

PREFACE

This Complete Solutions Manual for PreCalculus, Fifth Edition, by Faires and DeFranza, contains solutions to all the exercises in the book. Our approach to PreCalculus places a heavy emphasis on graphing techniques to help students visualize and master important topics. So in this manual you will find more solutions accompanied by illustrations than you would find in typical books at this level.

Also available for your students is a Student Solutions Manual and Study Guide. This Guide provides students with access to more extensive algebra, geometry and trigonometry review material. The algebra and trigonometry review is interwoven through the text making the presentation in the Guide valuable for those students who need extra review and practice.

The Student Study Guide also contains supplemental examples for each section of the book, detailed solutions to all the odd-numbered exercises, and solutions for all of the exercises in the Chapter Tests.

Two short examinations that students can use to test their readiness for Precalculus and for Calculus can be downloaded in Adobe PDF form at the book web site

<http://www.math.ysu.edu/~fares/PreCalculus>

We suggest that students work one of the examinations at the start of the PreCalculus course and the second at completion of the course. Students scoring approximately 16 or higher on the 40 question examinations are likely prepared to take a PreCalculus course based on this book. A score of approximately 28 or higher indicates that the student is ready for a College or University Calculus sequence.

If you have suggestions for improvements in future editions of the PreCalculus book or either of these supplements, we would be most grateful for your comments. Additional information about the book and any updates will be placed at the book web site.

We would like to thank Jena Baun, a second-year student at Youngstown State University, who did much of the technical preparation of this manual, and Krista Foster, a graduate student at Penn State University, for checking the accuracy of the results.

Doug Faires
fares@math.ysu.edu

Youngstown State University

Jim DeFranza
jdefranza@stlawu.edu
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St. Lawrence University

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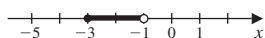
Solutions to All Exercises

Exercise Set 1.2 (Page 12)

1. $-1 \leq x \leq 5$



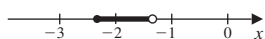
2. $-3 \leq x < -1$



3. $-\sqrt{3} < x \leq \sqrt{2}$



4. $-\sqrt{5} \leq x < -\sqrt{2}$



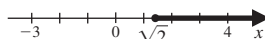
5. $x < 3$



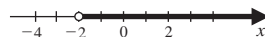
6. $x \leq 0$



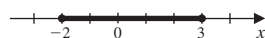
7. $x \geq \sqrt{2}$



8. $x > -2$



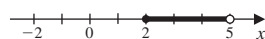
9. $[-2, 3]$



10. $(2, 6]$



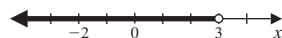
11. $[2, 5)$



12. $[-2, 4)$



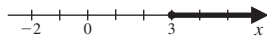
13. $(-\infty, 3)$



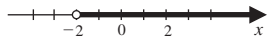
14. $(-\infty, -2]$



15. $[3, \infty)$



16. $(-2, \infty)$



17. (a) $|3 - 7| = |-4| = 4$

(b) $\frac{1}{2}(3 + 7) = 5$

18. (a) $|4 - 7| = |-3| = 3$

(b) $\frac{1}{2}(4 + 7) = \frac{11}{2}$

19. (a) $|-3 - 5| = |-8| = 8$

(b) $\frac{1}{2}(-3 + 5) = 1$

20. (a) $|-4 - (-1)| = |-3| = 3$

(b) $\frac{1}{2}(-4 + (-1)) = -\frac{5}{2}$

21. $x^2 + 3x + 2 = (x + 1)(x + 2)$

22. $x^2 + 7x + 6 = (x + 1)(x + 6)$

23. $x^2 + 5x + 6 = (x + 2)(x + 3)$

24. $x^2 - 9x + 8 = (x - 1)(x - 8)$

25. $x^2 + 4x - 12 = (x - 2)(x + 6)$

26. $x^2 - 4x - 12 = (x + 2)(x - 6)$

27. (a) $(0, 3]$

(b) $[-1, 4)$

28. (a) $[-2, -1]$

(b) $[-3, 2)$

29. (a) $(-2, 0)$

(b) $(-\infty, 3]$

30. (a) $[1, 3)$

(b) $[-3, \infty)$

31. The inequality $x + 3 < 5$ implies that $x < 2$, so in interval notation $(-\infty, 2)$.

32. The inequality $x - 4 < 9$ implies that $x < 13$, so in interval notation $(-\infty, 13)$.

33. The inequality $2x - 2 \geq 8$ implies that $2x \geq 10$, so $x \geq 5$, and in interval notation $[5, \infty)$.

34. The inequality $3x + 2 \geq 8$ implies that $3x \geq 6$, so $x \geq 2$, and in interval notation $[2, \infty)$.

35. The inequality $-3x + 4 < 5$ implies that $-3x < 1$, so $x > -\frac{1}{3}$, and in interval notation $(-\frac{1}{3}, \infty)$.

36. The inequality $-2x - 4 \geq 10$ implies that $-2x \geq 14$, so $x \leq -7$, and in interval notation $(-\infty, -7]$.

37. The inequality $2x + 9 \leq 5 + x$ implies that $x \leq -4$, so in interval notation $(-\infty, -4]$.

38. The inequality $-3x - 2 < 3 - x$ implies that $-2x < 5$, so $x > -\frac{5}{2}$, and in interval notation $(-\frac{5}{2}, \infty)$.

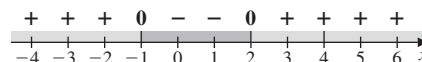
39. The inequality $-1 < 3x - 3 < 6$ implies that $2 < 3x < 9$, so $\frac{2}{3} < x < 3$, and in interval notation $(\frac{2}{3}, 3)$.

40. The inequality $-3 < 2x + 1 \leq 2$ implies that $-4 < 2x \leq 1$, so $-2 < x \leq \frac{1}{2}$, and in interval notation $(-2, \frac{1}{2}]$.

41. The inequality

$$(x + 1)(x - 2) \geq 0$$

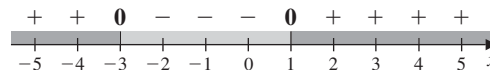
is satisfied for x in $(-\infty, -1] \cup [2, \infty)$



42. The inequality

$$(x - 1)(x + 3) < 0$$

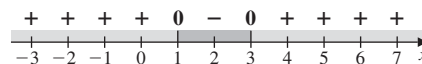
is satisfied for x in $(-3, 1)$.



43. The inequality

$$x^2 - 4x + 3 = (x - 1)(x - 3) \leq 0$$

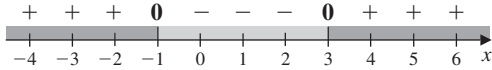
is satisfied for x in $[1, 3]$.



44. The inequality

$$x^2 - 2x - 3 = (x + 1)(x - 3) > 0$$

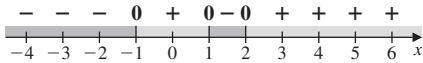
is satisfied for x in $(-\infty, -1) \cup (3, \infty)$.



45. The inequality

$$(x - 1)(x - 2)(x + 1) \leq 0$$

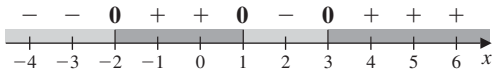
is satisfied for x in $(-\infty, -1] \cup [1, 2]$.



46. The inequality

$$(x - 1)(x + 2)(x - 3) \geq 0$$

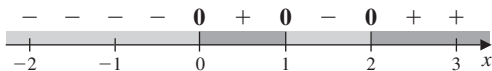
is satisfied for x in $[-2, 1] \cup [3, \infty)$.



47. The inequality

$$x^3 - 3x^2 + 2x = x(x - 1)(x - 2) \geq 0$$

is satisfied for x in $[0, 1] \cup [2, \infty)$.



48. The inequality

$$x^3 - 3x^2 - 4x = x(x - 4)(x + 1) < 0$$

is satisfied for x in $(-\infty, -1) \cup (0, 4)$.



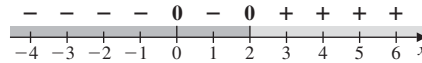
49. The inequality

$$x^3 - 2x^2 = x^2(x - 2) < 0$$

implies that

$$(x - 2) < 0 \text{ and } x \neq 0,$$

which is satisfied for x in $(-\infty, 0) \cup (0, 2)$.



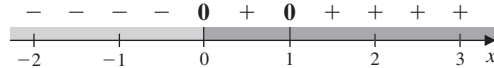
50. The inequality

$$x^3 - 2x^2 + x = x(x^2 - 2x + 1) > 0$$

implies that

$$x(x - 1)^2 > 0 \text{ and } x \neq 0,$$

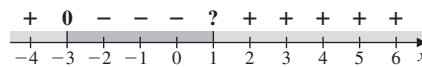
which is satisfied for x in $(0, 1) \cup (1, \infty)$.



51. The inequality

$$\frac{x + 3}{x - 1} \geq 0$$

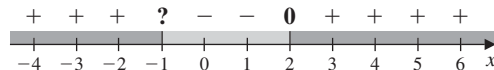
is satisfied for x in $(-\infty, -3] \cup (1, \infty)$.



52. The inequality

$$\frac{x - 2}{x + 1} \leq 0$$

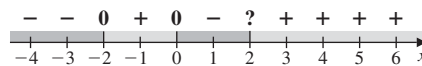
is satisfied for x in $(-1, 2]$.



53. The inequality

$$\frac{x(x + 2)}{x - 2} \leq 0$$

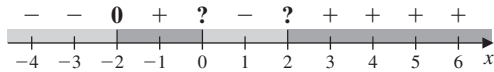
is satisfied for x in $(-\infty, -2] \cup [0, 2)$.



54. The inequality

$$\frac{x+2}{x(x-2)} > 0$$

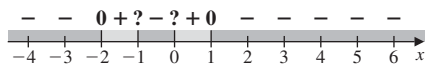
is satisfied for x in $(-2, 0) \cup (2, \infty)$.



55. The inequality

$$\frac{(1-x)(x+2)}{x(x+1)} > 0$$

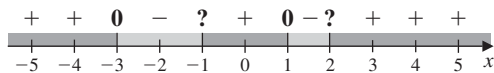
is satisfied for x in $(-2, -1) \cup (0, 1)$.



56. The inequality

$$\frac{(1-x)(x+3)}{(x+1)(2-x)} \leq 0$$

is satisfied for x in $[-3, -1) \cup [1, 2)$.



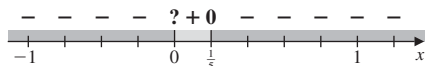
57. The inequality

$$\frac{1}{x} \leq 5 \quad \text{implies that} \quad \frac{1}{x} - 5 \leq 0,$$

which can be written as

$$\frac{1-5x}{x} \leq 0,$$

so in interval notation $(-\infty, 0) \cup \left[\frac{1}{5}, \infty\right)$.



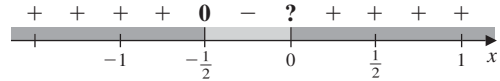
58. The inequality

$$-2 \leq \frac{1}{x} \quad \text{implies that} \quad \frac{1}{x} + 2 \geq 0,$$

which can be written as

$$\frac{1+2x}{x} \geq 0,$$

so in interval notation $\left(-\infty, -\frac{1}{2}\right] \cup (0, \infty)$.



59. The inequality

$$\frac{2}{x-1} \geq \frac{3}{x+2}$$

implies that

$$\frac{2}{x-1} - \frac{3}{x+2} \geq 0,$$

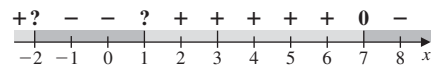
and

$$\frac{2(x+2) - 3(x-1)}{(x-1)(x+2)} \geq 0,$$

which can be written as

$$\frac{7-x}{(x-1)(x+2)} \geq 0,$$

so in interval notation $(-\infty, -2) \cup (1, 7]$.



60. The inequality

$$\frac{2}{x-1} - \frac{x}{x+1} \leq -1$$

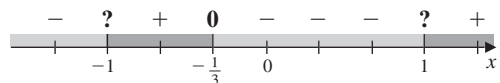
implies that

$$0 \geq \frac{2(x+1) - x(x-1)}{(x-1)(x+1)} + 1,$$

which can be written as

$$0 \geq \frac{3x+1}{(x-1)(x+1)},$$

so in interval notation $(-\infty, -1) \cup \left[-\frac{1}{3}, 1\right)$.



61. We have $|5x - 3| = 2$ when $5x - 3 = 2$ or $5x - 3 = -2$, so $x = 1$ or $x = \frac{1}{5}$.
62. We have $|2x + 3| = 1$ when $2x + 3 = 1$ or $2x + 3 = -1$, so $x = -1$ or $x = -2$.
63. We have $\left| \frac{x-1}{2x+3} \right| = 2$ when $\frac{x-1}{2x+3} = 2$ or $\frac{x-1}{2x+3} = -2$, which implies that $x - 1 = 2x + 3$ or $x - 1 = -4x - 6$ which implies $-7 = 3x$ or $5x = -5$, so $x = -\frac{7}{3}$ or $x = -1$.
64. We have $\left| \frac{2x+1}{x-3} \right| = 4$ when $\frac{2x+1}{x-3} = 4$ or $\frac{2x+1}{x-3} = -4$, which implies that $2x + 1 = 4x - 12$ or $2x + 1 = -4x + 12$ which implies $13 = 2x$ or $6x = 11$, so $x = \frac{13}{2}$ or $x = \frac{11}{6}$.
65. We have $|x - 4| \leq 1$, which implies $-1 \leq x - 4 \leq 1$, so $3 \leq x \leq 5$, and in interval notation we have $[3, 5]$.
66. We have $|4x - 1| < 0.01$, which implies $-0.01 < 4x - 1 < 0.01$, so $0.99 < 4x < 1.01$ which implies $0.2475 < x < 0.2525$, and in interval notation we have $(0.2475, 0.2525)$.
67. We have $|3 - x| \geq 2$, which implies $3 - x \geq 2$ or $3 - x \leq -2$, so $-x \geq -1$ or $-x \leq -5$, which implies $x \leq 1$ or $x \geq 5$, and in interval notation we have $(-\infty, 1] \cup [5, \infty)$.
68. We have $|2x - 1| > 5$, which implies $2x - 1 > 5$ or $2x - 1 < -5$, so $x > 3$ or $x < -2$, and in interval notation we have $(-\infty, -2) \cup (3, \infty)$.
69. We have $\frac{1}{|x+5|} > 2$, which implies $|x+5| < \frac{1}{2}$, so $-\frac{1}{2} < x + 5 < \frac{1}{2}$, which implies $-\frac{11}{2} < x < -\frac{9}{2}$, and in interval notation we have $\left(-\frac{11}{2}, -\frac{9}{2}\right) \cup \left(-5, -\frac{9}{2}\right)$. We omit $x = -5$ since this is where $|x + 5| = 0$.
70. We have $\left| \frac{3}{2x+1} \right| < 1$, which implies $\frac{3}{|2x+1|} < 1$, so $|2x+1| > 3$ which implies $2x + 1 > 3$ or $2x + 1 < -3$, therefore $x > 1$ or $x < -2$, and in interval notation we have $(-\infty, -2) \cup (1, \infty)$.
71. We have $|x^2 - 4| > 0$, which implies $x^2 - 4 \neq 0$, so $x \neq \pm 2$, and in interval notation we have $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$.
72. We have $|x^2 - 4| \leq 1$, which implies $-1 \leq x^2 - 4 \leq 1$, so $x^2 - 3 \geq 0$ which implies $x \leq -\sqrt{3}$ or $x \geq \sqrt{3}$ and $x^2 - 5 \leq 0$ which implies $-\sqrt{5} \leq x \leq \sqrt{5}$, so $-\sqrt{5} \leq x \leq -\sqrt{3}$ or $\sqrt{3} \leq x \leq \sqrt{5}$. In interval notation we have $[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$.
73. Since $0 < a < b$ we have both $0 < a^2 < ab$ and $0 < ab < b^2$. Hence, $0 < a^2 < ab < b^2$.
74. To solve $|x - 1| = |2x + 2|$, there are four possibilities:
- $x - 1 = 2x + 2$ which implies $x = -3$;
 - $x - 1 = -(2x + 2)$ so $x - 1 = -2x - 2$ which implies $x = -\frac{1}{3}$;
 - $-(x - 1) = 2x + 2$ which implies $x = -\frac{1}{3}$;
 - $-(x - 1) = -(2x + 2)$ which implies $x = -3$;
- So the solutions are $x = -3$ and $x = -\frac{1}{3}$. Note that $|-3 - 1| = 4 = |-6 + 2|$ and $|\frac{1}{3} - 1| = \frac{2}{3} = |2(-\frac{1}{3}) + 2|$.
75. (a) Since $20 \leq F \leq 50$ implies that $-12 \leq F - 32 \leq 18$ we have $-12 \left(\frac{5}{9}\right) \leq \frac{5}{9}(F - 32) \leq 18 \left(\frac{5}{9}\right)$, so $-\frac{20}{3} \leq C \leq 10$.
- (b) Since $20 \leq C \leq 50$ we have $20 \leq \frac{5}{9}(F - 32) \leq 50$, which implies $\frac{9}{5}(20) \leq F - 32 \leq \frac{9}{5}(50)$ so $68 \leq F \leq 122$.
76. We have $-16t^2 + 48t + 128 \geq 64$ which implies $-16t^2 + 48t + 64 = -16(t^2 - 3t - 4) \geq 0$, so $(t - 4)(t + 1) \leq 0$ and $-1 \leq t \leq 4$. Since t is time, it is positive and the solution is $0 \leq t \leq 4$.

77. Suppose that amount A is invested at 5% and amount $50000 - A$ is invested at 9%. The total amount needed from the investments is $0.075 \cdot 50000 = 3750$ dollars. So we need to

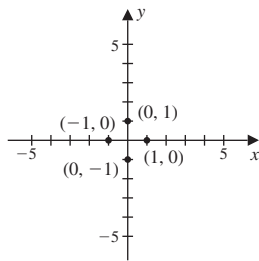
find A satisfying

$$3750 = 0.05A + 0.09(50000 - A) = 4500 - 0.04A.$$

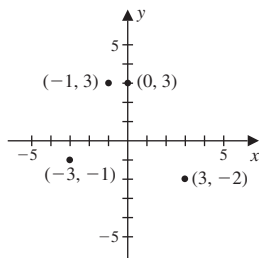
Solving the equation gives $A = \$18,750$

Exercise Set 1.3 (Page 20)

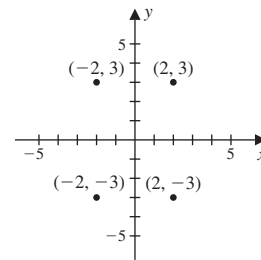
1. The points are on the graph.



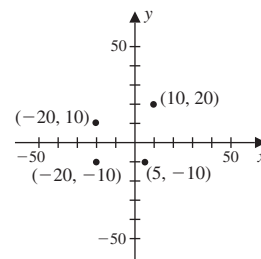
2. The points are on the graph.



3. The points are on the graph.



4. The points are on the graph.



5. (a) The distance is

$$\sqrt{(2 - (-1))^2 + (4 - 3)^2} = \sqrt{10}.$$

- (b) The midpoint is

$$\left(\frac{2 - 1}{2}, \frac{4 + 3}{2} \right) = \left(\frac{1}{2}, \frac{7}{2} \right).$$

6. (a) The distance is

$$\sqrt{(-3 - 5)^2 + (8 - 4)^2} = \sqrt{80} = 4\sqrt{5}.$$

- (b) The midpoint is

$$\left(\frac{-3 + 5}{2}, \frac{8 + 4}{2} \right) = (1, 6).$$

7. (a) The distance is

$$\begin{aligned} & \sqrt{(\pi - (-1))^2 + (0 - 2)^2} \\ &= \sqrt{(\pi + 1)^2 + 4} \\ &= \sqrt{\pi^2 + 2\pi + 5}. \end{aligned}$$

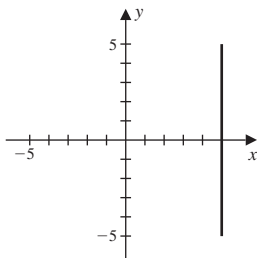
- (b) The midpoint is $\left(\frac{\pi - 1}{2}, \frac{0 + 2}{2}\right) = \left(\frac{\pi - 1}{2}, 1\right)$.

8. (a) The distance is

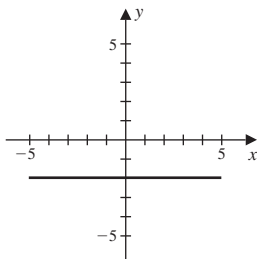
$$\begin{aligned} & \sqrt{(\sqrt{3} - \sqrt{2})^2 + (\sqrt{2} - \sqrt{3})^2} \\ &= \sqrt{2(\sqrt{3} - \sqrt{2})^2} \\ &= \sqrt{2}(\sqrt{3} - \sqrt{2}) \\ &= \sqrt{6} - 2. \end{aligned}$$

- (b) The midpoint is $\left(\frac{\sqrt{3} + \sqrt{2}}{2}, \frac{\sqrt{3} + \sqrt{2}}{2}\right)$.

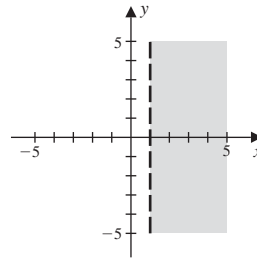
9. The values when $x = 5$ are on the graph.



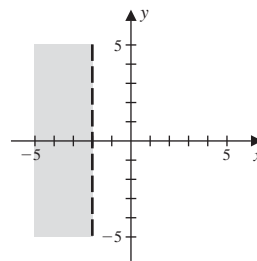
10. The values when $y = -2$ are on the graph.



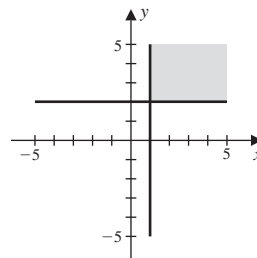
11. The values when $x > 1$ are on the graph.



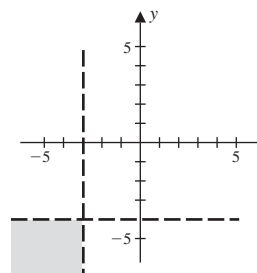
12. The values when $x < -2$ are on the graph.



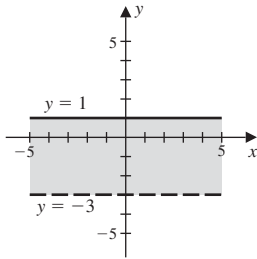
13. The values when $x \geq 1$ and $y \geq 2$ are on the graph.



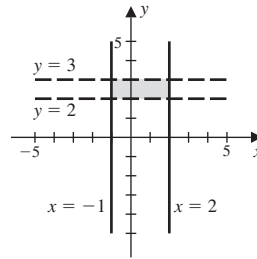
14. The values when $x < -3$ and $y < -4$ are on the graph.



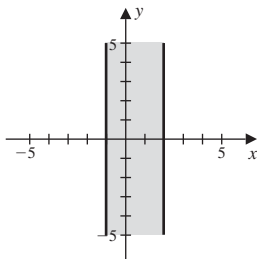
15. The values when $-3 < y \leq 1$ are on the graph.



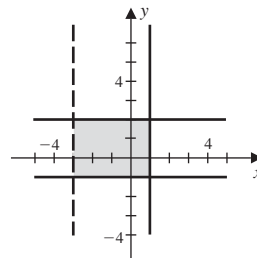
16. The values when $-1 \leq x \leq 2$ are on the graph.



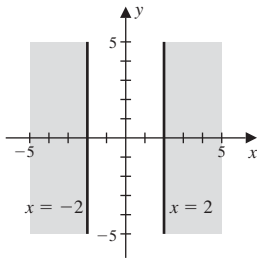
20. The values when $-3 < x \leq 1$ and $-1 \leq y \leq 2$ are on the graph.



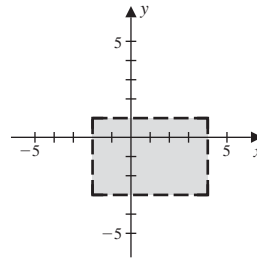
17. The values when $2 \leq |x|$, that is, when $x \geq 2$ or $x \leq -2$, are on the graph.



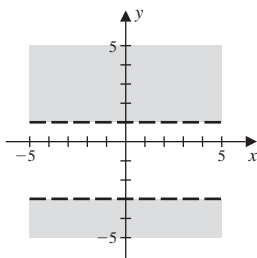
21. The values when $|x - 1| < 3$ and $|y + 1| < 2$, that is, when $-2 < x < 4$ and $-3 < y < 1$ are on the graph.



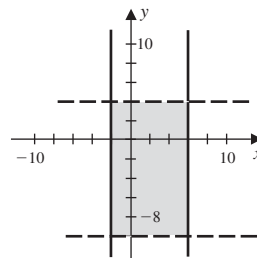
18. The values when $|y + 1| > 2$, that is, when $y > 1$ or $y < -3$ are on the graph.



22. The values when $|x - 2| \leq 4$ and $|y + 3| < 7$, that is, when $-2 \leq x \leq 6$ and $-10 < y < 4$, are on the graph.

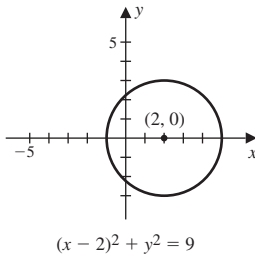


19. The values when $-1 \leq x \leq 2$ and $2 < y < 3$ are on the graph.



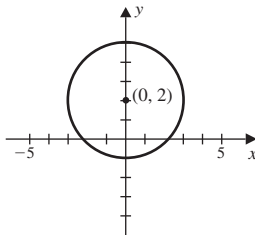
23. The circle with center $(2, 0)$ and radius 3 has equation

$$9 = (x - 2)^2 + y^2.$$



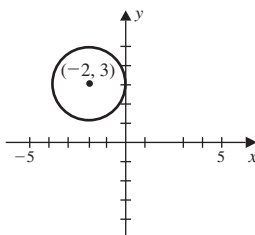
24. The circle with center $(0, 2)$ and radius 3 has equation

$$9 = x^2 + (y - 2)^2.$$



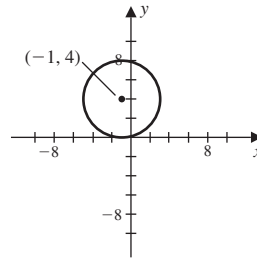
25. The circle with center $(-2, 3)$ and radius 2 has equation

$$4 = (x + 2)^2 + (y - 3)^2.$$



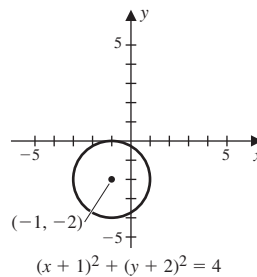
26. The circle with center $(-1, 4)$ and radius 4 has equation

$$16 = (x + 1)^2 + (y - 4)^2.$$



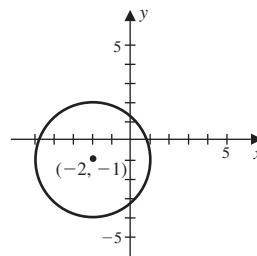
27. The circle with center $(-1, -2)$ and radius 2 has equation

$$4 = (x + 1)^2 + (y + 2)^2.$$



28. The circle with center $(-2, -1)$ and radius 3 has equation

$$9 = (x + 2)^2 + (y + 1)^2.$$



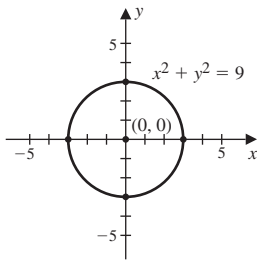
29. The circle has equation

$$4 = (x - 2)^2 + (y - 2)^2.$$

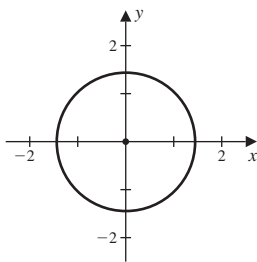
30. The circle has equation

$$25 = (x - 2)^2 + (y - 1)^2.$$

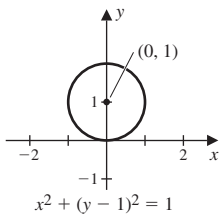
31. The circle has center $(0, 0)$ and radius $\sqrt{9} = 3$.



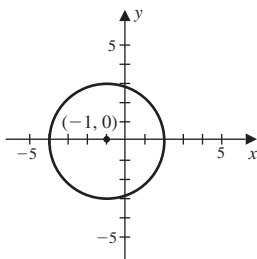
32. The circle has center $(0, 0)$ and radius $\sqrt{2}$.



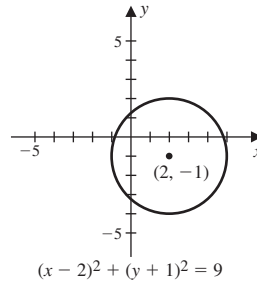
33. The circle has center $(0, 1)$ and radius $\sqrt{1} = 1$.



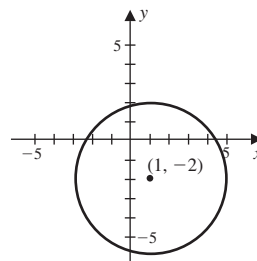
34. The circle with center $(-1, 0)$ and radius $\sqrt{9} = 3$.



35. The circle has center $(2, -1)$ and radius $\sqrt{9} = 3$.



36. The circle has center $(1, -2)$ and radius $\sqrt{16} = 4$.



37. The circle has center $(1, 0)$ and radius 2 since

$$3 = x^2 - 2x + y^2 = (x^2 - 2x + 1) - 1 + y^2,$$

$$\text{so } (x - 1)^2 + y^2 = 4.$$

38. The circle has center $(0, -2)$ and radius 1 since

$$-3 = x^2 + y^2 + 4y = x^2 + (y^2 + 4y + 4) - 4,$$

$$\text{so } x^2 + (y + 2)^2 = 1.$$

39. The circle has center $(-1, 2)$ and radius 1 since

$$-4 = x^2 + 2x + y^2 - 4y$$

$$= (x^2 + 2x + 1) - 1 + (y^2 - 4y + 4) - 4,$$

$$\text{so } (x + 1)^2 + (y - 2)^2 = 1.$$

40. The circle has center $(1, -2)$ and radius 3 since

$$4 = x^2 - 2x + y^2 + 4y$$

$$= (x^2 - 2x + 1) - 1 + (y^2 + 4y + 4) - 4,$$

$$\text{so } (x - 1)^2 + (y + 2)^2 = 9.$$

41. The circle has center $(2, 1)$ and radius 3 since

$$\begin{aligned} 0 &= x^2 - 4x + y^2 - 2y - 4 \\ &= (x^2 - 4x + 4) - 4 + (y^2 - 2y + 1) - 1 - 4, \end{aligned}$$

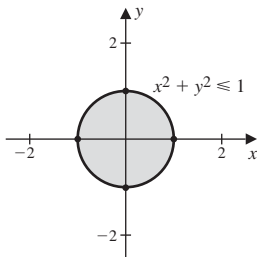
$$\text{so } (x - 2)^2 + (y - 1)^2 = 9.$$

42. The circle has center $(-2, -3)$ and radius 2 since

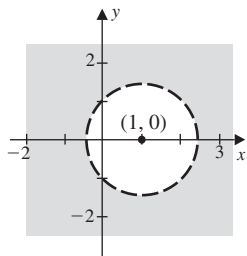
$$\begin{aligned} 0 &= x^2 + 4x + y^2 + 6y + 9 \\ &= (x^2 + 4x + 4) - 4 + (y^2 + 6y + 9), \end{aligned}$$

$$\text{so } (x + 2)^2 + (y + 3)^2 = 4.$$

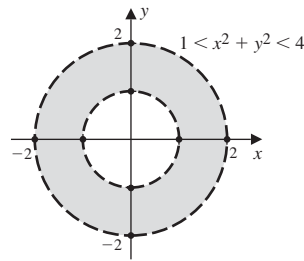
43. The values when $x^2 + y^2 \leq 1$ are on the graph.



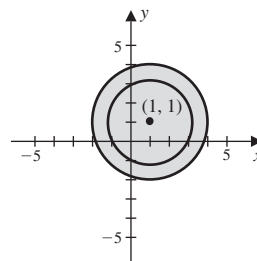
44. The values when $(x - 1)^2 + y^2 > 2$ are on the graph.



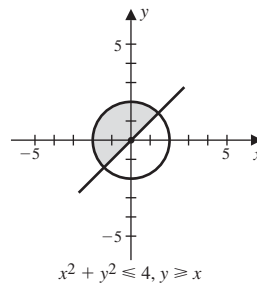
45. The values when $1 < x^2 + y^2 < 4$ are on the graph.



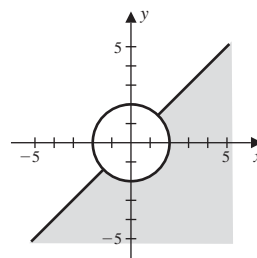
46. The values when $4 \leq (x - 1)^2 + (y - 1)^2 \leq 9$ are on the graph.



47. The values when $x^2 + y^2 \leq 4$ and $y \geq x$ are on the graph.



48. The values when $x^2 + y^2 \geq 4$ and $y \leq x$ are on the graph.



49. The point $(6, 3)$ is closer to the origin, since $d((6, 3), (0, 0)) = \sqrt{45}$ and $d((-7, 2), (0, 0)) = \sqrt{53}$.

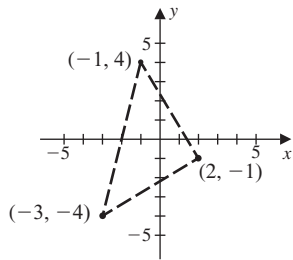
50. The points are the same distance from $(3, 2)$, since

$$\begin{aligned} d((1, 6), (3, 2)) &= \sqrt{(1-3)^2 + (6-2)^2} \\ &= \sqrt{4 + 16} \\ &= \sqrt{20} \end{aligned}$$

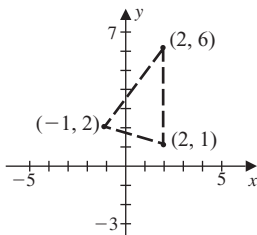
and

$$\begin{aligned} d((7, 4), (3, 2)) &= \sqrt{(7-3)^2 + (4-2)^2} \\ &= \sqrt{16 + 4} \\ &= \sqrt{20}. \end{aligned}$$

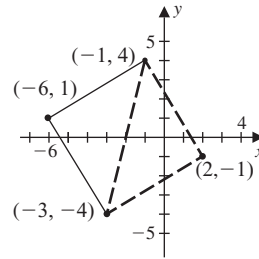
51. (a) Let $a = d((-3, -4), (2, -1)) = \sqrt{34}$, $b = d((-1, 4), (2, -1)) = \sqrt{34}$, and $c = d((-1, 4), (-3, -4)) = \sqrt{68}$. Since $a^2 + b^2 = c^2$, the points are the vertices of a right triangle.
- (b) The right angle is located at vertex $(2, -1)$.



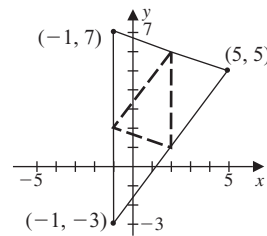
52. Since $d((2, 6), (-1, 2)) = d((2, 6), (2, 1)) = 5$, the triangle is isosceles.



53. The point $(-3, -4)$ is 5 units left and 3 units down from the point $(2, -1)$. The unique point producing a square is the same distance from $(-1, 4)$, that is, $(-1-5, 4-3) = (-6, 1)$.

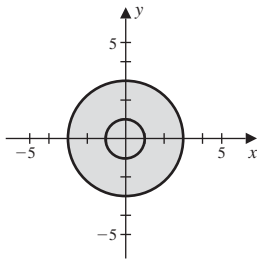


54. The points $(5, 5)$, $(-1, 7)$ and $(-1, -3)$ all work.

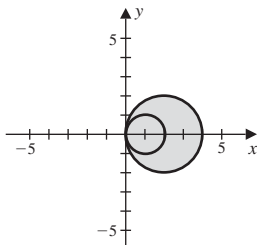


55. The radius $r = d((0, 0), (2, 3)) = \sqrt{13}$, so the equation of the circle is $x^2 + y^2 = 13$.
56. The radius $r = d((1, 3), (-2, 4)) = \sqrt{10}$, so the equation of the circle is $(x-1)^2 + (y-3)^2 = 10$.
57. The radius $r = d((3, 7), (0, 7)) = \sqrt{9} = 3$, so the equation of the circle is $(x-3)^2 + (y-7)^2 = 9$.
58. A point on the y -axis has the form $(0, y)$ and is equidistant from $(2, 1)$ and $(4, -3)$ provided $\sqrt{(0-2)^2 + (y-1)^2} = \sqrt{(0-4)^2 + (y+3)^2}$ or $4 + y^2 - 2y + 1 = 16 + y^2 + 6y + 9$, which simplifies to $-20 = 8y$ and $y = -\frac{5}{2}$. The point is $(0, -\frac{5}{2})$.
59. A circle with radius 3, tangent to both the x - and y -axes, and center in the second quadrant, has center $(-3, 3)$. The equation is $(x+3)^2 + (y-3)^2 = 9$.
60. The point $(3, 4)$ is in the first quadrant and is a horizontal distance of 2 units from the line $x = 1$ and a vertical distance of 2 units from the line $y = 2$. So $(3, 4)$ is the center of the circle, and since the radius is 2, the equation is $(x-3)^2 + (y-4)^2 = 4$.

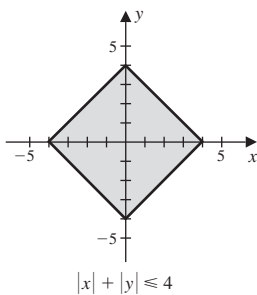
61. The area of the shaded region in the figure is $\pi(3)^2 - \pi(1)^2 = 8\pi$.



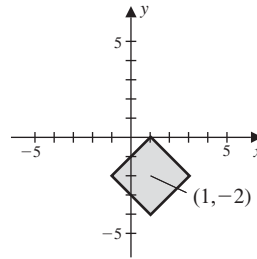
62. The area of the shaded region in the figure is $\pi(2)^2 - \pi(1)^2 = 3\pi$.



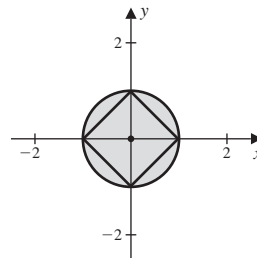
63. The values when $|x| + |y| \leq 4$ are on the graph.



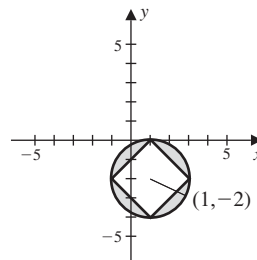
64. The values when $|x - 1| + |y + 2| \leq 2$ are on the graph.



65. The area of the shaded region is the area of the circle minus the area of the square, that is, $\pi(1)^2 - \sqrt{2}\sqrt{2} = \pi - 2$.



66. The area of the shaded region is the area of the circle minus the area of the square, that is, $\pi(2)^2 - (2\sqrt{2})(2\sqrt{2}) = 4\pi - 8$.



67. If c is the total cost, then

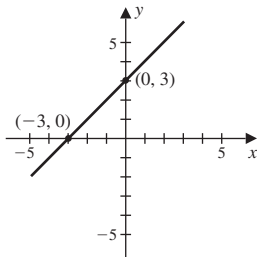
$$C = \sqrt{13}(200,000) + 50000 \approx \$771,110.03.$$

68. Ship A has gone 75 miles and Ship B has gone 24 miles, so the distance between them is $\sqrt{24^2 + 75^2} \approx 78$ miles.

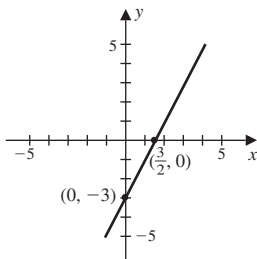
69. The direct route would cost $3(200,000) + \sqrt{20}(150,000)$ dollars or approximately 127,082,000 dollars. Going from A to C and then from C to B would cost $9(200,000) = 1,800,000$ dollars.

Exercise Set 1.4 (Page 28)

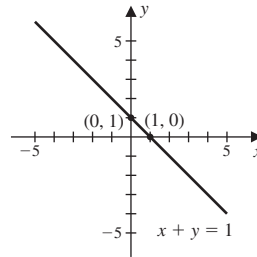
1. Symmetry with respect to the x -axis.
2. Symmetry with respect to the origin.
3. Symmetry with respect to the x -axis, y -axis, and origin.
4. No axis or origin symmetry.
5. Symmetry with respect to the origin.
6. Symmetry with respect to the y -axis.
7. Symmetry with respect to the x -axis.
8. Symmetry with respect to the origin.
9. For $y = x + 3$:
Intercepts: $(-3, 0), (0, 3)$;
Symmetry: No axis or origin symmetry.



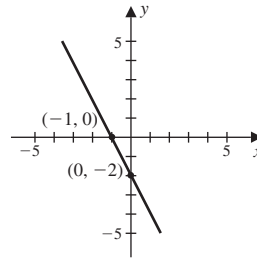
10. For $y = 2x - 3$:
Intercepts: $(\frac{3}{2}, 0), (0, -3)$;
Symmetry: No axis or origin symmetry.



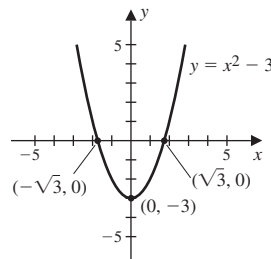
11. For $x + y = 1$, or $y = 1 - x$:
Intercepts: $(1, 0), (0, 1)$;
Symmetry: No axis or origin symmetry.



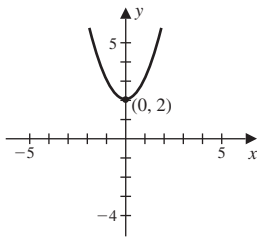
12. For $-2x - y = 2$, or $y = -2x - 2$:
Intercepts: $(-1, 0), (0, -2)$;
Symmetry: No axis or origin symmetry.



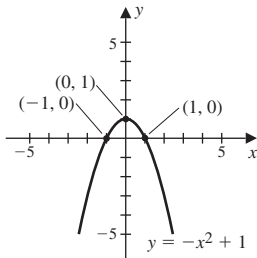
13. For $y = x^2 - 3$:
Intercepts: $(-\sqrt{3}, 0), (\sqrt{3}, 0), (0, -3)$;
Symmetry: y -axis.



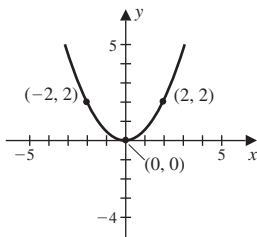
14. For $y = x^2 + 2$:
Intercepts: $(0, 2)$;
Symmetry: y -axis.



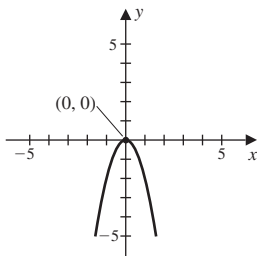
15. For $y = 1 - x^2$:
Intercepts: $(-1, 0)$, $(1, 0)$, $(0, 1)$;
Symmetry: y -axis.



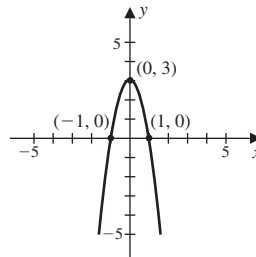
16. For $2y = x^2$, or $y = \frac{1}{2}x^2$:
Intercepts: $(0, 0)$;
Symmetry: y -axis.



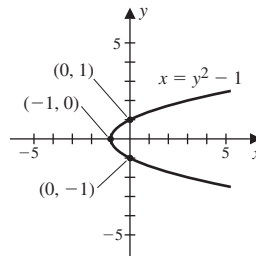
17. For $y = -2x^2$:
Intercepts: $(0, 0)$;
Symmetry: y -axis.



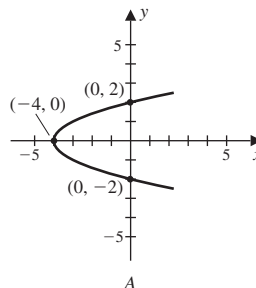
18. For $y = 3 - 3x^2$:
Intercepts: $(-1, 0)$, $(1, 0)$, $(0, 3)$;
Symmetry: y -axis.



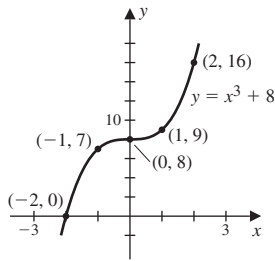
19. For $x = y^2 - 1$:
Intercepts: $(-1, 0)$, $(0, 1)$, $(0, -1)$;
Symmetry: x -axis.



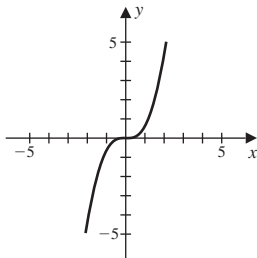
20. For $x = y^2 - 4$:
Intercepts: $(-4, 0)$, $(0, 2)$, $(0, -2)$;
Symmetry: x -axis.



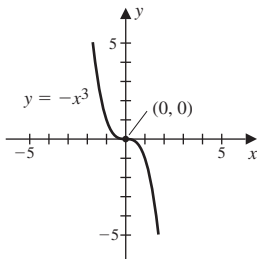
21. For $y = x^3 + 8$:
Intercepts: $(-2, 0)$, $(0, 8)$;
Symmetry: No axis or origin symmetry.



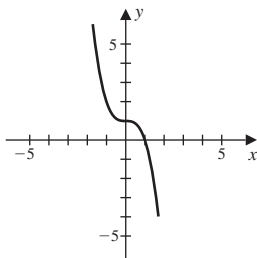
22. For $2y = x^3$, or $y = \frac{1}{2}x^3$:
Intercepts: $(0, 0)$;
Symmetry: origin.



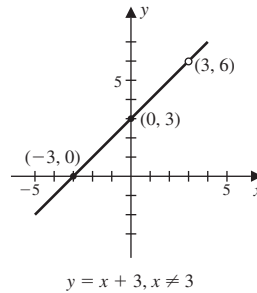
23. For $y = -x^3$:
Intercepts: $(0, 0)$;
Symmetry: origin.



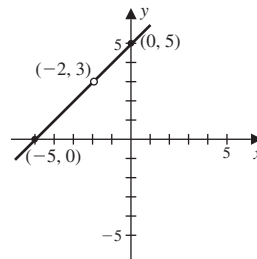
24. For $y = -x^3 + 1$:
Intercepts: $(1, 0)$, $(0, 1)$;
Symmetry: No axis or origin symmetry.



25. For $y = \frac{(x+3)(x-3)}{x-3} = x+3$, when $x \neq 3$:
Intercepts: $(-3, 0)$, $(0, 3)$;
Symmetry: No axis or origin symmetry.



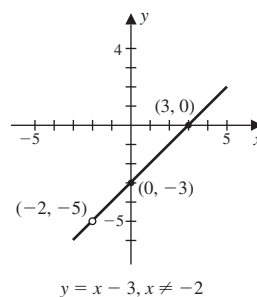
26. For $y = \frac{(x+5)(x+2)}{x+2} = x+5$, when $x \neq -2$:
Intercepts: $(-5, 0)$, $(0, 5)$;
Symmetry: No axis or origin symmetry.



27. For $y = \frac{x^2 - x - 6}{x + 2}$,
$$= \frac{(x-3)(x+2)}{x+2}$$

$$= x - 3$$

when $x \neq -2$:
Intercepts: $(3, 0)$, $(0, -3)$;
Symmetry: No axis or origin symmetry.

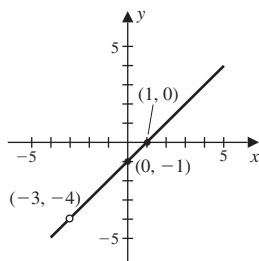


28. For $y = \frac{x^2 + 2x - 3}{x + 3}$

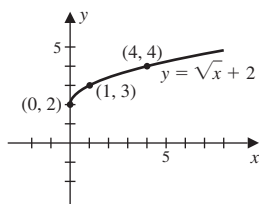
$$= \frac{(x + 3)(x - 1)}{x + 3}$$

$$= x - 1,$$

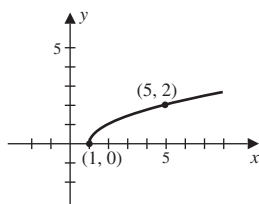
 when $x \neq -3$:
 Intercepts: $(1, 0), (0, -1)$;
 Symmetry: No axis or origin symmetry.



29. For $y = \sqrt{x} + 2$:
 Intercepts: $(0, 2)$;
 Symmetry: No axis or origin symmetry.

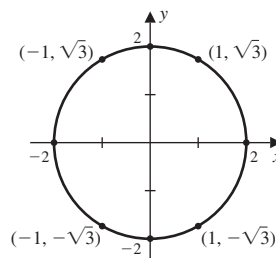


30. For $y = \sqrt{x - 1}$:
 Intercepts: $(1, 0)$;
 Symmetry: No axis or origin symmetry.

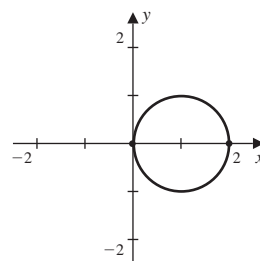


31. For $x^2 + y^2 = 4$:
 Intercepts: $(-2, 0), (2, 0), (0, -2), (0, 2)$;

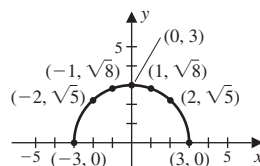
Symmetry: x -axis, y -axis, and origin.



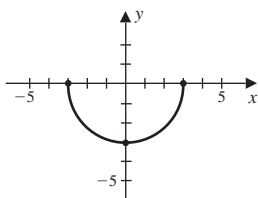
32. For $(x - 1)^2 + y^2 = 1$:
 Intercepts: $(0, 0), (2, 0)$;
 Symmetry: x -axis.



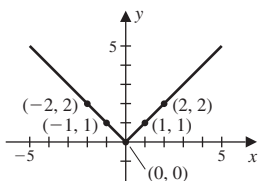
33. For $y = \sqrt{9 - x^2}$:
 Intercepts: $(-3, 0), (3, 0), (0, 3)$;
 Symmetry: y -axis.



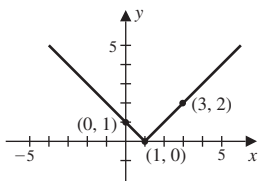
34. For $y = -\sqrt{9 - x^2}$:
 Intercepts: $(-3, 0), (3, 0), (0, -3)$;
 Symmetry: y -axis.



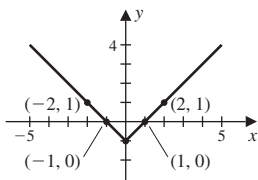
35. For $y = |x|$:
Intercepts: $(0, 0)$;
Symmetry: y -axis.



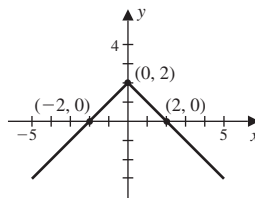
36. For $y = |x - 1|$:
Intercepts: $(1, 0)$, $(0, 1)$;
Symmetry: No axis or origin symmetry.



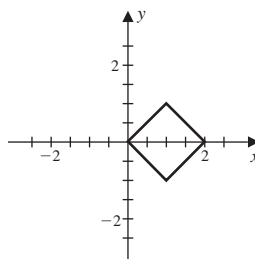
37. For $y = |x| - 1$:
Intercepts: $(-1, 0)$, $(1, 0)$, $(0, -1)$;
Symmetry: y -axis.



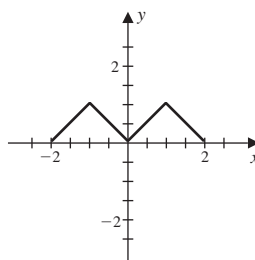
38. For $y = 2 - |x|$:
Intercepts: $(-2, 0)$, $(2, 0)$, $(0, 2)$;
Symmetry: y -axis.



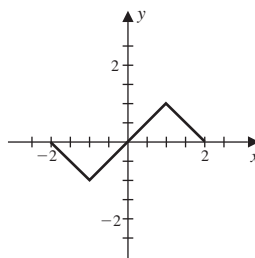
39. This graph has x -axis symmetry.



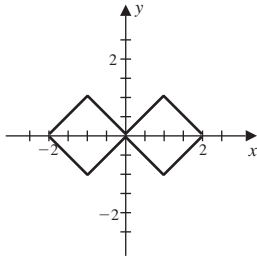
40. This graph has y -axis symmetry.



41. This graph has origin symmetry.

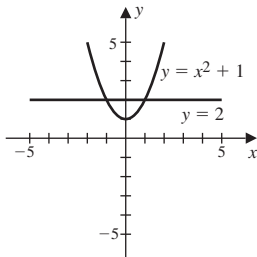


42. This graph has x -axis and y -axis symmetry.



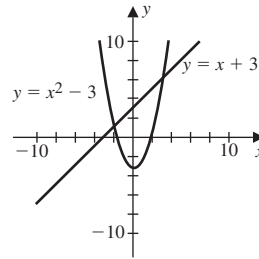
43. From the graph we can see the points of intersection of $y = x^2 + 1$ and $y = 2$ are $(1, 2)$ and $(-1, 2)$. Algebraically we have $x^2 + 1 = 2$ so $x^2 = 1$ and $x = \pm 1$. The distance between the two points of intersection is

$$\begin{aligned} d((1, 2), (-1, 2)) &= \sqrt{(1 - (-1))^2 + (2 - 2)^2} \\ &= \sqrt{4} = 2. \end{aligned}$$



44. From the graph we can see the points of intersection of $y = x^2 - 3$ and $y = x + 3$ are $(3, 6)$ and $(-2, 1)$. Algebraically we have $x^2 - 3 = x + 3$ so $x^2 - x - 6 = (x - 3)(x + 2) = 0$ and $x = 3, x = -2$. The distance between the two points of intersection is

$$\begin{aligned} d((3, 6), (-2, 1)) &= \sqrt{(3 - (-2))^2 + (6 - 1)^2} \\ &= \sqrt{25 + 25} = \sqrt{50} = 5\sqrt{2}. \end{aligned}$$



45. If the three consecutive numbers are $x, x + 1$, and $x + 2$, then $x + x + 1 + x + 2 = 156$ which implies $3x + 3 = 156$ so $x = 51$. The three numbers are 51, 52, and 53.
46. Solving for n in the equation $n^2 + (n + 1)^2 = 925$ gives the two consecutive integers whose squares sum to 925 as 21 and 22.
47. Let x denote the length of the rectangle and y denote the width. Then $xy = 12$ and $2x + 2y = 14$, which implies $x + y = 7$. So $y = 7 - x$ which implies $x(7 - x) = 12$ giving $x^2 - 7x + 12 = 0$. Factoring the quadratic we have $(x - 4)(x - 3) = 0$, so $x = 4$ or $x = 3$. The rectangle has dimension 3 by 4.
48. Solving for x in the equation $\frac{1}{4}x^2 = x$ gives $x = 4$, so area of the square is 16.
49. Let x denote the number of quarts of 10% solution that are required. Then $0.1x + (10 - x) = 0.3(10)$, which implies $x = \frac{7}{0.9} \approx 7.8$ quarts. Approximately $10 - 7.8 = 2.2$ quarts must be drained and replaced with pure antifreeze.
50. If we use x lb of alloy A and $100 - x$ lb of alloy B we will have $35 = 0.2x + 0.45(100 - x)$ pounds of copper. Since this must total 35 pounds we must choose x so that
- $$35 = 0.2x + 0.45(100 - x) = 45 - 0.25x.$$
- So
- $$x = \frac{10}{0.25} = 40.$$
- Hence we should choose 40 lb of alloy A and 60 lb of alloy B.
51. Assume the graph has symmetry with respect to both the x - and y -axes. To show the graph is symmetric with respect to the origin, we

need to show that if (x, y) is on the graph, then $(-x, -y)$ is also on the graph. If (x, y) is on the graph and the graph is symmetric with respect to the x -axis, then $(x, -y)$ is also on the graph. But if $(x, -y)$ is on the graph and the graph is symmetric with respect to the y -axis, then $(-x, -y)$ is also on the graph. Hence the graph is symmetric with respect to the origin.

It is also true that if the graph has symmetry with respect to the origin and to one of the axes, then it has symmetry with respect to the other axis. Consider, for example, the situation of symmetry with respect to the origin and the x -axis. To show the graph is symmetric with respect to the y -axis, we need to show that if (x, y) is on the graph then $(-x, y)$ is also on the graph. If (x, y) is on the graph and the graph is symmetric with respect to

the x -axis, then $(x, -y)$ is also on the graph. But if $(x, -y)$ is on the graph and the graph is symmetric with respect to the origin, then $(-x, -(-y)) = (-x, y)$ is also on the graph. Hence the graph is symmetric with respect to the y -axis.

52. The total return should be $0.06(10000) = 600$. If an amount A is invested at 5%, then the amount $10000 - A$ is invested at 8%. Hence we need to have

$$600 = 0.05A + 0.08(10000 - A) = 800 - 0.03A$$

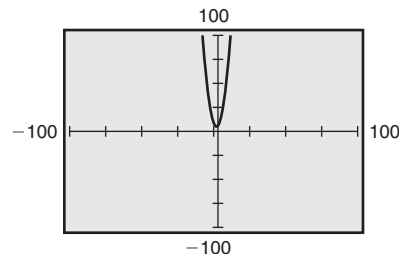
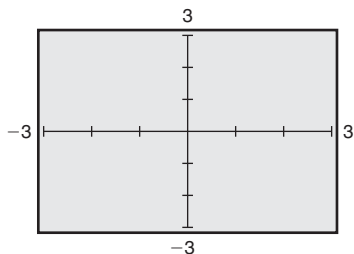
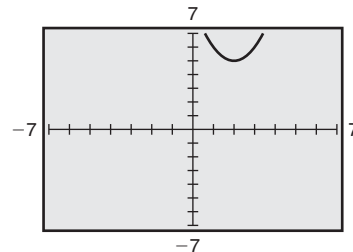
which implies that

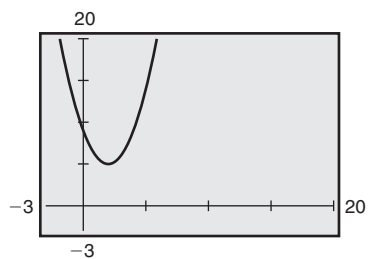
$$A = \frac{-200}{-0.03} = 6666.67.$$

So \$6666.67 should be invested at 5% and the remainder at 8%.

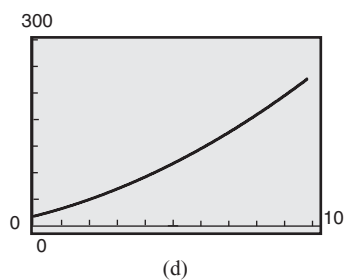
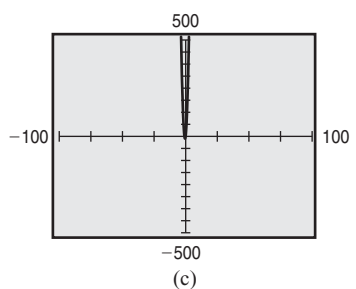
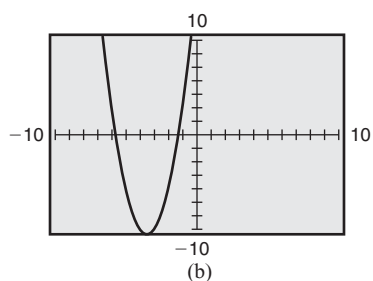
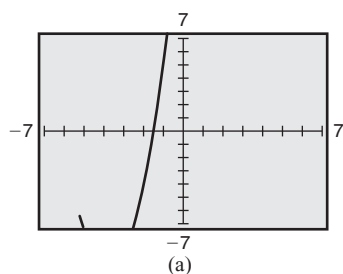
Exercise Set 1.5 (Page 35)

1. Part (d) gives the best representation.

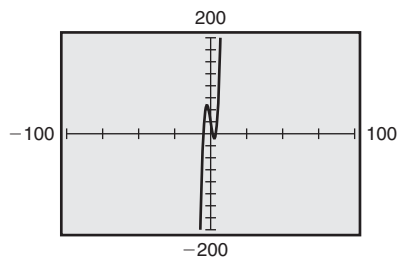
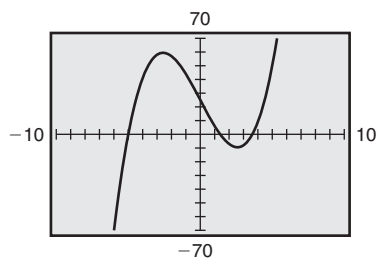
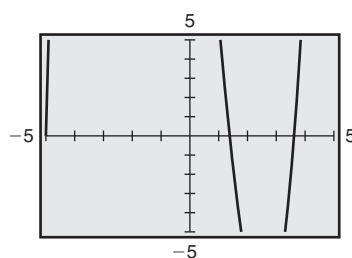
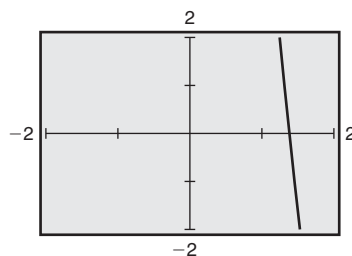




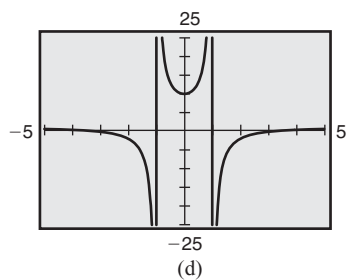
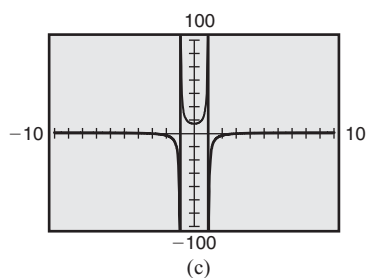
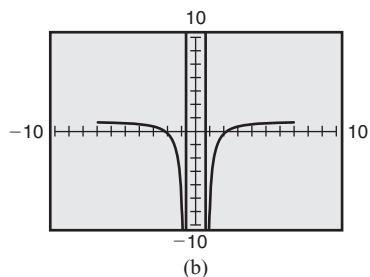
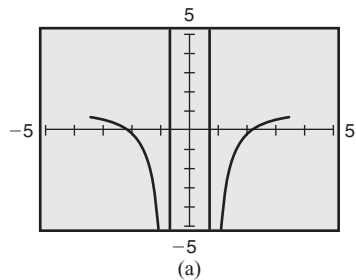
2. Part (b) gives the best representation.



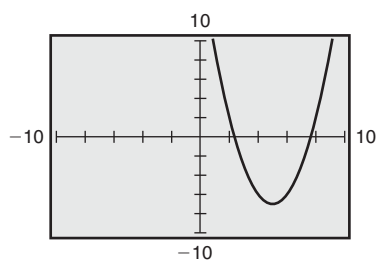
3. Part (c) gives the best representation.



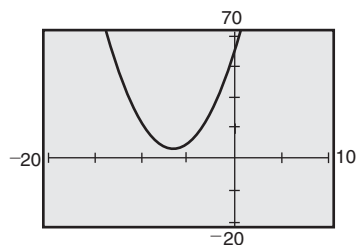
4. Part (d) gives the best representation.



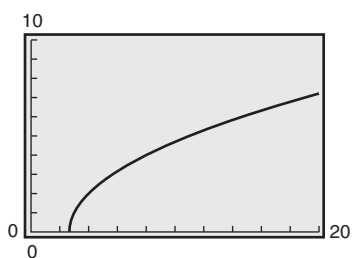
5. $[-10, 10] \times [-10, 10]$



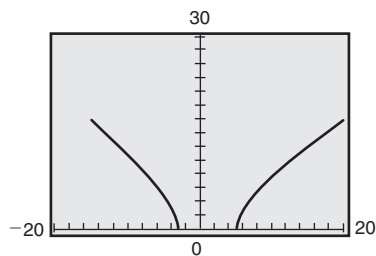
6. $[-20, 10] \times [-20, 70]$



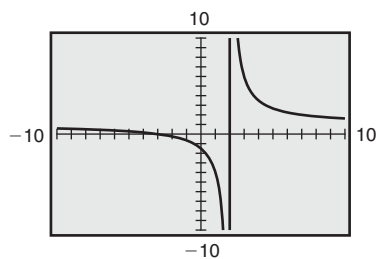
7. $[0, 20] \times [0, 10]$



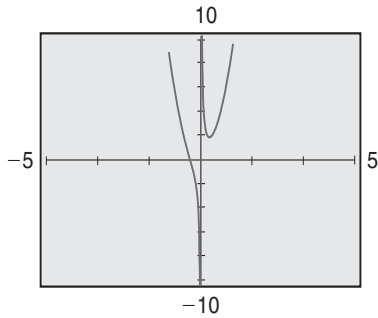
8. $[-20, 20] \times [0, 30]$



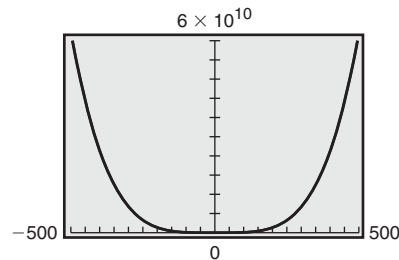
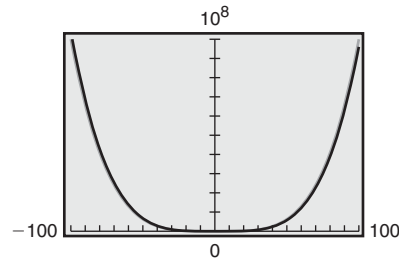
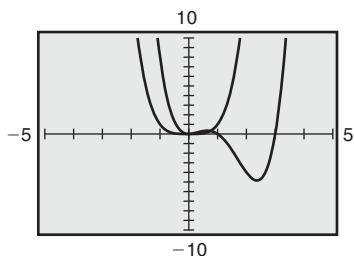
9. $[-10, 10] \times [-10, 10]$



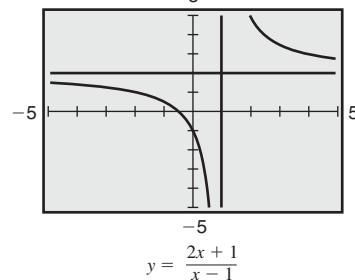
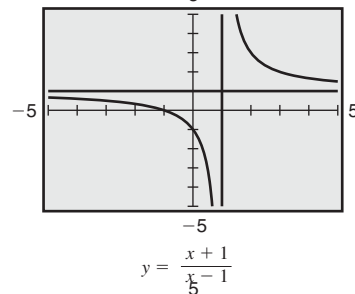
10. $[-5, 5] \times [-10, 10]$



11. The graph of $y = x^3 + x^2 + 3x - 4$ crosses the x -axis at $x \approx 0.86$.
12. The graph of $y = x^3 - x^2 - 4x - 2$ crosses the x -axis at $x \approx -1$, $x \approx -0.7$, and $x \approx 2.7$.
13. The graph of $y = x^4 - 3x^3 + x^2 - 4$ crosses the x -axis at $x \approx -0.93$ and $x \approx 2.82$.
14. The graph of $y = x^4 + x^3 - 2x^2 - x + 1$ crosses the x -axis at $x \approx -1.6$, $x \approx -1$, $x \approx 0.6$ and $x \approx 1$.
15. The points of intersection are approximately $(1, 6)$ and $(-1, 0)$.
16. The points of intersection are approximately $(-2.73, -15.35)$, $(0.73, 5.39)$ and $(2.5, 20.63)$.
17. The points of intersection are approximately $(0, -1)$ and $(0.46, -0.79)$.
18. The points of intersection are approximately $(-2.3, -4.9)$ and $(0.95, -1.6)$.
19. a. We have $x \leq -3.56$ or $x \geq 0.56$, and
b. $x < -2.3$ or $1.3 < x < 3$.
20. The inequality $-x^2 + 2x + 1 \geq x^3 - 2x^2 - x + 2$ is satisfied for $0.3 \leq x \leq 2.2$ or $x \leq -1.5$.
21. For x very large the graphs are almost identical. That is, as x grows without bound $x^4 - 4x^3 + 3x^2$ is approximately x^4 .

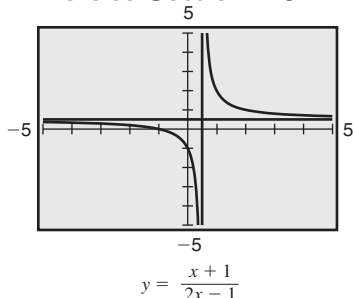


22. If $c > 0$, the graph has an appearance similar to that of $y = x^2$. If $c < 0$, the graph crosses the x -axis three times, just touching the point $(0, 0)$. As c increases in magnitude the other two points where the graph crosses the x -axis move further from the origin and are always symmetric on either side of the origin. The graph has a local high point at the origin and two local low points symmetric about the origin.
23. The graph appears to approach the horizontal line $y = \frac{a}{b}$ and the vertical line $x = \frac{1}{b}$.



24 Exercise Section 1.6

Solutions to All Exercises



24. The value of a affects the inclination of the line. If $a > 0$, the line is increasing from left to right, and if $a < 0$ the line is decreasing. The larger the magnitude of a , the steeper the inclination. The constant b determines where the line crosses the y -axis.
25. If $a = b = 0$, the graph is the standard parabola $y = x^2$. The constants a and b shift $y = x^2$ either horizontally or vertically. If $a > 0$, the shift is to the right a units. If

$a < 0$, the shift is to the left $|a|$ units. If $b > 0$, the shift is upward b units, and if $b < 0$, the shift is downward $|b|$ units.

26. (a) When $0 < x < 1$: $\frac{1}{x^n} < \frac{1}{x^{n+2}}$.
- (b) When $x > 1$: $\frac{1}{x^n} > \frac{1}{x^{n+2}}$.
- (c) When $-1 < x < 0$ and n is even we have $\frac{1}{x^n} < \frac{1}{x^{n+2}}$, but when n is odd we have $\frac{1}{x^n} > \frac{1}{x^{n+2}}$.
- (d) When $x < -1$ and n is even we have $\frac{1}{x^n} > \frac{1}{x^{n+2}}$, but when n is odd we have $\frac{1}{x^n} < \frac{1}{x^{n+2}}$.

Exercise Set 1.6 (Page 50)

1. For $f(x) = 2x^2 + 3$ we have:

- (a) $f(2) = 2(2)^2 + 3 = 11$
- (b) $f(\sqrt{3}) = 2(\sqrt{3})^2 + 3 = 9$
- (c) $f(2 + \sqrt{3}) = 2(2 + \sqrt{3})^2 + 3$
 $= 2(7 + 4\sqrt{3}) + 3$
 $= 17 + 8\sqrt{3}$
- (d) $f(2) + f(\sqrt{3}) = 11 + 9 = 20$
- (e) $f(2x) = 2(2x)^2 + 3$
 $= 2(4x^2) + 3 = 8x^2 + 3$
- (f) $f(1 - x) = 2(1 - x)^2 + 3$
 $= 2(1 - 2x + x^2) + 3$
 $= 2x^2 - 4x + 5$
- (g) $f(x + h) = 2(x + h)^2 + 3$
 $= 2(x^2 + 2hx + h^2) + 3$
 $= 2x^2 + 4hx + 2h^2 + 3$
- (h) $f(x + h) - f(x)$

$$= 2x^2 + 4hx + 2h^2 + 3 - (2x^2 + 3)$$

$$= 4hx + 2h^2$$

2. For $f(x) = \sqrt{x+4}$ we have:

- (a) $f(-1) = \sqrt{-1+4} = \sqrt{3}$
- (b) $f(0) = \sqrt{4} = 2$
- (c) $f(4) = \sqrt{8} = 2\sqrt{2}$
- (d) $f(5) = \sqrt{9} = 3$
- (e) $f(a) = \sqrt{a+4}$
- (f) $f(2a-1) = \sqrt{(2a-1)+4} = \sqrt{2a+3}$
- (g) $f(x+h) = \sqrt{x+h+4}$
- (h) $f(x+h) - f(x) = \sqrt{x+h+4} - \sqrt{x+4}$

3. For $f(t) = |t-2|$ we have:

- (a) $f(4) = |4-2| = |2| = 2$
- (b) $f(1) = |1-2| = |-1| = 1$
- (c) $f(0) = |0-2| = |-2| = 2$
- (d) $f(t+2) = |t+2-2| = |t|$

- (e) $f(2 - t^2) = |2 - t^2 - 2|$
 $= |-t^2| = |t^2| = t^2$
- (f) $f(-t) = |-t - 2| = |-(t + 2)| = |t + 2|$
4. For $f(x) = \begin{cases} x^2, & \text{if } x \geq 0 \\ 2x - 1, & \text{if } x < 0 \end{cases}$
 we have:
- (a) $f(1) = 1^2 = 1$
 (b) $f(-1) = 2(-1) - 1 = -3$
 (c) $f(-2) = 2(-2) - 1 = -5$
 (d) $f(3) = 3^2 = 9$
 (e) For $a > 1$, $1 - a < 0$ and $f(1 - a) = 2(1 - a) - 1 = 1 - 2a$. If $a = 1$, $1 - a = 0$ and $f(0) = 0$.
 (f) For $0 < a < 1$, $1 < 1 + a < 2$ and $f(1 + a) = (1 + a)^2 = 1 + 2a + a^2$.
5. Reading from the graph given in the exercise we have:
- (a) $f(-1) = 0$, $f(0) = -\frac{1}{2}$, $f(1) = 0$, and $f(3) = -2$
 (b) $\text{Domain}(f) = [-2, 3]$ and $\text{Range}(f) = [-2, 3]$
6. Reading from the graph given in the exercise we have:
- (a) $f(-1) = -1.5$, $f(0) = -0.5$, $f(1) = 2$ and $f(3) = 0$
 (b) $\text{Domain}(f) = [-3.5, 3]$ and $\text{Range}(f) = [-4, -0.5] \cup (0, 2.25]$
7. The graph satisfies the vertical line test, that is, each vertical line crosses the curve in at most one place, so it is the graph of a function.
8. There are vertical lines that cross the curve twice, so it is not the graph of a function.
9. The graph satisfies the vertical line test, so it is the graph of a function.
10. There are vertical lines that cross the curve twice, so it is not the graph of a function.
11. (a) Since the only vertical line that does not intersect the curve is $x = 0$, the domain is $(-\infty, 0) \cup (0, \infty)$.
 (b) Since the only horizontal line that does not intersect the curve in at least one place is $y = 0$, the range is $(-\infty, 0) \cup (0, \infty)$.
12. (a) Since the only vertical lines that do not intersect the curve are $x = -4$ and $x = 4$, the domain is all real numbers except $x = 4$ and $x = -4$, that is, $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$.
 (b) Since any horizontal line above $y = 0$ and below and including the line $y = 2$, fails to cross the curve, the range is $(-\infty, 0] \cup (2, \infty)$.
13. (a) $\text{Domain}(f) = (-\infty, \infty)$
 (b) $\text{Range}(f) = (1, \infty) \cup \{-1\}$
14. (a) $\text{Domain}(f) = (-\infty, 2)$
 (b) $\text{Range}(f) = (0, \infty)$
15. For $f(x) = x^2 - 1$ we have:
 (a) $\text{Domain}(f) = (-\infty, \infty)$.
 (b) $\text{Range}(f) = [-1, \infty)$.
16. For $f(x) = x^2 + 1$ we have:
 (a) $\text{Domain}(f) = (-\infty, \infty)$.
 (b) $\text{Range}(f) = [1, \infty)$.
17. For $f(x) = \sqrt{x} + 4$ we have:
 (a) $\text{Domain}(f) = [0, \infty)$.
 (b) $\text{Range}(f) = [4, \infty)$.
18. For $f(x) = \sqrt{x} - 2$ we have:
 (a) $\text{Domain}(f) = [0, \infty)$.
 (b) $\text{Range}(f) = [-2, \infty)$.
19. For $f(x) = x^2 - 2x + 1 = (x - 1)^2$ we have:
 (a) $\text{Domain}(f) = (-\infty, \infty)$.
 (b) $\text{Range}(f) = [0, \infty)$.
20. For $f(x) = x^2 - 2x + 2 = (x - 1)^2 + 1$ we have:
 (a) $\text{Domain}(f) = (-\infty, \infty)$.
 (b) $\text{Range}(f) = [1, \infty)$.

21. For $f(x) = \begin{cases} 1, & \text{if } x \geq 0 \\ -1, & \text{if } x < 0 \end{cases}$

we have:

(a) $\text{Domain}(f) = (-\infty, \infty)$.

(b) $\text{Range}(f) = \{-1, 1\}$.

22. For $f(x) = \begin{cases} x^3, & \text{if } x \geq 0 \\ -2x, & \text{if } x < 0 \end{cases}$

we have:

(a) $\text{Domain}(f) = (-\infty, \infty)$.

(b) $\text{Range}(f) = [0, \infty)$.

23. For $f(x) = x^2 - 1$, $b = 0$, and $f(a) = a^2 - 1$
 $f(a) = 0$ implies $a^2 - 1 = 0$
 so $a^2 = 1$ and $a = \pm 1$.

24. For $f(x) = x^2 - 1$, $b = 2$, and $f(a) = a^2 - 1$
 $f(a) = 2$ implies $a^2 - 1 = 2$
 so $a^2 = 3$ and $a = \pm\sqrt{3}$.

25. For $f(x) = \sqrt{x-1}$, $b = \frac{1}{2}$, and
 $f(a) = \sqrt{a-1}$

$$f(a) = \frac{1}{2} \text{ implies } \sqrt{a-1} = \frac{1}{2}$$

so $a - 1 = \frac{1}{4}$ and $a = \frac{5}{4}$.

26. For $f(x) = \sqrt{x} - 1$, $b = \frac{1}{2}$, and $f(a) = \sqrt{a} - 1$

$$f(a) = \frac{1}{2} \text{ implies } \sqrt{a} - 1 = \frac{1}{2}$$

so $\sqrt{a} = \frac{3}{2}$ and $a = \frac{9}{4}$.

27. (a) $\text{Domain}(f) = (-\infty, \infty)$

(b) $\text{Domain}(f) = \{x : 2 - x \neq 0\}$
 $= \{x : x \neq 2\}$
 $= (-\infty, 2) \cup (2, \infty)$

(c) $\text{Domain}(f) = \{x : 2 - x \geq 0\}$
 $= \{x : x \leq 2\}$
 $= (-\infty, 2]$

(d) $\text{Domain}(f) = \{x : 2 - x > 0\} = (-\infty, 2)$

28. (a) $\text{Domain}(f) = (-\infty, \infty)$

(b) $\text{Domain}(f) = \{x : 3x + 1 \neq 0\}$
 $= (-\infty, -\frac{1}{3}) \cup (-\frac{1}{3}, \infty)$

(c) $\text{Domain}(f) = \{x : 3x + 1 \geq 0\}$
 $= [-\frac{1}{3}, \infty)$

(d) $\text{Domain}(f) = \{x : 3x + 1 > 0\}$
 $= (-\frac{1}{3}, \infty)$

29. (a) $\text{Domain}(f)$
 $= \{x : x^2 - 2 \neq 0\}$
 $= \{x : x \neq \pm\sqrt{2}\}$
 $= (-\infty, -\sqrt{2}) \cup (-\sqrt{2}, \sqrt{2}) \cup (\sqrt{2}, \infty)$

(b) For $f(x) = \frac{x-2}{x^2-2}$, the domain is still
 $(-\infty, -\sqrt{2}) \cup (-\sqrt{2}, \sqrt{2}) \cup (\sqrt{2}, \infty)$

(c) We have

$$\begin{aligned} \text{Domain}(f) &= \left\{x : \frac{2x^2}{x^2-2} \geq 0\right\} \\ &= \{0\} \cup \left\{x : \frac{2x^2}{x^2-2} > 0\right\} \\ &= \{0\} \cup \{x : x^2 - 2 > 0\} \\ &= \{0\} \cup (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty). \end{aligned}$$

30. (a) For $f(x) = \frac{x-5}{x^2+3x-10}$

$$= \frac{x-5}{(x+5)(x-2)},$$

$\text{Domain}(f)$

$$\begin{aligned} &= \{x : (x+5)(x-2) \neq 0\} \\ &= \{x : x \neq -5, x \neq 2\} \\ &= (-\infty, -5) \cup (-5, 2) \cup (2, \infty). \end{aligned}$$

(b) For $\frac{x+5}{x^2+3x-10} = \frac{x+5}{(x+5)(x-2)}$
 $= \frac{1}{x-2},$
 $\text{Domain}(f)$

$$\begin{aligned} &= \{x : (x+5)(x-2) \neq 0\} \\ &= \{x : x \neq -5, x \neq 2\} \\ &= (-\infty, -5) \cup (-5, 2) \cup (2, \infty). \end{aligned}$$

(c) For $f(x) = \sqrt{\frac{(x-5)^2}{x^2+3x-10}}$, the domain
 is all x with

$$\frac{(x-5)^2}{x^2+3x-10} \geq 0,$$

so $x = 5$ or $x^2 + 3x - 10 > 0$

which implies that $x = 5, x < -5$, or
 $x > 2$, and the domain is
 $(-\infty, -5) \cup (2, \infty)$.

31. (a) For $f(x) = \sqrt{x(x-2)}$,
 $\text{Domain}(f) = \{x : x(x-2) \geq 0\}$
 $= (-\infty, 0] \cup [2, \infty)$.
- (b) For $f(x) = \sqrt{x(2-x)}$,
 $\text{Domain}(f) = \{x : x(2-x) \geq 0\}$
 $= \{x : x(x-2) \leq 0\} = [0, 2]$.
- (c) For $f(x) = \frac{x^2}{x^2-2x} = \frac{x^2}{x(x-2)}$,
 $\text{Domain}(f) = \{x : x(x-2) \neq 0\}$
 $= \{x : x \neq 0, x \neq 2\}$
 $= (-\infty, 0) \cup (0, 2) \cup (2, \infty)$.
- (d) For $f(x) = \sqrt{\frac{x^2}{x^2-2x}} = \sqrt{\frac{x^2}{x(x-2)}}$,
 $\text{Domain}(f) = \{x : x(x-2) > 0\}$
 $= (-\infty, 0) \cup (2, \infty)$.
32. (a) For $f(x) = \sqrt{(x+1)(x-1)}$,
 $\text{Domain}(f) = \{x : (x+1)(x-1) \geq 0\}$
 $= (-\infty, -1] \cup [1, \infty)$.
- (b) For $f(x) = \sqrt{(x+1)(1-x)}$,
 $\text{Domain}(f) = \{x : (x+1)(1-x) \geq 0\}$
 $= \{x : (x+1)(x-1) \leq 0\}$
 $= [-1, 1]$.
- (c) For $f(x) = \frac{x^4-x^2}{x^2-1}$,
 $\text{Domain}(f)$
 $= \{x : x^2-1 \neq 0\}$
 $= \{x : x \neq \pm 1\}$
 $= (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$.
- (d) For $f(x) = \sqrt{\frac{x^4-x^2}{x^2-1}}$,
 $\text{Domain}(f)$
 $= \left\{x : \frac{x^4-x^2}{x^2-1} \geq 0\right\}$
 $= \{x : x \neq \pm 1\}$
 $= (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$.

33. For $f(x) = x^2 + 2$, we have

$$\begin{aligned} f(-x) &= (-x)^2 + 2 = x^2 + 2; \\ -f(x) &= -(x^2 + 2) = -x^2 - 2; \\ f\left(\frac{1}{x}\right) &= \left(\frac{1}{x}\right)^2 + 2 = \frac{1}{x^2} + 2; \\ \frac{1}{f(x)} &= \frac{1}{x^2 + 2}; \\ f(\sqrt{x}) &= (\sqrt{x})^2 + 2 = x + 2; \\ \sqrt{f(x)} &= \sqrt{x^2 + 2}. \end{aligned}$$

34. For $f(x) = x^2 + 4x$ we have
 $f(-x) = (-x)^2 + 4(-x) = x^2 - 4x$;
 $-f(x) = -(x^2 + 4x) = -x^2 - 4x$;
 $f\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)^2 + 4\left(\frac{1}{x}\right) = \frac{1}{x^2} + \frac{4}{x}$;
 $\frac{1}{f(x)} = \frac{1}{x^2 + 4x}$;
 $f(\sqrt{x}) = (\sqrt{x})^2 + 4\sqrt{x} = x + 4\sqrt{x}$;
 $\sqrt{f(x)} = \sqrt{x^2 + 4x}$.
35. For $f(x) = \frac{1}{x}$ we have
 $f(-x) = \frac{1}{-x} = -\frac{1}{x}$;
 $-f(x) = -\frac{1}{x}$;
 $f\left(\frac{1}{x}\right) = \frac{1}{\frac{1}{x}} = x$;
 $\frac{1}{f(x)} = \frac{1}{\frac{1}{x}} = x$;
 $f(\sqrt{x}) = \frac{1}{\sqrt{x}} = \frac{1}{\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{\sqrt{x}}{x}$;
 $\sqrt{f(x)} = \sqrt{\frac{1}{x}} = \frac{1}{\sqrt{x}} = \frac{\sqrt{x}}{x}$.
36. For $f(x) = \sqrt{x}$ we have
 $f(-x) = \sqrt{-x}$;
 $-f(x) = -\sqrt{x}$;
 $f\left(\frac{1}{x}\right) = \sqrt{\frac{1}{x}} = \frac{1}{\sqrt{x}} = \frac{1}{\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{\sqrt{x}}{x}$;
 $\frac{1}{f(x)} = \frac{1}{\sqrt{x}} = \frac{\sqrt{x}}{x}$;
 $f(\sqrt{x}) = \sqrt{\sqrt{x}} = (x^{1/2})^{1/2} = x^{1/4}$;
 $\sqrt{f(x)} = \sqrt{\sqrt{x}} = x^{1/4}$.
37. For $f(x) = 3x - 2$:
 (a) $f(x+h) = 3(x+h) - 2 = 3x + 3h - 2$
 (b) $f(x+h) - f(x) = 3x + 3h - 2 - 3x + 2 = 3h$
 (c) The difference quotient is

$$\frac{f(x+h) - f(x)}{h} = \frac{3h}{h} = 3.$$

 (d) As h approaches 0,
 $\frac{f(x+h) - f(x)}{h}$ approaches 3. In fact,
 it is always 3 if $h \neq 0$.
38. For $f(x) = \frac{3}{2}x + \frac{1}{4}$:
 (a) $f(x+h) = \frac{3}{2}(x+h) + \frac{1}{4} = \frac{3}{2}x + \frac{3}{2}h + \frac{1}{4}$

(b) $f(x+h) - f(x) = \frac{3}{2}x + \frac{3}{2}h + \frac{1}{4} - \frac{3}{2}x - \frac{1}{4} = \frac{3}{2}h$

(c) The difference quotient is

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{3}{2}h}{h} = \frac{3}{2}.$$

(d) As h approaches 0, $\frac{f(x+h) - f(x)}{h}$ approaches $\frac{3}{2}$.

39. For $f(x) = x^2$:

(a) $f(x+h) = (x+h)^2 = x^2 + 2hx + h^2$

(b) $f(x+h) - f(x) = x^2 + 2hx + h^2 - x^2 = 2hx + h^2$

(c) The difference quotient is

$$\frac{f(x+h) - f(x)}{h} = \frac{2hx + h^2}{h} = \frac{h(2x + h)}{h} = 2x + h.$$

(d) As h approaches 0, $\frac{f(x+h) - f(x)}{h}$ approaches $2x$.

40. For $f(x) = -x^2$:

(a) $f(x+h) = -(x+h)^2 = -(x^2 + 2hx + h^2) = -x^2 - 2hx - h^2$

(b) $f(x+h) - f(x) = -x^2 - 2hx - h^2 + x^2 = -2hx - h^2$

(c) The difference quotient is

$$\frac{f(x+h) - f(x)}{h} = \frac{-2hx - h^2}{h} = \frac{h(-2x - h)}{h} = -2x - h.$$

(d) As h approaches 0, $\frac{f(x+h) - f(x)}{h}$ approaches $-2x$.

41. For $f(x) = 2 - x - x^2$:

(a) $f(x+h) = 2 - (x+h) - (x+h)^2 = 2 - x - h - x^2 - 2hx - h^2$

(b) $f(x+h) - f(x) = 2 - x - h - x^2 - 2hx - h^2 - 2 + x + x^2 = -h - 2hx - h^2$

(c) The difference quotient is

$$\frac{f(x+h) - f(x)}{h} = \frac{-h - 2hx - h^2}{h} = \frac{h(-1 - 2x - h)}{h} = -1 - 2x - h.$$

(d) As h approaches 0, $\frac{f(x+h) - f(x)}{h}$ approaches $-1 - 2x$.

42. For $f(x) = 3x^2 + 2x + 1$:

(a) $f(x+h) = 3(x+h)^2 + 2(x+h) + 1 = 3(x^2 + 2hx + h^2) + 2x + 2h + 1 = 3x^2 + 6hx + 3h^2 + 2x + 2h + 1$

(b) $f(x+h) - f(x) = 3x^2 + 6hx + 3h^2 + 2x + 2h + 1 - 3x^2 - 2x - 1 = 6hx + 3h^2 + 2h$

(c) The difference quotient is

$$\frac{f(x+h) - f(x)}{h} = \frac{6hx + 3h^2 + 2h}{h} = \frac{h(6x + 3h + 2)}{h} = 6x + 3h + 2.$$

(d) As h approaches 0, $\frac{f(x+h) - f(x)}{h}$ approaches $6x + 2$.

43. For $f(x) = \frac{1}{x}$:

(a) $f(x+h) = \frac{1}{x+h}$

(b) $f(x+h) - f(x) = \frac{1}{x+h} - \frac{1}{x} = \frac{x - (x+h)}{x(x+h)} = -\frac{h}{x(x+h)}$

(c) The difference quotient is

$$\frac{f(x+h) - f(x)}{h} = \frac{-\frac{h}{x(x+h)}}{h} = \frac{-h}{hx(x+h)} = -\frac{1}{x(x+h)}.$$

(d) As h approaches 0, $\frac{f(x+h) - f(x)}{h}$ approaches $-\frac{1}{x^2}$.

44. For $f(x) = x + \frac{1}{x}$:

(a) $f(x+h) = x+h + \frac{1}{x+h}$

(b) $f(x+h) - f(x)$
 $= x+h + \frac{1}{x+h} - x - \frac{1}{x}$
 $= h - \frac{h}{x(x+h)}$

(c) The difference quotient is

$$\frac{f(x+h) - f(x)}{h} = \frac{h + \frac{1}{x+h} - \frac{1}{x}}{h} = \frac{h}{h} + \frac{-\frac{h}{x(x+h)}}{h} = 1 + \frac{-h}{hx(x+h)} = 1 - \frac{1}{x(x+h)}.$$

(d) As h approaches 0,
 $\frac{f(x+h) - f(x)}{h}$ approaches $1 - \frac{1}{x^2}$.

45. For $f(x) = \frac{x}{x-3}$:

(a) $f(x+h) = \frac{x+h}{x+h-3}$

(b) $f(x+h) - f(x) = \frac{x+h}{x+h-3} - \frac{x}{x-3} = \frac{x^2 - 3x + hx - 3h - x^2 - xh + 3x}{(x+h-3)(x-3)}$
 $= -\frac{3h}{(x+h-3)(x-3)}$

(c) The difference quotient is

$$\frac{f(x+h) - f(x)}{h} = \frac{-\frac{3h}{(x+h-3)(x-3)}}{h} = \frac{-3h}{h(x+h-3)(x-3)} = -\frac{3}{(x+h-3)(x-3)}.$$

(d) As h approaches 0, $\frac{f(x+h) - f(x)}{h}$ approaches $-\frac{3}{(x-3)^2}$.

46. For $f(x) = \frac{3-5x}{2x}$:

(a) $f(x+h) = \frac{3-5(x+h)}{2(x+h)} = \frac{3-5x-5h}{2x+2h}$

$$\begin{aligned}
 \text{(b)} \quad f(x+h) - f(x) &= \frac{3-5x-5h}{2x+2h} - \frac{3-5x}{2x} = \frac{3x-5x^2-5x-3x-3h+5x^2+5hx}{2x(x+h)} \\
 &= -\frac{3h}{2x(x+h)}
 \end{aligned}$$

(c) The difference quotient is

$$\frac{f(x+h) - f(x)}{h} = \frac{-\frac{3h}{2x(x+h)}}{h} = \frac{-3h}{2hx(x+h)} = \frac{-3}{2x(x+h)}.$$

(d) As h approaches 0, $\frac{f(x+h) - f(x)}{h}$ approaches $-\frac{3}{2x^2}$.

47. For $f(x) = x^3$:

$$\text{(a)} \quad f(x+h) = (x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$$

$$\text{(b)} \quad f(x+h) - f(x) = x^3 + 3x^2h + 3xh^2 + h^3 - x^3 = 3x^2h + 3xh^2 + h^3$$

(c) The difference quotient is

$$\begin{aligned}
 \frac{f(x+h) - f(x)}{h} &= \frac{3x^2h + 3xh^2 + h^3}{h} = \frac{h(3x^2 + 3xh + h^2)}{h} \\
 &= 3x^2 + 3xh + h^2.
 \end{aligned}$$

(d) As h approaches 0, $\frac{f(x+h) - f(x)}{h}$ approaches $3x^2$.

48. For $f(x) = 2x - x^3$:

$$\text{(a)} \quad f(x+h) = 2(x+h) - (x+h)^3 = 2x + 2h - x^3 - 3x^2h - 3xh^2 - h^3$$

$$\text{(b)} \quad f(x+h) - f(x) = 2x + 2h - x^3 - 3x^2h - 3xh^2 - h^3 - 2x + x^3 = 2h - 3x^2h - 3xh^2 - h^3$$

(c) The difference quotient is

$$\frac{f(x+h) - f(x)}{h} = \frac{2h - 3x^2h - 3xh^2 - h^3}{h} = \frac{h(2 - 3x^2 - 3xh - h^2)}{h} = 2 - 3x^2 - 3xh - h^2.$$

(d) As h approaches 0, $\frac{f(x+h) - f(x)}{h}$ approaches $2 - 3x^2$.

49. (a) For $f(x) = \sqrt{x}$, we have $f(x+h) = \sqrt{x+h}$ and

$$\begin{aligned}
 \frac{f(x+h) - f(x)}{h} &= \frac{\sqrt{x+h} - \sqrt{x}}{h} \\
 &= \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} = \frac{(x+h) - x}{h(\sqrt{x+h} + \sqrt{x})} \\
 &= \frac{h}{h(\sqrt{x+h} + \sqrt{x})} = \frac{1}{\sqrt{x+h} + \sqrt{x}}.
 \end{aligned}$$

(b) As h approaches 0, the value of the difference quotient $\frac{f(x+h) - f(x)}{h}$ approaches

$$\frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}} = \frac{\sqrt{x}}{2x}.$$

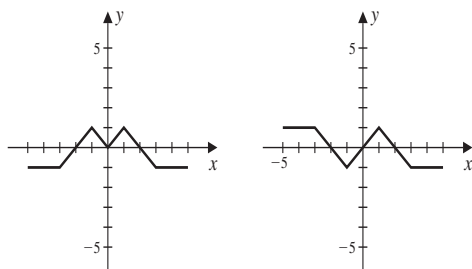
50. (a) For $f(x) = \frac{1}{\sqrt{x}}$ we have

$$\begin{aligned} & \frac{f(x+h) - f(x)}{h} \\ &= \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h} = \frac{\frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x+h}\sqrt{x}}}{h} \\ &= \frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x+h}\sqrt{x}} \cdot \frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}} \\ &= \frac{x - (x+h)}{h\sqrt{x+h}\sqrt{x}(\sqrt{x} + \sqrt{x+h})} \\ &= \frac{-1}{\sqrt{x+h}\sqrt{x}(\sqrt{x} + \sqrt{x+h})}. \end{aligned}$$

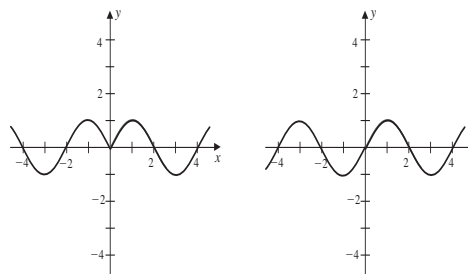
- (b) As h approaches 0,

$$\begin{aligned} & \frac{f(x+h) - f(x)}{h} \\ & \rightarrow \frac{-1}{\sqrt{x}\sqrt{x}(\sqrt{x} + \sqrt{x})} \\ &= -\frac{1}{2x\sqrt{x}} = -\frac{\sqrt{x}}{2x^2}. \end{aligned}$$

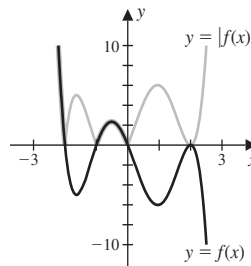
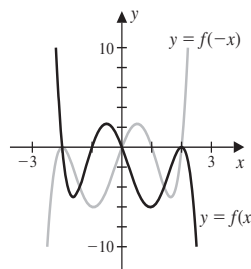
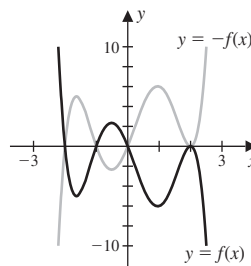
51. The function is even, since the graph is symmetric with respect to the y -axis.
52. The function is even, since the graph is symmetric with respect to the y -axis.
53. The function is neither even nor odd, since the graph is not symmetric with respect to either the x -axis or the origin.
54. The function is odd, since the graph is symmetric with respect to the origin.
55. The left figure is even; the right figure is odd.



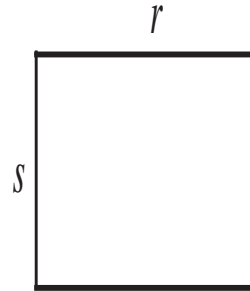
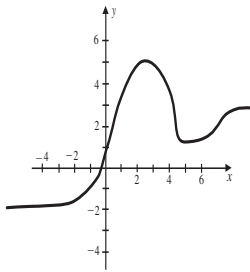
56. The left figure is even; the right figure is odd.



57. The transformations are shown in the figures.

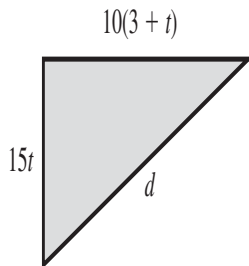


58. The following graph satisfies all the conditions.

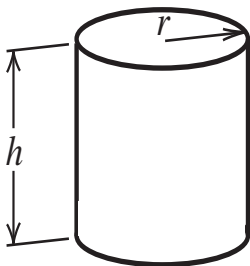


59. The distance between the ships is

$$\begin{aligned} d &= \sqrt{(10(3+t))^2 + (15t)^2} \\ &= \sqrt{325t^2 + 600t + 900}. \end{aligned}$$



60. The volume of a cylinder is $V = \pi r^2 h$. So when $h = 2r$, $V(r) = 2\pi r^3$.



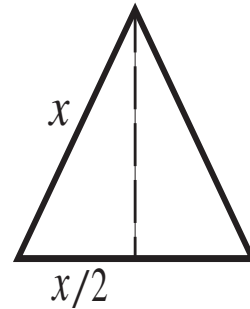
61. A rectangle with sides of length s and r has area $A = rs$ and perimeter $P = 2s + 2r$. Since $A = 64$, $r = 64/s$ and $P(s) = 2s + 128/s$.

62. The area of a triangle with base b and altitude a is $A = \frac{1}{2}ab$. Since the triangle is equilateral, the base is x and the Pythagorean Theorem implies that the altitude is

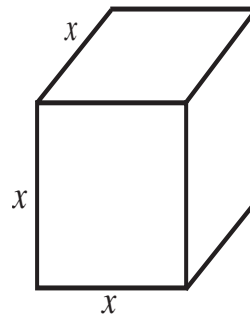
$$\sqrt{x^2 - \left(\frac{x}{2}\right)^2} = \frac{\sqrt{3}}{2}x,$$

so the area is

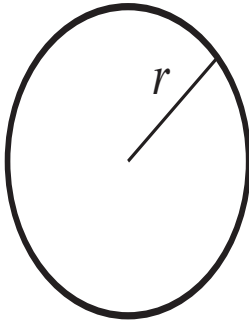
$$A(x) = \frac{\sqrt{3}}{4}x^2.$$



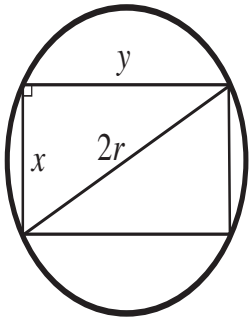
63. Let the length of the side of the cube be x . Since there are six faces of a cube, and each face of the cube has area x^2 , the total surface area is $S = 6x^2$. The cube has volume $V = x^3$, so $x = \sqrt[3]{V}$. Hence $S(V) = 6(\sqrt[3]{V})^2 = 6V^{2/3}$.



64. Let the radius of the circle be r . Then the area is $A = \pi r^2$ and the circumference is $C = 2\pi r$, or $r = C/2\pi$. Hence $A(C) = \pi(C/2\pi)^2 = C^2/(4\pi)$.



65. Let r be the radius of the circle and x and y be the lengths of the sides of the rectangle. Then $x^2 + y^2 = (2r)^2 = 4r^2$, so $y = \sqrt{4r^2 - x^2}$. Hence $A = xy = x\sqrt{4r^2 - x^2}$.



66. (a) Let ℓ denote the length, w the width, P the perimeter, and A the area of the plot of land. Then

$$P = 2\ell + 2w,$$

and

$$A = \ell w = 432, \text{ so } \ell = \frac{432}{w}$$

and

$$P(w) = 2\left(\frac{432}{w}\right) + 2w = 2w + \frac{864}{w}.$$

The domain is $(0, \infty)$.

- (b) $w \approx 20.8$ feet, $\ell \approx 20.8$ feet.

67. For $P(x) = 300x - 2x^2$:

- (a) Average rate of change:

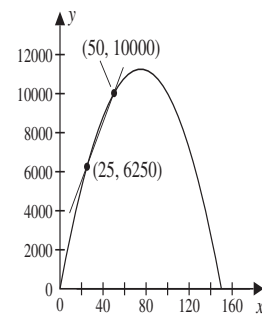
$$\begin{aligned} & \frac{P(x+h) - P(x)}{h} \\ &= \frac{300(x+h) - 2(x+h)^2 - (300x - 2x^2)}{h} \\ &= \frac{300x + 300h - 2(x^2 + 2hx + h^2) - 300x + 2x^2}{h} \\ &= \frac{300h - 4hx - 2h^2}{h} \\ &= \frac{h(300 - 4x - 2h)}{h} \\ &= 300 - 4x - 2h \end{aligned}$$

- (b) If $h = 25$ and $x = 25$, then $x + h = 50$, and the average rate of change in the profit as the number of units changes from 25 to 50 is

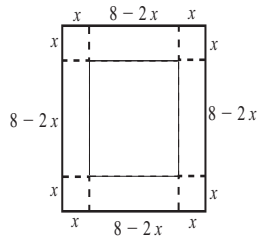
$$300 - 4(25) - 2(25) = 150.$$

- (c) As h approaches 0, $\frac{P(x+h) - P(x)}{h}$ approaches $300 - 4x$, which is the instantaneous rate of change. And when $x = 25$, the instantaneous rate of change is $300 - 4(25) = 200$.

- (d) The graph shows $P(x)$ and the line joining $(25, P(25))$ and $(50, P(50))$.



68. (a) Let l , w , and h be the length, width, and height, respectively, of the box. Then $l = 8 - 2x$, $w = 8 - 2x$, and $h = x$, so the volume is $V(x) = x(8 - 2x)^2$, for $0 < x < 4$.



- (b) A graphing device shows that the maximum volume occurs when $x \approx 1.3$ and $V(1.3) \approx 37.9$.
69. (a) Let r and h be the radius and height of the cylinder, respectively. Since the volume is 900 and $V = \pi r^2 h$, we have $900 = \pi r^2 h$ and $h = 900/(\pi r^2)$. The amount of material needed to construct the can is $M = 2(\pi r^2) + 2\pi r h$, so

$$M(r) = 2\pi r^2 + 2\pi r(900/\pi r^2)$$

$$= 2\pi r^2 + 1800/r.$$
- (b) A graphing device shows that the minimum material occurs when $r \approx 5.2$, $h \approx 10.6$, and in this case $M \approx 516.1$.
70. (a) The situation here is similar to that in Exercise 69(a), except that now $M = 2(2r)^2 + 2\pi r h$. Since $h = 900/(\pi r^2)$, we now have

$$M(r) = 8r^2 + 1800/r.$$
- (b) A graphing device shows that the minimum material used occurs when $r \approx 4.8$, $h \approx 12.4$, and in this case $M \approx 559.3$. It is reasonable that when the top and bottom of the can become more relatively inexpensive, the optimal solution should result in a decreased radius, an increased height, and the amount of material increases.
71. (a) The combined area of the semi-circle regions is πr^2 , and the area of the rectangle making up the remainder is $2rl$, so the total area is $A = \pi r^2 + 2rl$. Since the

perimeter is 1 mile and is also $2\pi r + 2l$, we have $1 = 2\pi r + 2l$ and $l = (1 - 2\pi r)/2$. Hence

$$A(r) = \pi r^2 + 2r(1 - 2\pi r)/2$$

$$= \pi r^2 + r - 2\pi r^2 = r - \pi r^2.$$

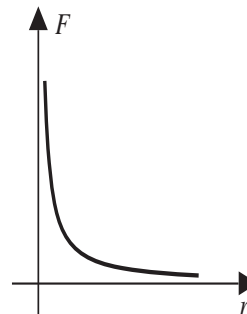
- (b) A graphing device shows that the maximum area occurs when $r \approx 0.16$ and $A(0.16) \approx 0.08$.
72. (a) Let x be the number of tickets exceeding 10. Then for $0 \leq x$, the revenue $R(x)$ is

$$R(x) = (10 + x)(10 - 0.25x)$$

$$= 100 + 7.5x - 0.25x^2.$$
- (b) The maximum revenue occurs when $x \approx 15$ and $R(15) = 25(6.25) = 156.25$.
73. (a) Let x be the number of tickets exceeding 10. Then for $0 \leq x$, the revenue $R(x)$ is

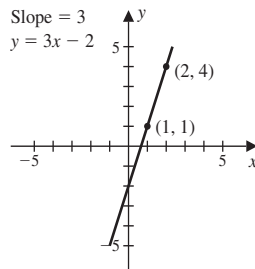
$$R(x) = (10 + x)(80 - 2x)$$

$$= 800 + 60x - 2x^2.$$
- (b) The maximum revenue occurs when $x = 15$, and the maximum revenue is $R(15) = 25 \cdot 50 = \$1250$.
74. (a) The conditions imply that $F = \frac{Gm_1m_2}{r^2}$, where G is the constant of proportionality.
- (b) The physical situation requires the objects be some positive distance apart, so the domain is $(0, \infty)$.
- (c) Depending on the values of G, m_1 , and m_2 , the graph appears as shown.

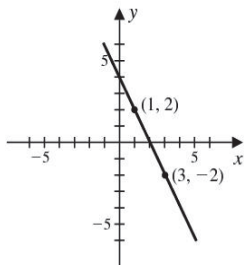


Exercise Set 1.7 (Page 62)

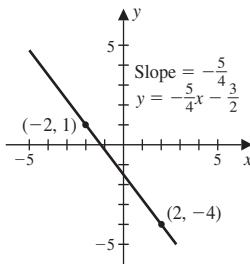
1. Slope: $m = \frac{4-1}{2-1} = 3$;
Equation: $y = 3x - 2$



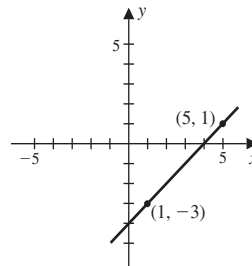
2. Slope: $m = \frac{-2-2}{3-1} = -2$;
Equation: $y - 2 = -2(x - 1)$ or $y = -2x + 4$



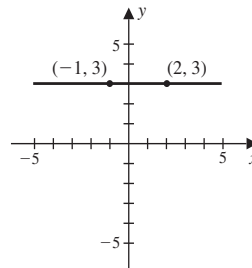
3. Slope: $m = \frac{-4-1}{2-(-2)} = -\frac{5}{4}$;
Equation: $y - 1 = -\frac{5}{4}(x + 2)$ or $y = -\frac{5}{4}x - \frac{3}{2}$



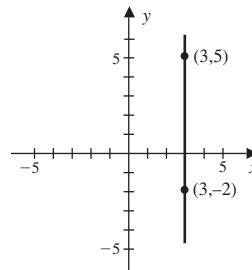
4. Slope: $m = \frac{1-(-3)}{5-1} = 1$;
Equation: $y - (-3) = x - 1$ or $y = x - 4$



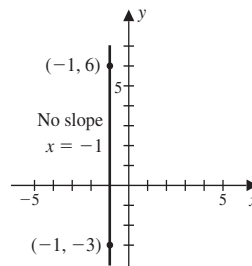
5. Slope: $m = \frac{3-3}{-1-2} = 0$;
Equation: $y = 3$



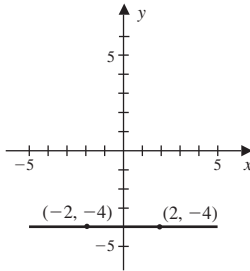
6. Slope: $m = \frac{-2-5}{3-3}$ which is undefined;
Equation: $x = 3$



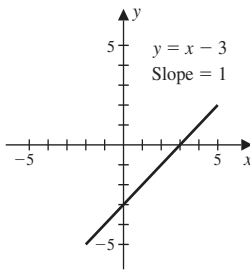
7. Slope: $m = \frac{6-(-3)}{-1-(-1)}$ which is undefined;
Equation: $x = -1$



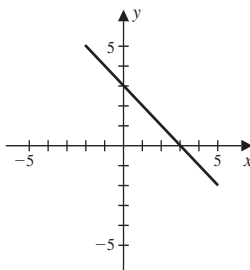
8. Slope: $m = \frac{-4 - (-4)}{-2 - 2} = 0$;
Equation: $y = -4$



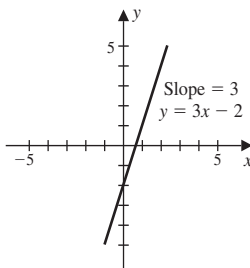
9. For $y = x - 3$ the slope is $m = 1$.



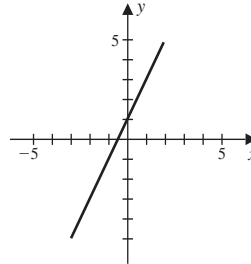
10. For $x + y = 3$ or $y = -x + 3$, the slope is $m = -1$.



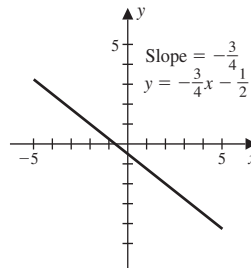
11. For $3x - y = 2$, or $y = 3x - 2$, the slope is $m = 3$.



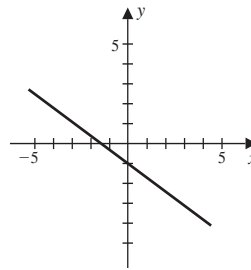
12. For $2x - y = -1$, or $-y = -2x - 1$ so $y = 2x + 1$, the slope is $m = 2$.



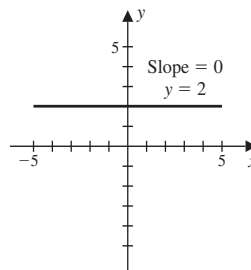
13. For $-3x - 4y = 2$, or $-4y = 3x + 2$ so $y = -\frac{3}{4}x - \frac{1}{2}$, the slope is $m = -\frac{3}{4}$.



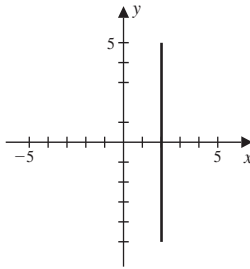
14. For $-2x - 3y = 3$, or $y = -\frac{2}{3}x - 1$, the slope is $m = -\frac{2}{3}$.



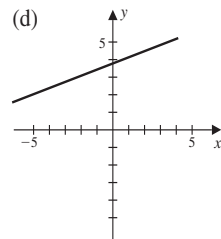
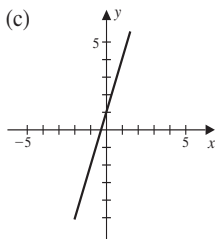
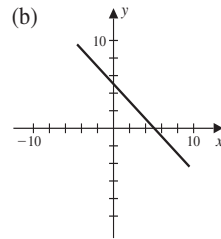
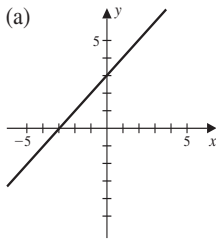
15. For $y = 2$ the slope is 0.



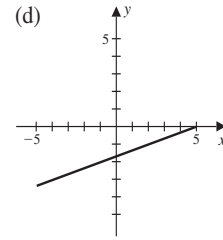
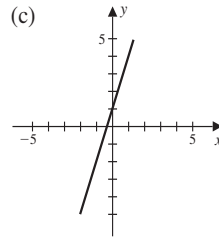
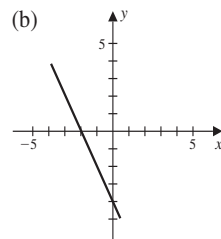
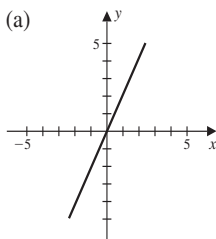
16. For $x = 2$ the slope is undefined.



17. (a) $y - 4 = x - 1$ implies $y = x + 3$
 (b) $y - 4 = -(x - 1)$ implies $y = -x + 5$
 (c) $y - 4 = 3(x - 1)$ implies $y = 3x + 1$
 (d) $y - 4 = \frac{1}{3}(x - 1)$ implies $y = \frac{1}{3}x + \frac{11}{3}$



18. (a) $y - (-2) = 2(x - (-1))$ implies $y = 2x$
 (b) $y + 2 = -2(x + 1)$ implies $y = -2x - 4$
 (c) $y + 2 = 3(x + 1)$ implies $y = 3x + 1$
 (d) $y + 2 = \frac{1}{3}(x + 1)$ implies $y = \frac{1}{3}x - \frac{5}{3}$



19. The slopes of the lines are:
 (a) 0 (b) 1 (c) -1 (d) 1 (e) 2
 (f) 0 (g) -1 (h) 2 (i) 1 (j) 1
 Parallel Lines: (a) and (f); (c) and (g);
 (b), (d), (i), and (j); (e) and (h).
20. The slopes of the lines are:
 (a) 0 (b) undefined (c) 2 (d) 3 (e) $\frac{1}{3}$
 (f) 1 (g) -1 (h) -3 (i) $-\frac{1}{3}$ (j) $-\frac{1}{2}$
 Perpendicular Lines: (a) and (b); (c) and (j);
 (d) and (i); (e) and (h); (f) and (g).
21. For $y = 2x + 1$:
- (a) The line parallel through $(0, 0)$ has same slope $m = 2$, so the equation is $y = 2x$.
 (b) The line perpendicular through $(0, 0)$ has slope negative reciprocal $m = -\frac{1}{2}$, so the equation is $y = -\frac{1}{2}x$.
22. For $x + y = -2$, that is, $y = -x - 2$:
- (a) The line parallel through $(0, 0)$ has same slope $m = -1$, so the equation is $y = -x$.
 (b) The line perpendicular through $(0, 0)$ has slope negative reciprocal $m = 1$, so the equation is $y = x$.
23. For $y = -2x + 3$:
- (a) The line parallel through $(-1, 2)$ has same slope $m = -2$, so the equation is $y - 2 = -2(x + 1)$ or $y = -2x$.
 (b) The line perpendicular through $(-1, 2)$ has slope negative reciprocal $m = \frac{1}{2}$, so the equation is $y - 2 = \frac{1}{2}(x + 1)$ or $y = \frac{1}{2}x + \frac{5}{2}$.
24. For $y = 3x - 2$:

- (a) The line parallel through $(1, 2)$ has same slope $m = 3$, so the equation is $y - 2 = 3(x - 1)$ or $y = 3x - 1$.
- (b) The line perpendicular through $(1, 2)$ has slope negative reciprocal $m = -\frac{1}{3}$, so the equation is $y - 2 = -\frac{1}{3}(x - 1)$ or $y = -\frac{1}{3}x + \frac{7}{3}$.
25. Since the y -intercept is 2, and the slope is -1 , the equation is $y = -x + 2$.
26. We have $y = -4x + 1$.
27. We have $y - (-2) = 3(x - 1)$ or $y = 3x - 5$.
28. We have $y - (-3) = -2(x - (-1))$, which implies $y + 3 = -2(x + 1)$, so $y = -2x - 5$.
29. The line passes through $(1, 0)$ and $(0, 3)$, so the slope is $m = \frac{3-0}{0-1} = -3$, and the equation is $y - 0 = -3(x - 1)$ or $y = -3x + 3$.
30. The line passes through $(-2, 0)$ and $(0, -3)$, so the slope is $m = \frac{-3-0}{0-(-2)} = -\frac{3}{2}$, and the equation is $y + 3 = -\frac{3}{2}x$ or $y = -\frac{3}{2}x - 3$.
31. Since the line is parallel to the x -axis, its slope is 0, so the line is horizontal with equation $y = -1$.
32. Since the line is parallel to the y -axis, it is vertical with undefined slope and equation $x = -2$.
33. We have $2x - 3y = 2$, which implies $-3y = -2x + 2$, so $y = \frac{2}{3}x - \frac{2}{3}$. The parallel line also has slope $\frac{2}{3}$ and equation $y - 3 = \frac{2}{3}(x - 4)$ or $y = \frac{2}{3}x + \frac{1}{3}$.
34. We have $2x + y = 3$, which implies $y = -2x + 3$. The parallel line also has slope -2 and equation $y - (-2) = -2(x - 3)$ or $y = -2x + 4$.
35. A line perpendicular to $y = 2 - x$ has slope 1, and if it passes through $(1, 1)$, has equation $y - 1 = x - 1$ or $y = x$.
36. We have $x - 2y = 4$, which implies $-2y = -x + 4$, so $y = \frac{1}{2}x - 2$. A line perpendicular to this line has slope -2 , and if it passes through $(-3, 5)$ it has equation $y - 5 = -2(x + 3)$ or $y = -2x - 1$.

37. (a) The line tangent to the circle $x^2 + y^2 = 3$ at the point $(1, \sqrt{2})$ is perpendicular to the radius line passing through $(0, 0)$ and $(1, \sqrt{2})$. The slope of the radius line is $m = \frac{\sqrt{2}-0}{1-0} = \sqrt{2}$. A line tangent to the circle is perpendicular to this radius, so it has slope $-\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$, and if it passes through $(1, \sqrt{2})$ has equation

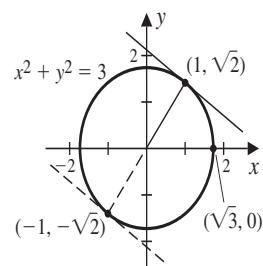
$$y - \sqrt{2} = -\frac{\sqrt{2}}{2}(x - 1)$$

$$\text{or } y = -\frac{\sqrt{2}}{2}x + \frac{3\sqrt{2}}{2}.$$

- (b) The other point on the circle where the tangent line is parallel to this line is at the point diametrically opposite the point $(1, \sqrt{2})$, which is the point $(-1, -\sqrt{2})$. The tangent line at this point has equation

$$y + \sqrt{2} = -\frac{\sqrt{2}}{2}(x + 1)$$

$$\text{or } y = -\frac{\sqrt{2}}{2}x - \frac{3\sqrt{2}}{2}.$$

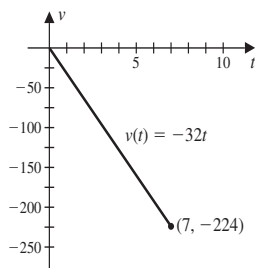


38. The line passes through $(a, 0)$ and $(0, b)$, so it has slope $m = \frac{b-0}{0-a} = -\frac{b}{a}$ and equation

$$y - 0 = -\frac{b}{a}(x - a) \quad \text{so} \quad y = -\frac{b}{a}x + b,$$

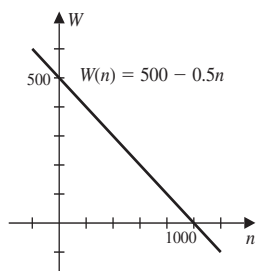
$$\text{that is, } \frac{b}{a}x + y = b \quad \text{and} \quad \frac{x}{a} + \frac{y}{b} = 1.$$

39. Since $v(t) = -32t$, $v(7) = -224$, and the rock is traveling downward at a speed of 224 feet/second.



40. At sea level, water freezes at 32°F (0°C) and boils at 212°F (100°C), so the line plotting Fahrenheit against Celsius passes through the points $(0, 32)$ and $(100, 212)$. The slope of the line is $m = \frac{212-32}{100-0} = \frac{9}{5}$ and the equation is $F - 32 = \frac{9}{5}(C - 0)$ or $F = \frac{9}{5}C + 32$. The Fahrenheit temperature corresponding to 30°C is $F = \frac{9}{5}30 + 32 = 86^\circ$.

41. (a) For $W(n) = 500 - 0.5n$ we have the following graph.



- (b) The total fish weight is
 $T(n) = n \cdot W(n)$
 $= n(500 - 0.5n) = 500n - 0.5n^2$.

- (c) At $n = 1000$ the total weight is 0, so the fish population has disappeared. This is the predicted limiting value of the number of fish in the pond. For $n > 1000$, the total weight given by the equation is negative, which is physically unreasonable.

42. (a) The linear model passes through the points $(0, 10000)$ and $(5, 2000)$. The slope of the line is

$$m = \frac{10000 - 2000}{0 - 5} = -1600$$

and the equation for the linear depreciation is

$$y - 10000 = -1600t$$

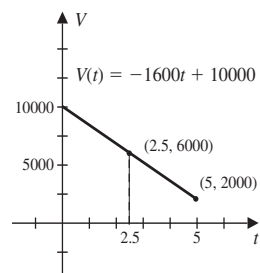
$$\text{or } y = -1600t + 10000.$$

So the value of the equipment at time t is $V(t) = -1600t + 10000$.

- (b) After 2.5 years the equipment is worth $V(2.5) = -1600(2.5) + 10000 = 6000$.
 (c) We have

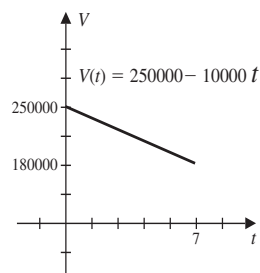
$$\frac{V(3) - V(1)}{3 - 1} = \frac{5200 - 8400}{2} = -1600,$$

and the negative sign indicates that the value is depreciating.



43. (a) $V(t) = 28,000 - 4000t$ dollars.
 (b) $V(5) \approx \$8000$.
 (c) In 7 years.

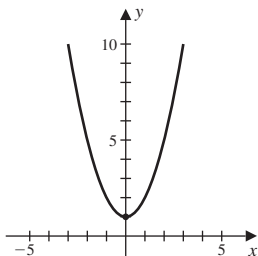
44. (a) $V(t) = 250000 - \frac{t}{7}(250000 - 180000) = 250000 - 10000t$, for $0 \leq t \leq 7$.



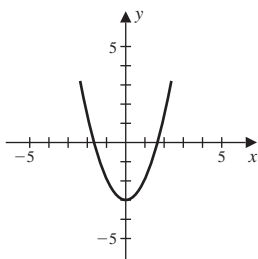
- (b) They will keep the equipment while $250000 - 10000t \geq 70000$, that is until $t = 18$.

Exercise Set 1.8 (Page 73)

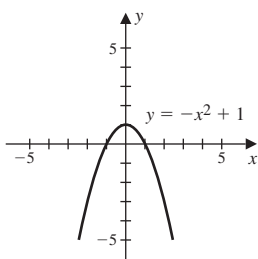
1. $y = x^2 + 1$



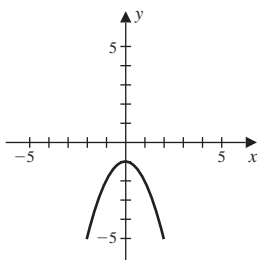
2. $y = x^2 - 3$



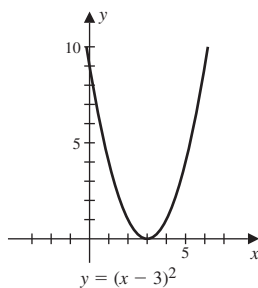
3. $y = -x^2 + 1$



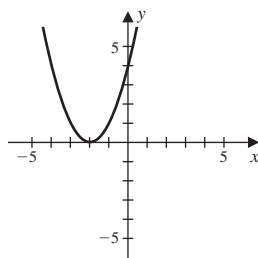
4. $y = -x^2 - 1$



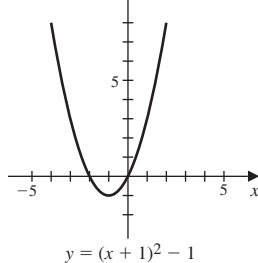
5. $y = (x - 3)^2$



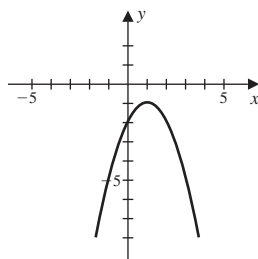
6. $y = (x + 2)^2$



7. $y = (x + 1)^2 - 1$

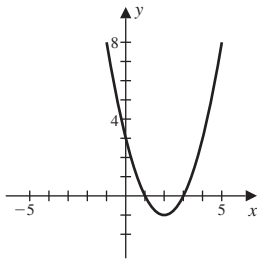


8. $y = -(x - 1)^2 - 1$



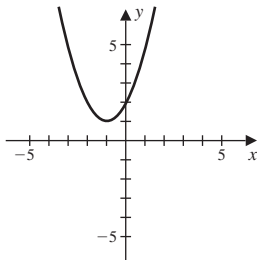
9. Completing the square gives

$$\begin{aligned} y &= x^2 - 4x + 3 = x^2 - 4x + 4 - 4 + 3 \\ &= (x - 2)^2 - 1. \end{aligned}$$



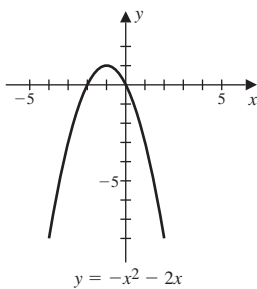
10. Completing the square gives

$$\begin{aligned} y &= x^2 + 2x + 2 \\ &= x^2 + 2x + 1 - 1 + 2 \\ &= (x + 1)^2 + 1. \end{aligned}$$



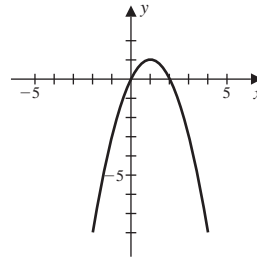
11. Completing the square gives

$$\begin{aligned} y &= -x^2 - 2x \\ &= -(x^2 + 2x) \\ &= -(x^2 + 2x + 1 - 1) \\ &= -(x + 1)^2 + 1. \end{aligned}$$



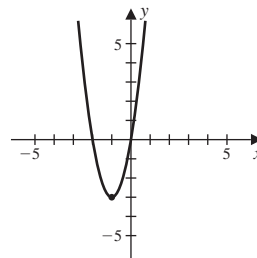
12. Completing the square gives

$$\begin{aligned} y &= -x^2 + 2x \\ &= -(x^2 - 2x) \\ &= -(x^2 - 2x + 1 - 1) \\ &= -(x - 1)^2 + 1. \end{aligned}$$



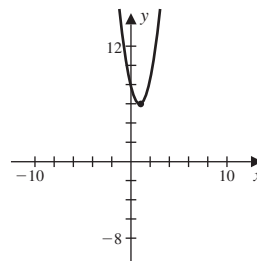
13. Completing the square gives

$$\begin{aligned} y &= 3x^2 + 6x \\ &= 3(x^2 + 2x) \\ &= 3(x^2 + 2x + 1 - 1) \\ &= 3(x + 1)^2 - 3. \end{aligned}$$



14. Completing the square gives

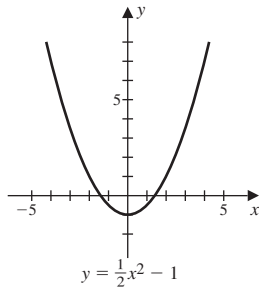
$$\begin{aligned} y &= 2x^2 - 4x + 8 \\ &= 2(x^2 - 2x + 1 - 1) + 8 \\ &= 2(x - 1)^2 + 6. \end{aligned}$$



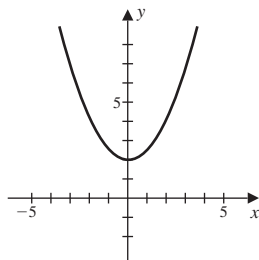
15. $y = \frac{1}{2}x^2 - 1$

42 Exercise Section 1.8

Solutions to All Exercises

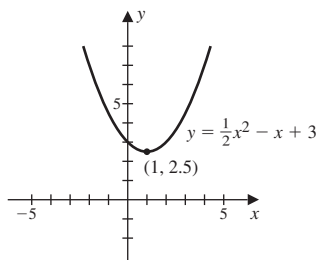


16. $y = \frac{1}{2}x^2 + 2$



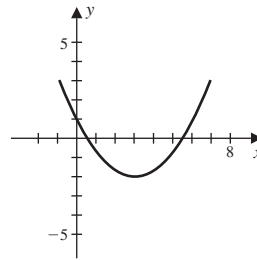
17. Completing the square gives

$$\begin{aligned} y &= \frac{1}{2}x^2 - x + 3 \\ &= \frac{1}{2}(x^2 - 2x) + 3 \\ &= \frac{1}{2}(x^2 - 2x + 1 - 1) + 3 \\ &= \frac{1}{2}(x - 1)^2 + \frac{5}{2}. \end{aligned}$$



18. Completing the square gives

$$\begin{aligned} y &= \frac{1}{3}x^2 - 2x + 1 \\ &= \frac{1}{3}(x^2 - 6x + 9 - 9) + 1 \\ &= \frac{1}{3}(x - 3)^2 - 2. \end{aligned}$$



19. (a) $f(x) = x^2 - 6x + 7 = (x^2 - 6x + 9 - 9) + 7 = (x - 3)^2 - 2$
 (b) To find the x -intercepts solve $(x - 3)^2 - 2 = 0$, that is $(x - 3)^2 = 2$, so $x = \pm\sqrt{2} + 3$, and the x -intercepts are $(3 + \sqrt{2}, 0)$ and $(3 - \sqrt{2}, 0)$. The y -intercept is $(0, 7)$.
 (c) Since the parabola opens upward the vertex $(3, -2)$ is a minimum point. So the minimum value of the function is $f(3) = -2$.
20. (a) $f(x) = x^2 + 4x + 5 = (x^2 + 4x + 4 - 4) + 5 = (x + 2)^2 + 1$
 (b) The graph does not cross the x -axis and so there are no x -intercepts. The y -intercept is $(0, 5)$.
 (c) Since the parabola opens upward, the vertex $(-2, 1)$ is a minimum point. So the minimum value of the function is $f(-2) = 1$.
21. (a) We have $f(x) = -x^2 + 4x + 6 = -(x^2 - 4x + 4 - 4) + 6 = -(x - 2)^2 + 10$.
 (b) To find the x -intercepts solve $-(x - 2)^2 + 10 = 0$, that is $(x - 2)^2 = 10$ so $x = \pm\sqrt{10} + 2$, and the x -intercepts are $(\sqrt{10} + 2, 0)$ and $(-\sqrt{10} + 2, 0)$. The y -intercept is $(0, 6)$.
 (c) Since the parabola opens downward the vertex $(2, 10)$ is a maximum point. So the maximum value of the function is $f(2) = 10$.
22. (a) $f(x) = -x^2 - 4x - 4 = -(x^2 + 4x + 4) = -(x + 2)^2$
 (b) To find the x -intercepts solve $-(x + 2)^2 = 0$, that is $(x + 2)^2 = 0$ so $x = -2$, and the x -intercept is $(-2, 0)$. The y -intercept is $(0, -4)$.

(c) Since the parabola opens downward the vertex $(-2, 0)$ is a maximum point. So the maximum value of the function is $f(-2) = 0$.

23. For $6x^2 - 5x + 1 = 0$,

$$\begin{aligned} x &= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(6)(1)}}{2(6)} \\ &= \frac{5 \pm \sqrt{25 - 24}}{12} = \frac{5 \pm 1}{12} \end{aligned}$$

so the x -intercepts are $x = \frac{1}{2}$ and $x = \frac{1}{3}$.

24. For $12x^2 + x - 1 = 0$,

$$\begin{aligned} x &= \frac{-1 \pm \sqrt{(1)^2 - 4(12)(-1)}}{2(12)} \\ &= \frac{-1 \pm \sqrt{1 + 48}}{24} = \frac{-1 \pm 7}{24} \end{aligned}$$

so the x -intercepts are $x = \frac{1}{4}$ and $x = -\frac{1}{3}$.

25. For $2x^2 - 4x + 1 = 0$,

$$\begin{aligned} x &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(1)}}{2(2)} \\ &= \frac{4 \pm \sqrt{16 - 8}}{4} \\ &= \frac{4 \pm \sqrt{8}}{4} = \frac{4 \pm 2\sqrt{2}}{4} \end{aligned}$$

so the x -intercepts are

$$x = 1 + \frac{\sqrt{2}}{2} \quad \text{and} \quad x = 1 - \frac{\sqrt{2}}{2}.$$

26. For $2x^2 - 6x + 3 = 0$,

$$\begin{aligned} x &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(3)}}{2(2)} \\ &= \frac{6 \pm \sqrt{36 - 24}}{4} \\ &= \frac{6 \pm \sqrt{12}}{4} = \frac{6 \pm 2\sqrt{3}}{4} \end{aligned}$$

so the x -intercepts are

$$x = \frac{3}{2} + \frac{\sqrt{3}}{2} \quad \text{and} \quad x = \frac{3}{2} - \frac{\sqrt{3}}{2}.$$

27. For $2x^2 + 4x + 3 = 0$,

$$\begin{aligned} x &= \frac{-4 \pm \sqrt{(4)^2 - 4(2)(3)}}{2(2)} \\ &= \frac{-4 \pm \sqrt{16 - 24}}{4} \\ &= \frac{-4 \pm \sqrt{-8}}{4}. \end{aligned}$$

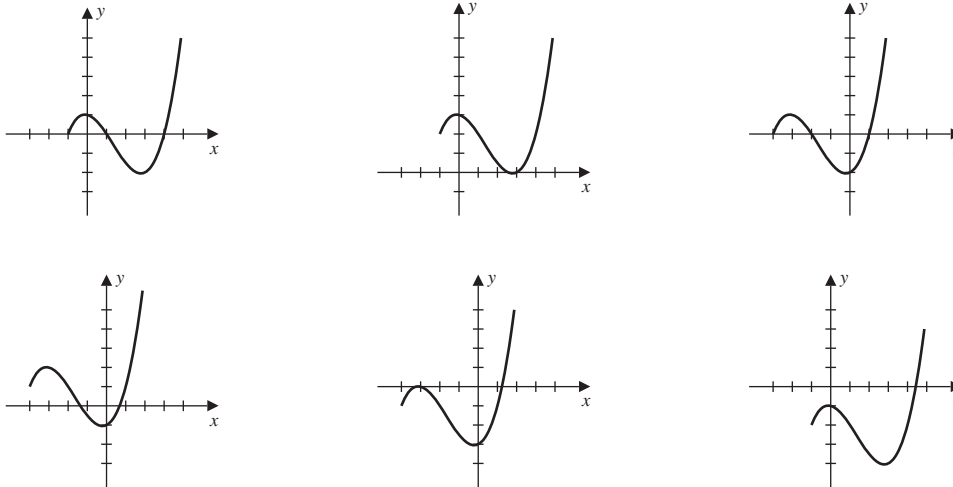
Since the square root of a negative number is not a real number, the quadratic has no solutions and there are no x -intercepts.

28. For $4x^2 - 8x + 7 = 0$,

$$\begin{aligned} x &= \frac{-(-8) \pm \sqrt{(-8)^2 - 4(4)(7)}}{2(4)} \\ &= \frac{8 \pm \sqrt{64 - 112}}{8} \\ &= \frac{8 \pm \sqrt{-48}}{8}. \end{aligned}$$

Since the square root of a negative number is not a real number, the quadratic has no solutions and there are no x -intercepts.

29. The various graphs are shown.



30. (a) An upward shift by 2 units of $y = x^2$, so it is (v).
 (b) Since $y = x^2 - 4x + 4 = (x - 2)^2$, the curve is a right shift by 2 units of $y = x^2$, so it is (ii).
 (c) A left shift by 2 units of $y = x^2$, so it is (iv).
 (d) A left shift by 1 unit of $y = x^2$, followed by a downward shift of 2 units so it is (iii).
 (e) Since $y = x^2 - 2x + 3 = (x - 1)^2 + 2$, the curve is a right shift by 1 unit of $y = x^2$, followed by an upward shift of 2 units so it is (vi).
 (f) A reflection of $y = x^2$ about the x -axis followed by a downward shift of 1 unit so it is (i).
31. The function will have the form $f(x) = a(x - 1)^2 + 3$ and if the graph passes through the point $(-2, 5)$ we have

$$5 = a(-2 - 1)^2 + 3, \quad \text{that is} \quad 5 = 9a + 3 \quad \text{so} \quad a = \frac{2}{9},$$

$$\text{and } f(x) = \frac{2}{9}(x - 1)^2 + 3.$$

32. The function will have the form $f(x) = a(x + 2)^2 + 2$ and if the graph passes through the point $(1, -4)$ we have

$$-4 = a(1 + 2)^2 + 2, \quad \text{that is} \quad -4 = 9a + 2 \quad \text{so} \quad a = -\frac{6}{9} = -\frac{2}{3},$$

$$\text{and } f(x) = -\frac{2}{3}(x + 2)^2 + 2.$$

33. (a) If x is in the domain of $f(x) = \sqrt{x^2 - 3}$, then

$$x^2 - 3 = (x - \sqrt{3})(x + \sqrt{3}) \geq 0 \quad \text{so} \quad x \leq -\sqrt{3} \quad \text{or} \quad x \geq \sqrt{3},$$

$$\text{which can be written } (-\infty, -\sqrt{3}] \cup [\sqrt{3}, \infty).$$

(b) The domain of $f(x) = \sqrt{x^2 - \frac{1}{2}x} = \sqrt{x\left(x - \frac{1}{2}\right)}$ is all x with $x\left(x - \frac{1}{2}\right) \geq 0$ so $x \leq 0$ or $x \geq \frac{1}{2}$, which can be written $(-\infty, 0] \cup \left[\frac{1}{2}, \infty\right)$.

34. For $f(x) = 100x^2$ and $g(x) = 0.1x^3$ we have

$$f(x) \geq g(x), \quad \text{that is } 100x^2 \geq 0.1x^3 \quad \text{so } \frac{1}{10}x^3 - 100x^2 \leq 0 \quad \text{and } \frac{1}{10}x^2(x - 1000) \leq 0$$

if and only if

$$x - 1000 \leq 0 \quad \text{or } x \leq 1000, \quad \text{that is } x \text{ is in } (-\infty, 1000].$$

35. Let $f(x) = ax^2 + bx + c$.

(a) Since $(1, 1)$ is on the graph, we have $1 = f(1) = a + b + c$. So $a + b + c = 1$.

(b) The y -intercept is 6, which implies $6 = f(0) = c$. So $c = 6$.

(c) The vertex of a parabola is $\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$. The vertex is $(1, 1)$, which implies $-\frac{b}{2a} = 1$ and $\frac{4ac - b^2}{4a} = 1$, so $b = -2a$ and

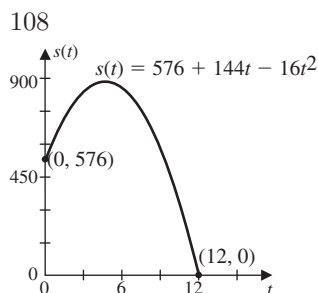
$$1 = \frac{4ac - b^2}{4a} = \frac{4ac - (-2a)^2}{4a} = \frac{4ac - 4a^2}{4a} = \frac{4a(c - a)}{4a} = c - a, \quad \text{when } a \neq 0.$$

Since $c - a = 1$, we have $c = a + 1$.

(d) $a + b + c = 1$ and $c = 6$ so $a + b = -5$. But $c = a + 1$ so $a = 5$ and $b = -10$. So $a = 5, b = -10, c = 6$ and $f(x) = 5x^2 - 10x + 6$.

36. (a) Let $s(t) = 576 + 144t - 16t^2$. Completing the square gives

$$\begin{aligned} -16t^2 + 144t + 576 &= -16(t^2 - 9t) + 576 \\ &= -16\left(t^2 - 9t + \frac{81}{4} - \frac{81}{4}\right) + 576 \\ &= -16\left(t - \frac{9}{2}\right)^2 + 324 + 576 \\ &= -16\left(t - \frac{9}{2}\right)^2 + 900. \end{aligned}$$



- (b) When the rock is on the ground we have

$$s(t) = 576 + 144t - 16t^2 = 0 \quad \text{implies} \quad 16t^2 - 144t - 576 = 16(t^2 - 9t - 36) = 0$$

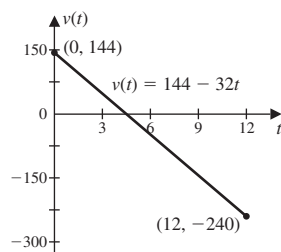
if and only if

$$16(t - 12)(t + 3) = 0 \quad \text{so} \quad t = 12 \quad \text{or} \quad t = -3.$$

It takes the rock 12 seconds to reach the ground.

- (c) The rock reaches its maximum height at the vertex of the parabolic path. From part (a) the vertex of the parabola is $\left(\frac{9}{2}, 900\right)$, so the rock reaches a maximum height of 900 feet $\frac{9}{2}$ seconds after it has been thrown.
- (d) The domain consists of only real numbers greater than or equal to 0 since t represents time, and should not exceed the value of t for which the rock strikes the ground. This implies that the domain is $[0, 12]$. The range is all real numbers greater than or equal to 0, and less than the maximum height the rock reaches. Hence the range is $[0, 900]$.

37. (a) Let $v(t) = 144 - 32t$.



- (b) From Exercise 36(b), the rock strikes the ground at $t = 12$, so the velocity of the rock is $v(12) = 144 - 32(12) = -240$ feet/second. The minus sign indicates the rock is falling to the earth.
- (c) $\text{Domain}(v) = [0, 12]$; $\text{Range}(v) = [-240, 144]$

38. Let $C(x) = x^2 - 120x + 4000$.

The parabola opens upward, so the vertex is the minimum point on the curve. To find the number of items to produce in order to minimize the unit cost, complete the square on the quadratic equation to find the vertex.

$$x^2 - 120x + 4000 = x^2 - 120x + 3600 - 3600 + 4000 = (x - 60)^2 + 400$$

The vertex is at $(60, 400)$, so the number of items to produce is 60 per day with a minimum cost of \$400.00.

39. Let $P(x) = -0.1x^2 + 160x - 20000$.

- (a) The parabola opens downward, so the vertex is the maximum point on the curve. To find the number of terminals to produce in order to maximize the profit, complete the square and find

the vertex.

$$\begin{aligned}
 -0.1x^2 + 160x - 20000 &= -0.1(x^2 - 1600x) - 20000 \\
 &= -0.1(x^2 - 1600x + 640000 - 640000) - 20000 \\
 &= -0.1(x - 800)^2 + 64000 - 20000 \\
 &= -0.1(x - 800)^2 + 44000
 \end{aligned}$$

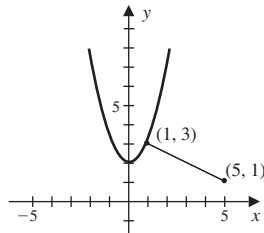
The vertex is at $(800, 44000)$, so the company should produce 800 terminals to maximize the profit.

(b) The maximum profit is \$44,000.00.

40. (a) The distance from (x, y) to $(5, 1)$ is

$$d = \sqrt{(x - 5)^2 + (y - 1)^2} = \sqrt{(x - 5)^2 + (x^2 + 2 - 3)^2} = \sqrt{(x - 5)^2 + (x^2 - 1)^2}.$$

(b) A graphing device indicates that the point on the graph of $y = x^2 + 2$ that is closest to $(5, 1)$ is approximately $(1, 3)$.



41. (a) The area of the rectangle, for $x > 0$, is $A(x) = 2x(4 - x^2) = 8x - 2x^3$.

(b) A graphing device indicates that the maximum area occurs when the width, w , and the height, h , satisfy $w \approx 2.3$, $h \approx 2.7$, and $A(w/2) \approx 6.2$.

42. Let $y = a(x - 1970)^2 + b(x - 1970) + c$.

(a) Substituting the values 1970, 1990, and 2000 into the equation gives the three equations

$$c = 80, \quad 400a + 20b + c = 690, \quad \text{and} \quad 900a + 30b + c = 1299$$

and solving the system of two equations in two unknowns

$$400a + 20b = 610 \quad \text{and} \quad 900a + 30b = 1219,$$

gives $a = \frac{76}{75}$ and $b = \frac{307}{30}$. The parabola fitting the data is

$$y = \frac{76}{75}(x - 1970)^2 + \frac{307}{30}(x - 1970) + 80.$$

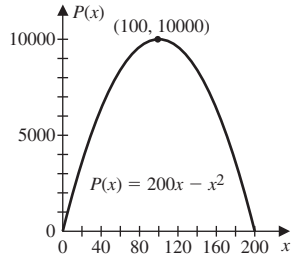
(b) Substituting 2010 into the equation gives

$$\frac{76}{75}(40)^2 + \frac{307}{30}(40) + 80 \approx 2111 \text{ billion.}$$

43. $R = kP(M - P)$, where k is the constant of proportionality.

44. (a) Let $P(x) = 200x - x^2$. Completing the square gives

$$\begin{aligned} P(x) &= -x^2 + 200x = -(x^2 - 200x) \\ &= -(x^2 - 200x + 10000 - 10000) = -(x - 100)^2 + 10000. \end{aligned}$$



(b) The vertex of the parabola will give the maximum profit, so from part (a) the vertex is at $(100, 10000)$ and hence 100 units should be produced giving a maximum profit is \$10,000.00.

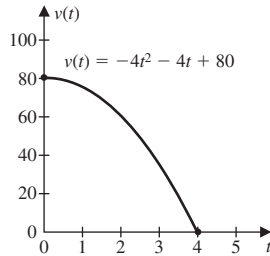
(c) We have

$$\begin{aligned} \frac{P(x+h) - P(x)}{h} &= \frac{200(x+h) - (x+h)^2 - (200x - x^2)}{h} \\ &= \frac{200x + 200h - (x^2 + 2hx + h^2) - 200x + x^2}{h} \\ &= \frac{200h - 2hx - h^2}{h} = \frac{h(200 - 2x - h)}{h} \\ &= 200 - 2x - h. \end{aligned}$$

As h approaches 0, the difference quotient approaches $200 - 2x$.

45. (a) Let $v(t) = -4t^2 - 4t + 80$. Completing the square gives

$$-4t^2 - 4t + 80 = -4(t^2 + t) + 80 = -4\left(t^2 + t + \frac{1}{4} - \frac{1}{4}\right) + 80 = -4\left(t + \frac{1}{2}\right)^2 + 81.$$



(b) The car is at rest when

$$v(t) = -4t^2 - 4t + 80 = -4(t^2 + t - 20) = -4(t+5)(t-4) = 0 \quad \text{so} \quad t = -5, t = 4.$$

Since t is time, $t = -5$ is not a solution, and the car comes to rest after $t = 4$ seconds.

(c)

t	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4
$v(t)$	80	77	72	65	56	45	32	17	0

(d) slowest: 77 feet/second; fastest: 80 feet/second

(e)

	slowest	fastest
second $1/2$ sec	72	77
third $1/2$ sec	65	72

(f)

	minimum distance	maximum distance
first $1/2$ sec	$77 \times 1/2 = 38.5$ ft	$80 \times 1/2 = 40$ ft
second $1/2$ sec	$72 \times 1/2 = 36$ ft	$77 \times 1/2 = 38.5$ ft

(g) Lower bound:

$$\frac{1}{2}[77 + 72 + 65 + 56 + 45 + 32 + 17] = \frac{1}{2} \cdot 364 = 182$$

Upper bound:

$$\frac{1}{2}[80 + 77 + 72 + 65 + 56 + 45 + 32 + 17] = \frac{1}{2} \cdot 444 = 222$$

Review Exercises For Chapter 1 (Page 76)

1. (a) $-1 \leq x \leq 7$

2. $1 < x < \sqrt{3}$

(b)



3. (a) $x < 7$

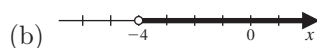
(b)



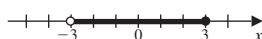
4. $-5 \leq x$



5. (a) $(-4, \infty)$



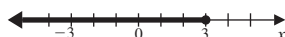
6. $(-3, 3]$



7. (a) $[2, 10]$



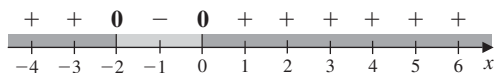
8. $(-\infty, 3]$



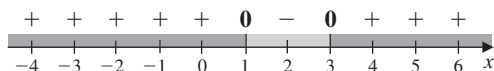
9. $2x + 3 \geq 4$ implies $2x \geq 1$ so $x \geq \frac{1}{2}$

10. $-(2x + 1) \leq 4$, implies $2x + 1 \geq -4$, which implies $2x \geq -5$. So $x \geq -\frac{5}{2}$

11. $x^2 + 2x + 1 \geq 1$ implies $x^2 + 2x \geq 0$, so $x(x + 2) \geq 0$. The solution set is $(-\infty, -2] \cup [0, \infty)$. $x(x + 2)$

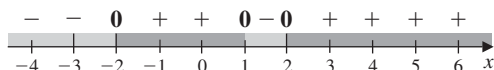


12. $x^2 - 4x > -3$ implies $x^2 - 4x + 3 > 0$ so $(x - 1)(x - 3) > 0$. The solution set is $(-\infty, 1) \cup (3, \infty)$. $(x - 1)(x - 3)$



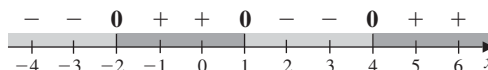
13. $(x - 1)(x + 2)(x - 2) \geq 0$

The solution set is $[-2, 1] \cup [2, \infty)$. $(x - 1)(x + 2)(x - 2)$



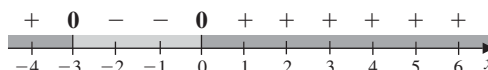
14. $(x - 1)(x - 4)(x + 2) < 0$

The solution set is $(-\infty, -2) \cup (1, 4)$. $(x - 1)(x - 4)(x + 2)$

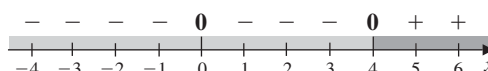


15. $x^2 + 3x > 0$ implies $x(x + 3) > 0$.

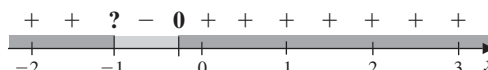
The solution set is $(-\infty, -3) \cup (0, \infty)$. $x(x + 3)$



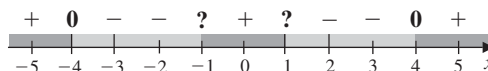
16. $x^3 - 4x^2 \leq 0$ implies $x^2(x - 4) \leq 0$, so $x - 4 \leq 0$ and $x \leq 4$. $x^2(x - 4)$



17. $\frac{2x - 1}{x + 1} \leq -2$ implies $\frac{2x - 1}{x + 1} + 2 \leq 0$ so $\frac{2x - 1 + 2(x + 1)}{x + 1} \leq 0$ and $\frac{4x + 1}{x + 1} \leq 0$. The solution set is $\left(-1, -\frac{1}{4}\right]$. $\frac{4x + 1}{x + 1}$



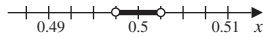
18. $\frac{x^2 - 16}{x^2 - 1} \leq 0$ implies $\frac{(x + 4)(x - 4)}{(x + 1)(x - 1)} \leq 0$. The solution set is $[-4, -1) \cup (1, 4]$. $\frac{(x + 4)(x - 4)}{(x + 1)(x - 1)}$



19. $|2x - 3| < 5$ implies $-5 < 2x - 3 < 5$ so $-2 < 2x < 8$ and $-1 < x < 4$



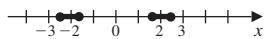
20. $|4x - 2| < 0.01$ implies $-0.01 < 4x - 2 < 0.01$
so $1.99 < 4x < 2.01$ and $0.4975 < x < 0.5025$



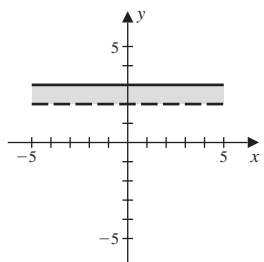
21. $|3 - x| \leq 4$ implies $-4 \leq 3 - x \leq 4$ so $-x \leq 1$
and $-x \geq -7$ or $x \geq -1$ and $x \leq 7$. That is,
 $[-1, 7]$.



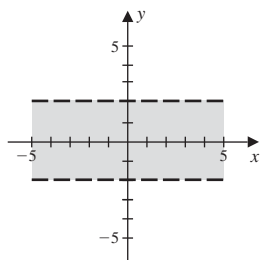
22. $|x^2 - 4| \leq 1$ so $-1 \leq x^2 - 4 \leq 1$.
That is, $x^2 - 3 \geq 0$ and $x^2 - 5 \leq 0$,
which implies $(x - \sqrt{3})(x + \sqrt{3}) \geq 0$ and
 $(x - \sqrt{5})(x + \sqrt{5}) \leq 0$. Hence $x \geq \sqrt{3}$ or
 $x \leq -\sqrt{3}$ and $-\sqrt{5} \leq x \leq \sqrt{5}$. In interval
notation the solution set is
 $[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$.



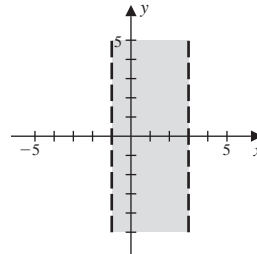
23. $2 < y \leq 3$



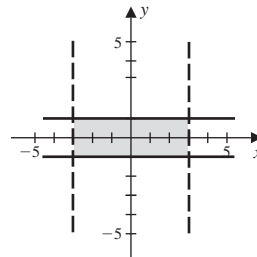
24. $|y| < 2$ implies $-2 < y < 2$



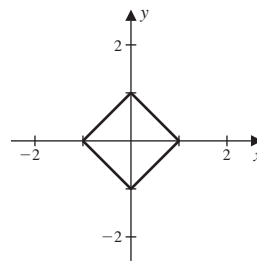
25. $|x - 1| < 2$ implies $-2 < x - 1 < 2$ so
 $-1 < x < 3$



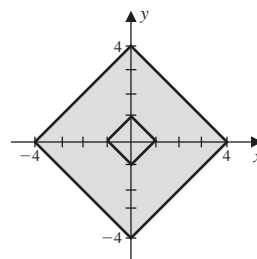
26. $|x| < 3$ and $|y| \leq 1$ implies $-3 < x < 3$ and
 $-1 \leq y \leq 1$



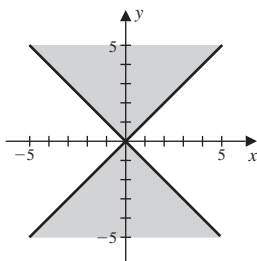
27. $|x| + |y| = 1$



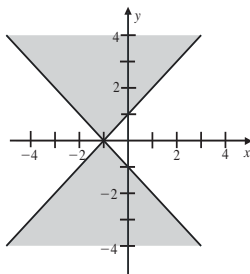
28. $1 \leq |x| + |y| \leq 4$



29. $|x| \leq |y|$



30. $|x + 1| \leq |y|$



31. (a) For $f(x) = x^2 - 3$ the domain is the set of all real numbers.

(b) Since the graph is obtained from the basic graph of $y = x^2$, shifted 3 units downward, the vertex is $(0, -3)$, so the range is $[-3, \infty)$.

32. (a) For $f(x) = \frac{1}{2x - 5}$ the domain is the set of all real numbers for which the denominator is not zero, so the domain is $\{x : x \neq 5/2\} = (-\infty, 5/2) \cup (5/2, \infty)$.

(b) The range is all real numbers except $x = 0$, so the range is $(-\infty, 0) \cup (0, \infty)$.

33. (a) For $f(x) = \sqrt{x - 2} + 2$ the domain is the set of all real numbers for which the expression under the radical is nonnegative, so the domain is $\{x : x - 2 \geq 0\} = [2, \infty)$.

(b) The range is $[2, \infty)$.

34. (a) Since the function $f(x) = \begin{cases} -x + 3, & \text{if } x < 0 \\ -2x^2, & \text{if } x \geq 0 \end{cases}$ is defined at every real number, the domain is the set of all real numbers.

(b) When $x \geq 0$, $y = -2x^2$, which takes on all non-positive values. When $x < 0$,

$y = -x + 3$ takes on all values greater than or equal to 3. So the range is $(-\infty, 0] \cup (3, \infty)$.

35. (a) For $f(x) = \frac{1}{x^2 - 6x + 8}$ the domain is $\{x \mid x^2 - 6x + 8 \neq 0\}$
 $= \{x : (x - 4)(x - 2) \neq 0\}$
 $= (-\infty, 2) \cup (2, 4) \cup (4, \infty)$.

(b) For

$$f(x) = \frac{x - 2}{x^2 - 6x + 8} = \frac{x - 2}{(x - 4)(x - 2)} = \frac{1}{x - 4}$$

the domain is still

$(-\infty, 2) \cup (2, 4) \cup (4, \infty)$, since $x = 2$ can not be substituted into the original equation.

- (c) For $f(x) = \sqrt{\frac{x^2}{x^2 - 6x + 8}}$ the domain is $\{x \mid (x - 4)(x - 2) > 0 \text{ or } x = 0\}$
 $= (-\infty, 2) \cup (4, \infty) \cup \{0\}$.



36. (a) For

$$f(x) = \frac{x - 1}{x^2 + 3x + 2} = \frac{x - 1}{(x + 2)(x + 1)}$$

the domain is $\{x : (x + 2)(x + 1) \neq 0\}$
 $= (-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$.

- (b) For $f(x) = \frac{x + 1}{x^2 + 3x + 2} = \frac{x + 1}{(x + 2)(x + 1)} = \frac{1}{x + 2}$, the domain is still

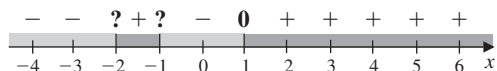
$(-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$,

since $x = -1$ can not be substituted into the original equation.

(c) For

$$f(x) = \sqrt{\frac{x-1}{x^2+3x+2}}$$

$$\begin{aligned} \text{the domain is } & \left\{ x \mid \frac{x-1}{(x+2)(x+1)} \geq 0 \right\} \\ & = (-2, -1) \cup [1, \infty). \end{aligned}$$



37. For $f(x) = 5x + 3$ we have:

$$(a) f(x+h) = 5(x+h) + 3 = 5x + 5h + 3$$

$$\begin{aligned} (b) \frac{f(x+h) - f(x)}{h} &= \frac{5x + 5h + 3 - (5x + 3)}{h} \\ &= \frac{5h}{h} = 5. \end{aligned}$$

38. For $f(x) = -\frac{1}{3}x + 2$ we have:

$$(a) f(x+h) = -\frac{1}{3}(x+h) + 2 = -\frac{1}{3}x - \frac{1}{3}h + 2$$

$$\begin{aligned} (b) \frac{f(x+h) - f(x)}{h} &= \frac{-\frac{1}{3}x - \frac{1}{3}h + 2 - \left(-\frac{1}{3}x + 2\right)}{h} \\ &= \frac{-\frac{1}{3}h}{h} = -\frac{1}{3}. \end{aligned}$$

39. For $f(x) = x^2 - 1$ we have:

$$(a) f(x+h) = (x+h)^2 - 1 = x^2 + 2hx + h^2 - 1$$

$$\begin{aligned} (b) \frac{f(x+h) - f(x)}{h} &= \frac{x^2 + 2hx + h^2 - 1 - (x^2 - 1)}{h} \\ &= \frac{2hx + h^2}{h} = \frac{h(2x + h)}{h} = 2x + h. \end{aligned}$$

40. For $f(x) = 2x^2 - x$ we have:

$$(a) f(x+h) = 2(x+h)^2 - (x+h) = 2x^2 + 4hx + 2h^2 - x - h$$

$$\begin{aligned} (b) \frac{f(x+h) - f(x)}{h} &= \frac{2x^2 + 4hx + 2h^2 - x - h - (2x^2 - x)}{h} \\ &= \frac{4hx + 2h^2 - h}{h} \\ &= \frac{h(4x + 2h - 1)}{h} = 4x + 2h - 1. \end{aligned}$$

41. For $f(x) = \frac{1}{x-1}$ we have:

$$(a) f(x+h) = \frac{1}{x+h-1}$$

$$\begin{aligned} (b) \frac{f(x+h) - f(x)}{h} &= \frac{\frac{1}{x+h-1} - \frac{1}{x-1}}{h} = \frac{\frac{x-1-(x+h-1)}{(x+h-1)(x-1)}}{h} \\ &= \frac{-h}{h(x+h-1)(x-1)} = \frac{-1}{(x+h-1)(x-1)}. \end{aligned}$$

42. For $f(x) = \frac{2}{1-x}$ we have:

$$(a) f(x+h) = \frac{2}{1-x-h}$$

$$\begin{aligned} (b) \frac{f(x+h) - f(x)}{h} &= \frac{\frac{2}{1-x-h} - \frac{2}{1-x}}{h} \\ &= \frac{\frac{2 - 2x - 2(1-x-h)}{(1-x-h)(1-x)}}{h} \\ &= \frac{2h}{h(1-x-h)(1-x)} \\ &= \frac{2}{(1-x-h)(1-x)}. \end{aligned}$$

43. (a) (ii)

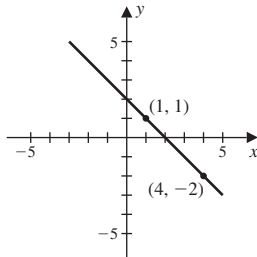
(b) (v)

(c) (vi)

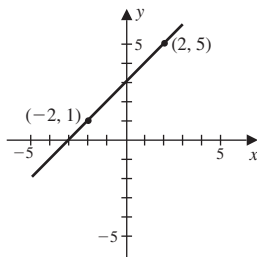
(d) (iv)

(e) (i)

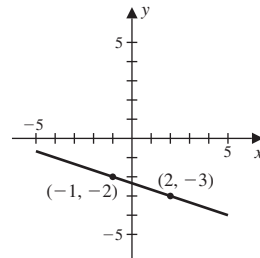
- (f) (iii)
44. (a) (ii)
- (b) (iii) $x^2 + x = x^2 + x + \frac{1}{4} - \frac{1}{4}$
 $= \left(x + \frac{1}{2}\right)^2 - \frac{1}{4}$
- (c) (iv) $x^2 - 2x - 1 = x^2 - 2x + 1 - 1 - 1$
 $= (x - 1)^2 - 2$
- (d) (i) $x^2 + 2x + 3 = x^2 + 2x + 1 - 1 + 3 =$
 $(x + 1)^2 + 2$
45. (a), (d)



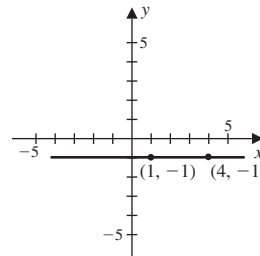
- (b) $d = \sqrt{(4 - 1)^2 + (-2 - 1)^2}$
 $= \sqrt{18} = 3\sqrt{2}$
- (c) $\left(\frac{4 + 1}{2}, \frac{-2 + 1}{2}\right) = \left(\frac{5}{2}, -\frac{1}{2}\right)$
- (e) $m = \frac{-2 - 1}{4 - 1} = -\frac{3}{3} = -1;$
 $y - 1 = -(x - 1)$ implies $y = -x + 2$
46. (a), (d)



- (b) $d = \sqrt{(-2 - 2)^2 + (1 - 5)^2}$
 $= \sqrt{32} = 4\sqrt{2}$
- (c) $\left(\frac{-2 + 2}{2}, \frac{1 + 5}{2}\right) = (0, 3)$
- (e) $m = \frac{5 - 1}{2 - (-2)} = \frac{4}{4} = 1;$
 $y - 5 = (x - 2)$ implies $y = x + 3$
47. (a), (d)



- (b) $d = \sqrt{(-1 - 2)^2 + (-2 - (-3))^2}$
 $= \sqrt{10}$
- (c) $\left(\frac{-1 + 2}{2}, \frac{-2 - 3}{2}\right) = \left(\frac{1}{2}, -\frac{5}{2}\right)$
- (e) $m = \frac{-3 - (-2)}{2 - (-1)} = -\frac{1}{3};$
 $y - (-2) = -\frac{1}{3}(x - (-1))$ implies
 $y = -\frac{1}{3}x - \frac{7}{3}$
48. (a), (d)

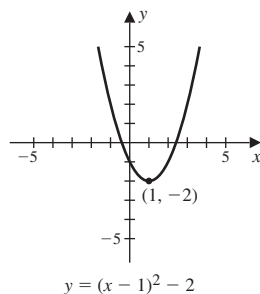


- (b) $d = \sqrt{(1 - 4)^2 + (-1 - (-1))^2}$
 $= \sqrt{9} = 3$
- (c) $\left(\frac{1 + 4}{2}, \frac{-1 - 1}{2}\right) = \left(\frac{5}{2}, -1\right)$

- (e) $m = \frac{-1 - (-1)}{4 - 1} = 0$; $y = -1$
49. (a) $m = 4$; $y - 0 = 4(x - 0)$ implies $y = 4x$
 (b) $m = -\frac{1}{4}$; $y = -\frac{1}{4}x$
50. (a) $m = \frac{1}{3}$; $y - 2 = \frac{1}{3}(x - 1)$ implies
 $y = \frac{1}{3}x + \frac{5}{3}$
 (b) $m = -3$; $y - 2 = -3(x - 1)$ implies
 $y = -3x + 5$

51. $-7x - 5y = -1$ implies $y = -\frac{7}{5}x + \frac{1}{5}$

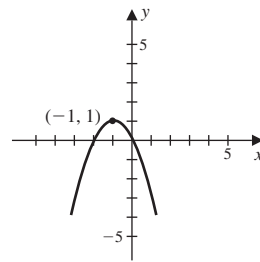
- (a) $m = -\frac{7}{5}$; $y - (-3) = -\frac{7}{5}(x - (-1))$
 implies $y = -\frac{7}{5}x - \frac{22}{5}$
 (b) $m = \frac{5}{7}$; $y + 3 = \frac{5}{7}(x + 1)$
 implies $y = \frac{5}{7}x - \frac{16}{7}$
52. (a) $m = \frac{2}{3}$; $y - (-1) = \frac{2}{3}(x - 1)$ implies
 $y = \frac{2}{3}x - \frac{5}{3}$
 (b) $m = -\frac{3}{2}$; $y - (-1) = -\frac{3}{2}(x - 1)$ implies
 $y = -\frac{3}{2}x + \frac{1}{2}$
53. (a) For $y = (x - 1)^2 - 2$, we have the following graph.



- (b) The range is $[-2, \infty)$.
 (c) The x -intercepts occur when
 $0 = (x - 1)^2 - 2$ so $x = 1 \pm \sqrt{2}$. The
 y -intercept is $(0, -1)$.

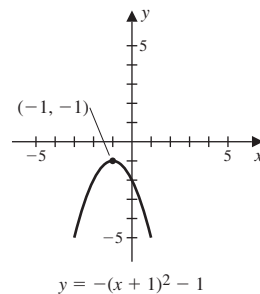
- (d) The minimum value of y is -2 when
 $x = 1$.

54. (a) For $y = -(x + 1)^2 + 1$, we have the following graph.



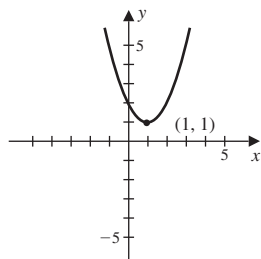
- (b) The range is $(-\infty, 1]$.
 (c) The x -intercepts occur when
 $0 = -(x + 1)^2 + 1$ so $x = -2$ or $x = 0$.
 The y -intercept is $(0, 0)$.
 (d) The maximum value of y is 1 when
 $x = -1$.

55. (a) For $y = -(x + 1)^2 - 1$, we have the following graph.



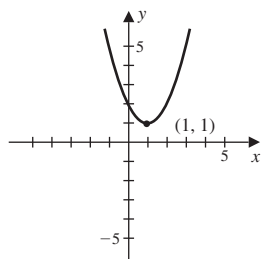
- (b) The range is $(-\infty, -1]$.
 (c) There are no x -intercepts.
 The y -intercept is $(0, -2)$.
 (d) The maximum value of y is -1 when
 $x = -1$.

56. (a) For $y = (x - 1)^2 + 1$, we have the following graph.



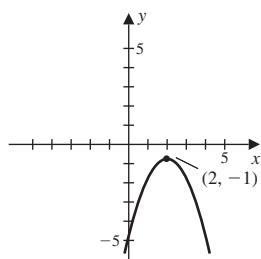
- (b) The range is $[1, \infty)$.
 (c) There are no x -intercepts. The y -intercept is $(0, 2)$.
 (d) The minimum value of y is 1 when $x = 1$.

57. (a) We have
 $y = x^2 - 4x$
 $= x^2 - 4x + 4 - 4 = (x - 2)^2 - 4$.



- (b) The range is $[-4, \infty)$.
 (c) x -intercepts: $(0, 0)$, $(4, 0)$ since
 $0 = x^2 - 4x = x(x - 4)$ implies $x = 0, x = 4$;
 y -intercept: $(0, 0)$;
 (d) A minimum occurs at the point $(2, -4)$.

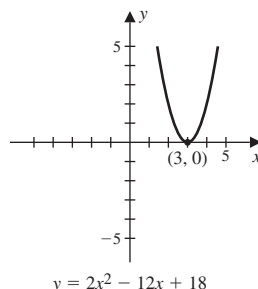
58. (a) We have $y = -x^2 + 4x - 5$
 $= -(x^2 - 4x) - 5$
 $= -(x^2 - 4x + 4 - 4) - 5 = -(x - 2)^2 - 1$.



- (b) The range is $(-\infty, -1]$.

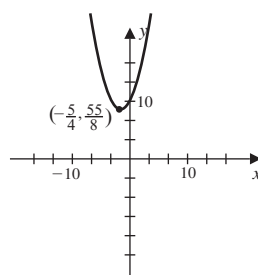
- (c) x -intercepts: none; y -intercept: $(0, -5)$;
 (d) A maximum occurs at the point $(2, -1)$.

59. (a) We have
 $y = 2x^2 - 12x + 18$
 $= 2(x^2 - 6x) + 18$
 $= 2(x^2 - 6x + 9 - 9) + 18 = 2(x - 3)^2$.



- (b) The range is $[0, \infty)$.
 (c) x -intercepts: $(3, 0)$; y -intercept: $(0, 18)$;
 (d) A minimum occurs at the point $(3, 0)$.

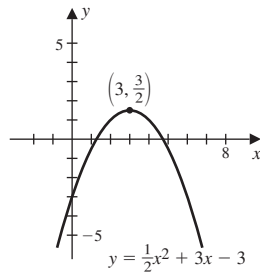
60. (a) We have
 $y = 2x^2 + 5x + 10$
 $= 2\left(x^2 + \frac{5}{2}x\right) + 10$
 $= 2\left(x^2 + \frac{5}{2}x + \frac{25}{16} - \frac{25}{16}\right) + 10$
 $= 2\left(x + \frac{5}{4}\right)^2 + \frac{55}{8}$.



- (b) The range is $\left[\frac{55}{8}, \infty\right)$.
 (c) x -intercepts: none; y -intercept: $(0, 10)$;
 (d) A minimum occurs at the point
 $\left(-\frac{5}{4}, \frac{55}{8}\right)$.

61. (a) We have
 $y = -\frac{1}{2}x^2 + 3x - 3$

$$\begin{aligned}
 &= -\frac{1}{2}(x^2 - 6x) - 3 \\
 &= -\frac{1}{2}(x^2 - 6x + 9 - 9) - 3 \\
 &= -\frac{1}{2}(x - 3)^2 + \frac{3}{2}.
 \end{aligned}$$



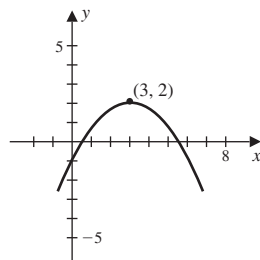
(b) The range is $\left(-\infty, \frac{3}{2}\right]$.

(c) x -intercepts: $(3 + \sqrt{3}, 0)$, $(3 - \sqrt{3}, 0)$,
 since $-\frac{1}{2}(x - 3)^2 = -\frac{3}{2}$ implies
 $(x - 3)^2 = 3$ and $x = 3 \pm \sqrt{3}$.
 y -intercept: $(0, -3)$;

(d) A maximum occurs at the point
 $\left(3, \frac{3}{2}\right)$.

62. (a) We have

$$\begin{aligned}
 y &= -\frac{1}{3}x^2 + 2x - 1 = -\frac{1}{3}(x^2 - 6x) - 1 \\
 &= -\frac{1}{3}(x^2 - 6x + 9 - 9) - 1 \\
 &= -\frac{1}{3}(x - 3)^2 + 2.
 \end{aligned}$$

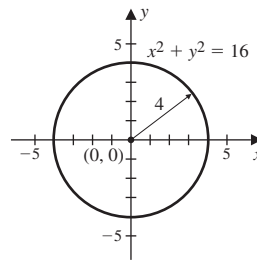


(b) The range is $(-\infty, 2]$.

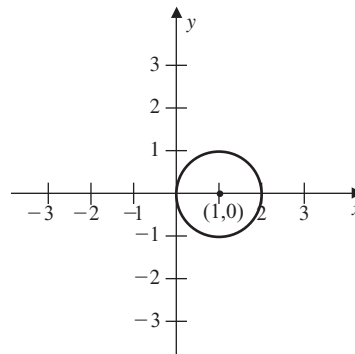
(c) x -intercepts: $(3 + \sqrt{6}, 0)$, $(3 - \sqrt{6}, 0)$,
 since $-\frac{1}{3}(x - 3)^2 = -2$ implies
 $(x - 3)^2 = 6$ so $x = 3 \pm \sqrt{6}$
 y -intercept: $(0, -1)$;

(d) A maximum occurs at the point $(3, 2)$.

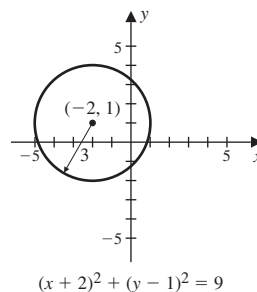
63. center: $(0, 0)$; radius: 4



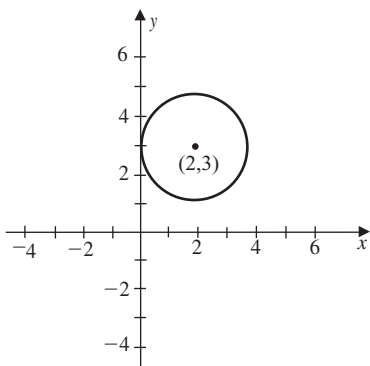
64. center: $(1, 0)$; radius: 1



65. center: $(-2, 1)$; radius: 3



66. center: $(2, 3)$; radius 2

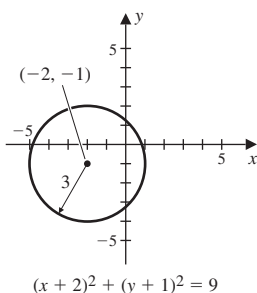


67. Completing the square on the x - and y - terms gives

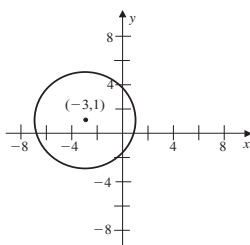
$$4 = x^2 + 4x + 4 - 4 + y^2 + 2y + 1 - 1,$$

which implies $(x + 2)^2 + (y + 1)^2 = 9$.

center: $(-2, -1)$; radius: 3



68. Completing the square on the x - and y - terms gives $0 = x^2 + 6x + 9 - 9 + y^2 - 2y + 1 - 1 - 6$, which implies $(x + 3)^2 + (y - 1)^2 = 16$. center: $(-3, 1)$; radius: 4

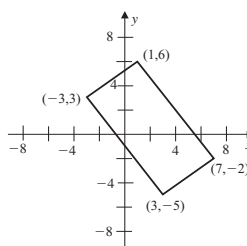


69. To show that the points $(-3, 3)$, $(3, -5)$, and $(7, -2)$ are vertices of a rectangle, show that the line passing through $(-3, 3)$ and $(3, -5)$ is perpendicular to the line passing through $(3, -5)$ and $(7, -2)$. Computing the slopes of the lines gives

$$m_1 = \frac{3 - (-5)}{-3 - 3} = -\frac{4}{3} \text{ and } m_2 =$$

$$\frac{-5 - (-2)}{3 - 7} = \frac{3}{4}.$$

Since the slopes are negative reciprocals of one another, the lines are perpendicular. To find the fourth vertex, first note that $(7, -2)$ is 4 to the right and 3 units upward from $(3, -5)$. Moving this distance from $(-3, 3)$ places the other vertex of the rectangle at $(-3 + 4, 3 + 3) = (1, 6)$.



70. The line passes through $(3, 0)$ and $(0, 2)$, so the slope is $m = \frac{2 - 0}{0 - 3} = -\frac{2}{3}$ and the equation is $y = -\frac{2}{3}x + 2$.

71. The line perpendicular to $y = 2x - 2$ and passing through the point $(-2, -1)$ has slope $m = -\frac{1}{2}$ and equation $y + 1 = -\frac{1}{2}(x + 2)$, or $y = -\frac{1}{2}x - 2$. The point of intersection of $y = 2x - 2$ and $y = -\frac{1}{2}x - 2$ is found from the equation $2x - 2 = -\frac{1}{2}x - 2$, so $\frac{5}{2}x = 0$ and $x = 0, y = -2$. Then
- $$d((-2, -1), (0, -2)) = \sqrt{(-2 - 0)^2 + (-1 - (-2))^2} = \sqrt{5}.$$

72. A radius for the circle is the line segment between $(-3, -1)$ and $(-5, -3)$, so the radius is $r = \sqrt{(-3 - (-5))^2 + (-1 - (-3))^2} = \sqrt{4 + 4} = \sqrt{8}$. The equation of the circle is $(x + 3)^2 + (y + 1)^2 = 8$.

73. If the circle has center $(4, 2)$ and is tangent to the x -axis, then the circle also passes through $(2, 0)$. So the radius is 2 and the equation is $(x - 4)^2 + (y - 2)^2 = 4$.

74. We have

$$\begin{aligned} 0 &= x^2 - 4x + y^2 - 2y - 1 \\ &= x^2 - 4x + 4 - 4 + y^2 - 2y + 1 - 1 - 1 \end{aligned}$$

$$\text{so } (x-2)^2 + (y-1)^2 = 6.$$

The circle has center $(2, 1)$ so the line passing through $(2, 1)$ and $(3, 4)$ has slope

$$m = \frac{4-1}{3-2} = 3 \text{ and equation}$$

$$y-1 = 3(x-2) \text{ implying } y = 3x-5.$$

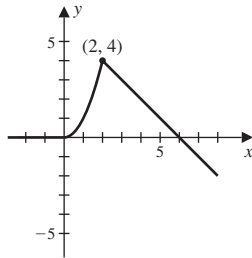
75. We need to find $(x, 0)$ so that

$$\begin{aligned} &\sqrt{(x-(-2))^2 + (0-(-3))^2} \\ &= \sqrt{(x-3)^2 + (0-5)^2} \text{ so} \\ &\sqrt{(x+2)^2 + 9} = \sqrt{(x-3)^2 + 25} \text{ and} \\ &x^2 + 4x + 13 = x^2 - 6x + 34. \end{aligned}$$

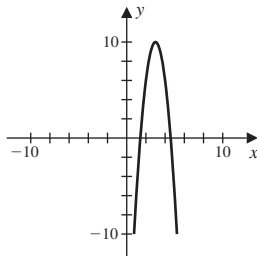
$$\text{This reduces to } 10x = 21, \text{ so } x = \frac{21}{10}, \text{ and the}$$

$$\text{point is } \left(\frac{21}{10}, 0\right).$$

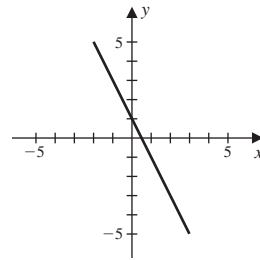
76. The graph of $y = g(x)$ is shown.



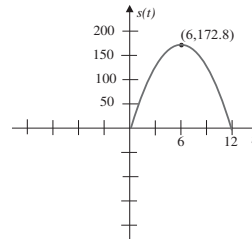
77. (a) The data points appear to lie on a parabola with vertex $(3, 10)$ that opens downward and has been vertically elongated by a factor of 4. This implies that the equation of the parabola is $y = -4(x-3)^2 + 10$.



(b) For each single unit change in x , the change in y is -2 , so the data points represent a line with slope -2 . The line passes through the point $(0, 1)$, so the equation is $y-1 = -2(x-0)$, that is, $y = -2x + 1$.



78. (a) From the graph of $s(t) = -4.8t^2 + 57.6t$ the maximum height is approximately 172.8 meters, occurring when $t = 6$ seconds.



(b) We need to solve $-4.8t^2 + 57.6t \geq 120$ so $0 \leq -4.8t^2 + 57.6t - 120 = -4.8(t^2 - 12t + 25)$, which implies that $t^2 - 12t + 25 \leq 0$. Solving $t^2 - 12t + 25 = 0$ gives

$$t = \frac{12 \pm \sqrt{144 - 4(1)(25)}}{2}$$

$$= \frac{12 \pm \sqrt{44}}{2}$$

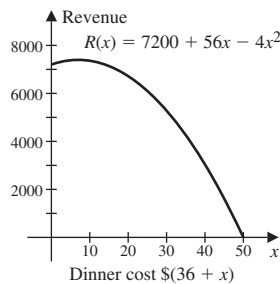
$$= \frac{12 \pm 2\sqrt{11}}{2} = 6 \pm \sqrt{11}.$$

$$\text{So } 2.7 \approx 6 - \sqrt{11} \leq t \leq 6 + \sqrt{11} \approx 9.3.$$

(c) The ball strikes the ground when $0 = -4.8t^2 + 57.6t = -4.8t(t-12)$ so $t = 0, t = 12$. Since $t > 0$, we have $t = 12$.

79. Let x denote the price over \$36.00 for a complete dinner and let $P(x)$ denote the amount

taken in by the restaurant, so that $P(x) = (36 + x)(200 - 4x) = 7200 + 56x - 4x^2$. From the graph of $P(x) = 7200 + 56x - 4x^2$ we see that the maximum return occurs at approximately $x = 7$. So the owner should charge $36 + 7 = \$43.00$ per dinner. By completing the square on the quadratic we have $-4x^2 + 56x + 7200 = -4(x^2 - 14x) + 7200 = -4(x^2 - 14x + 49 - 49) + 7200 = -4(x - 7)^2 + 7396$, so the exact maximum return is when $x = 7$.



80. The cost per day to rent from agency A is $21 + 0.21x$ dollars and from agency B is $32 + 0.18x$ dollars, where x is the number of miles the car is driven.

- (a) If the car is driven 320 miles, the cost from agency A is $21 + 0.21(320) = \$88.20$ and the cost from agency B is $32 + 0.18(320) = \$89.60$. This implies that a driver should rent from agency A .
- (b) Since the slope of the line describing agency A 's cost is greater than the slope of the line describing agency B 's cost, it will eventually overtake the line with the smaller positive slope. The point of intersection determines the number of miles driven where the cost to both agencies is the same and for any number of miles greater agency B 's cost is less. So solve $21 + 0.21x = 32 + 0.18x$ which implies that $0.03x = 11$ and $x = \frac{11}{0.03} \approx 367$ miles.

81. (a) The amount of fence required is the perimeter of the plot plus the length of fence down the middle. If ℓ and w are

the length and width of the plot, then the amount of fence, F , is $F = 3\ell + 2w$, where the assumption is the length of the middle section of fence is ℓ . Since the area of the plot is 432, we have $432 = \ell w$. This implies that $w = \frac{432}{\ell}$ and that

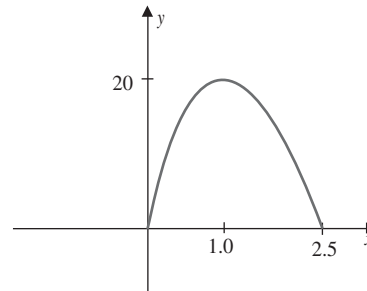
$$F(\ell) = 3\ell + 2 \cdot \frac{432}{\ell} = 3\ell + \frac{864}{\ell}.$$

- (b) The domain of the function is $(0, \infty)$.

82. The volume of the box is

$$V(x) = x(5 - 2x)(9 - 2x) = 45x - 28x^2 + 4x^3.$$

The graph indicates that the box of maximum volume is constructed when a square of about 1 centimeter is removed from each corner of the sheet of cardboard.

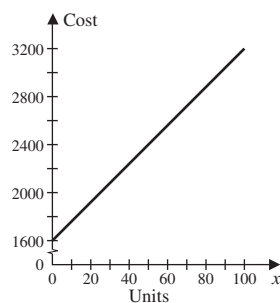


83. (a) If the relationship between cost and number of units produced per day is linear, then the points $(100, 3200)$ and $(4024, 9600)$ are on the line. The slope of the line is

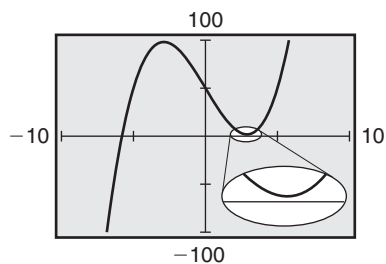
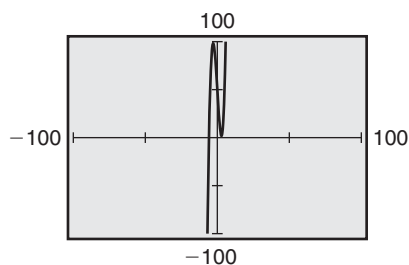
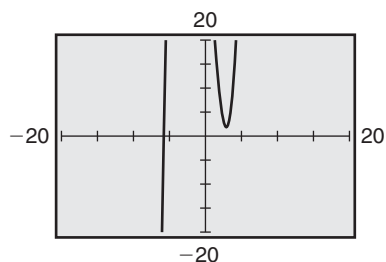
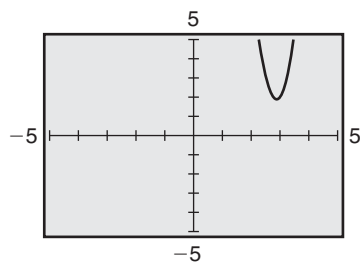
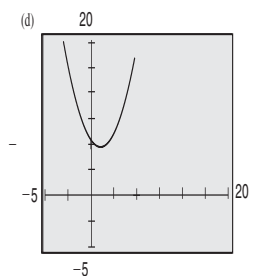
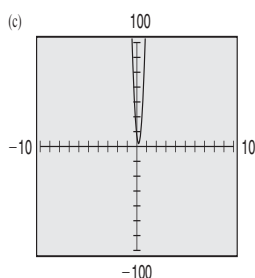
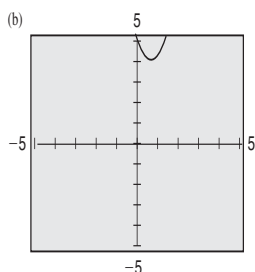
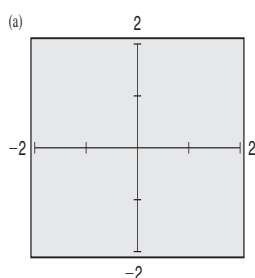
$$m = \frac{9600 - 3200}{4024 - 100} = \frac{6400}{400} = 16$$

and the equation of the line is $y - 3200 = 16(x - 100) = 16x - 1600$. Hence $y = 16x + 1600$.

- (b) The slope of line indicates that for each one unit increase in the number of units produced, results in an increase of \$16.00 in the cost.
- (c) The y -intercept is 1600 and represents the fixed costs of production.

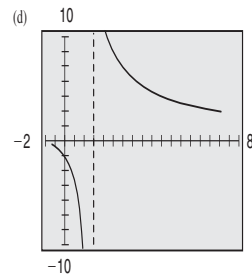
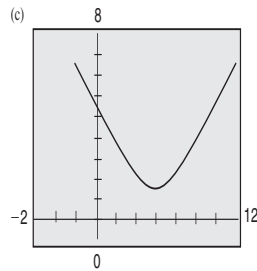
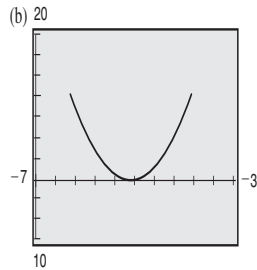
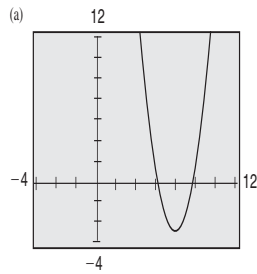


84. Part (d) gives the best representation.



85. Part (d) gives the best representation.

86. (a) $y = x^2 - 16x + 61$
 (b) $y = x^2 + 10x + 38$
 (c) $y = \sqrt{x^2 - 10x + 29}$
 (d) $y = \frac{x + 4}{x - 3}$



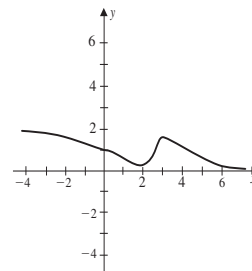
87. (a) Between A and B and to the right of E .

(b) Between C and D .

(c) Between B and C .

(d) Between D and E .

88. This graph has the required properties.



Chapter 1 Exercises for Calculus (Page 79)

1. For each increase in x of 4 units, the y -coordinate $y = g(x)$ always increases by 12 units, so g is linear and f is quadratic. The slope of the line $y = g(x)$ is $m = \frac{12}{4} = 3$ and the line passes through $(-2, 2)$, so $y - 2 = 3(x + 2)$ implies $y = g(x) = 3x + 8$. The quadratic $y = f(x) = ax^2$ and passes through $(-2, 2)$, so $2 = f(-2) = 4a$, which implies $a = \frac{1}{2}$ and $f(x) = \frac{1}{2}x^2$.

2. Let $3x + ay = -2a$.

(a) $ay = -3x - 2a$ implies $y = -\frac{3}{a}x - 2$. If the slope is 2, then $-\frac{3}{a} = 2$ so $a = -\frac{3}{2}$.

(b) A line is horizontal provided the slope is 0, but a can not be zero, so there are no values of a .

(c) The line is vertical provided the slope is undefined, which holds when $a = 0$.

- (d) If the line passes through $(0, -2)$, then $3(0) + a(-2) = -2a$ implies $-2a = -2a$, which is a true statement for all values of a . The line passes through $(0, -2)$ for all $a \neq 0$.

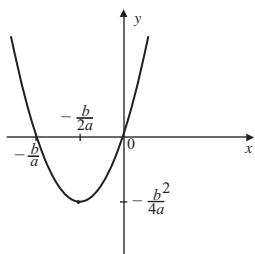
3. (a) The table shows that the solution is $14 \leq n \leq 16$.

n	12	13	14	15	16	17
$\frac{n(n+1)}{2}$	78	91	105	120	136	153

- (b) The table shows that the solution is $8 \leq n \leq 9$.

n	5	6	7	8	9	10
$\frac{n(n+1)(2n+1)}{6}$	55	91	140	204	285	385

4. (a) The graph of $y = ax^2 + bx$ has x -intercepts at $x = 0$ and $x = -b/a$, and a vertex at $(-b/(2a), -b^2/(4a))$.



- (b) To show that $y = ax^2 + bx$ is symmetric about the line $x = -\frac{b}{2a}$, select values equidistant on either side of the line and show they yield the same y -coordinate for the corresponding point on the curve. Let $z > 0$. Then

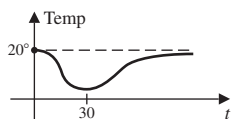
$$a\left(-\frac{b}{2a} + z\right)^2 + b\left(-\frac{b}{2a} + z\right) = a\left(\frac{b^2}{4a^2} - \frac{b}{a}z + z^2\right) - \frac{b^2}{2a} + bz = az^2 + \frac{b^2}{4a} - \frac{b^2}{2a} = az^2 - \frac{b^2}{4a}$$

$$a\left(-\frac{b}{2a} - z\right)^2 + b\left(-\frac{b}{2a} - z\right) = a\left(\frac{b^2}{4a^2} + \frac{b}{a}z + z^2\right) - \frac{b^2}{2a} - bz = az^2 + \frac{b^2}{4a} - \frac{b^2}{2a} = az^2 - \frac{b^2}{4a},$$

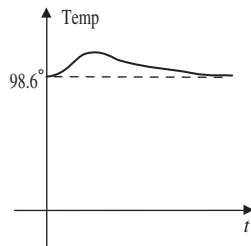
so the graph is symmetric about the vertical line $y = -\frac{b}{2a}$.

- (c) Since the graph of $y = ax^2 + bx + c$ is simply a vertical shift of the graph of $y = ax^2 + bx$, the result in part (b) implies that it also is symmetric about the vertical line $x = -\frac{b}{2a}$.

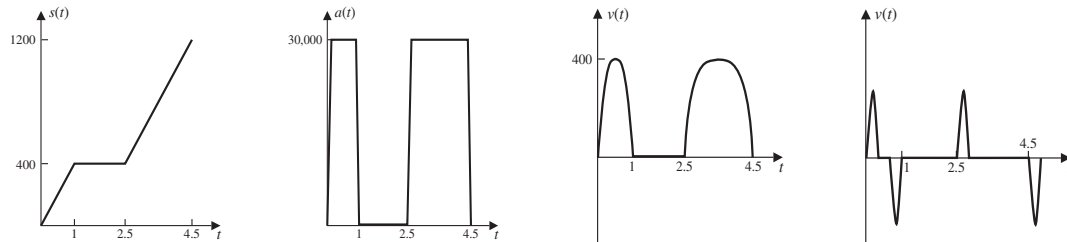
5. The graph of the temperature might be as follows.



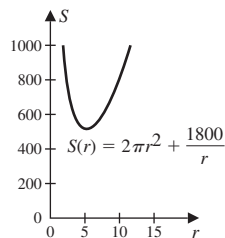
6. The graph of the child's temperature might be as follows.



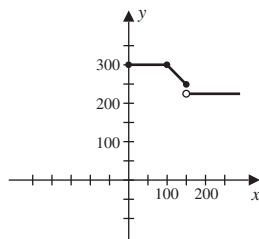
7. Graphs for the airplane are shown below.



8. (a) The surface area of the can is the sum of the area of the top and bottom plus the area of the walls. Hence the surface area is $S = 2\pi r^2 + 2\pi r h$. Since the volume of the can is 900 centimeters³, we have $900 = V = \pi r^2 h \Rightarrow h = \frac{900}{\pi r^2}$. So $S = 2\pi r^2 + 2\pi r \left(\frac{900}{\pi r^2} \right) = 2\pi r^2 + \frac{1800}{r}$.
- (b) From the graph we see that the radius of the can that yields the minimum surface area is approximately $r = 5.2$ centimeters. The height is then $h = \frac{900}{\pi(5.2)^2} \approx 10.6$ centimeters.



9. (a) Let x be the number purchased. Then the price per item is
- $$P(x) = \begin{cases} 300, & \text{if } 0 \leq x \leq 100, \\ 300 - (x - 100) = 400 - x, & \text{if } 100 < x \leq 150, \\ 225, & \text{if } 150 < x. \end{cases}$$
- (b) The graph of the price per item is given below.



10. (a) Let x be the number of trees planted per acre. Then the yield of a one acre plot is
- $$Y(x) = \begin{cases} 300x, & \text{if } 0 \leq x \leq 30 \\ (300 - 5(x - 30))x, & \text{if } 30 < x \leq 90 \end{cases} = \begin{cases} 300x, & \text{if } 0 \leq x \leq 30 \\ 450x - 5x^2, & \text{if } 30 < x \leq 90 \end{cases}$$
- The bound of 90 is given since if 90 trees are planted, there will be no fruit per tree.
- (b) The maximum yield occurs when $x = 45$ and $Y(45) = 450(45) - 5(45)^2 = 10,125$ lb.
11. (a) The length of the rod at a temperature t above 0° C, is $L(t) = \ell + alt$, where ℓ is the original length of the rod and $a = 11 \times 10^{-6}$ is the coefficient of linear expansion. Since the length of the rod is 2 meters at 0° C, the length of the rod is $L(t) = 2 + (22 \times 10^{-6})t$.
- (b) $L(1000) = 2 + (22 \times 10^{-6})(1000) = 2 + 22 \times 10^{-3}$.
12. (a) The slope of the radius is $\frac{\sqrt{3}/2}{1/2} = \sqrt{3}$. The tangent line to the point is perpendicular to the radius through the point, so the slope of the tangent line is $-\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$. The equation of the tangent line is $y - \frac{\sqrt{3}}{2} = -\frac{\sqrt{3}}{3} \left(x - \frac{1}{2} \right)$ so $y = -\frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{6} + \frac{\sqrt{3}}{2} = -\frac{\sqrt{3}}{3}x + \frac{2\sqrt{3}}{3}$.
- (b) For $y = f(x) = \sqrt{1 - x^2}$, we have
- $$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{\sqrt{1 - (x+h)^2} - \sqrt{1 - x^2}}{h} \\ &= \frac{\sqrt{1 - (x+h)^2} - \sqrt{1 - x^2}}{h} \left(\frac{\sqrt{1 - (x+h)^2} + \sqrt{1 - x^2}}{\sqrt{1 - (x+h)^2} + \sqrt{1 - x^2}} \right) \\ &= \frac{-h(2x+h)}{h(\sqrt{1 - (x+h)^2} + \sqrt{1 - x^2})}. \end{aligned}$$
- As $h \rightarrow 0$ this reduces to $\frac{-2x}{2\sqrt{1 - x^2}} = -\frac{x}{\sqrt{1 - x^2}}$ and when
- $x = 1/2$ $-\frac{1/2}{\sqrt{1 - 1/4}} = -\frac{1}{2\sqrt{3/4}} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$. This value agrees, of course, with the slope of the tangent line found in part (a).

Chapter 1 Chapter Test (Page 81)

1. False. Since $3x - 2 = 4$ implies $3x = 6$, the solution is $x = 2$.
2. False. Since $x^2 - 3x + 2 = (x - 1)(x - 2)$, the solutions are $x = 2$ and $x = 1$.
3. True.
4. True.
5. True.
6. True.
7. False. Another solution is $x = 2$.
8. True.
9. True.
10. True.
11. False. Since $x - 3 \geq 0$ implies $x \geq 3$, the domain is $[3, \infty)$.
12. False. Since the denominator cannot be 0, $x = 3$ must be excluded, resulting in the domain $(3, \infty)$.
13. True.
14. True.
15. False. The center of the circle $(x - 2)^2 + y^2 = 4$ is $(2, 0)$, but the radius $\sqrt{4} = 2$.
16. False. Completing the square on x and on y gives

$$x^2 + 2x + 1 + y^2 - 4y + 4 = 4 + 1 + 4$$
 or

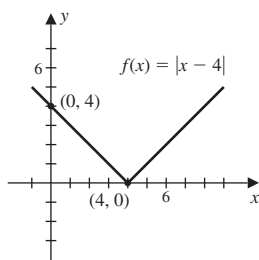
$$(x + 1)^2 + (y - 2)^2 = 9.$$
 This is the equation of a circle with radius 3, but the center is at $(-1, 2)$.
17. False. Since $y = \frac{2}{3}x - \frac{4}{3}$, the slope of the line is $2/3$.
18. False. The lines $x + y = 2$ and $3x - 2y = 1$ intersect at the point $(1, 1)$.
19. False. Since $-3x + 2y = 5$ has slope $3/2$ and $4y = 6x + 7$ has slope $3/2$, the slopes are the same and the lines are parallel.
20. False. Since $x - 3y = 3$ has slope $1/3$ and $4x - 6y = 5$ has slope $2/3$, the slopes differ and the lines are not parallel.
21. False. Since $2x + y = 2$ has slope -2 and $2y + x = -1$ has slope $-1/2$, the products of the slopes $(-2)(-1/2) = 1 \neq -1$, so the lines are not perpendicular.
22. False. Since $x + 2y = 1$ has slope $-1/2$ and $-2x + y = 3$ has slope 2 , the slopes differ and the lines are not parallel.
23. False. The equation of the line that has slope -3 and passes through the point $(0, 1)$ is $y - 1 = -3(x - 0)$, or $y = -3x + 1$.
24. False. The equation of the line that passes through the two points $(2, 1)$ and $(-3, 2)$ has slope $(2 - 1)/(-3 - 2) = -1/5$ and equation

$$-\frac{1}{5} = \frac{y - 1}{x - 2}$$
 which simplifies to $5y + x = 7$.
25. False. Since $y = 1$ is not included, the shaded region is described by $y < 1$.
26. False. Since $y = 1$ is included, the region outside the shaded region is described by $y \geq 1$.
27. True.
28. True.
29. True.
30. True.
31. True.
32. False. The parabola $y = -(x + 1)^2 - 2$ has a maximum point at $(-1, -2)$.
33. True.
34. True.

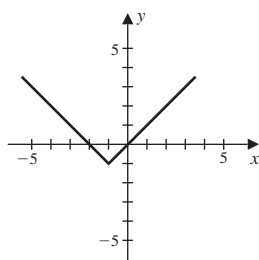
35. True.
36. True.
37. False. The graph of an even function is symmetric with respect to the y -axis and the graph of an odd function is symmetric with respect to the origin.
38. False. The graph of $y = -f(x)$ is the reflection of the graph of $y = f(x)$ about the x -axis.
39. False. A curve will describe the graph of a function provided every vertical line crosses the curve at most one time.
40. False. The graph of $y = (x - 1)^2 + 2$ is obtained by shifting the graph of $y = x^2$ to the right 1 unit and upward 2 units.
41. True.
42. True.
43. True.
44. True.
45. True.

Exercise Set 2.2 (Page 91)

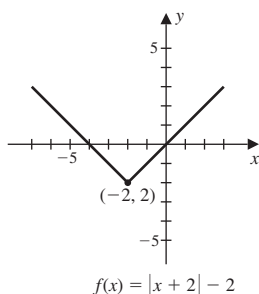
1. $f(x) = |x - 4|$



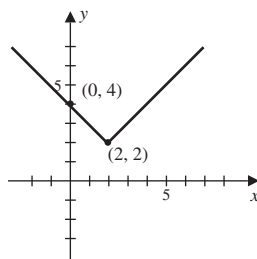
2. $f(x) = |x + 1| - 1$



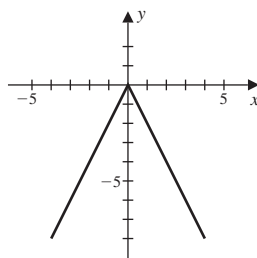
3. $f(x) = |x + 2| - 2$



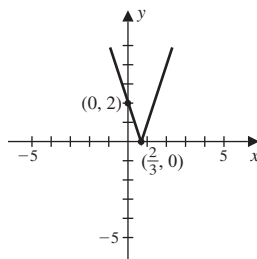
4. $f(x) = |x - 2| + 2$



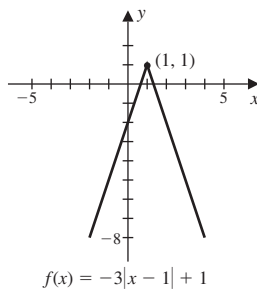
5. $f(x) = -2|x|$



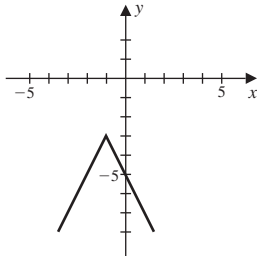
6. $f(x) = |3x - 2| = \left| 3 \left(x - \frac{2}{3} \right) \right| = 3 \left| x - \frac{2}{3} \right|$



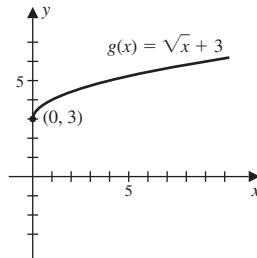
7. $f(x) = -3|x - 1| + 1$



8. $f(x) = -2|x + 1| - 3$

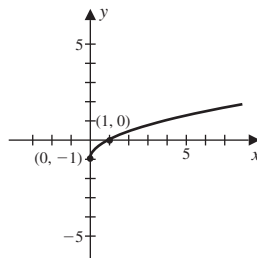


9. (a) $g(x) = \sqrt{x} + 3$



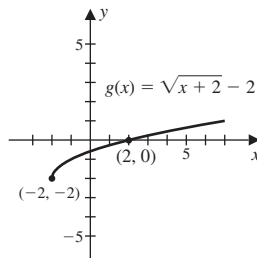
- (b) Domain: $g(x) = \sqrt{x} + 3$ is defined when $x \geq 0$, so the domain is $[0, \infty)$.
Range: Since $\sqrt{x} + 3 \geq 3$, the range is $[3, \infty)$.

10. (a) $g(x) = \sqrt{x} - 1$



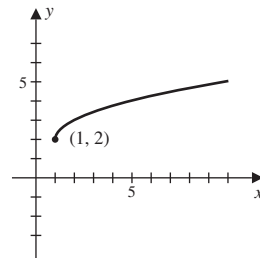
- (b) Domain: $g(x) = \sqrt{x} - 1$ is defined when $x \geq 0$, so the domain is $[0, \infty)$.
Range: Since $\sqrt{x} - 1 \geq -1$, the range is $[-1, \infty)$.

11. (a) $g(x) = \sqrt{x+2} - 2$



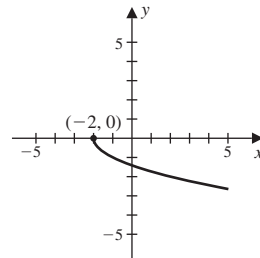
- (b) Domain: $g(x) = \sqrt{x+2} - 2$ is defined when $x+2 \geq 0$, so $x \geq -2$ and the domain is $[-2, \infty)$.
Range: Since $\sqrt{x+2} - 2 \geq -2$, the range is $[-2, \infty)$.

12. (a) $g(x) = \sqrt{x-1} + 2$



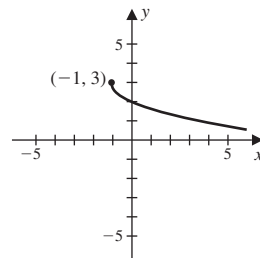
- (b) Domain: $g(x) = \sqrt{x-1} + 2$ is defined when $x-1 \geq 0$, so $x \geq 1$ and the domain is $[1, \infty)$. Range: Since $\sqrt{x-1} + 2 \geq 2$, the range is $[2, \infty)$.

13. (a) $g(x) = -\sqrt{x+2}$



- (b) Domain: $g(x) = -\sqrt{x+2}$ is defined when $x+2 \geq 0$, so $x \geq -2$ and the domain is $[-2, \infty)$.
Range: Since $-\sqrt{x+2} \leq 0$, the range is $(-\infty, 0]$.

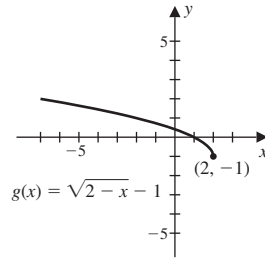
14. (a) $g(x) = 3 - \sqrt{x+3}$



- (b) Domain: $g(x) = 3 - \sqrt{x+3}$ is defined when $x+3 \geq 0$, so $x \geq -3$ and the domain is $[-3, \infty)$.

Range: Since $3 - \sqrt{x+3} \leq 3$, the range is $(-\infty, 3]$.

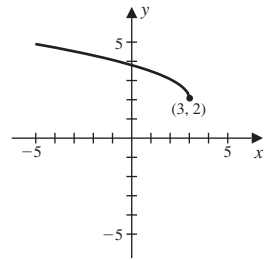
15. (a) $g(x) = \sqrt{2-x} - 1 = \sqrt{-(x-2)} - 1$



- (b) Domain: $g(x) = \sqrt{2-x} - 1$ is defined when $2-x \geq 0$, that is, $x-2 \leq 0$, so $x \leq 2$ and the domain is $(-\infty, 2]$.

Range: Since $\sqrt{2-x} - 1 \geq -1$, the range is $[-1, \infty)$.

16. (a) $g(x) = \sqrt{3-x} + 2 = \sqrt{-(x-3)} + 2$

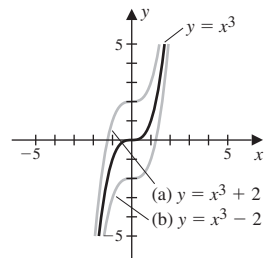


- (b) Domain: $g(x) = \sqrt{3-x} + 2$ is defined when $3-x \geq 0$, that is, $x-3 \leq 0$, so $x \leq 3$ and the domain is $(-\infty, 3]$.

Range: Since $\sqrt{3-x} + 2 \geq 2$, the range is $[2, \infty)$.

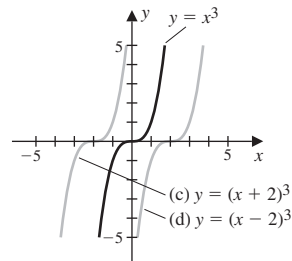
17. (a) $g(x) = x^3 + 2$

(b) $g(x) = x^3 - 2$



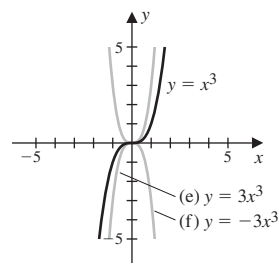
(c) $g(x) = (x+2)^3$

(d) $g(x) = (x-2)^3$

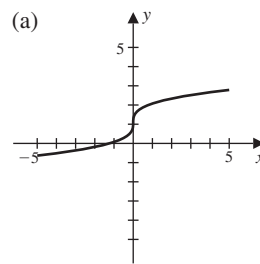


(e) $g(x) = 3x^3$

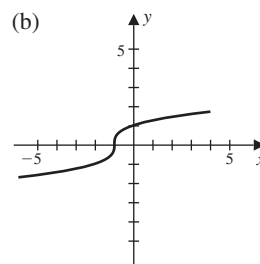
(f) $g(x) = -3x^3$



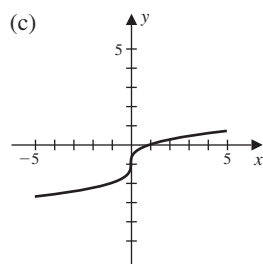
18. (a) $g(x) = \sqrt[3]{x} + 1$



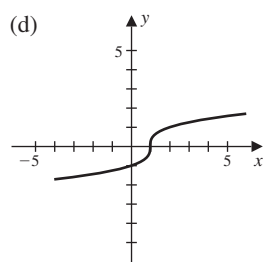
(b) $g(x) = \sqrt[3]{x+1}$



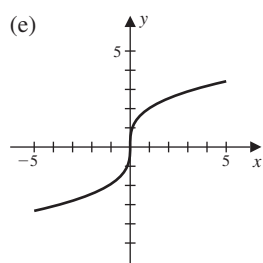
(c) $g(x) = \sqrt[3]{x} - 1$



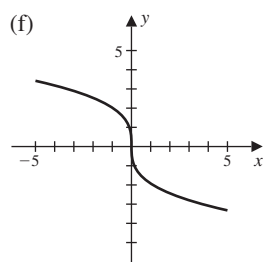
(d) $g(x) = \sqrt[3]{x-1}$



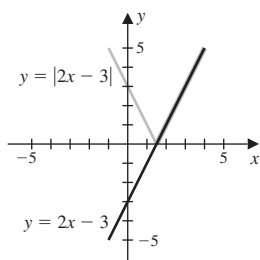
(e) $g(x) = 2\sqrt[3]{x}$



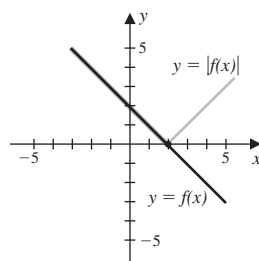
(f) $g(x) = -2\sqrt[3]{x}$



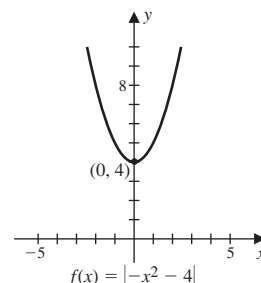
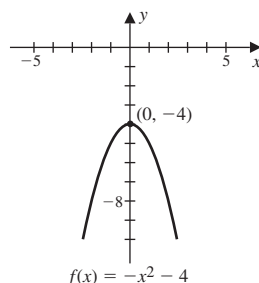
19. $f(x) = 2x - 3$



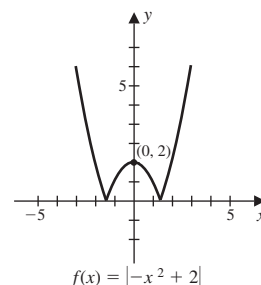
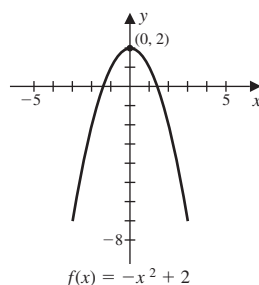
20. $f(x) = -x + 2$



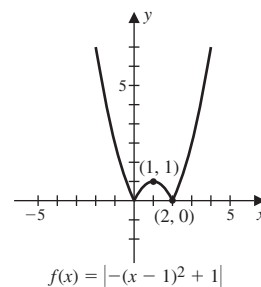
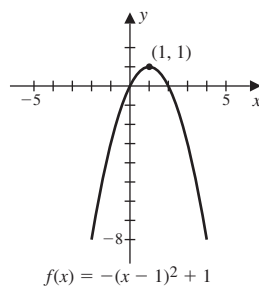
21. $f(x) = -x^2 - 4$



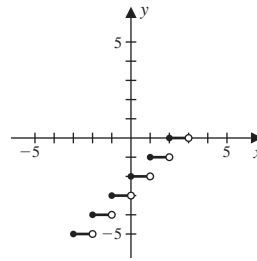
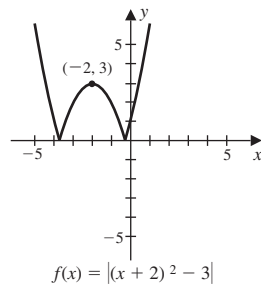
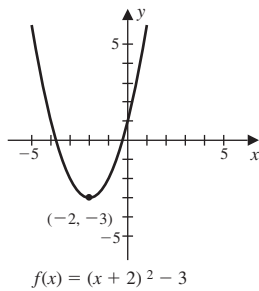
22. $f(x) = -x^2 + 2$



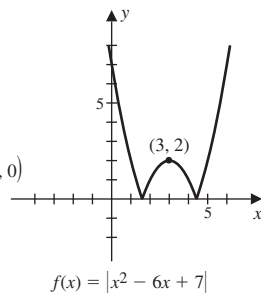
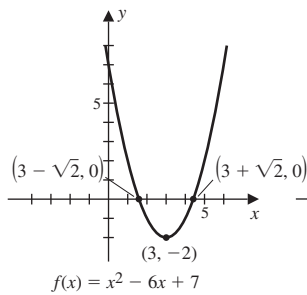
23. $f(x) = -(x-1)^2 + 1$



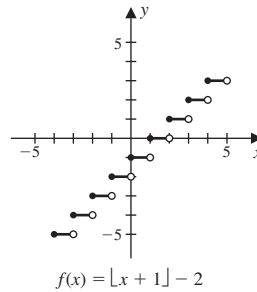
24. $f(x) = (x+2)^2 - 3$



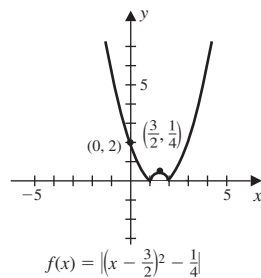
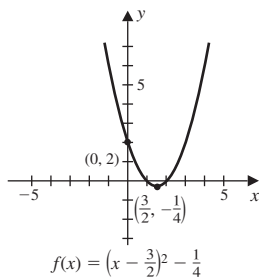
$$\begin{aligned} 25. \quad f(x) &= x^2 - 6x + 7 \\ &= x^2 - 6x + 9 - 9 + 7 \\ &= (x - 3)^2 - 2 \end{aligned}$$



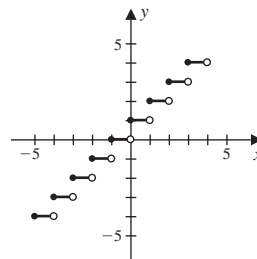
$$29. \quad f(x) = \lfloor x + 1 \rfloor - 2$$



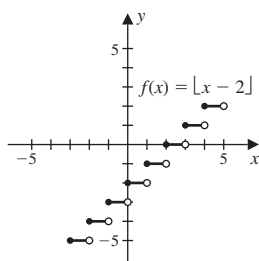
$$\begin{aligned} 26. \quad f(x) &= x^2 - 3x + 2 \\ &= x^2 - 3x + \frac{9}{4} - \frac{9}{4} + 2 \\ &= \left(x - \frac{3}{2}\right)^2 - \frac{1}{4} \end{aligned}$$



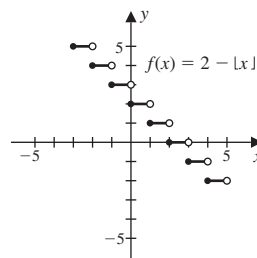
$$30. \quad f(x) = \lfloor x - 1 \rfloor + 2$$



$$27. \quad f(x) = \lfloor x - 2 \rfloor$$

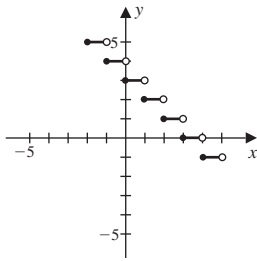


$$31. \quad f(x) = 2 - \lfloor x \rfloor$$

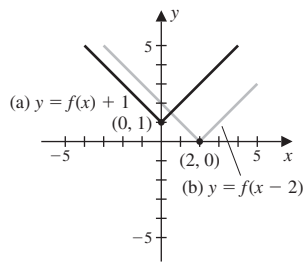


$$28. \quad f(x) = \lfloor x \rfloor - 2$$

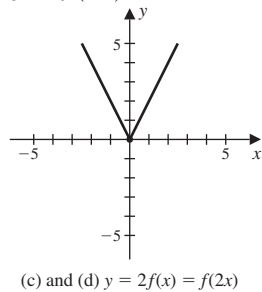
$$32. \quad f(x) = 2 - \lfloor x - 1 \rfloor$$



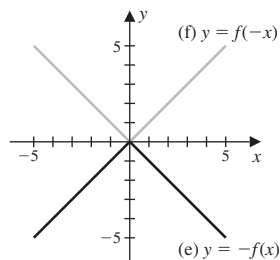
33. (a) $y = f(x) + 1$
(b) $y = f(x - 2)$



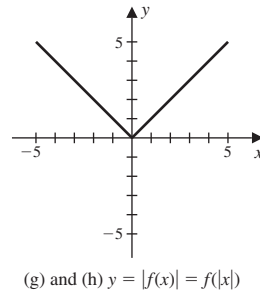
- (c) $y = 2f(x)$
(d) $y = f(2x)$



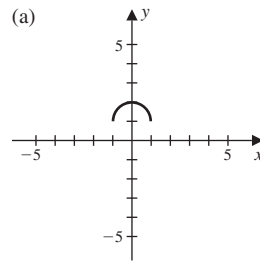
- (e) $y = -f(x)$
(f) $y = f(-x)$



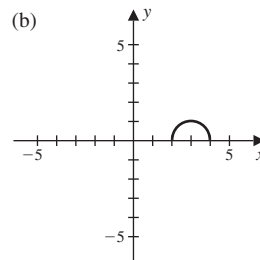
- (g) $y = |f(x)|$
(h) $y = f(|x|)$



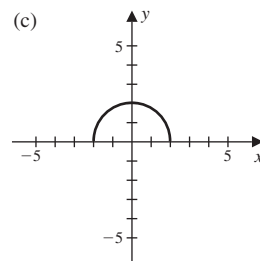
34. (a) $y = f(x) + 1$



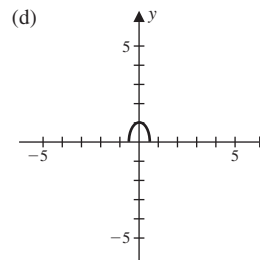
- (b) $y = f(x - 2)$



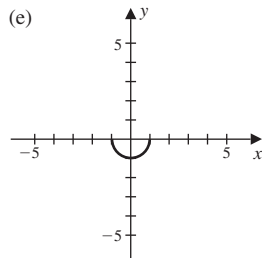
- (c) $y = 2f(x)$



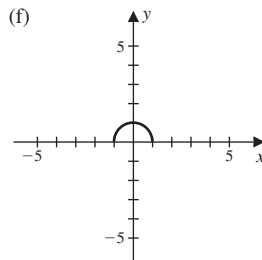
- (d) $y = f(2x)$



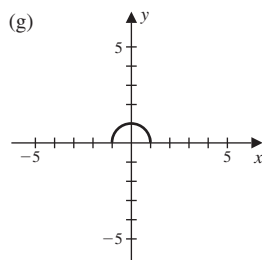
(e) $y = -f(x)$



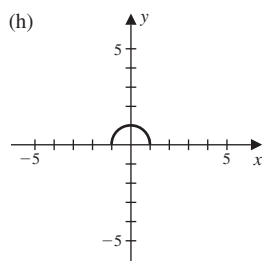
(f) $y = f(-x)$



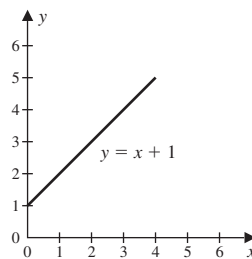
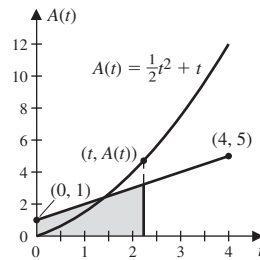
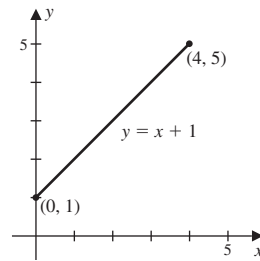
(g) $y = |f(x)|$



(h) $y = f(|x|)$



35. (a) The graphs of parts (a), (b), and (c) are shown in left, center, and right, respectively.

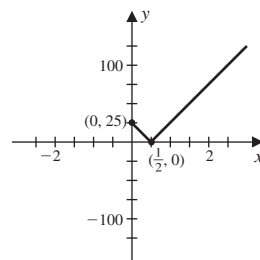


- (b) The area under the curve is the area of a trapezoid, which implies

$$A(t) = \frac{1}{2}(1 + (t + 1)) \cdot t = t + \frac{1}{2}t^2.$$

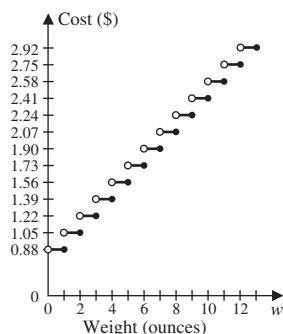
(c) $d(x) = d((0, 0), (x, f(x)))$
 $= d((0, 0), (x, x + 1))$
 $= \sqrt{(x - 0)^2 + (x + 1 - 0)^2}$
 $= \sqrt{x^2 + x^2 + 2x + 1}$
 $= \sqrt{2x^2 + 2x + 1}$

36. (a) The graph of the distance from C is shown in the figure.



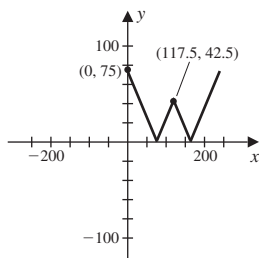
- (b) The line on the interval $\left[0, \frac{1}{2}\right]$ passes through the points $(0, 25)$ and $\left(\frac{1}{2}, 0\right)$ so the slope is $m = \frac{25}{-\frac{1}{2}} = -50$. The equation of the line is $y = -50x + 25$. On the interval $\left[\frac{1}{2}, 2\right]$ the line passes through $\left(\frac{1}{2}, 0\right)$ and $(2, 75)$ so the slope is $m = \frac{75}{\frac{3}{2}} = 50$. The equation of the line is $y = 50\left(x - \frac{1}{2}\right) = 50x - 25$. The distance can then be expressed as $|50x - 25|$.

37. Domain(f) = $(0, 13]$;
Range(f) = $\{0.88, 1.05, 1.22, \dots, 2.92\}$
 = $\{0.88 + 0.17, n = 0, 1, \dots, 13\}$



$$38. f(x) = \begin{cases} -x + 75, & \text{if } 0 \leq x \leq 75, \\ x - 75, & \text{if } 75 \leq x \leq 117.5, \\ -x + 160, & \text{if } 117.5 \leq x \leq 160, \\ x - 160, & \text{if } 160 \leq x \leq 241. \end{cases}$$

$$= \begin{cases} |x - 75|, & \text{if } 0 \leq x \leq 117.5, \\ |x - 160|, & \text{if } 117.5 \leq x \leq 214. \end{cases}$$



39. (a) For approximating $\sqrt{2}$ start with $a = 1.4$. The table gives four iterations.

a	$b = \frac{1}{2}\left(a + \frac{2}{a}\right)$
1.4	1.414285714
1.414285714	1.414213564
1.414213564	1.414213562
1.414213562	1.414213562

So $\sqrt{2} \approx 1.414313562 \approx 1.414$.

- (b) For approximating $\sqrt{13}$ start with $a = 3.5$. The table gives four iterations.

a	$b = \frac{1}{2}\left(a + \frac{13}{a}\right)$
3.5	3.607142857
3.607142857	3.605551627
3.605551627	3.605551276
3.605551276	3.605551276

So $\sqrt{13} \approx 3.605551276 \approx 3.606$.

Notice that the number of decimal places of accuracy approximately doubles with each iteration of the technique. This provides an extremely efficient technique for determining square roots.

40. Let n be the unique integer satisfying $n \leq x < n + 1$. Then $n + 1 \leq x + 1 < n + 2$ and $n + 3 \leq x + 3 < n + 4$, so $\lfloor x + 3 \rfloor - 2 = n + 3 - 2 = n + 1 = \lfloor x + 1 \rfloor$.
41. (a) Let $f_0(x) = \lfloor x + 0.5 \rfloor$ and n be an integer.
If $n \leq x < n + 0.5$, then $n + 0.5 \leq x + 0.5 < n + 1$, so $f_0(x) = n$.
If $n + 0.5 \leq x < n + 1$, then $n + 1 \leq x + 0.5 < n + 1.5$, so $f_0(x) = n + 1$.
- (b) Let $f_1(x) = (0.1)\lfloor 10x + 0.5 \rfloor$ and n and m be integers, with $0 \leq m \leq 9$. If $n + m/10 \leq x < n + m/10 + 0.05$, then $10n + m \leq 10x < 10n + m + 0.5$, so

$$10n+m \leq 10x+0.5 < 10n+m+1 \quad \text{and} \\ \lfloor 10x+0.5 \rfloor = 10n+m. \text{ If}$$

$$n+m/10+0.05 \leq x \\ < n+(m+1)/10.$$

$$\text{then } 10n+m+0.5 \leq 10x < 10n+m+1, \\ \text{so}$$

$$10n+m+1 \leq 10x+0.5 \\ < 10n+m+1.5$$

$$\text{and } \lfloor 10x+0.5 \rfloor = 10n+m+1.$$

Hence, in either case,

$$f_1(x) = \frac{1}{10} \lfloor 10x+0.5 \rfloor \text{ rounds to the} \\ \text{nearest decimal point.}$$

- (c) Let $f_n(x) = (0.1)^n \lfloor 10^n x + 0.5 \rfloor$. By an argument similar to that in (b), this will round an real number to n decimal places.

42. (a) Let $f_0(x) = \lfloor x - 0.5 \rfloor$ and n be an integer. If

$$n \leq x \leq n+0.5, \text{ then} \\ n-0.5 \leq x-0.5 \leq n, \text{ so } f_0(x) = n.$$

$$\text{If } n+0.5 < x < n+1, \text{ then} \\ n < x-0.5 < n+0.5, \text{ so } f_0(x) = n+1.$$

- (b) Let $f_1(x) = (0.1) \lfloor 10x - 0.5 \rfloor$ and n and m be integers, with $0 \leq m \leq 9$. If

$$n+m/10 \leq x \leq n+m/10+0.05, \quad \text{then} \\ 10n+m \leq 10x \leq 10n+m+0.5, \text{ so}$$

$$10n+m-0.5 \leq 10x-0.5 \\ \leq 10n+m$$

$$\text{and } \lfloor 10x-0.5 \rfloor = 10n+m. \text{ If}$$

$$n+m/10+0.05 < x \\ < n+(m+1)/10,$$

$$\text{then } 10n+m < 10x < 10n+m+0.5, \\ \text{so}$$

$$10n+m < 10x+0.5 \\ < 10n+m+0.5$$

$$\text{and } \lfloor 10x+0.5 \rfloor = 10n+m+1.$$

Hence, in either case,

$$f_1(x) = \frac{1}{10} \lfloor 10x-0.5 \rfloor \text{ rounds to the} \\ \text{nearest decimal point, with the mid-} \\ \text{point rounding downward.}$$

- (c) Let $f_n(x) = (0.1)^n \lfloor 10^n x - 0.5 \rfloor$. By an argument similar to that in (b), this will round an real number to n decimal places, with the midpoint rounding downward.

43. (a) \$7.97, rounded to nearest cent.
(b) \$9.11, rounded to nearest cent.
(c) \$12.26, rounded to nearest cent.

Exercise Set 2.3 (Page 100)

1. For $f(x) = 2x$ and $g(x) = x^2 + 1$ we have

$$(f+g)(x) = f(x) + g(x) = 2x + (x^2 + 1) \\ = x^2 + 2x + 1;$$

$$(f-g)(x) = f(x) - g(x) = 2x - (x^2 + 1) \\ = 2x - x^2 - 1 = -x^2 + 2x - 1;$$

$$(f \cdot g)(x) = f(x)g(x) = 2x(x^2 + 1) = 2x^3 + 2x;$$

$$(f/g)(x) = \frac{f(x)}{g(x)} = \frac{2x}{x^2 + 1}.$$

The domain of $f+g$, $f-g$, $f \cdot g$, and f/g is $\text{Domain}(f) \cap \text{Domain}(g)$

$$= (-\infty, \infty) \cap (-\infty, \infty) = (-\infty, \infty).$$

2. For $f(x) = x^2$ and $g(x) = x + 1$ we have

$$(f+g)(x) = f(x) + g(x) = x^2 + x + 1;$$

$$(f-g)(x) = f(x) - g(x) = x^2 - (x + 1) \\ = x^2 - x - 1;$$

$$(f \cdot g)(x) = f(x)g(x) = x^2(x + 1) = x^3 + x^2;$$

$$(f/g)(x) = \frac{f(x)}{g(x)} = \frac{x^2}{x + 1}.$$

The domain of $f+g$, $f-g$, and $f \cdot g$ is

$$\begin{aligned}\text{Domain}(f) \cap \text{Domain}(g) \\ = (-\infty, \infty) \cap (-\infty, \infty) = (-\infty, \infty).\end{aligned}$$

The domain of f/g is

$\text{Domain}(f) \cap \text{Domain}(g)$ excluding any values that make the denominator 0. The denominator is 0 when $x+1=0$ implies $x=-1$, so $D(f/g) = (-\infty, -1) \cup (-1, \infty)$.

3. For $f(x) = \frac{1}{x}$ and $g(x) = \frac{x}{x-2}$ we have

$$\begin{aligned}(f+g)(x) &= f(x) + g(x) = \frac{1}{x} + \frac{x}{x-2} \\ &= \frac{(x-2) + x^2}{x(x-2)} = \frac{x^2 + x - 2}{x^2 - 2x};\end{aligned}$$

$$\begin{aligned}(f-g)(x) &= f(x) - g(x) = \frac{1}{x} - \frac{x}{x-2} \\ &= \frac{(x-2) - x^2}{x(x-2)} = \frac{-x^2 + x - 2}{x^2 - 2x};\end{aligned}$$

$$(f \cdot g)(x) = f(x)g(x) = \frac{x}{x(x-2)} = \frac{1}{x-2};$$

$$\begin{aligned}(f/g)(x) &= \frac{f(x)}{g(x)} = \frac{\frac{1}{x}}{\frac{x}{x-2}} = \frac{1}{x} \cdot \frac{x-2}{x} \\ &= \frac{x-2}{x^2}.\end{aligned}$$

Domain of $f : x \neq 0$; Domain of $g : x-2 \neq 0$ implies $x \neq 2$

$$\begin{aligned}\text{So } \text{Domain}(f) \cap \text{Domain}(g) \\ = \{x \mid x \neq 0 \text{ and } x \neq 2\} \\ = (-\infty, 0) \cup (0, 2) \cup (2, \infty).\end{aligned}$$

The domain of $f+g$, $f-g$, and $f \cdot g$ is $\text{Domain}(f) \cap \text{Domain}(g) = (-\infty, 0) \cup (0, 2) \cup (2, \infty)$.

The domain of f/g is

$\text{Domain}(f) \cap \text{Domain}(g)$ excluding any values that make the denominator 0, which is $x=0$, which has already been excluded.

So $\text{Domain}(f/g) = (-\infty, 0) \cup (0, 2) \cup (2, \infty)$.

4. For $f(x) = \frac{1}{x-1}$ and $g(x) = \frac{1}{x+1}$ we have

$$\begin{aligned}(f+g)(x) &= f(x) + g(x) = \frac{1}{x-1} + \frac{1}{x+1} \\ &= \frac{(x+1) + (x-1)}{(x-1)(x+1)} = \frac{2x}{x^2 - 1};\end{aligned}$$

$$\begin{aligned}(f-g)(x) &= f(x) - g(x) = \frac{1}{x-1} - \frac{1}{x+1} \\ &= \frac{(x+1) - (x-1)}{(x-1)(x+1)} = \frac{2}{x^2 - 1};\end{aligned}$$

$$\begin{aligned}(f \cdot g)(x) &= f(x)g(x) = \frac{1}{x-1} \cdot \frac{1}{x+1} \\ &= \frac{1}{x^2 - 1};\end{aligned}$$

$$\begin{aligned}(f/g)(x) &= \frac{f(x)}{g(x)} = \frac{\frac{1}{x-1}}{\frac{1}{x+1}} = \frac{1}{x-1} \cdot \frac{x+1}{1} \\ &= \frac{x+1}{x-1}.\end{aligned}$$

The domain of f is all real numbers so the denominator is not 0, that is, all $x \neq 1$, and the domain of g is all x so the denominator is not 0, so is all $x \neq -1$. Then

$$\begin{aligned}\text{Domain}(f) \cap \text{Domain}(g) \\ = \{x \mid x \neq 1 \text{ and } x \neq -1\} \\ = (-\infty, -1) \cup (-1, 1) \cup (1, \infty).\end{aligned}$$

The domain of $f+g$, $f-g$, and $f \cdot g$ is

$$\begin{aligned}\text{Domain}(f) \cap \text{Domain}(g) \\ = (-\infty, -1) \cup (-1, 1) \cup (1, \infty).\end{aligned}$$

The domain of f/g is

$\text{Domain}(f) \cap \text{Domain}(g)$ excluding any values that make the denominator 0. The denominator is 0 when $x=1$, which has already been excluded from the intersection, so

$$\begin{aligned}\text{Domain}(f/g) \\ = (-\infty, -1) \cup (-1, 1) \cup (1, \infty).\end{aligned}$$

5. For $f(x) = \sqrt{x+1}$ and $g(x) = \sqrt{3-x}$ we have

$$(f+g)(x) = f(x) + g(x) = \sqrt{x+1} + \sqrt{3-x};$$

$$(f-g)(x) = f(x) - g(x) = \sqrt{x+1} - \sqrt{3-x};$$

$$\begin{aligned}(f \cdot g)(x) &= f(x)g(x) = \sqrt{x+1} \cdot \sqrt{3-x} \\ &= \sqrt{(x+1)(3-x)} \\ &= \sqrt{3+2x-x^2};\end{aligned}$$

$$(f/g)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x+1}}{\sqrt{3-x}}.$$

Domain of $f : x+1 \geq 0$ implies $x \geq -1$;

Domain of $g : 3-x \geq 0$ implies $x \leq 3$

So $\text{Domain}(f) \cap \text{Domain}(g) = [-1, 3]$.

The domain of $f+g$, $f-g$, and $f \cdot g$ is

$$\text{Domain}(f) \cap \text{Domain}(g) = [-1, 3].$$

The domain of f/g is

$\text{Domain}(f) \cap \text{Domain}(g)$, excluding any values that make the denominator 0, which is $x=3$, so $\text{Domain}(f/g) = [-1, 3)$.

6. For $f(x) = \frac{1}{x}$ and $g(x) = \sqrt{x-1}$ we have

$$\begin{aligned}(f+g)(x) &= f(x) + g(x) = \frac{1}{x} + \sqrt{x-1} \\ &= \frac{1 + x\sqrt{x-1}}{x};\end{aligned}$$

$$\begin{aligned}(f-g)(x) &= f(x) - g(x) = \frac{1}{x} - \sqrt{x-1} \\ &= \frac{1 - x\sqrt{x-1}}{x};\end{aligned}$$

$$(f \cdot g)(x) = f(x)g(x) = \frac{1}{x} \cdot \sqrt{x-1} = \frac{\sqrt{x-1}}{x};$$

$$(f/g)(x) = \frac{f(x)}{g(x)} = \frac{\frac{1}{x}}{\sqrt{x-1}} = \frac{1}{x\sqrt{x-1}}.$$

The domain of f is all real numbers so the denominator is not 0, that is, all $x \neq 0$. The domain of g is all x so the radical is defined, that is, all x satisfying $x-1 \geq 0$ implies $x \geq 1$. Since 0 is not in the domain of g , $\text{Domain}(f) \cap \text{Domain}(g) = \text{Domain}(g) = [1, \infty)$.

The domain of $f+g$, $f-g$, and $f \cdot g$ is $\text{Domain}(g) \cap \text{Domain}(f) = [1, \infty)$.

The domain of f/g is

$\text{Domain}(g) \cap \text{Domain}(f)$ excluding any values that make the denominator 0. The denominator is 0 when $x = 0$, or $x-1 = 0$ implies $x = 0$, or $x = 1$, so $\text{Domain}(f/g) = (1, \infty)$.

7. For $f(x) = \begin{cases} -1, & \text{if } x < 0 \\ 1, & \text{if } x \geq 0 \end{cases}$ and

$$g(x) = \begin{cases} 1, & \text{if } x < 0 \\ 0, & \text{if } x \geq 0 \end{cases}$$

we perform the operations separately for $x < 0$ and for $x \geq 0$.

$$\begin{aligned}(f+g)(x) &= f(x) + g(x) \\ &= \begin{cases} 0, & \text{if } x < 0 \\ 1, & \text{if } x \geq 0 \end{cases};\end{aligned}$$

$$\begin{aligned}(f-g)(x) &= f(x) - g(x) \\ &= \begin{cases} -2, & \text{if } x < 0 \\ 1, & \text{if } x \geq 0 \end{cases};\end{aligned}$$

$$(f \cdot g)(x) = f(x)g(x) = \begin{cases} -1, & \text{if } x < 0 \\ 0, & \text{if } x \geq 0 \end{cases}$$

The quotient is defined only for $x < 0$, since $g(x) = 0$ for $x \geq 0$. So $(f/g)(x) = -1$, for $x < 0$.

The domain of $f+g$, $f-g$, $f \cdot g$ is $(-\infty, \infty)$ and the domain of f/g is $(-\infty, 0)$.

8. For $f(x) = \begin{cases} 0, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$ and

$$g(x) = \begin{cases} -x, & \text{if } x < 0 \\ 0, & \text{if } x \geq 0 \end{cases}$$

we perform the operations separately for $x < 0$ and for $x \geq 0$.

$$\begin{aligned}(f+g)(x) &= f(x) + g(x) \\ &= \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases} = |x|;\end{aligned}$$

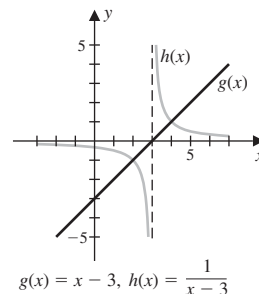
$$\begin{aligned}(f-g)(x) &= f(x) - g(x) \\ &= \begin{cases} x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases} = x;\end{aligned}$$

$$\begin{aligned}(f \cdot g)(x) &= f(x)g(x) \\ &= \begin{cases} 0, & \text{if } x < 0 \\ 0, & \text{if } x \geq 0 \end{cases} = 0;\end{aligned}$$

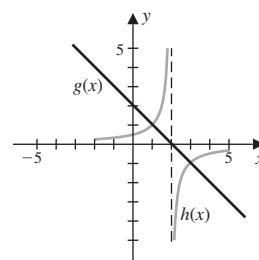
The quotient is defined only for $x < 0$ since $g(x) = 0$ for $x \geq 0$. So $(f/g)(x) = 0$ for $x < 0$.

The domain of $f+g$, $f-g$, $f \cdot g$ is $(-\infty, \infty)$ and the domain of f/g is $(-\infty, 0)$.

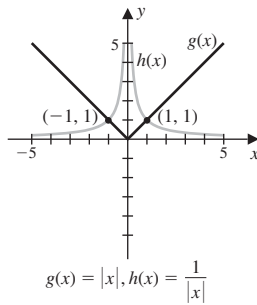
9. $g(x) = x-3$; $h(x) = \frac{1}{x-3}$



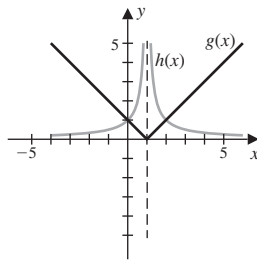
10. $g(x) = 2-x$; $h(x) = \frac{1}{2-x}$



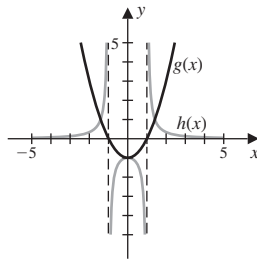
11. $g(x) = |x|$; $h(x) = \frac{1}{|x|}$



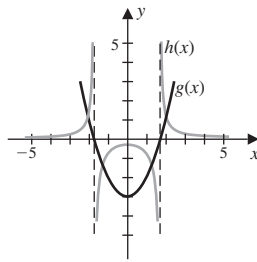
12. $g(x) = |x - 1|; h(x) = \frac{1}{|x - 1|}$



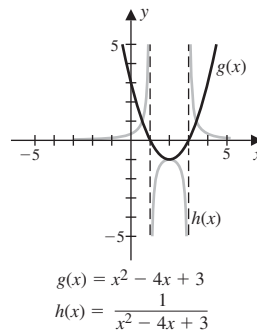
13. $g(x) = x^2 - 1; h(x) = \frac{1}{x^2 - 1}$



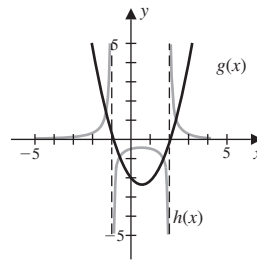
14. $g(x) = x^2 - 3; h(x) = \frac{1}{x^2 - 3}$



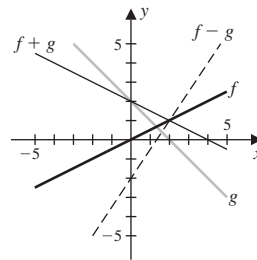
15. $g(x) = x^2 - 4x + 3$
 $= x^2 - 4x + 4 - 4 + 3$
 $= (x - 2)^2 - 1$
 and
 $h(x) = \frac{1}{(x - 2)^2 - 1}$



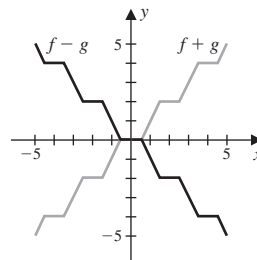
16. $g(x) = x^2 - x - 2$
 $= x^2 - x + \frac{1}{4} - \frac{1}{4} - 2$
 $= \left(x - \frac{1}{2}\right)^2 - \frac{9}{4}$
 and
 $h(x) = \frac{1}{\left(x - \frac{1}{2}\right)^2 - \frac{9}{4}}$



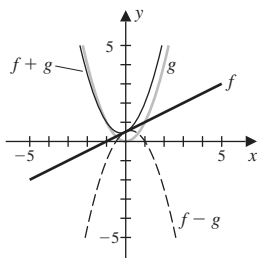
17. The figure shows the graphs of $f + g$ and $f - g$.



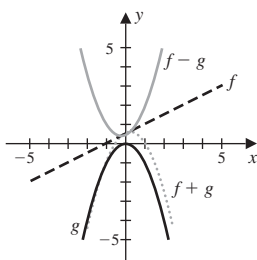
18. The figure shows the graphs of $f + g$ and $f - g$.



19. The figure shows the graphs of $f+g$ and $f-g$.



20. The figure shows the graphs of $f+g$ and $f-g$.



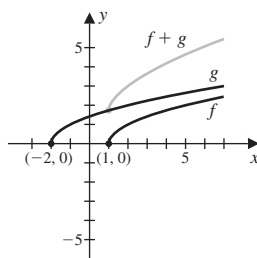
21. We have $f(x) = \frac{x^2 - 4}{x - 2} = \frac{(x - 2)(x + 2)}{x - 2} = x + 2$, for $x \neq 2$. The denominator of the original expression is 0 at $x = 2$, making the quotient undefined.

To graph $y = f(x) = \frac{x^2 - 4}{x - 2}$, simply graph $y = x + 2$ and remove the point with x -coordinate 2. That is, remove the point $(2, 4)$. So the graphs of $f(x) = \frac{x^2 - 4}{x - 2}$ and $g(x) = x + 2$ are identical except for the point $(2, 4)$ removed from the graph of $y = f(x)$.

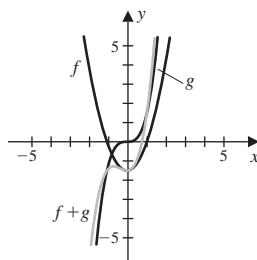
22. We have $f(x) = \frac{x^3 - 2x^2}{x} = \frac{x^2(x - 2)}{x} = x(x - 2)$, for $x \neq 0$. The denominator of the original expression is 0 at $x = 0$, making the quotient undefined.

To graph $y = f(x) = \frac{x^3 - 2x^2}{x}$, simply graph $y = x^2 - 2x$ and remove the point with x -coordinate 0. That is, remove the point $(0, 0)$. So the graphs of $f(x) = \frac{x^3 - 2x^2}{x}$ and $g(x) = x^2 - 2x$ are identical except for the point $(0, 0)$ removed from the graph of $y = f(x)$.

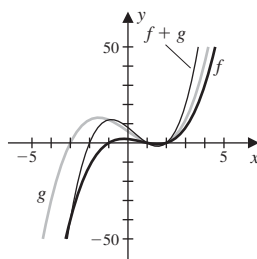
23. The figure shows the graphs of $y = \sqrt{x - 1}$, $y = \sqrt{x + 2}$, and $y = \sqrt{x - 1} + \sqrt{x + 2}$.



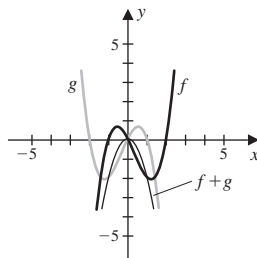
24. The figure shows the graphs of $y = x^2 - 1$, $y = x^3$, and $y = x^3 + x^2 - 1$.



25. The figure shows the graphs of $y = x^3 - 2x^2 - x + 2$, $y = x^3 - 7x + 6$, and $y = 2x^3 - 2x^2 - 8x + 8$.



26. The figure shows the graphs of $y = -x^3 - x^2 + 2x$, $y = x^3 - x^2 - 2x$, and $y = -2x^2$.



27. Let $R(t)$ denote the rate of spread of the disease at time t , $I(t)$ the number of infected, and $H(t)$ the number healthy. Then $R(t) = \frac{1}{74550}(70)(4930) \approx 4.6$.
28. Assume that $P(t)$ is the number of bacteria at time t , where t is a nonnegative integer.

(a) Then $P(0) = I$, $P(1) = 2I - A$,

$$\begin{aligned} P(2) &= 4I - 2A - A = 4I - 3A, \\ P(3) &= 8I - 6A - A = 8I - 7A, \\ \text{and } P(4) &= 16I - 15A. \end{aligned}$$

(b) For any nonnegative integer t we have $P(t) = 2^t I - (1 + 2 + 4 + \cdots + 2^{t-1})A$.

(c) Because $1 + 2 + 4 + \cdots + 2^{t-1} = \frac{1 - 2^t}{1 - 2} = 2^t - 1$, we have $H(t) = (2^t - 1)A$.

Exercise Set 2.4 (Page 109)

- For $f(x) = 2x - 3$ and $g(x) = x^2 + 2$ we have $(f \circ g)(3) = f(g(3)) = f(3^2 + 2) = f(11) = 2(11) - 3 = 19$.
- For $f(x) = 2x - 3$ and $g(x) = x^2 + 2$ we have $(g \circ f)(-1) = g(f(-1)) = g(2(-1) - 3) = g(-5) = (-5)^2 + 2 = 27$.
- For $f(x) = 2x - 3$ and $g(x) = x^2 + 2$ we have $(f \circ g)(-3) = f(g(-3)) = f((-3)^2 + 2) = f(11) = 2(11) - 3 = 19$.
- For $f(x) = 2x - 3$ and $g(x) = x^2 + 2$ we have $(g \circ f)(5) = g(f(5)) = g(2(5) - 3) = g(7) = (7)^2 + 2 = 51$.
- For $f(x) = 2x - 3$ and $g(x) = x^2 + 2$ we have $(f \circ f)(-2) = f(f(-2)) = f(2(-2) - 3) = f(-7) = 2(-7) - 3 = -17$.
- For $f(x) = 2x - 3$ and $g(x) = x^2 + 2$ we have $(g \circ g)(1/2) = g(g(1/2)) = g\left(\left(\frac{1}{2}\right)^2 + 2\right) = g\left(\frac{9}{4}\right) = \left(\frac{9}{4}\right)^2 + 2 = \frac{113}{16}$.

- For $f(x) = 2x + 1$ and $g(x) = 3x - 1$ we have $(f \circ g)(x) = f(g(x)) = f(3x - 1) = 2(3x - 1) + 1 = 6x - 1$; $(g \circ f)(x) = g(f(x)) = g(2x + 1) = 3(2x + 1) - 1 = 6x + 2$.

Since the domain of f and g is the set of all real numbers, the domain of each of the compositions is also the set of all real numbers.

- For $f(x) = x^2 + 1$ and $g(x) = x - 1$ we have $(f \circ g)(x) = f(g(x)) = f(x - 1) = (x - 1)^2 + 1 = x^2 - 2x + 2$; $(g \circ f)(x) = g(f(x)) = g(x^2 + 1) = x^2 + 1 - 1 = x^2$.

Since the domain of f and g is the set of all real numbers, the domain of each of the compositions is also the set of all real numbers.

- For $f(x) = \frac{1}{x}$ and $g(x) = x^2 + 2x$ we have $(f \circ g)(x) = f(g(x)) = f(x^2 + 2x) = \frac{1}{x^2 + 2x}$ and

$$(g \circ f)(x) = g(f(x)) = g\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)^2 + 2\left(\frac{1}{x}\right) = \frac{1}{x^2} + \frac{2}{x} = \frac{1+2x}{x^2}.$$

$$\text{Domain } f : \{x \mid x \neq 0\} = (-\infty, 0) \cup (0, \infty); \text{ Domain } g : (-\infty, \infty);$$

$$\text{Domain } f \circ g : \{x \mid x^2 + 2x = x(x+2) \neq 0\} = \{x \mid x \neq 0, x \neq -2\} = (-\infty, -2) \cup (-2, 0) \cup (0, \infty);$$

$$\text{Domain } g \circ f : \{x \neq 0 \mid 1/x \text{ is defined}\} = \{x \mid x \neq 0\} = (-\infty, 0) \cup (0, \infty).$$

10. For $f(x) = \frac{2}{x-5}$ and $g(x) = x^2 + 4x$ we have

$$(f \circ g)(x) = f(g(x)) = f(x^2 + 4x) = \frac{2}{x^2 + 4x - 5};$$

$$\begin{aligned} (g \circ f)(x) &= g(f(x)) = g\left(\frac{2}{x-5}\right) = \left(\frac{2}{x-5}\right)^2 + 4 \cdot \frac{2}{x-5} \\ &= \frac{4}{(x-5)^2} + \frac{8}{x-5} = \frac{4+8(x-5)}{(x-5)^2} = \frac{8x-36}{(x-5)^2}. \end{aligned}$$

$$\text{Domain } f : \{x \mid x \neq 5\} = (-\infty, 5) \cup (5, \infty); \text{ Domain } g : (-\infty, \infty);$$

$$\text{Domain } f \circ g : \{x \mid x^2 + 4x \neq 5\} = \{x \mid (x-1)(x+5) \neq 0\} = (-\infty, -5) \cup (-5, 1) \cup (1, \infty);$$

$$\text{Domain } g \circ f : \{x \neq 5 \mid f(x) \text{ is defined}\} = (-\infty, 5) \cup (5, \infty).$$

11. For $f(x) = \sqrt{x-1}$ and $g(x) = x^2 - 3$ we have

$$(f \circ g)(x) = f(g(x)) = f(x^2 - 3) = \sqrt{x^2 - 3 - 1} = \sqrt{x^2 - 4};$$

$$(g \circ f)(x) = g(f(x)) = f(\sqrt{x-1}) = (\sqrt{x-1})^2 - 3 = x - 4.$$

$$\text{Domain } f : \{x \mid x - 1 \geq 0\} = [1, \infty); \text{ Domain } g : (-\infty, \infty);$$

$$\text{Domain } f \circ g : \{x \mid x^2 - 3 \geq 1\} = \{x \mid (x-2)(x+2) \geq 0\} = (-\infty, -2] \cup [2, \infty);$$

$$\text{Domain } g \circ f : \{x \mid x \geq 1 \text{ and } f(x) \text{ is defined}\} = [1, \infty).$$

12. For $f(x) = \sqrt{x-9}$ and $g(x) = x^2$ we have

$$(f \circ g)(x) = f(g(x)) = f(x^2) = \sqrt{x^2 - 9};$$

$$(g \circ f)(x) = g(f(x)) = g(\sqrt{x-9}) = (\sqrt{x-9})^2 = x - 9.$$

$$\text{Domain } f : \{x \mid x - 9 \geq 0\} = [9, \infty); \text{ Domain } g : (-\infty, \infty);$$

$$\text{Domain } f \circ g : \{x \mid x^2 \geq 9\} = \{x \mid (x-3)(x+3) \geq 0\} = (-\infty, -3] \cup [3, \infty);$$

$$\text{Domain } g \circ f : \{x \mid x \geq 9 \text{ and } f(x) \text{ is defined}\} = [9, \infty).$$

13. For $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{x+1}$ we have

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{x+1}\right) = \frac{1}{\frac{1}{x+1}} = x+1;$$

$$(g \circ f)(x) = g(f(x)) = g\left(\frac{1}{x}\right) = \frac{1}{\frac{1}{x} + 1} = \frac{1}{\frac{1+x}{x}} = \frac{x}{x+1}.$$

$$\text{Domain } f : \{x \mid x \neq 0\} = (-\infty, 0) \cup (0, \infty);$$

$$\text{Domain } g : \{x \mid x+1 \neq 0\} = (-\infty, -1) \cup (-1, \infty);$$

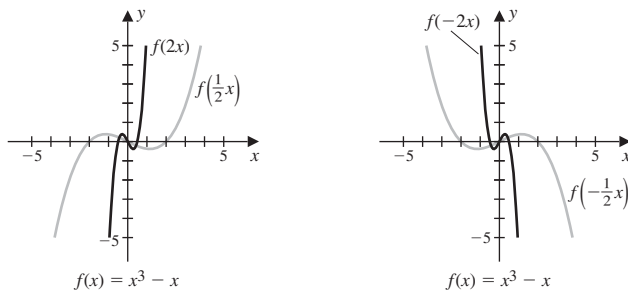
$$\text{Domain } f \circ g : \{x \neq -1 \mid \frac{1}{x+1} \neq 0\} = (-\infty, -1) \cup (-1, \infty);$$

$$\text{Domain } g \circ f : \{x \neq 0 \mid \frac{1}{x} \neq -1\} = (-\infty, -1) \cup (-1, 0) \cup (0, \infty).$$

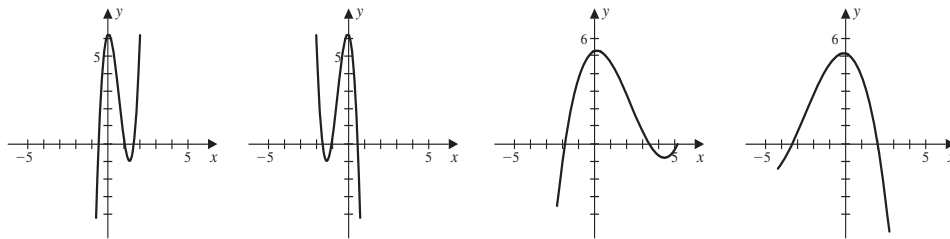
14. For $f(x) = \frac{1}{x-1}$ and $g(x) = \frac{x+1}{x-1}$ we have
- $$(f \circ g)(x) = f(g(x)) = f\left(\frac{x+1}{x-1}\right) = \frac{1}{\frac{x+1}{x-1} - 1} = \frac{1}{\frac{x+1-x+1}{x-1}} = \frac{1}{\frac{2}{x-1}} = \frac{x-1}{2};$$
- $$(g \circ f)(x) = g(f(x)) = g\left(\frac{1}{x-1}\right) = \frac{\frac{1}{x-1} + 1}{\frac{1}{x-1} - 1} = \frac{1+x-1}{1-x+1} = \frac{x}{2-x}.$$
- Domain $f : \{x \mid x-1 \neq 0\} = (-\infty, 1) \cup (1, \infty)$;
 Domain $g : \{x \mid x-1 \neq 0\} = (-\infty, 1) \cup (1, \infty)$;
 Domain $f \circ g : \{x \mid x-1 \neq 0\} = (-\infty, 1) \cup (1, \infty)$;
 Domain $g \circ f : \{x \neq 1 \mid 2-x \neq 0\} = \{x \neq 1 : -x \neq -2\} = \{x \neq 1 \mid x \neq 2\} = (-\infty, 1) \cup (1, 2) \cup (2, \infty).$
15. The inside operation of $h(x) = (3x^2 - 2)^4$ is $3x^2 - 2$ and the outside operation is raising to the fourth power, x^4 . So define $g(x) = 3x^2 - 2$ and $f(x) = x^4$. This gives
- $$(f \circ g)(x) = f(g(x)) = f(3x^2 - 2) = (3x^2 - 2)^4 = h(x).$$
16. The inside operation of $h(x) = (x^2 - x + 1)^8$ is $x^2 - x + 1$ and the outside operation is raising to the eighth power, x^8 . So define $g(x) = x^2 - x + 1$ and $f(x) = x^8$. This gives
- $$(f \circ g)(x) = f(g(x)) = f(x^2 - x + 1) = (x^2 - x + 1)^8 = h(x).$$
17. The inside operation of $h(x) = \sqrt[3]{x-4}$ is $x-4$ and the outside operation is the cube root of a number, $\sqrt[3]{x}$. So define $g(x) = x-4$ and $f(x) = \sqrt[3]{x}$. This gives
- $$(f \circ g)(x) = f(g(x)) = f(x-4) = \sqrt[3]{x-4} = h(x).$$
18. The inside operation of $h(x) = \sqrt[3]{x+\sqrt{x}}$ is $x+\sqrt{x}$ and the outside operation is taking the cube root, $\sqrt[3]{x}$. So define $g(x) = x+\sqrt{x}$ and $f(x) = \sqrt[3]{x}$. This gives
- $$(f \circ g)(x) = f(g(x)) = f(x+\sqrt{x}) = \sqrt[3]{x+\sqrt{x}} = h(x).$$
19. Define the first operation to be $x+2$ and then take the reciprocal, so let $g(x) = x+2$ and $f(x) = \frac{1}{x}$. Then
- $$(f \circ g)(x) = f(g(x)) = f(x+2) = \frac{1}{x+2} = h(x).$$
20. The inside operation of $h(x) = |x^2 + x + 1|$ is $x^2 + x + 1$ and the outside operation is the absolute value of a number, $|x|$. So define $g(x) = x^2 + x + 1$ and $f(x) = |x|$. This gives
- $$(f \circ g)(x) = f(g(x)) = f(x^2 + x + 1) = |x^2 + x + 1| = h(x).$$
21. (a) (i) $(f \circ g)(-2) = f(g(-2)) = f(1) = 2$
 (ii) $(g \circ f)(-2) = g(f(-2)) = g(0) = 0$
 (iii) $(g \circ f)(2) = g(f(2)) = g(0) = 0$
 (iv) $(f \circ g)(2) = f(g(2)) = f(-1) = -2$
- (b) (i) $(f \circ g)(x) = f(g(x)) = 0$ whenever $g(x) = -2, 0, 2$, that is, $x = -4, 0, 4$
 (ii) $(g \circ f)(x) = g(f(x)) = 0$ whenever $f(x) = 0$, that is, $x = -2, 0, 2$
22. The completed table assumes f is even and g is odd.

x	$f(x)$	$g(x)$	$(f \circ g)(x)$	$(g \circ f)(x)$
-3	-2	-2	3	3
-2	3	3	-2	2
-1	0	0	1	1
0	1	1	0	0
1	0	0	1	1
2	3	-3	-2	2
3	-2	2	3	3

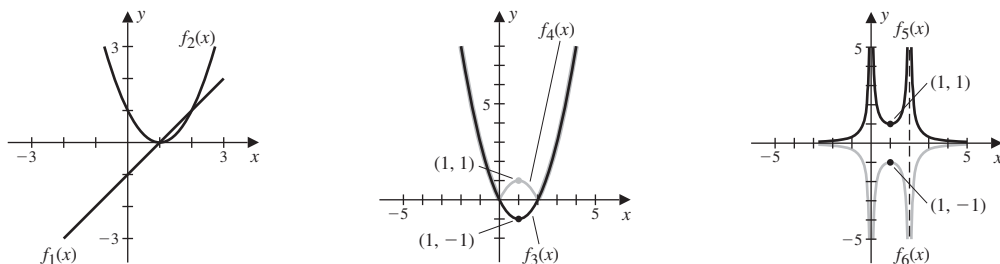
23. The figure shows the graphs of $y = f(2x)$, $y = f(-2x)$, $y = f\left(\frac{1}{2}x\right)$, and $y = f\left(-\frac{1}{2}x\right)$.



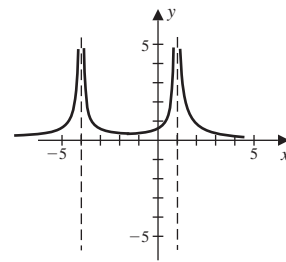
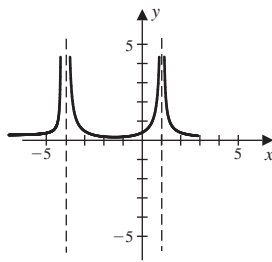
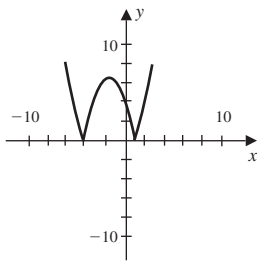
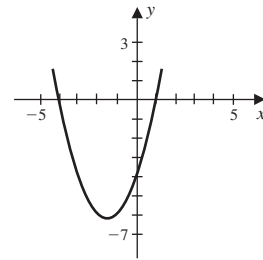
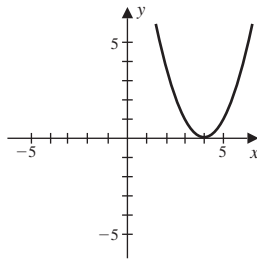
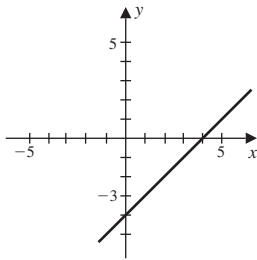
24. The figure shows the graphs of $y = f(2x)$, $y = f(-2x)$, $y = f\left(\frac{1}{2}x\right)$, and $y = f\left(-\frac{1}{2}x\right)$.



25. (a) $y = x - 1$ (b) $y = (x - 1)^2$ (c) $y = x^2 - 2x = (x - 1)^2 - 1$ (d) $y = |x^2 - 2x|$
 (e) $y = -\frac{1}{|x^2 - 2x|}$



26. (a) $y = x - 4$ (b) $y = (x - 4)^2$ (c) $y = x^2 - 8x + 13 = (x - 4)^2 - 3$
 (d) $y = |x^2 - 8x + 13|$ (e) $y = \frac{1}{|x^2 - 8x + 13|}$ (f) $y = \frac{2}{|x^2 - 8x + 13|}$



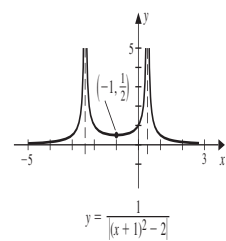
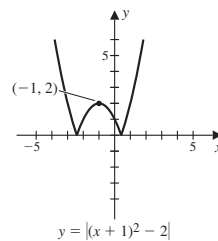
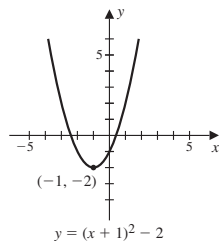
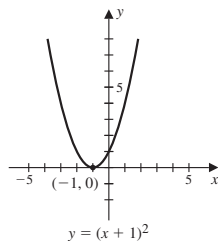
27. (a) For $f(x) = \frac{1}{|x^2 + 2x - 1|}$, completing the square on the quadratic in the denominator gives

$$f(x) = \frac{1}{|x^2 + 2x + 1 - 1 - 1|} = \frac{1}{|(x + 1)^2 - 2|}.$$

Then letting $g_1(x) = (x + 1)^2$, $g_2(x) = x - 2$, $g_3(x) = |x|$ and $g_4(x) = \frac{1}{x}$ we have

$$\begin{aligned} g_4(g_3(g_2(g_1(x)))) &= g_4(g_3(g_2((x + 1)^2))) = g_4(g_3((x + 1)^2 - 2)) \\ &= g_4(|(x + 1)^2 - 2|) = \frac{1}{|(x + 1)^2 - 2|} = f(x). \end{aligned}$$

- (b) The graphs of $y = g_1(x)$, $y = g_2(g_1(x))$, $y = g_3(g_2(g_1(x)))$, and $y = f(x)$ are shown in the figures.



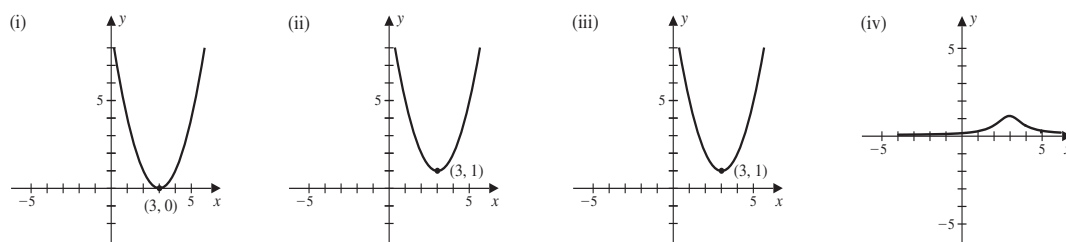
28. (a) For $f(x) = \frac{1}{|x^2 - 6x + 10|}$, completing the square on the quadratic in the denominator gives

$$f(x) = \frac{1}{|x^2 - 6x + 9 - 9 + 10|} = \frac{1}{|(x - 3)^2 + 1|}.$$

Then letting $g_1(x) = (x - 3)^2$, $g_2(x) = x + 1$, $g_3(x) = |x|$ and $g_4(x) = \frac{1}{x}$ we have

$$\begin{aligned} g_4(g_3(g_2(g_1(x)))) &= g_4(g_3(g_2((x - 3)^2))) = g_4(g_3((x - 3)^2 + 1)) \\ &= g_4(|(x - 3)^2 + 1|) = \frac{1}{|(x - 3)^2 + 1|} = f(x). \end{aligned}$$

(b) The graphs of $y = g_1(x)$, $y = g_2(g_1(x))$, $y = g_3(g_2(g_1(x)))$, and $y = f(x)$ are shown in the figures.



29. Let $f(x) = x$ and $g(x) = x + 1$. Then $(f \circ g)(x) = f(g(x)) = f(x + 1) = x + 1$ and $(g \circ f)(x) = g(f(x)) = g(x) = x + 1$. There are many other examples.

30. If f and g are odd functions, then $f(-x) = -f(x)$ and $g(-x) = -g(x)$. So $(f \circ g)(-x) = f(g(-x)) = f(-g(x)) = -f(g(x)) = -(f \circ g)(x)$ and $(g \circ f)(-x) = g(f(-x)) = g(-f(x)) = -g(f(x)) = -(g \circ f)(x)$.

31. If f is an odd function and g is an even function, then $f(-x) = -f(x)$ and $g(-x) = g(x)$. So

$$(f \circ g)(-x) = f(g(-x)) = f(g(x)) = (f \circ g)(x)$$

and

$$(g \circ f)(-x) = g(f(-x)) = g(-f(x)) = g(f(x)) = (g \circ f)(x).$$

32. Let $f(x) = ax + b$. Then

$$(f \circ f)(x) = f(f(x)) = a(ax + b) + b = a^2x + b(a + 1),$$

so $(f \circ f)(x) = 4x + 3$ which implies $a^2x + b(a + 1) = 4x + 3$. Then $a^2 = 4$, $b(a + 1) = 3$, so $a = 2, 3b = 3$ or $a = -2, -b = 3$ which means $a = 2, b = 1$ or $a = -2, b = -3$. So two possible linear functions are $f(x) = 2x + 1$ or $f(x) = -2x - 3$.

33. Let $f(x) = ax + b$ and $g(x) = cx + d$. Then

$$(f \circ g)(x) = a(cx + d) + b = acx + ad + b$$

$$(g \circ f)(x) = c(ax + b) + d = acx + bc + d.$$

(a) $(f \circ g)(x) = (g \circ f)(x)$ implies $ad + b = bc + d$.

(b) $(f \circ g)(x) = f(x)$ implies $ac = a$ and $ad + b = b$ so $c = 1$ and $d = 0$.

(c) $(f \circ g)(x) = g(x)$ implies $ac = c$ and $ad + b = d$ so $a = 1, b = 0$.

34. (a) A linear function. This is seen from the equations in Exercise 33.

(b) Let $f(x) = ax + b$ and $g(x) = cx^2 + dx + e$. Then

$$f(g(x)) = a(cx^2 + dx + e) + b = acx^2 + adx + ae + b$$

and

$$g(f(x)) = c(ax + b)^2 + d(ax + b) + e = ca^2x^2 + (2abc + ad)x + cb^2 + bd + e$$

both of which are quadratic.

(c) Let $f(x) = ax^2 + bx + c$ and $g(x) = dx^2 + ex + f$. Then

$$f(g(x)) = a(dx^2 + ex + f)^2 + b(dx^2 + ex + f) + c$$

has highest power x^4 with leading coefficient ad^2 . Similarly $(g \circ f)(x)$ is a fourth degree polynomial.

35. $g(x) = 3f(x - 1) + 2$

36. $g(x) = -f\left(\frac{1}{2}(x + 2)\right) = -f\left(\frac{1}{2}x + 1\right)$

37. The volume of a sphere of radius r is $V(r) = \frac{4}{3}\pi r^3$. So $V(t) = \frac{4}{3}\pi(3 + 0.01t)^3$.

38. The volume of a cone of radius r and height h is $V = \frac{\pi}{3}r^2h$, and if the radius and height are equal we have

$$V(h) = \frac{\pi}{3}h^2h = \frac{\pi}{3}h^3 \text{ and } V(t) = \frac{\pi}{3}(10 + 0.25t)^3.$$

39. (a) The volume of a sphere of radius r is $V(r) = \frac{4}{3}\pi r^3$ so

$$V(t) = V(r(t)) = \frac{4}{3}\pi \left(3\sqrt{t} + 5\right)^3 \text{ centimeters}^3.$$

(b) The surface area is $S(r) = 4\pi r^2$ so

$$S(t) = S(r(t)) = 4\pi \left(3\sqrt{t} + 5\right)^2 \text{ centimeters}^2.$$

40. Since $s(t) = 576 + 144t - 16t^2$ we have

$$\begin{aligned} \bar{s}(t) &= s(t - 2) \\ &= 576 + 144(t - 2) - 16(t - 2)^2 \\ &= 576 + 144t - 288 - 16(t^2 - 4t + 4) = 224 + 208t - 16t^2. \end{aligned}$$

To find the domain, determine the time t when the ball strikes the ground. Factoring we have

$$224 + 208t - 16t^2 = -16(t + 1)(t - 14)$$

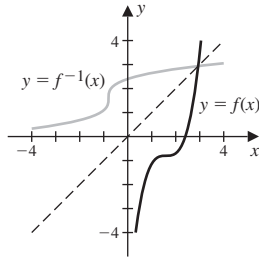
and the rock strikes the ground at time $t = 14$ seconds and the domain is $[2, 14]$. Since the parabola is the parabola of Exercise 36 of Exercise Set 1.8, shifted 2 units to the right, the vertex has the same height of 900. So the range is $[0, 900]$.

41. (a) The value after 2 years is $(1.04)^2x + P(1 + 1.04)$, and the value after 3 years is $(1.04)^3x + P(1 + 1.04 + (1.04)^2)$.
- (b) Let $V_n(x)$ be the value of the account after n years. Then $V_n(x) = V^n(x)$, that is, V composed with itself n times.

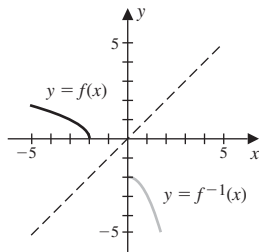
Exercise Set 2.5 (Page 119)

- The function is one-to-one since every horizontal line crosses the curve in only one point.
- The function is not one-to-one since the horizontal line $y = 1$ crosses the curve in many places.
- The function is not one-to-one since many horizontal lines cross the curve in more than one point.
- The function is one-to-one since every horizontal line crosses the curve in only one point.
- The function is not one-to-one since every horizontal line that crosses the curve intersects the curve in two points.
- The function is not one-to-one since every horizontal line that crosses the curve intersects the curve in two points.
- The function defined by $f(x) = 3x - 4$ is one-to-one, since $f(x_1) = f(x_2)$ implies $3x_1 - 4 = 3x_2 - 4$ so $3x_1 = 3x_2$ and $x_1 = x_2$.
- The function defined by $f(x) = \sqrt{x-1}$ is one-to-one, since $f(x_1) = f(x_2)$ implies $\sqrt{x_1-1} = \sqrt{x_2-1}$ so $x_1 - 1 = x_2 - 1$ and $x_1 = x_2$.
- The function defined by $f(x) = |x - 1| + 1$ is not one-to-one. For example, if $x_1 = 0$ and $x_2 = 2$, then $f(x_1) = f(0) = |0 - 1| + 1 = 2$ and $f(x_2) = f(2) = |2 - 1| + 1 = 2$. So we have found x_1 and x_2 , with $x_1 \neq x_2$ and $f(x_1) = f(x_2)$.
- The function defined by $f(x) = x^2 - 6x + 5$ is not one-to-one, since $f(x) = (x - 5)(x - 1) = 0$ implies $x = 1$ or $x = 5$. That is $f(1) = f(5) = 0$.
- The function defined by $f(x) = x^4 + 1$ is not one-to-one, since, for example, $f(-1) = 2 = f(1)$.
- The function defined by $f(x) = x^6 - 2$ is not one-to-one, since, for example, $f(-1) = -1 = f(1)$.
- The function defined by $f(x) = \begin{cases} 2x^2, & \text{if } x \geq 0 \\ x - 2 & \text{if } x < 0 \end{cases}$ is one-to-one, since the graph satisfies the horizontal line test. That is, every horizontal line that crosses the graph does so in only one point.
- The function defined by $f(x) = \begin{cases} \sqrt{x}, & \text{if } x \geq 0 \\ x + 1 & \text{if } x < 0 \end{cases}$ is not one-to-one, since the graph does not satisfy the horizontal line test.

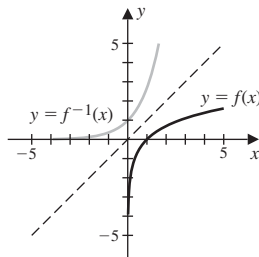
15. The figure shows the graph of the function and its inverse.



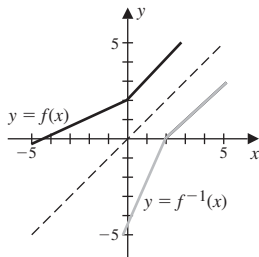
16. The figure shows the graph of the function and its inverse.



17. The figure shows the graph of the function and its inverse.



18. The figure shows the graph of the function and its inverse.

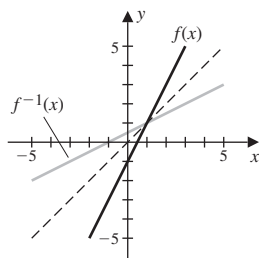


19. Since $f(x) = 2$, $f^{-1}(2) = x$. Solving $x + 1 = 2$ implies $x = 1$. So $f^{-1}(2) = 1$.
20. Since $f(x) = 0$, $f^{-1}(0) = x$. Solving $x - 2 = 0$ implies $x = 2$. So $f^{-1}(0) = 2$.
21. Since $f(x) = 1$, $f^{-1}(1) = x$. Solving $2x - 2 = 1$ implies $x = \frac{3}{2}$. So $f^{-1}(1) = \frac{3}{2}$.
22. Since $f(x) = 2$, $f^{-1}(2) = x$. Solving $3x - 2 = 2$ implies $x = \frac{4}{3}$. So $f^{-1}(2) = \frac{4}{3}$.

23. Since $f(x) = 2$, $f^{-1}(2) = x$. Solving $\frac{1}{x} = 2$ implies $x = \frac{1}{2}$. So $f^{-1}(2) = \frac{1}{2}$.
24. Since $f(x) = -8$, $f^{-1}(-8) = x$. Solving $x^3 = -8$ implies $x = -2$. So $f^{-1}(-8) = -2$.
25. Let $f(x) = 2x - 1$. Then f is one-to-one since

$$f(x_1) = f(x_2) \quad \text{implies} \quad 2x_1 - 1 = 2x_2 - 1 \quad \text{so} \quad x_1 = x_2.$$

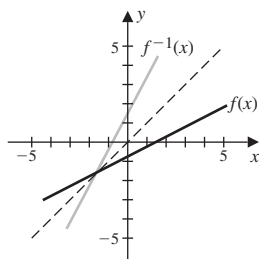
To find $f^{-1} : y = 2x - 1$ implies $2x = y + 1$ so $x = \frac{y+1}{2}$, and $f^{-1}(x) = \frac{x+1}{2} = \frac{x}{2} + \frac{1}{2}$.



26. Let $f(x) = \frac{2x-3}{4}$. Then f is one-to-one since

$$f(x_1) = f(x_2) \quad \text{implies} \quad \frac{2x_1-3}{4} = \frac{2x_2-3}{4} \quad \text{so} \quad 2x_1-3 = 2x_2-3 \quad \text{and} \quad x_1 = x_2.$$

To find $f^{-1} : y = \frac{2x-3}{4}$ implies $4y = 2x-3$ or $2x = 4y+3$ so $x = \frac{4y+3}{2}$, and $f^{-1}(x) = \frac{4x+3}{2}$.

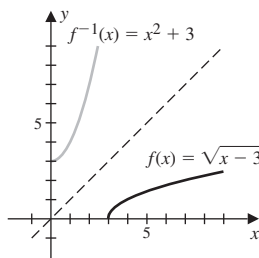


27. Let $f(x) = \sqrt{x-3}$. Then f is one-to-one since

$$f(x_1) = f(x_2) \quad \text{implies} \quad \sqrt{x_1-3} = \sqrt{x_2-3} \quad \text{so} \quad x_1-3 = x_2-3 \quad \text{and} \quad x_1 = x_2.$$

To find $f^{-1} : y = \sqrt{x-3}$ implies $x-3 = y^2$ so $x = y^2+3$, and $f^{-1}(x) = x^2+3$.

Note: The domain of f is $[3, \infty)$, which is also the range of f^{-1} and the range of f is $[0, \infty)$, which is also the domain of f^{-1} .

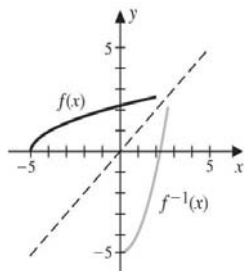


28. Let $f(x) = \sqrt{x+2}$. Then f is one-to-one since

$$f(x_1) = f(x_2) \text{ implies } \sqrt{x_1+2} = \sqrt{x_2+2} \text{ so } x_1+2 = x_2+2 \text{ and } x_1 = x_2.$$

To find $f^{-1} : y = \sqrt{x+2}$ implies $x+2 = y^2$ so $x = y^2 - 2$, and $f^{-1}(x) = x^2 - 2$.

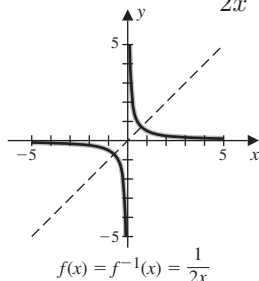
Note: The domain of f is $[-2, \infty)$, which is also the range of f^{-1} and the range of f is $[0, \infty)$, which is also the domain of f^{-1} .



29. Let $f(x) = \frac{1}{2x}$. Then f is one-to-one since

$$f(x_1) = f(x_2) \text{ implies } \frac{1}{2x_1} = \frac{1}{2x_2} \text{ so } x_1 = x_2.$$

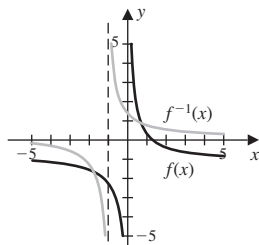
To find $f^{-1} : y = \frac{1}{2x}$ implies $2xy = 1$ so $x = \frac{1}{2y}$, and $f^{-1}(x) = \frac{1}{2x}$.



30. Let $f(x) = \frac{1}{x+1}$. Then f is one-to-one since

$$f(x_1) = f(x_2) \text{ implies } \frac{1}{x_1+1} = \frac{1}{x_2+1} \text{ so } x_2+1 = x_1+1 \text{ and } x_1 = x_2.$$

To find $f^{-1} : y = \frac{1}{x+1}$ implies $(x+1)y = 1$ so $x+1 = \frac{1}{y}$ or $x = \frac{1}{y} - 1 = \frac{1-y}{y}$, and $f^{-1}(x) = \frac{1-x}{x}$.

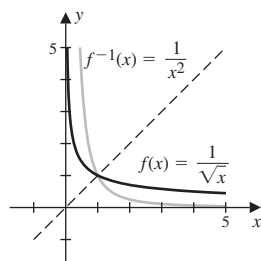


31. Let $f(x) = \frac{1}{\sqrt{x}}$. Then f is one-to-one since

$$f(x_1) = f(x_2) \quad \text{implies} \quad \frac{1}{\sqrt{x_1}} = \frac{1}{\sqrt{x_2}} \quad \text{so} \quad \sqrt{x_2} = \sqrt{x_1} \quad \text{and} \quad x_1 = x_2.$$

To find $f^{-1} : y = \frac{1}{\sqrt{x}}$ implies $\sqrt{x} = \frac{1}{y}$ so $x = \frac{1}{y^2}$, and $f^{-1}(x) = \frac{1}{x^2}$.

Note: The domain of f is $(0, \infty)$, which is also the range of f^{-1} and the range of f is $(0, \infty)$, which is also the domain of f^{-1} .

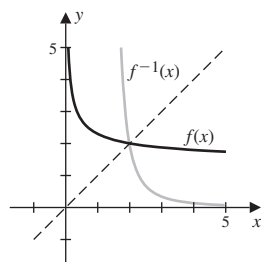


32. Let $f(x) = \frac{1}{\sqrt{x}} + 1$. Then f is one-to-one since

$$f(x_1) = f(x_2) \quad \text{implies} \quad \frac{1}{\sqrt{x_1}} + 1 = \frac{1}{\sqrt{x_2}} + 1 \quad \text{so} \quad \sqrt{x_2} = \sqrt{x_1} \quad \text{and} \quad x_1 = x_2.$$

To find $f^{-1} : y = \frac{1}{\sqrt{x}} + 1$ implies $\frac{1}{\sqrt{x}} = y - 1$ so $\sqrt{x} = \frac{1}{y - 1}$ implies $x = \frac{1}{(y - 1)^2}$,
and $f^{-1}(x) = \frac{1}{(x - 1)^2}$.

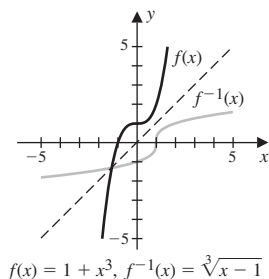
Note: The domain of f is $(0, \infty)$, which is also the range of f^{-1} and the range of f is $(1, \infty)$, which is also the domain of f^{-1} .



33. Let $f(x) = 1 + x^3$. Then f is one-to-one since

$$f(x_1) = f(x_2) \quad \text{implies} \quad 1 + x_1^3 = 1 + x_2^3 \quad \text{so} \quad x_1^3 = x_2^3 \quad \text{and} \quad x_1 = x_2.$$

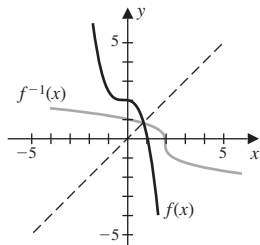
To find $f^{-1} : y = 1 + x^3$ implies $x^3 = y - 1$ so $x = (y - 1)^{\frac{1}{3}}$, and $f^{-1}(x) = (x - 1)^{\frac{1}{3}}$.



34. Let $f(x) = 2 - x^3$. Then f is one-to-one since

$$f(x_1) = f(x_2) \text{ implies } 2 - x_1^3 = 2 - x_2^3 \text{ so } x_1^3 = x_2^3 \text{ and } x_1 = x_2.$$

To find $f^{-1} : y = 2 - x^3$ implies $x^3 = 2 - y$ so $x = (2 - y)^{\frac{1}{3}}$, and $f^{-1}(x) = (2 - x)^{\frac{1}{3}}$.



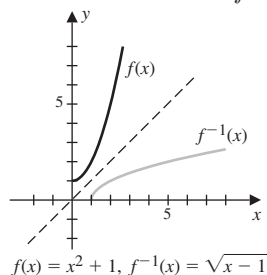
35. Let $f(x) = x^2 + 1, x \geq 0$. Then f is one-to-one since

$$f(x_1) = f(x_2) \text{ implies } x_1^2 + 1 = x_2^2 + 1 \text{ so } x_1^2 = x_2^2 \text{ and } x_1 = x_2,$$

since $x \geq 0$ (otherwise we could only conclude that $|x_1| = |x_2|$).

To find $f^{-1} : y = x^2 + 1$ implies $y - 1 = x^2$ so $x = \sqrt{y - 1}$, and $f^{-1}(x) = \sqrt{x - 1}$.

Note: The domain of f is $[0, \infty)$, which is also the range of f^{-1} and the range of f is $[1, \infty)$, which is also the domain of f^{-1} .



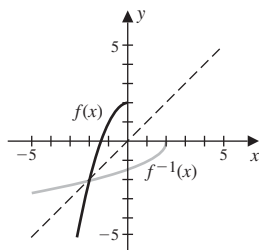
36. Let $f(x) = 1 - x^2, x \leq 0$. Then f is one-to-one since

$$f(x_1) = f(x_2) \text{ implies } 1 - x_1^2 = 1 - x_2^2 \text{ so } x_1^2 = x_2^2 \text{ and } x_1 = x_2,$$

since $x \leq 0$ (otherwise we could only conclude that $|x_1| = |x_2|$).

To find $f^{-1} : y = 1 - x^2$ implies $y - 1 = -x^2$ so $x^2 = 1 - y$ implies $x = -\sqrt{1 - y}$, and $f^{-1}(x) = -\sqrt{1 - x}$.

Note: The domain of f is $(-\infty, 0]$, which is also the range of f^{-1} and the range of f is $(-\infty, 1]$, which is also the domain of f^{-1} .



37. (a) The graph of the function $f(x) = |2 - x|$ is V-shaped, opens upward and the point is at $(2, 0)$. The line $y = 1$, for example, must cross the curve two times, and $|2 - x| = 1$ implies $2 - x = 1$ or $2 - x = -1$ so $x = 1$ or $x = 3$. So the function is not one-to-one.

- (b) The function is one-to-one on the interval $[2, \infty)$.

f^{-1} for f restricted on $[2, \infty)$: Since $2 - x \leq 0$ for $x \geq 2$,

$$y = -(2 - x) = x - 2 \text{ implies } x = y + 2, \text{ so } f^{-1}(x) = x + 2.$$

The domain of f^{-1} is the range of f , which equals $[0, \infty)$.

$$f^{-1}(x) = x + 2, \text{ for } x \geq 0$$

38. (a) For $f(x) = -|x - 3| - 2$, $f(2) = -|-1| - 2 = -3$ and $f(4) = -|1| - 2 = -3$ so the function is not one-to-one.

- (b) The function is one-to-one on the interval $[3, \infty)$.

f^{-1} for f restricted on $[3, \infty)$: Since $x - 3 \geq 0$ for $x \geq 3$,

$$y = -(x - 3) - 2 = -x + 1 \text{ implies } x = 1 - y, \text{ so } f^{-1}(x) = 1 - x.$$

The domain of f^{-1} is the range of f , which equals $(-\infty, -2]$.

39. (a) We have $f(x) = x^2 - 2x = x(x - 2)$, and $f(x) = 0$ for both $x = 0$ and $x = 2$, so f is not one-to-one. The graph is a parabola that opens upward and has vertex $(1, -1)$.

- (b) The function is one-to-one on the interval $[1, \infty)$.

f^{-1} for f restricted on $[1, \infty)$:

$$y = x^2 - 2x = x^2 - 2x + 1 - 1 = (x - 1)^2 - 1 \text{ implies } y + 1 = (x - 1)^2$$

if and only if

$$x - 1 = \sqrt{y + 1} \text{ implies } x = \sqrt{y + 1} + 1, \text{ so } f^{-1}(x) = \sqrt{x + 1} + 1.$$

The domain of f^{-1} is $[-1, \infty)$ which is also the range of f .

$$f^{-1}(x) = 1 + \sqrt{x + 1}, \text{ for } x \geq -1.$$

40. (a) For the function $f(x) = x^2 - 2x + 3 = (x + 1)(x - 3)$ we have both $f(0) = 3$ and $f(2) = 3$, so f is not one-to-one.

- (b) Since

$$\begin{aligned} f(x) &= x^2 - 2x + 3 \\ &= x^2 - 2x + 1 - 1 + 3 \\ &= (x - 1)^2 + 2 \end{aligned}$$