Section 2.6

Check Point Exercises

1. **a.** $x-5=0$

$x=5$

$\{x \mid x \neq 5\}$

b. $x^2-25=0$

$x^2=25$

$x=\pm 5$

$\{x \mid x \neq 5, x \neq -5\}$

c. The denominator cannot equal zero.
All real numbers.

2. **a.** $x^2-1=0$

$x^2=1$

$x=1, x=-1$

b. $g(x)=\dfrac{x-1}{x^2-1}=\dfrac{x-1}{(x-1)(x+1)}=\dfrac{1}{x+1}$

$x=-1$

c. The denominator cannot equal zero.
No vertical asymptotes.

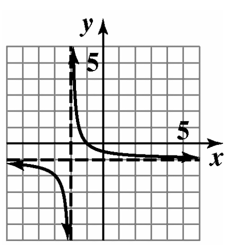
3. **a.** Since $n=m$, $y=\dfrac{9}{3}=3$

$y=3$ is a horizontal asymptote.

b. Since $n<m$, $y=0$ is a horizontal asymptote.

c. Since $n>m$, there is no horizontal asymptote.

4. Begin with the graph of $f(x)=\dfrac{1}{x}$.



$$g(x)=\frac{1}{x+2}-1$$

Shift the graph 2 units to the left by subtracting 2 from each x-coordinate. Shift the graph 1 unit down by subtracting 1 from each y-coordinate.

5. $f(x)=\dfrac{3x-3}{x-2}$

$f(-x)=\dfrac{3(-x)-3}{-x-2}=\dfrac{-3x-3}{-x-2}=\dfrac{3x+3}{x+2}$

no symmetry

$f(0)=\dfrac{3(0)-3}{0-2}=\dfrac{3}{2}$

The y-intercept is $\dfrac{3}{2}$.

$3x-3=0$

$3x=3$

$x=1$

The x-intercept is 1.

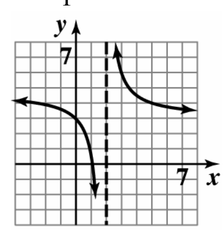
Vertical asymptote:

$x-2=0$

$x=2$

Horizontal asymptote:

$y=\dfrac{3}{1}=3$



$$f(x)=\frac{3x-3}{x-2}$$

6. $f(x)=\dfrac{2x^2}{x^2-9}$

$f(-x)=\dfrac{2(-x)^2}{(-x)^2-9}=\dfrac{2x^2}{x^2-9}=f(x)$

The y-axis symmetry.

$f(0)=\dfrac{2(0)^2}{0^2-9}=0$

The y-intercept is 0.

$2x^2=0$

$x=0$

The x-intercept is 0.

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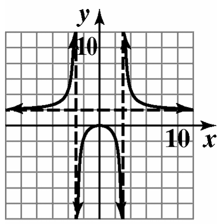
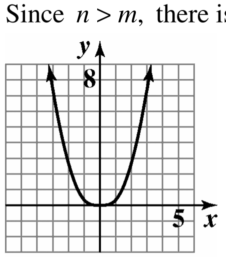
vertical asymptotes:
$x^2 - 9 = 0$
$x = 3, x = -3$
horizontal asymptote:
$y = \frac{2}{1} = 2$



$f(x) = \frac{2x^2}{x^2 - 9}$

7. $f(x) = \frac{x^4}{x^2 + 2}$

$f(-x) = \frac{(-x)^4}{(-x)^2 + 2} = \frac{x^4}{x^2 + 2} = f(x)$
y-axis symmetry
$f(0) = \frac{0^4}{0^2 + 2} = 0$
The y-intercept is 0.
$x^4 = 0$
$x = 0$
The x-intercept is 0.
vertical asymptotes:
$x^2 + 2 = 0$
$x^2 = -2$
no vertical asymptotes
horizontal asymptote:
Since $n > m$, there is no horizontal asymptote.



$f(x) = \frac{x^4}{x^2 + 2}$

8.
$$\begin{array}{r|rrr} 2 & 2 & -5 & 7 \\ & & 4 & -2 \\ \hline & 2 & -1 & 5 \end{array}$$
the equation of the slant asymptote is
$y = 2x - 1$.

9. **a.** $C(x) = 500{,}000 + 400x$

b. $\bar{C}(x) = \frac{500{,}000 + 400x}{x}$

c.
$$\bar{C}(1000) = \frac{500{,}000 + 400(1000)}{1000} = 900$$
$$\bar{C}(10{,}000) = \frac{500{,}000 + 400(10{,}000)}{10{,}000} = 450$$
$$\bar{C}(100{,}000) = \frac{500{,}000 + 400(100{,}000)}{100{,}000} = 405$$
The average cost per wheelchair of producing 1000, 10,000, and 100,000 wheelchairs is \$900, \$450, and \$405, respectively.

d. $y = \frac{400}{1} = 400$
The cost per wheelchair approaches \$400 as more wheelchairs are produced.

10. $x - 10$ = the average velocity on the return trip.
The function that expresses the total time required to complete the round trip is
$T(x) = \frac{20}{x} + \frac{20}{x - 10}$.

Exercise Set 2.6

1. $f(x) = \frac{5x}{x - 4}$
$\{x | x \neq 4\}$

2. $f(x) = \frac{7x}{x - 8}$
$\{x | x \neq 8\}$

3. $g(x) = \frac{3x^2}{(x - 5)(x + 4)}$
$\{x | x \neq 5, x \neq -4\}$

4. $g(x) = \frac{2x^2}{(x - 2)(x + 6)}$
$\{x | x \neq 2, x \neq -6\}$

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5. $h(x)=\dfrac{x+7}{x^2-49}$
$x^2-49=(x-7)(x+7)$
$\{x|x\neq 7, x\neq -7\}$

6. $h(x)=\dfrac{x+8}{x^2-64}$
$x^2-64=(x-8)(x+8)$
$\{x|x\neq 8, x\neq -8\}$

7. $f(x)=\dfrac{x+7}{x^2+49}$
all real numbers

8. $f(x)=\dfrac{x+8}{x^2+64}$
all real numbers

9. $-\infty$

10. $+\infty$

11. $-\infty$

12. $+\infty$

13. 0

14. 0

15. $+\infty$

16. $-\infty$

17. $-\infty$

18. $+\infty$

19. 1

20. 1

21. $f(x)=\dfrac{x}{x+4}$
$x+4=0$
$x=-4$
vertical asymptote: $x=-4$

22. $f(x)=\dfrac{x}{x-3}$
$x-3=0$
$x=3$
vertical asymptote: $x=3$

23. $g(x)=\dfrac{x+3}{x(x+4)}$
$x(x+4)=0$
$x=0, x=-4$
vertical asymptotes: $x=0,\ x=-4$

24. $g(x)=\dfrac{x+3}{x(x-3)}$
$x(x-3)=0$
$x=0, x=3$
vertical asymptotes: $x=0, x=3$

25. $h(x)=\dfrac{x}{x(x+4)}=\dfrac{1}{x+4}$
$x+4=0$
$x=-4$
vertical asymptote: $x=-4$

26. $h(x)=\dfrac{x}{x(x-3)}=\dfrac{1}{x-3}$
$x-3=0$
$x=3$
vertical asymptote: $x=3$

27. $r(x)=\dfrac{x}{x^2+4}$
x^2+4 has no real zeros
There are no vertical asymptotes.

28. $r(x)=\dfrac{x}{x^2+3}$
x^2+3 has no real zeros
There is no vertical asymptotes.

29. $f(x)=\dfrac{12x}{3x^2+1}$
$n<m$
horizontal asymptote: $y=0$

30. $f(x)=\dfrac{15x}{3x^2+1}$
$n<m$
horizontal asymptote: $y=0$

31. $g(x)=\dfrac{12x^2}{3x^2+1}$
$n=m$,
horizontal asymptote: $y=\dfrac{12}{3}=4$

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32. $g(x) = \dfrac{15x^2}{3x^2 + 1}$

$n = m$

horizontal asymptote: $y = \dfrac{15}{3} = 5$

33. $h(x) = \dfrac{12x^3}{3x^2 + 1}$

$n > m$

no horizontal asymptote

34. $h(x) = \dfrac{15x^3}{3x^2 + 1}$

$n > m$

no horizontal asymptote

35. $f(x) = \dfrac{-2x + 1}{3x + 5}$

$n = m$

horizontal asymptote: $y = -\dfrac{2}{3}$

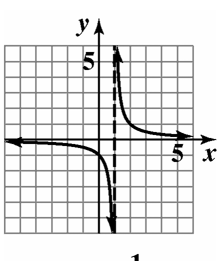
36. $f(x) = \dfrac{-3x + 7}{5x - 2}$

$n = m$ s

horizontal asymptote: $y = -\dfrac{3}{5}$

37. $g(x) = \dfrac{1}{x - 1}$

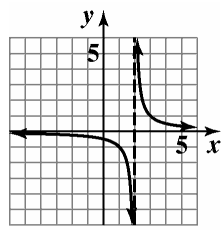
Shift the graph of $f(x) = \dfrac{1}{x}$ 1 unit to the right.



$g(x) = \dfrac{1}{x - 1}$

38. $g(x) = \dfrac{1}{x - 2}$

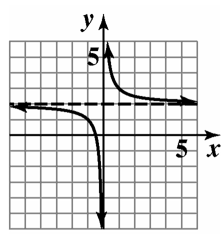
Shift the graph of $f(x) = \dfrac{1}{x}$ 2 units to the right.



$g(x) = \dfrac{1}{x - 2}$

39. $h(x) = \dfrac{1}{x} + 2$

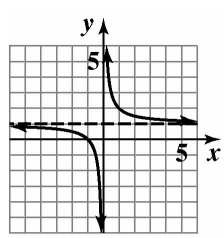
Shift the graph of $f(x) = \dfrac{1}{x}$ 2 units up.



$h(x) = \dfrac{1}{x} + 2$

40. $h(x) = \dfrac{1}{x} + 1$

Shift the graph of $f(x) = \dfrac{1}{x}$ 1 unit up.



$h(x) = \dfrac{1}{x} + 1$

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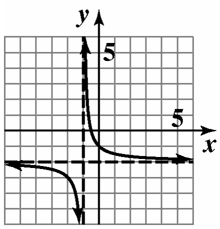
41. $g(x)=\dfrac{1}{x+1}-2$

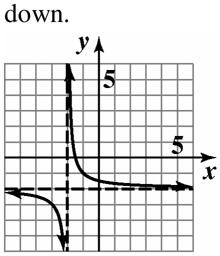
Shift the graph of $f(x)=\dfrac{1}{x}$ 1 unit left and 2 units down.



$\mathbf{g(x) = \dfrac{1}{x+1} - 2}$

42. $g(x)=\dfrac{1}{x+2}-2$

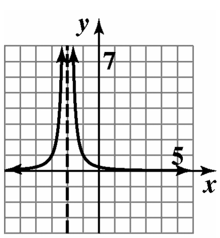
Shift the graph of $f(x)=\dfrac{1}{x}$ 2 units left and 2 units down.



$\mathbf{g(x) = \dfrac{1}{x+2} - 2}$

43. $g(x)=\dfrac{1}{(x+2)^2}$

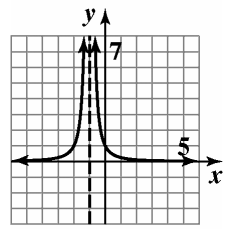
Shift the graph of $f(x)=\dfrac{1}{x^2}$ 2 units left.



$\mathbf{g(x) = \dfrac{1}{(x+2)^2}}$

44. $g(x)=\dfrac{1}{(x+1)^2}$

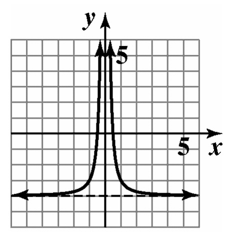
Shift the graph of $f(x)=\dfrac{1}{x^2}$ 1 unit left.



$\mathbf{g(x) = \dfrac{1}{(x+1)^2}}$

45. $h(x)=\dfrac{1}{x^2}-4$

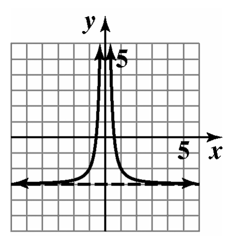
Shift the graph of $f(x)=\dfrac{1}{x^2}$ 4 units down.



$\mathbf{h(x) = \dfrac{1}{x^2} - 4}$

46. $h(x)=\dfrac{1}{x^2}-3$

Shift the graph of $f(x)=\dfrac{1}{x^2}$ 3 units down.



$\mathbf{h(x) = \dfrac{1}{x^2} - 3}$

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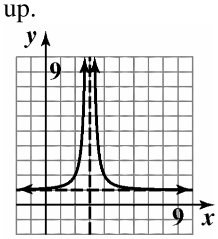
47. $h(x) = \dfrac{1}{(x-3)^2} + 1$

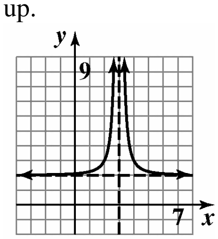
Shift the graph of $f(x) = \dfrac{1}{x^2}$ 3 units right and 1 unit up.



$\mathbf{h(x) = \dfrac{1}{(x-3)^2} + 1}$

48. $h(x) = \dfrac{1}{(x-3)^2} + 2$

Shift the graph of $f(x) = \dfrac{1}{x^2}$ 3 units right and 2 units up.



$\mathbf{h(x) = \dfrac{1}{(x-3)^2} + 2}$

49. $f(x) = \dfrac{4x}{x-2}$

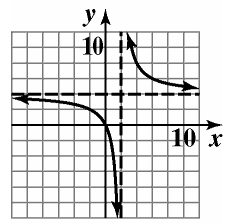
$f(-x) = \dfrac{4(-x)}{(-x)-2} = \dfrac{4x}{x+2}$

$f(-x) \neq f(x), f(-x) \neq -f(x)$
no symmetry

y-intercept: $y = \dfrac{4(0)}{0-2} = 0$

x-intercept: $4x = 0$
$x = 0$

vertical asymptote:
$x - 2 = 0$
$x = 2$
horizontal asymptote:
$n = m$, so $y = \dfrac{4}{1} = 4$



$\mathbf{f(x) = \dfrac{4x}{x-2}}$

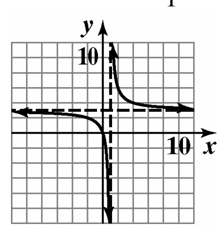
50. $f(x) = \dfrac{3x}{x-1}$

$f(-x) = \dfrac{3(-x)}{(-x)-1} = \dfrac{3x}{x+1}$

$f(-x) \neq f(x), f(-x) \neq -f(x)$
no symmetry

y-intercept: $y = \dfrac{3(0)}{0-1} = 0$

x-intercept: $3x = 0$
$x = 0$
vertical asymptote:
$x - 1 = 0$
$x = 1$
horizontal asymptote:
$n = m$, so $y = \dfrac{3}{1} = 3$



$\mathbf{f(x) = \dfrac{3x}{x-1}}$

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51. $f(x)=\dfrac{2x}{x^2-4}$

$f(-x)=\dfrac{2(-x)}{(-x)^2-4}=-\dfrac{2x}{x^2-4}=-f(x)$

Origin symmetry

y-intercept: $\dfrac{2(0)}{0^2-4}=\dfrac{0}{-4}=0$

x-intercept:

$2x=0$

$x=0$

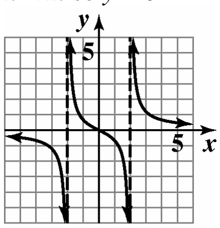
vertical asymptotes:

$x^2-4=0$

$x=\pm 2$

horizontal asymptote:

$n<m$ so $y=0$



$f(x)=\dfrac{2x}{x^2-4}$

52. $f(x)=\dfrac{4x}{x^2-1}$

$f(-x)=\dfrac{4(-x)}{(-x)^2-1}=-\dfrac{4x}{x^2-1}=-f(x)$

Origin symmetry

y-intercept: $\dfrac{4(0)}{0^2-1}=0$

x-intercept:$4x=0$

$x=0$

vertical asymptotes:

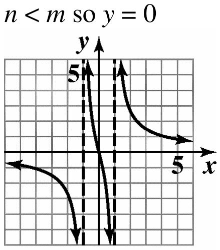
$x^2-1=0$

$(x-1)(x+1)=0$

$x=\pm 1$

horizontal asymptote:

$n<m$ so $y=0$



$f(x)=\dfrac{4x}{x^2-1}$

53. $f(x)=\dfrac{2x^2}{x^2-1}$

$f(-x)=\dfrac{2(-x)^2}{(-x)^2-1}=\dfrac{2x^2}{x^2-1}=f(x)$

y-axis symmetry

y-intercept: $y=\dfrac{2(0)^2}{0^2-1}=\dfrac{0}{1}=0$

x-intercept:

$2x^2=0$

$x=0$

vertical asymptote:

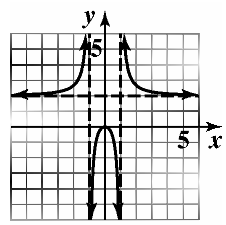
$x^2-1=0$

$x^2=1$

$x=\pm 1$

horizontal asymptote:

$n=m$, so $y=\dfrac{2}{1}=2$



$f(x)=\dfrac{2x^2}{x^2-1}$

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54. $f(x)=\dfrac{4x^2}{x^2-9}$

$f(-x)=\dfrac{4(-x)^2}{(-x)^2-9}=\dfrac{4x^2}{x^2-9}=f(x)$

y-axis symmetry

y-intercept: $y=\dfrac{4(0)^2}{0^2-9}=0$

x-intercept:

$4x^2=0$

$x=0$

vertical asymptotes:

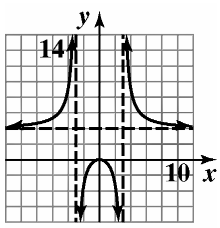
$x^2-9=0$

$(x-3)(x+3)=0$

$x=\pm 3$

horizontal asymptote:

$n=m$, so $y=\dfrac{4}{1}=4$



$f(x)=\dfrac{4x^2}{x^2-9}$

55. $f(x)=\dfrac{-x}{x+1}$

$f(-x)=\dfrac{-(-x)}{(-x)+1}=\dfrac{x}{-x+1}$

$f(-x)\neq f(x),\ f(-x)\neq -f(x)$

no symmetry

y-intercept: $y=\dfrac{-(0)}{0+1}=\dfrac{0}{1}=0$

x-intercept:

$-x=0$

$x=0$

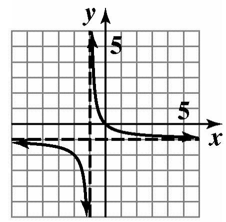
vertical asymptote:

$x+1=0$

$x=-1$

horizontal asymptote:

$n=m$, so $y=\dfrac{-1}{1}=-1$



$f(x)=\dfrac{-x}{x+1}$

56. $f(x)=\dfrac{-3x}{x+2}$

$f(-x)=\dfrac{-3(-x)}{(-x)+2}=\dfrac{3x}{-x+2}$

$f-x)\neq f(x),\ f(-x)\neq -f(x)$

no symmetry

y-intercept:

$y=\dfrac{-3(0)}{0+2}=0$

x-intercept:

$-3x=0$

$x=0$

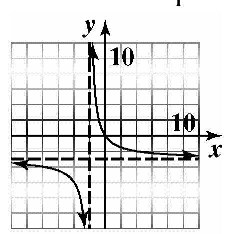
vertical asymptote:

$x+2=0$

$x=-2$

horizontal asymptote:

$n=m$, so $y=\dfrac{-3}{1}=-3$



$f(x)=\dfrac{-3x}{x+2}$

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57. $f(x) = -\dfrac{1}{x^2-4}$

$f(-x) = -\dfrac{1}{(-x)^2-4} = -\dfrac{1}{x^2-4} = f(x)$

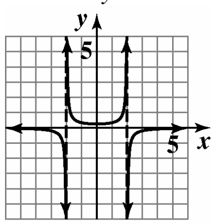
y-axis symmetry

y-intercept: $y = -\dfrac{1}{0^2-4} = \dfrac{1}{4}$

x-intercept: $-1 \neq 0$
no x-intercept
vertical asymptotes:

$x^2-4=0$
$x^2=4$
$x=\pm 2$

horizontal asymptote:
$n < m$ or $y = 0$



$f(x) = -\dfrac{1}{x^2-4}$

58. $f(x) = -\dfrac{2}{x^2-1}$

$f(-x) = -\dfrac{2}{(-x)^2-1} = -\dfrac{2}{x^2-1} = f(x)$

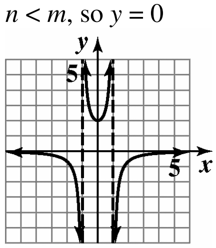
y-axis symmetry
y-intercept:

$y = -\dfrac{2}{0^2-1} = -\dfrac{2}{-1} = 2$

x-intercept:
$-2 = 0$
no x-intercept
vertical asymptotes:

$x^2-1=0$
$(x-1)(x+1)$
$x=\pm 1$

horizontal asymptote:
$n < m$, so $y = 0$



$f(x) = -\dfrac{2}{x^2-1}$

59. $f(x) = \dfrac{2}{x^2+x-2}$

$f(-x) = -\dfrac{2}{(-x)^2-x-2} = \dfrac{2}{x^2-x-2}$

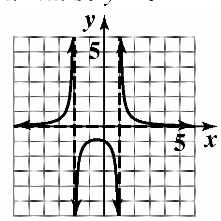
$f(-x) \neq f(x), f(-x) \neq -f(x)$
no symmetry

y-intercept: $y = \dfrac{2}{0^2+0-2} = \dfrac{2}{-2} = -1$

x-intercept: none
vertical asymptotes:

$x^2+x-2=0$
$(x+2)(x-1)=0$
$x=-2, x=1$

horizontal asymptote:
$n < m$ so $y = 0$



$f(x) = \dfrac{2}{x^2+x-2}$

60. $f(x) = \dfrac{-2}{x^2-x-2}$

$f(-x) = \dfrac{-2}{(-x)^2-(-x)-2} = \dfrac{-2}{x^2+x-2}$

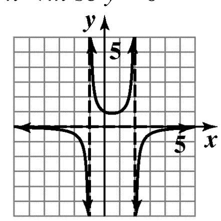
$f(-x) \neq f(x), f(-x) \neq -f(x)$
no symmetry

y-intercept: $y = \dfrac{-2}{0^2-0-2} = 1$

x-intercept: none
vertical asymptotes:

$x^2-x-2=0$
$(x-2)(x+1)=0$
$x=2, x=-1$

horizontal asymptote:
$n < m$ so $y = 0$



$f(x) = \dfrac{-2}{x^2-x-2}$

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61. $f(x)=\dfrac{2x^2}{x^2+4}$

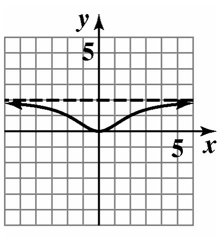
$f(-x)=\dfrac{2(-x)^2}{(-x)^2+4}=\dfrac{2x^2}{x^2+4}=f(x)$

y axis symmetry

y-intercept: $y=\dfrac{2(0)^2}{0^2+4}=0$

x-intercept: $2x^2=0$
$x=0$
vertical asymptote: none
horizontal asymptote:

$n=m,$ so $y=\dfrac{2}{1}=2$



$f(x)=\dfrac{2x^2}{x^2+4}$

62. $f(x)=\dfrac{4x^2}{x^2+1}$

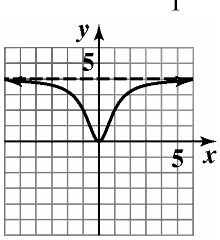
$f(-x)=\dfrac{4(-x)^2}{(-x)^2+1}=\dfrac{4x^2}{x^2+1}=f(x)$

y axis symmetry

y-intercept: $y=\dfrac{4(0)^2}{0^2+1}=0$

x-intercept: $4x^2=0$
$x=0$
vertical asymptote: none
horizontal asymptote:

$n=m,$ so $y=\dfrac{4}{1}=4$



$f(x)=\dfrac{4x^2}{x^2+1}$

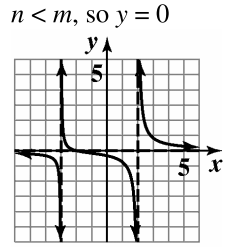
63. $f(x)=\dfrac{x+2}{x^2+x-6}$

$f(-x)=\dfrac{-x+2}{(-x)^2-(-x)-6}=\dfrac{-x+2}{x^2+x-6}$

$f(-x)\neq f(x), f(-x)\neq -f(x)$
no symmetry

y-intercept: $y=\dfrac{0+2}{0^2+0-6}=-\dfrac{2}{6}=-\dfrac{1}{3}$

x-intercept:
$x+2=0$
$x=-2$
vertical asymptotes:
$x^2+x-6=0$
$(x+3)(x-2)$
$x=-3, x=2$
horizontal asymptote:
$n<m$, so $y=0$



$f(x)=\dfrac{x+2}{x^2+x-6}$

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64. $f(x) = \dfrac{x-4}{x^2-x-6}$

$f(-x) = \dfrac{-x-4}{(-x)^2-(-x)-6} = -\dfrac{x+4}{x^2+x-6}$

$f(-x) \neq f(x), f(-x) \neq -f(x)$

no symmetry

y-intercept: $y = \dfrac{0-4}{0^2-0-6} = \dfrac{2}{3}$

x-intercept:

$x-4=0, x=4$

vertical asymptotes:

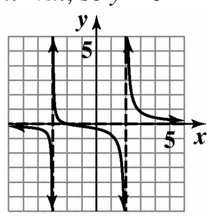
$x^2-x-6=0$

$(x-3)(x+2)$

$x=3, x=-2$

horizontal asymptote:

$n < m$, so $y = 0$



$f(x) = \dfrac{x+2}{x^2-x-6}$

65. $f(x) = \dfrac{x^4}{x^2+2}$

$f(-x) = \dfrac{(-x)^4}{(-x)^2+2} = \dfrac{x^4}{x^2+2} = f(x)$

y-axis symmetry

y-intercept: $y = \dfrac{0^4}{0^2+2} = 0$

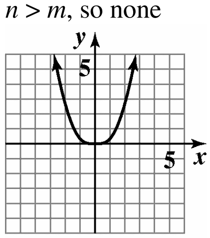
x-intercept: $x^4 = 0$

$x = 0$

vertical asymptote: none

horizontal asymptote:

$n > m$, so none



$f(x) = \dfrac{x^4}{x^2+2}$

66. $f(x) = \dfrac{2x^4}{x^2+1}$

$f(-x) = \dfrac{2(-x)^4}{(-x)^2+1} = \dfrac{2x^4}{x^2+1} = f(x)$

y-axis symmetry

y-intercept: $y = \dfrac{2(0^4)}{0^2+2} = 0$

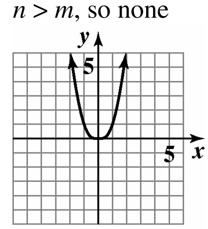
x-intercept: $2x^4 = 0$

$x = 0$

vertical asymptote: none

horizontal asymptote:

$n > m$, so none



$f(x) = \dfrac{2x^4}{x^2+1}$

67. $f(x) = \dfrac{x^2+x-12}{x^2-4}$

$f(-x) = \dfrac{(-x)^2-x-12}{(-x)^2-4} = \dfrac{x^2-x-12}{x^2-4}$

$f(-x) \neq f(x), f(-x) \neq -f(x)$

no symmetry

y-intercept: $y = \dfrac{0^2+0-12}{0^2-4} = 3$

x-intercept: $x^2+x-12=0$

$(x-3)(x+4)=0$

$x=3, x=-4$

vertical asymptotes:

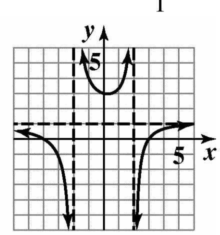
$x^2-4=0$

$(x-2)(x+2)=0$

$x=2, x=-2$

horizontal asymptote:

$n = m$, so $y = \dfrac{1}{1} = 1$



$f(x) = \dfrac{x^2+x-12}{x^2-4}$

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68. $f(x)=\dfrac{x^2}{x^2+x-6}$

$f(-x)=\dfrac{(-x)^2}{(-x)^2-x-6}=\dfrac{x^2}{x^2-x-6}$

$f(-x)\neq f(x), f(-x)\neq -f(x)$

no symmetry

y-intercept: $y=\dfrac{0^2}{0^2+0-6}=0$

x-intercept: $x^2=0, x=0$

vertical asymptotes:

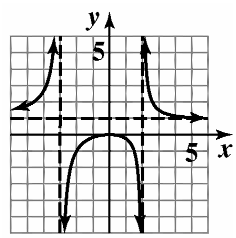
$x^2+x-6=0$

$(x+3)(x-2)=0$

$x=-3, x=2$

horizontal asymptote:

$n=m$, so $y=\dfrac{1}{1}=1$



$f(x)=\dfrac{x^2}{x^2+x-6}$

69. $f(x)=\dfrac{3x^2+x-4}{2x^2-5x}$

$f(-x)=\dfrac{3(-x)^2-x-4}{2(-x)^2+5x}=\dfrac{3x^2-x-4}{2x^2+5x}$

$f(-x)\neq f(x), f(-x)\neq -f(x)$

no symmetry

y-intercept: $y=\dfrac{3(0)^2+0-4}{2(0)^2-5(0)}=\dfrac{-4}{0}$

no y-intercept

x-intercepts:

$3x^2+x-4=0$

$(3x+4)(x-1)=0$

$3x+4=0 \quad x-1=0$

$3x=-4$

$x=-\dfrac{4}{3}, x=1$

vertical asymptotes:

$2x^2-5x=0$

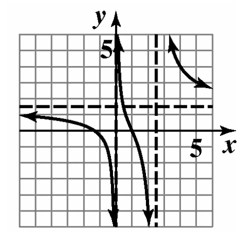
$x(2x-5)=0$

$x=0, 2x=5$

$x=\dfrac{5}{2}$

horizontal asymptote:

$n=m$, so $y=\dfrac{3}{2}$



$f(x)=\dfrac{3x^2+x-4}{2x^2-5x}$

70. $f(x)=\dfrac{x^2-4x+3}{(x+1)^2}$

$f(-x)=\dfrac{(-x)^2-4(-x)+3}{(-x+1)^2}=\dfrac{x^2+4x+3}{(-x+1)^2}$

$f(-x)\neq f(x), f(-x)\neq -f(x)$

no symmetry

y-intercept: $y=\dfrac{0^2-4(0)+3}{(0+1)^2}=\dfrac{3}{1}=3$

x-intercept:

$x^2-4x+3=0$

$(x-3)(x-1)=0$

$x=3$ and $x=1$

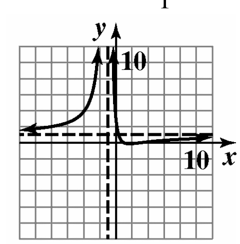
vertical asymptote:

$(x+1)^2=0$

$x=-1$

horizontal asymptote:

$n=m$, so $y=\dfrac{1}{1}=1$



$f(x)=\dfrac{x^2-4x+3}{(x+1)^2}$

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71. a. Slant asymptote:

$f(x) = x - \frac{1}{x}$

$y = x$

b. $f(x) = \frac{x^2 - 1}{x}$

$f(-x) = \frac{(-x)^2 - 1}{(-x)} = \frac{x^2 - 1}{-x} = -f(x)$

Origin symmetry

y-intercept: $y = \frac{0^2 - 1}{0} = \frac{-1}{0}$

no y-intercept

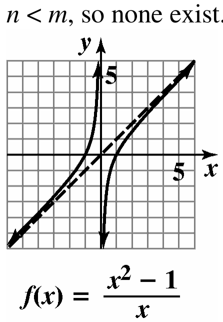
x-intercepts: $x^2 - 1 = 0$

$x = \pm 1$

vertical asymptote: $x = 0$

horizontal asymptote:

$n < m$, so none exist.



72. $f(x) = \frac{x^2 - 4}{x}$

a. slant asymptote:

$f(x) = x - \frac{4}{x}$

$y = x$

b. $f(x) = \frac{x^2 - 4}{x}$

$f(-x) = \frac{(-x)^2 - 4}{-x} = \frac{x^2 - 4}{-x} = -f(x)$

origin symmetry

y-intercept: $y = \frac{0^2 - 4}{0} = -\frac{4}{0}$

no y-intercept

x-intercept:

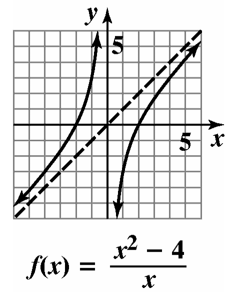
$x^2 - 4 = 0$

$x = \pm 2$

vertical asymptote: $x = 0$

horizontal asymptote:

$n > m$, so none exist.



73. a. Slant asymptote:

$f(x) = x + \frac{1}{x}$

$y = x$

b. $f(x) = \frac{x^2 + 1}{x}$

$f(-x) = \frac{(-x)^2 + 1}{-x} = \frac{x^2 + 1}{-x} = -f(x)$

Origin symmetry

y-intercept: $y = \frac{0^2 + 1}{0} = \frac{1}{0}$

no y-intercept

x-intercept:

$x^2 + 1 = 0$

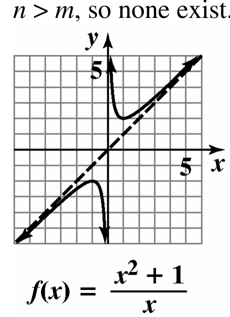
$x^2 = -1$

no x-intercept

vertical asymptote: $x = 0$

horizontal asymptote:

$n > m$, so none exist.



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74. $f(x) = \dfrac{x^2+4}{x}$

a. slant asymptote:

$g(x) = x + \dfrac{4}{x}$

$y = x$

b. $f(x) = \dfrac{x^2+4}{x}$

$f(-x) = \dfrac{(-x)^2+4}{-x} = \dfrac{x^2+4}{-x} = -f(x)$

origin symmetry

y-intercept: $y = \dfrac{0^2+4}{0} = \dfrac{4}{0}$

no y-intercept

$x^2+4=0$

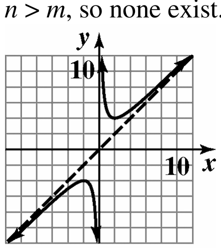
$x^2=-4$

no x-intercept

vertical asymptote: $x = 0$

horizontal asymptote:

$n > m$, so none exist.



$f(x) = \dfrac{x^2+4}{x}$

75. a. Slant asymptote:

$f(x) = x + 4 + \dfrac{6}{x-3}$

$y = x + 4$

b. $f(x) = \dfrac{x^2+x-6}{x-3}$

$f(-x) = \dfrac{(-x)^2+(-x)-6}{-x-3} = \dfrac{x^2-x-6}{-x-3}$

$f(-x) \neq g(x)$, $g(-x) \neq -g(x)$

No symmetry

y-intercept: $y = \dfrac{0^2+0-6}{0-3} = \dfrac{-6}{-3} = 2$

x-intercept:

$x^2+x-6=0$

$(x+3)(x-2)=0$

$x=-3$ and $x=2$

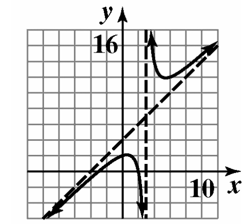
vertical asymptote:

$x-3=0$

$x=3$

horizontal asymptote:

$n > m$, so none exist.



$f(x) = \dfrac{x^2+x-6}{x-3}$

76. $f(x) = \dfrac{x^2-x+1}{x-1}$

a. slant asymptote:

$g(x) = x + \dfrac{1}{x-1}$

$y = x$

b. $f(x) = \dfrac{x^2-x-1}{x-1}$

$f(-x) = \dfrac{(-x)^2-(-x)+1}{-x-1} = \dfrac{x^2+x+1}{-x-1}$

no symmetry

$f(-x) \neq f(x), f(-x) \neq -g(x)$

y-intercept: $y = \dfrac{0^2-0+1}{0-1} = \dfrac{1}{-1} = -1$

x-intercept:

$x^2-x+1=0$

no x-intercept

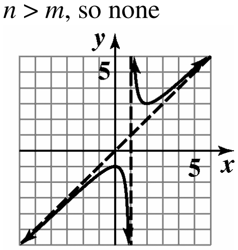
vertical asymptote:

$x-1=0$

$x=1$

horizontal asymptote:

$n > m$, so none



$f(x) = \dfrac{x^2-x+1}{x-1}$

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77. $f(x) = \dfrac{x^3+1}{x^2+2x}$

a. slant asymptote:

$$\begin{array}{r|l} & x-2 \\ \hline x^2+2x & x^3 \qquad +1 \\ & \underline{x^3+2x^2} \\ & -2x^2 \\ & \underline{-2x^2+4x} \\ & -4x+1 \end{array}$$

$y = x-2$

b. $f(-x) = \dfrac{(-x)^3+1}{(-x)^2+2(-x)} = \dfrac{-x^3+1}{x^2-2x}$

$f(-x) \neq f(x),\ f(-x) \neq -f(x)$

no symmetry

y-intercept: $y = \dfrac{0^3+1}{0^2+2(0)} = \dfrac{1}{0}$

no y-intercept

x-intercept: $x^3 + 1 = 0$

$x^3 = -1$

$x = -1$

vertical asymptotes:

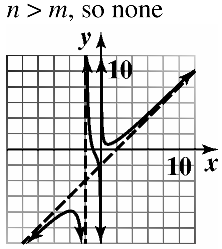
$x^2+2x = 0$

$x(x+2) = 0$

$x = 0, x = -2$

horizontal asymptote:

$n > m$, so none



$f(x) = \dfrac{x^3+1}{x^2+2x}$

78. $f(x) = \dfrac{x^3-1}{x^2-9}$

a. slant asymptote:

$$\begin{array}{r|l} & x+\dfrac{9x-1}{x^2-9} \\ \hline x^2-9 & x^3 \qquad -1 \\ & \underline{x^3-9x} \\ & 9x-1 \end{array}$$

$y = x$

b. $f(-x) = \dfrac{(-x)^3-1}{(-x)^2-9} = \dfrac{-x^3-1}{x^2-9}$

$f(-x) \neq f(x), f(-x) \neq -f(x)$

no symmetry

y-intercept: $y = \dfrac{0^3-1}{0^2-9} = \dfrac{1}{9}$

x-intercept: $x^3 - 1 = 0$

$x^3 = 1$

$x = 1$

vertical asymptotes:

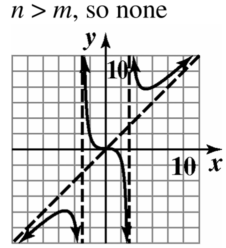
$x^2-9 = 0$

$(x-3)(x+3) = 0$

$x = 3, x = -3$

horizontal asymptote:

$n > m$, so none



$f(x) = \dfrac{x^3-1}{x^2-9}$

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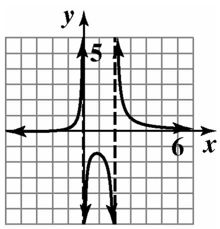
79. $\dfrac{5x^2}{x^2-4}\cdot\dfrac{x^2+4x+4}{10x^3}$

$=\dfrac{\cancel{5}\,\cancel{x^2}}{\cancel{(x+2)}\,(x-2)}\cdot\dfrac{(x+2)^{\cancel{2}}}{\underset{2}{\cancel{10}}\,x^{\cancel{3}\,1}}$

$=\dfrac{x+2}{2x(x-2)}$

So, $f(x)=\dfrac{x+2}{2x(x-2)}$



$f(x) = \dfrac{x+2}{2x(x-2)}$

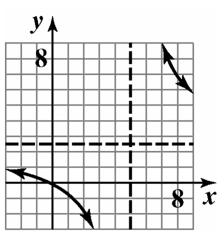
80. $\dfrac{x-5}{10x-2}\div\dfrac{x^2-10x+25}{25x^2-1}$

$=\dfrac{x-5}{10x-2}\cdot\dfrac{25x^2-1}{x^2-10x+25}$

$=\dfrac{\cancel{x-5}}{2\cancel{(5x-1)}}\cdot\dfrac{(5x+1)\,\cancel{(5x-1)}}{(x-5)^{\cancel{2}}}$

$=\dfrac{5x+1}{2(x-5)}$

So, $f(x)=\dfrac{5x+1}{2(x-5)}$



$f(x) = \dfrac{5x+1}{2(x-5)}$

81. $\dfrac{x}{2x+6}-\dfrac{9}{x^2-9}$

$\dfrac{x}{2x+6}-\dfrac{9}{x^2-9}$

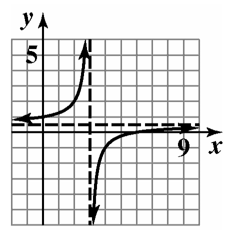
$=\dfrac{x}{2(x+3)}-\dfrac{9}{(x+3)(x-3)}$

$=\dfrac{x(x-3)-9(2)}{2(x+3)(x-3)}$

$=\dfrac{x^2-3x-18}{2(x+3)(x-3)}$

$=\dfrac{(x-6)\,\cancel{(x+3)}}{2\cancel{(x+3)}\,(x-3)}=\dfrac{x-6}{2(x-3)}$

So, $f(x)=\dfrac{x-6}{2(x-3)}$



$f(x) = \dfrac{x-6}{2(x-3)}$

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82. $\dfrac{2}{x^2+3x+2}-\dfrac{4}{x^2+4x+3}$

$=\dfrac{2}{(x+2)(x+1)}-\dfrac{4}{(x+3)(x+1)}$

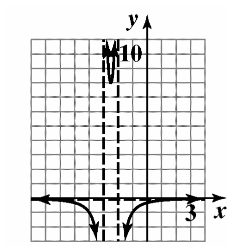
$=\dfrac{2(x+3)-4(x+2)}{(x+2)(x+1)(x+3)}$

$=\dfrac{2x+6-4x-8}{(x+2)(x+1)(x+3)}$

$=\dfrac{-2x-2}{(x+2)(x+1)(x+3)}$

$=\dfrac{-2\cancel{(x+1)}}{(x+2)\cancel{(x+1)}(x+3)}=\dfrac{-2}{(x+2)(x+3)}$

So, $f(x)=\dfrac{-2}{(x+2)(x+3)}$



$f(x)=\dfrac{-2}{(x+2)(x+3)}$

83. $\dfrac{1-\dfrac{3}{x+2}}{1+\dfrac{1}{x-2}}=\dfrac{1-\dfrac{3}{x+2}}{1+\dfrac{1}{x-2}}\cdot\dfrac{(x+2)(x-2)}{(x+2)(x-2)}$

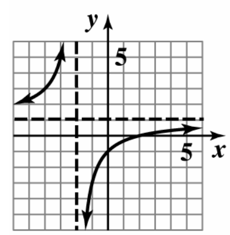
$=\dfrac{(x+2)(x-2)-3(x-2)}{(x+2)(x-2)+(x+2)}$

$=\dfrac{x^2-4-3x+6}{x^2-4+x+2}$

$=\dfrac{x^2-3x+2}{x^2+x-2}$

$=\dfrac{(x-2)\cancel{(x-1)}}{(x+2)\cancel{(x-1)}}=\dfrac{x-2}{x+2}$

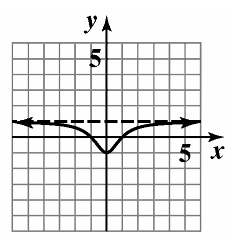
So, $f(x)=\dfrac{x-2}{x+2}$



$f(x)=\dfrac{x-2}{x+2}$

84. $\dfrac{x-\dfrac{1}{x}}{x+\dfrac{1}{x}}\cdot\dfrac{x}{x}=\dfrac{x^2-1}{x^2+1}=\dfrac{(x-1)(x+1)}{x^2+1}$

So, $f(x)=\dfrac{(x-1)(x+1)}{x^2+1}$



$f(x)=\dfrac{(x-1)(x+1)}{x^2+1}$

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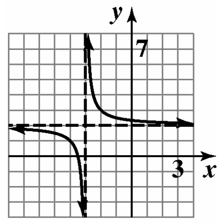
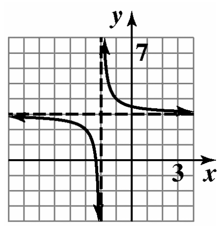
85. $g(x) = \dfrac{2x+7}{x+3} = \dfrac{1}{x+3} + 2$



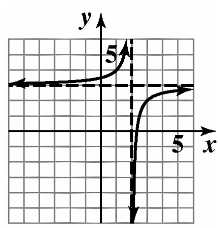
$\boldsymbol{f(x) = \dfrac{1}{x+3} + 2}$

86. $g(x) = \dfrac{3x+7}{x+2} = \dfrac{1}{x+2} + 3$



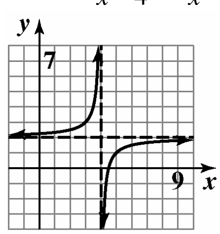
$\boldsymbol{f(x) = \dfrac{1}{x+2} + 3}$

87. $g(x) = \dfrac{3x-7}{x-2} = \dfrac{-1}{x-2} + 3$



$\boldsymbol{f(x) = \dfrac{-1}{x-2} + 3}$

88. $g(x) = \dfrac{2x-9}{x-4} = \dfrac{-1}{x-4} + 2$



$\boldsymbol{f(x) = \dfrac{-1}{x-4} + 2}$

89. **a.** $C(x) = 100x + 100{,}000$

b. $\overline{C}(x) = \dfrac{100x + 100{,}000}{x}$

c. $\overline{C}(500) = \dfrac{100(500) + 100{,}000}{500} = \$300$

When 500 bicycles are manufactured, it costs \$300 to manufacture each.

$\overline{C}(1000) = \dfrac{100(1000) + 100{,}000}{1000} = \$200$

When 1000 bicycles are manufactured, it costs \$200 to manufacture each.

$\overline{C}(2000) = \dfrac{100(2000) + 100{,}000}{2000} = \$150$

When 2000 bicycles are manufactured, it costs \$150 to manufacture each.

$\overline{C}(4000) = \dfrac{100(4000) + 100{,}000}{4000} = \$125$

When 4000 bicycles are manufactured, it costs \$125 to manufacture each.

The average cost decreases as the number of bicycles manufactured increases.

d. $n = m$, so $y = \dfrac{100}{1} = 100$.

As greater numbers of bicycles are manufactured, the average cost approaches \$100.

90. **a.** $C(x) = 30x + 300{,}000$

b. $\overline{C} = \dfrac{300{,}000 + 30x}{x}$

c. $\overline{C}(1000) = \dfrac{300000 + 30(1000)}{1000} = 330$

When 1000 shoes are manufactured, it costs \$330 to manufacture each.

$\overline{C}(10000) = \dfrac{300000 + 30(10000)}{10000} = 60$

When 10,000 shoes are manufactured, it costs \$60 to manufacture each.

$\overline{C}(100{,}00) = \dfrac{300{,}000 + 30(100{,}000)}{100{,}000} = 33$

When 100,000 shoes are manufactured, it costs \$33 to manufacture each.

The average cost decreases as the number of shoes manufactured increases.

d. $n = m$, so $y = \dfrac{30}{1} = 30$.

As greater numbers of shoes are manufactured, the average cost approaches \$30.

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91. a. From the graph the pH level of the human mouth 42 minutes after a person eats food containing sugar will be about 6.0.

b. From the graph, the pH level is lowest after about 6 minutes.

$$f(6)=\frac{6.5(6)^2-20.4(6)+234}{6^2+36}$$
$$=4.8$$

The pH level after 6 minutes (i.e. the lowest pH level) is 4.8.

c. From the graph, the pH level appears to approach 6.5 as time goes by. Therefore, the normal pH level must be 6.5.

d. $y=6.5$

Over time, the pH level rises back to the normal level.

e. During the first hour, the pH level drops quickly below normal, and then slowly begins to approach the normal level.

92. a. From the graph, the drug's concentration after three hours appears to be about 1.5 milligrams per liter.

$$C(3)=\frac{5(3)}{3^2+1}=\frac{15}{10}=1.5$$

This verifies that the drug's concentration after 3 hours will be 1.5 milligrams per liter.

b. The degree of the numerator, 1, is less than the degree of the denominator, 2, so the the horizontal asymptote is $y=0$.

Over time, the drug's concentration will approach 0 milligrams per liter.

93. $P(10)=\dfrac{100(10-1)}{10}=90 \quad (10, 90)$

For a disease that smokers are 10 times more likely to contact than non-smokers, 90% of the deaths are smoking related.

94. $P(9)=\dfrac{100(9-1)}{9}=89 \quad (9, 89)$

For a disease that smokers are 9 times more likely to have than non-smokers, 89% of the deaths are smoking related.

95. $y = 100$ As incidence of the diseases increases, the percent of death approaches, but never gets to be, 100%.

96. No, the percentage approaches 100%, but never reaches 100%.

97. a. $f(x)=\dfrac{11x^2+40x+1040}{12x^2+230x+2190}$

b. According to the graph, $\dfrac{1707.2}{2708.7}$ or about 63% of federal expenditures were spent on human resources in 2006.

c. According to the function,

$$f(36)=\frac{11(36)^2+40(36)+1040}{12(36)^2+230(36)+2190}=\frac{16736}{26022} \text{ or}$$

about 64% of federal expenditures were spent on human resources in 2006. This overestimates the actual percent found in the graph by 1%.

d. The horizontal asymptote is $y=\dfrac{11}{12}$.

If trends continue, $\dfrac{11}{12}$ or about 92% of federal expenditures will spent on human resources over time.

98. $x - 10$ = the average velocity on the return trip.
The function that expresses the total time required to complete the round trip is

$$T(x)=\frac{600}{x}+\frac{600}{x-10}.$$

99. $T(x)=\dfrac{90}{9x}+\dfrac{5}{x}=\dfrac{10}{x}+\dfrac{5}{x}$

The function that expresses the total time for driving and hiking is $T(x)=\dfrac{10}{x}+\dfrac{5}{x}$.

100. $A = xy = 2500$

$$y=\frac{2500}{x}$$

$$P=2x+2y=2x+2\left(\frac{2500}{x}\right)=2x+\frac{5000}{x}$$

The perimeter of the floor, P, as a function of the width, x is $P(x)=2x+\dfrac{5000}{x}$.

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101. $A = lw$

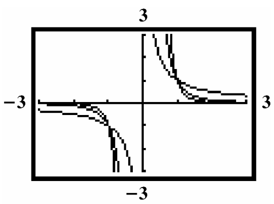
$$xy = 50$$
$$l = y + 2 = \frac{50}{x} + 2$$
$$w = x + 1$$
$$A = \frac{50}{x} + 2\ (x+1)$$
$$= 50 + \frac{50}{x} + 2x + 2$$
$$= 2x + \frac{50}{x} + 52$$

The total area of the page is

$$A(x) = 2x + \frac{50}{x} + 52.$$

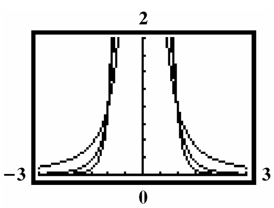
102. – 111. Answers may vary.

112.



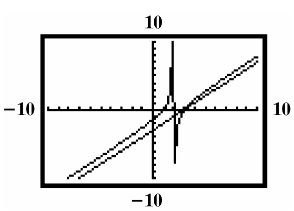
The graph approaches the horizontal asymptote faster and the vertical asymptote slower as n increases.

113.



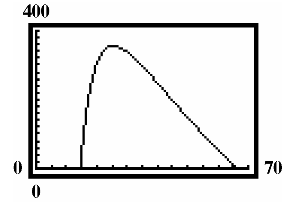
The graph approaches the horizontal asymptote faster and the vertical asymptote slower as n increases.

114.



$g(x)$ is the graph of a line where $f(x)$ is the graph of a rational function with a slant asymptote.
In $g(x)$, $x - 2$ is a factor of $x^2 - 5x + 6$.

115. a. $f(x) = \dfrac{27725(x-14)}{x^2+9} - 5x$



b. The graph increases from late teens until about the age of 25, and then the number of arrests decreases.

c. At age 25 the highest number arrests occurs. There are about 356 arrests for every 100,000 drivers.

116. does not make sense; Explanations will vary. Sample explanation: A rational function can have at most one horizontal asymptote.

117. does not make sense; Explanations will vary. Sample explanation: The function has one vertical asymptote, $x = 2$.

118. makes sense

119. does not make sense; Explanations will vary. Sample explanation: As production level increases, the average cost for a company to produce each unit of its product decreases.

120. false; Changes to make the statement true will vary. A sample change is: The graph of a rational function may have both a vertical asymptote and a horizontal asymptote.

121. true

122. true

123. true

124. – 127. Answers may vary.

128.

$$2x^2 + x = 15$$
$$2x^2 + x - 15 = 0$$
$$(2x-5)(x+3) = 0$$
$$2x - 5 = 0 \quad \text{or} \quad x + 3 = 0$$
$$x = \frac{5}{2} \qquad\qquad x = -3$$

The solution set is $\left\{-3, \frac{5}{2}\right\}$.

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129.
$$x^3 + x^2 = 4x + 4$$
$$x^3 + x^2 - 4x - 4 = 0$$
$$x^2(x+1) - 4(x+1) = 0$$
$$(x+1)(x^2 - 4) = 0$$
$$(x+1)(x+2)(x-2) = 0$$
The solution set is $\{-2, -1, 2\}$.

130.
$$\frac{x+1}{x+3} - 2 = \frac{x+1}{x+3} - \frac{2(x+3)}{x+3}$$
$$= \frac{x+1}{x+3} - \frac{2x+6}{x+3}$$
$$= \frac{x+1-2x-6}{x+3}$$
$$= \frac{-x-5}{x+3} \text{ or } -\frac{x+5}{x+3}$$

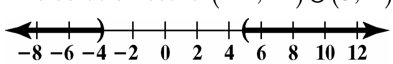
Section 2.7

Check Point Exercises

1.
$$x^2 - x > 20$$
$$x^2 - x - 20 > 0$$
$$(x+4)(x-5) > 0$$
Solve the related quadratic equation.
$$(x+4)(x-5) = 0$$
Apply the zero product principle.
$$x + 4 = 0 \quad \text{or} \quad x - 5 = 0$$
$$x = -4 \qquad\qquad x = 5$$
The boundary points are –2 and 4.

Test Interval	Test Number	Test	Conclusion
$(-\infty, -4)$	–5	$(-5)^2 - (-5) > 20$ $30 > 20$, true	$(-\infty, -4)$ belongs to the solution set.
$(-4, 5)$	0	$(0)^2 - (0) > 20$ $0 > 20$, false	$(-4, 5)$ does not belong to the solution set.
$(5, \infty)$	10	$(10)^2 - (10) > 20$ $90 > 20$, true	$(5, \infty)$ belongs to the solution set.

The solution set is $(-\infty, -4) \cup (5, \infty)$ or $\{x \mid x < -4 \text{ or } x > 5\}$.



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2.

$$x^3+3x^2 \le x+3$$
$$x^3+3x^2-x-3 \le 0$$
$$(x+1)(x-1)(x+3) \le 0$$
$$(x+1)(x-1)(x+3) = 0$$
$$x+1=0 \quad \text{or} \quad x-1=0 \quad \text{or} \quad x+3=0$$
$$x=-1 \qquad x=1 \qquad x=-3$$

Test Interval	Test Number	Test	Conclusion
$(-\infty,-3)$	-4	$(-4)^3+3(-4)^2 \le (-4)+3$ $-16 \le -1$ true	$(-\infty,-3)$ belongs to the solution set.
$(-3,-1]$	-2	$(-2)^3+3(-2)^2 \le (-2)+3$ $4 \le 1$ false	$(-3,-1]$ does not belong to the solution set.
$[-1,1]$	0	$(0)^3+3(0)^2 \le (0)+3$ $0 \le 3$ true	$[-1,1]$ belongs to the solution set.
$[1,\infty)$	2	$(6+3)(6-5)>0$ true	$[1,\infty)$ does not belong to the solution set.

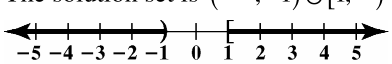
The solution set is $(-\infty,-3]\cup[-1,1]$ or $\{x \mid x \le -3 \text{ or } -1 \le x \le 1\}$.

3.

$$\frac{2x}{x+1} \ge 1$$
$$\frac{2x}{x+1} - 1 \ge 0$$
$$\frac{x-1}{x+1} \ge 0$$
$$x-1=0 \quad \text{or} \quad x+1=0$$
$$x=1 \qquad x=-1$$

Test Interval	Test Number	Test	Conclusion
$(-\infty,-1)$	-2	$\frac{2(-2)}{-2+1} \ge 1$ $4 \ge 1$, true	$(-\infty,-1)$ belongs to the solution set.
$(-1,1]$	0	$\frac{2(0)}{0+1} \ge 1$ $0 \ge 1$, false	$(-1,1]$ does not belong to the solution set.
$[1,\infty)$	2	$\frac{2(2)}{2+1} \ge 1$ $\frac{4}{3} \ge 1$, true	$[1,\infty)$ belongs to the solution set.

The solution set is $(-\infty,-1)\cup[1,\infty)$ or $\{x \mid x < -1 \text{ or } x \ge 1\}$.



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4.
$$-16t^2 + 80t > 64$$
$$-16t^2 + 80t - 64 > 0$$
$$-16(t-1)(t-4) > 0$$
$$t - 1 = 0 \quad \text{or} \quad t - 4 = 0$$
$$t = 1 \qquad\qquad t = 4$$

Test Interval	Test Number	Test	Conclusion
$(-\infty, 1)$	0	$-16(0)^2 + 80(0) > 64$ $0 > 64$, false	$(-\infty, 1)$ does not belong to the solution set.
$(1, 4)$	2	$-16(2)^2 + 80(2) > 64$ $96 > 64$, true	$(1, 4)$ belongs to the solution set.
$(4, \infty)$	5	$-16(5)^2 + 80(5) > 64$ $0 > 64$, false	$(4, \infty)$ does not belong to the solution set.

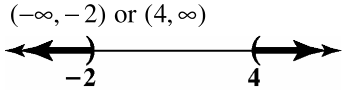
The object will be more than 64 feet above the ground between 1 and 4 seconds.

Exercise Set 2.7

1. $(x-4)(x+2) > 0$
$x = 4$ or $x = -2$

T	F	T
	-2	4

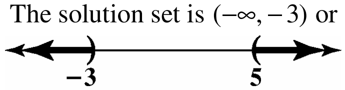
Test –3: $(-3-4)(-3+2) > 0$
$7 > 0$ True
Test 0: $(0-4)(0+2) > 0$
$-8 > 0$ False
Test 5: $(5-4)(5+2) > 0$
$7 > 0$ True
$(-\infty, -2)$ or $(4, \infty)$



2. $(x+3)(x-5) > 0$
$x = -3$ or $x = 5$

T	F	T
	-3	5

Test –4: $(-4+3)(-4-5) > 0$
$9 > 0$ True
Test 0: $(0+3)(0-5) > 0$
$-15 > 0$ False
Test 6: $(6+3)(6-5) > 0$
$18 > 0$ True
The solution set is $(-\infty, -3)$ or $(5, \infty)$.



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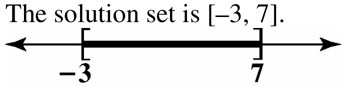
3. $(x-7)(x+3) \le 0$
$x = 7$ or $x = -3$

F	T	F

$-3 \quad 7$

Test –4: $(-4-7)(-4+3) \le 0$
$11 \le 0$ False
Test 0: $(0-7)(0+3) \le 0$
$-21 \le 0$ True
Test 8: $(8-7)(8+3) \le 0$
$11 \le 0$ False
The solution set is $[-3, 7]$.

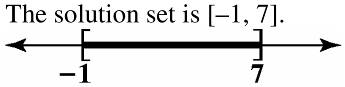


4. $(x+1)(x-7) \le 0$
$x = -1$ or $x = 7$

F	T	F

$-1 \quad 7$

Test –2: $(-2+1)(-2-7) \le 0$
$9 \le 0$ False
Test 0: $(0+1)(0-7) \le 0$
$-7 \le 0$ True
Test 8: $(8+1)(8-7) \le 0$
$9 \le 0$ False
The solution set is $[-1, 7]$.

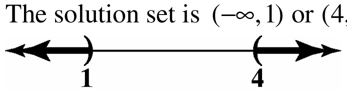


5. $x^2 - 5x + 4 > 0$
$(x-4)(x-1) > 0$
$x = 4$ or $x = 1$

T	F	T

$1 \quad 4$

Test 0: $0^2 - 5(0) + 4 > 0$
$4 > 0$ True
Test 2: $2^2 - 5(2) + 4 > 0$
$-2 > 0$ False
Test 5: $5^2 - 5(5) + 4 > 0$
$4 > 0$ True
The solution set is $(-\infty, 1)$ or $(4, \infty)$.

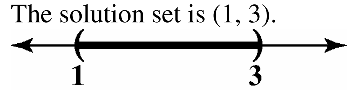


6. $x^2 - 4x + 3 < 0$
$(x-1)(x-3) < 0$
$x = 1$ or $x = 3$

F	T	F

$1 \quad 3$

Test 0: $0^2 - 4(0) + 3 < 0$
$3 < 0$ False
Test 2: $2^2 - 4(2) + 3 < 0$
$-1 < 0$ True
Test 4: $4^2 - 4(4) + 3 < 0$
$3 < 0$ False
The solution set is $(1, 3)$.

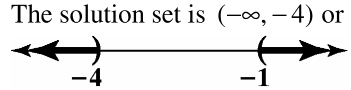


7. $x^2 + 5x + 4 > 0$
$(x+1)(x+4) > 0$
$x = -1$ or $x = -4$

T	F	T

$-4 \quad -1$

Test –5: $(-5)^2 + 5(-5) + 4 > 0$
$4 > 0$ True
Test –3: $(-3)^2 + 5(-3) + 4 > 0$
$-2 > 0$ False
Test 0: $0^2 + 5(0) + 4 > 0$
$4 > 0$ True
The solution set is $(-\infty, -4)$ or $(-1, \infty)$.

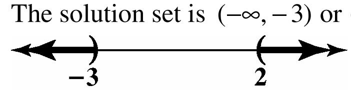


8. $x^2 + x - 6 > 0$
$(x+3)(x-2) > 0$
$x = -3$ or $x = 2$

T	F	T

$-3 \quad 2$

Test –4: $(-4)^2 - 4 - 6 > 0$
$6 > 0$ True
Test 0: $(0)^2 + 0 - 6 > 0$
$-6 > 0$ False
Test 3: $3^2 + 3 - 6 > 0$
$6 > 0$ True
The solution set is $(-\infty, -3)$ or $(2, \infty)$.



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9. $x^2 - 6x + 9 < 0$
$(x-3)(x-3) < 0$
$x = 3$

F	F

3

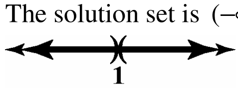
Test 0: $0^2 - 6(0) + 9 < 0$
$9 < 0$ False
Test 4: $4^2 - 6(4) + 9 < 0$
$1 < 0$ False
The solution set is the empty set, $\varnothing$.

10. $x^2 - 2x + 1 > 0$
$(x-1)(x-1) > 0$
$x = 1$

T	T

1

Test 0: $0^2 - 2(0) + 1 > 0$
$1 > 0$ True
Test 2: $2^2 - 2(2) + 1 > 0$
$1 > 0$ True
The solution set is $(-\infty, 1)$ or $(1, \infty)$.



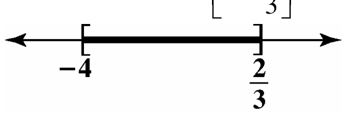
11. $3x^2 + 10x - 8 \le 0$
$(3x-2)(x+4) \le 0$
$x = \frac{2}{3}$ or $x = -4$

F	T	F

-4 $\frac{2}{3}$

Test –5: $3(-5)^2 + 10(-5) - 8 \le 0$
$17 \le 0$ False
Test 0: $3(0)^2 + 10(0) - 8 \le 0$
$8 \le 0$ True
Test 1: $3(1)^2 + 10(1) - 8 \le 0$
$5 \le 0$ False

The solution set is $\left[-4, \frac{2}{3}\right]$.

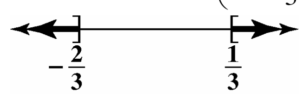


12. $9x^2 + 3x - 2 \ge 0$
$(3x-1)(3x+2) \ge 0$
$3x = 1 \quad 3x = -2$
$x = \frac{1}{3} \quad x = \frac{-2}{3}$

T	F	T

$\frac{-2}{3}$ $\frac{1}{3}$

Test –1: $9(-1)^2 + 3(-1) - 2 \ge 0$
$4 \ge 0$ True
Test 0: $9(0)^2 + 3(0) - 2 \ge 0$
$-2 \ge 0$ False
Test 1: $9(1)^2 + 3(1) - 2 \le 0$
$10 \ge 0$ True
The solution set is $\left(-\infty, \frac{-2}{3}\right]$ or $\left[\frac{1}{3}, \infty\right)$.



13. $2x^2 + x < 15$
$2x^2 + x - 15 < 0$
$(2x-5)(x+3) < 0$
$2x - 5 = 0$ or x+3=0
$2x = 5$
$x = \frac{5}{2}$ or $x = -3$

F	T	F

-3 $\frac{5}{2}$

Test –4: $2(-4)^2 + (-4) < 15$
28 < 15 False
Test 0: $2(0)^2 + 0 < 15$
0 < 15 True
Test 3: $2(3)^2 + 3 < 15$
21 < 15 False
The solution set is $\left(-3, \frac{5}{2}\right)$.

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14. $6x^2 + x > 1$

$6x^2 + x - 1 > 0$

$(2x+1)(3x-1) > 0$

$2x+1=0$ or $3x-1=0$

$2x=-1$ $\quad$ $3x=1$

$x=-\frac{1}{2}$ $\quad$ $x=\frac{1}{3}$

T	F	T

$-\frac{1}{2}$ $\quad$ $\frac{1}{3}$

Test –1: $6(-1)^2 + (-1) > 1$

$5 > 1$ True

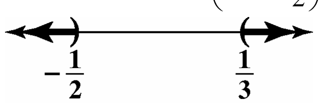
Test 0: $6(0)^2 + 0 > 1$

$0 > 1$ False

Test 1: $6(1)^2 + 1 > 1$

$7 > 1$ True

The solution set is $\left(-\infty, -\frac{1}{2}\right)$ or $\left(\frac{1}{3}, \infty\right)$.



15. $4x^2 + 7x < -3$

$4x^2 + 7x + 3 < 0$

$(4x+3)(x+1) < 0$

$4x+3=0$ or $x+1=0$

$4x$-3=0

$x=-\frac{3}{4}$ or $x=-1$

F	T	F

-1 $\quad$ $-\frac{3}{4}$

Test –2: $4(-2)^2 + 7(-2) < -3$

$2 < -3$ False

Test $-\frac{7}{8}$: $4\left(-\frac{7}{8}\right)^2 + 7\left(-\frac{7}{8}\right) < -3$

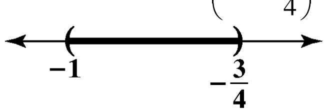
$\frac{49}{16} - \frac{49}{8} < -3$

$-\frac{49}{16} < -3$ True

Test 0: $4(0)^2 + 7(0) < -3$

$0 < -3$ False

The solution set is $\left(-1, -\frac{3}{4}\right)$.



16. $3x^2 + 16x < -5$

$3x^2 + 16x + 5 < 0$

$(3x+1)(x+5) < 0$

$3x+1=0$ or $x+5=0$

$3x=-1$

$x=-\frac{1}{3}$ $\quad$ $x=-5$

F	T	F

-5 $\quad$ $-\frac{1}{3}$

Test –6: $3(-6)^2 + 16(-6) < -5$

$12 < -5$ False

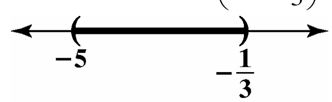
Test –2: $3(-2)^2 + 16(-2) < -5$

$-20 < -5$ True

Test 0: $3(0)^2 + 16(0) < -5$

$0 < -5$ False

The solution set is $\left(-5, -\frac{1}{3}\right)$.



17. $5x \le 2 - 3x^2$

$3x^2 + 5x - 2 \le 0$

$(3x-1)(x+2) \le 0$

$3x - 1 = 0$ or $x + 2 = 0$

$3x = 1$

$3x-1=0$ or $x+2=0$

$3x=1$

$x=\frac{1}{3}$ or $x=-2$

F	T	F

-2 $\quad$ $\frac{1}{3}$

Test –3: $5(-3) \le 2 - 3(-3)^2$

$-15 \le -25$ False

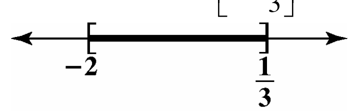
Test 0: $5(0) \le 2 - 3(0)^2$

$0 \le 2$ True

Test 1: $5(1) \le 2 - 3(1)^2$

$5 \le -1$ False

The solution set is $\left[-2, \frac{1}{3}\right]$.



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18. $4x^2+1\geq 4x$

$4x^2-4x+1\geq 0$

$(2x-1)(2x-1)\geq 0$

$2x-1=0$

$x=\frac{1}{2}$

T	T

$\frac{1}{2}$

Test 0: $4(0)^2+1\geq 4(0)$

$1\geq 0$ True

Test 1: $4(1)^2+1\geq 4(1)$

$5\geq 4$ True

The solution set is $(-\infty,\infty)$.

19. $x^2-4x\geq 0$

$x(x-4)\geq 0$

$x=0$ or x - 4 = 0

$x=4$

T	F	T

0 4

Test –1: $(-1)^2-4(-1)\geq 0$

$5\geq 0$ True

Test 1: $(1)^2-4(1)\geq 0$

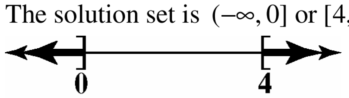
$-3\geq 0$ False

$0\leq 2$ True

Test 5: $5^2-4(5)\geq 0$

$5\geq 0$ True

The solution set is $(-\infty,0]$ or $[4,\infty)$.



20. $x^2+2x<0$

$x(x+2)<0$

$x=0$ or $x=-2$

F	T	F

–2 0

Test –3: $(-3)^2+2(-3)<0$

3 < 0 False

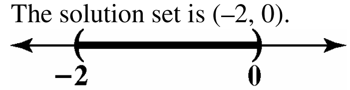
Test –1: $(-1)^2+2(-1)<0$

–1 < 0 True

Test 1: $(1)^2+2(1)<0$

3 < 0 False

The solution set is (–2, 0).



21. $2x^2+3x>0$

$x(2x+3)>0$

$x=0$ or $x=-\frac{3}{2}$

T	F	T

$-\frac{3}{2}$ 0

Test –2: $2(-2)^2+3(-2)>0$

2 > 0 True

Test –1: $2(-1)^2+3(-1)>0$

–1 > 0 False

Test 1: $2(1)^2+3(1)>0$

5 > 0 True

The solution set is $\left(-\infty,-\frac{3}{2}\right)$ or $(0,\infty)$.

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22. $3x^2 - 5x \le 0$

$x(3x-5) \le 0$

$x = 0$ or $x = \frac{5}{3}$

F	T	F
	0	$\frac{5}{3}$

Test –1: $3(-1)^2 - 5(-1) \le 0$

$8 \le 0$ False

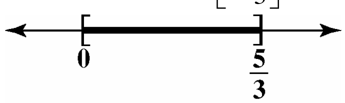
Test 1: $3(1)^2 - 5(1) \le 0$

$-2 \le 0$ True

Test 2: $3(2)^2 - 5(2) \le 0$

$2 \le 0$ False

The solution set is $\left[0, \frac{5}{3}\right]$.



23. $-x^2 + x \ge 0$

$x^2 - x \le 0$

$x(x-1) \le 0$

$x = 0$ or $x = 1$

F	T	F
	0	1

Test –1: $-(-1)^2 + (-1) \ge 0$

$-2 \ge 0$ False

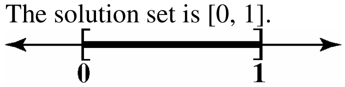
Test $\frac{1}{2}$: $-\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right) \ge 0$

$\frac{1}{4} \ge 0$ True

Test 2: $-(2)^2 + 2 \ge 0$

$-2 \ge 0$ False

The solution set is [0, 1].



24. $-x^2 + 2x \ge 0$

$x(-x+2) \ge 0$

$x = 0$ or $x = 2$

F	T	F
	0	2

Test –1: $-(-1)^2 + 2(-1) \ge 0$

$-3 \ge 0$ False

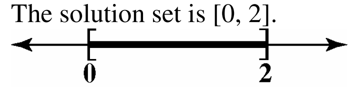
Test 1: $-(1)^2 + 2(1) \ge 0$

$1 \ge 0$ True

Test 3: $-(3)^2 + 2(3) \ge 0$

$-3 \ge 0$ False

The solution set is [0, 2].



25. $x^2 \le 4x - 2$

$x^2 - 4x + 2 \le 0$

Solve $x^2 - 4x + 2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)}$$

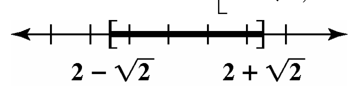
$$= \frac{4 \pm \sqrt{8}}{2}$$

$$= 2 \pm \sqrt{2}$$

$x \approx 0.59$ or $x \approx 3.41$

F	T	F
	0.59	3.41

The solution set is $\left[2-\sqrt{2}, 2+\sqrt{2}\right]$ or $[0.59, 3.41]$.



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26. $x^2 \le 2x+2$

$x^2-2x-2 \le 0$

Solve $x^2-2x-2=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(-2)}}{2(1)}$$

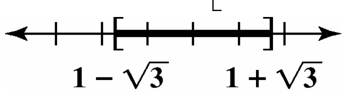
$$=\frac{2\pm\sqrt{12}}{2}$$

$$=1\pm\sqrt{3}$$

$x\approx -0.73$ or $x\approx 2.73$

F	T	F
	–0.73	2.73

The solution set is $\left[1-\sqrt{3},1+\sqrt{3}\right]$ or $\left[-0.73, 2.73\right]$.



27. $x^2-6x+9<0$

Solve $x^2-6x+9=0$

$(x-3)(x-3)=0$

$(x-3)^2=0$

$x=3$

F	F
	3

The solution set is the empty set, $\varnothing$.

28. $4x^2-4x+1\ge 0$

Solve $4x^2-4x+1=0$

$(2x-1)(2x-1)=0$

$(2x-1)^2=0$

$$x=\frac{1}{2}$$

T	T
	$\frac{1}{2}$

The solution set is $(-\infty,\infty)$.

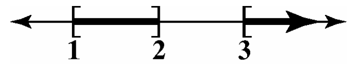
29. $(x-1)(x-2)(x-3)\ge 0$

Boundary points: 1, 2, and 3

Test one value in each interval.

F	T	F	T
	1	2	3

The solution set is [1, 2] ∪ [3, ∞).



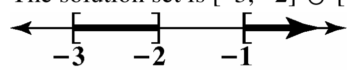
30. $(x+1)(x+2)(x+3)\ge 0$

Boundary points: –1, –2, and –3

Test one value in each interval.

F	T	F	T
	–3	–2	–1

The solution set is [–3, –2] ∪ [–1, ∞).



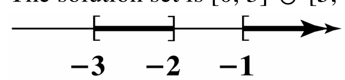
31. $x(3-x)(x-5)\le 0$

Boundary points: 0, 3, and 5

Test one value in each interval.

F	T	F	T
	0	3	5

The solution set is [0, 3] ∪ [5, ∞).



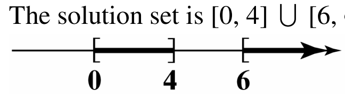
32. $x(4-x)(x-6)\le 0$

Boundary points: 0, 3, and 5

Test one value in each interval.

F	T	F	T
	0	4	6

The solution set is [0, 4] ∪ [6, ∞).



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33. $(2-x)^2\left(x-\frac{7}{2}\right)<0$

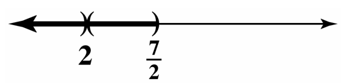
Boundary points: 2, and $\frac{7}{2}$

Test one value in each interval.

T	T	F

$2 \qquad \frac{7}{2}$

The solution set is $(-\infty,2)\cup\left(2,\frac{7}{2}\right)$.



34. $(5-x)^2\left(x-\frac{13}{2}\right)<0$

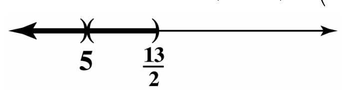
Boundary points: 5, and $\frac{13}{2}$

Test one value in each interval.

T	T	F

$5 \qquad \frac{13}{2}$

The solution set is $(-\infty,5)\cup\left(5,\frac{13}{2}\right)$.



35. $x^3+2x^2-x-2\geq 0$

$x^2(x+2)-1(x+2)\geq 0$

$(x+2)(x^2-1)\geq 0$

$(x+2)(x-1)(x+1)\geq 0$

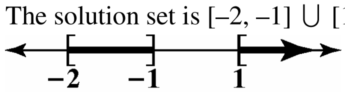
Boundary points: –2, –1, and 2

Test one value in each interval.

F	T	F	T

$-2 \qquad -1 \qquad 2$

The solution set is [–2, –1] ∪ [1, ∞).



36. $x^3+2x^2-4x-8\geq 0$

$x^2(x+2)-4(x+1)\geq 0$

$(x+2)(x^2-4)\geq 0$

$(x+2)(x+2)(x-2)\geq 0$

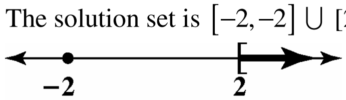
Boundary points: –2, and 2

Test one value in each interval.

F	F	T

$-2 \qquad 2$

The solution set is $[-2,-2]$ ∪ [2, ∞).



37. $x^3+2x^2-x-2\geq 0$

$x^2(x-3)-9(x-3)\geq 0$

$(x-3)(x^2-9)\geq 0$

$(x-3)(x+3)(x-3)\geq 0$

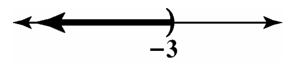
Boundary points: –3 and 3

Test one value in each interval.

T	F	F

$-3 \qquad 3$

The solution set is $(-\infty,-3]$.



38. $x^3+7x^2-x-7<0$

$x^2(x+7)-(x+7)<0$

$(x+7)(x^2-1)<0$

$(x+7)(x+1)(x-1)<0$

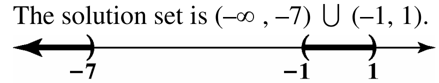
Boundary points: –7, –1 and 1

Test one value in each interval.

T	F	T	F

$-7 \qquad -1 \qquad 1$

The solution set is (–∞ , –7) ∪ (–1, 1).



39. $x^3+x^2+4x+4>0$

$x^2(x+1)+4(x+1)\geq 0$

$(x+1)(x^2+4)\geq 0$

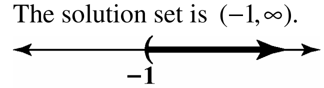
Boundary point: –1

Test one value in each interval.

F	T

-1

The solution set is $(-1,\infty)$.



40. $x^3-x^2+9x-9>0$

$x^2(x-1)+9(x-1)\geq 0$

$(x-1)(x^2+9)\geq 0$

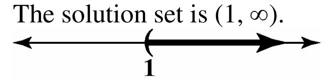
Boundary point: 1.

Test one value in each interval.

F	T

1

The solution set is (1, ∞).



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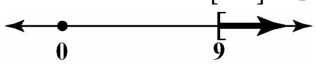
41. $x^3 - 9x^2 \geq 0$

$x^2(x-9) \geq 0$

Boundary points: 0 and 9
Test one value in each interval.

F	F	T
	0	9

The solution set is $[0,0] \cup [9, \infty)$.



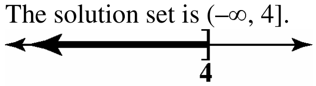
42. $x^3 - 4x^2 \leq 0$

$x^2(x-4) \leq 0$

Boundary points: 0 and 4.
Test one value in each interval.

T	T	F
	0	4

The solution set is $(-\infty, 4]$.



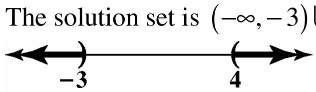
43. $\frac{x-4}{x+3} > 0$

$x - 4 = 0 \quad x + 3 = 0$

$x = 4 \qquad x = -3$

T	F	T
	−3	4

The solution set is $(-\infty, -3) \cup (4, \infty)$.

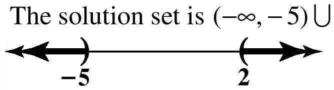


44. $\frac{x+5}{x-2} > 0$

$x = -5$ or $x = 2$

T	F	T
	−5	2

The solution set is $(-\infty, -5) \cup (2, \infty)$.

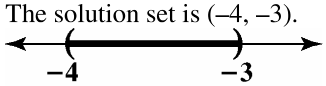


45. $\frac{x+3}{x+4} < 0$

$x = -3$ or $x = -4$

F	T	F
	−4	−3

The solution set is $(-4, -3)$.

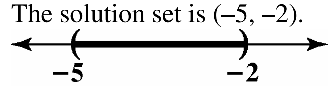


46. $\frac{x+5}{x+2} < 0$

$x = -5$ or $x = -2$

F	T	F
	−5	−2

The solution set is $(-5, -2)$.

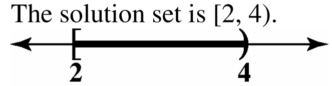


47. $\frac{-x+2}{x-4} \geq 0$

$x = 2$ or $x = 4$

F	T	F
	2	4

The solution set is $[2, 4)$.

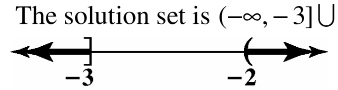


48. $\frac{-x-3}{x+2} \leq 0$

$x = -3$ or $x = -2$

T	F	T
	−3	−2

The solution set is $(-\infty, -3] \cup (-2, \infty)$.

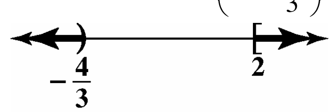


49. $\frac{4-2x}{3x+4} \leq 0$

$x = 2$ or $x = -\frac{4}{3}$

T	F	T
	$-\frac{4}{3}$	2

The solution set is $\left(-\infty, \frac{-4}{3}\right) \cup [2, \infty)$.



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50. $\dfrac{3x+5}{6-2x} \geq 0$

$x = -\dfrac{5}{3}$ or $x = 3$

F	T	F
	$-\frac{5}{3}$	3

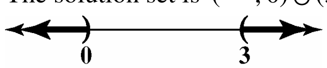
The solution set is $\left[-\dfrac{5}{3}, 3\right)$.

51. $\dfrac{x}{x-3} > 0$

$x = 0$ or $x = 3$

T	F	T
	0	3

The solution set is $(-\infty, 0) \cup (3, \infty)$.

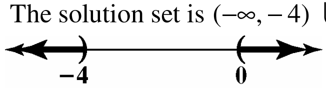


52. $\dfrac{x+4}{x} > 0$

$x = -4$ or $x = 0$

T	F	T
	–4	0

The solution set is $(-\infty, -4) \cup (0, \infty)$.



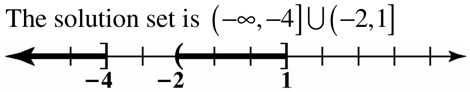
53. $\dfrac{(x+4)(x-1)}{x+2} \leq 0$

$x = -4$ or $x = -2$ or $x = 1$.

T	F	T	F
	–4	–2	1

Values of $x = -4$ or $x = 1$ result in $f(x) = 0$ and, therefore must be included in the solution set.

The solution set is $(-\infty, -4] \cup (-2, 1]$



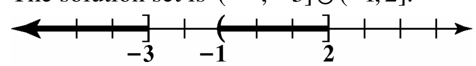
54. $\dfrac{(x+3)(x-2)}{x+1} \leq 0$

$x = -3$ or $x = -1$ or $x = 2$.

T	F	T	F
	–3	–1	2

Values of $x = -3$ or $x = 2$ result in $f(x) = 0$ and, therefore must be included in the solution set.

The solution set is $(-\infty, -3] \cup (-1, 2]$.

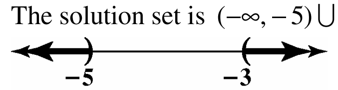


55.
$$\frac{x+1}{x+3} < 2$$
$$\frac{x+1}{x+3} - 2 < 0$$
$$\frac{x+1-2(x+3)}{x+3} < 0$$
$$\frac{x+1-2x-6}{x+3} < 0$$
$$\frac{-x-5}{x+3} < 0$$

$x =$ or $x = -3$

T	F	T
	–5	–3

The solution set is $(-\infty, -5) \cup (-3, \infty)$.

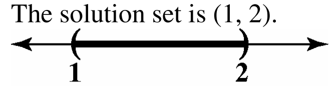


56.
$$\frac{x}{x-1} > 2$$
$$\frac{x}{x-1} - 2 > 0$$
$$\frac{x}{x-1} - \frac{2(x-1)}{x-1} > 0$$
$$\frac{x-2x+2}{x-1} > 0$$
$$\frac{-x+2}{x-1} > 0$$

$x = 2$ or $x = 1$

F	T	F
	1	2

The solution set is (1, 2).



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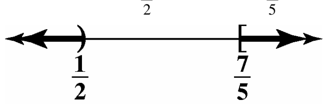
57.
$$\frac{x+4}{2x-1} \le 3$$
$$\frac{x+4}{2x-1} - 3 \le 0$$
$$\frac{x+4-3(2x-1)}{2x-1} \le 0$$
$$\frac{x+4-6x+3}{2x-1} \le 0$$
$$\frac{-5x+7}{2x-1} \le 0$$

$x = \frac{7}{5}$ or $x = \frac{1}{2}$

T	F	T
	$\frac{1}{2}$	$\frac{7}{5}$

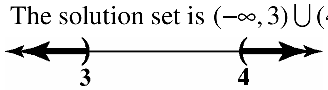


58.
$$\frac{1}{x-3} < 1$$
$$\frac{1}{x-3} - 1 < 0$$
$$\frac{1}{x-3} - \frac{x-3}{x-3} < 0$$
$$\frac{1-x+3}{x-3} < 0$$
$$\frac{-x+4}{x-3} < 0$$

$x = 4$ or $x = 3$

T	F	T
	3	4

The solution set is $(-\infty, 3) \cup (4, \infty)$.

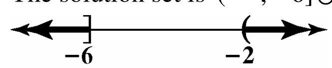


59.
$$\frac{x-2}{x+2} \le 2$$
$$\frac{x-2}{x+2} - 2 \le 0$$
$$\frac{x-2-2(x+2)}{x+2} \le 0$$
$$\frac{x-2-2x-4}{x+2} \le 0$$
$$\frac{-x-6}{x+2} \le 0$$

$x = -6$ or $x = -2$

T	F	T
	-6	-2

The solution set is $(-\infty, -6] \cup (-2, \infty)$.

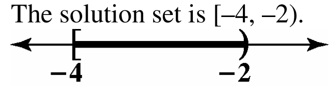


60.
$$\frac{x}{x+2} \ge 2$$
$$\frac{x}{x+2} - 2 \ge 0$$
$$\frac{x}{x+2} - \frac{2(x+2)}{x+2} \ge 0$$
$$\frac{x-2x-4}{x+2} \ge 0$$
$$\frac{-x-4}{x+2} \ge 0$$

$x = -4$ or $x = -2$

F	T	F
	-4	-2

The solution set is $[-4, -2)$.



61. $f(x) = \sqrt{2x^2 - 5x + 2}$

The domain of this function requires that
$2x^2 - 5x + 2 \ge 0$

Solve $2x^2 - 5x + 2 = 0$
$(x-2)(2x-1) = 0$
$x = \frac{1}{2}$ or $x = 2$

T	F	T
	$\frac{1}{2}$	2

The domain is $\left(-\infty, \frac{1}{2}\right] \cup [2, \infty)$.

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62. $f(x)=\dfrac{1}{\sqrt{4x^2-9x+2}}$

The domain of this function requires that $4x^2-9x+2>0$

Solve $4x^2-9x+2=0$

$(x-2)(4x-1)=0$

$x=\dfrac{1}{4}$ or $x=2$

T	F	T
	$\frac{1}{4}$	2

The domain is $\left(-\infty,\dfrac{1}{4}\right)\cup(2,\infty)$.

63. $f(x)=\sqrt{\dfrac{2x}{x+1}-1}$

The domain of this function requires that $\dfrac{2x}{x+1}-1\ge 0$ or $\dfrac{x-1}{x+1}\ge 0$

$x=-1$ or $x=1$

T	F	T
	-1	1

The value $x=1$ results in 0 and, thus, it must be included in the domain.

The domain is $(-\infty,-1)\cup[1,\infty)$.

64. $f(x)=\sqrt{\dfrac{x}{2x-1}-1}$

The domain of this function requires that $\dfrac{x}{2x-1}-1\ge 0$ or $\dfrac{-x+1}{2x-1}\ge 0$

$x=\dfrac{1}{2}$ or $x=1$

F	T	F
	$\frac{1}{2}$	1

The value $x=1$ results in 0 and, thus, it must be included in the domain.

The domain is $\left(\dfrac{1}{2},1\right]$.

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65. $\left|x^2+2x-36\right|>12$

Express the inequality without the absolute value symbol:

$x^2+2x-36<-12$ or $x^2+2x-36>12$

$x^2+2x-24<0$ $\quad$ $x^2+2x-48>0$

Solve the related quadratic equations.

$x^2+2x-24=0$ or $x^2+2x-48=0$

$(x+6)(x-4)=0$ $\quad$ $(x+8)(x-6)=0$

Apply the zero product principle.

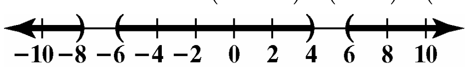
$x+6=0$ or $x-4=0$ or $x+8=0$ or $x-6=0$

$x=-6$ $\quad$ $x=4$ $\quad$ $x=-8$ $\quad$ $x=6$

The boundary points are $-8,\ -6,\ 4$ and 6.

Test Interval	Test Number	Test	Conclusion
$(-\infty,-8)$	-9	$\left\|(-9)^2+2(-9)-36\right\|>12$ $27>12$, True	$(-\infty,-8)$ belongs to the solution set.
$(-8,-6)$	-7	$\left\|(-7)^2+2(-7)-36\right\|>12$ $1>12$, False	$(-8,-6)$ does not belong to the solution set.
$(-6,4)$	0	$\left\|0^2+2(0)-36\right\|>12$ $36>12$, True	$(-6,4)$ belongs to the solution set.
$(4,6)$	5	$\left\|5^2+2(5)-36\right\|>12$ $1>12$, False	$(4,6)$ does not belong to the solution set.
$(6,\infty)$	7	$\left\|7^2+2(7)-36\right\|>12$ $27>12$, True	$(6,\infty)$ belongs to the solution set.

The solution set is $(-\infty,-8)\cup(-6,4)\cup(6,\infty)$ or $\{x \mid x<-8 \text{ or } -6<x<4 \text{ or } x>6\}$.



66. $\left|x^2+6x+1\right|>8$

Express the inequality without the absolute value symbol:

$x^2+6x+1<-8$ or $x^2+6x+1>8$

$x^2+6x+9<0$ $\quad$ $x^2+6x-7>0$

Solve the related quadratic equations.

$x^2+6x+9=0$ or $x^2+6x-7=0$

$(x+3)^2=0$ $\quad$ $(x+7)(x-1)=0$

$x+3=\pm\sqrt{0}$ or $x+7=0$ or $x-1=0$

$x+3=0$ $\quad$ $x=-7$ $\quad$ $x=1$

$x=-3$

The boundary points are $-7,-3,$ and 1.

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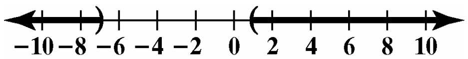
Test Interval	Test Number	Test	Conclusion
$(-\infty,-7)$	-8	$\lvert(-8)^2+6(-8)+1\rvert>8$ $17\ge 8$, True	$(-\infty,-7)$ belongs to the solution set.
$(-7,-3)$	-5	$\lvert(-5)^2+6(-5)+1\rvert>8$ $4\ge 8$, False	$(-7,-3)$ does not belong to the solution set.
$(-3,1)$	0	$\lvert 0^2+6(0)+1\rvert>8$ $1\ge 8$, False	$(-3,1)$ does not belong to the solution set.
$(1,\infty)$	2	$\lvert 2^2+6(2)+1\rvert>8$ $17\ge 8$, True	$(1,\infty)$ belongs to the solution set.

The solution set is $(-\infty,-7)\cup(1,\infty)$ or $\{x \mid x<-7 \text{ or } x>1\}$.



67. $\dfrac{3}{x+3}>\dfrac{3}{x-2}$

Express the inequality so that one side is zero.

$$\frac{3}{x+3}-\frac{3}{x-2}>0$$

$$\frac{3(x-2)}{(x+3)(x-2)}-\frac{3(x+3)}{(x+3)(x-2)}>0$$

$$\frac{3x-6-3x-9}{(x+3)(x-2)}<0$$

$$\frac{-15}{(x+3)(x-2)}<0$$

Find the values of x that make the denominator zero.

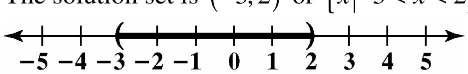
$x+3=0 \qquad x-2=0$

$x=-3 \qquad x=2$

The boundary points are -3 and 2.

Test Interval	Test Number	Test	Conclusion
$(-\infty,-3)$	-4	$\frac{3}{-4+3}>\frac{3}{-4-2}$ $-3>\frac{1}{2}$, False	$(-\infty,-3)$ does not belong to the solution set.
$(-3,2)$	0	$\frac{3}{0+3}>\frac{3}{0-2}$ $1>-\frac{3}{2}$, True	$(-3,2)$ belongs to the solution set.
$(2,\infty)$	3	$\frac{3}{3+3}>\frac{3}{3-2}$ $\frac{1}{2}>3$, False	$(2,\infty)$ does not belong to the solution set.

The solution set is $(-3,2)$ or $\{x \mid -3<x<2\}$.



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68. $\frac{1}{x+1} > \frac{2}{x-1}$

Express the inequality so that one side is zero.

$$\frac{1}{x+1} - \frac{2}{x-1} > 0$$

$$\frac{x-1}{(x+1)(x-1)} - \frac{2(x+1)}{(x+1)(x-1)} > 0$$

$$\frac{x-1-2x-2}{(x+1)(x-1)} < 0$$

$$\frac{-x-3}{(x+1)(x-1)} < 0$$

Find the values of x that make the numerator and denominator zero.

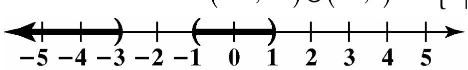
$-x-3=0$ $\quad$ $x+1=0$ $\quad$ $x-1=0$

$-3=x$ $\quad$ $x=-1$ $\quad$ $x=1$

The boundary points are -3, -1, and 1.

Test Interval	Test Number	Test	Conclusion
$(-\infty,-3)$	-4	$\frac{1}{-4+1} > \frac{2}{-3-1}$ $-\frac{1}{3} > -\frac{1}{2}$, True	$(-\infty,-3)$ belongs to the solution set.
$(-3,-1)$	-2	$\frac{1}{-2+1} > \frac{2}{-2-1}$ $-1 > -\frac{2}{3}$, False	$(-3,-1)$ does not belong to the solution set.
$(-1,1)$	0	$\frac{1}{0+1} > \frac{2}{0-1}$ $1 > -2$, True	$(-3,1)$ belongs to the solution set.
$(1,\infty)$	2	$\frac{1}{2+1} > \frac{2}{2-1}$ $\frac{1}{3} > 1$, False	$(1,\infty)$ does not belong to the solution set.

The solution set is $(-\infty,-3)\cup(-1,1)$ or $\{x \mid x<-3 \text{ or } -1<x<1\}$.



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69. $\dfrac{x^2-x-2}{x^2-4x+3}>0$

Find the values of x that make the numerator and denominator zero.

$x^2-x-2=0 \qquad x^2-4x+3=0$

$(x-2)(x+1)=0 \qquad (x-3)(x-1)=0$

Apply the zero product principle.

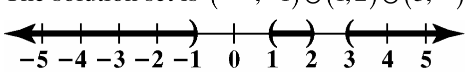
$x-2=0$ or $x+1=0 \qquad x-3=0$ or $x-1=0$

$x=2 \qquad x=-1 \qquad x=3 \qquad x=1$

The boundary points are -1, 1, 2 and 3.

Test Interval	Test Number	Test	Conclusion
$(-\infty,-1)$	-2	$\dfrac{(-2)^2-(-2)-2}{(-2)^2-4(-2)+3}>0$ $\dfrac{4}{15}>0$, True	$(-\infty,-1)$ belongs to the solution set.
$(-1,1)$	0	$\dfrac{0^2-0-2}{0^2-4(0)+3}>0$ $-\dfrac{2}{3}>0$, False	$(-1,1)$ does not belong to the solution set.
$(1,2)$	1.5	$\dfrac{1.5^2-1.5-2}{1.5^2-4(1.5)+3}>0$ $\dfrac{5}{3}>0$, True	$(1,2)$ belongs to the solution set.
$(2,3)$	2.5	$\dfrac{2.5^2-2.5-2}{2.5^2-4(2.5)+3}>0$ $-\dfrac{7}{3}>0$, False	$(2,3)$ does not belong to the solution set.
$(3,\infty)$	4	$\dfrac{4^2-4-2}{4^2-4(4)+3}>0$ $\dfrac{10}{3}>0$, True	$(3,\infty)$ belongs to the solution set.

The solution set is $(-\infty,-1)\cup(1,2)\cup(3,\infty)$ or $\{x \mid x<-1 \text{ or } 1<x<2 \text{ or } x>3\}$.



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70. $\dfrac{x^2-3x+2}{x^2-2x-3}>0$

Find the values of x that make the numerator and denominator zero.

$x^2-3x+2=0 \qquad x^2-2x-3=0$

$(x-2)(x-1)=0 \qquad (x-3)(x+1)=0$

Apply the zero product principle.

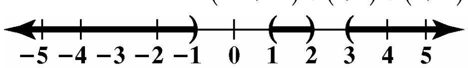
$x-2=0$ or $x-1=0 \qquad x-3=0$ or $x+1=0$

$x=2 \qquad x=1 \qquad x=3 \qquad x=-1$

The boundary points are -1 , 1, 2 and 3.

Test Interval	Test Number	Test	Conclusion
$(-\infty,-1)$	-2 $\dfrac{x^2-3x+2}{x^2-2x-3}>0$	$\dfrac{(-2)^2-3(-2)+2}{(-2)^2-2(-2)-3}>0$ $\dfrac{12}{5}>0$, True	$(-\infty,-1)$ belongs to the solution set.
$(-1,1)$	0	$\dfrac{0^2-3(0)+2}{0^2-2(0)-3}>0$ $-\dfrac{2}{3}>0$, False	$(-1,1)$ does not belong to the solution set.
$(1,2)$	1.5	$\dfrac{1.5^2-3(1.5)+2}{1.5^2-2(1.5)-3}>0$ $\dfrac{1}{15}>0$, True	$(1,2)$ belongs to the solution set.
$(2,3)$	2.5	$\dfrac{2.5^2-3(2.5)+2}{2.5^2-2(2.5)-3}>0$ $-\dfrac{3}{7}>0$, False	$(2,3)$ does not belong to the solution set.
$(3,\infty)$	4	$\dfrac{4^2-3(4)+2}{4^2-2(4)-3}>0$ $\dfrac{6}{5}>0$, True	$(3,\infty)$ belongs to the solution set.

The solution set is $(-\infty,-1)\cup(1,2)\cup(3,\infty)$ or $\{x|x<-1 \text{ or } 1<x<2 \text{ or } x>3\}$.



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71.

$$2x^3 + 11x^2 \ge 7x + 6$$
$$2x^3 + 11x^2 - 7x - 6 \ge 0$$

The graph of $f(x) = 2x^3 + 11x^2 - 7x - 6$ appears to cross the *x*-axis at -6, $-\frac{1}{2}$, and 1. We verify this numerically by substituting these values into the function:

$$f(-6) = 2(-6)^3 + 11(-6)^2 - 7(-6) - 6 = 2(-216) + 11(36) - (-42) - 6 = -432 + 396 + 42 - 6 = 0$$
$$f\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)^3 + 11\left(-\frac{1}{2}\right)^2 - 7\left(-\frac{1}{2}\right) - 6 = 2\left(-\frac{1}{8}\right) + 11\left(\frac{1}{4}\right) - \left(-\frac{7}{2}\right) - 6 = -\frac{1}{4} + \frac{11}{4} + \frac{7}{2} - 6 = 0$$
$$f(1) = 2(1)^3 + 11(1)^2 - 7(1) - 6 = 2(1) + 11(1) - 7 - 6 = 2 + 11 - 7 - 6 = 0$$

Thus, the boundaries are -6, $-\frac{1}{2}$, and 1. We need to find the intervals on which $f(x) \ge 0$. These intervals are indicated on the graph where the curve is above the *x*-axis. Now, the curve is above the *x*-axis when $-6 < x < -\frac{1}{2}$ and when $x > 1$. Thus, the solution set is $\left\{x \,\middle|\, -6 \le x \le -\frac{1}{2} \text{ or } x \ge 1\right\}$ or $\left[-6, -\frac{1}{2}\right] \cup [1, \infty)$.

72.

$$2x^3 + 11x^2 < 7x + 6$$
$$2x^3 + 11x^2 - 7x - 6 < 0$$

In Problem 63, we verified that the boundaries are -6, $-\frac{1}{2}$, and 1. We need to find the intervals on which $f(x) < 0$. These intervals are indicated on the graph where the curve is below the *x*-axis. Now, the curve is below the *x*-axis when $x < -6$ and when $-\frac{1}{2} < x < 1$. Thus, the solution set is $\left\{x \,\middle|\, x < -6 \text{ or } -\frac{1}{2} < x < 1\right\}$ or $(-\infty, -6) \cup \left(-\frac{1}{2}, 1\right)$.

73.

$$\frac{1}{4(x+2)} \le -\frac{3}{4(x-2)}$$
$$\frac{1}{4(x+2)} + \frac{3}{4(x-2)} \le 0$$

Simplify the left side of the inequality:

$$\frac{x-2}{4(x+2)} + \frac{3(x+2)}{4(x-2)} = \frac{x-2+3x+6}{4(x+2)(x-2)} = \frac{4x+4}{4(x+2)(x-2)} = \frac{4(x+1)}{4(x+2)(x-2)} = \frac{x+1}{x^2-4}.$$

The graph of $f(x) = \frac{x+1}{x^2-4}$ crosses the *x*-axis at -1, and has vertical asymptotes at $x = -2$ and $x = 2$. Thus, the boundaries are -2, -1, and 1. We need to find the intervals on which $f(x) \le 0$. These intervals are indicated on the graph where the curve is below the *x*-axis. Now, the curve is below the *x*-axis when $x < -2$ and when $-1 < x < 2$. Thus, the solution set is $\{x \mid x < -2 \text{ or } -1 \le x < 2\}$ or $(-\infty, -2) \cup [-1, 2)$.

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74.
$$\frac{1}{4(x+2)} > -\frac{3}{4(x-2)}$$
$$\frac{1}{4(x+2)} + \frac{3}{4(x-2)} > 0$$
$$\frac{x+1}{(x+2)(x-2)} > 0$$

The boundaries are -2, -1, and 2. We need to find the intervals on which $f(x) > 0$. These intervals are indicated on the graph where the curve is above the *x*-axis. The curve is above the *x*-axis when $-2 < x < -1$ and when $x > 2$. Thus, the solution set is $\{x \mid -2 < x < -1 \text{ or } x > 2\}$ or $(-2,-1) \cup (2,\infty)$.

75. $s(t) = -16t^2 + 8t + 87$

The diver's height will exceed that of the cliff when $s(t) > 87$
$$-16t^2 + 8t + 87 > 87$$
$$-16t^2 + 8t > 0$$
$$-8t(2t-1) > 0$$

The boundaries are 0 and $\frac{1}{2}$. Testing each interval shows that the diver will be higher than the cliff for the first half second after beginning the jump. The interval is $\left(0, \frac{1}{2}\right)$.

76. $s(t) = -16t^2 + 48t + 160$

The ball's height will exceed that of the rooftop when $s(t) > 160$
$$-16t^2 + 48t + 160 > 160$$
$$-16t^2 + 48t > 0$$
$$-16t(t-3) > 0$$

The boundaries are 0 and 3. Testing each interval shows that the ball will be higher than the rooftop for the first three seconds after the throw. The interval is $(0,3)$.

77. $f(x) = 0.0875x^2 - 0.4x + 66.6$

$g(x) = 0.0875x^2 + 1.9x + 11.6$

a. $f(35) = 0.0875(35)^2 - 0.4(35) + 66.6 \approx 160$ feet

$g(35) = 0.0875(35)^2 + 1.9(35) + 11.6 \approx 185$ feet

b. Dry pavement: graph (b)
Wet pavement: graph (a)

c. The answers to part (a) model the actual stopping distances shown in the figure extremely well. The function values and the data are identical.

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d. $0.0875x^2 - 0.4x + 66.6 > 540$

$0.0875x^2 - 0.4x + 473.4 > 0$

Solve the related quadratic equation.

$0.0875x^2 - 0.4x + 473.4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-0.4) \pm \sqrt{(-0.4)^2 - 4(0.0875)(473.4)}}{2(0.0875)}$$

$x \approx -71$ or 76

Since the function's domain is $x \geq 30$, we must test the following intervals.

Interval	Test Value	Test	Conclusion
$(30, 76)$	50	$0.0875(50)^2 - 0.4(50) + 66.6 > 540$ $265.35 > 540$, False	$(30, 76)$ does not belong to the solution set.
$(76, \infty)$	100	$0.0875(100)^2 - 0.4(100) + 66.6 > 540$ $901.6 > 540$, True	$(76, \infty)$ belongs to the solution set.

On dry pavement, stopping distances will exceed 540 feet for speeds exceeding 76 miles per hour. This is represented on graph (b) to the right of point (76, 540).

78. $f(x) = 0.0875x^2 - 0.4x + 66.6$

$g(x) = 0.0875x^2 + 1.9x + 11.6$

a. $f(55) = 0.0875(55)^2 - 0.4(55) + 66.6 \approx 309$ feet

$g(55) = 0.0875(55)^2 + 1.9(55) + 11.6 \approx 381$ feet

b. Dry pavement: graph (b)
Wet pavement: graph (a)

c. The answers to part (a) model the actual stopping distances shown in the figure extremely well.

d. $0.0875x^2 + 1.9x + 11.6 > 540$

$0.0875x^2 + 1.9x + 528.4 > 0$

Solve the related quadratic equation.

$0.0875x^2 + 1.9x + 528.4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(1.9) \pm \sqrt{(1.9)^2 - 4(0.0875)(528.4)}}{2(0.0875)}$$

$x \approx -89$ or 68

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Since the function's domain is $x \geq 30,$ we must test the following intervals.

Interval	Test Value	Test	Conclusion
$(30,68)$	50	$0.0875(50)^2 + 1.9(50) + 11.6 > 540$ $325.35 > 540$, False	$(30,68)$ does not belong to the solution set.
$(68,\infty)$	100	$0.0875(100)^2 + 1.9(100) + 11.6 > 540$ $1076.6 > 540$, True	$(68,\infty)$ belongs to the solution set.

On wet pavement, stopping distances will exceed 540 feet for speeds exceeding 68 miles per hour. This is represented on graph (a) to the right of point (68, 540).

79. Let x = the length of the rectangle.
Since $\text{Perimeter} = 2(\text{length}) + 2(\text{width})$, we know

$$50 = 2x + 2(\text{width})$$
$$50 - 2x = 2(\text{width})$$
$$\text{width} = \frac{50-2x}{2} = 25 - x$$

Now, $A = (\text{length})(\text{width})$, so we have that

$$A(x) \leq 114$$
$$x(25-x) \leq 114$$
$$25x - x^2 \leq 114$$

Solve the related equation

$$25x - x^2 = 114$$
$$0 = x^2 - 25x + 114$$
$$0 = (x-19)(x-6)$$

Apply the zero product principle:

$x - 19 = 0$ or $x - 6 = 0$

$x = 19$ $\qquad$ $x = 6$

The boundary points are 6 and 19.

Test Interval	Test Number	Test	Conclusion
$(-\infty,6)$	0	$25(0) - 0^2 \leq 114$ $0 \leq 114$, True	$(-\infty,6)$ belongs to the solution set.
$(6,19)$	10	$25(10) - 10^2 \leq 114$ $150 \leq 114$, False	$(6,19)$ does not belong to the solution set.
$(19,\infty)$	20	$25(20) - 20^2 \leq 114$ $100 \leq 114$, True	$(19,\infty)$ belongs to the solution set.

If the length is 6 feet, then the width is 19 feet. If the length is less than 6 feet, then the width is greater than 19 feet. Thus, if the area of the rectangle is not to exceed 114 square feet, the length of the shorter side must be 6 feet or less.

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80. $2l + 2w = P$

$2l + 2w = 180$

$2l = 180 - 2w$

$l = 90 - w$

We want to restrict the area to 800 square feet. That is,

$$A \le 800$$
$$l \cdot w \le 800$$
$$(90 - w)w \le 800$$
$$90w - w^2 \le 800$$
$$-w^2 + 90w - 800 \le 0$$
$$w^2 - 90w + 800 \ge 0$$
$$w^2 - 90w + 800 = 0$$
$$(w - 80)(w - 10) = 0$$
$$w - 80 = 0 \quad \text{or} \quad w - 10 = 0$$
$$w = 80 \qquad\qquad w = 10$$

Assuming the width is the shorter side, we ignore the larger solution.

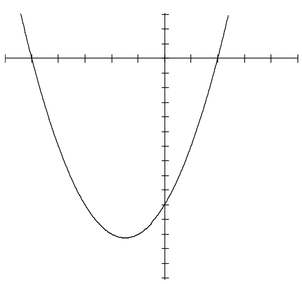
Test Interval	Test Number	Test	Conclusion
$(0,10)$	5	$90(5)-(5)^2 \le 800$ true	$(0,10)$ is part of the solution set
$(10,45)$	20	$90(20)-(20)^2 \le 800$ false	$(10,45)$ is not part of the solution set

The solution set is $\{w \mid 0 < w \le 10\}$ or $(0,10]$.

The length of the shorter side cannot exceed 10 feet.

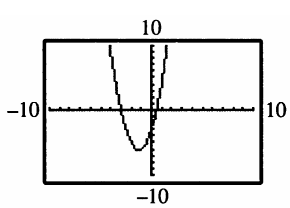
81. – 85. Answers may vary.

86.



The solution set is $(-\infty, -5) \cup (2, \infty)$.

87. Graph $y_1 = 2x^2 + 5x - 3$ in a standard window. The graph is below or equal to the x-axis for $-3 \le x \le \frac{1}{2}$.



The solution set is $\left\{x \,\middle|\, -3 \le x \le \frac{1}{2}\right\}$ or $\left[-3, \frac{1}{2}\right]$.

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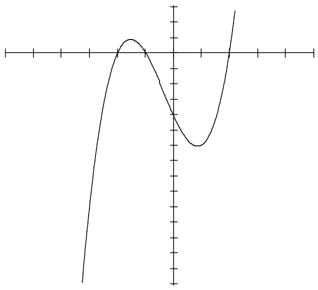
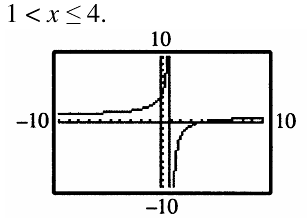
88. 

The solution set is (–2, –1) or (2, ∞).

89. Graph $y_1 = \dfrac{x-4}{x-1}$ in a standard viewing window. The graph is below the *x*-axis for $1 < x \le 4$.



The solution set is (1, 4].

90. Graph $y_1 = \dfrac{x+2}{x-3}$ and $y_2 = 2$

y_1 less than or equal to y_2 for $x < 3$ or $x \ge 8$.

The solution set is $(-\infty, 3) \cup [8, \infty)$

91. Graph $y_1 = \dfrac{1}{x+1}$ and $y_2 = \dfrac{2}{x+4}$

y_1 less than or equal to y_2 for $-4 < x < -1$ or $x \ge 2$.

The solution set is $(-4, -1) \cup [2, \infty)$

92. **a.** $f(x) = 0.1125x^2 - 0.1x + 55.9$

b. $0.1125x^2 - 0.1x + 55.9 > 455$

$0.1125x^2 - 0.1x + 399.1 > 0$

Solve the related quadratic equation.

$0.1125x^2 - 0.1x + 399.1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-0.1) \pm \sqrt{(-0.1)^2 - 4(0.1125)(399.1)}}{2(0.1125)}$$

$x \approx -59$ or 60

Since the function's domain must be $x \ge 0$, we must test the following intervals.

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Interval	Test Value	Test	Conclusion
$(0,60)$	50	$0.1125(50)^2-0.1(50)+55.9>455$ $332.15>455$, False	$(0,60)$ does not belong to the solution set.
$(60,\infty)$	100	$0.1125(100)^2-0.1(100)+55.9>455$ $1170.9>455$, True	$(60,\infty)$ belongs to the solution set.

On dry pavement, stopping distances will exceed 455 feet for speeds exceeding 60 miles per hour.

93. **a.** $f(x)=0.1375x^2+0.7x+37.8$

b. $0.1375x^2+0.7x+37.8>446$

$0.1375x^2+0.7x+408.2>0$

Solve the related quadratic equation.

$0.1375x^2+0.7x+408.2=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(0.7)\pm\sqrt{(0.7)^2-4(0.1375)(408.2)}}{2(0.1375)}$$

$x\approx-57$ or 52

Since the function's domain must be $x\geq0$, we must test the following intervals.

Interval	Test Value	Test	Conclusion
$(0,52)$	10	$0.1375(10)^2+0.7(10)+37.8>446$ $58.55>446$, False	$(0,52)$ does not belong to the solution set.
$(52,\infty)$	100	$0.1375(100)^2+0.7(100)+37.8>446$ $1482.8>446$, True	$(52,\infty)$ belongs to the solution set.

On wet pavement, stopping distances will exceed 446 feet for speeds exceeding 52 miles per hour.

94. makes sense

95. does not make sense; Explanations will vary. Sample explanation: Polynomials are defined for all values.

96. makes sense

97. does not make sense; Explanations will vary. Sample explanation: To solve this inequality you must first subtract 2 from both sides.

98. false; Changes to make the statement true will vary. A sample change is: The solution set is $\{x|x<-5 \text{ or } x>5\}$ or $(-\infty,-5)\cup(5,\infty)$.

99. false; Changes to make the statement true will vary. A sample change is: The inequality cannot be solved by multiplying both sides by $x+3$. We do not know if $x+3$ is positive or negative. Thus, we would not know whether or not to reverse the order of the inequality.

100. false; Changes to make the statement true will vary. A sample change is: The inequalities have different solution sets. The value, 1, is included in the domain of the first inequality, but not included in the domain of the second inequality.

101. true

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102. One possible solution: $x^2 - 2x - 15 \le 0$

103. One possible solution: $\dfrac{x-3}{x+4} \ge 0$

104. Because any non-zero number squared is positive, the solution is all real numbers except 2.

105. Because any number squared other than zero is positive, the solution includes only 2.

106. Because any number squared is positive, the solution is the empty set, $\varnothing$.

107. Because any number squared other than zero is positive, and the reciprocal of zero is undefined, the solution is all real numbers except 2.

108. a. The solution set is all real numbers.

b. The solution set is the empty set, $\varnothing$.

c.
$$4x^2 - 8x + 7 > 0$$
$$x = \frac{8 \pm \sqrt{(-8)^2 - 4(4)(7)}}{2(4)}$$
$$x = \frac{8 \pm \sqrt{64 - 112}}{8}$$
$$x = \frac{8 \pm \sqrt{-48}}{8} \Rightarrow \text{imaginary}$$

no critical values

Test 0: $4(0)^2 - 8(0) + 7 > 0$

$7 > 0$ True

The inequality is true for all numbers.

$$4x^2 - 8x + 7 < 0$$

no critical values

Test 0: $4(0)^2 - 8(0) + 7 = 7 < 0$ False

The solution set is the empty set.

109.
$$\sqrt{27 - 3x^2} \ge 0$$
$$27 - 3x^2 \ge 0$$
$$9 - x^2 \ge 0$$
$$(3-x)(3+x) \ge 0$$
$$3 - x = 0 \qquad 3 + x = 0$$
$$x = 3 \text{ or } \qquad x = -3$$

	T	
	–3	3

Test –4: $\sqrt{27 - 3(-4)^2} \ge 0$
$$\sqrt{27 - 48} \ge 0$$
$$\sqrt{-21} \ge 0$$

no graph- imaginary

Test 0: $\sqrt{27 - 3(0)^2} \ge 0$
$$\sqrt{27} \ge 0 \text{ True}$$

Test 4: $\sqrt{27 - 3(4)^2} \ge 0$
$$\sqrt{27 - 48} \ge 0$$
$$\sqrt{-21} \ge 0$$

no graph -imaginary

The solution set is [–3, 3].

110. a.
$$y = kx^2$$
$$64 = k \cdot 2^2$$
$$64 = 4k$$
$$16 = k$$

b.
$$y = kx^2$$
$$y = 16x^2$$

c.
$$y = kx^2$$
$$y = 16x^2$$
$$y = 16 \cdot 5^2$$
$$y = 400$$

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111. **a.** $y = \frac{k}{x}$

$12 = \frac{k}{8}$

$96 = k$

b. $y = \frac{k}{x}$

$y = \frac{96}{x}$

c. $y = \frac{96}{x}$

$y = \frac{96}{3}$

$y = 32$

112. $S = \frac{kA}{P}$

$12,000 = \frac{k \cdot 60,000}{40}$

$\frac{12,000 \cdot 40}{60,000} = k$

$8 = k$

Section 2.8

Check Point Exercises

1. y varies directly as x is expressed as $y = kx$.
The volume of water, W, varies directly as the time, t can be expressed as $W = kt$.
Use the given values to find k.

$W = kt$

$30 = k(5)$

$6 = k$

Substitute the value of k into the equation.

$W = kt$

$W = 6t$

Use the equation to find W when $t = 11$.

$W = 6t$

$= 6(11)$

$= 66$

A shower lasting 11 minutes will use 66 gallons of water.

2. y varies directly as the cube of x is expressed as $y = kx^3$.
The weight, w, varies directly as the cube of the length, l can be expressed as $w = kl^2$.
Use the given values to find k.

$w = kl^3$

$2025 = k(15)^3$

$0.6 = k$

Substitute the value of k into the equation.

$w = kl^3$

$w = 0.6l^3$

Use the equation to find w when $l = 25$.

$w = 0.6l^3$

$= 0.6(25)^3$

$= 9375$

The 25-foot long shark was 9375 pounds.

3. y varies inversely as x is expressed as $y = \frac{k}{x}$.
The length, L, varies inversely as the frequency, f can be expressed as $L = \frac{k}{f}$.
Use the given values to find k.

$L = \frac{k}{f}$

$8 = \frac{k}{640}$

$5120 = k$

Substitute the value of k into the equation.

$L = \frac{k}{f}$

$L = \frac{5120}{f}$

Use the equation to find f when $L = 10$.

$L = \frac{5120}{f}$

$10 = \frac{5120}{f}$

$10f = 5120$

$f = 512$

A 10 inch violin string will have a frequency of 512 cycles per second.

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4. let M represent the number of minutes
let Q represent the number of problems
let P represent the number of people
M varies directly as Q and inversely as P is expressed as $M = \frac{kQ}{P}$.
Use the given values to find k.

$$M = \frac{kQ}{P}$$
$$32 = \frac{k(16)}{4}$$
$$8 = k$$

Substitute the value of k into the equation.

$$M = \frac{kQ}{P}$$
$$M = \frac{8Q}{P}$$

Use the equation to find M when $P = 8$ and $Q = 24$.

$$M = \frac{8Q}{P}$$
$$M = \frac{8(24)}{8}$$
$$M = 24$$

It will take 24 minutes for 8 people to solve 24 problems.

5. V varies jointly with h and r^2 and can be modeled as $V = khr^2$.
Use the given values to find k.

$$V = khr^2$$
$$120\pi = k(10)(6)^2$$
$$\frac{\pi}{3} = k$$

Therefore, the volume equation is $V = \frac{1}{3}hr^2$.

$$V = \frac{\pi}{3}(2)(12)^2 = 96\pi \text{ cubic feet}$$

Exercise Set 2.8

1. Use the given values to find k.

$$y = kx$$
$$65 = k \cdot 5$$
$$\frac{65}{5} = \frac{k \cdot 5}{5}$$
$$13 = k$$

The equation becomes $y = 13x$.
When $x = 12$, $y = 13x = 13 \cdot 12 = 156$.

2.

$$y = kx$$
$$45 = k \cdot 5$$
$$9 = k$$

$$y = 9x = 9 \cdot 13 = 117$$

3. Since y varies inversely with x, we have $y = \frac{k}{x}$.
Use the given values to find k.

$$y = \frac{k}{x}$$
$$12 = \frac{k}{5}$$
$$5 \cdot 12 = 5 \cdot \frac{k}{5}$$
$$60 = k$$

The equation becomes $y = \frac{60}{x}$.

When $x = 2$, $y = \frac{60}{2} = 30$.

4.

$$y = \frac{k}{x}$$
$$6 = \frac{k}{3}$$
$$18 = k$$

$$y = \frac{18}{9} = 2$$

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5. Since y varies inversely as x and inversely as the square of z, we have $y = \frac{kx}{z^2}$.

Use the given values to find k.

$$y = \frac{kx}{z^2}$$
$$20 = \frac{k(50)}{5^2}$$
$$20 = \frac{k(50)}{25}$$
$$20 = 2k$$
$$10 = k$$

The equation becomes $y = \frac{10x}{z^2}$.

When $x = 3$ and $z = 6$,

$$y = \frac{10x}{z^2} = \frac{10(3)}{6^2} = \frac{10(3)}{36} = \frac{30}{36} = \frac{5}{6}.$$

6.

$$a = \frac{kb}{c^2}$$
$$7 = \frac{k(9)}{(6)^2}$$
$$7 = \frac{k(9)}{36}$$
$$7 = \frac{k}{4}$$
$$28 = k$$

$$a = \frac{28(4)}{(8)^2} = \frac{28(4)}{64} = \frac{7}{4}$$

7. Since y varies jointly as x and z, we have $y = kxz$.

Use the given values to find k.

$$y = kxz$$
$$25 = k(2)(5)$$
$$25 = k(10)$$
$$\frac{25}{10} = \frac{k(10)}{10}$$
$$\frac{5}{2} = k$$

The equation becomes $y = \frac{5}{2}xz$.

When $x = 8$ and $z = 12$, $y = \frac{5}{2}(8)(12) = 240$.

8.

$$C = kAT$$
$$175 = k(2100)(4)$$
$$175 = k(8400)$$
$$\frac{1}{48} = k$$

$$C = \frac{1}{48}(2400)(6) = \frac{14400}{48} = 300$$

9. Since y varies jointly as a and b and inversely as the square root of c, we have $y = \frac{kab}{\sqrt{c}}$.

Use the given values to find k.

$$y = \frac{kab}{\sqrt{c}}$$
$$12 = \frac{k(3)(2)}{\sqrt{25}}$$
$$12 = \frac{k(6)}{5}$$
$$12(5) = \frac{k(6)}{5}(5)$$
$$60 = 6k$$
$$\frac{60}{6} = \frac{6k}{6}$$
$$10 = k$$

The equation becomes $y = \frac{10ab}{\sqrt{c}}$.

When $a = 5$, $b = 3$, $c = 9$,

$$y = \frac{10ab}{\sqrt{c}} = \frac{10(5)(3)}{\sqrt{9}} = \frac{150}{3} = 50.$$

10.

$$y = \frac{kmn^2}{p}$$
$$15 = \frac{k(2)(1)^2}{6}$$
$$15 = \frac{2k}{6}$$
$$15(6) = \frac{2k}{6}(6)$$
$$90 = 2k$$
$$k = 45$$

$$y = \frac{45mn^2}{p} = \frac{45(3)(4)^2}{10} = \frac{2160}{10} = 216$$

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11. $x = kyz$;
Solving for *y*:
$$x = kyz$$
$$\frac{x}{kz} = \frac{kyz}{yz}.$$
$$y = \frac{x}{kz}$$

12. $x = kyz^2$;
Solving for y:
$$x = kyz^2$$
$$\frac{x}{kz^2} = \frac{kyz^2}{kz^2}$$
$$y = \frac{x}{kz^2}$$

13. $x = \frac{kz^3}{y}$;
Solving for *y*
$$x = \frac{kz^3}{y}$$
$$xy = y \cdot \frac{kz^3}{y}$$
$$xy = kz^3$$
$$\frac{xy}{x} = \frac{kz^3}{x}$$
$$y = \frac{kz^3}{x}$$

14.
$$x = \frac{k\sqrt[3]{z}}{y}$$
$$yx = y \cdot \frac{k\sqrt[3]{z}}{y}$$
$$yx = k\sqrt[3]{z}$$
$$\frac{yx}{x} = \frac{k\sqrt[3]{z}}{x}$$
$$y = \frac{k\sqrt[3]{z}}{x}$$

15. $x = \frac{kyz}{\sqrt{w}}$;
Solving for *y*:
$$x = \frac{kyz}{\sqrt{w}}$$
$$x\left(\sqrt{w}\right) = \left(\sqrt{w}\right)\frac{kyz}{\sqrt{w}}$$
$$x\sqrt{w} = kyz$$
$$\frac{x\sqrt{w}}{kz} = \frac{kyz}{kz}$$
$$y = \frac{x\sqrt{w}}{kz}$$

16.
$$x = \frac{kyz}{w^2}$$
$$\left(\frac{w^2}{kz}\right)x = \frac{w^2}{kz}\frac{kyz}{w^2}$$
$$y = \frac{xw^2}{kz}$$

17. $x = kz(y + w)$;
Solving for *y*:
$$x = kz(y + w)$$
$$x = kzy + kzw$$
$$x - kzw = kzy$$
$$\frac{x - kzw}{kz} = \frac{kzy}{kz}$$
$$y = \frac{x - kzw}{kz}$$

18.
$$x = kz(y - w)$$
$$x = kzy - kzw$$
$$x + kzw = kzy$$
$$\frac{x + kzw}{kz} = \frac{kzy}{kz}$$
$$y = \frac{x + kzw}{kz}$$

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19. $x = \dfrac{kz}{y-w}$;

Solving for *y:*

$$x = \frac{kz}{y-w}$$
$$(y-w)x = (y-w)\frac{kz}{y-w}$$
$$xy - wx = kz$$
$$xy = kz + wx$$
$$\frac{xy}{x} = \frac{kz+wx}{x}$$
$$y = \frac{xw+kz}{x}$$

20.
$$x = \frac{kz}{y+w}$$
$$(y+w)x = (y+w)\frac{kz}{y+w}$$
$$yx + xw = kz$$
$$yx = kz - xw$$
$$\frac{yx}{x} = \frac{kz-xw}{x}$$
$$y = \frac{kz-xw}{x}$$

21. Since T varies directly as B, we have $T = kB$.
Use the given values to find k.

$$T = kB$$
$$3.6 = k(4)$$
$$\frac{3.6}{4} = \frac{k(4)}{4}$$
$$0.9 = k$$

The equation becomes $T = 0.9B$.
When $B = 6$, $T = 0.9(6) = 5.4$.
The tail length is 5.4 feet.

22.
$$M = kE$$
$$60 = k(360)$$
$$\frac{60}{360} = \frac{k(360)}{360}$$
$$\frac{1}{6} = k$$

$$M = \frac{1}{6}(186) = 31$$

A person who weighs 186 pounds on Earth will weigh 31 pounds on the moon.

23. Since B varies directly as D, we have $B = kD$.
Use the given values to find k.

$$B = kD$$
$$8.4 = k(12)$$
$$\frac{8.4}{12} = \frac{k(12)}{12}$$
$$k = \frac{8.4}{12} = 0.7$$

The equation becomes $B = 0.7D$.
When $B = 56$,

$$56 = 0.7D$$
$$\frac{56}{0.7} = \frac{0.7D}{0.7}$$
$$D = \frac{56}{0.7} = 80$$

It was dropped from 80 inches.

24.
$$d = kf$$
$$9 = k(12)$$
$$\frac{9}{12} = \frac{k(12)}{12}$$
$$0.75 = k$$
$$d = 0.75f$$
$$15 = 0.75f$$
$$\frac{15}{0.75} = \frac{0.75f}{0.75}$$
$$20 = f$$

A force of 20 pounds is needed.

25. Since a man's weight varies directly as the cube of his height, we have $w = kh^3$.
Use the given values to find k.

$$w = kh^3$$
$$170 = k(70)^3$$
$$170 = k(343{,}000)$$
$$\frac{170}{343{,}000} = \frac{k(343{,}000)}{343{,}000}$$
$$0.000496 = k$$

The equation becomes $w = 0.000496h^3$.
When $h = 107$,

$$w = 0.000496(107)^3$$
$$= 0.000496(1{,}225{,}043) \approx 607.$$

Robert Wadlow's weight was approximately 607 pounds.

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26.
$$h = kd^2$$
$$50 = k \cdot 10^2$$
$$0.5 = k$$
$$h = 0.5d^2$$

a.
$$h = 0.5d^2$$
$$h = 0.5(30)^2$$
$$h = 450$$
A water pipe with a 30 centimeter diameter can serve 450 houses.

b.
$$h = 0.5d^2$$
$$1250 = 0.5d^2$$
$$d^2 = 625$$
$$d = \sqrt{625}$$
$$d = 25$$
A water pipe with a 25 centimeter diameter can serve 1250 houses.

27. Since the banking angle varies inversely as the turning radius, we have $B = \frac{k}{r}$.

Use the given values to find *k*.
$$B = \frac{k}{r}$$
$$28 = \frac{k}{4}$$
$$28(4) = 28\left(\frac{k}{4}\right)$$
$$112 = k$$
The equation becomes $B = \frac{112}{r}$.

When $r = 3.5$, $B = \frac{112}{r} = \frac{112}{3.5} = 32$.

The banking angle is 32° when the turning radius is 3.5 feet.

28.
$$t = \frac{k}{d}$$
$$4.4 = \frac{k}{1000}$$
$$(1000)4.4 = (1000)\frac{k}{1000}$$
$$4400 = k$$
$$t = \frac{4400}{d} = \frac{4400}{5000} = 0.88$$
The water temperature is 0.88° Celsius at a depth of 5000 meters.

29. Since intensity varies inversely as the square of the distance, we have pressure, we have $I = \frac{k}{d}$.

Use the given values to find *k*.
$$I = \frac{k}{d^2}.$$
$$62.5 = \frac{k}{3^2}$$
$$62.5 = \frac{k}{9}$$
$$9(62.5) = 9\left(\frac{k}{9}\right)$$
$$562.5 = k$$
The equation becomes $I = \frac{562.5}{d^2}$.

When $d = 2.5$, $I = \frac{562.5}{2.5^2} = \frac{562.5}{6.25} = 90$

The intensity is 90 milliroentgens per hour.

30.
$$i = \frac{k}{d^2}$$
$$3.75 = \frac{k}{40^2}$$
$$3.75 = \frac{k}{1600}$$
$$(1600)3.75 = (1600)\frac{k}{1600}$$
$$6000 = k$$
$$i = \frac{6000}{d^2} = \frac{6000}{50^2} = \frac{6000}{2500} = 2.4$$
The illumination is 2.4 foot-candles at a distance of 50 feet.

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31. Since index varies directly as weight and inversely as the square of one's height, we have $I = \frac{kw}{h^2}$.

Use the given values to find *k*.

$$I = \frac{kw}{h^2}$$
$$35.15 = \frac{k(180)}{60^2}$$
$$35.15 = \frac{k(180)}{3600}$$
$$(3600)35.15 = \frac{k(180)}{3600}$$
$$126540 = k(180)$$
$$k = \frac{126540}{180} = 703$$

The equation becomes $I = \frac{703w}{h^2}$.

When $w = 170$ and $h = 70$,

$I = \frac{703(170)}{(70)^2} \approx 24.4$.

This person has a BMI of 24.4 and is not overweight.

32.

$$i = \frac{km}{c}$$
$$125 = \frac{k(25)}{20}$$
$$20(125) = (20)\frac{k(25)}{20}$$
$$2500 = 25k$$
$$\frac{2500}{25} = \frac{25k}{25}$$
$$100 = k$$

$$i = \frac{100m}{c}$$
$$80 = \frac{100(40)}{c}$$
$$80 = \frac{4000}{c}$$
$$80c = c \cdot \frac{4000}{c}$$
$$80c = 4000$$
$$\frac{80c}{80} = \frac{4000}{80}$$
$$c = 50$$

The chronological age is 50.

33. Since heat loss varies jointly as the area and temperature difference, we have $L = kAD$. Use the given values to find *k*.

$$L = kAD$$
$$1200 = k(3 \cdot 6)(20)$$
$$1200 = 360k$$
$$\frac{1200}{360} = \frac{360k}{360}$$
$$k = \frac{10}{3}$$

The equation becomes $L = \frac{10}{3}AD$

When $A = 6 \cdot 9 = 54$, $D = 10$,

$L = \frac{10}{3}(9 \cdot 6)(10) = 1800$.

The heat loss is 1800 Btu .

34.

$$e = kmv^2$$
$$36 = k(8)(3)^2$$
$$36 = k(8)(9)$$
$$36 = 72k$$
$$\frac{36}{72} = \frac{72k}{72}$$
$$k = 0.5$$

$e = 0.5mv^2 = 0.5(4)(6)^2 = 0.5(4)(36) = 72$

A mass of 4 grams and velocity of 6 centimeters per second has a kinetic energy of 72 ergs.

35. Since intensity varies inversely as the square of the distance from the sound source, we have $I = \frac{k}{d^2}$. If you move to a seat twice as far, then $d = 2d$. So we have $I = \frac{k}{(2d)^2} = \frac{k}{4d^2} = \frac{1}{4} \cdot \frac{k}{d^2}$. The intensity will be multiplied by a factor of $\frac{1}{4}$. So the sound intensity is $\frac{1}{4}$ of what it was originally.

36.

$$t = \frac{k}{a}$$
$$t = \frac{k}{3a} = \frac{1}{3} \cdot \frac{k}{a}$$

A year will seem to be $\frac{1}{3}$ of a year.

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37. **a.** Since the average number of phone calls varies jointly as the product of the populations and inversely as the square of the distance, we have $C = \frac{kP_1P_2}{d^2}$.

b. Use the given values to find k.

$$C = \frac{kP_1P_2}{d^2}$$
$$326,000 = \frac{k(777,000)(3,695,000)}{(420)^2}$$
$$326,000 = \frac{k(2.87\times 10^{12})}{176,400}$$
$$326,000 = 16269841.27k$$
$$0.02 \approx k$$

The equation becomes $C = \frac{0.02P_1P_2}{d^2}$.

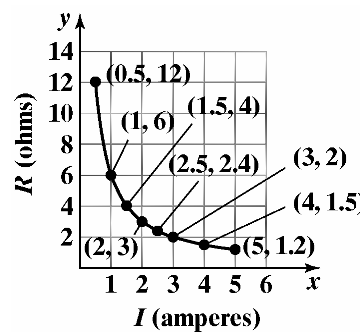
c. $C = \frac{0.02(650,000)(220,000)}{(400)^2}$
$= 17,875$

There are approximately 17,875 daily phone calls.

38.
$$f = kas^2$$
$$150 = k(4\cdot 5)(30)^2$$
$$150 = k(20)(900)$$
$$150 = 18000k$$
$$\frac{150}{18000} = \frac{18000k}{150}$$
$$\frac{1}{120} = k$$
$$f = \frac{1}{120}as^2 = \frac{1}{120}(3\cdot 4)(60)^2$$
$$= \frac{1}{120}(12)(3600)$$
$$= 360$$

Yes, the wind will exert a force of 360 pounds on the window.

39. **a.**



b. Current varies inversely as resistance. Answers will vary.

c. Since the current varies inversely as resistance we have $R = \frac{k}{I}$. Using one of the given ordered pairs to find k.

$$12 = \frac{k}{0.5}$$
$$12(0.5) = \frac{k}{0.5}(0.5)$$
$$k = 6$$

The equation becomes $R = \frac{6}{I}$.

40. – 48. Answers may vary.

49. does not make sense; Explanations will vary. Sample explanation: For an inverse variation, the independent variable can not be zero.

50. does not make sense; Explanations will vary. Sample explanation: A direct variation with a positive constant of variation will have both variables increase simultaneously.

51. makes sense

52. makes sense

53. Pressure, P, varies directly as the square of wind velocity, v, can be modeled as $P = kv^2$.
If $v = x$ then $P = k(x)^2 = kx^2$
If $v = 2x$ then $P = k(2x)^2 = 4kx^2$
If the wind speed doubles the pressure is 4 times more destructive.

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54. Illumination, I, varies inversely as the square of the distance, d, can be modeled as $I = \frac{k}{d^2}$.

If $d = 15$ then $I = \frac{k}{15^2} = \frac{k}{225}$

If $d = 30$ then $I = \frac{k}{30^2} = \frac{k}{900}$

Note that $\frac{900}{225} = 4$

If the distance doubles the illumination is 4 times less intense.

55. The Heat, H, varies directly as the square of the voltage, v, and inversely as the resistance, r.

$H = \frac{kv^2}{r}$

If the voltage remains constant, to triple the heat the resistant must be reduced by a multiple of 3.

56. Illumination, I, varies inversely as the square of the distance, d, can be modeled as $I = \frac{k}{d^2}$.

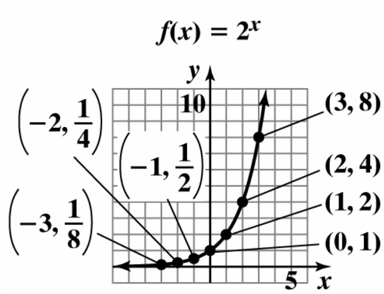
If $I = x$ then $x = \frac{k}{d^2} \Rightarrow d = \sqrt{\frac{k}{x}}$.

If $I = \frac{1}{50}x$ then $\frac{1}{50}x = \frac{k}{d^2} \Rightarrow d = \sqrt{\frac{50k}{x}} = \sqrt{50}\sqrt{\frac{k}{x}}$.

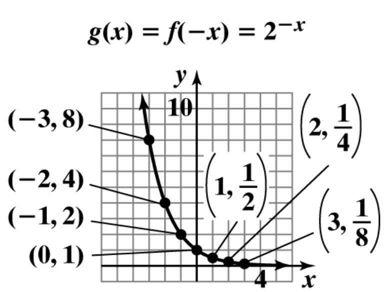
Since $\sqrt{50} \approx 7$, the Hubble telescope is able to see about 7 times farther than a ground-based telescope.

57. Answers may vary.

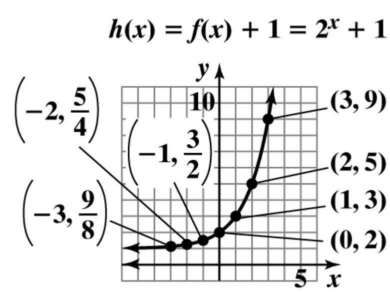
58.



59.



60.



Chapter 2 Review Exercises

1. $(8 - 3i) - (17 - 7i) = 8 - 3i - 17 + 7i$
$= -9 + 4i$

2. $4i(3i-2) = (4i)(3i) + (4i)(-2)$
$= 12i^2 - 8i$
$= -12 - 8i$

3. $(7-i)(2+3i)$
$= 7 \cdot 2 + 7(3i) + (-i)(2) + (-i)(3i)$
$= 14 + 21i - 2i + 3$
$= 17 + 19i$

4. $(3-4i)^2 = 3^2 + 2 \cdot 3(-4i) + (-4i)^2$
$= 9 - 24i - 16$
$= -7 - 24i$

5. $(7+8i)(7-8i) = 7^2 + 8^2 = 49 + 64 = 113$

6. $\frac{6}{5+i} = \frac{6}{5+i} \cdot \frac{5-i}{5-i}$
$= \frac{30-6i}{25+1}$
$= \frac{30-6i}{26}$
$= \frac{15-3i}{13}$
$= \frac{15}{13} - \frac{3}{13}i$

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7. $\frac{3+4i}{4-2i} = \frac{3+4i}{4-2i} \cdot \frac{4+2i}{4+2i}$

$= \frac{12+6i+16i+8i^2}{16-4i^2}$

$= \frac{12+22i-8}{16+4}$

$= \frac{4+22i}{20}$

$= \frac{1}{5} + \frac{11}{10}i$

8. $\sqrt{-32} - \sqrt{-18} = i\sqrt{32} - i\sqrt{18}$

$= i\sqrt{16 \cdot 2} - i\sqrt{9 \cdot 2}$

$= 4i\sqrt{2} - 3i\sqrt{2}$

$= (4i - 3i)\sqrt{2}$

$= i\sqrt{2}$

9. $(-2+\sqrt{-100})^2 = (-2+i\sqrt{100})^2$

$= (-2+10i)^2$

$= 4 - 40i + (10i)^2$

$= 4 - 40i - 100$

$= -96 - 40i$

10. $\frac{4+\sqrt{-8}}{2} = \frac{4+i\sqrt{8}}{2} = \frac{4+2i\sqrt{2}}{2} = 2+i\sqrt{2}$

11. $x^2 - 2x + 4 = 0$

$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)}$

$x = \frac{2 \pm \sqrt{4-16}}{2}$

$x = \frac{2 \pm \sqrt{-12}}{2}$

$x = \frac{2 \pm 2i\sqrt{3}}{2}$

$x = 1 \pm i\sqrt{3}$

The solution set is $\left\{1 - i\sqrt{3},\ 1 + i\sqrt{3}\right\}$

12. $2x^2 - 6x + 5 = 0$

$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(5)}}{2(2)}$

$x = \frac{6 \pm \sqrt{36-40}}{4}$

$x = \frac{6 \pm \sqrt{-4}}{4}$

$x = \frac{6 \pm 2i}{4}$

$x = \frac{6}{4} \pm \frac{2i}{4}$

$= \frac{3}{2} \pm \frac{1}{2}i$

The solution set is $\left\{\frac{3}{2} - \frac{1}{2}i,\ \frac{3}{2} + \frac{1}{2}i\right\}$.

13. $f(x) = -(x+1)^2 + 4$

vertex: (–1, 4)

x-intercepts:

$0 = -(x+1)^2 + 4$

$(x+1)^2 = 4$

$x + 1 = \pm 2$

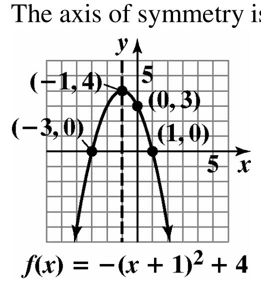
$x = -1 \pm 2$

$x = -3$ or $x = 1$

y-intercept:

$f(0) = -(0+1)^2 + 4 = 3$

The axis of symmetry is $x = -1$.



domain: $(-\infty, \infty)$ range: $(-\infty, 4]$

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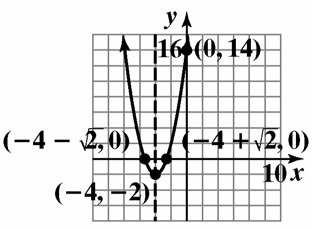
14. $f(x) = (x+4)^2 - 2$
vertex: (–4, –2)
x-intercepts:

$$0 = (x+4)^2 - 2$$
$$(x+4)^2 = 2$$
$$x+4 = \pm\sqrt{2}$$
$$x = -4 \pm \sqrt{2}$$

y-intercept:

$$f(0) = (0+4)^2 - 2 = 14 \ = -1$$

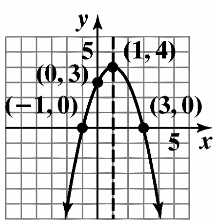


***f*(*x*) = (*x* + 4)² − 2**

The axis of symmetry is $x = -4$.
domain: $(-\infty, \infty)$ range: $[-2, \infty)$

15. $f(x) = -x^2 + 2x + 3$

$$= -\left(x^2 - 2x + 1\right) + 3 + 1$$
$$f(x) = -(x-1)^2 + 4$$

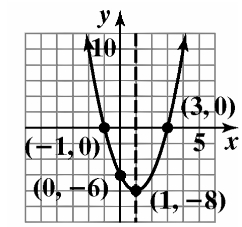


***f*(*x*) = − *x*² + 2*x* + 3**

domain: $(-\infty, \infty)$ range: $(-\infty, 4]$

16. $f(x) = 2x^2 - 4x - 6$

$$f(x) = 2\left(x^2 - 2x + 1\right) - 6 - 2$$
$$2(x-1)^2 - 8$$



***f*(*x*) = 2*x*² − 4*x* − 6**
axis of symmetry: $x = 1$
domain: $(-\infty, \infty)$ range: $[-8, \infty)$

17. $f(x) = -x^2 + 14x - 106$

a. Since $a < 0$ the parabola opens down with the maximum value occurring at

$$x = -\frac{b}{2a} = -\frac{14}{2(-1)} = 7 .$$

The maximum value is $f(7)$.

$$f(7) = -(7)^2 + 14(7) - 106 = -57$$

b. domain: $(-\infty, \infty)$ range: $(-\infty, -57]$

18. $f(x) = 2x^2 + 12x + 703$

a. Since $a > 0$ the parabola opens up with the minimum value occurring at

$$x = -\frac{b}{2a} = -\frac{12}{2(2)} = -3 .$$

The minimum value is $f(-3)$.

$$f(-3) = 2(-3)^2 + 12(-3) + 703 = 685$$

b. domain: $(-\infty, \infty)$ range: $[685, \infty)$

19. a. The maximum height will occur at the vertex.

$$f(x) = -0.025x^2 + x + 6$$
$$x = -\frac{b}{2a} = -\frac{1}{2(-0.025)} = 20$$
$$f(20) = -0.025(20)^2 + (20) + 6 = 16$$

The maximum height of 16 feet occurs when the ball is 20 yards downfield.

b. $f(x) = -0.025x^2 + x + 6$

$$f(0) = -0.025(0)^2 + (0) + 6 = 6$$

The ball was tossed at a height of 6 feet.

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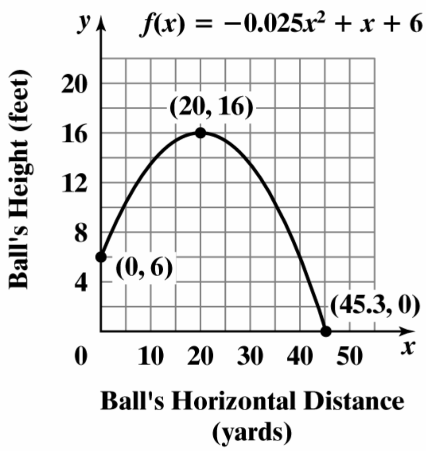
c. The ball is at a height of 0 when it hits the ground.

$$f(x) = -0.025x^2 + x + 6$$
$$0 = -0.025x^2 + x + 6$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(-0.025)(6)}}{2(-0.025)}$$
$$x \approx 45.3,\ -5.3 \text{(reject)}$$

The ball will hit the ground 45.3 yards downfield.

d. The football's path:



20. Maximize the area using $A = lw$.

$$A(x) = x(1000 - 2x)$$
$$A(x) = -2x^2 + 1000x$$

Since $a = -2$ is negative, we know the function opens downward and has a maximum at

$$x = -\frac{b}{2a} = -\frac{1000}{2(-2)} = -\frac{1000}{-4} = 250.$$

The maximum area is achieved when the width is 250 yards. The maximum area is

$$A(250) = 250(1000 - 2(250))$$
$$= 250(1000 - 500)$$
$$= 250(500) = 125{,}000.$$

The area is maximized at 125,000 square yards when the width is 250 yards and the length is $1000 - 2 \cdot 250 = 500$ yards.

21. Let x = one of the numbers
Let $14 + x$ = the other number

We need to minimize the function

$$P(x) = x(14 + x)$$
$$= 14x + x^2$$
$$= x^2 + 14x.$$

The minimum is at

$$x = -\frac{b}{2a} = -\frac{14}{2(1)} = -\frac{14}{2} = -7.$$

The other number is $14 + x = 14 + (-7) = 7.$

The numbers which minimize the product are 7 and -7. The minimum product is $-7 \cdot 7 = -49$.

22. $3x + 4y = 1000$
$4y = 1000 - 3x$

$$y = \frac{1000 - 3x}{4}$$

$$A = x\ \frac{1000 - 3x}{4}$$
$$= -\frac{3}{4}x^2 + 250x$$

$$x = \frac{-b}{2a} = \frac{-250}{2\ -\frac{3}{4}} = 125$$

$$y = \frac{1000 - 3(125)}{4} = 166.7$$

125 feet by 166.7 feet will maximize the area.

23. $y = (35 + x)(150 - 4x)$

$$y = 5250 + 10x - 4x^2$$

$$x = \frac{-b}{2a} = \frac{-10}{2(-4)} = \frac{5}{4} = 1.25 \text{ or 1 tree}$$

The maximum number of trees should be 35 + 1 = 36 trees.
The maximum number of trees should be 35 + 1 = 36 trees.
$y = 36(150 - 4x) = 36(150 - 4\cdot1) = 5256$
The maximum yield will be 5256 pounds.

24. $f(x) = -x^3 + 12x^2 - x$

The graph rises to the left and falls to the right and goes through the origin, so graph (c) is the best match.

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25. $g(x) = x^6 - 6x^4 + 9x^2$

The graph rises to the left and rises to the right, so graph (b) is the best match.

26. $h(x) = x^5 - 5x^3 + 4x$

The graph falls to the left and rises to the right and crosses the y-axis at zero, so graph (a) is the best match.

27. $f(x) = -x^4 + 1$

$f(x)$ falls to the left and to the right so graph (d) is the best match.

28. The leading coefficient is –0.87 and the degree is 3. This means that the graph will fall to the right. This function is not useful in modeling the number of thefts over an extended period of time. The model predicts that eventually, the number of thefts would be negative. This is impossible.

29. In the polynomial, $f(x) = -x^4 + 21x^2 + 100,$ the leading coefficient is –1 and the degree is 4. Applying the Leading Coefficient Test, we know that even-degree polynomials with negative leading coefficient will fall to the left and to the right. Since the graph falls to the right, we know that the elk population will die out over time.

30. $f(x) = -2(x-1)(x+2)^2(x+5)^3$

$x = 1$, multiplicity 1, the graph crosses the x-axis
$x = -2$, multiplicity 2, the graph touches the x-axis
$x = -5$, multiplicity 5, the graph crosses the x-axis

31.
$$\begin{aligned} f(x) &= x^3 - 5x^2 - 25x + 125 \\ &= x^2(x-5) - 25(x-5) \\ &= (x^2 - 25)(x-5) \\ &= (x+5)(x-5)^2 \end{aligned}$$

$x = -5$, multiplicity 1, the graph crosses the x-axis
$x = 5$, multiplicity 2, the graph touches the x-axis

32. $f(x) = x^3 - 2x - 1$

$f(1) = (1)^3 - 2(1) - 1 = -2$

$f(2) = (2)^3 - 2(2) - 1 = 3$

The sign change shows there is a zero between the given values.

33. $f(x) = x^3 - x^2 - 9x + 9$

a. Since n is odd and $a_n > 0,$ the graph falls to the left and rises to the right.

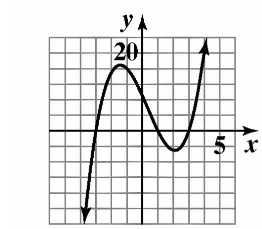
b.
$$\begin{aligned} f(-x) &= (-x)^3 - (-x)^2 - 9(-x) + 9 \\ &= -x^3 - x^2 + 9x + 9 \end{aligned}$$

$f(-x) \neq f(x), f(-x) \neq -f(x)$

no symmetry

c. $f(x) = (x-3)(x+3)(x-1)$

zeros: 3, −3, 1



$f(x) = x^3 - x^2 - 9x + 9$

34. $f(x) = 4x - x^3$

a. Since n is odd and $a_n < 0,$ the graph rises to the left and falls to the right.

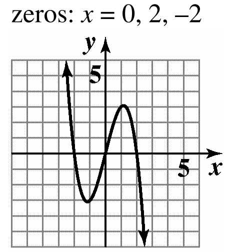
b. $f(-x) = -4x + x^3$

$f(-x) = -f(x)$

origin symmetry

c. $f(x) = x(x^2 - 4) = x(x-2)(x+2)$

zeros: $x = 0, 2, -2$



$f(x) = 4x - x^3$

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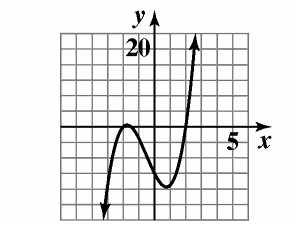
35. $f(x) = 2x^3 + 3x^2 - 8x - 12$

a. Since h is odd and $a_n > 0$, the graph falls to the left and rises to the right.

b. $f(-x) = -2x^3 + 3x^2 + 8x - 12$
$f(-x) \neq f(x),\ f(-x) = -f(x)$
no symmetry

c. $f(x) = (x - 2)(x + 2)(2x + 3)$
zeros: $x = 2, -2, -\frac{3}{2}$



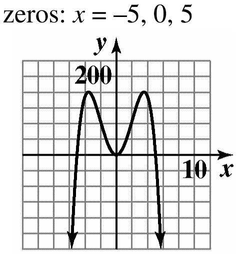
$f(x) = 2x^3 + 3x^2 - 8x - 12$

36. $g(x) = -x^4 + 25x^2$

a. The graph falls to the left and to the right.

b. $f(-x) = -(-x)^4 + 25(-x)^2$
$= -x^4 + 25x^2 = f(x)$
y-axis symmetry

c. $-x^4 + 25x^2 = 0$
$-x^2\left(x^2 - 25\right) = 0$
$-x^2\left(x - 5\right)\left(x + 5\right) = 0$
zeros: $x = -5, 0, 5$



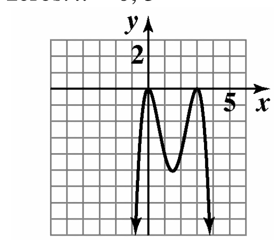
$f(x) = -x^4 + 25x^2$

37. $f\left(x\right) = -x^4 + 6x^3 - 9x^2$

a. The graph falls to the left and to the right.

b. $f(-x) = -(-x)^4 + 6(-x)^3 - 9(-x)$
$= -x^4 - 6x^3 - 9x^2 f(-x) \neq f(x)$
$f(-x) \neq -f(x)$
no symmetry

c. $= -x^2\left(x^2 - 6x + 9\right) = 0$
$-x^2\left(x - 3\right)\left(x - 3\right) = 0$
zeros: $x = 0, 3$



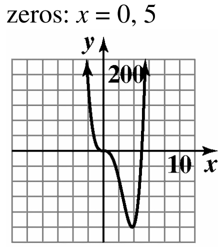
$f(x) = -x^4 + 6x^3 - 9x^2$

38. $f\left(x\right) = 3x^4 - 15x^3$

a. The graph rises to the left and to the right.

b. $f(-x) = 3(-x)^4 - 15(-x)^2 = 3x^4 + 15x^3$
$f(-x) \neq f(x),\ f(-x) \neq -f(x)$
no symmetry

c $3x^4 - 15x^3 = 0$
$3x^3\left(x - 5\right) = 0$
zeros: $x = 0, 5$



$f(x) = 3x^4 - 15x^3$

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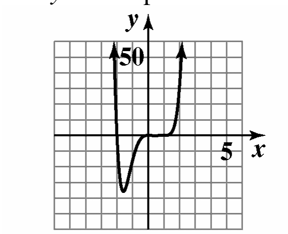
39. $f(x) = 2x^2(x-1)^3(x+2)$

Since $a_n > 0$ and n is even, $f(x)$ rises to the left and the right.
$x = 0, x = 1, x = -2$
The zeros at 1 and –2 have odd multiplicity so $f(x)$ crosses the x-axis at those points. The root at 0 has even multiplicity so $f(x)$ touches the axis at (0, 0)

$f(0) = 2(0)^2(0-1)^3(0+2) = 0$

The y-intercept is 0.



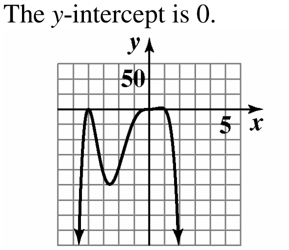
$\boldsymbol{f(x) = 2x^2(x-1)^3(x+2)}$

40. $f(x) = -x^3(x+4)^2(x-1)$

Since $a_n < 0$ and n is even, $f(x)$ falls to the left and the right.
$x = 0, x = -4, x = 1$
The roots at 0 and 1 have odd multiplicity so $f(x)$ crosses the x-axis at those points. The root at –4 has even multiplicity so $f(x)$ touches the axis at (–4, 0)

$f(0) = -(0)^3(0+4)^2(0-1) = 0$

The y-intercept is 0.



$\boldsymbol{f(x) = -x^3(x+4)^2(x-1)}$

41.

$$
\begin{array}{r|l}
 & 4x^2 - 7x + 5 \\
\hline
x+1 & 4x^3 - 3x^2 - 2x + 1 \\
 & \underline{4x^3 + 4x^2} \\
 & \quad -7x^2 - 2x \\
 & \quad \underline{-7x^2 - 7x} \\
 & \qquad 5x + 1 \\
 & \qquad \underline{5x + 5} \\
 & \qquad\quad -4
\end{array}
$$

Quotient: $4x^2 - 7x + 5 - \dfrac{4}{x+1}$

42.

$$
\begin{array}{r|l}
 & 2x^2 - 4x + 1 \\
\hline
5x-3 & 10x^3 - 26x^2 + 17x - 13 \\
 & \underline{10x^3 + 6x^2} \\
 & \quad -20x^2 + 17x \\
 & \quad \underline{-20x^2 + 12x} \\
 & \qquad 5x - 13 \\
 & \qquad \underline{5x - 3} \\
 & \qquad\quad -10
\end{array}
$$

Quotient: $2x^2 - 4x + 1 - \dfrac{10}{5x-3}$

43.

$$
\begin{array}{r|l}
 & 2x^2 + 3x - 1 \\
\hline
2x^2+1 & 4x^4 + 6x^3 + 3x - 1 \\
 & \underline{4x^2 + 2x^2} \\
 & \quad 6x^3 - 2x^2 + 3x \\
 & \quad \underline{6x^2 + 3x} \\
 & \qquad -2x^2 - 1 \\
 & \qquad \underline{-2x^2 - 1} \\
 & \qquad\quad 0
\end{array}
$$

44. $(3x^4 + 11x^3 - 20x^3 + 7x + 35) \div (x+5)$

$$
\begin{array}{r|rrrrr}
-5 & 3 & 11 & -20 & 7 & 35 \\
 & & -15 & 20 & 0 & -35 \\
\hline
 & 3 & -4 & 0 & 7 & 0
\end{array}
$$

Quotient: $3x^3 - 4x^2 + 7$

45. $(3x^4 - 2x^2 - 10x) \div (x-2)$

$$
\begin{array}{r|rrrrr}
2 & 3 & 0 & -2 & -10 & 0 \\
 & & 6 & 12 & 20 & 20 \\
\hline
 & 3 & 6 & 10 & 10 & 20
\end{array}
$$

Quotient: $3x^3 + 6x^2 + 10x + 10 + \dfrac{20}{x-2}$

46. $f(x) = 2x^3 - 7x^2 + 9x - 3$

$$
\begin{array}{r|rrrr}
-13 & 2 & -7 & 9 & -3 \\
 & & -26 & 429 & -5694 \\
\hline
 & 2 & -33 & 438 & -5697
\end{array}
$$

Quotient: $f(-13) = -5697$

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47. $f(x) = 2x^3 + x^2 - 13x + 6$

$$\begin{array}{r|rrrr} 2 & 2 & 1 & -13 & 6 \\ & & 4 & 10 & -6 \\ \hline & 2 & 5 & -3 & 0 \end{array}$$

$f(x) = (x-2)(2x^2 + 5x - 3)$
$\quad = (x-2)(2x-1)(x+3)$

Zeros: $x = 2, \frac{1}{2}, -3$

48. $x^3 - 17x + 4 = 0$

$$\begin{array}{r|rrrr} 4 & 1 & 0 & -17 & 4 \\ & & 4 & 16 & -4 \\ \hline & 1 & 4 & -1 & 0 \end{array}$$

$(x-4)(x^2 + 4x - 1) = 0$

$x = \frac{-4 \pm \sqrt{16+4}}{2} = \frac{-4 \pm 2\sqrt{5}}{2} = -2 \pm \sqrt{5}$

The solution set is $\{4, -2+\sqrt{5}, -2-\sqrt{5}\}$.

49. $f(x) = x^4 - 6x^3 + 14x^2 - 14x + 5$
$p: \pm 1, \pm 5$
$q: \pm 1$
$\frac{p}{q}: \pm 1, \pm 5$

50. $f(x) = 3x^5 - 2x^4 - 15x^3 + 10x^2 + 12x - 8$
$p: \pm 1, \pm 2, \pm 4, \pm 8$
$q: \pm 1, \pm 3$
$\frac{p}{q}: \pm 1, \pm 2, \pm 4, \pm 8, \pm\frac{8}{3}, \pm\frac{4}{3}, \pm\frac{2}{3}, \pm\frac{1}{3}$

51. $f(x) = 3x^4 - 2x^3 - 8x + 5$
$f(x)$ has 2 sign variations, so $f(x) = 0$ has 2 or 0 positive solutions.
$f(-x) = 3x^4 + 2x^3 + x + 5$
$f(-x)$ has no sign variations, so $f(x) = 0$ has no negative solutions.

52. $f(x) = 2x^5 - 3x^3 - 5x^2 + 3x - 1$
$f(x)$ has 3 sign variations, so $f(x) = 0$ has 3 or 1 positive real roots.
$f(-x) = -2x^5 + 3x^3 - 5x^2 - 3x - 1$
$f(-x)$ has 2 sign variations, so
$f(x) = 0$ has 2 or 0 negative solutions.

53. $f(x) = f(-x) = 2x^4 + 6x^2 + 8$
No sign variations exist for either $f(x)$ or $f(-x)$, so no real roots exist.

54. $f(x) = x^3 + 3x^2 - 4$

a. $p: \pm 1, \pm 2, \pm 4$
$q: \pm 1$
$\frac{p}{q}: \pm 1, \pm 2, \pm 4$

b. 1 sign variation $\Rightarrow$ 1 positive real zero
$f(-x) = -x^3 + 3x^2 - 4$
2 sign variations $\Rightarrow$ 2 or no negative real zeros

c.
$$\begin{array}{r|rrrr} 1 & 1 & 3 & 0 & -4 \\ & & 1 & 4 & -4 \\ \hline & 1 & 4 & 4 & 0 \end{array}$$

1 is a zero.
1, –2 are rational zeros.

d. $(x-1)(x^2 + 4x + 4) = 0$
$(x-1)(x+2)^2 = 0$
$x = 1$ or $x = -2$
The solution set is $\{1, -2\}$.

55. $f(x) = 6x^3 + x^2 - 4x + 1$

a. p: ±1
q: ±1, ±2, ±3, ±6
$\frac{p}{q}: \pm 1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}$

b. $f(x) = 6x^3 + x^2 - 4x + 1$
2 sign variations; 2 or 0 positive real zeros.
$f(-x) = -6x^3 + x^2 + 4x + 1$
1 sign variation; 1 negative real zero.

c.
$$\begin{array}{r|rrrr} -1 & 6 & 1 & -4 & 1 \\ & & -6 & 5 & -1 \\ \hline & 6 & -5 & 1 & 0 \end{array}$$

–1 is a zero.
$-1, \frac{1}{3}, \frac{1}{2}$ are rational zeros.

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d. $6x^3 + x^2 - 4x + 1 = 0$

$(x+1)(6x^2 - 5x + 1) = 0$

$(x+1)(3x-1)(2x-1) = 0$

$x = -1$ or $x = \frac{1}{3}$ or $x = \frac{1}{2}$

The solution set is $\left\{-1, \frac{1}{3}, \frac{1}{2}\right\}$.

56. $f(x) = 8x^3 - 36x^2 + 46x - 15$

a. p: ±1, ±3, ±5, ±15
q: ±1, ±2, ±4, ±8

$\frac{p}{q}: \pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm \frac{1}{8},$
$\pm \frac{3}{2}, \pm \frac{3}{4}, \pm \frac{3}{8}, \pm \frac{5}{2}, \pm \frac{5}{4},$
$\pm \frac{5}{8}, \pm \frac{15}{2}, \pm \frac{15}{4}, \pm \frac{15}{8}$

b. $f(x) = 8x^3 - 36x^2 + 46x - 15$
3 sign variations; 3 or 1 positive real solutions.
$f(-x) = -8x^3 - 36x^2 - 46x - 15$
0 sign variations; no negative real solutions.

c.

$\frac{1}{2}$	8	–36	46	–15
		4	–16	15
	8	–32	30	0

$\frac{1}{2}$ is a zero.
$\frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ are rational zeros.

d.

$8x^3 - 36x^2 + 46x - 15 = 0$

$\left(x - \frac{1}{2}\right)(8x^2 - 32x + 30) = 0$

$2\left(x - \frac{1}{2}\right)(4x - 16x + 15) = 0$

$2\left(x - \frac{1}{2}\right)(2x - 5)(2x - 3) = 0$

$x = \frac{1}{2}$ or $x = \frac{5}{2}$ or $x = \frac{3}{2}$

The solution set is $\left\{\frac{1}{2}, \frac{3}{2}, \frac{5}{2}\right\}$.

57. $2x^3 + 9x^2 - 7x + 1 = 0$

a. p: ±1
q: ±1, ±2
$\frac{p}{q}: \pm 1, \pm \frac{1}{2}$

b. $f(x) = 2x^3 + 9x^2 - 7x + 1$
2 sign variations; 2 or 0 positive real zeros.
$f(-x) = -2x^3 + 9x^2 + 7x + 1$
1 sign variation; 1 negative real zero.

c.

$\frac{1}{2}$	2	9	–7	1
		1	5	–1
	2	10	–2	0

$\frac{1}{2}$ is a rational zero.

d. $2x^3 + 9x^2 - 7x + 1 = 0$

$\left(x - \frac{1}{2}\right)(2x^2 + 10x - 2) = 0$

$2\left(x - \frac{1}{2}\right)(x^2 + 5x - 1) = 0$

Solving $x^2 + 5x - 1 = 0$ using the quadratic formula gives $x = \frac{-5 \pm \sqrt{29}}{2}$

The solution set is $\left\{\frac{1}{2}, \frac{-5+\sqrt{29}}{2}, \frac{-5-\sqrt{29}}{2}\right\}$.

58. $x^4 - x^3 - 7x^2 + x + 6 = 0$

a. $p = \frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 6$

b. $f(x) = x^4 - x^3 - 7x^2 + x + 6$
2 sign variations; 2 or 0 positive real zeros.
$f(-x) = x^4 + x^3 - 7x^2 - x + 6$
2 sign variations; 2 or 0 negative real zeros.

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c.

$$\begin{array}{r|rrrrr} 1 & 1 & -1 & -7 & 1 & 6 \\ & & 1 & 0 & -7 & -6 \\ \hline & 1 & 0 & -7 & -6 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -7 & -6 \\ & & -1 & 1 & 6 \\ \hline & 1 & -1 & -6 & 0 \end{array}$$

$-2, -1, 1, 3$ are rational zeros.

d.
$$x^4 - x^3 - 7x^2 + x + 6 = 0$$
$$(x-1)(x+1)(x^2 - x + 6) = 0$$
$$(x-1)(x+1)(x-3)(x+2) = 0$$
The solution set is $\{-2, -1, 1, 3\}$.

59. $4x^4 + 7x^2 - 2 = 0$

a. p: ±1, ±2
q: ±1, ±2, ±4
$\frac{p}{q}: \pm 1, \pm 2, \pm\frac{1}{2}, \pm\frac{1}{4}$

b. $f(x) = 4x^4 + 7x^2 - 2$
1 sign variation; 1 positive real zero.
$f(-x) = 4x^4 + 7x^2 - 2$
1 sign variation; 1 negative real zero.

c.

$$\begin{array}{r|rrrrr} \frac{1}{2} & 4 & 0 & 7 & 0 & -2 \\ & & 2 & 1 & 4 & 2 \\ \hline & 4 & 2 & 8 & 4 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -\frac{1}{2} & 4 & 2 & 8 & 4 \\ & & -2 & 0 & -4 \\ \hline & 4 & 0 & 8 & 0 \end{array}$$

$-\frac{1}{2}, \frac{1}{2}$ are rational zeros.

d.
$$4x^4 + 7x^2 - 2 = 0$$
$$\left(x - \frac{1}{2}\right)\left(x + \frac{1}{2}\right)(4x^2 + 8) = 0$$
$$4\left(x - \frac{1}{2}\right)\left(x + \frac{1}{2}\right)(x^2 + 2) = 0$$
Solving $x^2 + 2 = 0$ using the quadratic formula gives $x = \pm 2i$

The solution set is $\left\{-\frac{1}{2}, \frac{1}{2}, 2i, -2i\right\}$.

60. $f(x) = 2x^4 + x^3 - 9x^2 - 4x + 4$

a. p: ±1, ±2, ±4
q: ±1, ±2
$\frac{p}{q}: \pm 1, \pm 2, \pm 4, \pm\frac{1}{2}$

b. $f(x) = 2x^4 + x^3 - 9x^2 - 4x + 4$
2 sign variations; 2 or 0 positive real zeros.
$f(-x) = 2x^4 - x^3 - 9x^2 + 4x + 4$
2 sign variations; 2 or 0 negative real zeros.

c.

$$\begin{array}{r|rrrrr} 2 & 2 & 1 & -9 & -4 & 4 \\ & & 4 & 10 & 2 & -4 \\ \hline & 2 & 5 & 1 & -2 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -1 & 2 & 5 & 1 & -2 \\ & & -2 & -3 & 2 \\ \hline & 2 & 3 & -2 & 0 \end{array}$$

$-2, -1, \frac{1}{2}, 2$ are rational zeros.

d.
$$2x^2 + 3x - 2 = 0$$
$$(2x-1)(x+2) = 0$$
$$x = -2 \quad \text{or} \quad x = \frac{1}{2}$$
The solution set is $\left\{-2, -1, \frac{1}{2}, 2\right\}$.

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61. $f(x) = a_n(x-2)(x-2+3i)(x-2-3i)$

$f(x) = a_n(x-2)(x^2-4x+13)$

$f(1) = a_n(1-2)\left[1^2-4(1)+13\right]$

$-10 = -10a_n$

$a_n = 1$

$f(x) = 1(x-2)(x^2-4x+13)$

$f(x) = x^3-4x^2+13x-2x^2+8x-26$

$f(x) = x^3-6x^2+21x-26$

62. $f(x) = a_n(x-i)(x+i)(x+3)^2$

$f(x) = a_n(x^2+1)(x^2+6x+9)$

$f(-1) = a_n\left[(-1)^2+1\right]\left[(-1)^2+6(-1)+9\right]$

$16 = 8a_n$

$a_n = 2$

$f(x) = 2(x^2+1)(x^2+6x+9)$

$f(x) = 2(x^4+6x^3+9x^2+x^2+6x+9)$

$f(x) = 2x^4+12x^3+20x^2+12x+18$

63. $f(x) = 2x^4+3x^3+3x-2$

p: ±1, ±2

q: ±1, ±2

$\frac{p}{q}: \pm 1, \pm 2, \pm\frac{1}{2}$

$$\begin{array}{r|rrrrr} -2 & 2 & 3 & 0 & 3 & -2 \\ & & -4 & 2 & -4 & 2 \\ \hline & 2 & -1 & 2 & -1 & 0 \end{array}$$

$2x^4+3x^3+3x-2=0$

$(x+2)(2x^3-x^2+2x-1)=0$

$(x+2)[x^2(2x-1)+(2x-1)]=0$

$(x+2)(2x-1)(x^2+1)=0$

$x=-2,\ x=\frac{1}{2}$ or $x=\pm i$

The zeros are $-2,\ \frac{1}{2},\ \pm i$.

$f(x) = (x-i)(x+i)(x+2)(2x-1)$

64. $g(x) = x^4-6x^3+x^2+24x+16$

p: ±1, ±2, ±4, ±8, ±16

q: ±1

$\frac{p}{q}: \pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

$$\begin{array}{r|rrrrr} -1 & 1 & -6 & 1 & 24 & 16 \\ & & -1 & 7 & -8 & -16 \\ \hline & 1 & -7 & 8 & 16 & 0 \end{array}$$

$x^4-6x^3+x^2+24x+16=0$

$(x+1)(x^3-7x^2+8x+16)=0$

$$\begin{array}{r|rrrr} -1 & 1 & -7 & 8 & 16 \\ & & -1 & 8 & -16 \\ \hline & 1 & -8 & 16 & 0 \end{array}$$

$(x+1)^2(x^2-8x+16)=0$

$(x+1)^2(x-4)^2=0$

$x=-1$ or $x=4$

$g(x)=(x+1)^2(x-4)^2$

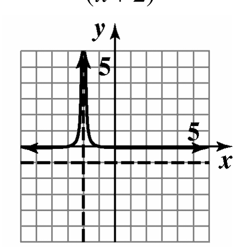
65. 4 real zeros, one with multiplicity two

66. 3 real zeros; 2 nonreal complex zeros

67. 2 real zeros, one with multiplicity two; 2 nonreal complex zeros

68. 1 real zero; 4 nonreal complex zeros

69. $g(x) = \frac{1}{(x+2)^2} - 1$



$\mathbf{g(x) = \frac{1}{(x+2)^2} - 1}$

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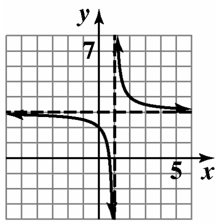
70. $h(x) = \dfrac{1}{x-1} + 3$



$h(x) = \dfrac{1}{x-1} + 3$

71. $f(x) = \dfrac{2x}{x^2-9}$

Symmetry: $f(-x) = -\dfrac{2x}{x^2-9} = -f(x)$

origin symmetry

x-intercept:

$$0 = \frac{2x}{x^2-9}$$
$$2x = 0$$
$$x = 0$$

y-intercept: $y = \dfrac{2(0)}{0^2-9} = 0$

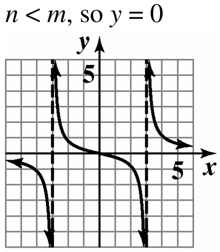
Vertical asymptote:

$$x^2 - 9 = 0$$
$$(x-3)(x+3) = 0$$

$x = 3$ and $x = -3$

Horizontal asymptote:

$n < m$, so $y = 0$



$f(x) = \dfrac{2x}{x^2-9}$

72. $g(x) = \dfrac{2x-4}{x+3}$

Symmetry: $g(-x) = \dfrac{-2x-4}{x+3}$

$g(-x) \neq g(x)$, $g(-x) \neq -g(x)$

No symmetry

x-intercept:

$2x - 4 = 0$

$x = 2$

y-intercept: $y = \dfrac{2(0)-4}{(0)+3} = -\dfrac{4}{3}$

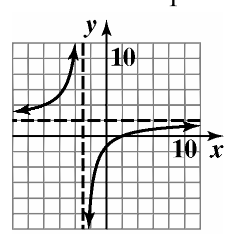
Vertical asymptote:

$x + 3 = 0$

$x = -3$

Horizontal asymptote:

$n = m$, so $y = \dfrac{2}{1} = 2$



$f(x) = \dfrac{2x-4}{x+3}$

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73. $h(x)=\dfrac{x^2-3x-4}{x^2-x-6}$

Symmetry: $h(-x)=\dfrac{x^2+3x-4}{x^2+x-6}$

$h(-x) \neq h(x)$, $h(-x) \neq -h(x)$

No symmetry

x-intercepts:

$x^2-3x-4=0$

$(x-4)(x+1)$

$x=4 \quad x=-1$

y-intercept: $y=\dfrac{0^2-3(0)-4}{0^2-0-6}=\dfrac{2}{3}$

Vertical asymptotes:

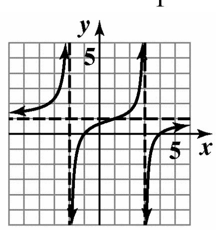
$x^2-x-6=0$

$(x-3)(x+2)=0$

$x=3, -2$

Horizontal asymptote:

$n=m$, so $y=\dfrac{1}{1}=1$



$\mathbf{h(x)=\dfrac{x^2-3x-4}{x^2-x-6}}$

74. $r(x)=\dfrac{x^2+4x+3}{(x+2)^2}$

Symmetry: $r(-x)=\dfrac{x^2-4x+3}{(-x+2)^2}$

$r(-x) \neq r(x)$, $r(-x) \neq -r(x)$

No symmetry

x-intercepts:

$x^2+4x+3=0$

$(x+3)(x+1)=0$

$x=-3, -1$

y-intercept: $y=\dfrac{0^2+4(0)+3}{(0+2)^2}=\dfrac{3}{4}$

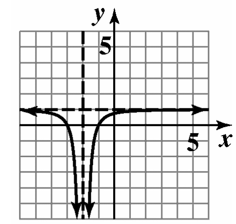
Vertical asymptote:

$x+2=0$

$x=-2$

Horizontal asymptote:

$n=m$, so $y=\dfrac{1}{1}=1$



$\mathbf{r(x)=\dfrac{x^2+4x+3}{(x+4)^2}}$

75. $y=\dfrac{x^2}{x+1}$

Symmetry: $f(-x)=\dfrac{x^2}{-x+1}$

$f(-x) \neq f(x)$, $f(-x) \neq -f(x)$

No symmetry

x-intercept:

$x^2=0$

$x=0$

y-intercept: $y=\dfrac{0^2}{0+1}=0$

Vertical asymptote:

$x+1=0$

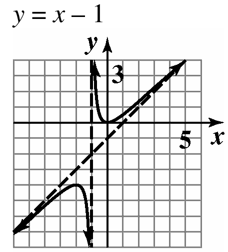
$x=-1$

$n>m$, no horizontal asymptote.

Slant asymptote:

$y=x-1+\dfrac{1}{x+1}$

$y=x-1$



$\mathbf{y=\dfrac{x^2}{x+1}}$

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76. $y = \dfrac{x^2 + 2x - 3}{x - 3}$

Symmetry: $f(-x) = \dfrac{x^2 - 2x - 3}{-x - 3}$

$f(-x) \neq f(x),\ f(-x) \neq -f(x)$

No symmetry

x-intercepts:

$x^2 + 2x - 3 = 0$

$(x + 3)(x - 1) = 0$

$x = -3, 1$

y-intercept: $y = \dfrac{0^2 + 2(0) - 3}{0 - 3} = \dfrac{-3}{-3} = 1$

Vertical asymptote:

$x - 3 = 0$

$x = 3$

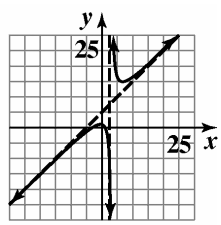
Horizontal asymptote:

$n > m$, so no horizontal asymptote.

Slant asymptote:

$y = x + 5 + \dfrac{12}{x - 3}$

$y = x + 5$



$\boldsymbol{f(x) = \dfrac{x^2 + 2x - 3}{x - 3}}$

77. $f(x) = \dfrac{-2x^3}{x^2 + 1}$

Symmetry: $f(-x) = \dfrac{2}{x^2 + 1} = -f(x)$

Origin symmetry

x-intercept:

$-2x^3 = 0$

$x = 0$

y-intercept: $y = \dfrac{-2(0)^3}{0^2 + 1} = \dfrac{0}{1} = 0$

Vertical asymptote:

$x^2 + 1 = 0$

$x^2 = -1$

No vertical asymptote.

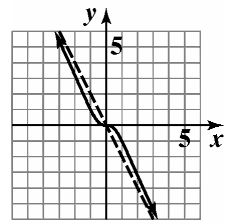
Horizontal asymptote:

$n > m$, so no horizontal asymptote.

Slant asymptote:

$f(x) = -2x + \dfrac{2x}{x^2 + 1}$

$y = -2x$



$\boldsymbol{f(x) = \dfrac{-2x^3}{x^2 + 1}}$

78. $g(x) = \dfrac{4x^2 - 16x + 16}{2x - 3}$

Symmetry: $g(-x) = \dfrac{4x^2 + 16x + 16}{-2x - 3}$

$g(-x) \neq g(x),\ g(-x) \neq -g(x)$

No symmetry

x-intercept:

$4x^2 - 16x + 16 = 0$

$4(x - 2)^2 = 0$

$x = 2$

y-intercept:

$y = \dfrac{4(0)^2 - 16(0) + 16}{2(0) - 3} = -\dfrac{16}{3}$

Vertical asymptote:

$2x - 3 = 0$

$x = \dfrac{3}{2}$

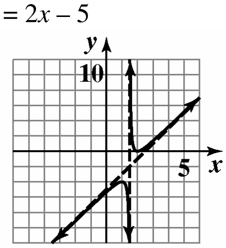
Horizontal asymptote:

$n > m$, so no horizontal asymptote.

Slant asymptote:

$g(x) = 2x - 5 + \dfrac{1}{2x - 3}$

$y = 2x - 5$



$\boldsymbol{g(x) = \dfrac{4x^2 - 16x + 16}{2x - 3}}$

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79. **a.** $C(x) = 50{,}000 + 25x$

b. $\overline{C}(x) = \dfrac{25x + 50{,}000}{x}$

c. $\overline{C}(50) = \dfrac{25(50) + 50{,}000}{50} = 1025$

When 50 calculators are manufactured, it costs \$1025 to manufacture each.

$\overline{C}(100) = \dfrac{25(100) + 50{,}000}{100} = 525$

When 100 calculators are manufactured, it costs \$525 to manufacture each.

$\overline{C}(1000) = \dfrac{25(1000) + 50{,}000}{1000} = 75$

When 1,000 calculators are manufactured, it costs \$75 to manufacture each.

$\overline{C}(100{,}000) = \dfrac{25(100{,}000) + 50{,}000}{100{,}000} = 25.5$

When 100,000 calculators are manufactured, it costs \$25.50 to manufacture each.

d. $n = m$, so $y = \dfrac{25}{1} = 25$ is the horizontal asymptote. Minimum costs will approach \$25.

80. $f(x) = \dfrac{150x + 120}{0.05x + 1}$

$n = m$, so $y = \dfrac{150}{0.05} = 3000$

The number of fish available in the pond approaches 3000.

81. $P(x) = \dfrac{72{,}900}{100x^2 + 729}$

$n < m$ so $y = 0$

As the number of years of education increases the percentage rate of unemployment approaches zero.

82. **a.**
$$\begin{aligned} P(x) &= M(x) + F(x) \\ &= 1.58x + 114.4 + 1.48x + 120.6 \\ &= 3.06x + 235 \end{aligned}$$

b. $R(x) = \dfrac{M(x)}{P(x)} = \dfrac{1.58x + 114.4}{3.06x + 235}$

c. $y = \dfrac{1.58}{3.06} \approx 0.52$

Over time, the percentage of men in the U.S. population will approach 52%.

83. $T(x) = \dfrac{4}{x+3} + \dfrac{2}{x}$

84. $1000 = lw$

$\dfrac{1000}{w} = l$

$P = 2x + 2\ \dfrac{1000}{x}$

$P = 2x + \dfrac{2000}{x}$

85. $2x^2 + 5x - 3 < 0$

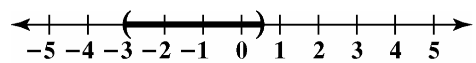
Solve the related quadratic equation.

$2x^2 + 5x - 3 = 0$

$(2x-1)(x+3) = 0$

The boundary points are -3 and $\dfrac{1}{2}$.

Testing each interval gives a solution set of $\left(-3, \dfrac{1}{2}\right)$



86. $2x^2 + 9x + 4 \ge 0$

Solve the related quadratic equation.

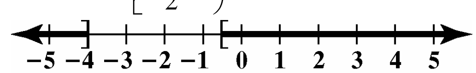
$2x^2 + 9x + 4 = 0$

$(2x+1)(x+4) = 0$

The boundary points are -4 and $-\frac{1}{2}$.

Testing each interval gives a solution set of

$(-\infty, -4] \cup \left[-\dfrac{1}{2}, \infty\right)$



87. $x^3 + 2x^2 > 3x$

Solve the related equation.

$x^3 + 2x^2 = 3x$

$x^3 + 2x^2 - 3x = 0$

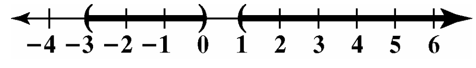
$x\left(x^2 + 2x - 3\right) = 0$

$x(x+3)(x-1) = 0$

The boundary points are -3, 0, and 1.

Testing each interval gives a solution set of

$(-3, 0) \cup (1, \infty)$



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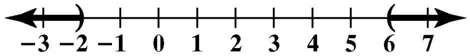
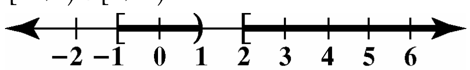
88. $\dfrac{x-6}{x+2} > 0$

Find the values of x that make the numerator and denominator zero.
The boundary points are –2 and 6.
Testing each interval gives a solution set of $(-\infty,-2)\cup(6,\infty)$.



89. $\dfrac{(x+1)(x-2)}{x-1} \ge 0$

Find the values of x that make the numerator and denominator zero.
The boundary points are –1, 1 and 2. We exclude 1 from the solution set, since this would make the denominator zero.
Testing each interval gives a solution set of $[-1,1)\cup[2,\infty)$.



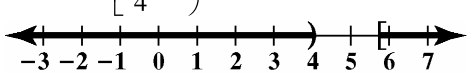
90. $\dfrac{x+3}{x-4} \le 5$

Express the inequality so that one side is zero.

$$\frac{x+3}{x-4} - 5 \le 0$$
$$\frac{x+3}{x-4} - \frac{5(x-4)}{x-4} \le 0$$
$$\frac{-4x+23}{x-4} \le 0$$

Find the values of x that make the numerator and denominator zero.

The boundary points are 4 and $\dfrac{23}{4}$. We exclude 4 from the solution set, since this would make the denominator zero.
Testing each interval gives a solution set of $(-\infty,4)\cup\left[\dfrac{23}{4},\infty\right)$.



91. **a.** $g(x) = 0.125x^2 + 2.3x + 27$

$g(35) = 0.125(35)^2 + 2.3(35) + 27 \approx 261$

The stopping distance on wet pavement for a motorcycle traveling 35 miles per hour is about 261 feet. This overestimates the distance shown in the graph by 1 foot.

b. $f(x) = 0.125x^2 - 0.8x + 99$

$0.125x^2 - 0.8x + 99 > 267$

$0.125x^2 - 0.8x - 168 > 0$

Solve the related quadratic equation.

$0.125x^2 - 0.8x - 168 = 0$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$$x = \frac{-(-0.8) \pm \sqrt{(-0.8)^2 - 4(0.125)(-168)}}{2(0.125)}$$

$x = -33.6,\ 40$

Testing each interval gives a solution set of $(-\infty,-33.6)\cup(40,\infty)$.

Thus, speeds exceeding 40 miles per hour on dry pavement will require over 267 feet of stopping distance.

92. $s = -16t^2 + v_0 t + s_0$

$32 < -16t^2 + 48t + 0$

$0 < -16t^2 + 48t - 32$

$0 < -16\left(t^2 - 3t + 2\right)$

$0 < -16(t-2)(t-1)$

F	T	F
	1	2

The projectile's height exceeds 32 feet during the time period from 1 to 2 seconds.

93. $w = ks$

$28 = k \cdot 250$

0.112=k

Thus, $w = 0.112s$.

$w = 0.112(1200) = 134.4$

1200 cubic centimeters of melting snow will produce 134.4 cubic centimeters of water.

94. $d = kt^2$

$144 = k(3)^2$

$k = 16$

$d = 16t^2$

$d = 16(10)^2 = 1{,}600$ ft

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95. $p = \frac{k}{w}$

$660 = \frac{k}{1.6}$

$1056 = k$

Thus, $p = \frac{1056}{w}$.

$p = \frac{1056}{2.4} = 440$

The pitch is 440 vibrations per second.

96. $l = \frac{k}{d^2}$

$28 = \frac{k}{8^2}$

$k = 1792$

$l = \frac{1792}{d^2}$

$l = \frac{1792}{4^2} = 112$ decibels

97. $t = \frac{kc}{w}$

$10 = \frac{k \cdot 30}{6}$

$10 = 5h$

$h = 2$

$t = \frac{2c}{w}$

$t = \frac{2(40)}{5} = 16$ hours

98. $V = khB$

$175 = k \cdot 15 \cdot 35$

$k = \frac{1}{3}$

$V = \frac{1}{3}hB$

$V = \frac{1}{3} \cdot 20 \cdot 120 = 800 \text{ ft}^3$

99. **a.** Use $L = \frac{k}{R}$ to find k.

$L = \frac{k}{R}$

$30 = \frac{k}{63}$

$63 \cdot 30 = 63 \cdot \frac{k}{63}$

$1890 = k$

Thus, $L = \frac{1890}{R}$.

b. This is an approximate model.

c. $L = \frac{1890}{R}$

$L = \frac{1890}{27} = 70$

The average life span of an elephant is 70 years.

Chapter 2 Test

1. $(6-7i)(2+5i) = 12+30i-14i-35i^2$

$= 12+16i+35$

$= 47+16i$

2. $\frac{5}{2-i} = \frac{5}{2-i} \cdot \frac{2+i}{2+i}$

$= \frac{5(2+i)}{4+1}$

$= \frac{5(2+i)}{5}$

$= 2+i$

3. $2\sqrt{-49} + 3\sqrt{-64} = 2(7i)+3(8i)$

$= 14i+24i$

$= 38i$

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4. $x^2 = 4x - 8$

$x^2 - 4x + 8 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(8)}}{2(1)}$

$x = \dfrac{4 \pm \sqrt{-16}}{2}$

$x = \dfrac{4 \pm 4i}{2}$

$x = 2 \pm 2i$

5. $f(x) = (x+1)^2 + 4$

vertex: (–1, 4)
axis of symmetry: $x = -1$
x-intercepts:

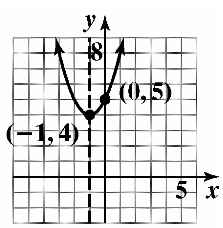
$(x+1)^2 + 4 = 0$

$x^2 + 2x + 5 = 0$

$x = \dfrac{-2 \pm \sqrt{4-20}}{2} = -1 \pm 2i$

no x-intercepts
y-intercept:

$f(0) = (0+1)^2 + 4 = 5$



***f*(*x*) = (*x* + 1)^2 + 4**

domain: $(-\infty, \infty)$; range: $[4, \infty)$

6. $f(x) = x^2 - 2x - 3$

$x = \dfrac{-b}{2a} = \dfrac{2}{2} = 1$

$f(1) = 1^2 - 2(1) - 3 = -4$

vertex: (1, –4)
axis of symmetry $x = 1$
x-intercepts:

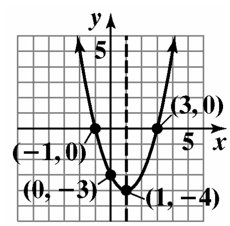
$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3$ or $x = -1$

y-intercept:

$f(0) = 0^2 - 2(0) - 3 = -3$



***f*(*x*) = *x*^{2} − 2*x* − 3**

domain: $(-\infty, \infty)$; range: $[-4, \infty)$

7. $f(x) = -2x^2 + 12x - 16$

Since the coefficient of x^2 is negative, the graph of $f(x)$ opens down and $f(x)$ has a maximum point.

$x = \dfrac{-12}{2(-2)} = 3$

$f(3) = -2(3)^2 + 12(3) - 16$
$= -18 + 36 - 16$
$= 2$

Maximum point: (3, 2)
domain: $(-\infty, \infty)$; range: $(-\infty, 2]$

8. $f(x) = -x^2 + 46x - 360$

$x = -\dfrac{b}{2a} = \dfrac{-46}{-2} = 23$

23 computers will maximize profit.

$f(23) = -(23)^2 + 46(23) - 360 = 169$

Maximum daily profit = $16,900.

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9. Let x = one of the numbers;
$14-x$ = the other number.
The product is $f(x)=x(14-x)$

$f(x)=x(14-x)=-x^2+14x$

The x-coordinate of the maximum is

$x=-\frac{b}{2a}=-\frac{14}{2(-1)}=-\frac{14}{-2}=7.$

$f(7)=-7^2+14(7)=49$

The vertex is (7, 49). The maximum product is 49. This occurs when the two number are 7 and $14-7=7$.

10. a. $f(x)=x^3-5x^2-4x+20$

$x^3-5x^2-4x+20=0$

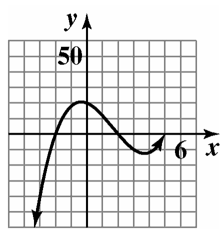
$x^2(x-5)-4(x-5)=0$

$(x-5)(x-2)(x+2)=0$

$x=5, 2, -2$

The solution set is {5, 2, –2}.

b. The degree of the polynomial is odd and the leading coefficient is positive. Thus the graph falls to the left and rises to the right.



11. $f(x)=x^5-x$

Since the degree of the polynomial is odd and the leading coefficient is positive, the graph of f should fall to the left and rise to the right. The x-intercepts should be –1 and 1.

12. a. The integral root is 2.

b.
$$\begin{array}{r|rrrr} 2 & 6 & -19 & 16 & -4 \\ & & 12 & -14 & 4 \\ \hline & 6 & -7 & 2 & 0 \end{array}$$

$6x^2-7x+2=0$

$(3x-2)(2x-1)=0$

$x=\frac{2}{3}$ or $x=\frac{1}{2}$

The other two roots are $\frac{1}{2}$ and $\frac{2}{3}$.

13. $2x^3+11x^2-7x-6=0$

p: ±1, ±2, ±3, ±6

q: ±1, ±2

$\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{2}, \pm\frac{3}{2}$

14. $f(x)=3x^5-2x^4-2x^2+x-1$

$f(x)$ has 3 sign variations.

$f(-x)=-3x^5-2x^4-2x^2-x-1$

$f(-x)$ has no sign variations.

There are 3 or 1 positive real solutions and no negative real solutions.

15. $x^3+9x^2+16x-6=0$

Since the leading coefficient is 1, the possible rational zeros are the factors of 6

$p=\frac{p}{q}: \pm 1, \pm 2, \pm 3, \pm 6$

$$\begin{array}{r|rrrr} -3 & 1 & 9 & 16 & -6 \\ & & -3 & -18 & 6 \\ \hline & 1 & 6 & -2 & 0 \end{array}$$

Thus $x=3$ is a root.

Solve the quotient $x^2+6x-2=0$ using the quadratic formula to find the remaining roots.

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(6)\pm\sqrt{(6)^2-4(1)(-2)}}{2(1)}$

$=\frac{-6\pm\sqrt{44}}{2}$

$=-3\pm\sqrt{11}$

The zeros are -3, $-3+\sqrt{11}$, and $-3-\sqrt{11}$.

16. $f(x)=2x^4-x^3-13x^2+5x+15$

a. Possible rational zeros are:

$p:\ \pm 1,\ \pm 3,\ \pm 5,\ \pm 15$

$q:\ \pm 1,\ \pm 2$

$\frac{p}{q}: \pm 1, \pm 3, \pm 5, \pm 15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}$

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b. Verify that -1 and $\frac{3}{2}$ are zeros as it appears in the graph:

$$\begin{array}{r|rrrrr} -1 & 2 & -1 & -13 & 5 & 15 \\ & & -2 & 3 & 10 & -15 \\ \hline & 2 & -3 & -10 & 15 & 0 \end{array}$$

$$\begin{array}{r|rrrr} \frac{3}{2} & 2 & -3 & -10 & 15 \\ & & 3 & 0 & -15 \\ \hline & 2 & 0 & -10 & 0 \end{array}$$

Thus, -1 and $\frac{3}{2}$ are zeros, and the polynomial factors as follows:

$$2x^4 - x^3 - 13x^2 + 5x + 15 = 0$$
$$(x+1)(2x^3 - 3x^2 - 10x + 15) = 0$$
$$(x+1)\left(x - \frac{3}{2}\right)(2x^2 - 10) = 0$$

Find the remaining zeros by solving:

$$2x^2 - 10 = 0$$
$$2x^2 = 10$$
$$x^2 = 5$$
$$x = \pm\sqrt{5}$$

The zeros are -1, $\frac{3}{2}$, and $\pm\sqrt{5}$.

17. $f(x)$ has zeros at –2 and 1. The zero at –2 has multiplicity of 2.
$x^3 + 3x^2 - 4 = (x-1)(x+2)^2$

18.
$$f(x) = a_0(x+1)(x-1)(x+i)(x-i)$$
$$= a_0(x^2-1)(x^2+1)$$
$$= a_0(x^4-1)$$

Since $f(3) = 160$, then

$$a_0(3^4 - 1) = 160$$
$$a_0(80) = 160$$
$$a_0 = \frac{160}{80}$$
$$a_0 = 2$$
$$f(x) = 2(x^4 - 1) = 2x^4 - 2$$

19. $f(x) = -3x^3 - 4x^2 + x + 2$
The graph shows a root at $x = -1$.
Use synthetic division to verify this root.

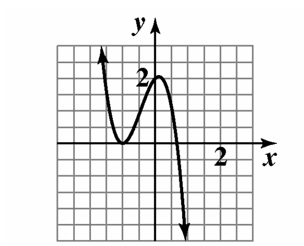
$$\begin{array}{r|rrrr} -1 & -3 & -4 & 1 & 2 \\ & & 3 & 1 & 4 \\ \hline & -3 & -1 & 2 & 0 \end{array}$$

Factor the quotient to find the remaining zeros.

$$-3x^2 - x + 2 = 0$$
$$-(3x-2)(x+1) = 0$$

The zeros (x-intercepts) are -1 and $\frac{2}{3}$.

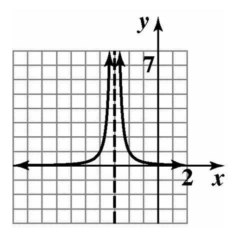
The y-intercept is $f(0) = 2$



$f(x) = -3x^3 - 4x^2 + x + 2$

20. $f(x) = \dfrac{1}{(x+3)^2}$

domain: $\{x \mid x \neq -3\}$ or $(-\infty, -3) \cup (-3, \infty)$



$f(x) = \dfrac{1}{(x+3)^2}$

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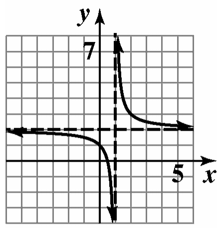
21. $f(x)=\dfrac{1}{x-1}+2$

domain: $\{x \mid x \neq 1\}$ or $(-\infty,1)\cup(1,\infty)$



$f(x) = \dfrac{1}{x-1} + 2$

22. $f(x)=\dfrac{x}{x^2-16}$

domain: $\{x \mid x \neq 4, x \neq -4\}$

Symmetry: $f(-x)=\dfrac{-x}{x^2-16}=-f(x)$

y-axis symmetry

x-intercept: $x = 0$

y-intercept: $y=\dfrac{0}{0^2-16}=0$

Vertical asymptotes:

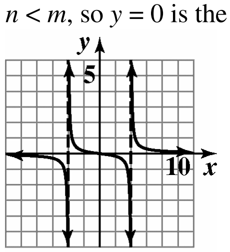
$x^2-16=0$

$(x-4)(x+4)=0$

$x = 4, -4$

Horizontal asymptote:

$n < m$, so $y = 0$ is the horizontal asymptote.



$f(x) = \dfrac{x}{x^2-16}$

23. $f(x)=\dfrac{x^2-9}{x-2}$

domain: $\{x \mid x \neq 2\}$

Symmetry: $f(-x)=\dfrac{x^2-9}{-x-2}$

$f(-x) \neq f(x), f(-x) \neq -f(x)$

No symmetry

x-intercepts:

$x^2-9=0$

$(x-3)(x+3)=0$

$x = 3, -3$

y-intercept: $y=\dfrac{0^2-9}{0-2}=\dfrac{9}{2}$

Vertical asymptote:

$x - 2 = 0$

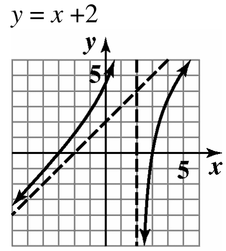
$x = 2$

Horizontal asymptote:

$n > m$, so no horizontal asymptote exists.

Slant asymptote: $f(x)=x+2-\dfrac{5}{x-2}$

$y = x + 2$



$f(x) = \dfrac{x^2-9}{x-2}$

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24. $f(x) = \dfrac{x+1}{x^2+2x-3}$

$x^2+2x-3=(x+3)(x-1)$

domain: $\{x \mid x \neq -3, x \neq 1\}$

Symmetry: $f(-x) = \dfrac{-x+1}{x^2-2x-3}$

$f(-x) \neq f(x), f(-x) \neq -f(x)$

No symmetry

x-intercept:

$x + 1 = 0$

$x = -1$

y-intercept: $y = \dfrac{0+1}{0^2+2(0)-3} = -\dfrac{1}{3}$

Vertical asymptotes:

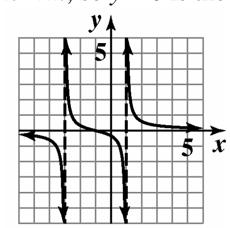
$x^2+2x-3=0$

$(x + 3)(x - 1) = 0$

x –3, 1

Horizontal asymptote:

$n < m$, so $y = 0$ is the horizontal asymptote.



$f(x) = \dfrac{x+1}{x^2+2x-3}$

25. $f(x) = \dfrac{4x^2}{x^2+3}$

domain: all real numbers

Symmetry: $f(-x) = \dfrac{4x^2}{x^2+3} = f(x)$

y-axis symmetry

x-intercept:

$4x^2=0$

$x = 0$

y-intercept: $y = \dfrac{4(0)^2}{0^2+3} = 0$

Vertical asymptote:

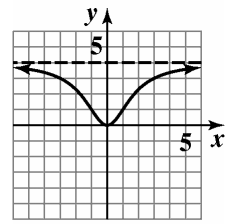
$x^2+3=0$

$x^2=-3$

No vertical asymptote.

Horizontal asymptote:

$n = m$, so $y = \dfrac{4}{1} = 4$ is the horizontal asymptote.



$f(x) = \dfrac{4x^2}{x^2+3}$

26. **a.** $\overline{C}(x) = \dfrac{300{,}000+10x}{x}$

b. Since the degree of the numerator equals the degree of the denominator, the horizontal asymptote is $x = \dfrac{10}{1} = 10$.

This represents the fact that as the number of satellite radio players produced increases, the production cost approaches \$10 per radio.

27.

$x^2 < x+12$

$x^2-x-12<0$

$(x+3)(x-4)<0$

Boundary values: –3 and 4

Solution set: $(-3, 4)$

28.

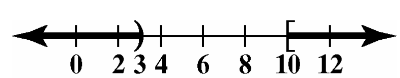
$\dfrac{2x+1}{x-3} \leq 3$

$\dfrac{2x+1}{x-3} - 3 \leq 0$

$\dfrac{10-x}{x-3} \leq 0$

Boundary values: 3 and 10

Solution set: $(-\infty, 3) \cup [10, \infty)$



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29. $i = \frac{k}{d^2}$

$20 = \frac{k}{15^2}$

$4500 = k$

$i = \frac{4500}{d^2} = \frac{4500}{10^2} = 45$ foot-candles

Cumulative Review Exercises (Chapters P–2)

1. domain: $(-2, 2)$ range: $[0, \infty)$

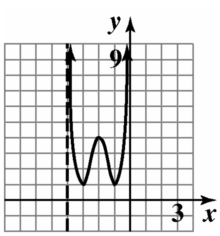
2. The zero at –1 touches the x-axis at turns around so it must have a minimum multiplicity of 2.
The zero at 1 touches the x-axis at turns around so it must have a minimum multiplicity of 2.

3. There is a relative maximum at the point (0, 3).

4. $(f \circ f)(-1) = f\left(f(-1)\right) = f(0) = 3$

5. $f(x) \to \infty$ as $\underline{x \to -2^+}$ or as $\underline{x \to 2^-}$

6.



7. $|2x - 1| = 3$

$2x - 1 = 3$

$2x = 4$

$x = 2$

$2x - 1 = -3$

$2x = -2$

$x = -1$

The solution set is {2, –1}.

8. $3x^2 - 5x + 1 = 0$

$x = \frac{5 \pm \sqrt{25 - 12}}{6} = \frac{5 \pm \sqrt{13}}{6}$

The solution set is $\left\{\frac{5+\sqrt{13}}{6}, \frac{5-\sqrt{13}}{6}\right\}$.

9. $9 + \frac{3}{x} = \frac{2}{x^2}$

$9x^2 + 3x = 2$

$9x^2 + 3x - 2 = 0$

$(3x - 1)(3x + 2) = 0$

$3x - 1 = 0 \quad 3x + 2 = 0$

$x = \frac{1}{3}$ or $x = -\frac{2}{3}$

The solution set is $\left\{\frac{1}{3}, -\frac{2}{3}\right\}$.

10. $x^3 + 2x^2 - 5x - 6 = 0$

p: ±1, ±2, ±3, ±6

q: ±1

$\frac{p}{q}$: ±1, ±2, ±3, ±6

$$\begin{array}{r|rrrr} -3 & 1 & 2 & -5 & -6 \\ & & -3 & 3 & 6 \\ \hline & 1 & -1 & -2 & 0 \end{array}$$

$x^3 + 2x^2 - 5x - 6 = 0$

$(x + 3)(x^2 - x - 2) = 0$

$(x + 3)(x + 1)(x - 2) = 0$

$x = -3$ or $x = -1$ or $x = 2$

The solution set is {–3, –1, 2}.

11. $|2x - 5| > 3$

$2x - 5 > 3$

$2x > 8$

$x > 4$

$2x - 5 < -3$

$2x < 2$

$x < 1$

$(-\infty, 1)$ or $(4, \infty)$

12. $3x^2 > 2x + 5$

$3x^2 - 2x - 5 > 0$

$3x^2 - 2x - 5 = 0$

$(3x - 5)(x + 1) = 0$

$x = \frac{5}{3}$ or $x = -1$

Test intervals are $(-\infty, -1)$, $\left(-1, \frac{5}{3}\right)$, $\left(\frac{5}{3}, \infty\right)$.

Testing points, the solution is $(-\infty, -1)$ or $\left(\frac{5}{3}, \infty\right)$.

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13. $f\left(x\right)=x^3-4x^2-x+4$

x-intercepts:

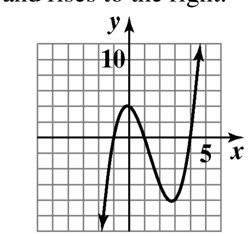
$$x^3-4x^2-x+4=0$$
$$x^2(x-4)-1(x-4)=0$$
$$(x-4)(x^2-1)=0$$
$$(x-4)(x+1)(x-1)=0$$

$x=-1,\ 1,\ 4$

x-intercepts:

$f\left(0\right)=0^3-4(0)^2-0+4=4$

The degree of the polynomial is odd and the leading coefficient is positive. Thus the graph falls to the left and rises to the right.



***f*(*x*) = *x*³ − 4*x*² − *x* + 4**

14. $f(x)=x^2+2x-8$

$$x=\frac{-b}{2a}=\frac{-2}{2}=-1$$
$$f(-1)=(-1)^2+2(-1)-8$$
$$=1-2-8=-9$$

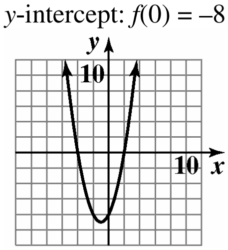
vertex: $(-1, -9)$

x-intercepts:

$$x^2+2x-8=0$$
$$(x+4)(x-2)=0$$

$x=-4$ or $x=2$

y-intercept: $f(0)=-8$



***f*(*x*) = *x*² + 2*x* − 8**

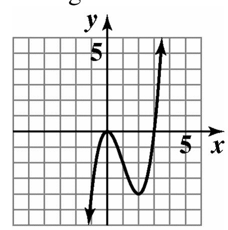
15. $f(x)=x^2(x-3)$

zeros: $x=0$ (multiplicity 2) and $x=3$

y-intercept: $y=0$

$f(x)=x^3-3x^2$

$n=3, a_n=0$ so the graph falls to the left and rises to the right.



***f*(*x*) = *x*²(*x* − 3)**

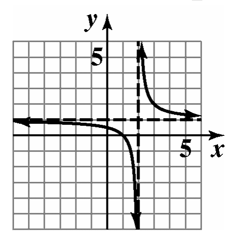
16. $f(x)=\dfrac{x-1}{x-2}$

vertical asymptote: $x=2$

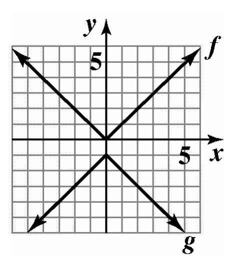
horizontal asymptote: $y=1$

x-intercept: $x=1$

y-intercept: $y=\dfrac{1}{2}$



17.



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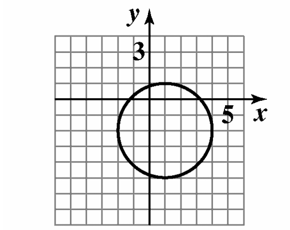
18.



$x^2 + y^2 - 2x + 4y - 4 = 0$

19. $(f \circ g)(x) = f\left(g(x)\right)$

$$= 2(4x-1)^2 - (4x-1) - 1$$
$$= 32x^2 - 20x + 2$$

20. $\dfrac{f(x+h) - f(x)}{h}$

$$= \frac{\left[2(x+h)^2 - (x+h) - 1\right] - \left[2x^2 - x - 1\right]}{h}$$
$$= \frac{2x^2 + 4hx - x + 2h^2 - h - 1 - 2x^2 + x + 1}{h}$$
$$= \frac{4hx + 2h^2 - h}{h}$$
$$= 4x + 2h - 1$$

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