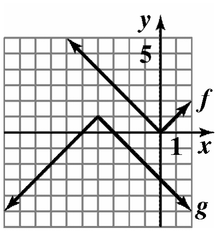

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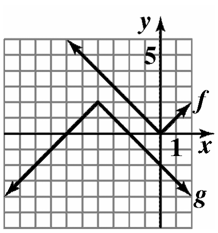

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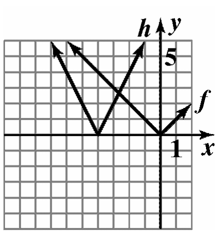

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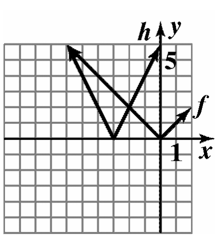

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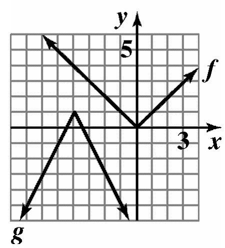

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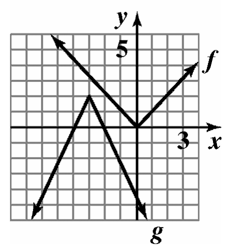

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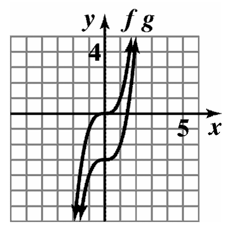

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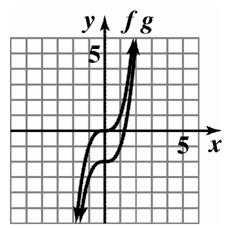

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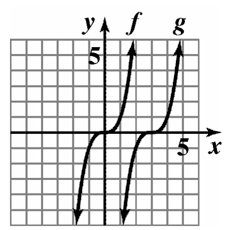

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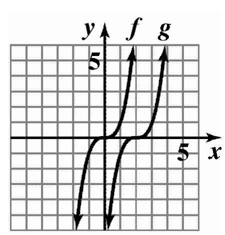

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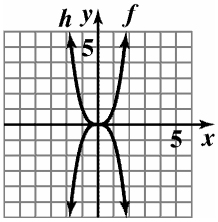

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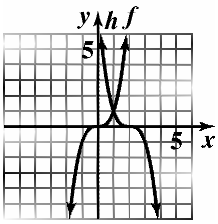

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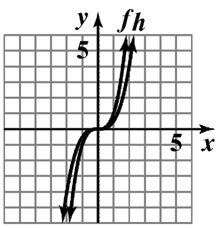

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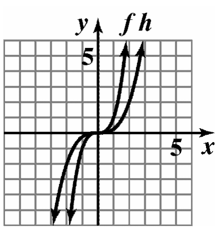

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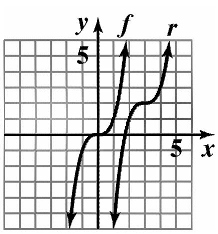

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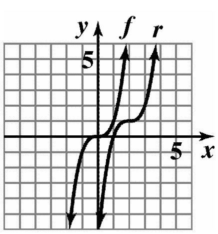

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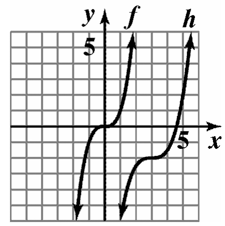

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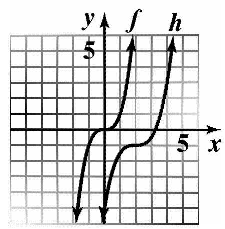

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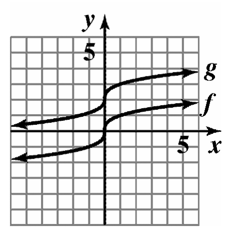

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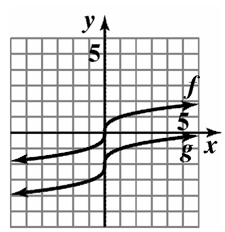

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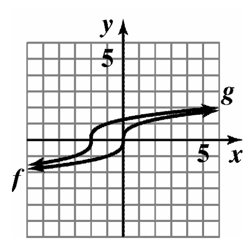

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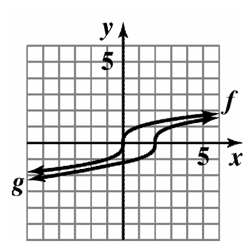

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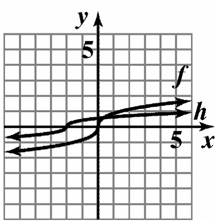

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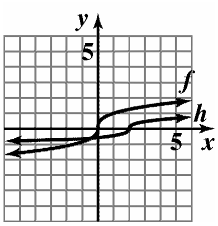

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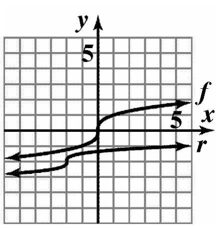

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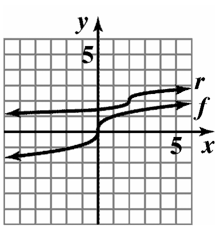

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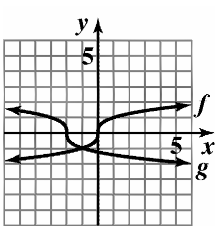

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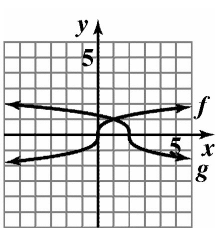

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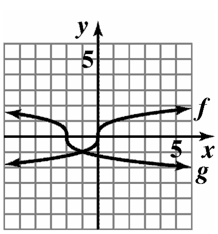

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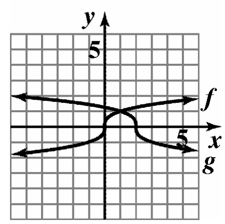

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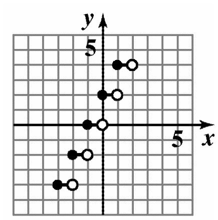
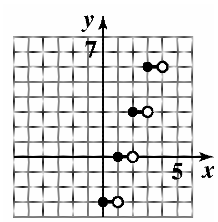
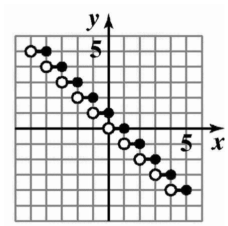


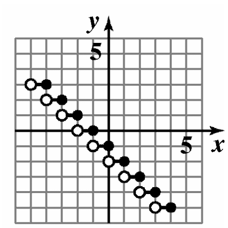
$g(x) = 2\ \text{int}\ (x + 1)$

120.


$g(x) = 3\ \text{int}\ (x - 1)$

121.


$h(x) = \text{int}\ (-x) + 1$

122.


$h(x) = \text{int}\ (-x) - 1$

123. $y = \sqrt{x-2}$

124. $y = -x^3 + 2$

125. $y = (x+1)^2 - 4$

126. $y = \sqrt{x-2} + 1$

127. a. First, vertically stretch the graph of $f(x)=\sqrt{x}$ by the factor 2.9; then shift the result up 20.1 units.

b. $f(x)=2.9\sqrt{x}+20.1$

$f(48)=2.9\sqrt{48}+20.1\approx 40.2$

The model describes the actual data very well.

c. $\dfrac{f(x_2)-f(x_1)}{x_2-x_1}$

$=\dfrac{f(10)-f(0)}{10-0}$

$=\dfrac{\left(2.9\sqrt{10}+20.1\right)-\left(2.9\sqrt{0}+20.1\right)}{10-0}$

$=\dfrac{29.27-20.1}{10}$

≈ 0.9

0.9 inches per month

d. $\dfrac{f(x_2)-f(x_1)}{x_2-x_1}$

$=\dfrac{f(60)-f(50)}{60-50}$

$=\dfrac{\left(2.9\sqrt{60}+20.1\right)-\left(2.9\sqrt{50}+20.1\right)}{60-50}$

$=\dfrac{42.5633-40.6061}{10}$

≈ 0.2

This rate of change is lower than the rate of change in part (c). The relative leveling off of the curve shows this difference.

128. a. First, vertically stretch the graph of $f(x)=\sqrt{x}$ by the factor 3.1; then shift the result up 19 units.

b. $f(x)=3.1\sqrt{x}+19$

$f(48)=3.1\sqrt{48}+19\approx 40.5$

The model describes the actual data very well.

c. $\dfrac{f(x_2)-f(x_1)}{x_2-x_1}$

$=\dfrac{f(10)-f(0)}{10-0}$

$=\dfrac{\left(3.1\sqrt{10}+19\right)-\left(3.1\sqrt{0}+19\right)}{10-0}$

$=\dfrac{28.8031-19}{10}$

≈ 1.0

1.0 inches per month

d. $\dfrac{f(x_2)-f(x_1)}{x_2-x_1}$

$=\dfrac{f(60)-f(50)}{60-50}$

$=\dfrac{\left(3.1\sqrt{60}+19\right)-\left(3.1\sqrt{50}+19\right)}{60-50}$

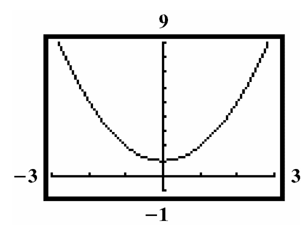
$=\dfrac{43.0125-40.9203}{10}$

≈ 0.2

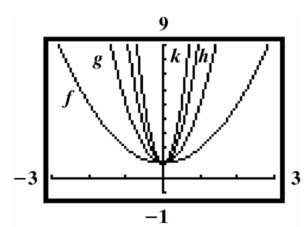
This rate of change is lower than the rate of change in part (c). The relative leveling off of the curve shows this difference.

129. – 134. Answers may vary.

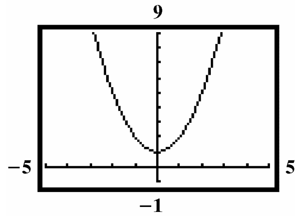
135. a.



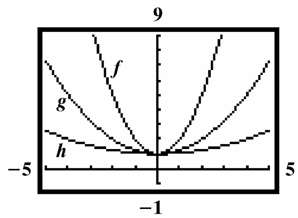
b.



136. a.



b.



137. makes sense

138. makes sense

139. does not make sense; Explanations will vary. Sample explanation: The reprogram should be $y = f(t+1)$.

140. does not make sense; Explanations will vary. Sample explanation: The reprogram should be $y = f(t-1)$.

141. false; Changes to make the statement true will vary. A sample change is: The graph of *g* is a translation of *f* three units to the left and three units upward.

142. false; Changes to make the statement true will vary. A sample change is: The graph of *f* is a reflection of the graph of $y=\sqrt{x}$ in the *x*-axis, while the graph of *g* is a reflection of the graph of $y=\sqrt{x}$ in the *y*-axis.

143. false; Changes to make the statement true will vary. A sample change is: The stretch will be 5 units and the downward shift will be 10 units.

144. true

145. $g(x) = -(x+4)^2$

146. $g(x) = -|x-5|+1$

147. $g(x) = -\sqrt{x-2}+2$

148. $g(x) = -\frac{1}{4}\sqrt{16-x^2}-1$

149. $(-a, b)$

150. $(a, 2b)$

151. $(a+3, b)$

152. $(a, b-3)$

153.
$$\begin{aligned}(2x-1)(x^2+x-2) &= 2x(x^2+x-2)-1(x^2+x-2)\\ &= 2x^3+2x^2-4x-x^2-x+2\\ &= 2x^3+2x^2-x^2-4x-x+2\\ &= 2x^3+x^2-5x+2\end{aligned}$$

154.
$$\begin{aligned}\left(f(x)\right)^2-2f(x)+6 &= (3x-4)^2-2(3x-4)+6\\ &= 9x^2-24x+16-6x+8+6\\ &= 9x^2-24x-6x+16+8+6\\ &= 9x^2-30x+30\end{aligned}$$

155. $\dfrac{2}{\frac{3}{x}-1} = \dfrac{2x}{\frac{3x}{x}-x} = \dfrac{2x}{3-x}$

Section 1.7

Check Point Exercises

1. **a.** The function $f(x)=x^2+3x-17$ contains neither division nor an even root. The domain of *f* is the set of all real numbers or $(-\infty,\infty)$.

b. The denominator equals zero when $x = 7$ or $x = -7$. These values must be excluded from the domain.
domain of $g = (-\infty,-7)\cup(-7,7)\cup(7,\infty)$.

c. Since $h(x)=\sqrt{9x-27}$ contains an even root; the quantity under the radical must be greater than or equal to 0.
$$\begin{aligned}9x-27 &\ge 0\\ 9x &\ge 27\\ x &\ge 3\end{aligned}$$
Thus, the domain of *h* is $\{x|x\ge 3\}$, or the interval $[3,\infty)$.

2. **a.**
$$\begin{aligned}(f+g)(x) &= f(x)+g(x)\\ &= x-5+\left(x^2-1\right)\\ &= x-5+x^2-1\\ &= -x^2+x-6\end{aligned}$$

b. $(f-g)(x) = f(x) - g(x)$
$= x - 5 - (x^2 - 1)$
$= x - 5 - x^2 + 1$
$= -x^2 + x - 4$

c. $(fg)(x) = (x-5)(x^2-1)$
$= x(x^2-1) - 5(x^2-1)$
$= x^3 - x - 5x^2 + 5$
$= x^3 - 5x^2 - x + 5$

d. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$
$= \frac{x-5}{x^2-1},\ x \neq \pm 1$

3. **a.** $(f+g)(x) = f(x) + g(x)$
$= \sqrt{x-3} + \sqrt{x+1}$

b. domain of *f*: $x - 3 \geq 0$
$x \geq 3$
$[3, \infty)$
domain of g: $x + 1 \geq 0$
$x \geq -1$
$[-1, \infty)$
The domain of $f + g$ is the set of all real numbers that are common to the domain of f and the domain of g. Thus, the domain of $f + g$ is $[3, \infty)$.

4. **a.** $(f \circ g)(x) = f(g(x))$
$= 5(2x^2 - x - 1) + 6$
$= 10x^2 - 5x - 5 + 6$
$= 10x^2 - 5x + 1$

b. $(g \circ f)(x) = g(f(x))$
$= 2(5x+6)^2 - (5x+6) - 1$
$= 2(25x^2 + 60x + 36) - 5x - 6 - 1$
$= 50x^2 + 120x + 72 - 5x - 6 - 1$
$= 50x^2 + 115x + 65$

c. $(f \circ g)(x) = 10x^2 - 5x + 1$
$(f \circ g)(-1) = 10(-1)^2 - 5(-1) + 1$
$= 10 + 5 + 1$
$= 16$

5. **a.** $(f \circ g)(x) = \frac{4}{\frac{1}{x} + 2} = \frac{4x}{1 + 2x}$

b. domain: $\left\{x \middle| x \neq 0,\ x \neq -\frac{1}{2}\right\}$

6. $h(x) = f \circ g$ where $f(x) = \sqrt{x}$; $g(x) = x^2 + 5$

Exercise Set 1.7

1. The function contains neither division nor an even root. The domain $= (-\infty, \infty)$

2. The function contains neither division nor an even root. The domain $= (-\infty, \infty)$

3. The denominator equals zero when $x = 4$. This value must be excluded from the domain.
domain: $(-\infty, 4) \cup (4, \infty)$.

4. The denominator equals zero when $x = -5$. This value must be excluded from the domain.
domain: $(-\infty, -5) \cup (-5, \infty)$.

5. The function contains neither division nor an even root. The domain $= (-\infty, \infty)$

6. The function contains neither division nor an even root. The domain $= (-\infty, \infty)$

7. The values that make the denominator equal zero must be excluded from the domain.
domain: $(-\infty, -3) \cup (-3, 5) \cup (5, \infty)$

8. The values that make the denominator equal zero must be excluded from the domain.
domain: $(-\infty, -4) \cup (-4, 3) \cup (3, \infty)$

9. The values that make the denominators equal zero must be excluded from the domain.
domain: $(-\infty, -7) \cup (-7, 9) \cup (9, \infty)$

10. The values that make the denominators equal zero must be excluded from the domain.
domain: $(-\infty, -8) \cup (-8, 10) \cup (10, \infty)$

11. The first denominator cannot equal zero. The values that make the second denominator equal zero must be excluded from the domain.
domain: $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

12. The first denominator cannot equal zero. The values that make the second denominator equal zero must be excluded from the domain.
domain: $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$

13. Exclude x for $x=0$.
Exclude x for $\frac{3}{x}-1=0$.

$$\frac{3}{x}-1=0$$
$$x\left(\frac{3}{x}-1\right)=x(0)$$
$$3-x=0$$
$$-x=-3$$
$$x=3$$

domain: $(-\infty,0)\cup(0,3)\cup(3,\infty)$

14. Exclude x for $x=0$.
Exclude x for $\frac{4}{x}-1=0$.

$$\frac{4}{x}-1=0$$
$$x\left(\frac{4}{x}-1\right)=x(0)$$
$$4-x=0$$
$$-x=-4$$
$$x=4$$

domain: $(-\infty,0)\cup(0,4)\cup(4,\infty)$

15. Exclude x for $x-1=0$.

$$x-1=0$$
$$x=1$$

Exclude x for $\frac{4}{x-1}-2=0$.

$$\frac{4}{x-1}-2=0$$
$$(x-1)\left(\frac{4}{x-1}-2\right)=(x-1)(0)$$
$$4-2(x-1)=0$$
$$4-2x+2=0$$
$$-2x+6=0$$
$$-2x=-6$$
$$x=3$$

domain: $(-\infty,1)\cup(1,3)\cup(3,\infty)$

16. Exclude x for $x-2=0$.

$$x-2=0$$
$$x=2$$

Exclude x for $\frac{4}{x-2}-3=0$.

$$\frac{4}{x-2}-3=0$$
$$(x-2)\left(\frac{4}{x-2}-3\right)=(x-2)(0)$$
$$4-3(x-2)=0$$
$$4-3x+6=0$$
$$-3x+10=0$$
$$-3x=-10$$
$$x=\frac{10}{3}$$

domain: $(-\infty,2)\cup\left(2,\frac{10}{3}\right)\cup\left(\frac{10}{3},\infty\right)$

17. The expression under the radical must not be negative.

$$x-3\geq 0$$
$$x\geq 3$$

domain: $[3,\infty)$

18. The expression under the radical must not be negative.

$$x+2\geq 0$$
$$x\geq -2$$

domain: $[-2,\infty)$

19. The expression under the radical must be positive.
$x-3>0$
$x>3$
domain: $(3,\infty)$

20. The expression under the radical must be positive.
$x+2>0$
$x>-2$
domain: $(-2,\infty)$

21. The expression under the radical must not be negative.
$5x+35\ge 0$
$5x\ge -35$
$x\ge -7$
domain: $[-7,\infty)$

22. The expression under the radical must not be negative.
$7x-70\ge 0$
$7x\ge 70$
$x\ge 10$
domain: $[10,\infty)$

23. The expression under the radical must not be negative.
$24-2x\ge 0$
$-2x\ge -24$
$$\frac{-2x}{-2}\le\frac{-24}{-2}$$
$x\le 12$
domain: $(-\infty,12]$

24. The expression under the radical must not be negative.
$84-6x\ge 0$
$-6x\ge -84$
$$\frac{-6x}{-6}\le\frac{-84}{-6}$$
$x\le 14$
domain: $(-\infty,14]$

25. The expressions under the radicals must not be negative.
$x-2\ge 0$ and $x+3\ge 0$
$x\ge 2$ $\quad$ $x\ge -3$
To make both inequalities true, $x\ge 2$.
domain: $[2,\infty)$

26. The expressions under the radicals must not be negative.
$x-3\ge 0$ and $x+4\ge 0$
$x\ge 3$ $\quad$ $x\ge -4$
To make both inequalities true, $x\ge 3$.
domain: $[3,\infty)$

27. The expression under the radical must not be negative.
$x-2\ge 0$
$x\ge 2$
The denominator equals zero when $x=5$.
domain: $[2,5)\cup(5,\infty)$.

28. The expression under the radical must not be negative.
$x-3\ge 0$
$x\ge 3$
The denominator equals zero when $x=6$.
domain: $[3,6)\cup(6,\infty)$.

29. Find the values that make the denominator equal zero and must be excluded from the domain.
$$x^3-5x^2-4x+20$$
$$=x^2(x-5)-4(x-5)$$
$$=(x-5)(x^2-4)$$
$$=(x-5)(x+2)(x-2)$$
–2, 2, and 5 must be excluded.
domain: $(-\infty,-2)\cup(-2,2)\cup(2,5)\cup(5,\infty)$

30. Find the values that make the denominator equal zero and must be excluded from the domain.
$$x^3-2x^2-9x+18$$
$$=x^2(x-2)-9(x-2)$$
$$=(x-2)(x^2-9)$$
$$=(x-2)(x+3)(x-3)$$
–3, 2, and 3 must be excluded.
domain: $(-\infty,-3)\cup(-3,2)\cup(2,3)\cup(3,\infty)$

31. $(f+g)(x) = 3x + 2$
domain: $(-\infty, \infty)$
$$(f-g)(x) = f(x) - g(x) = (2x+3) - (x-1) = x + 4$$
domain: $(-\infty, \infty)$
$$(fg)(x) = f(x)\cdot g(x) = (2x+3)\cdot(x-1) = 2x^2 + x - 3$$
domain: $(-\infty, \infty)$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{2x+3}{x-1}$$
domain: $(-\infty, 1)\cup(1, \infty)$

32. $(f+g)(x) = 4x - 2$
domain: $(-\infty, \infty)$
$(f-g)(x) = (3x-4) - (x+2) = 2x - 6$
domain: $(-\infty, \infty)$
$(fg)(x) = (3x-4)(x+2) = 3x^2 + 2x - 8$
domain: $(-\infty, \infty)$
$$\left(\frac{f}{g}\right)(x) = \frac{3x-4}{x+2}$$
domain: $(-\infty, -2)\cup(-2, \infty)$

33. $(f+g)(x) = 3x^2 + x - 5$
domain: $(-\infty, \infty)$
$(f-g)(x) = -3x^2 + x - 5$
domain: $(-\infty, \infty)$
$(fg)(x) = (x-5)(3x^2) = 3x^3 - 15x^2$
domain: $(-\infty, \infty)$
$$\left(\frac{f}{g}\right)(x) = \frac{x-5}{3x^2}$$
domain: $(-\infty, 0)\cup(0, \infty)$

34. $(f+g)(x) = 5x^2 + x - 6$
domain: $(-\infty, \infty)$
$(f-g)(x) = -5x^2 + x - 6$
domain: $(-\infty, \infty)$
$(fg)(x) = (x-6)(5x^2) = 5x^3 - 30x^2$
domain: $(-\infty, \infty)$
$$\left(\frac{f}{g}\right)(x) = \frac{x-6}{5x^2}$$
domain: $(-\infty, 0)\cup(0, \infty)$

35. $(f+g)(x) = 2x^2 - 2$
domain: $(-\infty, \infty)$
$(f-g)(x) = 2x^2 - 2x - 4$
domain: $(-\infty, \infty)$
$$(fg)(x) = (2x^2 - x - 3)(x+1) = 2x^3 + x^2 - 4x - 3$$
domain: $(-\infty, \infty)$
$$\left(\frac{f}{g}\right)(x) = \frac{2x^2 - x - 3}{x+1} = \frac{(2x-3)(x+1)}{(x+1)} = 2x - 3$$
domain: $(-\infty, -1)\cup(-1, \infty)$

36. $(f+g)(x) = 6x^2 - 2$
domain: $(-\infty, \infty)$
$(f-g)(x) = 6x^2 - 2x$
domain: $(-\infty, \infty)$
$(fg)(x) = (6x^2 - x - 1)(x-1) = 6x^3 - 7x^2 + 1$
domain: $(-\infty, \infty)$
$$\left(\frac{f}{g}\right)(x) = \frac{6x^2 - x - 1}{x-1}$$
domain: $(-\infty, 1)\cup(1, \infty)$

37. $$(f+g)(x) = (3 - x^2) + (x^2 + 2x - 15) = 2x - 12$$
domain: $(-\infty, \infty)$
$$(f-g)(x) = (3 - x^2) - (x^2 + 2x - 15) = -2x^2 - 2x + 18$$
domain: $(-\infty, \infty)$
$$(fg)(x) = (3 - x^2)(x^2 + 2x - 15) = -x^4 - 2x^3 + 18x^2 + 6x - 45$$
domain: $(-\infty, \infty)$
$$\left(\frac{f}{g}\right)(x) = \frac{3 - x^2}{x^2 + 2x - 15}$$
domain: $(-\infty, -5)\cup(-5, 3)\cup(3, \infty)$

38. $(f+g)(x) = (5-x^2)+(x^2+4x-12)$
$= 4x-7$
domain: $(-\infty, \infty)$
$(f-g)(x) = (5-x^2)-(x^2+4x-12)$
$= -2x^2-4x+17$
domain: $(-\infty, \infty)$
$(fg)(x) = (5-x^2)(x^2+4x-12)$
$= -x^4-4x^3+17x^2+20x-60$
domain: $(-\infty, \infty)$
$\left(\frac{f}{g}\right)(x) = \frac{5-x^2}{x^2+4x-12}$
domain: $(-\infty,-6)\cup(-6,2)\cup(2,\infty)$

39. $(f+g)(x) = \sqrt{x}+x-4$
domain: $[0,\infty)$
$(f-g)(x) = \sqrt{x}-x+4$
domain: $[0,\infty)$
$(fg)(x) = \sqrt{x}(x-4)$
domain: $[0,\infty)$
$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x}}{x-4}$
domain: $[0,4)\cup(4,\infty)$

40. $(f+g)(x) = \sqrt{x}+x-5$
domain: $[0,\infty)$
$(f-g)(x) = \sqrt{x}-x+5$
domain: $[0,\infty)$
$(fg)(x) = \sqrt{x}(x-5)$
domain: $[0,\infty)$
$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x}}{x-5}$
domain: $[0,5)\cup(5,\infty)$

41. $(f+g)(x) = 2+\frac{1}{x}+\frac{1}{x} = 2+\frac{2}{x} = \frac{2x+2}{x}$
domain: $(-\infty,0)\cup(0,\infty)$
$(f-g)(x) = 2+\frac{1}{x}-\frac{1}{x} = 2$
domain: $(-\infty,0)\cup(0,\infty)$
$(fg)(x) = \left(2+\frac{1}{x}\right)\cdot\frac{1}{x} = \frac{2}{x}+\frac{1}{x^2} = \frac{2x+1}{x^2}$
domain: $(-\infty,0)\cup(0,\infty)$
$\left(\frac{f}{g}\right)(x) = \frac{2+\frac{1}{x}}{\frac{1}{x}} = \left(2+\frac{1}{x}\right)\cdot x = 2x+1$
domain: $(-\infty,0)\cup(0,\infty)$

42. $(f+g)(x) = 6-\frac{1}{x}+\frac{1}{x} = 6$
domain: $(-\infty,0)\cup(0,\infty)$
$(f-g)(x) = 6-\frac{1}{x}-\frac{1}{x} = 6-\frac{2}{x} = \frac{6x-2}{x}$
domain: $(-\infty,0)\cup(0,\infty)$
$(fg)(x) = \left(6-\frac{1}{x}\right)\cdot\frac{1}{x} = \frac{6}{x}-\frac{1}{x^2} = \frac{6x-1}{x^2}$
domain: $(-\infty,0)\cup(0,\infty)$
$\left(\frac{f}{g}\right)(x) = \frac{6-\frac{1}{x}}{\frac{1}{x}} = \left(6-\frac{1}{x}\right)\cdot x = 6x-1$
domain: $(-\infty,0)\cup(0,\infty)$

43. $(f+g)(x) = f(x)+g(x)$

$$= \frac{5x+1}{x^2-9}+\frac{4x-2}{x^2-9}$$
$$= \frac{9x-1}{x^2-9}$$

domain: $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

$(f-g)(x) = f(x)-g(x)$

$$= \frac{5x+1}{x^2-9}-\frac{4x-2}{x^2-9}$$
$$= \frac{x+3}{x^2-9}$$
$$= \frac{1}{x-3}$$

domain: $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

$(fg)(x) = f(x)\cdot g(x)$

$$= \frac{5x+1}{x^2-9}\cdot\frac{4x-2}{x^2-9}$$
$$= \frac{(5x+1)(4x-2)}{\left(x^2-9\right)^2}$$

domain: $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

$$\left(\frac{f}{g}\right)(x) = \frac{\frac{5x+1}{x^2-9}}{\frac{4x-2}{x^2-9}}$$
$$= \frac{5x+1}{x^2-9}\cdot\frac{x^2-9}{4x-2}$$
$$= \frac{5x+1}{4x-2}$$

The domain must exclude –3, 3, and any values that make $4x-2=0$.

$4x-2=0$

$4x=2$

$x=\frac{1}{2}$

domain: $(-\infty,-3)\cup(-3,\tfrac{1}{2})\cup(\tfrac{1}{2},3)\cup(3,\infty)$

44. $(f+g)(x) = f(x)+g(x)$

$$= \frac{3x+1}{x^2-25}+\frac{2x-4}{x^2-25}$$
$$= \frac{5x-3}{x^2-25}$$

domain: $(-\infty,-5)\cup(-5,5)\cup(5,\infty)$

$(f-g)(x) = f(x)-g(x)$

$$= \frac{3x+1}{x^2-25}-\frac{2x-4}{x^2-25}$$
$$= \frac{x+5}{x^2-25}$$
$$= \frac{1}{x-5}$$

domain: $(-\infty,-5)\cup(-5,5)\cup(5,\infty)$

$(fg)(x) = f(x)\cdot g(x)$

$$= \frac{3x+1}{x^2-25}\cdot\frac{2x-4}{x^2-25}$$
$$= \frac{(3x+1)(2x-4)}{\left(x^2-25\right)^2}$$

domain: $(-\infty,-5)\cup(-5,5)\cup(5,\infty)$

$$\left(\frac{f}{g}\right)(x) = \frac{\frac{3x+1}{x^2-25}}{\frac{2x-4}{x^2-25}}$$
$$= \frac{3x+1}{x^2-25}\cdot\frac{x^2-25}{2x-4}$$
$$= \frac{3x+1}{2x-4}$$

The domain must exclude –5, 5, and any values that make $2x-4=0$.

$2x-4=0$

$2x=4$

$x=2$

domain: $(-\infty,-5)\cup(-5,2)\cup(2,5)\cup(5,\infty)$

45. $(f+g)(x) = \sqrt{x+4}+\sqrt{x-1}$

domain: $[1,\infty)$

$(f-g)(x) = \sqrt{x+4}-\sqrt{x-1}$

domain: $[1,\infty)$

$(fg)(x) = \sqrt{x+4}\cdot\sqrt{x-1} = \sqrt{x^2+3x-4}$

domain: $[1,\infty)$

$$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x+4}}{\sqrt{x-1}}$$

domain: $(1,\infty)$

46. $(f+g)(x)=\sqrt{x+6}+\sqrt{x-3}$
domain: $[3, \infty)$
$(f-g)(x)=\sqrt{x+6}-\sqrt{x-3}$
domain: $[3, \infty)$
$(fg)(x)=\sqrt{x+6}\cdot\sqrt{x-3}=\sqrt{x^2+3x-18}$
domain: $[3, \infty)$
$\left(\frac{f}{g}\right)(x)=\frac{\sqrt{x+6}}{\sqrt{x-3}}$
domain: $(3, \infty)$

47. $(f+g)(x)=\sqrt{x-2}+\sqrt{2-x}$
domain: $\{2\}$
$(f-g)(x)=\sqrt{x-2}-\sqrt{2-x}$
domain: $\{2\}$
$(fg)(x)=\sqrt{x-2}\cdot\sqrt{2-x}=\sqrt{-x^2+4x-4}$
domain: $\{2\}$
$\left(\frac{f}{g}\right)(x)=\frac{\sqrt{x-2}}{\sqrt{2-x}}$
domain: $\varnothing$

48. $(f+g)(x)=\sqrt{x-5}+\sqrt{5-x}$
domain: $\{5\}$
$(f-g)(x)=\sqrt{x-5}-\sqrt{5-x}$
domain: $\{5\}$
$(fg)(x)=\sqrt{x-5}\cdot\sqrt{5-x}=\sqrt{-x^2+10x-25}$
domain: $\{5\}$
$\left(\frac{f}{g}\right)(x)=\frac{\sqrt{x-5}}{\sqrt{5-x}}$
domain: $\varnothing$

49. $f(x) = 2x$; $g(x) = x + 7$

a. $(f\circ g)(x)=2(x+7)=2x+14$

b. $(g\circ f)(x)=2x+7$

c. $(f\circ g)(2)=2(2)+14=18$

50. $f(x) = 3x$; $g(x) = x - 5$

a. $(f\circ g)(x)=3(x-5)=3x-15$

b. $(g\circ f)(x)=3x-5$

c. $(f\circ g)(2)=3(2)-15=-9$

51. $f(x) = x + 4$; $g(x) = 2x + 1$

a. $(f\circ g)(x)=(2x+1)+4=2x+5$

b. $(g\circ f)(x)=2(x+4)+1=2x+9$

c. $(f\circ g)(2)=2(2)+5=9$

52. $f(x) = 5x + 2$; $g(x) = 3x - 4$

a. $(f\circ g)(x)=5(3x-4)+2=15x-18$

b. $(g\circ f)(x)=3(5x+2)-4=15x+2$

c. $(f\circ g)(2)=15(2)-18=12$

53. $f(x) = 4x - 3$; $g(x)=5x^2-2$

a. $(f\circ g)(x)=4(5x^2-2)-3$
$=20x^2-11$

b. $(g\circ f)(x)=5(4x-3)^2-2$
$=5(16x^2-24x+9)-2$
$=80x^2-120x+43$

c. $(f\circ g)(2)=20(2)^2-11=69$

54. $f(x)=7x+1; g(x)=2x^2-9$

a. $(f\circ g)(x)=7(2x^2-9)+1=14x^2-62$

b. $(g\circ f)(x)=2(7x+1)^2-9$
$=2(49x^2+14x+1)-9$
$=98x^2+28x-7$

c. $(f\circ g)(2)=14(2)^2-62=-6$

55. $f(x)=x^2+2; g(x)=x^2-2$

a. $(f\circ g)(x)=(x^2-2)^2+2$
$=x^4-4x^2+4+2$
$=x^4-4x^2+6$

b. $(g\circ f)(x)=(x^2+2)^2-2$
$=x^4+4x^2+4-2$
$=x^4+4x^2+2$

c. $(f\circ g)(2)=2^4-4(2)^2+6=6$

56. $f(x) = x^2 + 1; g(x) = x^2 - 3$

a. $$(f \circ g)(x) = (x^2 - 3)^2 + 1 \\ = x^4 - 6x^2 + 9 + 1 \\ = x^4 - 6x^2 + 10$$

b. $$(g \circ f)(x) = (x^2 + 1)^2 - 3 \\ = x^4 + 2x^2 + 1 - 3 \\ = x^4 + 2x^2 - 2$$

c. $(f \circ g)(2) = 2^4 - 6(2)^2 + 10 = 2$

57. $f(x) = 4 - x; \quad g(x) = 2x^2 + x + 5$

a. $$(f \circ g)(x) = 4 - \left(2x^2 + x + 5\right) \\ = 4 - 2x^2 - x - 5 \\ = -2x^2 - x - 1$$

b. $$(g \circ f)(x) = 2\left(4 - x\right)^2 + \left(4 - x\right) + 5 \\ = 2(16 - 8x + x^2) + 4 - x + 5 \\ = 32 - 16x + 2x^2 + 4 - x + 5 \\ = 2x^2 - 17x + 41$$

c. $(f \circ g)(2) = -2(2)^2 - 2 - 1 = -11$

58. $f(x) = 5x - 2; \quad g(x) = -x^2 + 4x - 1$

a. $$(f \circ g)(x) = 5\left(-x^2 + 4x - 1\right) - 2 \\ = -5x^2 + 20x - 5 - 2 \\ = -5x^2 + 20x - 7$$

b. $$(g \circ f)(x) = -\left(5x - 2\right)^2 + 4\left(5x - 2\right) - 1 \\ = -(25x^2 - 20x + 4) + 20x - 8 - 1 \\ = -25x^2 + 20x - 4 + 20x - 8 - 1 \\ = -25x^2 + 40x - 13$$

c. $(f \circ g)(2) = -5(2)^2 + 20(2) - 7 = 13$

59. $f(x) = \sqrt{x}; \; g(x) = x - 1$

a. $(f \circ g)(x) = \sqrt{x - 1}$

b. $(g \circ f)(x) = \sqrt{x} - 1$

c. $(f \circ g)(2) = \sqrt{2 - 1} = \sqrt{1} = 1$

60. $f(x) = \sqrt{x}; g(x) = x + 2$

a. $(f \circ g)(x) = \sqrt{x + 2}$

b. $(g \circ f)(x) = \sqrt{x} + 2$

c. $(f \circ g)(2) = \sqrt{2 + 2} = \sqrt{4} = 2$

61. $f(x) = 2x - 3; \; g(x) = \dfrac{x + 3}{2}$

a. $$(f \circ g)(x) = 2\left(\frac{x + 3}{2}\right) - 3 \\ = x + 3 - 3 \\ = x$$

b. $(g \circ f)(x) = \dfrac{(2x - 3) + 3}{2} = \dfrac{2x}{2} = x$

c. $(f \circ g)(2) = 2$

62. $f(x) = 6x - 3; g(x) = \dfrac{x + 3}{6}$

a. $(f \circ g)(x) = 6\left(\dfrac{x + 3}{6}\right) - 3 = x + 3 - 3 = x$

b. $(g \circ f)(x) = \dfrac{6x - 3 + 3}{6} = \dfrac{6x}{6} = x$

c. $(f \circ g)(2) = 2$

63. $f(x) = \dfrac{1}{x}; \quad g(x) = \dfrac{1}{x}$

a. $(f \circ g)(x) = \dfrac{1}{\frac{1}{x}} = x$

b. $(g \circ f)(x) = \dfrac{1}{\frac{1}{x}} = x$

c. $(f \circ g)(2) = 2$

64. $f(x)=\frac{2}{x};\quad g(x)=\frac{2}{x}$

a. $(f\circ g)(x)=\frac{2}{\frac{2}{x}}=x$

b. $(g\circ f)(x)=\frac{2}{\frac{2}{x}}=x$

c. $(f\circ g)(2)=2$

65. a. $(f\circ g)(x)=f\left(\frac{1}{x}\right)=\frac{2}{\frac{1}{x}+3},\ x\neq 0$

$$=\frac{2(x)}{\left(\frac{1}{x}+3\right)(x)}$$

$$=\frac{2x}{1+3x}$$

b. We must exclude 0 because it is excluded from g.
We must exclude $-\frac{1}{3}$ because it causes the denominator of $f\circ g$ to be 0.
domain: $\left(-\infty,-\frac{1}{3}\right)\cup\left(-\frac{1}{3},0\right)\cup(0,\infty)$.

66. a. $f\circ g(x)=f\left(\frac{1}{x}\right)=\frac{5}{\frac{1}{x}+4}=\frac{5x}{1+4x}$

b. We must exclude 0 because it is excluded from g.
We must exclude $-\frac{1}{4}$ because it causes the denominator of $f\circ g$ to be 0.
domain: $\left(-\infty,-\frac{1}{4}\right)\cup\left(-\frac{1}{4},0\right)\cup(0,\infty)$.

67. a. $(f\circ g)(x)=f\left(\frac{4}{x}\right)=\frac{\frac{4}{x}}{\frac{4}{x}+1}$

$$=\frac{\left(\frac{4}{x}\right)(x)}{\left(\frac{4}{x}+1\right)(x)}$$

$$=\frac{4}{4+x},\ x\neq -4$$

b. We must exclude 0 because it is excluded from g.
We must exclude -4 because it causes the denominator of $f\circ g$ to be 0.
domain: $(-\infty,-4)\cup(-4,0)\cup(0,\infty)$.

68. a. $f\circ g(x)=f\left(\frac{6}{x}\right)=\frac{\frac{6}{x}}{\frac{6}{x}+5}=\frac{6}{6+5x}$

b. We must exclude 0 because it is excluded from g.
We must exclude $-\frac{6}{5}$ because it causes the denominator of $f\circ g$ to be 0.
domain: $\left(-\infty,-\frac{6}{5}\right)\cup\left(-\frac{6}{5},0\right)\cup(0,\infty)$.

69. a. $f\circ g(x)=f(x-2)=\sqrt{x-2}$

b. The expression under the radical in $f\circ g$ must not be negative.
$x-2\geq 0$
$x\geq 2$
domain: $[2,\infty)$.

70. a. $f\circ g(x)=f(x-3)=\sqrt{x-3}$

b. The expression under the radical in $f\circ g$ must not be negative.
$x-3\geq 0$
$x\geq 3$
domain: $[3,\infty)$.

71. **a.** $(f \circ g)(x) = f(\sqrt{1-x})$
$= \left(\sqrt{1-x}\right)^2 + 4$
$= 1 - x + 4$
$= 5 - x$

b. The domain of $f \circ g$ must exclude any values that are excluded from g.
$1 - x \geq 0$
$-x \geq -1$
$x \leq 1$
domain: $(-\infty, 1]$.

72. **a.** $(f \circ g)(x) = f(\sqrt{2-x})$
$= \left(\sqrt{2-x}\right)^2 + 1$
$= 2 - x + 1$
$= 3 - x$

b. The domain of $f \circ g$ must exclude any values that are excluded from g.
$2 - x \geq 0$
$-x \geq -2$
$x \leq 2$
domain: $(-\infty, 2]$.

73. $f(x) = x^4$; $g(x) = 3x - 1$

74. $f(x) = x^3$; $g(x) = 2x - 5$

75. $f(x) = \sqrt[3]{x}$; $g(x) = x^2 - 9$

76. $f(x) = \sqrt{x}$; $g(x) = 5x^2 + 3$

77. $f(x) = |x|$; $g(x) = 2x - 5$

78. $f(x) = |x|$; $g(x) = 3x - 4$

79. $f(x) = \dfrac{1}{x}$; $g(x) = 2x - 3$

80. $f(x) = \dfrac{1}{x}$; $g(x) = 4x + 5$

81. $(f + g)(-3) = f(-3) + g(-3) = 4 + 1 = 5$

82. $(g - f)(-2) = g(-2) - f(-2) = 2 - 3 = -1$

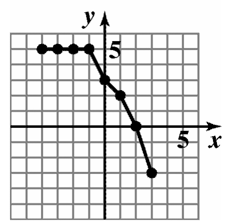
83. $(fg)(2) = f(2)\,g(2) = (-1)(1) = -1$

84. $\left(\dfrac{g}{f}\right)(3) = \dfrac{g(3)}{f(3)} = \dfrac{0}{-3} = 0$

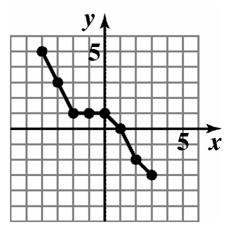
85. The domain of $f + g$ is $[-4, 3]$.

86. The domain of $\dfrac{f}{g}$ is $(-4, 3)$

87. The graph of $f + g$



88. The graph of $f - g$



89. $(f \circ g)(-1) = f(g(-1)) = f(-3) = 1$

90. $(f \circ g)(1) = f(g(1)) = f(-5) = 3$

91. $(g \circ f)(0) = g(f(0)) = g(2) = -6$

92. $(g \circ f)(-1) = g(f(-1)) = g(1) = -5$

93.
$$(f \circ g)(x) = 7$$
$$2\left(x^2 - 3x + 8\right) - 5 = 7$$
$$2x^2 - 6x + 16 - 5 = 7$$
$$2x^2 - 6x + 11 = 7$$
$$2x^2 - 6x + 4 = 0$$
$$x^2 - 3x + 2 = 0$$
$$(x-1)(x-2) = 0$$
$$x - 1 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = 1 \qquad\qquad x = 2$$

94.
$$
\begin{aligned}
(f \circ g)(x) &= -5\\
1-2\left(3x^2+x-1\right) &= -5\\
1-6x^2-2x+2 &= -5\\
-6x^2-2x+3 &= -5\\
-6x^2-2x+8 &= 0\\
3x^2+x-4 &= 0\\
(3x+4)(x-1) &= 0
\end{aligned}
$$
$3x+4=0$ or $x-1=0$
$3x=-4$ $\quad x=1$
$x=-\dfrac{4}{3}$

95. **a.** $(B-D)(x)$
$= B(x)-D(x)$
$= (7.4x^2-15x+4046)-(-3.5x^2+20x+2405)$
$= 7.4x^2-15x+4046+3.5x^2-20x-2405$
$= 10.9x^2-35x+1641$

b. $(B-D)(x)=10.9x^2-35x+1641$
$(B-D)(3)=10.9(3)^2-35(3)+1641$
$=1634.1$
The change in population in the U.S. in 2003 was 1634.1 thousand.

c. $(B-D)(x)$ overestimates the actual change in population in the U.S. in 2003 by 0.1 thousand.

96. **a.** $(B+D)(x)$
$= B(x)+D(x)$
$= (7.4x^2-15x+4046)+(-3.5x^2+20x+2405)$
$= 7.4x^2-15x+4046-3.5x^2+20x+2405$
$= 3.9x^2+5x+6451$

b. $(B+D)(x)=3.9x^2+5x+6451$
$(B+D)(5)=3.9(5)^2+5(5)+6451$
$=6573.5$
The number of births and deaths in the U.S. in 2005 is 6573.5 thousand.

c. $(B+D)(x)$ underestimates the actual number of births and deaths in 2005 by 1.5 thousand.

97. $(R-C)(20{,}000)$
$= 65(20{,}000)-(600{,}000+45(20{,}000))$
$= -200{,}000$
The company lost \$200,000 since costs exceeded revenues.
$(R - C)(30{,}000)$
$= 65(30{,}000) - (600{,}000 + 45(30{,}000))$
$= 0$
The company broke even.

98. **a.** The slope for *f* is -0.44 This is the decrease in profits for the first store for each year after 2004.

b. The slope of *g* is 0.51 This is the increase in profits for the second store for each year after 2004.

c. $f + g = -.044x + 13.62 + 0.51x + 11.14$
$= 0.07x + 24.76$
The slope for $f + g$ is 0.07 This is the profit for the two stores combined for each year after 2004.

99. **a.** *f* gives the price of the computer after a \$400 discount. *g* gives the price of the computer after a 25% discount.

b. $(f \circ g)(x)=0.75x-400$
This models the price of a computer after first a 25% discount and then a \$400 discount.

c. $(g \circ f)(x)=0.75(x-400)$
This models the price of a computer after first a \$400 discount and then a 25% discount.

d. The function $f \circ g$ models the greater discount, since the 25% discount is taken on the regular price first.

100. **a.** *f* gives the cost of a pair of jeans for which a \$5 rebate is offered.
g gives the cost of a pair of jeans that has been discounted 40%.

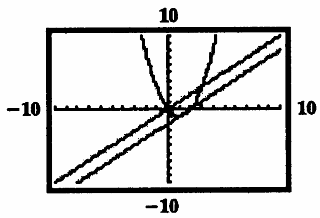
b. $(f \circ g)(x)=0.6x-5$
The cost of a pair of jeans is 60% of the regular price minus a \$5 rebate.

c. $(g \circ f)(x)=0.6(x-5)$
$= 0.6x - 3$
The cost of a pair of jeans is 60% of the regular price minus a \$3 rebate.

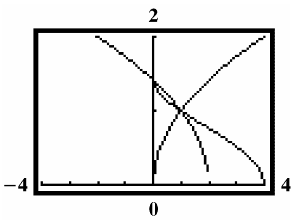
d. $f \circ g$ because of a \$5 rebate.

101. – 105. Answers may vary.

106. When your trace reaches $x = 0$, the y value disappears because the function is not defined at $x = 0$.



107.



$(f \circ g)(x) = \sqrt{2-\sqrt{x}}$

The domain of g is $[0,\infty)$.

The expression under the radical in $f \circ g$ must not be negative.

$$2-\sqrt{x} \ge 0$$
$$-\sqrt{x} \ge -2$$
$$\sqrt{x} \le 2$$
$$x \le 4$$

domain: $[0,4]$

108. makes sense

109. makes sense

110. does not make sense; Explanations will vary. Sample explanation: It is common that $f \circ g$ and $g \circ f$ are not the same.

111. does not make sense; Explanations will vary. Sample explanation: The diagram illustrates $g(f(x)) = x^2 + 4$.

112. false; Changes to make the statement true will vary. A sample change is: $(f \circ g)(x) = f\left(\sqrt{x^2-4}\right)$

$$= \left(\sqrt{x^2-4}\right)^2 - 4$$
$$= x^2 - 4 - 4$$
$$= x^2 - 8$$

113. false; Changes to make the statement true will vary. A sample change is:

$f(x) = 2x; g(x) = 3x$

$(f \circ g)(x) = f(g(x)) = f(3x) = 2(3x) = 6x$

$(g \circ f)(x) = g(f(x)) = g(f(x)) = 3(2x) = 6x$

114. false; Changes to make the statement true will vary. A sample change is:

$(f \circ g)(4) = f(g(4)) = f(7) = 5$

115. true

116. $(f \circ g)(x) = (f \circ g)(-x)$

$f(g(x)) = f(g(-x))$ since g is even

$f(g(x)) = f(g(x))$ so $f \circ g$ is even

117. Answers may vary.

118. $\{(4,-2),(1,-1),(1,1),(4,2)\}$

The element 1 in the domain corresponds to two elements in the range.

Thus, the relation is not a function.

119.

$$x = \frac{5}{y} + 4$$
$$y(x) = y\left(\frac{5}{y} + 4\right)$$
$$xy = 5 + 4y$$
$$xy - 4y = 5$$
$$y(x-4) = 5$$
$$y = \frac{5}{x-4}$$

120.

$$x = y^2 - 1$$
$$x + 1 = y^2$$
$$\sqrt{x+1} = \sqrt{y^2}$$
$$\sqrt{x+1} = y$$
$$y = \sqrt{x+1}$$

Section 1.8

Check Point Exercises

1. $f\left(g(x)\right)=4\left(\frac{x+7}{4}\right)-7=x$

$g\left(f(x)\right)=\frac{(4x-7)+7}{4}=x$

$f\left(g(x)\right)=g\left(f(x)\right)=x$

2. $f(x)=2x+7$

Replace $f(x)$ with *y*:

$y=2x+7$

Interchange *x* and *y*:

$x=2y+7$

Solve for *y*:

$$x=2y+7$$
$$x-7=2y$$
$$\frac{x-7}{2}=y$$

Replace y with $f^{-1}(x)$:

$f^{-1}(x)=\frac{x-7}{2}$

3. $f(x)=4x^3-1$

Replace $f(x)$ with *y*:

$y=4x^3-1$

Interchange *x* and *y*:

$x=4y^3-1$

Solve for *y*:

$$x=4y^3-1$$
$$x+1=4y^3$$
$$\frac{x+1}{4}=y^3$$
$$\sqrt[3]{\frac{x+1}{4}}=y$$

Replace *y* with $f^{-1}(x)$:

$f^{-1}(x)=\sqrt[3]{\frac{x+1}{4}}$

Alternative form for answer:

$$f(x)^{-1}=\sqrt[3]{\frac{x+1}{4}}=\frac{\sqrt[3]{x+1}}{\sqrt[3]{4}}$$
$$=\frac{\sqrt[3]{x+1}}{\sqrt[3]{4}}\cdot\frac{\sqrt[3]{2}}{\sqrt[3]{2}}=\frac{\sqrt[3]{2x+2}}{\sqrt[3]{8}}$$
$$=\frac{\sqrt[3]{2x+2}}{2}$$

4. $f(x)=\frac{3}{x}-1$

Replace $f(x)$ with *y*:

$y=\frac{3}{x}-1$

Interchange *x* and *y*:

$x=\frac{3}{y}-1$

Solve for *y*:

$$x=\frac{3}{y}-1$$
$$xy=3-y$$
$$xy+y=3$$
$$y(x+1)=3$$
$$y=\frac{3}{x+1}$$

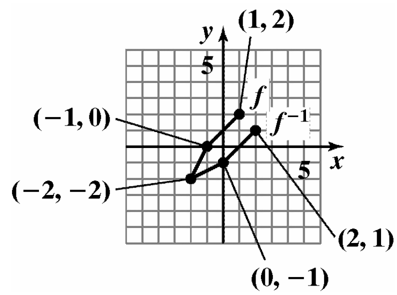
Replace y with $f^{-1}(x)$:

$f^{-1}(x)=\frac{3}{x+1}$

5. The graphs of (b) and (c) pass the horizontal line test and thus have an inverse.

6. Find points of f^{-1}.

$f(x)$	$f^{-1}(x)$
$(-2,-2)$	$(-2,-2)$
$(-1,0)$	$(0,-1)$
$(1,2)$	$(2,1)$



7. $f(x)=x^2+1$

Replace $f(x)$ with y:

$y=x^2+1$

Interchange x and y:

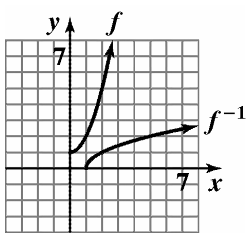
$x=y^2+1$

Solve for y:

$$x=y^2+1$$
$$x-1=y^2$$
$$\sqrt{x-1}=y$$

Replace y with $f^{-1}(x)$:

$f^{-1}(x)=\sqrt{x-1}$



Exercise Set 1.8

1. $f(x)=4x;\ g(x)=\dfrac{x}{4}$

$$f(g(x))=4\left(\frac{x}{4}\right)=x$$

$$g(f(x))=\frac{4x}{4}=x$$

f and *g* are inverses.

2. $f(x)=6x;\ g(x)=\dfrac{x}{6}$

$$f\left(g(x)\right)=6\left(\frac{x}{6}\right)=x$$

$$g\left(f(x)\right)=\frac{6x}{6}=x$$

f and *g* are inverses.

3. $f(x)=3x+8;\ g(x)=\dfrac{x-8}{3}$

$$f(g(x))=3\left(\frac{x-8}{3}\right)+8=x-8+8=x$$

$$g(f(x))=\frac{(3x+8)-8}{3}=\frac{3x}{3}=x$$

f and *g* are inverses.

4. $f(x)=4x+9;\ g(x)=\dfrac{x-9}{4}$

$$f\left(g(x)\right)=4\left(\frac{x-9}{4}\right)+9=x-9+9=x$$

$$g\left(f(x)\right)=\frac{(4x+9)-9}{4}=\frac{4x}{4}=x$$

f and *g* are inverses.

5. $f(x)=5x-9;\ g(x)=\dfrac{x+5}{9}$

$$f(g(x))=5\left(\frac{x+5}{9}\right)-9$$
$$=\frac{5x+25}{9}-9$$
$$=\frac{5x-56}{9}$$

$$g(f(x))=\frac{5x-9+5}{9}=\frac{5x-4}{9}$$

f and *g* are not inverses.

6. $f(x)=3x-7;\ g(x)=\dfrac{x+3}{7}$

$$f\left(g(x)\right)=3\left(\frac{x+3}{7}\right)-7=\frac{3x+9}{7}-7=\frac{3x-40}{7}$$

$$g\left(f(x)\right)=\frac{3x-7+3}{7}=\frac{3x-4}{7}$$

f and *g* are not inverses.

7. $f(x)=\dfrac{3}{x-4};\ g(x)=\dfrac{3}{x}+4$

$$f(g(x))=\frac{3}{\frac{3}{x}+4-4}=\frac{3}{\frac{3}{x}}=x$$

$$g(f(x))=\frac{3}{\frac{3}{x-4}}+4$$
$$=3\cdot\left(\frac{x-4}{3}\right)+4$$
$$=x-4+4$$
$$=x$$

f and *g* are inverses.

8. $f(x)=\dfrac{2}{x-5};\ g(x)=\dfrac{2}{x}+5$

$$f(g(x))=\frac{2}{\left(\frac{2}{x}+5\right)-5}=\frac{2x}{2}=x$$

$$g\left(f(x)\right)=\frac{2}{\frac{2}{x-5}}+5=2\left(\frac{x-5}{2}\right)+5=x-5+5=x$$

f and *g* are inverses.

9. $f(x) = -x;\ g(x) = -x$
$f(g(x)) = -(-x) = x$
$g(f(x)) = -(-x) = x$
f and g are inverses.

10. $f(x) = \sqrt[3]{x-4};\ g(x) = x^3 + 4$
$f\left(g(x)\right) = \sqrt[3]{x^3+4-4} = \sqrt[3]{x^3} = x$
$g\left(f(x)\right) = \left(\sqrt[3]{x-4}\right)^3 + 4 = x - 4 + 4 = x$
f and g are inverses.

11. a.
$$\begin{aligned} f(x) &= x + 3 \\ y &= x + 3 \\ x &= y + 3 \\ y &= x - 3 \\ f^{-1}(x) &= x - 3 \end{aligned}$$

b. $f(f^{-1}(x)) = x - 3 + 3 = x$
$f^{-1}(f(x)) = x + 3 - 3 = x$

12. a.
$$\begin{aligned} f(x) &= x+5 \\ y &= x+5 \\ x &= y+5 \\ y &= x-5 \\ f^{-1}(x) &= x-5 \end{aligned}$$

b. $f\left(f^{-1}(x)\right) = x - 5 + 5 = x$
$f^{-1}\left(f(x)\right) = x + 5 - 5 = x$

13. a.
$$\begin{aligned} f(x) &= 2x \\ y &= 2x \\ x &= 2y \\ y &= \frac{x}{2} \\ f^{-1}(x) &= \frac{x}{2} \end{aligned}$$

b. $f(f^{-1}(x)) = 2\left(\frac{x}{2}\right) = x$
$f^{-1}(f(x)) = \frac{2x}{2} = x$

14. a.
$$\begin{aligned} f(x) &= 4x \\ y &= 4x \\ x &= 4y \\ y &= \frac{x}{4} \\ f^{-1}(x) &= \frac{x}{4} \end{aligned}$$

b. $f\left(f^{-1}(x)\right) = 4\left(\frac{x}{4}\right) = x$
$f^{-1}\left(f(x)\right) = \frac{4x}{4} = x$

15. a.
$$\begin{aligned} f(x) &= 2x+3 \\ y &= 2x+3 \\ x &= 2y+3 \\ x-3 &= 2y \\ y &= \frac{x-3}{2} \\ f^{-1}(x) &= \frac{x-3}{2} \end{aligned}$$

b.
$$\begin{aligned} f(f^{-1}(x)) &= 2\left(\frac{x-3}{2}\right)+3 \\ &= x-3+3 \\ &= x \end{aligned}$$
$f^{-1}(f(x)) = \frac{2x+3-3}{2} = \frac{2x}{2} = x$

16. a.
$$\begin{aligned} f(x) &= 3x-1 \\ y &= 3x-1 \\ x &= 3y-1 \\ x+1 &= 3y \\ y &= \frac{x+1}{3} \\ f^{-1}(x) &= \frac{x+1}{3} \end{aligned}$$

b. $f\left(f^{-1}(x)\right) = 3\left(\frac{x+1}{3}\right) - 1 = x + 1 - 1 = x$
$f^{-1}\left(f(x)\right) = \frac{3x-1+1}{3} = \frac{3x}{3} = x$

17. a.

$$\begin{aligned} f(x) &= x^3+2 \\ y &= x^3+2 \\ x &= y^3+2 \\ x-2 &= y^3 \\ y &= \sqrt[3]{x-2} \\ f^{-1}(x) &= \sqrt[3]{x-2} \end{aligned}$$

b.

$$\begin{aligned} f(f^{-1}(x)) &= \left(\sqrt[3]{x-2}\right)^3+2 \\ &= x-2+2 \\ &= x \end{aligned}$$

$$f^{-1}(f(x)) = \sqrt[3]{x^3+2-2} = \sqrt[3]{x^3} = x$$

18. a.

$$\begin{aligned} f(x) &= x^3-1 \\ y &= x^3-1 \\ x &= y^3-1 \\ x+1 &= y^3 \\ y &= \sqrt[3]{x+1} \\ f^{-1}(x) &= \sqrt[3]{x+1} \end{aligned}$$

b.

$$\begin{aligned} f(f^{-1}(x)) &= \left(\sqrt[3]{x+1}\right)^3-1 \\ &= x+1-1 \\ &= x \end{aligned}$$

$$f^{-1}(f(x)) = \sqrt[3]{x^3-1+1} = \sqrt[3]{x^3} = x$$

19. a.

$$\begin{aligned} f(x) &= (x+2)^3 \\ y &= (x+2)^3 \\ x &= (y+2)^3 \\ \sqrt[3]{x} &= y+2 \\ y &= \sqrt[3]{x}-2 \\ f^{-1}(x) &= \sqrt[3]{x}-2 \end{aligned}$$

b.

$$f(f^{-1}(x)) = \left(\sqrt[3]{x}-2+2\right)^3 = \left(\sqrt[3]{x}\right)^3 = x$$

$$\begin{aligned} f^{-1}(f(x)) &= \sqrt[3]{(x+2)^3}-2 \\ &= x+2-2 \\ &= x \end{aligned}$$

20. a.

$$\begin{aligned} f(x) &= (x-1)^3 \\ y &= (x-1)^3 \\ x &= (y-1)^3 \\ \sqrt[3]{x} &= y-1 \\ y &= \sqrt[3]{x}+1 \end{aligned}$$

b.

$$f\left(f^{-1}(x)\right) = \left(\sqrt[3]{x}+1-1\right)^3 = \left(\sqrt[3]{x}\right)^3 = x$$

$$f^{-1}\left(f(x)\right) = \sqrt[3]{(x-1^3}+1 = x-1+1 = x$$

21. a.

$$\begin{aligned} f(x) &= \frac{1}{x} \\ y &= \frac{1}{x} \\ x &= \frac{1}{y} \\ xy &= 1 \\ y &= \frac{1}{x} \\ f^{-1}(x) &= \frac{1}{x} \end{aligned}$$

b.

$$f(f^{-1}(x)) = \frac{1}{\frac{1}{x}} = x$$

$$f^{-1}(f(x)) = \frac{1}{\frac{1}{x}} = x$$

22. a.

$$\begin{aligned} f(x) &= \frac{2}{x} \\ y &= \frac{2}{x} \\ x &= \frac{2}{y} \\ xy &= 2 \\ y &= \frac{2}{x} \\ f^{-1}(x) &= \frac{2}{x} \end{aligned}$$

b.

$$f(f^{-1}(x)) = \frac{2}{\frac{2}{x}} = 2\cdot\frac{x}{2} = x$$

$$f^{-1}(f(x)) = \frac{2}{\frac{2}{x}} = 2\cdot\frac{x}{2} = x$$

23. **a.**
$$f(x)=\sqrt{x}$$
$$y=\sqrt{x}$$
$$x=\sqrt{y}$$
$$y=x^2$$
$$f^{-1}(x)=x^2,\ x\geq 0$$

b. $f(f^{-1}(x))=\sqrt{x^2}=|x|=x$ for $x\geq 0.$
$$f^{-1}(f(x))=(\sqrt{x})^2=x$$

24. **a.**
$$f(x)=\sqrt[3]{x}$$
$$y=\sqrt[3]{x}$$
$$x=\sqrt[3]{y}$$
$$y=x^3$$
$$f^{-1}(x)=x^3$$

b.
$$f\left(f^{-1}(x)\right)=\sqrt[3]{x^3}=x$$
$$f^{-1}\left(f(x)\right)=\left(\sqrt[3]{x}\right)^3=x$$

25. **a.**
$$f(x)=\frac{7}{x}-3$$
$$y=\frac{7}{x}-3$$
$$x=\frac{7}{y}-3$$
$$xy=7-3y$$
$$xy+3y=7$$
$$y\left(x+3\right)=7$$
$$y=\frac{7}{x+3}$$
$$f^{-1}(x)=\frac{7}{x+3}$$

b.
$$f\left(f^{-1}(x)\right)=\frac{7}{\frac{7}{x+3}}-3=x$$
$$f^{-1}\left(f(x)\right)=\frac{7}{\frac{7}{x}-3+3}=x$$

26. **a.**
$$f(x)=\frac{4}{x}+9$$
$$y=\frac{4}{x}+9$$
$$x=\frac{4}{y}+9$$
$$xy=4+9y$$
$$xy-9y=4$$
$$y\left(x-9\right)=4$$
$$y=\frac{4}{x-9}$$
$$f^{-1}(x)=\frac{4}{x-9}$$

b.
$$f\left(f^{-1}(x)\right)=\frac{4}{\frac{4}{x-9}}+9=x$$
$$f^{-1}\left(f(x)\right)=\frac{4}{\frac{4}{x}+9-9}=x$$

27. **a.**
$$f\left(x\right)=\frac{2x+1}{x-3}$$
$$y=\frac{2x+1}{x-3}$$
$$x=\frac{2y+1}{y-3}$$
$$x(y-3)=2y+1$$
$$xy-3x=2y+1$$
$$xy-2y=3x+1$$
$$y(x-2)=3x+1$$
$$y=\frac{3x+1}{x-2}$$
$$f^{-1}\left(x\right)=\frac{3x+1}{x-2}$$

b. $f(f^{-1}(x)) = \dfrac{2\left(\dfrac{3x+1}{x-2}\right)+1}{\dfrac{3x+1}{x-2}-3}$

$= \dfrac{2(3x+1)+x-2}{3x+1-3(x-2)} = \dfrac{6x+2+x-2}{3x+1-3x+6}$

$= \dfrac{7x}{7} = x$

$f^{-1}(f(x)) = \dfrac{3\left(\dfrac{2x+1}{x-3}\right)+1}{\dfrac{2x+1}{x-3}-2}$

$= \dfrac{3(2x+1)+x-3}{2x+1-2(x-3)}$

$= \dfrac{6x+3+x-3}{2x+1-2x+6} = \dfrac{7x}{7} = x$

28. a. $f(x) = \dfrac{2x-3}{x+1}$

$y = \dfrac{2x-3}{x+1}$

$x = \dfrac{2y-3}{y+1}$

$xy + x = 2y - 3$

$y(x-2) = -x-3$

$y = \dfrac{-x-3}{x-2}$

$f^{-1}(x) = \dfrac{-x-3}{x-2}, \quad x \neq 2$

b. $f\left(f^{-1}(x)\right) = \dfrac{2\left(\dfrac{-x-3}{x-2}\right)-3}{\dfrac{-x-3}{x-2}+1}$

$= \dfrac{-2x-6-3x+6}{-x-3+x-2} = \dfrac{-5x}{-5} = x$

$f^{-1}(f(x)) = \dfrac{-\left(\dfrac{2x-3}{x+1}\right)-3}{\dfrac{2x-3}{x+1}-2}$

$= \dfrac{-2x+3-3x-3}{2x-3-2x-2} = \dfrac{-5x}{-5} = x$

29. The function fails the horizontal line test, so it does not have an inverse function.

30. The function passes the horizontal line test, so it does have an inverse function.

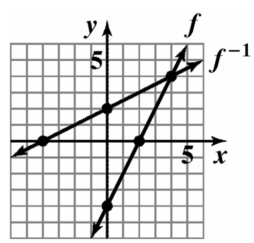
31. The function fails the horizontal line test, so it does not have an inverse function.

32. The function fails the horizontal line test, so it does not have an inverse function.

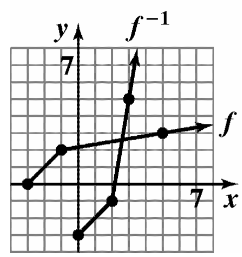
33. The function passes the horizontal line test, so it does have an inverse function.

34. The function passes the horizontal line test, so it does have an inverse function.

35.



36.



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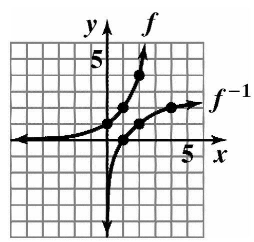

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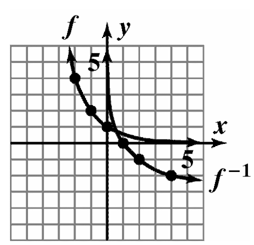
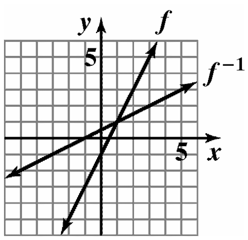


39. a.
$$\begin{aligned} f(x) &= 2x-1 \\ y &= 2x-1 \\ x &= 2y-1 \\ x+1 &= 2y \\ \frac{x+1}{2} &= y \\ f^{-1}(x) &= \frac{x+1}{2} \end{aligned}$$

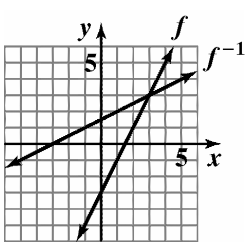
b.



c. domain of f: $(-\infty,\infty)$
range of f: $(-\infty,\infty)$
domain of f^{-1}: $(-\infty,\infty)$
range of f^{-1}: $(-\infty,\infty)$

40. a.
$$\begin{aligned} f(x) &= 2x-3 \\ y &= 2x-3 \\ x &= 2y-3 \\ x+3 &= 2y \\ \frac{x+3}{2} &= y \\ f^{-1}(x) &= \frac{x+3}{2} \end{aligned}$$

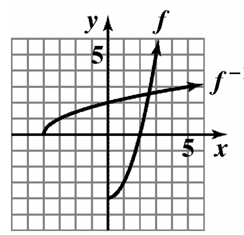
b.



c. domain of f: $(-\infty,\infty)$
range of f: $(-\infty,\infty)$
domain of f^{-1}: $(-\infty,\infty)$
range of f^{-1}: $(-\infty,\infty)$

41. a.
$$\begin{aligned} f(x) &= x^2-4 \\ y &= x^2-4 \\ x &= y^2-4 \\ x+4 &= y^2 \\ \sqrt{x+4} &= y \\ f^{-1}(x) &= \sqrt{x+4} \end{aligned}$$

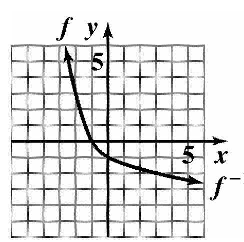
b.



c. domain of f: $[0,\infty)$
range of f: $[-4,\infty)$
domain of f^{-1}: $[-4,\infty)$
range of f^{-1}: $[0,\infty)$

42. a.
$$\begin{aligned} f(x) &= x^2-1 \\ y &= x^2-1 \\ x &= y^2-1 \\ x+1 &= y^2 \\ -\sqrt{x+1} &= y \\ f^{-1}(x) &= -\sqrt{x+1} \end{aligned}$$

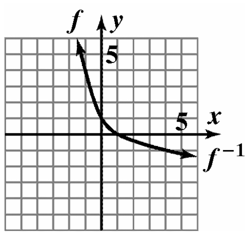
b.



c. domain of f: $(-\infty,0]$
range of f: $[-1,\infty)$
domain of f^{-1}: $[-1,\infty)$
range of f^{-1}: $(-\infty,0]$

43. a.
$$f(x) = (x-1)^2$$
$$y = (x-1)^2$$
$$x = (y-1)^2$$
$$-\sqrt{x} = y-1$$
$$-\sqrt{x}+1 = y$$
$$f^{-1}(x) = 1-\sqrt{x}$$

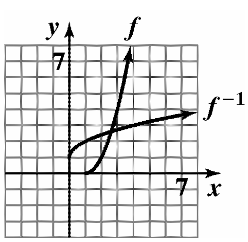
b.



c. domain of f: $(-\infty, 1]$
range of f: $[0, \infty)$
domain of f^{-1}: $[0, \infty)$
range of f^{-1}: $(-\infty, 1]$

44. a.
$$f(x) = (x-1)^2$$
$$y = (x-1)^2$$
$$x = (y-1)^2$$
$$\sqrt{x} = y-1$$
$$\sqrt{x}+1 = y$$
$$f^{-1}(x) = 1+\sqrt{x}$$

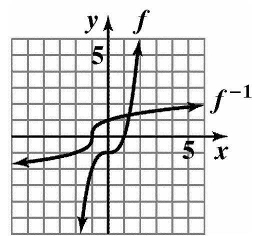
b.



c. domain of f: $[1, \infty)$
range of f: $[0, \infty)$
domain of f^{-1}: $[0, \infty)$
range of f^{-1}: $[1, \infty)$

45. a.
$$f(x) = x^3-1$$
$$y = x^3-1$$
$$x = y^3-1$$
$$x+1 = y^3$$
$$\sqrt[3]{x+1} = y$$
$$f^{-1}(x) = \sqrt[3]{x+1}$$

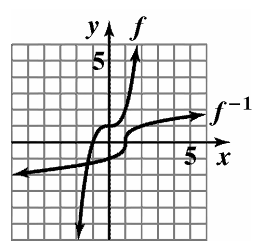
b.



c. domain of f: $(-\infty, \infty)$
range of f: $(-\infty, \infty)$
domain of f^{-1}: $(-\infty, \infty)$
range of f^{-1}: $(-\infty, \infty)$

46. a.
$$f(x) = x^3+1$$
$$y = x^3+1$$
$$x = y^3+1$$
$$x-1 = y^3$$
$$\sqrt[3]{x-1} = y$$
$$f^{-1}(x) = \sqrt[3]{x-1}$$

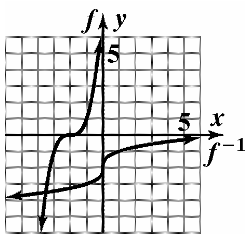
b.



c. domain of f: $(-\infty, \infty)$
range of f: $(-\infty, \infty)$
domain of f^{-1}: $(-\infty, \infty)$
range of f^{-1}: $(-\infty, \infty)$

47. **a.**
$$f(x)=(x+2)^3$$
$$y=(x+2)^3$$
$$x=(y+2)^3$$
$$\sqrt[3]{x}=y+2$$
$$\sqrt[3]{x}-2=y$$
$$f^{-1}(x)=\sqrt[3]{x}-2$$

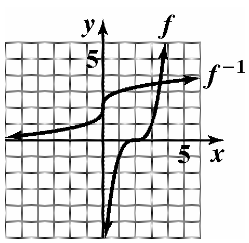
b.



c. domain of f : $(-\infty,\infty)$
range of f : $(-\infty,\infty)$
domain of f^{-1}: $(-\infty,\infty)$
range of f^{-1}: $(-\infty,\infty)$

48. **a.**
$$f(x)=(x-2)^3$$
$$y=(x-2)^3$$
$$x=(y-2)^3$$
$$\sqrt[3]{x}=y-2$$
$$\sqrt[3]{x}+2=y$$
$$f^{-1}(x)=\sqrt[3]{x}+2$$

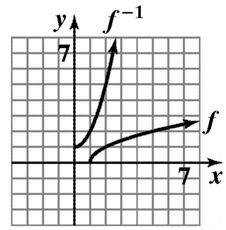
b.



c. domain of f : $(-\infty,\infty)$
range of f : $(-\infty,\infty)$
domain of f^{-1}: $(-\infty,\infty)$
range of f^{-1}: $(-\infty,\infty)$

49. **a.**
$$f(x)=\sqrt{x-1}$$
$$y=\sqrt{x-1}$$
$$x=\sqrt{y-1}$$
$$x^2=y-1$$
$$x^2+1=y$$
$$f^{-1}(x)=x^2+1$$

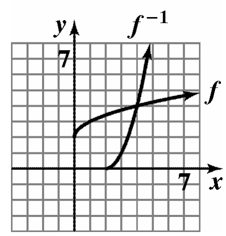
b.



c. domain of f : $[1,\infty)$
range of f : $[0,\infty)$
domain of f^{-1}: $[0,\infty)$
range of f^{-1}: $[1,\infty)$

50. **a.**
$$f(x)=\sqrt{x}+2$$
$$y=\sqrt{x}+2$$
$$x=\sqrt{y}+2$$
$$x-2=\sqrt{y}$$
$$(x-2)^2=y$$
$$f^{-1}(x)=(x-2)^2$$

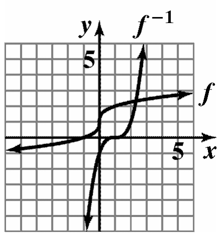
b.



c. domain of f : $[0,\infty)$
range of f : $[2,\infty)$
domain of f^{-1}: $[2,\infty)$
range of f^{-1}: $[0,\infty)$

51. **a.**
$$f(x)=\sqrt[3]{x}+1$$
$$y=\sqrt[3]{x}+1$$
$$x=\sqrt[3]{y}+1$$
$$x-1=\sqrt[3]{y}$$
$$(x-1)^3=y$$
$$f^{-1}(x)=(x-1)^3$$

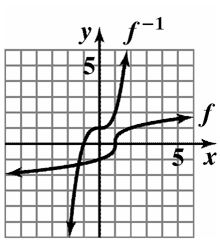
b.



c. domain of f: $(-\infty,\infty)$
range of f: $(-\infty,\infty)$
domain of f^{-1}: $(-\infty,\infty)$
range of f^{-1}: $(-\infty,\infty)$

52. **a.**
$$f(x)=\sqrt[3]{x-1}$$
$$y=\sqrt[3]{x-1}$$
$$x=\sqrt[3]{y-1}$$
$$x^3=y-1$$
$$x^3+1=y$$
$$f^{-1}(x)=x^3+1$$

b.



c. domain of f: $(-\infty,\infty)$
range of f: $(-\infty,\infty)$
domain of f^{-1}: $(-\infty,\infty)$
range of f^{-1}: $(-\infty,\infty)$

53. $f(g(1))=f(1)=5$

54. $f(g(4))=f(2)=-1$

55. $(g\circ f)(-1)=g(f(-1))=g(1)=1$

56. $(g\circ f)(0)=g(f(0))=g(4)=2$

57. $f^{-1}(g(10))=f^{-1}(-1)=2$, since $f(2)=-1$.

58. $f^{-1}(g(1))=f^{-1}(1)=-1$, since $f(-1)=1$.

59.
$$\begin{aligned}(f\circ g)(0)&=f(g(0))\\&=f(4\cdot 0-1)\\&=f(-1)=2(-1)-5=-7\end{aligned}$$

60.
$$\begin{aligned}(g\circ f)(0)&=g(f(0))\\&=g(2\cdot 0-5)\\&=g(-5)=4(-5)-1=-21\end{aligned}$$

61. Let $f^{-1}(1)=x$. Then
$$f(x)=1$$
$$2x-5=1$$
$$2x=6$$
$$x=3$$
Thus, $f^{-1}(1)=3$

62. Let $g^{-1}(7)=x$. Then
$$g(x)=7$$
$$4x-1=7$$
$$4x=8$$
$$x=2$$
Thus, $g^{-1}(7)=2$

63.
$$\begin{aligned}g(f[h(1)])&=g(f[1^2+1+2])\\&=g(f(4))\\&=g(2\cdot 4-5)\\&=g(3)\\&=4\cdot 3-1=11\end{aligned}$$

64.
$$\begin{aligned}f(g[h(1)])&=f(g[1^2+1+2])\\&=f(g(4))\\&=f(4\cdot 4-1)\\&=f(15)\\&=2\cdot 15-5=25\end{aligned}$$

65. **a.** $\{(17,9.7),(22,8.7),(30,8.4),(40,8.3),(50,8.2),(60,8.3)\}$

b. $\{(9.7,17),(8.7,22),(8.4,30),(8.3,40),(8.2,50),(8.3,60)\}$
f is not a one-to-one function because the inverse of *f* is not a function.

66. **a.** $\{(17,9.3),(22,9.1),(30,8.8),(40,8.5),(50,8.4),(60,8.5)\}$

b. $\{(9.3,17),(9.1,22),(8.8,30),(8.5,40),(8.4,50),(8.5,60)\}$
g is not a one-to-one function because the inverse of *g* is not a function.

67. **a.** It passes the horizontal line test and is one-to-one.

b. $f^{-1}(0.25) = 15$ If there are 15 people in the room, the probability that 2 of them have the same birthday is 0.25.
$f^{-1}(0.5) = 21$ If there are 21 people in the room, the probability that 2 of them have the same birthday is 0.5.
$f^{-1}(0.7) = 30$ If there are 30 people in the room, the probability that 2 of them have the same birthday is 0.7.

68. **a.** This function fails the horizontal line test. Thus, this function does not have an inverse.

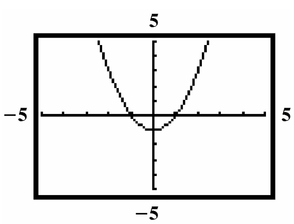
b. The average happiness level is 3 at 12 noon and at 7 p.m. These values can be represented as $(12,3)$ and $(19,3)$.

c. The graph does not represent a one-to-one function. $(12,3)$ and $(19,3)$ are an example of two *x*-values that correspond to the same *y*-value.

69.
$$\begin{aligned} f(g(x)) &= \frac{9}{5}\left[\frac{5}{9}(x-32)\right]+32 \\ &= x-32+32 \\ &= x \end{aligned}$$
f and *g* are inverses.

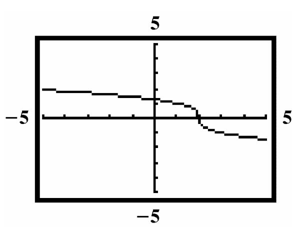
70. – 75. Answers may vary.

76.

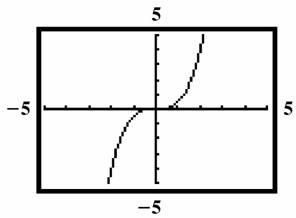


not one-to-one

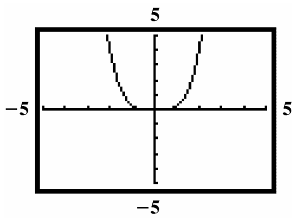
77.



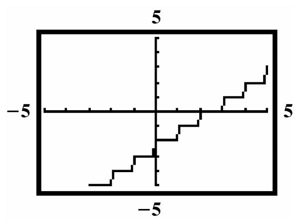
one-to-one

78.


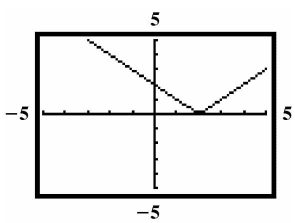
one-to-one

79.


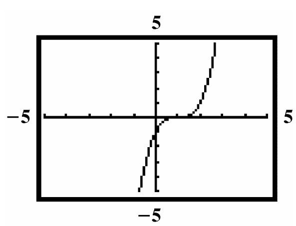
not one-to-one

80.


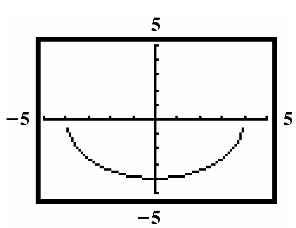
not one-to-one

81.


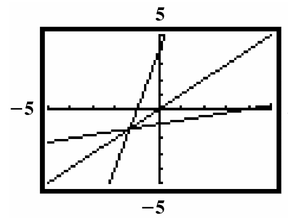
not one-to-one

82.


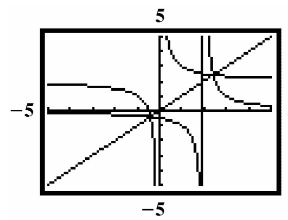
one-to-one

83.


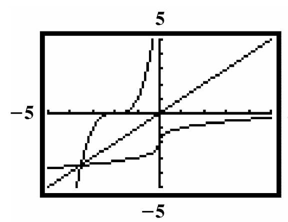
not one-to-one

84.


f and g are inverses

85.


f and g are inverses

86.


f and g are inverses

87. makes sense

88. makes sense

89. makes sense

90. makes sense

91. false; Changes to make the statement true will vary. A sample change is: The inverse is $\{(4,1), (7,2)\}$.

92. false; Changes to make the statement true will vary. A sample change is: $f(x) = 5$ is a horizontal line, so it does not pass the horizontal line test.

93. false; Changes to make the statement true will vary. A sample change is: $f^{-1}(x) = \dfrac{x}{3}$.

94. true

95. $(f \circ g)(x) = 3(x+5) = 3x+15.$

$$y = 3x+15$$
$$x = 3y+15$$
$$y = \frac{x-15}{3}$$
$$\left(f \circ g\right)^{-1}(x) = \frac{x-15}{3}$$
$$g(x) = x+5$$
$$y = x+5$$
$$x = y+5$$
$$y = x-5$$
$$g^{-1}(x) = x-5$$
$$f(x) = 3x$$
$$y = 3x$$
$$x = 3y$$
$$y = \frac{x}{3}$$
$$f^{-1}(x) = \frac{x}{3}$$
$$\left(g^{-1} \circ f^{-1}\right)(x) = \frac{x}{3} - 5 = \frac{x-15}{3}$$

96.

$$f(x) = \frac{3x-2}{5x-3}$$
$$y = \frac{3x-2}{5x-3}$$
$$x = \frac{3y-2}{5y-3}$$
$$x(5y-3) = 3y-2$$
$$5xy-3x = 3y-2$$
$$5xy-3y = 3x-2$$
$$y(5x-3) = 3x-2$$
$$y = \frac{3x-2}{5x-3}$$
$$f^{-1}(x) = \frac{3x-2}{5x-3}$$

Note: An alternative approach is to show that $(f \circ f)(x) = x.$

97. No, there will be 2 times when the spacecraft is at the same height, when it is going up and when it is coming down.

98. $8 + f^{-1}(x-1) = 10$

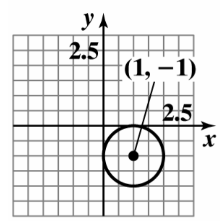
$$f^{-1}(x-1) = 2$$
$$f(2) = x-1$$
$$6 = x-1$$
$$7 = x$$
$$x = 7$$

99. Answers may vary.

100. $\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} = \sqrt{(1-7)^2 + (-1-2)^2}$

$$= \sqrt{(-6)^2 + (-3)^2}$$
$$= \sqrt{36+9}$$
$$= \sqrt{45}$$
$$= 3\sqrt{5}$$

101.



102. $y^2 - 6y - 4 = 0$

$$y^2 - 6y = 4$$
$$y^2 - 6y + 9 = 4 + 9$$
$$(y-3)^2 = 13$$
$$y - 3 = \pm\sqrt{13}$$
$$y = 3 \pm \sqrt{13}$$

Section 1.9

Check Point Exercises

1. $d=\sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2}$

$d=\sqrt{\left(1-(-4)\right)^2+\left(-3-9\right)^2}$

$=\sqrt{(5)^2+(-12)^2}$

$=\sqrt{25+144}$

$=\sqrt{169}$

$=13$

2. $\left(\frac{1+7}{2},\frac{2+(-3)}{2}\right)=\left(\frac{8}{2},\frac{-1}{2}\right)=\left(4,-\frac{1}{2}\right)$

3. $h=0,\ k=0,\ r=4;$

$(x-0)^2+(y-0)^2=4^2$

$x^2+y^2=16$

4. $h=0,\ k=-6,\ r=10;$

$(x-0)^2+[y-(-6)]^2=10^2$

$(x-0)^2+(y+6)^2=100$

$x^2+(y+6)^2=100$

5. **a.** $(x+3)^2+(y-1)^2=4$

$[x-(-3)]^2+(y-1)^2=2^2$

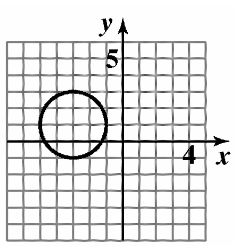
So in the standard form of the circle's equation $(x-h)^2+(y-k)^2=r^2$,

we have $h=-3,\ k=1,\ r=2.$

center: $(h,\ k)=(-3,\ 1)$

radius: $r=2$

b.



$\mathbf{(x+3)^2+(y-1)^2=4}$

c. domain: $[-5,-1]$

range: $[-1,3]$

6. $x^2+y^2+4x-4y-1=0$

$x^2+y^2+4x-4y-1=0$

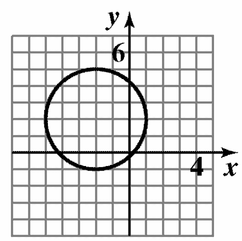
$\left(x^2+4x\quad\right)+\left(y^2-4y\quad\right)=0$

$\left(x^2+4x+4\right)+\left(y^2+4y+4\right)=1+4+4$

$(x+2)^2+(y-2)^2=9$

$[x-(-x)]^2+(y-2)^2=3^2$

So in the standard form of the circle's equation $(x-h)^2+(y-k)^2=r^2$, we have $h=-2,\ k=2,\ r=3$.



$\mathbf{x^2+y^2+4x-4y-1=0}$

Exercise Set 1.9

1. $d=\sqrt{(14-2)^2+(8-3)^2}$

$=\sqrt{12^2+5^2}$

$=\sqrt{144+25}$

$=\sqrt{169}$

$=13$

2. $d=\sqrt{(8-5)^2+(5-1)^2}$

$=\sqrt{3^2+4^2}$

$=\sqrt{9+16}$

$=\sqrt{25}$

$=5$

3. $d=\sqrt{\left(-6-4\right)^2+\left(3-(-1)\right)^2}$

$=\sqrt{(-10)^2+(4)^2}$

$=\sqrt{100+16}$

$=\sqrt{116}$

$=2\sqrt{29}$

≈ 10.77

4. $d=\sqrt{(-1-2)^2+(5-(-3))^2}$
$=\sqrt{(-3)^2+(8)^2}$
$=\sqrt{9+64}$
$=\sqrt{73}$
≈ 8.54

5. $d=\sqrt{(-3-0)^2+(4-0)^2}$
$=\sqrt{3^2+4^2}$
$=\sqrt{9+16}$
$=\sqrt{25}$
$=5$

6. $d=\sqrt{(3-0)^2+(-4-0)^2}$
$=\sqrt{3^2+(-4)^2}$
$=\sqrt{9+16}$
$=\sqrt{25}$
$=5$

7. $d=\sqrt{[3-(-2)]^2+[-4-(-6)]^2}$
$=\sqrt{5^2+2^2}$
$=\sqrt{25+4}$
$=\sqrt{29}$
≈ 5.39

8. $d=\sqrt{[2-(-4)]^2+[-3-(-1)]^2}$
$=\sqrt{6^2+(-2)^2}$
$=\sqrt{36+4}$
$=\sqrt{40}$
$=2\sqrt{10}$
≈ 6.32

9. $d=\sqrt{(4-0)^2+[1-(-3)]^2}$
$=\sqrt{4^2+4^2}$
$=\sqrt{16+16}$
$=\sqrt{32}$
$=4\sqrt{2}$
≈ 5.66

10. $d=\sqrt{(4-0)^2+[3-(-2)]^2}$
$=\sqrt{4^2+[3+2]^2}$
$=\sqrt{16+5^2}$
$=\sqrt{16+25}$
$=\sqrt{41}$
≈ 6.40

11. $d=\sqrt{(-.5-3.5)^2+(6.2-8.2)^2}$
$=\sqrt{(-4)^2+(-2)^2}$
$=\sqrt{16+4}$
$=\sqrt{20}$
$=2\sqrt{5}$
≈ 4.47

12. $d=\sqrt{(1.6-2.6)^2+(-5.7-1.3)^2}$
$=\sqrt{(-1)^2+(-7)^2}$
$=\sqrt{1+49}$
$=\sqrt{50}$
$=5\sqrt{2}$
≈ 7.07

13. $d=\sqrt{(\sqrt{5}-0)^2+[0-(-\sqrt{3})]^2}$
$=\sqrt{(\sqrt{5})^2+(\sqrt{3})^2}$
$=\sqrt{5+3}$
$=\sqrt{8}$
$=2\sqrt{2}$
≈ 2.83

14. $d=\sqrt{\left(\sqrt{7}-0\right)^2+\left[0-\left(-\sqrt{2}\right)\right]^2}$
$=\sqrt{\left(\sqrt{7}\right)^2+\left[-\sqrt{2}\right]^2}$
$=\sqrt{7+2}$
$=\sqrt{9}$
$=3$

15. $d=\sqrt{(-\sqrt{3}-3\sqrt{3})^2+(4\sqrt{5}-\sqrt{5})^2}$
$=\sqrt{(-4\sqrt{3})^2+(3\sqrt{5})^2}$
$=\sqrt{16(3)+9(5)}$
$=\sqrt{48+45}$
$=\sqrt{93}$
≈ 9.64

16. $d=\sqrt{\left(-\sqrt{3}-2\sqrt{3}\right)^2+\left(5\sqrt{6}-\sqrt{6}\right)^2}$
$=\sqrt{\left(-3\sqrt{3}\right)^2+\left(4\sqrt{6}\right)^2}$
$=\sqrt{9\cdot 3+16\cdot 6}$
$=\sqrt{27+96}$
$=\sqrt{123}$
≈ 11.09

17. $d=\sqrt{\left(\frac{1}{3}-\frac{7}{3}\right)^2+\left(\frac{6}{5}-\frac{1}{5}\right)^2}$
$=\sqrt{(-2)^2+1^2}$
$=\sqrt{4+1}$
$=\sqrt{5}$
≈ 2.24

18. $d=\sqrt{\left[\frac{3}{4}-\left(-\frac{1}{4}\right)\right]^2+\left[\frac{6}{7}-\left(-\frac{1}{7}\right)\right]^2}$
$=\sqrt{\left(\frac{3}{4}+\frac{1}{4}\right)^2+\left[\frac{6}{7}+\frac{1}{7}\right]^2}$
$=\sqrt{1^2+1^2}$
$=\sqrt{2}$
≈ 1.41

19. $\left(\frac{6+2}{2},\frac{8+4}{2}\right)=\left(\frac{8}{2},\frac{12}{2}\right)=(4,6)$

20. $\left(\frac{10+2}{2},\frac{4+6}{2}\right)=\left(\frac{12}{2},\frac{10}{2}\right)=(6,5)$

21. $\left(\frac{-2+(-6)}{2},\frac{-8+(-2)}{2}\right)$
$=\left(\frac{-8}{2},\frac{-10}{2}\right)=(-4,-5)$

22. $\left(\frac{-4+(-1)}{2},\frac{-7+(-3)}{2}\right)=\left(\frac{-5}{2},\frac{-10}{2}\right)$
$=\left(\frac{-5}{2},-5\right)$

23. $\left(\frac{-3+6}{2},\frac{-4+(-8)}{2}\right)$
$=\left(\frac{3}{2},\frac{-12}{2}\right)=\left(\frac{3}{2},-6\right)$

24. $\left(\frac{-2+(-8)}{2}\frac{-1+6}{2}\right)=\left(\frac{-10}{2},\frac{5}{2}\right)=\left(-5,\frac{5}{2}\right)$

25. $\left(\frac{\frac{-7}{2}+\left(-\frac{5}{2}\right)}{2},\frac{\frac{3}{2}+\left(-\frac{11}{2}\right)}{2}\right)$
$=\left(\frac{\frac{-12}{2}}{2},\frac{\frac{-8}{2}}{2}\right)=\left(-\frac{6}{2},\frac{-4}{2}\right)=(-3,-2)$

26. $\left(\frac{-\frac{2}{5}+\left(-\frac{2}{5}\right)}{2},\frac{\frac{7}{15}+\left(-\frac{4}{15}\right)}{2}\right)=\left(\frac{-\frac{4}{5}}{2},\frac{\frac{3}{15}}{2}\right)$
$=\left(-\frac{4}{5}\cdot\frac{1}{2},\frac{3}{15}\cdot\frac{1}{2}\right)=\left(-\frac{2}{5},\frac{1}{10}\right)$

27. $\left(\frac{8+(-6)}{2},\frac{3\sqrt{5}+7\sqrt{5}}{2}\right)$
$=\left(\frac{2}{2},\frac{10\sqrt{5}}{2}\right)=\left(1,5\sqrt{5}\right)$

28. $\left(\frac{7\sqrt{3}+3\sqrt{3}}{2},\frac{-6+(-2)}{2}\right)=\left(\frac{10\sqrt{3}}{2},\frac{-8}{2}\right)$
$=\left(5\sqrt{3},-4\right)$

29. $\left(\frac{\sqrt{18}+\sqrt{2}}{2},\frac{-4+4}{2}\right)$
$=\left(\frac{3\sqrt{2}+\sqrt{2}}{2},\frac{0}{2}\right)=\left(\frac{4\sqrt{2}}{2},0\right)=(2\sqrt{2},0)$

30. $\left(\dfrac{\sqrt{50}+\sqrt{2}}{2}, \dfrac{-6+6}{2}\right) = \left(\dfrac{5\sqrt{2}+\sqrt{2}}{2}, \dfrac{0}{2}\right)$

$= \left(\dfrac{6\sqrt{2}}{2}, 0\right) = \left(3\sqrt{2}, 0\right)$

31. $(x-0)^2 + (y-0)^2 = 7^2$

$x^2 + y^2 = 49$

32. $(x-0)^2 + (y-0)^2 = 8^2$

$x^2 + y^2 = 64$

33. $(x-3)^2 + (y-2)^2 = 5^2$

$(x-3)^2 + (y-2)^2 = 25$

34. $(x-2)^2 + [y-(-1)]^2 = 4^2$

$(x-2)^2 + (y+1)^2 = 16$

35. $[x-(-1)]^2 + (y-4)^2 = 2^2$

$(x+1)^2 + (y-4)^2 = 4$

36. $[x-(-3)]^2 + (y-5)^2 = 3^2$

$(x+3)^2 + (y-5)^2 = 9$

37. $[x-(-3)]^2 + [y-(-1)]^2 = \left(\sqrt{3}\right)^2$

$(x+3)^2 + (y+1)^2 = 3$

38. $[x-(-5)]^2 + [y-(-3)]^2 = \left(\sqrt{5}\right)^2$

$(x+5)^2 + (y+3)^2 = 5$

39. $[x-(-4)]^2 + (y-0)^2 = 10^2$

$(x+4)^2 + (y-0)^2 = 100$

40. $[x-(-2)]^2 + (y-0)^2 = 6^2$

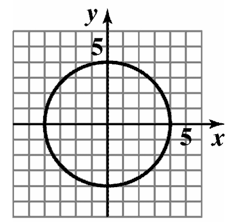
$(x+2)^2 + y^2 = 36$

41. $x^2 + y^2 = 16$

$(x-0)^2 + (y-0)^2 = y^2$

$h = 0,\ k = 0,\ r = 4;$

center = (0, 0); radius = 4



$\boldsymbol{x^2 + y^2 = 16}$

domain: $[-4, 4]$

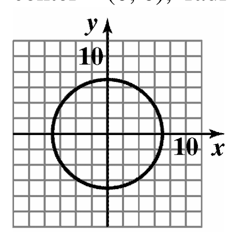
range: $[-4, 4]$

42. $x^2 + y^2 = 49$

$(x-0)^2 + (y-0)^2 = 7^2$

$h = 0, k = 0, r = 7;$

center = (0, 0); radius = 7



$\boldsymbol{x^2 + y^2 = 49}$

domain: $[-7, 7]$

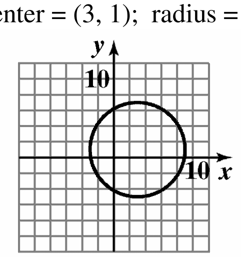
range: $[-7, 7]$

43. $(x-3)^2 + (y-1)^2 = 36$

$(x-3)^2 + (y-1)^2 = 6^2$

$h = 3,\ k = 1,\ r = 6;$

center = (3, 1); radius = 6



$\boldsymbol{(x-3)^2 + (y-1)^2 = 36}$

domain: $[-3, 9]$

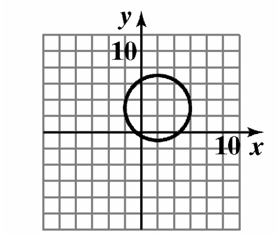
range: $[-5, 7]$

44. $(x-2)^2+(y-3)^2=16$

$(x-2)^2+(y-3)^2=4^2$

$h=2, k=3, r=4;$

center = (2, 3); radius = 4



$\mathbf{(x-2)^2+(y-3)^2=16}$

domain: $[-2,6]$

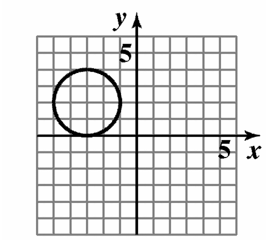
range: $[-1,7]$

45. $(x+3)^2+(y-2)^2=4$

$[x-(-3)]^2+(y-2)^2=2^2$

$h=-3,\ k=2,\ r=2$

center = (–3, 2); radius = 2



$\mathbf{(x+3)^2+(y-2)^2=4}$

domain: $[-5,-1]$

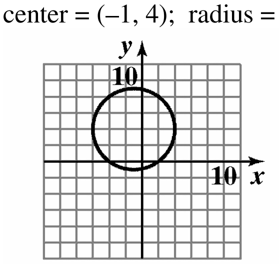
range: $[0,4]$

46. $(x+1)^2+(y-4)^2=25$

$[x-(-1)]^2+(y-4)^2=5^2$

$h=-1, k=4, r=5;$

center = (–1, 4); radius = 5



$\mathbf{(x+1)^2+(y-4)^2=25}$

domain: $[-6,4]$

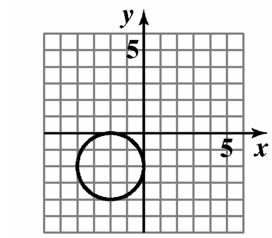
range: $[-1,9]$

47. $(x+2)^2+(y+2)^2=4$

$[x-(-2)]^2+[y-(-2)]^2=2^2$

$h=-2,\ k=-2,\ r=2$

center = (–2, –2); radius = 2



$\mathbf{(x+2)^2+(y+2)^2=4}$

domain: $[-4,0]$

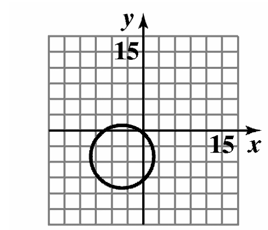
range: $[-4,0]$

48. $(x+4)^2+(y+5)^2=36$

$[x-(-4)]^2+[y-(-5)]^2=6^2$

$h=-4, k=-5, r=6;$

center = (–4, –5); radius = 6



$\mathbf{(x+4)^2+(y+5)^2=36}$

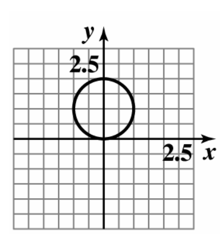
domain: $[-10,2]$

range: $[-11,1]$

49. $x^2+(y-1)^2=1$

$h=0, k=1, r=1;$

center = (0, 1); radius = 1



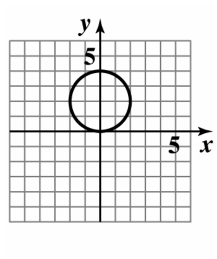
domain: $[-1,1]$

range: $[0,2]$

50. $x^2 + (y-2)^2 = 4$

$h = 0, k = 2, r = 2;$

center = (0,2); radius = 2



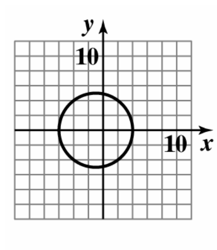
domain: $[-2, 2]$

range: $[0, 4]$

51. $(x+1)^2 + y^2 = 25$

$h = -1, k = 0, r = 5;$

center = (–1,0); radius = 5



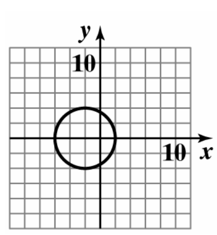
domain: $[-6, 4]$

range: $[-5, 5]$

52. $(x+2)^2 + y^2 = 16$

$h = -2, k = 0, r = 4;$

center = (–2,0); radius = 4

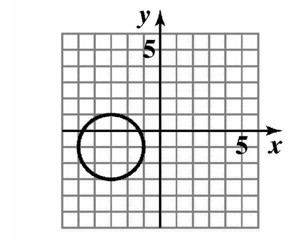


domain: $[-6, 2]$

range: $[-4, 4]$

53.
$$x^2 + y^2 + 6x + 2y + 6 = 0$$
$$(x^2 + 6x) + (y^2 + 2y) = -6$$
$$(x^2 + 6x + 9) + (y^2 + 2y + 1) = 9 + 1 - 6$$
$$(x+3)^2 + (y+1)^2 = 4$$
$$[x - (-3)]^2 + [9 - (-1)]^2 = 2^2$$

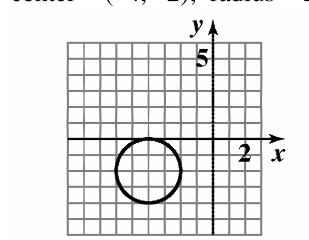
center = (–3, –1); radius = 2



$\boldsymbol{x^2 + y^2 + 6x + 2y + 6 = 0}$

54. $x^2 + y^2 + 8x + 4y + 16 = 0$
$$(x^2 + 8x) + (y^2 + 4y) = -16$$
$$(x^2 + 8x + 16) + (y^2 + 4y + 4) = 20 - 16$$
$$(x+4)^2 + (y+2)^2 = 4$$
$$[x - (-4)]^2 + [y - (-2)]^2 = 2^2$$

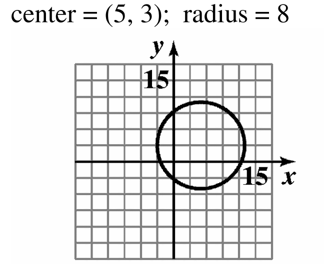
center = (–4, –2); radius = 2



$\boldsymbol{x^2 + y^2 + 8x + 4y + 16 = 0}$

55.
$$x^2 + y^2 - 10x - 6y - 30 = 0$$
$$(x^2 - 10x) + (y^2 - 6y) = 30$$
$$(x^2 - 10x + 25) + (y^2 - 6y + 9) = 25 + 9 + 30$$
$$(x-5)^2 + (y-3)^2 = 64$$
$$(x-5)^2 + (y-3)^2 = 8^2$$

center = (5, 3); radius = 8

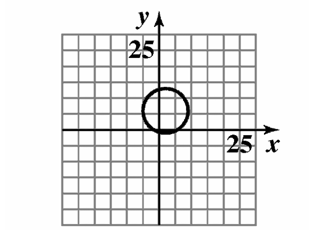


$\boldsymbol{x^2 + y^2 - 10x - 6y - 30 = 0}$

56. $x^2 + y^2 - 4x - 12y - 9 = 0$

$$\left(x^2 - 4x\right) + \left(y^2 - 12y\right) = 9$$
$$\left(x^2 - 4x + 4\right) + \left(y^2 - 12y + 36\right) = 4 + 36 + 9$$
$$\left(x - 2\right)^2 + \left(y - 6\right)^2 = 49$$
$$(x - 2)^2 + (y - 6)^2 = 7^2$$

center = (2, 6); radius = 7

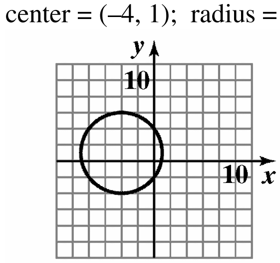


$\mathbf{x^2 + y^2 - 4x - 12y - 9 = 0}$

57. $x^2 + y^2 + 8x - 2y - 8 = 0$

$$\left(x^2 + 8x\right) + \left(y^2 - 2y\right) = 8$$
$$\left(x^2 + 8x + 16\right) + \left(y^2 - 2y + 1\right) = 16 + 1 + 8$$
$$\left(x + 4\right)^2 + \left(y - 1\right)^2 = 25$$
$$\left[x - (-4)\right]^2 + (y - 1)^2 = 5^2$$

center = (–4, 1); radius = 5

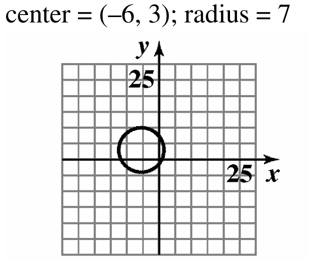


$\mathbf{x^2 + y^2 + 8x - 2y - 8 = 0}$

58. $x^2 + y^2 + 12x - 6y - 4 = 0$

$$\left(x^2 + 12x\right) + \left(y^2 - 6y\right) = 4$$
$$\left(x^2 + 12x + 36\right) + \left(y^2 - 6y + 9\right) = 36 + 9 + 4$$
$$\left[x - (-6)\right]^2 + \left(y - 3\right)^2 = 7^2$$

center = (–6, 3); radius = 7

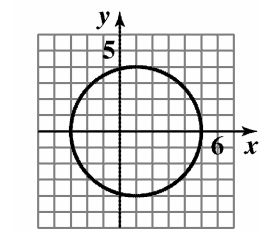


$\mathbf{x^2 + y^2 + 12x - 6y - 4 = 0}$

59. $x^2 - 2x + y^2 - 15 = 0$

$$\left(x^2 - 2x\right) + y^2 = 15$$
$$\left(x^2 - 2x + 1\right) + \left(y - 0\right)^2 = 1 + 0 + 15$$
$$\left(x - 1\right)^2 + \left(y - 0\right)^2 = 16$$
$$\left(x - 1\right)^2 + \left(y - 0\right)^2 = 4^2$$

center = (1, 0); radius = 4

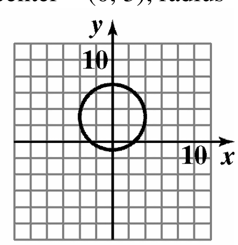


$\mathbf{x^2 - 2x + y^2 - 15 = 0}$

60. $x^2 + y^2 - 6y - 7 = 0$

$$x^2 + \left(y^2 - 6y\right) = 7$$
$$\left(x - 0\right)^2 = \left(y^2 - 6y + 9\right) = 0 + 9 + 7$$
$$\left(x - 0\right)^2 + \left(y - 3\right)^2 = 16$$
$$(x - 0)^2 + (y - 3)^2 = 4^2$$

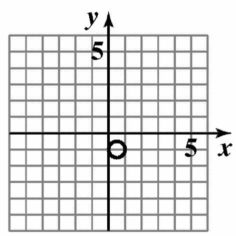
center = (0, 3); radius = 4



$\mathbf{x^2 + y^2 - 6y - 7 = 0}$

61.
$$\begin{aligned} x^2+y^2-x+2y+1&=0\\ x^2-x \quad +y^2+2y \quad &=-1\\ x^2-x+\frac{1}{4}+y^2+2y+1&=-1+\frac{1}{4}+1\\ \left(x-\frac{1}{2}\right)^2+(y+1)^2&=\frac{1}{4} \end{aligned}$$

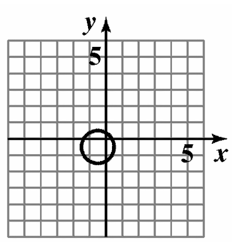
center = $\left(\frac{1}{2},-1\right)$; radius = $\frac{1}{2}$



$x^2 + y^2 - x + 2y + 1 = 0$

62.
$$\begin{aligned} x^2+y^2+x+y-\frac{1}{2}&=0\\ x^2+x \quad +y^2+y \quad &=\frac{1}{2}\\ x^2+x+\frac{1}{4}+y^2+y+\frac{1}{4}&=\frac{1}{2}+\frac{1}{4}+\frac{1}{4}\\ \left(x-\frac{1}{2}\right)^2+\left(y-\frac{1}{2}\right)^2&=1 \end{aligned}$$

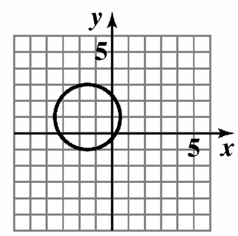
center = $\left(\frac{1}{2},\frac{1}{2}\right)$; radius = 1



$x^2 + y^2 + x + y - \frac{1}{2} = 0$

63.
$$\begin{aligned} x^2+y^2+3x-2y-1&=0\\ x^2+3x \quad +y^2-2y \quad &=1\\ x^2+3x+\frac{9}{4}+y^2-2y+1&=1+\frac{9}{4}+1\\ \left(x+\frac{3}{2}\right)^2+(y-1)^2&=\frac{17}{4} \end{aligned}$$

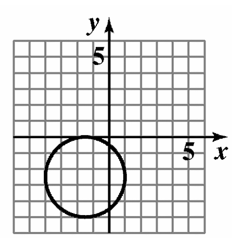
center = $\left(-\frac{3}{2},1\right)$; radius = $\frac{\sqrt{17}}{2}$



$x^2 + y^2 + 3x - 2y - 1 = 0$

64.
$$\begin{aligned} x^2+y^2+3x+5y+\frac{9}{4}&=0\\ x^2+3x \quad +y^2+5y \quad &=-\frac{9}{4}\\ x^2+3x+\frac{9}{4}+y^2+5y+\frac{25}{4}&=-\frac{9}{4}+\frac{9}{4}+\frac{25}{4}\\ \left(x+\frac{3}{2}\right)^2+\left(y+\frac{5}{2}\right)^2&=\frac{25}{4} \end{aligned}$$

center = $\left(-\frac{3}{2},-\frac{5}{2}\right)$; radius = $\frac{5}{2}$



$x^2 + y^2 + 3x + 5y + \frac{9}{4} = 0$

65. a. Since the line segment passes through the center, the center is the midpoint of the segment.

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$
$$= \left(\frac{3+7}{2}, \frac{9+11}{2}\right) = \left(\frac{10}{2}, \frac{20}{2}\right)$$
$$= (5,10)$$

The center is $(5,10)$.

b. The radius is the distance from the center to one of the points on the circle. Using the point $(3,9)$, we get:

$$d = \sqrt{(5-3)^2+(10-9)^2}$$
$$= \sqrt{2^2+1^2} = \sqrt{4+1}$$
$$= \sqrt{5}$$

The radius is $\sqrt{5}$ units.

c.
$$(x-5)^2+(y-10)^2 = \left(\sqrt{5}\right)^2$$
$$(x-5)^2+(y-10)^2 = 5$$

66. a. Since the line segment passes through the center, the center is the midpoint of the segment.

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$
$$= \left(\frac{3+5}{2}, \frac{6+4}{2}\right) = \left(\frac{8}{2}, \frac{10}{2}\right)$$
$$= (4,5)$$

The center is $(4,5)$.

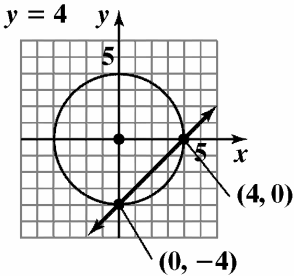
b. The radius is the distance from the center to one of the points on the circle. Using the point $(3,6)$, we get:

$$d = \sqrt{(4-3)^2+(5-6)^2}$$
$$= \sqrt{1^2+(-1)^2} = \sqrt{1+1}$$
$$= \sqrt{2}$$

The radius is $\sqrt{2}$ units.

c.
$$(x-4)^2+(y-5)^2 = \left(\sqrt{2}\right)^2$$
$$(x-4)^2+(y-5)^2 = 2$$

67. $x^2 + y^2 = 16$
$x - y = 4$



Intersection points: $(0,-4)$ and $(4,0)$

Check $(0,-4)$:

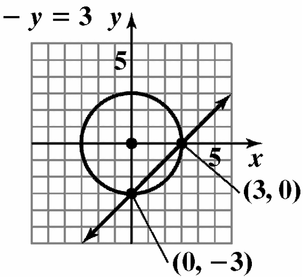
$0^2+(-4)^2=16$ $\quad$ $0-(-4)=4$
$16=16$ true $\quad$ $4=4$ true

Check $(4,0)$:

$4^2+0^2=16$ $\quad$ $4-0=4$
$16=16$ true $\quad$ $4=4$ true

The solution set is $\{(0,-4),(4,0)\}$.

68. $x^2 + y^2 = 9$
$x - y = 3$



Intersection points: $(0,-3)$ and $(3,0)$

Check $(0,-3)$:

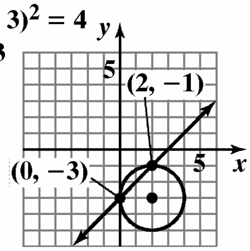
$0^2+(-3)^2=9$ $\quad$ $0-(-3)=3$
$9=9$ true $\quad$ $3=3$ true

Check $(3,0)$:

$3^2+0^2=9$ $\quad$ $3-0=3$
$9=9$ true $\quad$ $3=3$ true

The solution set is $\{(0,-3),(3,0)\}$.

69. $(x-2)^2 + (y+3)^2 = 4$
$y = x - 3$



Intersection points: $(0,-3)$ and $(2,-1)$

Check $(0,-3)$:

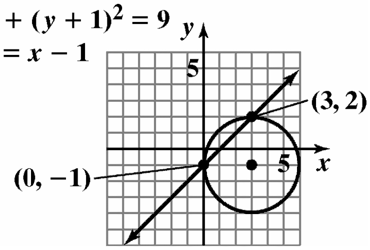
$$\begin{aligned}(0-2)^2+(-3+3)^2&=9 \qquad -3=0-3\\(-2)^2+0^2&=4 \qquad -3=-3 \text{ true}\\4&=4\\&\text{true}\end{aligned}$$

Check $(2,-1)$:

$$\begin{aligned}(2-2)^2+(-1+3)^2&=4 \qquad -1=2-3\\0^2+2^2&=4 \qquad -1=-1 \text{ true}\\4&=4\\&\text{true}\end{aligned}$$

The solution set is $\{(0,-3),(2,-1)\}$.

70. $(x-3)^2 + (y+1)^2 = 9$
$y = x - 1$



Intersection points: $(0,-1)$ and $(3,2)$

Check $(0,-1)$:

$$\begin{aligned}(0-3)^2+(-1+1)^2&=9 \qquad -1=0-1\\(-3)^2+0^2&=9 \qquad -1=-1 \text{ true}\\9&=9\\&\text{true}\end{aligned}$$

Check $(3,2)$:

$$\begin{aligned}(3-3)^2+(2+1)^2&=9 \qquad 2=3-1\\0^2+3^2&=9 \qquad 2=2 \text{ true}\\9&=9\\&\text{true}\end{aligned}$$

The solution set is $\{(0,-1),(3,2)\}$.

71. $d=\sqrt{(8495-4422)^2+(8720-1241)^2}\cdot\sqrt{0.1}$
$d=\sqrt{72{,}524{,}770}\cdot\sqrt{0.1}$
$d\approx 2693$
The distance between Boston and San Francisco is about 2693 miles.

72. $d=\sqrt{(8936-8448)^2+(3542-2625)^2}\cdot\sqrt{0.1}$
$d=\sqrt{1{,}079{,}033}\cdot\sqrt{0.1}$
$d\approx 328$
The distance between New Orleans and Houston is about 328 miles.

73. If we place L.A. at the origin, then we want the equation of a circle with center at $(-2.4,-2.7)$ and radius 30.

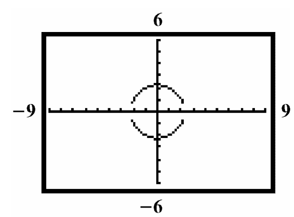
$$\begin{aligned}(x-(-2.4))^2+(y-(-2.7))^2&=30^2\\(x+2.4)^2+(y+2.7)^2&=900\end{aligned}$$

74. $C(0, 68 + 14) = (0, 82)$

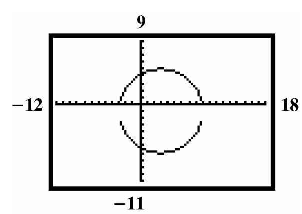
$$\begin{aligned}(x-0)^2+(y-82)^2&=68^2\\x^2+(y-82)^2&=4624\end{aligned}$$

75. – 82. Answers may vary.

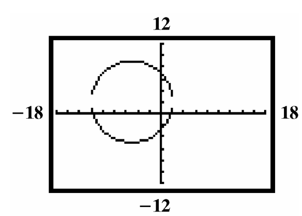
83.



84.



85.



86. makes sense

87. makes sense

88. does not make sense; Explanations will vary. Sample explanation: Since $r^2 = -4$ this is not the equation of a circle.

89. makes sense

90. false; Changes to make the statement true will vary. A sample change is: The equation would be $x^2 + y^2 = 256$.

91. false; Changes to make the statement true will vary. A sample change is: The center is at (3, –5).

92. false; Changes to make the statement true will vary. A sample change is: This is not an equation for a circle.

93. false; Changes to make the statement true will vary. A sample change is: Since $r^2 = -36$ this is not the equation of a circle.

94. The distance for A to B:

$$\overline{AB} = \sqrt{(3-1)^2 + [3+d-(1+d)]^2}$$
$$= \sqrt{2^2 + 2^2}$$
$$= \sqrt{4+4}$$
$$= \sqrt{8}$$
$$= 2\sqrt{2}$$

The distance from B to C:

$$\overline{BC} = \sqrt{(6-3)^2 + [3+d-(6+d)]^2}$$
$$= \sqrt{3^2 + (-3)^2}$$
$$= \sqrt{9+9}$$
$$= \sqrt{18}$$
$$= 3\sqrt{2}$$

The distance for A to C:

$$\overline{AC} = \sqrt{(6-1)^2 + [6+d-(1+d)]^2}$$
$$= \sqrt{5^2 + 5^2}$$
$$= \sqrt{25+25}$$
$$= \sqrt{50}$$
$$= 5\sqrt{2}$$
$$\overline{AB} + \overline{BC} = \overline{AC}$$
$$2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2}$$
$$5\sqrt{2} = 5\sqrt{2}$$

95. a. d_1 is distance from (x_1, x_2) to midpoint

$$d_1 = \sqrt{\left(\frac{x_1+x_2}{2} - x_1\right)^2 + \left(\frac{y_1+y_2}{2} - y_1\right)^2}$$
$$d_1 = \sqrt{\left(\frac{x_1+x_2-2x_1}{2}\right)^2 + \left(\frac{y_1+y_2-2y_1}{2}\right)^2}$$
$$d_1 = \sqrt{\left(\frac{x_2-x_1}{2}\right)^2 + \left(\frac{y_2-y_1}{2}\right)^2}$$
$$d_1 = \sqrt{\frac{x_2-2x_1x_2+x_1^2}{4} + \frac{y_2^2-2y_2y_1+y_1^2}{4}}$$
$$d_1 = \sqrt{\frac{1}{4}\left(x_2-2x_1x_2+x_1+y_2^2-2y_2y_1+y_1^2\right)}$$
$$d_1 = \frac{1}{2}\sqrt{x_2-2x_1x_2+x_1+y_2^2-2y_2y_1+y_1^2}$$

d_2 is distance from midpoint to (x_2, y_2)

$$d_2 = \sqrt{\left(\frac{x_1+x_2}{2} - x_2\right)^2 + \left(\frac{y_1+y_2}{2} - y_2\right)^2}$$
$$d_2 = \sqrt{\left(\frac{x_1+x_2-2x_2}{2}\right)^2 + \left(\frac{y_1+y_2-2y_2}{2}\right)^2}$$
$$d_2 = \sqrt{\left(\frac{x_1-x_2}{2}\right)^2 + \left(\frac{y_1-y_2}{2}\right)^2}$$
$$d_2 = \sqrt{\frac{x_1^2-2x_1x_2+x_2^2}{4} + \frac{y_1^2-2y_2y_1+y_2^2}{4}}$$
$$d_2 = \sqrt{\frac{1}{4}\left(x_1^2-2x_1x_2+x_2^2+y_1^2-2y_2y_1+y_2^2\right)}$$
$$d_2 = \frac{1}{2}\sqrt{x_1^2-2x_1x_2+x_2^2+y_1^2-2y_2y_1+y_2^2}$$
$$d_1 = d_2$$

b. d_3 is the distance from (x_1, y_1) to $(x_2 y_2)$

$$d_3 = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$
$$d_3 = \sqrt{x_2^2-2x_1x_2+x_1^2+y_2^2-2y_2y_1+y_1^2}$$
$$d_1 + d_2 = d_3 \text{ because } \frac{1}{2}\sqrt{a} + \frac{1}{2}\sqrt{a} = \sqrt{a}$$

96. Both circles have center (2, –3). The smaller circle has radius 5 and the larger circle has radius 6. The smaller circle is inside of the larger circle. The area between them is given by

$$\pi(6)^2 - \pi(5)^2 = 36\pi - 25\pi$$
$$= 11\pi$$
$$\approx 34.56 \text{ square units.}$$

97. The circle is centered at (0,0). The slope of the radius with endpoints (0,0) and (3,–4) is $m = -\frac{-4-0}{3-0} = -\frac{4}{3}$. The line perpendicular to the radius has slope $\frac{3}{4}$. The tangent line has slope $\frac{3}{4}$ and passes through (3,–4), so its equation is:

$$y + 4 = \frac{3}{4}(x-3).$$

98. $x - 200$

99. **a.** $p = 2l + 2w = 2(40) + 2(30) = 140$
$A = lw = (40)(30) = 1200$
The perimeter is 140 yd; the area is 1200 sq yd

b. $p = 2l + 2w = 2(50) + 2(20) = 140$
$A = lw = (50)(20) = 1000$
The perimeter is 140 yd; the area is 1000 sq yd

100. $\pi r^2 h = 22$

$$h = \frac{22}{\pi r^2}$$

$$2\pi r^2 + 2\pi r h = 2\pi r^2 + 2\pi r\left(\frac{22}{\pi r^2}\right) = 2\pi r^2 + \frac{44}{r}$$

Section 1.10

Check Point Exercises

1. **a.** $f(x) = 15 + 0.08x$

b. $g(x) = 3 + 0.12x$

c. $15 + 0.08x = 3 + 0.12x$
$12 = 0.04x$
$300 = x$
The plans cost the same for 300 minutes.

2. **a.** $N(x) = 8000 - 100(x - 100)$
$= 8000 - 100x + 10000$
$= 18{,}000 - 100x$

b. $R(x) = (18{,}000 - 100x)x$
$= -100x^2 + 18{,}000x$

3. $V(x) = (15 - 2x)(8 - 2x)x$
$= (120 - 46x + 4x^2)x$
$= 4x^3 - 46x^2 + 120x$
Since x represents the inches to be cut off, $x > 0$. The smallest side is 8, so must cut less than 4 off each side. The domain of V is $\{x \mid 0 < x < 4\}$ or, in interval notation, $(0, 4)$.

4. $2l + 2w = 200$
$2l = 200 - 2w$
$l = 100 - w$
Let x = width, then length = $100 - x$
$A(x) = x(100 - x)$
$= 100x - x^2$

5. $V = \pi r^2 h$

$$1000 = \pi r^2 h$$

$$\frac{1000}{\pi r^2} = h$$

$$A = 2\pi r^2 + 2\pi r h$$

$$= 2\pi r^2 + 2\pi r \frac{1000}{\pi r^2}$$

$$= 2\pi r^2 + \frac{2000}{r}$$

6. $I(x) = 0.07x + 0.09(25{,}000 - x)$

7. $d = \sqrt{(x-0)^2 + (y-0)^2}$

$$= \sqrt{x^2 + y^2}$$

$$y = x^3$$

$$d = \sqrt{x^2 + \left(x^3\right)^2}$$

$$= \sqrt{x^2 + x^6}$$

Exercise Set 1.10

1. **a.** $f(x) = 200 + 0.15x$

b. $320 = 200 + 0.15x$
$120 = 01.5x$
$800 = x$
800 miles

2. **a.** $f(x) = 180 + 0.25x$

b. $395 = 180 + 0.25x$
$215 = 0.25x$
$860 = x$
You drove 860 miles for $395.

3. **a.** $M(x) = 239.4 - 0.3x$

b. $180 = 239.4 - 0.3x$
$0.3x = 59.4$
$x = 198$
198 years after 1954, in 2152, someone will run a 3 minute mile.

4. **a.** $P(x) = 28 + 0.6x$

b. $40 = 28 + 06x$
$12 = 0.6x$
$20 = x$
20 years after 1990, in 2010, 40% of babies born will be out of wedlock.

5. **a.** $f(x) = 1.25x$

b. $g(x) = 21 + 0.5x$

c. $1.25x = 21 + 0.5x$
$0.75x = 21$
$x = 28$
$f(28) = 1.25(28) = 35$
$g(28) = 21 + 0.5(28) = 35$
If a person crosses the bridge 28 times the cost will be $35 for both options

6. **a.** $f(x) = 2.5x$

b. $g(x) = 21 + x$

c. $2.5x = 21 + x$
$1.5x = 21$
$x = 14$

$f(14) = 2.5(14) = 35$
$g(14) = 21 + 14 = 35$
To cross the bridge 14 times costs the same, $35, for either method.

7. **a.** $f(x) = 100 + 0.8x$

b. $g(x) = 40 + 0.9x$

c. $100 + 0.8x = 40 + 0.9x$
$60 = 0.1x$
$600 = x$
For $600 worth of merchandise, your cost is $580 for both plans

8. **a.** $f(x) = 300 + 0.7x$

b. $g(x) = 40 + 0.9x$

c. $300 + 0.7x = 40 + 0.9x$
$260 = 0.2x$
$1300 = x$

$f(1300) = 300 + 0.7(1300) = 1210$
$g(1300) = 40 + 0.9(1300) = 1210$
You would have to purchase $1300 in merchandise at a total cost of $1210.

9. **a.**
$$\begin{aligned} N(x) &= 30,000 - 500(x - 20) \\ &= 30,000 - 500x + 10000 \\ &= 40,000 - 500x \end{aligned}$$

b.
$$\begin{aligned} R(x) &= (40,000 - 500x)x \\ &= -500x^2 + 40,000x \end{aligned}$$

10. **a.**
$$\begin{aligned} N(x) &= 20,000 - 400(x - 15) \\ &= 20,000 - 400x + 6000 \\ &= 26,000 - 400x \end{aligned}$$

b.
$$\begin{aligned} R(x) &= (26,000 - 400x)x \\ &= -400x^2 + 26,000x \end{aligned}$$

11. a. $N(x) = 9000 + 50(150 - x)$
$= 9000 - 50x + 7500$
$= 16500 - 50x$

b. $R(x) = (16500 - 50x)x$
$= -50x^2 + 16500x$

12. a. $N(x) = 7,000 + 60(90 - x)$
$= 7000 - 60x + 5400$
$= 12400 - 60x$

b. $R(x) = (12400 - 60x)x$
$= -60x^2 + 12400x$

13. a. $Y(x) = 320 - 4(x - 50)$
$= 320 - 4x + 200$
$= 520 - 4x$

b. $T(x) = (520 - 4x)x$
$= -4x^2 + 520x$

14. a. $Y(x) = 270 - 3(x - 30)$
$= 270 - 3x + 90$
$= 360 - 3x$

b. $T(x) = (360 - 3x)x$
$= -3x^2 + 360x$

15. a. $V(x) = (24 - 2x)(24 - 2x)x$
$= (576 - 96x + 4x^2)x$
$= 4x^3 - 96x^2 + 576x$

b. $V(2) = 4(2)^3 - 96(2)^2 + 576(2) = 800$ If 2-inch squares are cut off each corner, the volume will be 800 square inches.

$V(3) = 4(3)^3 - 96(3)^2 + 576(3) = 972$ If 3-inch squares are cut off each corner, the volume will be 972 square inches.

$V(4) = 4(4)^3 - 96(4)^2 + 576(4) = 1024$ If 4-inch squares are cut off each corner, the volume will be 1024 square inches.

$V(5) = 4(5)^3 - 96(5)^2 + 576(5) = 980$ If 5-inch squares are cut off each corner, the volume will be 980 square inches.

$V(6) = 4(6)^3 - 96(6)^2 + 576(6) = 864$ If 6-inch squares are cut off each corner, the volume will be 864 square inches.

c. If x is the inches to be cut off, $x > 0$. Since each side is 24, you must cut less than 12 inches off each end.
$0 < x < 12$

16. a. $V(x) = (30 - 2x)(30 - 2x)x$
$= (900 - 120x + 4x^2)x$
$= 4x^3 - 120x^2 + 900x$

b. $V(3) = 4(3^3) - 120(3^2) + 900(3) = 1728$
If 3 inches are cut from each side, the volume will be 1728 square inches.

$V(4) = 4(4^3) - 120(4^2) + 900(4) = 1936$
If 4 inches are cut from each side, the volume will be 1936 square inches.

$V(5) = 4(5^3) - 120(5^2) + 900(5) = 2000$
If 5 inches are cut from each side, the volume will be 2000 square inches.

$V(6) = 4(6^3) - 120(6^2) + 900(6) = 1944$
If 6 inches are cut from each side, the volume will be 1944 square inches.

$V(7) = 4(7^3) - 120(7^2) + 900(7) = 1792$
If 7 inches are cut from each side, the volume will be 1792 square inches.

c. Since x is the number of inches to be cut from each side, $x > 0$. Since each side is 30 inches, you must cut less than 15 inches from each side.
$0 < x < 15$ or $(0, 15)$

17. $A(x) = x(20 - 2x)$
$= -2x^2 + 20x$

18.
$$A(x) = \left(\frac{x}{4}\right)^2 + \left(\frac{8-x}{4}\right)^2$$
$$= \frac{x^2}{16} + \frac{64 - 16x + x^2}{16}$$
$$= \frac{2x^2 - 16x + 64}{16}$$
$$= \frac{x^2 - 8x + 32}{8}$$

19. $P(x) = x(66 - x)$
$= -x^2 + 66x$

20. $P(x) = x(50 - x)$
$= -x^2 + 50x$

21. $A(x) = x(400 - x)$
$= -x^2 + 400x$

22. $A(x) = x(300 - x)$
$= -x^2 + 300x$

23. $2w + l = 800$
$l = 800 - 2w$
Let $x = w$
$A(x) = x(800 - 2x)$
$= -2x^2 + 800x$

24. $2w + l = 600$
$l = 600 - 2l$
let $x =$ width, $600 - 2x =$ length

$A(x) = (600 - 2x)x$
$= -2x^2 + 600x$

25. $2x + 3y = 1000$
$3y = 1000 - 2x$
$y = \frac{1000 - 2x}{3}$
$A(x) = x\left(\frac{1000 - 2x}{3}\right)$
$= \frac{x(1000 - 2x)}{3}$

26. $2x + 4y = 1200$
$4y = 1200 - 2x$
$y = \frac{1200 - 2x}{4}$

$A(x) = x\ \frac{1200 - 2x}{4}$
$= \frac{x(1200 - 2x)}{4}$
$= \frac{2x(600 - x)}{4}$
$= \frac{x(600 - x)}{2}$

27. $2x =$ distance around 2 straight sides
$\pi 2r =$ distance around 2 curved sides

$2x + 2\pi r = 440$
$2x = 440 - 2\pi r$
$x = 220 - \pi r$

$A(r) = (220 - \pi r)2r + \pi r^2$
$= 440r - 2\pi r^2 + \pi r^2$
$= 440r - \pi r^2$

28. $2x$ = distance around the 2 straight sides
$2\pi r$ = distance around the 2 curved sides
$2x + 2\pi r = 880$
$2x = 880 - 2\pi r$
$x = 440 - \pi r$

$A(x) = r(440 - \pi r) + \pi r^2$
$= 440r - \pi r^2 + \pi r^2$
$= 440r$

29. $xy = 4000$
$y = \frac{4000}{x}$

$C(x) = \left[2x + 2\left(\frac{4000}{x}\right)\right]175 + 125x$
$= 350x + \frac{1,400,000}{x} + 125x$
$= 475x + \frac{1,400,000}{x}$

30. $125 = lw$

$\frac{125}{l} = w;$ let $x = l$

$C(x) = 20\ \ 2\ \frac{125}{x}\ \ + x\ \ + 9x$

$= \frac{5000}{x} + 20x + 9x$

$= \frac{5000}{x} + 29x$

31. $10 = x^2 y$

$\frac{10}{x^2} = y$

$A(x) = x^2 + 4\left(x \cdot \frac{10}{x^2}\right)$

$= x^2 + \frac{40}{x}$

32. $400 = x^2 y$

$\frac{400}{x^2} = y$

$A = x^2 + 5\left(\frac{400}{x^2}\right)x$

$= x^2 + \frac{2000}{x}$

33. $300 = y + 4x$

$300 - 4x = y^2$

$A(x) = x^2(300 - 4x)$

$= -4x^3 + 300x^2$

34. $108 = y + 4x$

$108 - 4x = y$

$A = x^2(108 - 4x)$

$= -4x^3 + 108x^2$

35. **a.** Let x = amount invested at 15%
$50000 - x$ = amount invested at 7%
$I(x) = 0.15x + 0.07(50000 - x)$

b. $6000 = 0.15x + 0.07x(50000 - x)$

$6000 = 0.15x + 3500 - 0.07x$

$2500 = 0.08x$

$31250 = x$

$50000 - 31250 = 18750$

Invest \$31,250 at 15% and \$18,750 at 7%.

36. **a.** Let x = amount at 10%
$18{,}750 - x$ = amount at 12%
$I(x) = 0.10x + 0.12(18750 - x)$

b. $0.10x + 0.12(18750 - x) = 2117$

$0.1x + 2250 - 0.12x = 2117$

$-0.02x = -133$

$x = 6650$

The amount of money to be invested should be \$6650 at 10% and \$12100 at 12%.

37. Let x = amount invested at 12%
$8000 - x$ = amount invested at 5% loss
$I(x) = 0.12x - 0.05(8000 - x)$

38. Let x = amount at 14%
$12000 - x$ = amount at 6%
$I(x) = 0.14x + 0.06(12000 - x)$

$= 0.14x + 720 - 0.06x$

$= 0.08x + 720$

39. $d = \sqrt{(x-0)^2 + (y-0)^2}$

$= \sqrt{x^2 + y^2}$

$= \sqrt{x^2 + \left(x^2 - 4\right)^2}$

$= \sqrt{x^2 + x^4 - 8x^2 + 16}$

$= \sqrt{x^4 - 7x^2 + 16}$

40. $d = \sqrt{(x-0)^2 + (y-0)^2}$

$= \sqrt{x^2 + y^2}$

$= \sqrt{x^2 + \left(x^2 - 8\right)^2}$

$= \sqrt{x^2 + x^4 - 16x^2 + 64}$

$= \sqrt{x^4 - 15x^2 + 64}$

41. $$d = \sqrt{(x-1)^2 + y^2} = \sqrt{x^2 - 2x + 1 + \left(\sqrt{x}\right)^2} = \sqrt{x^2 - 2x + 1 + x} = \sqrt{x^2 - x + 1}$$

42. $$d = \sqrt{(x-2)^2 + y^2} = \sqrt{x^2 - 4x + 4 + \left(\sqrt{x}\right)^2} = \sqrt{x^2 - 3x + 4}$$

43. **a.** $A(x) = 2xy = 2x\sqrt{4 - x^2}$

b. $P(x) = 2(2x) + 2y = 4x + 2\sqrt{4 - x^2}$

44. **a.** $A(x) = 2xy = 2x\sqrt{9 - x^2}$

b. $P(x) = 2(2x) + 2y = 4x + 2\sqrt{9 - x^2}$

45. 6-foot pole
$c^2 = 6^2 + x^2$
$x = \sqrt{36 + x^2}$
8-foot pole
$c^2 = 8^2 + (10 - x)^2$
$c = \sqrt{64 + 100 - 20x + x^2}$
$c = \sqrt{x^2 - 20x + 164}$
total length
$f(x) = \sqrt{36 + x^2} + \sqrt{x^2 - 20x + 164}$

46. Road from Town A:
$c^2 = 6^2 + x^2$
$c = \sqrt{36 + x^2}$

Road from Town B:
$c^2 = 3^2 + (12 - x)^2$
$c = \sqrt{9 + 144 - 24x + x^2}$
$c = \sqrt{x^2 - 24x + 153}$

$f(x) = \sqrt{36 + x^2} + \sqrt{x^2 - 24x + 153}$

47. $A(x) = \frac{1}{2}x(x-5) + \frac{1}{2}x(x+3) + (x+2)\left[(x-5) + (x+3)\right]$
$A(x) = \frac{1}{2}x^2 - \frac{5}{2}x + \frac{1}{2}x^2 + \frac{3}{2}x + (x+2)\left[2x - 2\right]$
$A(x) = x^2 - x + 2x^2 + 2x - 4$
$A(x) = 3x^2 + x - 4$

48. $A(x) = \frac{1}{2}x(2x) + \frac{1}{2}(6x - 4x)(x+2) + (4x)(x+2) + 2x(8)$
$A(x) = x^2 + x(x+2) + 4x^2 + 8x + 16x$
$A(x) = x^2 + x^2 + 2x + 4x^2 + 8x + 16x$
$A(x) = 6x^2 + 26$

49. $V(x) = (x+5)(2x+1)(x+2) - (x+5)(3)(x)$
$V(x) = (x+5)(2x^2 + 5x + 2) - 3x(x+5)$
$V(x) = 2x^3 + 15x^2 + 27x + 10 - 3x^2 - 15x$
$V(x) = 2x^3 + 12x^2 + 12x + 10$

50. $V(x) = (x)(2x-1)(x+3) - (x)(x)\left[(2x-1) - (x+1)\right]$
$V(x) = (x)(2x^2 + 5x - 3) - x^2(x-2)$
$V(x) = 2x^3 + 5x^2 - 3x - x^3 + 2x^2$
$V(x) = x^3 + 7x^2 - 3x$

51. – 62. Answers may vary.

63. does not make sense; Explanations will vary. Sample explanation: This model is not reasonable, as it suggests a per minute charge of \$30.

64. does not make sense; Explanations will vary. Sample explanation: The decrease in passengers is modeled by $60(x - 300)$.

65. does not make sense; Explanations will vary. Sample explanation: The area of a rectangle is not solely determined by its perimeter. For example: A 4 by 6 rectangle and a 3 by 7 rectangle both have perimeters of 20 units, yet their areas are different from each other.

66. makes sense

67. Distance and time rowed:

$$d^2 = 2^2 + x^2$$
$$d = \sqrt{4+x^2}$$
$$rt = d$$
$$2t = \sqrt{4+x^2}$$
$$t = \frac{\sqrt{4+x^2}}{2}$$

Distance and time walked:

$$d = 6-x$$
$$rt = d$$
$$5t = 6-x$$
$$t = \frac{6-x}{5}$$

Total time:

$$T(x) = \frac{\sqrt{4+x^2}}{2} + \frac{6-x}{5}$$

68.
$$\begin{aligned} A(x) &= (20+2x)(10+2x) - 10(20) \\ &= 4x^2 + 60x + 200 - 200 \\ &= 4x^2 + 60x \end{aligned}$$

69.
$$P = 2h + 2r + \frac{1}{2}(\pi 2r)$$
$$12 = 2h + 2r + \pi r$$
$$12 - 2r - \pi r = 2h$$
$$\frac{12 - 2r - \pi r}{2} = h$$

$$\begin{aligned} A &= \left(\frac{12-2r-\pi r}{2}\right)2r + \frac{1}{2}\left(\pi r^2\right) \\ &= 12r - 2r^2 - \pi r^2 + \frac{1}{2}\pi r^2 \\ &= 12r - 2r^2 - \frac{1}{2}\pi r^2 \end{aligned}$$

70.
$$r = \frac{1}{2}h$$

$$\begin{aligned} V(h) &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h \\ &= \frac{1}{3}\pi\frac{1}{4}h^2 h \\ &= \frac{\pi}{12}h^3 \end{aligned}$$

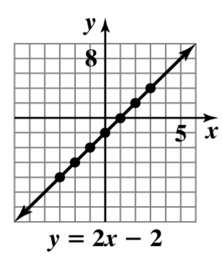
71.
$$\begin{aligned} (7-3x)(-2-5x) &= -14 - 35x + 6x + 15x^2 \\ &= -14 - 29x + 15x^2 \\ &\text{or} \\ &= 15x^2 - 29x - 14 \end{aligned}$$

72.
$$\begin{aligned} \sqrt{18} - \sqrt{8} &= \sqrt{9\cdot 2} - \sqrt{4\cdot 2} \\ &= 3\sqrt{2} - 2\sqrt{2} \\ &= \sqrt{2} \end{aligned}$$

73.
$$\begin{aligned} \frac{7+4\sqrt{2}}{2-5\sqrt{2}}\cdot\frac{2+5\sqrt{2}}{2+5\sqrt{2}} &= \frac{14+35\sqrt{2}+8\sqrt{2}+40}{4+10\sqrt{2}-10\sqrt{2}-50} \\ &= \frac{54+43\sqrt{2}}{-46} \\ &= -\frac{54+43\sqrt{2}}{46} \end{aligned}$$

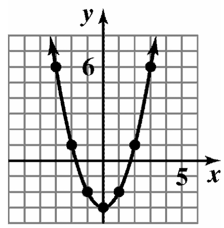
Chapter 1 Review Exercises

1.



$x = -3, y = -8$
$x = -2, y = -6$
$x = -1, y = -4$
$x = 0, y = -2$
$x = 1, y = 0$
$x = 2, y = 2$
$x = 3, y = 4$

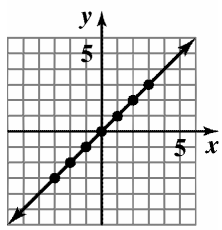
2.



$y = x^2 - 3$

$x = -3, y = 6$
$x = -2, y = 1$
$x = -1, y = -2$
$x = 0, y = -3$
$x = 1, y = -2$
$x = 2, y = 1$
$x = 3, y = 6$

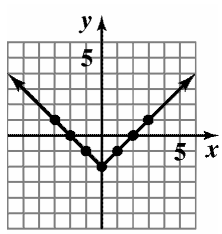
3.



$y = x$

$x = -3, y = -3$
$x = -2, y = -2$
$x = -1, y = -1$
$x = 0, y = 0$
$x = 1, y = 1$
$x = 2, y = 2$
$x = 3, y = 3$

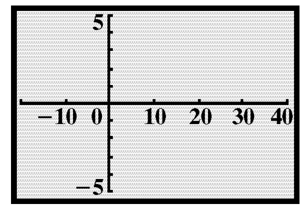
4.



$y = |x| - 2$

$x = -3, y = 1$
$x = -2, y = 0$
$x = -1, y = -1$
$x = 0, y = -2$
$x = 1, y = -1$
$x = 2, y = 0$
$x = 3, y = 1$

5. A portion of Cartesian coordinate plane with minimum x-value equal to –20, maximum x-value equal to 40, x-scale equal to 10 and with minimum y-value equal to –5, maximum y-value equal to 5, and y-scale equal to 1.



6. x-intercept: –2; The graph intersects the x-axis at (–2, 0).
y-intercept: 2; The graph intersects the y-axis at (0, 2).

7. x-intercepts: 2, –2; The graph intersects the x-axis at (–2, 0) and (2, 0).
y-intercept: –4; The graph intercepts the y-axis at (0, –4).

8. x-intercept: 5; The graph intersects the x-axis at (5, 0).
y-intercept: None; The graph does not intersect the y-axis.

9. The coordinates are (1985, 50%).

10. The top marginal tax rate in 2005 was 35%.

11. The highest marginal tax rate occurred in 1945 and was about 94%.

12. The lowest marginal tax rate occurred in 1990 and was about 28%.

13. During the ten-year period from 1950 to 1960, the top marginal tax rate remained constant at about 91%.

14. During the five-year period from 1930 to 1935, the top marginal tax rate increased about 38%.

15. function
domain: {2, 3, 5}
range: {7}

16. function
domain: {1, 2, 13}
range: {10, 500, π}

17. not a function
domain: {12, 14}
range: {13, 15, 19}

18. $2x + y = 8$
$$y = -2x + 8$$
Since only one value of y can be obtained for each value of x, y is a function of x.

19. $3x^2 + y = 14$
$$y = -3x^2 + 14$$
Since only one value of y can be obtained for each value of x, y is a function of x.

20. $2x + y^2 = 6$
$$y^2 = -2x + 6$$
$$y = \pm\sqrt{-2x+6}$$
Since more than one value of y can be obtained from some values of x, y is not a function of x.

21. $f(x) = 5 - 7x$

a. $f(4) = 5 - 7(4) = -23$

b.
$$\begin{aligned} f(x+3) &= 5 - 7(x+3) \\ &= 5 - 7x - 21 \\ &= -7x - 16 \end{aligned}$$

c. $f(-x) = 5 - 7(-x) = 5 + 7x$

22. $g(x) = 3x^2 - 5x + 2$

a. $g(0) = 3(0)^2 - 5(0) + 2 = 2$

b.
$$\begin{aligned} g(-2) &= 3(-2)^2 - 5(-2) + 2 \\ &= 12 + 10 + 2 \\ &= 24 \end{aligned}$$

c.
$$\begin{aligned} g(x-1) &= 3(x-1)^2 - 5(x-1) + 2 \\ &= 3(x^2 - 2x + 1) - 5x + 5 + 2 \\ &= 3x^2 - 11x + 10 \end{aligned}$$

d.
$$\begin{aligned} g(-x) &= 3(-x)^2 - 5(-x) + 2 \\ &= 3x^2 + 5x + 2 \end{aligned}$$

23. a. $g(13) = \sqrt{13-4} = \sqrt{9} = 3$

b. $g(0) = 4 - 0 = 4$

c. $g(-3) = 4 - (-3) = 7$

24. a. $f(-2) = \dfrac{(-2)^2 - 1}{-2-1} = \dfrac{3}{-3} = -1$

b. $f(1) = 12$

c. $f(2) = \dfrac{2^2 - 1}{2-1} = \dfrac{3}{1} = 3$

25. The vertical line test shows that this is not the graph of a function.

26. The vertical line test shows that this is the graph of a function.

27. The vertical line test shows that this is the graph of a function.

28. The vertical line test shows that this is not the graph of a function.

29. The vertical line test shows that this is not the graph of a function.

30. The vertical line test shows that this is the graph of a function.

31. a. domain: $[-3, 5)$

b. range: $[-5, 0]$

c. x-intercept: -3

d. y-intercept: -2

e. increasing: $(-2, 0)$ or $(3, 5)$
decreasing: $(-3, -2)$ or $(0, 3)$

f. $f(-2) = -3$ and $f(3) = -5$

32. **a.** domain: $(-\infty, \infty)$

b. range: $(-\infty, \infty)$

c. x-intercepts: –2 and 3

d. y-intercept: 3

e. increasing: (–5, 0)
decreasing: $(-\infty, -5)$ or $(0, \infty)$

f. $f(-2) = 0$ and $f(6) = -3$

33. **a.** domain: $(-\infty, \infty)$

b. range: [–2, 2]

c. x-intercept: 0

d. y-intercept: 0

e. increasing: (–2, 2)
constant: $(-\infty, -2)$ or $(2, \infty)$

f. $f(-9) = -2$ and $f(14) = 2$

34. **a.** 0, relative maximum –2

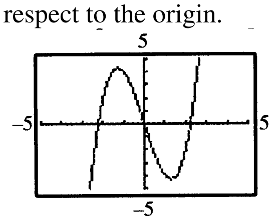
b. –2, 3, relative minimum –3, –5

35. **a.** 0, relative maximum 3

b. –5, relative minimum –6

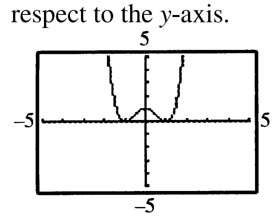
36.
$$f(x) = x^3 - 5x$$
$$f(-x) = (-x)^3 - 5(-x) = -x^3 + 5x = -f(x)$$

The function is odd. The function is symmetric with respect to the origin.



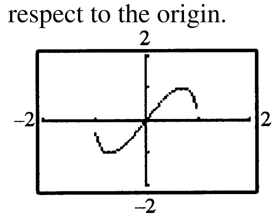
37.
$$f(x) = x^4 - 2x^2 + 1$$
$$f(-x) = (-x)^4 - 2(-x)^2 + 1 = x^4 - 2x^2 + 1 = f(x)$$

The function is even. The function is symmetric with respect to the y-axis.

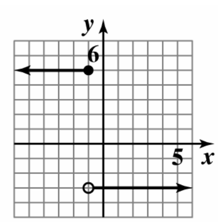


38.
$$f(x) = 2x\sqrt{1-x^2}$$
$$f(-x) = 2(-x)\sqrt{1-(-x)^2} = -2x\sqrt{1-x^2} = -f(x)$$

The function is odd. The function is symmetric with respect to the origin.

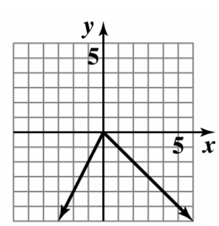


39. **a.**



b. range: {–3, 5}

40. **a.**



b. range: $\{y \mid y \le 0\}$

41. $\frac{8(x+h)-11-(8x-11)}{h}$
$=\frac{8x+8h-11-8x+11}{h}$
$=\frac{8h}{8}$
$=8$

42. $\frac{-2(x+h)^2+(x+h)+10-\left(-2x^2+x+10\right)}{h}$
$=\frac{-2\left(x^2+2xh+h^2\right)+x+h+10+2x^2-x-10}{h}$
$=\frac{-2x^2-4xh-2h^2+x+h+10+2x^2-x-10}{h}$
$=\frac{-4xh-2h^2+h}{h}$
$=\frac{h\left(-4x-2h+1\right)}{h}$
$-4x-2h+1$

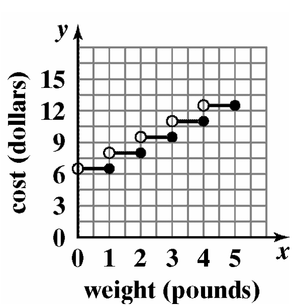
43. **a.** Yes, the eagle's height is a function of time since the graph passes the vertical line test.

b. Decreasing: (3, 12)
The eagle descended.

c. Constant: (0, 3) or (12, 17)
The eagle's height held steady during the first 3 seconds and the eagle was on the ground for 5 seconds.

d. Increasing: (17, 30)
The eagle was ascending.

44.



45. $m=\frac{1-2}{5-3}=\frac{-1}{2}=-\frac{1}{2}$; falls

46. $m=\frac{-4-(-2)}{-3-(-1)}=\frac{-2}{-2}=1$; rises

47. $m=\frac{\frac{1}{4}-\frac{1}{4}}{6-(-3)}=\frac{0}{9}=0$; horizontal

48. $m=\frac{10-5}{-2-(-2)}=\frac{5}{0}$ undefined; vertical

49. point-slope form: $y-2=-6(x+3)$
slope-intercept form: $y=-6x-16$

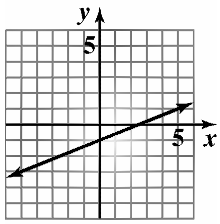
50. $m=\frac{2-6}{-1-1}=\frac{-4}{-2}=2$
point-slope form: $y-6=2(x-1)$
or $y-2=2(x+1)$
slope-intercept form: $y=2x+4$

51. $3x+y-9=0$
$y=-3x+9$
$m=-3$
point-slope form:
$y+7=-3(x-4)$
slope-intercept form:
$y=-3x+12-7$
$y=-3x+5$

52. perpendicular to $y=\frac{1}{3}x+4$
$m=-3$
point-slope form:
$y-6=-3(x+3)$
slope-intercept form:
$y=-3x-9+6$
$y=-3x-3$

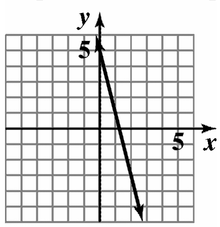
53. Write $6x-y-4=0$ in slope intercept form.
$6x-y-4=0$
$-y=-6x+4$
$y=6x-4$
The slope of the perpendicular line is 6, thus the slope of the desired line is $m=-\frac{1}{6}$.
$y-y_1=m(x-x_1)$
$y-(-1)=-\frac{1}{6}\left(x-(-12)\right)$
$y+1=-\frac{1}{6}(x+12)$
$y+1=-\frac{1}{6}x-2$
$6y+6=-x-12$
$x+6y+18=0$

54. slope: $\frac{2}{5}$; y-intercept: –1



$y = \frac{2}{5}x - 1$

55. slope: –4; y-intercept: 5



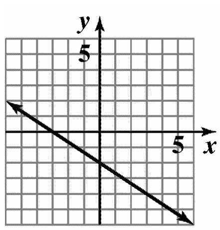
$f(x) = -4x + 5$

56. $2x+3y+6=0$

$$3y=-2x-6$$

$$y=-\frac{2}{3}x-2$$

slope: $-\frac{2}{3}$; y-intercept: –2



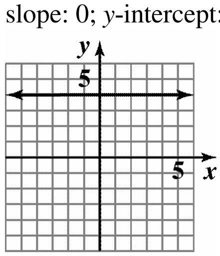
$2x + 3y + 6 = 0$

57. $2y-8=0$

$$2y=8$$

$$y=4$$

slope: 0; y-intercept: 4



$2y - 8 = 0$

58. $2x-5y-10=0$

Find x-intercept:

$$2x-5(0)-10=0$$

$$2x-10=0$$

$$2x=10$$

$$x=5$$

Find y-intercept:

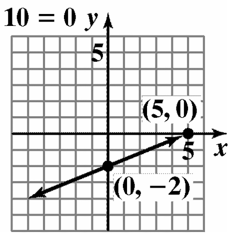
$$2(0)-5y-10=0$$

$$-5y-10=0$$

$$-5y=10$$

$$y=-2$$

$2x - 5y - 10 = 0$

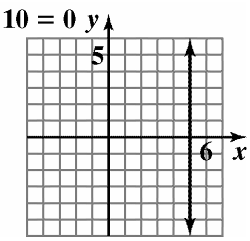


59. $2x-10=0$

$$2x=10$$

$$x=5$$

$2x - 10 = 0$



60. **a.** $m=\frac{11-2.3}{90-15}=\frac{8.7}{75}=0.116$

$$y-y_1=m(x-x_1)$$

$$y-11=0.116(x-90)$$

or

$$y-2.3=0.116(x-15)$$

b. $y-11=0.116(x-90)$

$$y-11=0.116x-10.44$$

$$y=0.116x+0.56$$

$$f(x)=0.116x+0.56$$

c. According to the graph, France has about 5 deaths per 100,000 persons.

d. $f(x) = 0.116x + 0.56$
$f(32) = 0.116(32) + 0.56$
$= 4.272$
≈ 4.3
According to the function, France has about 4.3 deaths per 100,000 persons.
This underestimates the value in the graph by 0.7 deaths per 100,000 persons.
The line passes below the point for France.

61. $m = \dfrac{1616-886}{2006-2002} = \dfrac{730}{4} = 182.5$
Corporate profits increased at a rate of $182.5 billion per year. The rate of change is $182.5 billion per year.

62. $\dfrac{f(x_2)-f(x_1)}{x_2-x_1} = \dfrac{[9^2-4(9)]-[4^2-4\cdot 5]}{9-5} = 10$

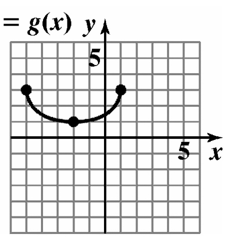
63. **a.** $S(0) = -16(0)^2 + 64(0) + 80 = 80$
$S(2) = -16(2)^2 + 64(2) + 80 = 144$

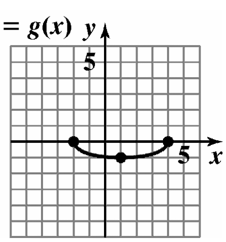
$\dfrac{144-80}{2-0} = 32$

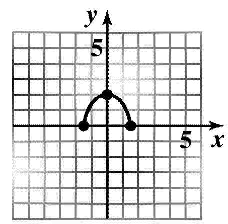
b. $S(4) = -16(4)^2 + 64(4) + 80 = 80$

$\dfrac{80-144}{4-2} = -32$

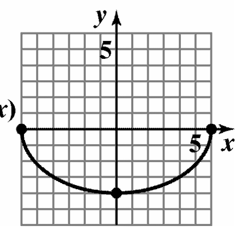
c. The ball is traveling up until 2 seconds, then it starts to come down.

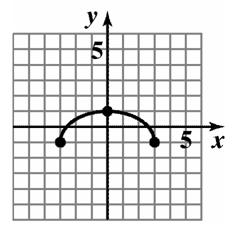
64. $\boldsymbol{y = g(x)}$


65. $\boldsymbol{y = g(x)}$


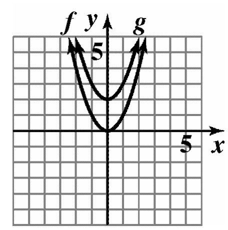
66.


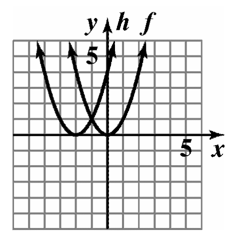
$\boldsymbol{y = g(x)}$

67.
$\boldsymbol{y = g(x)}$


68.


$\boldsymbol{y = g(x)}$

69.


70.


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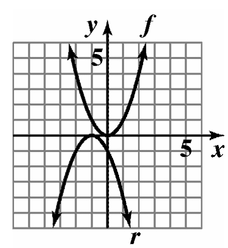
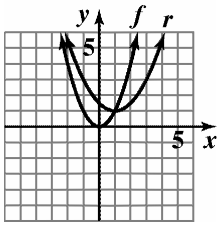


72.



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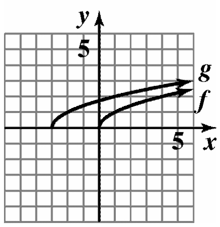

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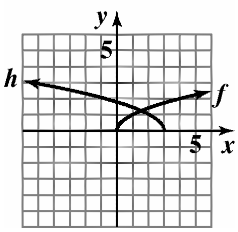
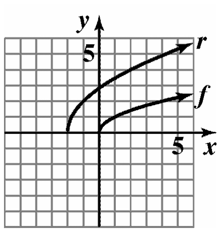
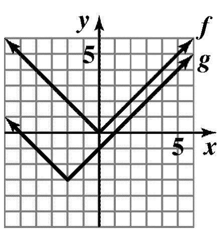


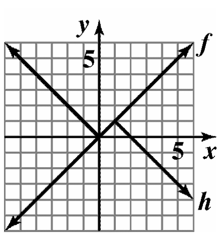
75.



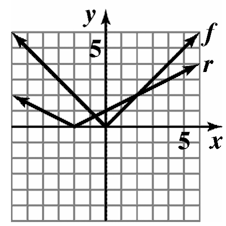
76.



77.



78.



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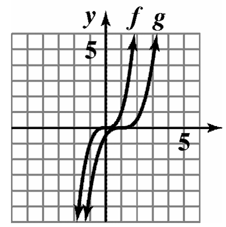
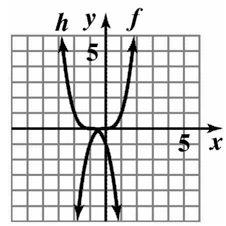
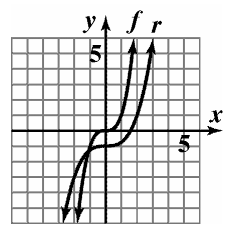


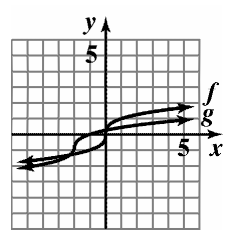
80.



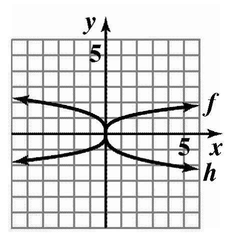
81.



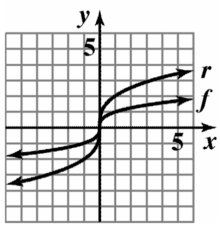
82.



83.



84.



85. domain: $(-\infty, \infty)$

86. The denominator is zero when $x = 7$. The domain is $(-\infty, 7) \cup (7, \infty)$.

87. The expressions under each radical must not be negative.
$8 - 2x \geq 0$
$-2x \geq -8$
$x \leq 4$
domain: $(-\infty, 4]$.

88. The denominator is zero when $x = -7$ or $x = 3$.
domain: $(-\infty, -7) \cup (-7, 3) \cup (3, \infty)$

89. The expressions under each radical must not be negative. The denominator is zero when $x = 5$.
$x - 2 \geq 0$
$x \geq 2$
domain: $[2, 5) \cup (5, \infty)$

90. The expressions under each radical must not be negative.
$x - 1 \geq 0$ and $x + 5 \geq 0$
$x \geq 1$ $x \geq -5$
domain: $[1, \infty)$

91. $f(x) = 3x - 1$; $g(x) = x - 5$
$(f + g)(x) = 4x - 6$
domain: $(-\infty, \infty)$
$(f - g)(x) = (3x - 1) - (x - 5) = 2x + 4$
domain: $(-\infty, \infty)$
$(fg)(x) = (3x - 1)(x - 5) = 3x^2 - 16x + 5$
domain: $(-\infty, \infty)$
$\left(\frac{f}{g}\right)(x) = \frac{3x - 1}{x - 5}$
domain: $(-\infty, 5) \cup (5, \infty)$

92. $f(x) = x^2 + x + 1$; $g(x) = x^2 - 1$
$(f + g)(x) = 2x^2 + x$
domain: $(-\infty, \infty)$
$(f - g)(x) = (x^2 + x + 1) - (x^2 - 1) = x + 2$
domain: $(-\infty, \infty)$
$(fg)(x) = (x^2 + x + 1)(x^2 - 1)$
$= x^4 + x^3 - x - 1$
$\left(\frac{f}{g}\right)(x) = \frac{x^2 + x + 1}{x^2 - 1}$
domain: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

93. $f(x) = \sqrt{x + 7}$; $g(x) = \sqrt{x - 2}$
$(f + g)(x) = \sqrt{x + 7} + \sqrt{x - 2}$
domain: $[2, \infty)$
$(f - g)(x) = \sqrt{x + 7} - \sqrt{x - 2}$
domain: $[2, \infty)$
$(fg)(x) = \sqrt{x + 7} \cdot \sqrt{x - 2}$
$= \sqrt{x^2 + 5x - 14}$
domain: $[2, \infty)$
$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x + 7}}{\sqrt{x - 2}}$
domain: $(2, \infty)$

94. $f(x) = x^2 + 3$; $g(x) = 4x - 1$

a. $(f \circ g)(x) = (4x - 1)^2 + 3$
$= 16x^2 - 8x + 4$

b. $(g \circ f)(x) = 4(x^2 + 3) - 1$
$= 4x^2 + 11$

c. $(f \circ g)(3) = 16(3)^2 - 8(3) + 4 = 124$

95. $f(x) = \sqrt{x}$; $g(x) = x + 1$

a. $(f \circ g)(x) = \sqrt{x + 1}$

b. $(g \circ f)(x) = \sqrt{x} + 1$

c. $(f \circ g)(3) = \sqrt{3 + 1} = \sqrt{4} = 2$

96. **a.** $(f \circ g)(x) = f\left(\frac{1}{x}\right)$

$$= \frac{\frac{1}{x}+1}{\frac{1}{x}-2} = \frac{\left(\frac{1}{x}+1\right)x}{\left(\frac{1}{x}-2\right)x} = \frac{1+x}{1-2x}$$

b. $x \neq 0$ $\quad 1-2x \neq 0$

$$x \neq \frac{1}{2}$$

$$(-\infty, 0) \cup \left(0, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$$

97. **a.** $(f \circ g)(x) = f(x+3) = \sqrt{x+3-1} = \sqrt{x+2}$

b. $x+2 \geq 0$ $\quad [-2, \infty)$

$x \geq -2$

98. $f(x) = x^4 \qquad g(x) = x^2 + 2x - 1$

99. $f(x) = \sqrt[3]{x} \qquad g(x) = 7x + 4$

100. $f(x) = \frac{3}{5}x + \frac{1}{2}; \; g(x) = \frac{5}{3}x - 2$

$$f(g(x)) = \frac{3}{5}\left(\frac{5}{3}x - 2\right) + \frac{1}{2}$$

$$= x - \frac{6}{5} + \frac{1}{2}$$

$$= x - \frac{7}{10}$$

$$g(f(x)) = \frac{5}{3}\left(\frac{3}{5}x + \frac{1}{2}\right) - 2$$

$$= x + \frac{5}{6} - 2$$

$$= x - \frac{7}{6}$$

f and *g* are not inverses of each other.

101. $f(x) = 2 - 5x; \; g(x) = \frac{2-x}{5}$

$$f(g(x)) = 2 - 5\left(\frac{2-x}{5}\right)$$

$$= 2 - (2 - x)$$

$$= x$$

$$g(f(x)) = \frac{2-(2-5x)}{5} = \frac{5x}{5} = x$$

f and *g* are inverses of each other.

102. **a.** $f(x) = 4x - 3$

$$y = 4x - 3$$

$$x = 4y - 3$$

$$y = \frac{x+3}{4}$$

$$f^{-1}(x) = \frac{x+3}{4}$$

b. $f(f^{-1}(x)) = 4\left(\frac{x+3}{4}\right) - 3$

$$= x + 3 - 3$$

$$= x$$

$$f^{-1}(f(x)) = \frac{(4x-3)+3}{4} = \frac{4x}{4} = x$$

103. **a.** $f(x) = 8x^3 + 1$

$$y = 8x^3 + 1$$

$$x = 8y^3 + 1$$

$$x - 1 = 8y^3$$

$$\frac{x-1}{8} = y^3$$

$$\sqrt[3]{\frac{x-1}{8}} = y$$

$$\frac{\sqrt[3]{x-1}}{2} = y$$

$$f^{-1}(x) = \frac{\sqrt[3]{x-1}}{2}$$

b.
$$f\left(f^{-1}(x)\right)=8\left(\frac{\sqrt[3]{x-1}}{2}\right)^3+1$$
$$=8\left(\frac{x-1}{8}\right)+1$$
$$=x-1+1$$
$$=x$$
$$f^{-1}\left(f(x)\right)=\frac{\sqrt[3]{\left(8x^3+1\right)-1}}{2}$$
$$=\frac{\sqrt[3]{8x^3}}{2}$$
$$=\frac{2x}{2}$$
$$=x$$

104. a.
$$f(x)=\frac{2}{x}+5$$
$$y=\frac{2}{x}+5$$
$$x=\frac{2}{y}+5$$
$$xy=2+5y$$
$$xy-5y=2$$
$$y(x-5)=2$$
$$y=\frac{2}{x-5}$$
$$f^{-1}(x)=\frac{2}{x-5}$$

b.
$$f\left(f^{-1}(x)\right)=\frac{2}{\frac{2}{x-5}}+5$$
$$=\frac{2(x-5)}{2}+5$$
$$=x-5+5$$
$$=x$$
$$f^{-1}\left(f(x)\right)=\frac{2}{\frac{2}{x}+5-5}$$
$$=\frac{2}{\frac{2}{x}}$$
$$=\frac{2x}{2}$$
$$=x$$

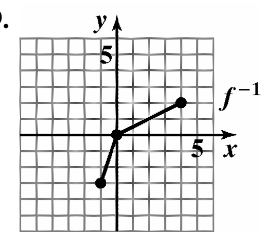
105. The inverse function exists.

106. The inverse function does not exist since it does not pass the horizontal line test.

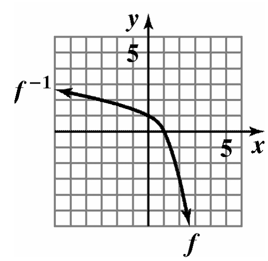
107. The inverse function exists.

108. The inverse function does not exist since it does not pass the horizontal line test.

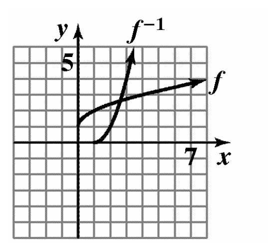
109.



110.
$$f(x)=1-x^2$$
$$y=1-x^2$$
$$x=1-y^2$$
$$y^2=1-x$$
$$y=\sqrt{1-x}$$
$$f^{-1}(x)=\sqrt{1-x}$$



111.
$$f(x)=\sqrt{x}+1$$
$$y=\sqrt{x}+1$$
$$x=\sqrt{y}+1$$
$$x-1=\sqrt{y}$$
$$(x-1)^2=y$$
$$f^{-1}(x)=(x-1)^2,\quad x\geq 1$$



$\boldsymbol{f(x)=\sqrt{x}+1}$
$\boldsymbol{g(x)=(x-1)^2, x\geq 1}$

112. $$\begin{aligned} d &= \sqrt{[3-(-2)]^2+[9-(-3)]^2} \\ &= \sqrt{5^2+12^2} \\ &= \sqrt{25+144} \\ &= \sqrt{169} \\ &= 13 \end{aligned}$$

113. $$\begin{aligned} d &= \sqrt{[-2-(-4)]^2+(5-3)^2} \\ &= \sqrt{2^2+2^2} \\ &= \sqrt{4+4} \\ &= \sqrt{8} \\ &= 2\sqrt{2} \\ &\approx 2.83 \end{aligned}$$

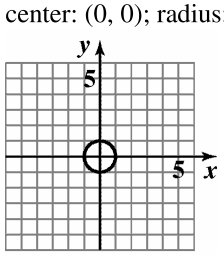
114. $$\left(\frac{2+(-12)}{2}, \frac{6+4}{2}\right) = \left(\frac{-10}{2}, \frac{10}{2}\right) = (-5, 5)$$

115. $$\left(\frac{4+(-15)}{2}, \frac{-6+2}{2}\right) = \left(\frac{-11}{2}, \frac{-4}{2}\right) = \left(\frac{-11}{2}, -2\right)$$

116. $$\begin{aligned} x^2+y^2 &= 3^2 \\ x^2+y^2 &= 9 \end{aligned}$$

117. $$\begin{aligned} (x-(-2))^2+(y-4)^2 &= 6^2 \\ (x+2)^2+(y-4)^2 &= 36 \end{aligned}$$

118. center: (0, 0); radius: 1

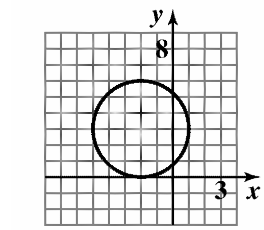


$\boldsymbol{x^2 + y^2 = 1}$

domain: $[-1, 1]$

range: $[-1, 1]$

119. center: (–2, 3); radius: 3



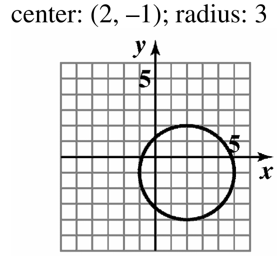
$\boldsymbol{(x+2)^2 + (y-3)^2 = 9}$

domain: $[-5, 1]$

range: $[0, 6]$

120. $$\begin{aligned} x^2+y^2-4x+2y-4 &= 0 \\ x^2-4x \quad + y^2+2y \quad &= 4 \\ x^2-4x+4+y^2+2y+1 &= 4+4+1 \\ (x-2)^2+(y+1)^2 &= 9 \end{aligned}$$

center: (2, –1); radius: 3



$\boldsymbol{x^2 + y^2 - 4x + 2y - 4 = 0}$

domain: $[-1, 5]$

range: $[-4, 2]$

121. **a.** $W(x) = 567 + 15x$

b. $$\begin{aligned} 702 &= 567 + 15x \\ 135 &= 15x \\ 9 &= x \end{aligned}$$

9 years after 2000, in 2009, the average weekly sales will be $702.

122. **a.** $f(x) = 15 + 0.05x$

b. $g(x) = 5 + 0.07x$

c. $$\begin{aligned} 15 + 0.05x &= 5 + 0.07x \\ 10 &= 0.02x \\ 500 &= x \end{aligned}$$

For 500 minutes, the two plans cost the same.

123. a.
$$\begin{aligned} N(x) &= 400-2(x-120) \\ &= 400-2x+240 \\ &= 640-2x \end{aligned}$$

b.
$$\begin{aligned} R(x) &= x(640-2x) \\ &= -2x^2+640x \end{aligned}$$

124. a.
$$w = 16-2x \qquad l = 24-2x$$
$$V(x) = (16-2x)(24-2x)x$$

b. $0 < x < 8$

125.
$$\begin{aligned} 2l+3w &= 400 \\ 2l &= 400-3w \\ l &= \frac{400-3w}{2} \end{aligned}$$
Let x = width
$$\begin{aligned} A(x) &= x\left(\frac{400-3w}{2}\right) \\ &= \frac{x(400-3w)}{2} \end{aligned}$$

126.
$$\begin{aligned} V &= lwh \\ 8 &= x \cdot x \cdot h \\ \frac{8}{x^2} &= h \end{aligned}$$
$$\begin{aligned} A(x) &= 2x \cdot x + 4hx \\ &= 2x^2+4\left(\frac{8}{x^2}\right)x \\ &= 2x^2+\frac{32}{x} \end{aligned}$$

127. $I = 0.08x + 0.12(10{,}000 - x)$

Chapter 1 Test

1. (b), (c), and (d) are not functions.

2. **a.** $f(4) - f(-3) = 3 - (-2) = 5$

b. domain: $(-5, 6]$

c. range: $[-4, 5]$

d. increasing: $(-1, 2)$

e. decreasing: $(-5, -1)$ or $(2, 6)$

f. $2, f(2) = 5$

g. $(-1, -4)$

h. x-intercepts: -4, 1, and 5.

i. y-intercept: -3

3. **a.** $-2, 2$

b. $-1, 1$

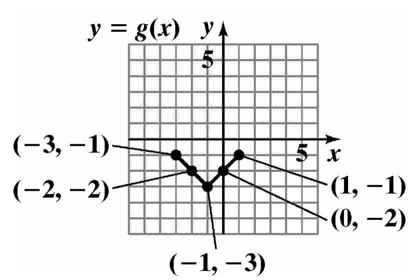
c. 0

d. even; $f(-x) = f(x)$

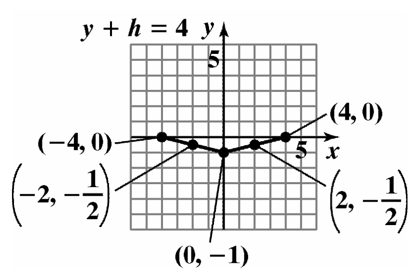
e. no; f fails the horizontal line test

f. $f(0)$ is a relative minimum.

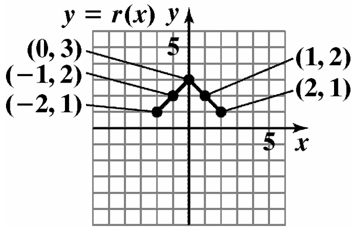
g.



h.

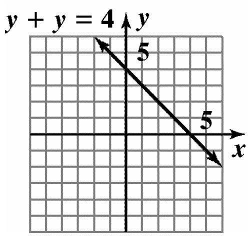


i.



j. $\dfrac{f(x_2)-f(x_1)}{x_2-x_1}=\dfrac{-1-0}{1-(-2)}=-\dfrac{1}{3}$

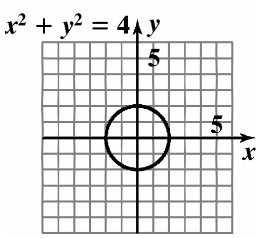
4.



domain: $(-\infty,\infty)$

range: $(-\infty,\infty)$

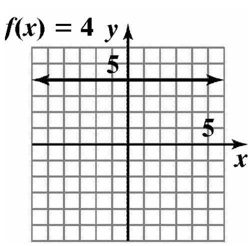
5.



domain: $[-2,2]$

range: $[-2,2]$

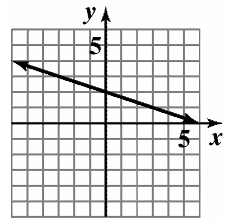
6.



domain: $(-\infty,\infty)$

range: $\{4\}$

7.

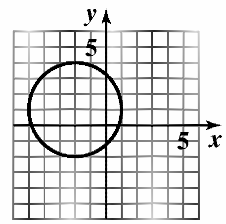


$\mathbf{f(x) = -\frac{1}{3}x + 2}$

domain: $(-\infty,\infty)$

range: $(-\infty,\infty)$

8.

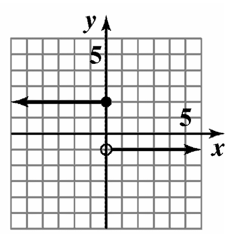


$\mathbf{(x+2)^2+(y-1)^2=9}$

domain: $[-5,1]$

range: $[-2,4]$

9.

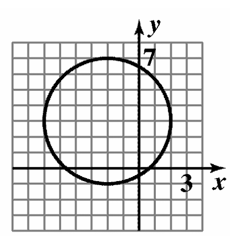


$f(x)=\begin{cases} 2 \text{ if } x\le 0 \\ -1 \text{ if } x>0 \end{cases}$

domain: $(-\infty,\infty)$

range: $\{-1,2\}$

10.

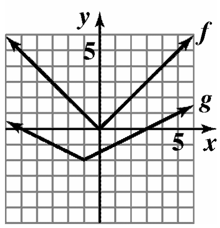


$\mathbf{x^2+y^2+4x+6y-3=0}$

domain: $[-6,2]$

range: $[-1,7]$

11.



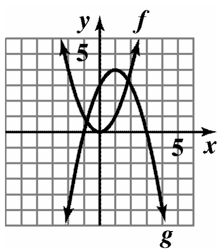
domain of *f*: $(-\infty, \infty)$

range of *f*: $[0, \infty)$

domain of *g*: $(-\infty, \infty)$

range of *g*: $[-2, \infty)$

12.



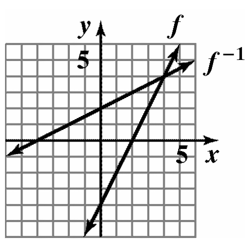
domain of *f*: $(-\infty, \infty)$

range of *f*: $[0, \infty)$

domain of *g*: $(-\infty, \infty)$

range of *g*: $(-\infty, 4]$

13.



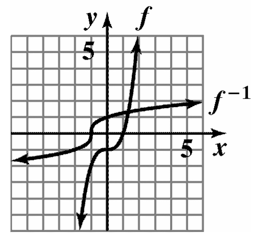
domain of *f*: $(-\infty, \infty)$

range of *f*: $(-\infty, \infty)$

domain of f^{-1}: $(-\infty, \infty)$

range of f^{-1}: $(-\infty, \infty)$

14.



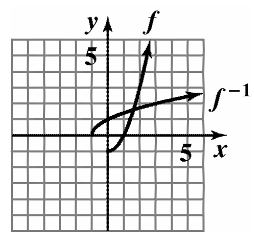
domain of *f*: $(-\infty, \infty)$

range of *f*: $(-\infty, \infty)$

domain of f^{-1}: $(-\infty, \infty)$

range of f^{-1}: $(-\infty, \infty)$

15.



domain of *f*: $[0, \infty)$

range of *f*: $[-1, \infty)$

domain of f^{-1}: $[-1, \infty)$

range of f^{-1}: $[0, \infty)$

16.

$$\begin{aligned} f(x) &= x^2 - x - 4 \\ f(x-1) &= (x-1)^2 - (x-1) - 4 \\ &= x^2 - 2x + 1 - x + 1 - 4 \\ &= x^2 - 3x - 2 \end{aligned}$$

17.

$$\begin{aligned} &\frac{f(x+h) - f(x)}{h} \\ &= \frac{(x+h)^2 - (x+h) - 4 - \left(x^2 - x - 4\right)}{h} \\ &= \frac{x^2 + 2xh + h^2 - x - h - 4 - x^2 + x + 4}{h} \\ &= \frac{2xh + h^2 - h}{h} \\ &= \frac{h\left(2x + h - 1\right)}{h} \\ &= 2x + h - 1 \end{aligned}$$

18.

$$\begin{aligned} (g - f)(x) &= 2x - 6 - \left(x^2 - x - 4\right) \\ &= 2x - 6 - x^2 + x + 4 \\ &= -x^2 + 3x - 2 \end{aligned}$$

19. $\left(\dfrac{f}{g}\right)(x)=\dfrac{x^2-x-4}{2x-6}$

domain: $(-\infty,3)\cup(3,\infty)$

20. $$\begin{aligned}(f\circ g)(x)&=f\left(g(x)\right)\\&=(2x-6)^2-(2x-6)-4\\&=4x^2-24x+36-2x+6-4\\&=4x^2-26x+38\end{aligned}$$

21. $$\begin{aligned}(g\circ f)(x)&=g\left(f(x)\right)\\&=2\left(x^2-x-4\right)-6\\&=2x^2-2x-8-6\\&=2x^2-2x-14\end{aligned}$$

22. $$\begin{aligned}g\left(f(-1)\right)&=2\left((-1)^2-(-1)-4\right)-6\\&=2\left(1+1-4\right)-6\\&=2\left(-2\right)-6\\&=-4-6\\&=-10\end{aligned}$$

23. $$\begin{aligned}f(x)&=x^2-x-4\\f(-x)&=(-x)^2-(-x)-4\\&=x^2+x-4\end{aligned}$$

f is neither even nor odd.

24. $m=\dfrac{-8-1}{-1-2}=\dfrac{-9}{-3}=3$

point-slope form: $y-1=3(x-2)$
or $y+8=3(x+1)$
slope-intercept form: $y=3x-5$

25. $y=-\dfrac{1}{4}x+5$ so $m=4$

point-slope form: $y-6=4(x+4)$
slope-intercept form: $y=4x+22$

26. Write $4x+2y-5=0$ in slope intercept form.

$$\begin{aligned}4x+2y-5&=0\\2y&=-4x+5\\y&=-2x+\frac{5}{2}\end{aligned}$$

The slope of the parallel line is –2, thus the slope of the desired line is $m=-2$.

$$\begin{aligned}y-y_1&=m(x-x_1)\\y-(-10)&=-2\left(x-(-7)\right)\\y+10&=-2(x+7)\\y+10&=-2x-14\\2x+y+24&=0\end{aligned}$$

27. **a.** First, find the slope using the points $(2,476)$ and $(4,486)$.

$$m=\frac{486-476}{4-2}=\frac{10}{2}=5$$

Then use the slope and a point to write the equation in point-slope form.

$$y-y_1=m\left(x-x_1\right)$$
$$y-486=5\left(x-4\right)$$
or
$$y-476=5\left(x-2\right)$$

b. $$\begin{aligned}y-486&=5\left(x-4\right)\\y-486&=5x-20\\y&=5x+466\\f\left(x\right)&=5x+466\end{aligned}$$

c. $f\left(10\right)=5(10)+466=516$

The function predicts that in 2010 the number of sentenced inmates in the U.S. will be 516 per 100,000 residents.

28. $$\begin{aligned}&\frac{3(10)^2-5-[3(6)^2-5]}{10-6}\\&=\frac{205-103}{4}\\&=\frac{192}{4}\\&=48\end{aligned}$$

29. $g(-1)=3-(-1)=4$
$g(7)=\sqrt{7-3}=\sqrt{4}=2$

30. The denominator is zero when $x = 1$ or $x = -5$.
domain: $(-\infty, -5) \cup (-5, 1) \cup (1, \infty)$

31. The expressions under each radical must not be negative.
$x + 5 \geq 0$ and $x - 1 \geq 0$
$x \geq -5$ $\quad x \geq 1$
domain: $[1, \infty)$

32. $(f \circ g)(x) = \dfrac{7}{\frac{2}{x} - 4} = \dfrac{7x}{2 - 4x}$

$x \neq 0, \quad 2 - 4x \neq 0$
$x \neq \dfrac{1}{2}$

domain: $(-\infty, 0) \cup \left(0, \dfrac{1}{2}\right) \cup \left(\dfrac{1}{2}, \infty\right)$

33. $f(x) = x^7 \qquad g(x) = 2x + 3$

34. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(5-2)^2 + (2-(-2))^2} \\ &= \sqrt{3^2 + 4^2} \\ &= \sqrt{9 + 16} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{2+5}{2}, \frac{-2+2}{2}\right) = \left(\frac{7}{2}, 0\right)$$

The length is 5 and the midpoint is $\left(\dfrac{7}{2}, 0\right)$.

35. **a.** $T(x) = 41.78 - 0.19x$

b.
$$\begin{aligned} 35.7 &= 41.78 - 0.19x \\ -6.08 &= -0.19x \\ 32 &= x \end{aligned}$$
32 years after 1980, in 2012, the winning time will be 35.7 seconds.

36. **a.**
$$\begin{aligned} Y(x) &= 50 - 1.5(x - 30) \\ &= 50 - 1.5x + 45 \\ &= 95 - 1.5x \end{aligned}$$

b.
$$\begin{aligned} T(x) &= x(95 - 1.5x) \\ &= -1.5x^2 + 95x \end{aligned}$$

37.
$$\begin{aligned} 2l + 2w &= 600 \\ 2l &= 600 - 2w \\ l &= 300 - w \end{aligned}$$
Let $x = w$

$$\begin{aligned} A(x) &= x(300 - x) \\ &= -x^2 + 300x \end{aligned}$$

38.
$$\begin{aligned} V &= lwh \\ 8000 &= x \cdot x \cdot h \\ \frac{8000}{x^2} &= h \end{aligned}$$

$$\begin{aligned} A(x) &= 2x^2 + 4x \ \frac{8000}{x^2} \\ &= 2x^2 + \frac{32{,}000}{x} \end{aligned}$$