Solutions to Exercises, Section 5.1

You should be able to do Exercises 1–4 without a calculator.

1 Evaluate $\cos^{-1}\frac{1}{2}$.

solution $\cos\frac{\pi}{3} = \frac{1}{2}$; thus $\cos^{-1}\frac{1}{2} = \frac{\pi}{3}$.

2 Evaluate $\sin^{-1}\frac{1}{2}$.

solution $\sin\frac{\pi}{6} = \frac{1}{2}$; thus $\sin^{-1}\frac{1}{2} = \frac{\pi}{6}$.

3 Evaluate $\tan^{-1}(-1)$.

solution $\tan\left(-\frac{\pi}{4}\right) = -1$; thus

$$\tan^{-1}(-1) = -\frac{\pi}{4}.$$

4 Evaluate $\tan^{-1}(-\sqrt{3})$.

solution $\tan(-\frac{\pi}{3}) = -\sqrt{3}$; thus $\tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3}$.

Exercises 5–16 emphasize the importance of understanding inverse notation as well as the importance of parentheses in determining the order of operations.

5 For $x = 0.3$, evaluate each of the following:

(a) $\cos^{-1} x$

(b) $(\cos x)^{-1}$

(c) $\cos(x^{-1})$

(d) $(\cos^{-1} x)^{-1}$

solution

(a) $\cos^{-1} 0.3 \approx 1.2661$

(b) $(\cos 0.3)^{-1} = \dfrac{1}{\cos 0.3} \approx 1.04675$

(c) $\cos(0.3^{-1}) = \cos \frac{1}{0.3} \approx -0.981674$

(d) $(\cos^{-1} 0.3)^{-1} = \dfrac{1}{\cos^{-1} 0.3} \approx 0.789825$

6 For $x = 0.4$, evaluate each of the following:

(a) $\cos^{-1} x$

(b) $(\cos x)^{-1}$

(c) $\cos(x^{-1})$

(d) $(\cos^{-1} x)^{-1}$

solution

(a) $\cos^{-1} 0.4 \approx 1.15928$

(b) $(\cos 0.4)^{-1} = \dfrac{1}{\cos 0.4} \approx 1.0857$

(c) $\cos(0.4^{-1}) = \cos \frac{1}{0.4} \approx -0.801144$

(d) $(\cos^{-1} 0.4)^{-1} = \dfrac{1}{\cos^{-1} 0.4} \approx 0.862605$

7 For $x = \frac{1}{7}$, evaluate each of the following:

(a) $\sin^{-1} x$

(b) $(\sin x)^{-1}$

(c) $\sin(x^{-1})$

(d) $(\sin^{-1} x)^{-1}$

solution

(a) $\sin^{-1} \frac{1}{7} \approx 0.143348$

(b) $(\sin \frac{1}{7})^{-1} = \dfrac{1}{\sin \frac{1}{7}} \approx 7.02387$

(c) $\sin\Big(\big(\frac{1}{7}\big)^{-1} \Big) = \sin 7 \approx 0.656987$

(d) $(\sin^{-1} \frac{1}{7})^{-1} = \dfrac{1}{\sin^{-1} \frac{1}{7}} \approx 6.97605$

8 For $x = \frac{1}{8}$, evaluate each of the following:

(a) $\sin^{-1} x$

(b) $(\sin x)^{-1}$

(c) $\sin(x^{-1})$

(d) $(\sin^{-1} x)^{-1}$

solution

(a) $\sin^{-1} \frac{1}{8} \approx 0.125328$

(b) $(\sin \frac{1}{8})^{-1} = \dfrac{1}{\sin \frac{1}{8}} \approx 8.02087$

(c) $\sin\Big(\big(\frac{1}{8}\big)^{-1} \Big) = \sin 8 \approx 0.989358$

(d) $(\sin^{-1} \frac{1}{8})^{-1} = \dfrac{1}{\sin^{-1} \frac{1}{8}} \approx 7.97907$

9 For $x = 2$, evaluate each of the following:

(a) $\tan^{-1} x$

(b) $(\tan x)^{-1}$

(c) $\tan(x^{-1})$

(d) $(\tan^{-1} x)^{-1}$

solution

(a) $\tan^{-1} 2 \approx 1.10715$

(b) $(\tan 2)^{-1} = \dfrac{1}{\tan 2} \approx -0.457658$

(c) $\tan(2^{-1}) = \tan \frac{1}{2} \approx 0.546302$

(d) $(\tan^{-1} 2)^{-1} = \dfrac{1}{\tan^{-1} 2} \approx 0.903221$

10 For $x = 3$, evaluate each of the following:

(a) $\tan^{-1} x$

(b) $(\tan x)^{-1}$

(c) $\tan(x^{-1})$

(d) $(\tan^{-1} x)^{-1}$

solution

(a) $\tan^{-1} 3 \approx 1.24905$

(b) $(\tan 3)^{-1} = \dfrac{1}{\tan 3} \approx -7.01525$

(c) $\tan(3^{-1}) = \tan \frac{1}{3} \approx 0.346254$

(d) $(\tan^{-1} 3)^{-1} = \dfrac{1}{\tan^{-1} 3} \approx 0.800611$

11 For $x = 4$, evaluate each of the following:

(a) $\big(\cos(x^{-1})\big)^{-1}$

(b) $\cos^{-1}(x^{-1})$

(c) $\big(\cos^{-1}(x^{-1})\big)^{-1}$

solution

(a) $\big(\cos(4^{-1})\big)^{-1} = \big(\cos \frac{1}{4}\big)^{-1} = \dfrac{1}{\cos \frac{1}{4}} \approx 1.03209$

(b) $\cos^{-1}(4^{-1}) = \cos^{-1} \dfrac{1}{4} \approx 1.31812$

(c) $\big(\cos^{-1}(4^{-1})\big)^{-1} = \big(\cos^{-1} \frac{1}{4}\big)^{-1} = \dfrac{1}{\cos^{-1} \frac{1}{4}} \approx 0.758659$

12 For $x = 5$, evaluate each of the following:

(a) $\big(\cos(x^{-1})\big)^{-1}$

(b) $\cos^{-1}(x^{-1})$

(c) $\big(\cos^{-1}(x^{-1})\big)^{-1}$

solution

(a) $\big(\cos(5^{-1})\big)^{-1} = \big(\cos\frac{1}{5}\big)^{-1} = \dfrac{1}{\cos\frac{1}{5}} \approx 1.02034$

(b) $\cos^{-1}(5^{-1}) = \cos^{-1}\frac{1}{5} \approx 1.36944$

(c) $\big(\cos^{-1}(5^{-1})\big)^{-1} = \big(\cos^{-1}\frac{1}{5}\big)^{-1} = \dfrac{1}{\cos^{-1}\frac{1}{5}} \approx 0.730226$

13 For $x = 6$, evaluate each of the following:

(a) $\left(\sin(x^{-1})\right)^{-1}$

(b) $\sin^{-1}(x^{-1})$

(c) $\left(\sin^{-1}(x^{-1})\right)^{-1}$

solution

(a) $\left(\sin(6^{-1})\right)^{-1} = \left(\sin\frac{1}{6}\right)^{-1} = \dfrac{1}{\sin\frac{1}{6}} \approx 6.02787$

(b) $\sin^{-1}(6^{-1}) = \sin^{-1}\frac{1}{6} \approx 0.167448$

(c) $\left(\sin^{-1}(6^{-1})\right)^{-1} = \left(\sin^{-1}\frac{1}{6}\right)^{-1} = \dfrac{1}{\sin^{-1}\frac{1}{6}} \approx 5.972$

14 For $x = 9$, evaluate each of the following:

(a) $\big(\sin(x^{-1})\big)^{-1}$

(b) $\sin^{-1}(x^{-1})$

(c) $\big(\sin^{-1}(x^{-1})\big)^{-1}$

solution

(a) $\big(\sin(9^{-1})\big)^{-1} = \big(\sin\frac{1}{9}\big)^{-1} = \dfrac{1}{\sin\frac{1}{9}} \approx 9.01855$

(b) $\sin^{-1}(9^{-1}) = \sin^{-1}\frac{1}{9} \approx 0.111341$

(c)

$$\begin{aligned}\big(\sin^{-1}(9^{-1})\big)^{-1} &= \Big(\sin^{-1}\frac{1}{9}\Big)^{-1}\\ &= \frac{1}{\sin^{-1}\frac{1}{9}}\\ &\approx 8.98142\end{aligned}$$

15 For $x = 0.1$, evaluate each of the following:

(a) $\big(\tan(x^{-1})\big)^{-1}$

(b) $\tan^{-1}(x^{-1})$

(c) $\big(\tan^{-1}(x^{-1})\big)^{-1}$

solution

(a) $\big(\tan(0.1^{-1})\big)^{-1} = \big(\tan 10\big)^{-1} = \dfrac{1}{\tan 10} \approx 1.54235$

(b) $\tan^{-1}(0.1^{-1}) = \tan^{-1} 10 \approx 1.47113$

(c)

$$\begin{aligned}\big(\tan^{-1}(0.1^{-1})\big)^{-1} &= \big(\tan^{-1} 10\big)^{-1} \\ &= \frac{1}{\tan^{-1} 10} \\ &\approx 0.679751\end{aligned}$$

16 For $x = 0.2$, evaluate each of the following:

(a) $\left(\tan(x^{-1})\right)^{-1}$

(b) $\tan^{-1}(x^{-1})$

(c) $\left(\tan^{-1}(x^{-1})\right)^{-1}$

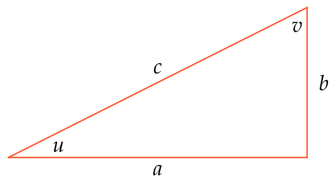
solution

(a) $\left(\tan(0.2^{-1})\right)^{-1} = \left(\tan 5\right)^{-1} = \dfrac{1}{\tan 5} \approx -0.295813$

(b) $\tan^{-1}(0.2^{-1}) = \tan^{-1} 5 \approx 1.3734$

(c)

$$\begin{aligned}\left(\tan^{-1}(0.2^{-1})\right)^{-1} &= \left(\tan^{-1} 5\right)^{-1} \\ &= \frac{1}{\tan^{-1} 5} \\ &\approx 0.72812\end{aligned}$$



Use the right triangle above for Exercises 17–24. This triangle is not drawn to scale corresponding to the data in the exercises.

17 Suppose $a = 2$ and $c = 3$. Evaluate u in radians.

solution Because the cosine of an angle in a right triangle equals the length of the adjacent side divided by the length of the hypotenuse, we have $\cos u = \frac{2}{3}$. Using a calculator working in radians, we then have

$$u = \cos^{-1} \frac{2}{3} \approx 0.841 \text{ radians.}$$

18 Suppose $a = 3$ and $c = 4$. Evaluate u in radians.

solution Because the cosine of an angle in a right triangle equals the length of the adjacent side divided by the length of the hypotenuse, we have $\cos u = \frac{3}{4}$. Using a calculator working in radians, we then have

$$u = \cos^{-1} \frac{3}{4} \approx 0.722734 \text{ radians.}$$

19 Suppose $a = 2$ and $c = 5$. Evaluate v in radians.

solution Because the sine of an angle in a right triangle equals the length of the opposite side divided by the length of the hypotenuse, we have $\sin v = \frac{2}{5}$. Using a calculator working in radians, we then have

$$v = \sin^{-1} \frac{2}{5} \approx 0.412 \text{ radians.}$$

20 Suppose $a = 3$ and $c = 5$. Evaluate v in radians.

solution Because the sine of an angle in a right triangle equals the length of the opposite side divided by the length of the hypotenuse, we have $\sin v = \frac{3}{5}$. Using a calculator working in radians, we then have

$$v = \sin^{-1} \frac{3}{5} \approx 0.643501 \text{ radians.}$$

21 Suppose $a = 5$ and $b = 4$. Evaluate u in degrees.

solution Because the tangent of an angle in a right triangle equals the length of the opposite side divided by the length of the adjacent side, we have $\tan u = \frac{4}{5}$. Using a calculator working in degrees, we then have

$$u = \tan^{-1}\frac{4}{5} \approx 38.7^\circ.$$

22 Suppose $a = 5$ and $b = 6$. Evaluate u in degrees.

solution Because the tangent of an angle in a right triangle equals the length of the opposite side divided by the length of the adjacent side, we have $\tan u = \frac{6}{5}$. Using a calculator working in degrees, we then have

$$u = \tan^{-1}\frac{6}{5} \approx 50.1944^\circ.$$

23 Suppose $a = 5$ and $b = 7$. Evaluate v in degrees.

solution Because the tangent of an angle in a right triangle equals the length of the opposite side divided by the length of the adjacent side, we have $\tan v = \frac{5}{7}$. Using a calculator working in degrees, we then have

$$v = \tan^{-1} \frac{5}{7} \approx 35.5°.$$

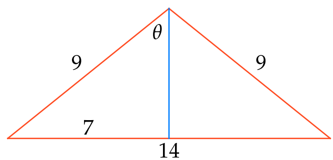
24 Suppose $a = 7$ and $b = 6$. Evaluate v in degrees.

solution Because the tangent of an angle in a right triangle equals the length of the opposite side divided by the length of the adjacent side, we have $\tan v = \frac{7}{6}$. Using a calculator working in degrees, we then have

$$v = \tan^{-1}\frac{7}{6} \approx 49.3987^\circ.$$

25 Find the angle between the two sides of length 9 in an isosceles triangle that has one side of length 14 and two sides of length 9.

solution Create a right triangle by dropping a perpendicular from the vertex to the base, as shown in the figure below.



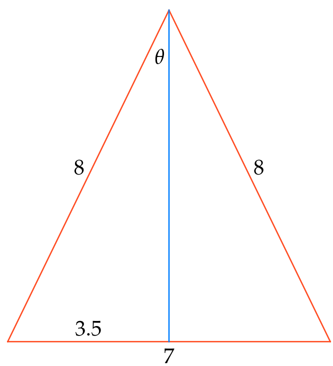
Let θ denote the angle between the perpendicular and a side of length 9. Because the base of the isosceles triangle has length 14, the side of the right triangle opposite the angle θ has length 7. Thus $\sin\theta = \frac{7}{9}$. Hence

$$\theta = \sin^{-1}\frac{7}{9} \approx 0.8911.$$

Thus the angle between the two sides of length 9 is approximately 1.7822 radians $(1.7822 = 2 \times 0.8911)$, which is approximately 102.1°.

26 Find the angle between the two sides of length 8 in an isosceles triangle that has one side of length 7 and two sides of length 8.

solution Create a right triangle by dropping a perpendicular from the vertex to the base, as shown in the figure below.



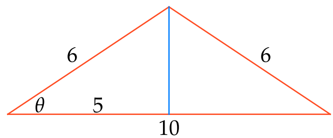
Let θ denote the angle between the perpendicular and a side of length 8. Because the base of the isosceles triangle has length 7, the side of the right triangle opposite the angle θ has length 3.5. Thus $\sin\theta = \frac{3.5}{8}$. Hence

$$\theta = \sin^{-1}\frac{3.5}{8} \approx 0.4528.$$

Thus the angle between the two sides of length 8 is approximately 0.9056 radians ($0.9056 = 2 \times 0.4528$), which is approximately 51.9°.

27 Find the angle between a side of length 6 and the side with length 10 in an isosceles triangle that has one side of length 10 and two sides of length 6.

solution Create a right triangle by dropping a perpendicular from the vertex to the base, as shown in the figure below.



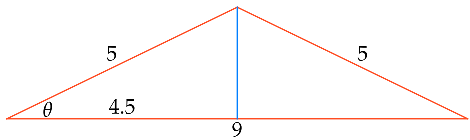
Let θ denote the angle between the side of length 10 and a side of length 6. Because the base of the isosceles triangle has length 10, the side of the right triangle adjacent to the angle θ has length 5. Thus $\cos\theta = \frac{5}{6}$. Hence

$$\theta = \cos^{-1}\frac{5}{6} \approx 0.58569.$$

Thus the angle between a side of length 6 and the side with length 10 is approximately 0.58569 radians, which is approximately $33.6°$.

28 Find the angle between a side of length 5 and the side with length 9 in an isosceles triangle that has one side of length 9 and two sides of length 5.

solution Create a right triangle by dropping a perpendicular from the vertex to the base, as shown in the figure below.



Let θ denote the angle between the side of length 9 and a side of length 5. Because the base of the isosceles triangle has length 9, the side of the right triangle adjacent to the angle θ has length 4.5. Thus $\cos\theta = \frac{4.5}{5} = 0.9$. Hence

$$\theta = \cos^{-1} 0.9 \approx 0.451027.$$

Thus the angle between a side of length 5 and the side with length 9 is approximately 0.451027 radians, which is approximately $25.8°$.

29 Find the smallest positive number θ such that $10^{\cos\theta} = 6$.

solution The equation above implies that $\cos\theta = \log 6$. Thus we take $\theta = \cos^{-1}(\log 6) \approx 0.67908$.

30 Find the smallest positive number θ such that $10^{\sin\theta} = 7$.

solution The equation above implies that $\sin\theta = \log 7$. Thus we take $\theta = \sin^{-1}(\log 7) \approx 1.00675$.

31 Find the smallest positive number θ such that $e^{\tan\theta} = 15$.

solution The equation above implies that $\tan\theta = \ln 15$. Thus we take $\theta = \tan^{-1}(\ln 15) \approx 1.21706$.

32 Find the smallest positive number θ such that $e^{\tan\theta} = 500$.

solution The equation above implies that $\tan\theta = \ln 500$. Thus we take $\theta = \tan^{-1}(\ln 500) \approx 1.41125$.

33 Find the second smallest positive number θ such that $4^{\sin\theta} = 3$.

solution Take the log of both sides of the equation above, getting

$$(\sin\theta)(\log 4) = \log 3,$$

which implies that

$$\sin\theta = \frac{\log 3}{\log 4}.$$

The smallest positive number θ satisfying this equation is $\sin^{-1}\frac{\log 3}{\log 4}$. The second smallest positive number satisfying this equation is $\pi - \sin^{-1}\frac{\log 3}{\log 4} \approx 2.22673$.

34 Find the second smallest positive number θ such that $7^{\cos\theta} = 5$.

solution Take the log of both sides of the equation above, getting

$$(\cos\theta)(\log 7) = \log 5,$$

which implies that

$$\cos\theta = \frac{\log 5}{\log 7}.$$

The smallest positive number θ satisfying this equation is $\cos^{-1}\frac{\log 5}{\log 7}$. The second smallest positive number satisfying this equation is $2\pi - \cos^{-1}\frac{\log 5}{\log 7} \approx 5.68629$.

35 Find the smallest positive number y such that $\cos(\tan y) = 0.2$.

solution The equation above implies that we should choose $\tan y = \cos^{-1} 0.2 \approx 1.36944$. Thus we should choose $y \approx \tan^{-1} 1.36944 \approx 0.94007$.

36 Find the smallest positive number y such that $\sin(\tan y) = 0.6$.

solution The equation above implies that we should choose $\tan y = \sin^{-1} 0.6 \approx 0.643501$. Thus we should choose $y \approx \tan^{-1} 0.643501 \approx 0.571793$.

37 Find the smallest positive number x such that

$$\sin^2 x - 3\sin x + 1 = 0.$$

solution Write $y = \sin x$. Then the equation above can be rewritten as

$$y^2 - 3y + 1 = 0.$$

Using the quadratic formula, we find that the solutions to this equation are

$$y = \frac{3+\sqrt{5}}{2} \approx 2.61803$$

and

$$y = \frac{3-\sqrt{5}}{2} \approx 0.381966.$$

Thus $\sin x \approx 2.61803$ or $\sin x \approx 0.381966$. However, there is no real number x such that $\sin x \approx 2.61803$ (because $\sin x$ is at most 1 for every real number x), and thus we must have $\sin x \approx 0.381966$. Thus $x \approx \sin^{-1} 0.381966 \approx 0.39192$.

38 Find the smallest positive number x such that

$$\sin^2 x - 4\sin x + 2 = 0.$$

solution Write $y = \sin x$. Then the equation above can be rewritten as

$$y^2 - 4y + 2 = 0.$$

Using the quadratic formula, we find that the solutions to this equation are

$$y = 2 + \sqrt{2} \approx 3.41421$$

and

$$y = 2 - \sqrt{2} \approx 0.585786.$$

Thus $\sin x \approx 3.41421$ or $\sin x \approx 0.585786$. However, there is no real number x such that $\sin x \approx 3.41421$ (because $\sin x$ is at most 1 for every real number x), and thus we must have $\sin x \approx 0.585786$. Thus $x \approx \sin^{-1} 0.585786 \approx 0.62585$.

39 Find the smallest positive number x such that

$$\cos^2 x - 0.5\cos x + 0.06 = 0.$$

solution Write $y = \cos x$. Then the equation above can be rewritten as

$$y^2 - 0.5y + 0.06 = 0.$$

Using the quadratic formula or factorization, we find that the solutions to this equation are

$$y = 0.2 \quad \text{and} \quad y = 0.3.$$

Thus $\cos x = 0.2$ or $\cos x = 0.3$, which suggests that we choose $x = \cos^{-1} 0.2$ or $x = \cos^{-1} 0.3$. Because arccosine is a decreasing function, $\cos^{-1} 0.3$ is smaller than $\cos^{-1} 0.2$. Because we want to find the smallest positive value of x satisfying the original equation, we choose $x = \cos^{-1} 0.3 \approx 1.2661$.

40 Find the smallest positive number x such that

$$\cos^2 x - 0.7\cos x + 0.12 = 0.$$

solution Write $y = \cos x$. Then the equation above can be rewritten as

$$y^2 - 0.7y + 0.12 = 0.$$

Using the quadratic formula or factorization, we find that the solutions to this equation are

$$y = 0.3 \quad \text{and} \quad y = 0.4.$$

Thus $\cos x = 0.3$ or $\cos x = 0.4$, which suggests that we choose $x = \cos^{-1} 0.3$ or $x = \cos^{-1} 0.4$. Because arccosine is a decreasing function, $\cos^{-1} 0.4$ is smaller than $\cos^{-1} 0.3$. Because we want to find the smallest positive value of x satisfying the original equation, we choose $x = \cos^{-1} 0.4 \approx 1.15928$.