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CHAPTER 1
Functions and Their Graphs

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Functions and Their Graphs

Section 1.1 Rectangular Coordinates

1. Cartesian

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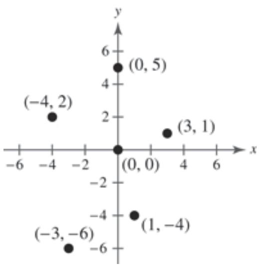
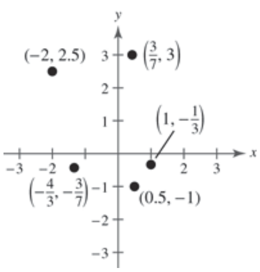
5.



6.



7. $(-3, 4)$

8. $(-12, 0)$

9. $x > 0$ and $y < 0$ in Quadrant IV.

10. $x < 0$ and $y < 0$ in Quadrant III.

11. $x = -4$ and $y > 0$ in Quadrant II.

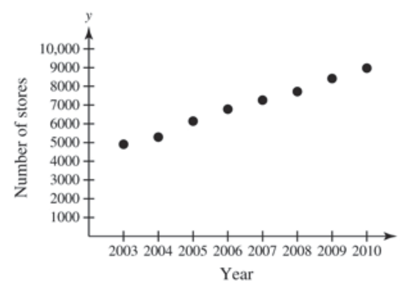
12. $y < -5$ in Quadrant III or IV.

13. $(x, -y)$ is in the second Quadrant means that (x, y) is in Quadrant III.

14. (x, y), $xy > 0$ means x and y have the same signs. This occurs in Quadrant I or III.

15.

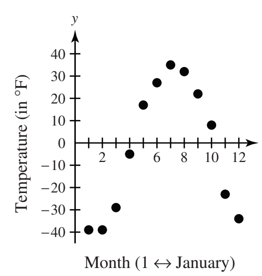
Year, x	Number of Stores, y
2003	4906
2004	5289
2005	6141
2006	6779
2007	7262
2008	7720
2009	8416
2010	8970



Year $(t = 3 \leftrightarrow 2003)$

16.

Month, x	Temperature, y
1	−39
2	−39
3	−29
4	−5
5	17
6	27
7	35
8	32
9	22
10	8
11	−23
12	−34



17. $$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3 - (-2))^2 + (-6 - 6)^2} \\ &= \sqrt{(5)^2 + (-12)^2} \\ &= \sqrt{25 + 144} \\ &= 13 \text{ units} \end{aligned}$$

18. $$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(0 - 8)^2 + (20 - 5)^2} \\ &= \sqrt{(-8)^2 + (15)^2} \\ &= \sqrt{64 + 225} \\ &= \sqrt{289} \\ &= 17 \text{ units} \end{aligned}$$

19. $$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-5 - 1)^2 + (-1 - 4)^2} \\ &= \sqrt{(-6)^2 + (-5)^2} \\ &= \sqrt{36 + 25} \\ &= \sqrt{61} \text{ units} \end{aligned}$$

20. $$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3 - 1)^2 + (-2 - 3)^2} \\ &= \sqrt{(2)^2 + (-5)^2} \\ &= \sqrt{4 + 25} \\ &= \sqrt{29} \text{ units} \end{aligned}$$

21. $$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{\left(2 - \frac{1}{2}\right)^2 + \left(-1 - \frac{4}{3}\right)^2} \\ &= \sqrt{\left(\frac{3}{2}\right)^2 + \left(-\frac{7}{3}\right)^2} \\ &= \sqrt{\frac{9}{4} + \frac{49}{9}} \\ &= \sqrt{\frac{277}{36}} \\ &= \frac{\sqrt{277}}{6} \text{ units} \end{aligned}$$

22. $$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-3.9 - 9.5)^2 + (8.2 - (-2.6))^2} \\ &= \sqrt{(-13.4)^2 + (10.8)^2} \\ &= \sqrt{179.56 + 116.64} \\ &= \sqrt{296.2} \\ &\approx 17.21 \text{ units} \end{aligned}$$

23. (a) $(1, 0), (13, 5)$

$$\text{Distance} = \sqrt{(13 - 1)^2 + (5 - 0)^2} = \sqrt{12^2 + 5^2} = \sqrt{169} = 13$$

$(13, 5), (13, 0)$

$$\text{Distance} = |5 - 0| = |5| = 5$$

$(1, 0), (13, 0)$

$$\text{Distance} = |1 - 13| = |-12| = 12$$

(b) $5^2 + 12^2 = 25 + 144 = 169 = 13^2$

24. (a) The distance between $(-1, 1)$ and $(9, 1)$ is 10.

The distance between $(9, 1)$ and $(9, 4)$ is 3.

The distance between $(-1, 1)$ and $(9, 4)$ is

$$\sqrt{(9 - (-1))^2 + (4 - 1)^2} = \sqrt{100 + 9} = \sqrt{109}.$$

(b) $10^2 + 3^2 = 109 = \left(\sqrt{109}\right)^2$

25. $$\begin{aligned} d_1 &= \sqrt{(4 - 2)^2 + (0 - 1)^2} = \sqrt{4 + 1} = \sqrt{5} \\ d_2 &= \sqrt{(4 + 1)^2 + (0 + 5)^2} = \sqrt{25 + 25} = \sqrt{50} \\ d_3 &= \sqrt{(2 + 1)^2 + (1 + 5)^2} = \sqrt{9 + 36} = \sqrt{45} \end{aligned}$$

$$\left(\sqrt{5}\right)^2 + \left(\sqrt{45}\right)^2 = \left(\sqrt{50}\right)^2$$

26. $$\begin{aligned} d_1 &= \sqrt{(3 - (-1))^2 + (5 - 3)^2} = \sqrt{16 + 4} = \sqrt{20} \\ d_2 &= \sqrt{(5 - 3)^2 + (1 - 5)^2} = \sqrt{4 + 16} = \sqrt{20} \\ d_3 &= \sqrt{(5 - (-1))^2 + (1 - 3)^2} = \sqrt{36 + 4} = \sqrt{40} \end{aligned}$$

$$\left(\sqrt{20}\right)^2 + \left(\sqrt{20}\right)^2 = \left(\sqrt{40}\right)^2$$

27. $d_1 = \sqrt{(1-3)^2 + (-3-2)^2} = \sqrt{4+25} = \sqrt{29}$

$d_2 = \sqrt{(3+2)^2 + (2-4)^2} = \sqrt{25+4} = \sqrt{29}$

$d_3 = \sqrt{(1+2)^2 + (-3-4)^2} = \sqrt{9+49} = \sqrt{58}$

$d_1 = d_2$

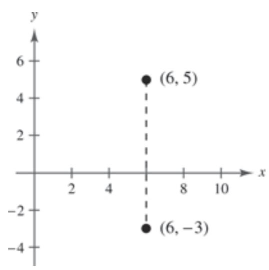
28. $d_1 = \sqrt{(4-2)^2 + (9-3)^2} = \sqrt{4+36} = \sqrt{40}$

$d_2 = \sqrt{(-2-4)^2 + (7-9)^2} = \sqrt{36+4} = \sqrt{40}$

$d_3 = \sqrt{(2-(-2))^2 + (3-7)^2} = \sqrt{16+16} = \sqrt{32}$

$d_1 = d_2$

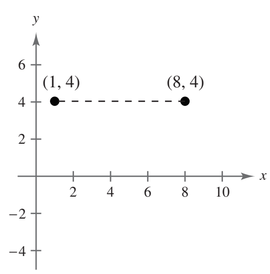
29. (a)



(b) $d = \sqrt{(5-(-3))^2 + (6-6)^2} = \sqrt{64} = 8$

(c) $\left(\dfrac{6+6}{2}, \dfrac{5+(-3)}{2}\right) = (6, 1)$

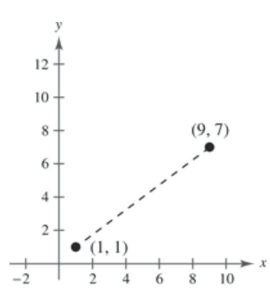
30. (a)



(b) $d = \sqrt{(4-4)^2 + (8-1)^2} = \sqrt{49} = 7$

(c) $\left(\dfrac{1+8}{2}, \dfrac{4+4}{2}\right) = \left(\dfrac{9}{2}, 4\right)$

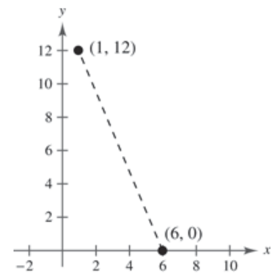
31. (a)



(b) $d = \sqrt{(9-1)^2 + (7-1)^2} = \sqrt{64+36} = 10$

(c) $\left(\dfrac{9+1}{2}, \dfrac{7+1}{2}\right) = (5, 4)$

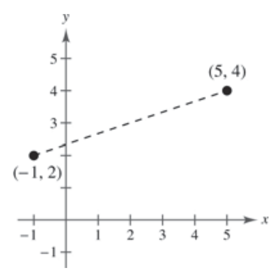
32. (a)



(b) $d = \sqrt{(1-6)^2 + (12-0)^2} = \sqrt{25+144} = 13$

(c) $\left(\dfrac{1+6}{2}, \dfrac{12+0}{2}\right) = \left(\dfrac{7}{2}, 6\right)$

33. (a)

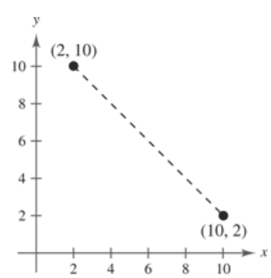


(b) $d = \sqrt{(5+1)^2 + (4-2)^2}$

$= \sqrt{36+4} = 2\sqrt{10}$

(c) $\left(\dfrac{-1+5}{2}, \dfrac{2+4}{2}\right) = (2, 3)$

34. (a)

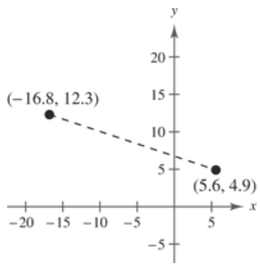


(b) $d = \sqrt{(2-10)^2 + (10-2)^2}$

$= \sqrt{64+64} = 8\sqrt{2}$

(c) $\left(\dfrac{2+10}{2}, \dfrac{10+2}{2}\right) = (6, 6)$

35. (a)

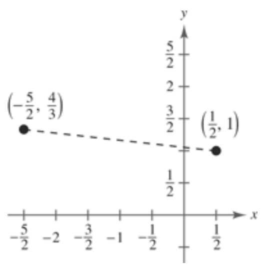


(b) $d = \sqrt{(-16.8 - 5.6)^2 + (12.3 - 4.9)^2}$

$= \sqrt{501.76 + 54.76} = \sqrt{556.52}$

(c) $\left(\dfrac{-16.8 + 5.6}{2}, \dfrac{12.3 + 4.9}{2}\right) = (-5.6, 8.6)$

36. (a)



(b) $d = \sqrt{\left(\dfrac{1}{2} + \dfrac{5}{2}\right)^2 + \left(1 - \dfrac{4}{3}\right)^2}$

$= \sqrt{9 + \dfrac{1}{9}} = \dfrac{\sqrt{82}}{3}$

(c) $\left(\dfrac{-(5/2) + (1/2)}{2}, \dfrac{(4/3) + 1}{2}\right) = \left(-1, \dfrac{7}{6}\right)$

37. $d = \sqrt{120^2 + 150^2}$

$= \sqrt{36{,}900}$

$= 30\sqrt{41}$

≈ 192.09

The plane flies about 192 kilometers.

38. $d = \sqrt{(42 - 18)^2 + (50 - 12)^2}$

$= \sqrt{24^2 + 38^2}$

$= \sqrt{2020}$

$= 2\sqrt{505}$

≈ 45

The pass is about 45 yards.

39. midpoint $= \left(\dfrac{2002 + 2010}{2}, \dfrac{19{,}564 + 35{,}123}{2}\right)$

$= (2006, 27{,}343.5)$

In 2006, the sales for the Coca-Cola Company were about $27,343.5 million.

40. midpoint $= \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

$= \left(\dfrac{2008 + 2010}{2}, \dfrac{1.89 + 2.83}{2}\right)$

$= (2009, 2.36)$

In 2009, the earnings per share for Big Lots, Inc. were about $2.36.

41. $(-2 + 2, -4 + 5) = (0, 1)$

$(2 + 2, -3 + 5) = (4, 2)$

$(-1 + 2, -1 + 5) = (1, 4)$

42. $(-3 + 6, 6 - 3) = (3, 3)$

$(-5 + 6, 3 - 3) = (1, 0)$

$(-3 + 6, 0 - 3) = (3, -3)$

$(-1 + 6, 3 - 3) = (5, 0)$

43. $(-7 + 4, -2 + 8) = (-3, 6)$

$(-2 + 4, 2 + 8) = (2, 10)$

$(-2 + 4, -4 + 8) = (2, 4)$

$(-7 + 4, -4 + 8) = (-3, 4)$

44. $(5 - 10, 8 - 6) = (-5, 2)$

$(3 - 10, 6 - 6) = (-7, 0)$

$(7 - 10, 6 - 6) = (-3, 0)$

45. (a) The minimum wage had the greatest increase from 2000 to 2010.

(b) Minimum wage in 1990: \$3.80

Minimum wage in 1995: \$4.25

Percent increase: $\left(\dfrac{4.25 - 3.80}{3.80}\right)(100) \approx 11.8\%$

Minimum wage in 1995: \$4.25

Minimum wage in 2011: \$7.25

Percent increase: $\left(\dfrac{7.25 - 4.25}{4.25}\right)(100) \approx 70.6\%$

So, the minimum wage increased 11.8% from 1990 to 1995 and 70.6% from 1995 to 2011.

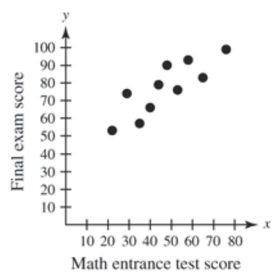
(c) $\begin{array}{l}\text{Minimum wage}\\ \text{in 2016}\end{array} = \begin{array}{l}\text{Minimum wage}\\ \text{in 2011}\end{array} + \begin{pmatrix}\text{Percent}\\ \text{increase}\end{pmatrix}\begin{pmatrix}\text{Minimum wage}\\ \text{in 2011}\end{pmatrix} \approx \$7.25 + 0.706(\$7.25) \approx \$12.37$

So, the minimum wage will be about \$12.37 in the year 2016.

(d) Answer will vary. *Sample answer:* No, the prediction is too high because it is likely that the percent increase over a 4-year period (2011–2016) will be less than the percent increase over a 16-year period (1995–2011).

46. (a)

x	y
22	53
29	74
35	57
40	66
44	79
48	90
53	76
58	93
65	83
76	99



(b) The point $(65, 83)$ represents an entrance exam score of 65.

(c) No. There are many variables that will affect the final exam score.

47. Because $x_m = \dfrac{x_1 + x_2}{2}$ and $y_m = \dfrac{y_1 + y_2}{2}$ we have:

$$2x_m = x_1 + x_2 \qquad 2y_m = y_1 + y_2$$
$$2x_m - x_1 = x_2 \qquad 2y_m - y_1 = y_2$$

So, $(x_2, y_2) = (2x_m - x_1, 2y_m - y_1)$.

48. (a) $(x_2, y_2) = (2x_m - x_1, 2y_m - y_1) = (2 \cdot 4 - 1, 2(-1) - (-2)) = (7, 0)$

(b) $(x_2, y_2) = (2x_m - x_1, 2y_m - y_1) = (2 \cdot 2 - (-5), 2 \cdot 4 - 11) = (9, -3)$

49. The midpoint of the given line segment is $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$.

The midpoint between (x_1, y_1) and $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$ is $\left(\dfrac{x_1 + \dfrac{x_1 + x_2}{2}}{2}, \dfrac{y_1 + \dfrac{y_1 + y_2}{2}}{2}\right) = \left(\dfrac{3x_1 + x_2}{4}, \dfrac{3y_1 + y_2}{4}\right)$.

The midpoint between $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$ and (x_2, y_2) is $\left(\dfrac{\dfrac{x_1 + x_2}{2} + x_2}{2}, \dfrac{\dfrac{y_1 + y_2}{2} + y_2}{2}\right) = \left(\dfrac{x_1 + 3x_2}{4}, \dfrac{y_1 + 3y_2}{4}\right)$.

So, the three points are $\left(\dfrac{3x_1 + x_2}{4}, \dfrac{3y_1 + y_2}{4}\right), \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$, and $\left(\dfrac{x_1 + 3x_2}{4}, \dfrac{y_1 + 3y_2}{4}\right)$.

50. (a) $\left(\dfrac{3x_1 + x_2}{4}, \dfrac{3y_1 + y_2}{4}\right) = \left(\dfrac{3 \cdot 1 + 4}{4}, \dfrac{3(-2) - 1}{4}\right) = \left(\dfrac{7}{4}, -\dfrac{7}{4}\right)$

$\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right) = \left(\dfrac{1 + 4}{2}, \dfrac{-2 - 1}{2}\right) = \left(\dfrac{5}{2}, -\dfrac{3}{2}\right)$

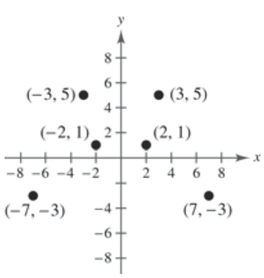
$\left(\dfrac{x_1 + 3x_2}{4}, \dfrac{y_1 + 3y_2}{4}\right) = \left(\dfrac{1 + 3 \cdot 4}{4}, \dfrac{-2 + 3(-1)}{4}\right) = \left(\dfrac{13}{4}, -\dfrac{5}{4}\right)$

(b) $\left(\dfrac{3x_1 + x_2}{4}, \dfrac{3y_1 + y_2}{4}\right) = \left(\dfrac{3(-2) + 0}{4}, \dfrac{3(-3) + 0}{4}\right) = \left(-\dfrac{3}{2}, -\dfrac{9}{4}\right)$

$\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right) = \left(\dfrac{-2 + 0}{2}, \dfrac{-3 + 0}{2}\right) = \left(-1, -\dfrac{3}{2}\right)$

$\left(\dfrac{x_1 + 3x_2}{4}, \dfrac{y_1 + 3y_2}{4}\right) = \left(\dfrac{-2 + 0}{4}, \dfrac{-3 + 0}{4}\right) = \left(-\dfrac{1}{2}, -\dfrac{3}{4}\right)$

51.



(a) The point is reflected through the y-axis.

(b) The point is reflected through the x-axis.

(c) The point is reflected through the origin.

52. (a)

First Set

$d(A, B) = \sqrt{(2 - 2)^2 + (3 - 6)^2} = \sqrt{9} = 3$

$d(B, C) = \sqrt{(2 - 6)^2 + (6 - 3)^2} = \sqrt{16 + 9} = 5$

$d(A, C) = \sqrt{(2 - 6)^2 + (3 - 3)^2} = \sqrt{16} = 4$

Because $3^2 + 4^2 = 5^2$, A, B, and C are the vertices of a right triangle.

Second Set

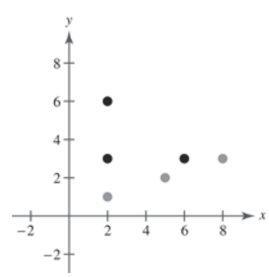
$d(A, B) = \sqrt{(8 - 5)^2 + (3 - 2)^2} = \sqrt{10}$

$d(B, C) = \sqrt{(5 - 2)^2 + (2 - 1)^2} = \sqrt{10}$

$d(A, C) = \sqrt{(8 - 2)^2 + (3 - 1)^2} = \sqrt{40}$

A, B, and C are the vertices of an isosceles triangle or are collinear: $\sqrt{10} + \sqrt{10} = 2\sqrt{10} = \sqrt{40}$.

(b)



First set: Not collinear

Second set: The points are collinear.

(c) If A, B, and C are collinear, then two of the distances will add up to the third distance.

53. No. It depends on the magnitude of the quantities measured.

54. The y-coordinate of a point on the x-axis is 0. The x-coordinates of a point on the y-axis is 0.

55. False, you would have to use the Midpoint Formula 15 times.

56. True. Two sides of the triangle have lengths $\sqrt{149}$ and the third side has a length of $\sqrt{18}$.

57. False. The polygon could be a rhombus. For example, consider the points $(4, 0)$, $(0, 6)$, $(-4, 0)$, and $(0, -6)$.

58. (a) Because (x_0, y_0) lies in Quadrant II, $(x_0, -y_0)$ must lie in Quadrant III. Matches (ii).

(b) Because (x_0, y_0) lies in Quadrant II, $(-2x_0, y_0)$ must lie in Quadrant I. Matches (iii).

(c) Because (x_0, y_0) lies in Quadrant II, $\left(x_0, \frac{1}{2}y_0\right)$ must lie in Quadrant II. Matches (iv).

(d) Because (x_0, y_0) lies in Quadrant II, $(-x_0, -y_0)$ must lie in Quadrant IV. Matches (i).

59. Use the Midpoint Formula to prove the diagonals of the parallelogram bisect each other.

$$\left(\frac{b+a}{2}, \frac{c+0}{2}\right) = \left(\frac{a+b}{2}, \frac{c}{2}\right)$$

$$\left(\frac{a+b+0}{2}, \frac{c+0}{2}\right) = \left(\frac{a+b}{2}, \frac{c}{2}\right)$$

Section 1.2 Graphs of Equations

1. solution or solution point

2. graph

3. intercepts

4. y-axis

5. circle; (h, k); r

6. numerical

7. (a) $(0, 2)$: $2 \stackrel{?}{=} \sqrt{0+4}$

$2 = 2$

Yes, the point *is* on the graph.

(b) $(5, 3)$: $3 \stackrel{?}{=} \sqrt{5+4}$

$3 \stackrel{?}{=} \sqrt{9}$

$3 = 3$

Yes, the point *is* on the graph.

8. (a) $(1, 2)$: $2 \stackrel{?}{=} \sqrt{5-1}$

$2 \stackrel{?}{=} \sqrt{4}$

$2 = 2$

Yes, the point *is* on the graph.

(b) $(5, 0)$: $0 \stackrel{?}{=} \sqrt{5-5}$

$0 = 0$

Yes, the point *is* on the graph.

9. (a) $(2, 0)$: $(2)^2 - 3(2) + 2 \stackrel{?}{=} 0$

$4 - 6 + 2 \stackrel{?}{=} 0$

$0 = 0$

Yes, the point *is* on the graph.

(b) $(-2, 8)$: $(-2)^2 - 3(-2) + 2 \stackrel{?}{=} 8$

$4 + 6 + 2 \stackrel{?}{=} 8$

$12 \neq 8$

No, the point *is not* on the graph.

10. (a) $(1, 5)$: $5 \stackrel{?}{=} 4 - |1 - 2|$

$5 \stackrel{?}{=} 4 - 1$

$5 \neq 3$

No, the point *is not* on the graph.

(b) $(6, 0)$: $0 \stackrel{?}{=} 4 - |6 - 2|$

$0 \stackrel{?}{=} 4 - 4$

$0 = 0$

Yes, the point *is* on the graph.

11. (a) $(2, 3)$: $3 \stackrel{?}{=} |2 - 1| + 2$

$3 \stackrel{?}{=} 1 + 2$

$3 = 3$

Yes, the point *is* on the graph.

(b) $(-1, 0)$: $0 \stackrel{?}{=} |-1 - 1| + 2$

$0 \stackrel{?}{=} 2 + 2$

$0 \neq 4$

No, the point *is not* on the graph.

12. (a) $(1, 2)$: $2(1) - 2 - 3 \stackrel{?}{=} 0$

$-3 \neq 0$

No, the point *is not* on the graph.

(b) $(1, -1)$: $2(1) - (-1) - 3 \stackrel{?}{=} 0$

$2 + 1 - 3 \stackrel{?}{=} 0$

$0 = 0$

Yes, the point *is* on the graph.

13. (a) $(3, -2)$: $(3)^2 + (-2)^2 \stackrel{?}{=} 20$

$9 + 4 \stackrel{?}{=} 20$

$13 \neq 20$

No, the point *is not* on the graph.

(b) $(-4, 2)$: $(-4)^2 + (2)^2 \stackrel{?}{=} 20$

$16 + 4 \stackrel{?}{=} 20$

$20 = 20$

Yes, the point *is* on the graph.

14. (a) $\left(2, -\frac{16}{3}\right)$: $\frac{1}{3}(2)^3 - 2(2)^2 \stackrel{?}{=} -\frac{16}{3}$

$\frac{1}{3} \cdot 8 - 2 \cdot 4 \stackrel{?}{=} -\frac{16}{3}$

$\frac{8}{3} - 8 \stackrel{?}{=} -\frac{16}{3}$

$\frac{8}{3} - \frac{24}{3} \stackrel{?}{=} -\frac{16}{3}$

$-\frac{16}{3} = -\frac{16}{3}$

Yes, the point *is* on the graph.

(b) $(-3, 9)$: $\frac{1}{3}(-3)^3 - 2(-3)^2 \stackrel{?}{=} 9$

$\frac{1}{3}(-27) - 2(9) \stackrel{?}{=} 9$

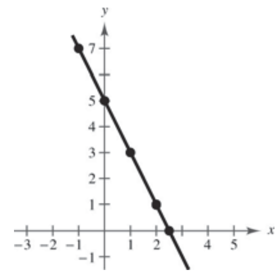
$-9 - 18 \stackrel{?}{=} 9$

$-27 \neq 9$

No, the point *is not* on the graph.

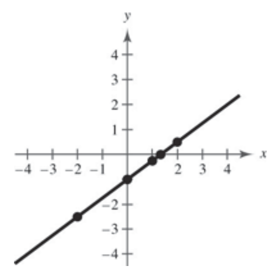
15. $y = -2x + 5$

x	-1	0	1	2	$\frac{5}{2}$
y	7	5	3	1	0
(x, y)	$(-1, 7)$	$(0, 5)$	$(1, 3)$	$(2, 1)$	$\left(\frac{5}{2}, 0\right)$



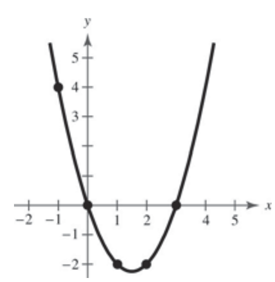
16. $y = \frac{3}{4}x - 1$

x	-2	0	1	$\frac{4}{3}$	2
y	$-\frac{5}{2}$	-1	$-\frac{1}{4}$	0	$\frac{1}{2}$
(x, y)	$\left(-2, -\frac{5}{2}\right)$	$(0, -1)$	$\left(1, -\frac{1}{4}\right)$	$\left(\frac{4}{3}, 0\right)$	$\left(2, \frac{1}{2}\right)$



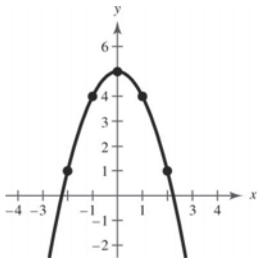
17. $y = x^2 - 3x$

x	-1	0	1	2	3
y	4	0	-2	-2	0
(x, y)	$(-1, 4)$	$(0, 0)$	$(1, -2)$	$(2, -2)$	$(3, 0)$



18. $y = 5 - x^2$

x	-2	-1	0	1	2
y	1	4	5	4	1
x, y	$(-2, 1)$	$(-1, 4)$	$(0, 5)$	$(1, 4)$	$(2, 1)$



19. x-intercept: $(3, 0)$

y-intercept: $(0, 9)$

20. x-intercepts: $(\pm 2, 0)$

y-intercept: $(0, 16)$

21. x-intercept: $(-2, 0)$

y-intercept: $(0, 2)$

22. x-intercept: $(4, 0)$

y-intercepts: $(0, \pm 2)$

23. x-intercept: $(1, 0)$

y-intercept: $(0, 2)$

24. x-intercepts: $(0, 0), (0, \pm 2)$

y-intercept: $(0, 0)$

25. $x^2 - y = 0$

$(-x)^2 - y = 0 \Rightarrow x^2 - y = 0 \Rightarrow$ y-axis symmetry

$x^2 - (-y) = 0 \Rightarrow x^2 + y = 0 \Rightarrow$ No x-axis symmetry

$(-x)^2 - (-y) = 0 \Rightarrow x^2 + y = 0 \Rightarrow$ No origin symmetry

26. $x - y^2 = 0$

$(-x) - y^2 = 0 \Rightarrow -x - y^2 = 0 \Rightarrow$ No y-axis symmetry

$x - (-y)^2 = 0 \Rightarrow x - y^2 = 0 \Rightarrow$ x-axis symmetry

$(-x) - (-y)^2 = 0 \Rightarrow -x - y^2 = 0 \Rightarrow$ No origin symmetry

27. $y = x^3$

$y = (-x)^3 \Rightarrow y = -x^3 \Rightarrow$ No y-axis symmetry

$-y = x^3 \Rightarrow y = -x^3 \Rightarrow$ No x-axis symmetry

$-y = (-x)^3 \Rightarrow -y = -x^3 \Rightarrow y = x^3 \Rightarrow$ Origin symmetry

28. $y = x^4 - x^2 + 3$

$y = (-x)^4 - (-x)^2 + 3 \Rightarrow y = x^4 - x^2 + 3 \Rightarrow$ y-axis symmetry

$-y = x^4 - x^2 + 3 \Rightarrow y = -x^4 + x^2 - 3 \Rightarrow$ No x-axis symmetry

$-y = (-x)^4 - (-x)^2 + 3 \Rightarrow y = -x^4 + x^2 - 3 \Rightarrow$ No origin symmetry

29. $y = \dfrac{x}{x^2 + 1}$

$y = \dfrac{-x}{(-x)^2 + 1} \Rightarrow y = \dfrac{-x}{x^2 + 1} \Rightarrow$ No y-axis symmetry

$-y = \dfrac{x}{x^2 + 1} \Rightarrow y = \dfrac{-x}{x^2 + 1} \Rightarrow$ No x-axis symmetry

$-y = \dfrac{-x}{(-x)^2 + 1} \Rightarrow -y = \dfrac{-x}{x^2 + 1} \Rightarrow y = \dfrac{x}{x^2 + 1} \Rightarrow$ Origin symmetry

30. $y = \dfrac{1}{1 + x^2}$

$y = \dfrac{1}{1 + (-x)^2} \Rightarrow y = \dfrac{1}{1 + x^2} \Rightarrow$ y-axis symmetry

$-y = \dfrac{1}{1 + x^2} \Rightarrow y = \dfrac{-1}{1 + x^2} \Rightarrow$ No x-axis symmetry

$-y = \dfrac{1}{1 + (-x)^2} \Rightarrow y = \dfrac{-1}{1 + x^2} \Rightarrow$ No origin symmetry

31. $xy^2 + 10 = 0$

$(-x)y^2 + 10 = 0 \Rightarrow -xy^2 + 10 = 0 \Rightarrow$ No y-axis symmetry

$x(-y)^2 + 10 = 0 \Rightarrow xy^2 + 10 = 0 \Rightarrow$ x-axis symmetry

$(-x)(-y)^2 + 10 = 0 \Rightarrow -xy^2 + 10 = 0 \Rightarrow$ No origin symmetry

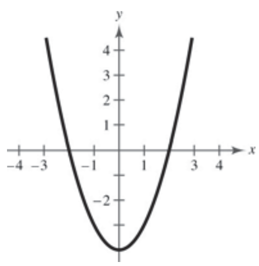
32. $xy = 4$

$(-x)y = 4 \Rightarrow xy = -4 \Rightarrow$ No y-axis symmetry

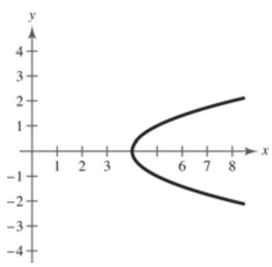
$x(-y) = 4 \Rightarrow xy = -4 \Rightarrow$ No x-axis symmetry

$(-x)(-y) = 4 \Rightarrow xy = 4 \Rightarrow$ Origin symmetry

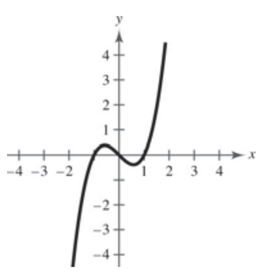
33.



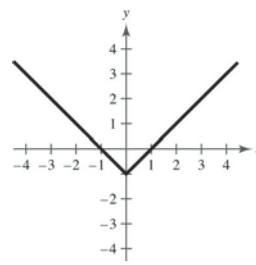
34.



35.



36.

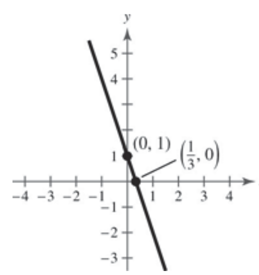


37. $y = -3x + 1$

x-intercept: $\left(\frac{1}{3}, 0\right)$

y-intercept: $(0, 1)$

No symmetry

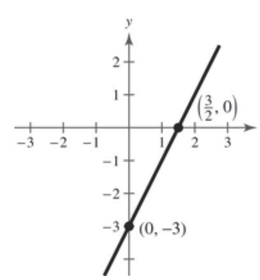


38. $y = 2x - 3$

x-intercept: $\left(\frac{3}{2}, 0\right)$

y-intercept: $(0, -3)$

No symmetry



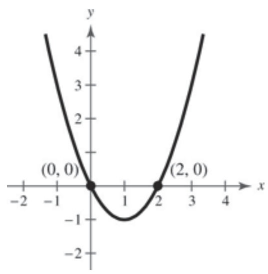
39. $y = x^2 - 2x$

x-intercepts: $(0, 0), (2, 0)$

y-intercept: $(0, 0)$

No symmetry

x	–1	0	1	2	3
y	3	0	–1	0	3

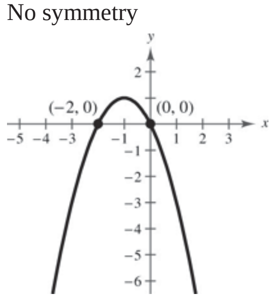


40. $y = -x^2 - 2x$

x-intercepts: $(-2, 0), (0, 0)$

y-intercept: $(0, 0)$

No symmetry



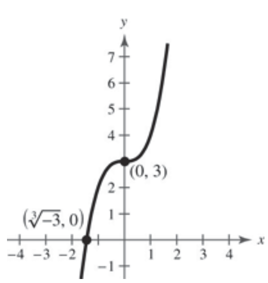
41. $y = x^3 + 3$

x-intercept: $\left(\sqrt[3]{-3}, 0\right)$

y-intercept: $(0, 3)$

No symmetry

x	–2	–1	0	1	2
y	–5	2	3	4	11

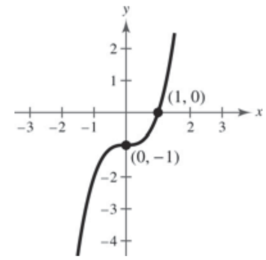


42. $y = x^3 - 1$

x-intercept: $(1, 0)$

y-intercept: $(0, -1)$

No symmetry



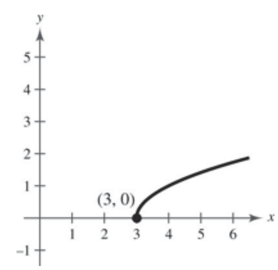
43. $y = \sqrt{x - 3}$

x-intercept: $(3, 0)$

y-intercept: none

No symmetry

x	3	4	7	12
y	0	1	2	3

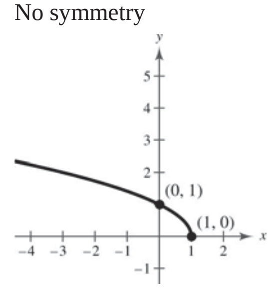


44. $y = \sqrt{1 - x}$

x-intercept: $(1, 0)$

y-intercept: $(0, 1)$

No symmetry



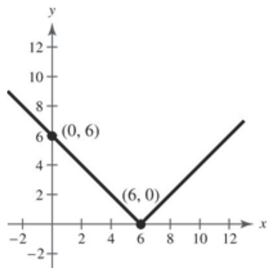
45. $y = |x - 6|$

x-intercept: $(6, 0)$

y-intercept: $(0, 6)$

No symmetry

x	–2	0	2	4	6	8	10
y	8	6	4	2	0	2	4

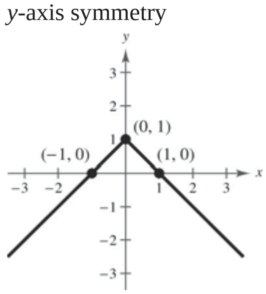


46. $y = 1 - |x|$

x-intercepts: $(1, 0), (-1, 0)$

y-intercept: $(0, 1)$

y-axis symmetry



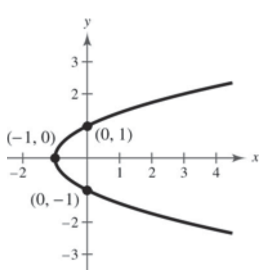
47. $x = y^2 - 1$

x-intercept: $(-1, 0)$

y-intercepts: $(0, -1), (0, 1)$

x-axis symmetry

x	–1	0	3
y	0	±1	±2

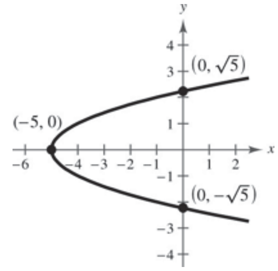


48. $x = y^2 - 5$

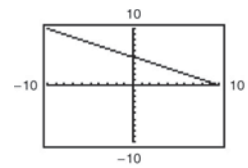
x-intercept: $(-5, 0)$

y-intercepts: $(0, \sqrt{5}), (0, -\sqrt{5})$

x-axis symmetry

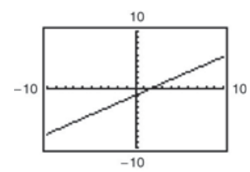


49. $y = 5 - \frac{1}{2}x$



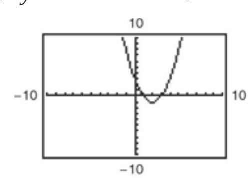
Intercepts: $(10, 0), (0, 5)$

50. $y = \frac{2}{3}x - 1$



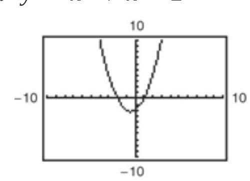
Intercepts: $(0, -1), \left(\frac{3}{2}, 0\right)$

51. $y = x^2 - 4x + 3$



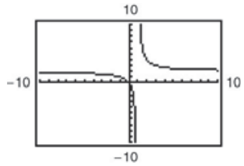
Intercepts: $(3, 0), (1, 0), (0, 3)$

52. $y = x^2 + x - 2$



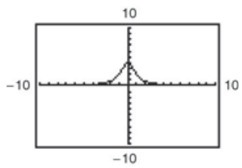
Intercepts: $(-2, 0), (1, 0), (0, -2)$

53. $y = \dfrac{2x}{x - 1}$



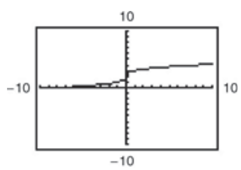
Intercept: $(0, 0)$

54. $y = \dfrac{4}{x^2 + 1}$



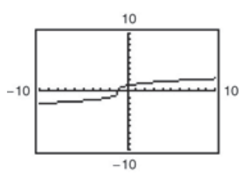
Intercept: $(0, 4)$

55. $y = \sqrt[3]{x} + 2$



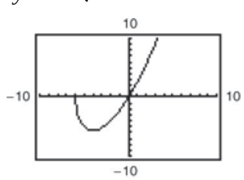
Intercepts: $(-8, 0), (0, 2)$

56. $y = \sqrt[3]{x + 1}$



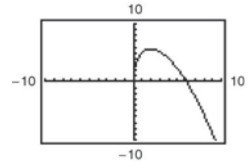
Intercepts: $(-1, 0), (0, 1)$

57. $y = x\sqrt{x + 6}$



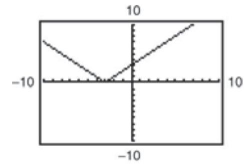
Intercepts: $(0, 0), (-6, 0)$

58. $y = (6 - x)\sqrt{x}$



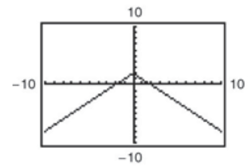
Intercepts: $(0, 0), (6, 0)$

59. $y = |x + 3|$



Intercepts: $(-3, 0), (0, 3)$

60. $y = 2 - |x|$



Intercepts: $(\pm 2, 0), (0, 2)$

61. Center: $(0, 0)$; Radius: 4

$$(x - 0)^2 + (y - 0)^2 = 4^2$$
$$x^2 + y^2 = 16$$

62. Center: $(0, 0)$; Radius: 5

$$(x - 0)^2 + (y - 0)^2 = 5^2$$
$$x^2 + y^2 = 25$$

63. Center: $(2, -1)$; Radius: 4

$$(x - 2)^2 + (y - (-1))^2 = 4^2$$
$$(x - 2)^2 + (y + 1)^2 = 16$$

64. Center: $(-7, -4)$; Radius: 7

$$(x - (-7))^2 + (y - (-4))^2 = 7^2$$
$$(x + 7)^2 + (y + 4)^2 = 49$$

65. Center: $(-1, 2)$; Solution point: $(0, 0)$

$$(x - (-1))^2 + (y - 2)^2 = r^2$$
$$(0 + 1)^2 + (0 - 2)^2 = r^2 \Rightarrow 5 = r^2$$
$$(x + 1)^2 + (y - 2)^2 = 5$$

66. Center: $(3, -2)$; Solution point: $(-1, 1)$

$$r = \sqrt{(3 - (-1))^2 + (-2 - 1)^2}$$
$$= \sqrt{4^2 + (-3)^2} = \sqrt{25} = 5$$
$$(x - 3)^2 + (y - (-2))^2 = 5^2$$
$$(x - 3)^2 + (y + 2)^2 = 25$$

67. Endpoints of a diameter: $(0, 0), (6, 8)$

Center: $\left(\dfrac{0 + 6}{2}, \dfrac{0 + 8}{2}\right) = (3, 4)$

$$(x - 3)^2 + (y - 4)^2 = r^2$$
$$(0 - 3)^2 + (0 - 4)^2 = r^2 \Rightarrow 25 = r^2$$
$$(x - 3)^2 + (y - 4)^2 = 25$$

68. Endpoints of a diameter: $(-4, -1), (4, 1)$

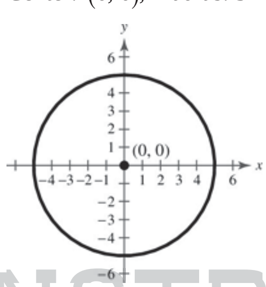
$$r = \frac{1}{2}\sqrt{(-4 - 4)^2 + (-1 - 1)^2}$$
$$= \frac{1}{2}\sqrt{(-8)^2 + (-2)^2}$$
$$= \frac{1}{2}\sqrt{64 + 4}$$
$$= \frac{1}{2}\sqrt{68} = \left(\frac{1}{2}\right)(2)\sqrt{17} = \sqrt{17}$$

Midpoint of diameter (center of circle):

$$\left(\frac{-4 + 4}{2}, \frac{-1 + 1}{2}\right) = (0, 0)$$
$$(x - 0)^2 + (y - 0)^2 = \left(\sqrt{17}\right)^2$$
$$x^2 + y^2 = 17$$

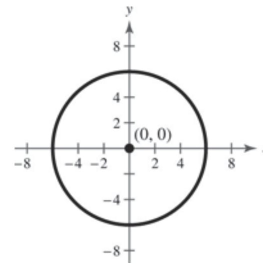
69. $x^2 + y^2 = 25$

Center: $(0, 0)$, Radius: 5



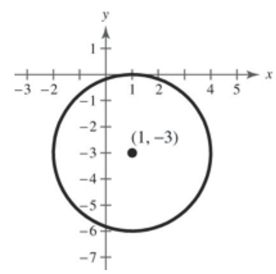
70. $x^2 + y^2 = 36$

Center: $(0, 0)$, Radius: 6



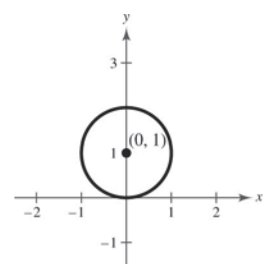
71. $(x - 1)^2 + (y + 3)^2 = 9$

Center: $(1, -3)$, Radius: 3



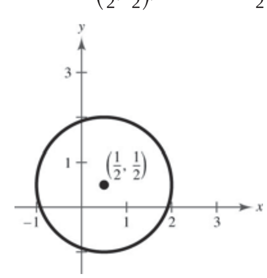
72. $x^2 + (y - 1)^2 = 1$

Center: $(0, 1)$, Radius: 1



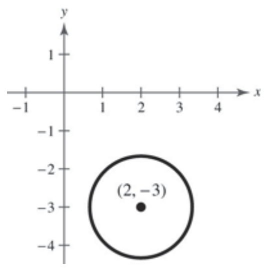
73. $\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{9}{4}$

Center: $\left(\frac{1}{2}, \frac{1}{2}\right)$, Radius: $\frac{3}{2}$

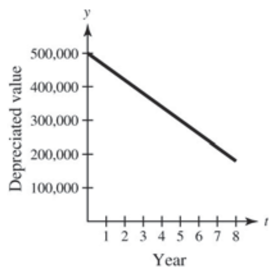


74. $(x - 2)^2 + (y + 3)^2 = \frac{16}{9}$

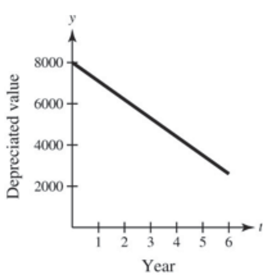
Center: $(2, -3)$, Radius: $\frac{4}{3}$



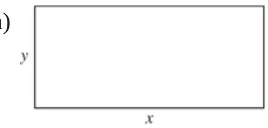
75. $y = 500{,}000 - 40{,}000t,\ 0 \le t \le 8$



76. $y = 8000 - 900t,\ 0 \le t \le 6$



77. (a)



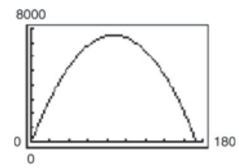
(b) $2x + 2y = \frac{1040}{3}$

$$2y = \frac{1040}{3} - 2x$$

$$y = \frac{520}{3} - x$$

$$A = xy = x\left(\frac{520}{3} - x\right)$$

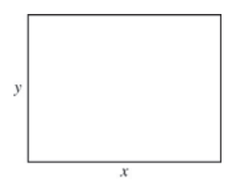
(c)



(d) When $x = y = 86\frac{2}{3}$ yards, the area is a maximum of $7511\frac{1}{9}$ square yards.

(e) A regulation NFL playing field is 120 yards long and $53\frac{1}{3}$ yards wide. The actual area is 6400 square yards.

78. (a)



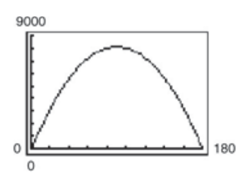
(b) $P = 360$ meters so:

$$2x + 2y = 360$$

$$w = y = 180 - x$$

$$A = lw = x(180 - x)$$

(c)

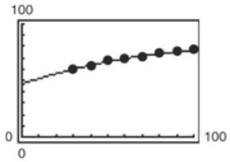


(d) $x = 90$ and $y = 90$

A square will give the maximum area of 8100 square meters.

(e) Answers will vary. *Sample answer:* The dimensions of a Major League Soccer field can vary between 110 and 120 yards in length and between 70 and 80 yards in width. A field of length 115 yards and width 75 yards would have an area of 8625 square yards.

79. (a)



Because the line is close to the points, the model fits the data well.

(b) Graphically: The point $(90, 75.4)$ represents a life expectancy of 75.4 years in 1990.

Algebraically: $$\begin{aligned} y &= -0.002t^2 + 0.5t + 46.6 \\ &= -0.002(90)^2 + 0.5(90) + 46.6 \\ &= 75.4 \end{aligned}$$

So, the life expectancy in 1990 was about 75.4 years.

(c) Graphically: The point $(94.6, 76.0)$ represents a life expectancy of 76 years during the year 1994.

Algebraically: $$\begin{aligned} y &= -0.002t^2 + 0.5t + 46.6 \\ 76.0 &= -0.002t^2 + 0.5t + 46.6 \\ 0 &= -0.002t^2 + 0.5t - 29.4 \end{aligned}$$

Use the quadratic formula to solve.

$$\begin{aligned} t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(0.5) \pm \sqrt{(0.5)^2 - 4(-0.002)(-29.4)}}{2(-0.002)} \\ &= \frac{-0.5 \pm \sqrt{0.0148}}{-0.004} \\ &= 125 \pm 30.4 \end{aligned}$$

So, $t = 94.6$ or $t = 155.4$. Since 155.4 is not in the domain, the solution is $t = 94.6$, which is the year 1994.

(d) When $t = 115$:

$$\begin{aligned} y &= -0.002t^2 + 0.5t + 46.6 \\ &= -0.002(115)^2 + (0.5)(115) + 46.6 \\ &= 77.65 \end{aligned}$$

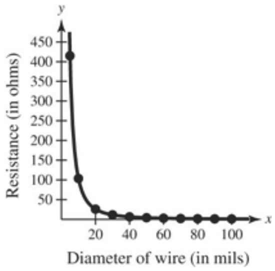
The life expectancy using the model is 77.65 years, which is slightly less than the given projection of 78.9 years.

(e) Answers will vary. *Sample answer:* No. Because the model is quadratic, the life expectancies begin to decrease after a certain point.

80. (a)

x	5	10	20	30	40	50	60	70	80	90	100
y	414.8	103.7	25.9	11.5	6.5	4.1	2.9	2.1	1.6	1.3	1.0

(b)



When $x = 85.5$, the resistance is about 1.4 ohms.

(c) When $x = 85.5$,

$$y = \frac{10{,}370}{(85.5)^2} = 1.4 \text{ ohms.}$$

(d) As the diameter of the copper wire increases, the resistance decreases.

81. $y = ax^2 + bx^3$

(a) $y = a(-x)^2 + b(-x)^3$

$= ax^2 - bx^3$

To be symmetric with respect to the y-axis; a can be any non-zero real number, b must be zero.

(b) $-y = a(-x)^2 + b(-x)^3$

$-y = ax^2 - bx^3$

$y = -ax^2 + bx^3$

To be symmetric with respect to the origin; a must be zero, b can be any non-zero real number.

82. x-axis symmetry:

$$x^2 + y^2 = 1$$
$$x^2 + (-y)^2 = 1$$
$$x^2 + y^2 = 1$$

y-axis symmetry:

$$x^2 + y^2 = 1$$
$$(-x)^2 + y^2 = 1$$
$$x^2 + y^2 = 1$$

Origin symmetry:

$$x^2 + y^2 = 1$$
$$(-x)^2 + (-y)^2 = 1$$
$$x^2 + y^2 = 1$$

So, the graph of the equation is symmetric with respect to the x-axis, the y-axis, and the origin.

Section 1.3 Linear Equations in Two Variables

1. linear

2. slope

3. point-slope

4. parallel

5. perpendicular

6. rate or rate of change

7. linear extrapolation

8. general

9. (a) $m = \frac{2}{3}$. Because the slope is positive, the line rises. Matches L_2.

(b) m is undefined. The line is vertical. Matches L_3.

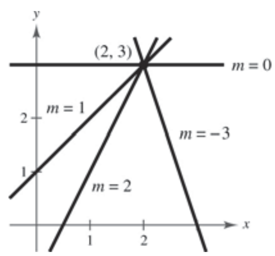
(c) $m = -2$. The line falls. Matches L_1.

10. (a) $m = 0$. The line is horizontal. Matches L_2.

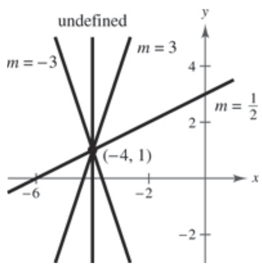
(b) $m = -\frac{3}{4}$. Because the slope is negative, the line falls. Matches L_1.

(c) $m = 1$. Because the slope is positive, the line rises. Matches L_3.

11.



12.



13. Two points on the line: $(0, 0)$ and $(4, 6)$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6}{4} = \frac{3}{2}$$

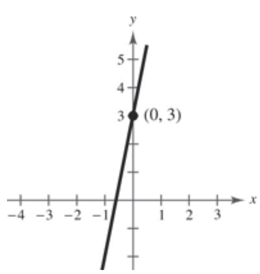
14. The line appears to go through $(0, 7)$ and $(7, 0)$.

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 7}{7 - 0} = -1$$

15. $y = 5x + 3$

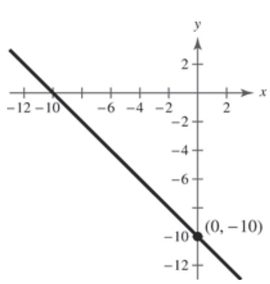
Slope: $m = 5$

y-intercept: $(0, 3)$



16. Slope: $m = -1$

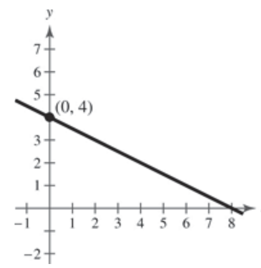
y-intercept: $(0, -10)$



17. $y = -\frac{1}{2}x + 4$

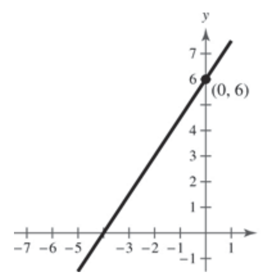
Slope: $m = -\frac{1}{2}$

y-intercept: $(0, 4)$



18. Slope: $m = \frac{3}{2}$

y-intercept: $(0, 6)$

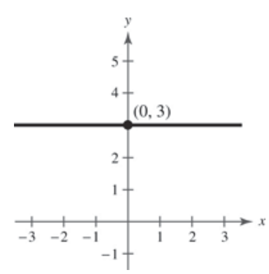


19. $y - 3 = 0$

$y = 3$, horizontal line

Slope: $m = 0$

y-intercept: $(0, 3)$

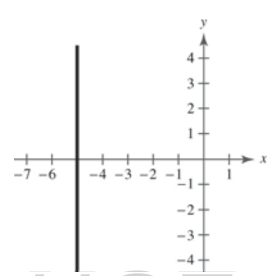


20. $x + 5 = 0$

$x = -5$

Slope: undefined (vertical line)

No y-intercept

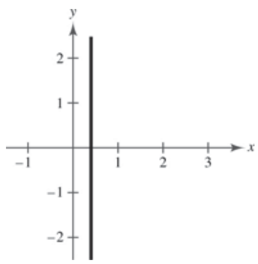


21. $5x - 2 = 0$

$x = \frac{2}{5}$, vertical line

Slope: undefined

No y-intercept



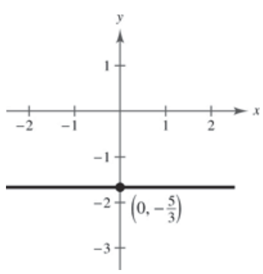
22. $3y + 5 = 0$

$3y = -5$

$y = -\frac{5}{3}$

Slope: $m = 0$

y-intercept: $\left(0, -\frac{5}{3}\right)$



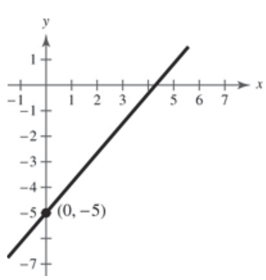
23. $7x - 6y = 30$

$-6y = -7x + 30$

$y = \frac{7}{6}x - 5$

Slope: $m = \frac{7}{6}$

y-intercept: $(0, -5)$



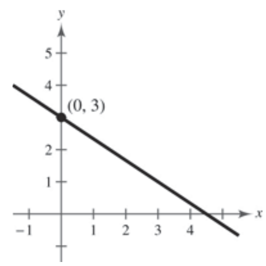
24. $2x + 3y = 9$

$3y = -2x + 9$

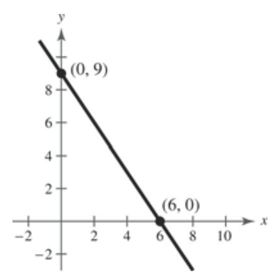
$y = -\frac{2}{3}x + 3$

Slope: $m = -\frac{2}{3}$

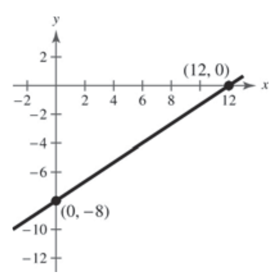
y-intercept: $(0, 3)$



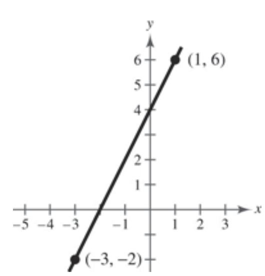
25. $m = \dfrac{0 - 9}{6 - 0} = \dfrac{-9}{6} = -\dfrac{3}{2}$



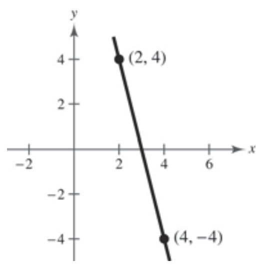
26. $m = \dfrac{-8 - 0}{0 - 12} = \dfrac{8}{12} = \dfrac{2}{3}$



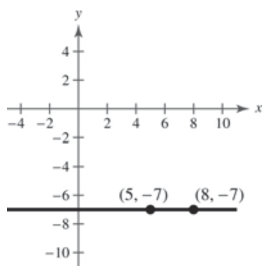
27. $m = \dfrac{6 - (-2)}{1 - (-3)} = \dfrac{8}{4} = 2$



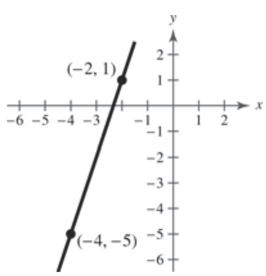
28. $m = \dfrac{-4 - 4}{4 - 2} = -4$



29. $m = \dfrac{-7 - (-7)}{8 - 5} = \dfrac{0}{3} = 0$

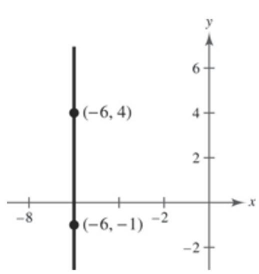


30. $m = \dfrac{-5 - 1}{-4 - (-2)} = 3$

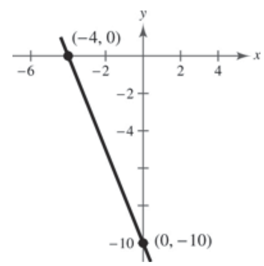


31. $m = \dfrac{4 - (-1)}{-6 - (-6)} = \dfrac{5}{0}$

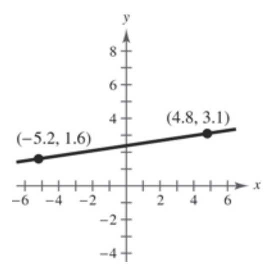
m is undefined.



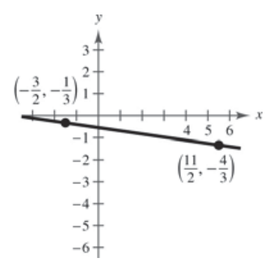
32. $m = \dfrac{0 - (-10)}{-4 - 0} = -\dfrac{5}{2}$



33. $m = \dfrac{1.6 - 3.1}{-5.2 - 4.8} = \dfrac{-1.5}{-10} = 0.15$



34. $m = \dfrac{-\frac{1}{3} - \left(-\frac{4}{3}\right)}{-\frac{3}{2} - \frac{11}{2}} = -\dfrac{1}{7}$



35. Point: $(2, 1)$, Slope: $m = 0$

Because $m = 0$, y does not change. Three points are $(0, 1)$, $(3, 1)$, and $(-1, 1)$.

36. Point: $(3, -2)$, Slope: $m = 0$

Because $m = 0$, y does not change. Three other points are $(-4, -2)$, $(0, -2)$, and $(5, -2)$.

37. Point: $(-8, 1)$, Slope is undefined.

Because m is undefined, x does not change. Three points are $(-8, 0)$, $(-8, 2)$, and $(-8, 3)$.

38. Point: $(1, 5)$, Slope is undefined.

Because m is undefined, x does not change. Three other points are $(1, -3)$, $(1, 1)$, and $(1, 7)$.

39. Point: $(-5, 4)$, Slope: $m = 2$

Because $m = 2 = \frac{2}{1}$, y increases by 2 for every one unit increase in x. Three additional points are $(-4, 6)$, $(-3, 8)$, and $(-2, 10)$.

40. Point: $(0, -9)$, Slope: $m = -2$

Because $m = -2$, y decreases by 2 for every one unit increase in x. Three other points are $(-2, -5)$, $(1, -11)$, and $(3, -15)$.

41. Point: $(-1, -6)$, Slope: $m = -\frac{1}{2}$

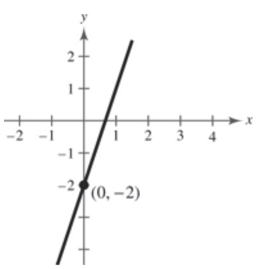
Because $m = -\frac{1}{2}$, y decreases by 1 unit for every two unit increase in x. Three additional points are $(1, -7)$, $(3, -8)$, and $(-13, 0)$.

42. Point: $(7, -2)$, Slope: $m = \frac{1}{2}$

Because $m = \frac{1}{2}$, y increases by 1 unit for every two unit increase in x. Three additional points are $(9, -1)$, $(11, 0)$, and $(13, 1)$.

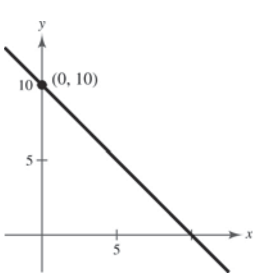
43. Point: $(0, -2)$; $m = 3$

$$y + 2 = 3(x - 0)$$
$$y = 3x - 2$$



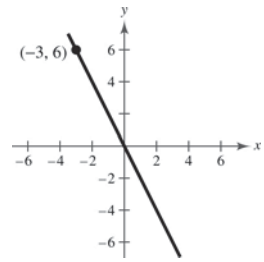
44. Point: $(0, 10)$; $m = -1$

$$y - 10 = -1(x - 0)$$
$$y - 10 = -x$$
$$y = -x + 10$$



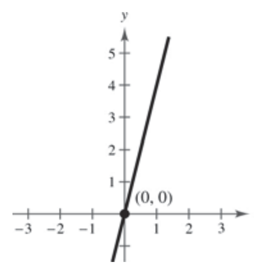
45. Point: $(-3, 6)$; $m = -2$

$$y - 6 = -2(x + 3)$$
$$y = -2x$$



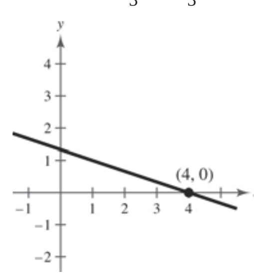
46. Point: $(0, 0)$; $m = 4$

$$y - 0 = 4(x - 0)$$
$$y = 4x$$



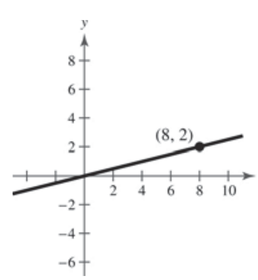
47. Point: $(4, 0)$; $m = -\frac{1}{3}$

$$y - 0 = -\frac{1}{3}(x - 4)$$
$$y = -\frac{1}{3}x + \frac{4}{3}$$



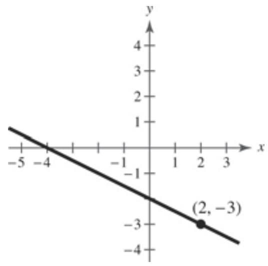
48. Point: $(8, 2)$; $m = \frac{1}{4}$

$$y - 2 = \frac{1}{4}(x - 8)$$
$$y - 2 = \frac{1}{4}x - 2$$
$$y = \frac{1}{4}x$$



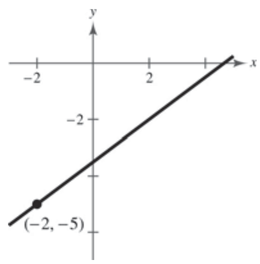
49. Point: $(2, -3)$; $m = -\frac{1}{2}$

$$y - (-3) = -\frac{1}{2}(x - 2)$$
$$y + 3 = -\frac{1}{2}x + 1$$
$$y = -\frac{1}{2}x - 2$$



50. Point: $(-2, -5)$; $m = \frac{3}{4}$

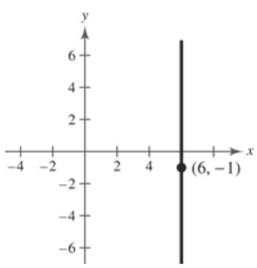
$$y + 5 = \tfrac{3}{4}(x + 2)$$
$$4y + 20 = 3x + 6$$
$$4y = 3x - 14$$
$$y = \tfrac{3}{4}x - \tfrac{7}{2}$$



51. Point: $(6, -1)$; m is undefined.

Because the slope is undefined, the line is a vertical line.

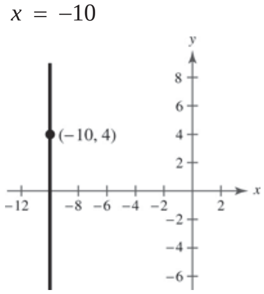
$x = 6$



52. Point: $(-10, 4)$; m is undefined.

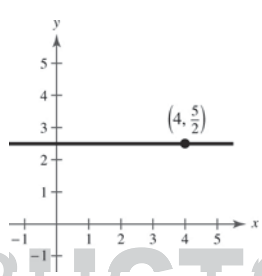
Because the slope is undefined, the line is a vertical line.

$x = -10$



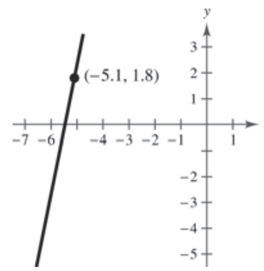
53. Point: $\left(4, \frac{5}{2}\right)$; $m = 0$

$$y - \tfrac{5}{2} = 0(x - 4)$$
$$y - \tfrac{5}{2} = 0$$
$$y = \tfrac{5}{2}$$



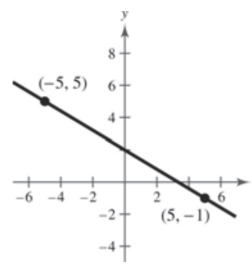
54. Point: $(-5.1, 1.8)$; $m = 5$

$$y - 1.8 = 5(x - (-5.1))$$
$$y = 5x + 27.3$$



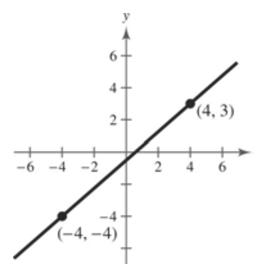
55. $(5, -1), (-5, 5)$

$$y + 1 = \frac{5 + 1}{-5 - 5}(x - 5)$$
$$y = -\frac{3}{5}(x - 5) - 1$$
$$y = -\frac{3}{5}x + 2$$



56. $(4, 3), (-4, -4)$

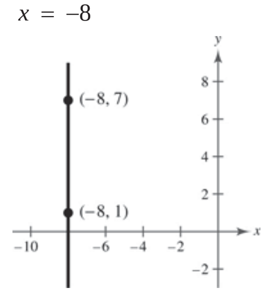
$$y - 3 = \frac{-4 - 3}{-4 - 4}(x - 4)$$
$$y - 3 = \frac{7}{8}(x - 4)$$
$$y - 3 = \frac{7}{8}x - \frac{7}{2}$$
$$y = \frac{7}{8}x - \frac{1}{2}$$



57. $(-8, 1), (-8, 7)$

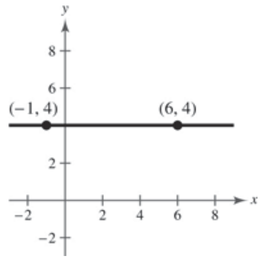
Because both points have $x = -8$, the slope is undefined, and the line is vertical.

$x = -8$



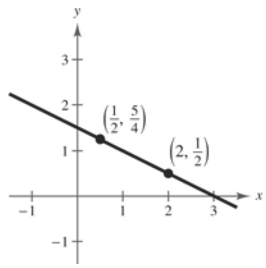
58. $(-1, 4), (6, 4)$

$$y - 4 = \frac{4 - 4}{6 - (-1)}(x + 1)$$
$$y - 4 = 0(x + 1)$$
$$y - 4 = 0$$
$$y = 4$$



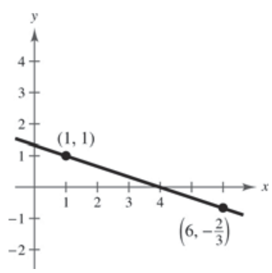
59. $\left(2, \frac{1}{2}\right), \left(\frac{1}{2}, \frac{5}{4}\right)$

$$y - \frac{1}{2} = \frac{\frac{5}{4} - \frac{1}{2}}{\frac{1}{2} - 2}(x - 2)$$
$$y = -\frac{1}{2}(x - 2) + \frac{1}{2}$$
$$y = -\frac{1}{2}x + \frac{3}{2}$$



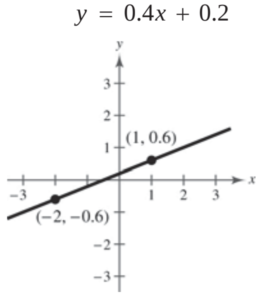
60. $(1, 1), \left(6, -\frac{2}{3}\right)$

$$y - 1 = \frac{-\frac{2}{3} - 1}{6 - 1}(x - 1)$$
$$y - 1 = -\frac{1}{3}(x - 1)$$
$$y - 1 = -\frac{1}{3}x + \frac{1}{3}$$
$$y = -\frac{1}{3}x + \frac{4}{3}$$



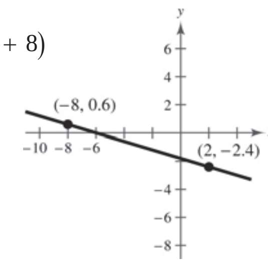
61. $(1, 0.6), (-2, -0.6)$

$$y - 0.6 = \frac{-0.6 - 0.6}{-2 - 1}(x - 1)$$
$$y = 0.4(x - 1) + 0.6$$
$$y = 0.4x + 0.2$$



62. $(-8, 0.6), (2, -2.4)$

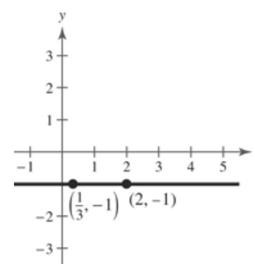
$$y - 0.6 = \frac{-2.4 - 0.6}{2 - (-8)}(x + 8)$$
$$y - 0.6 = -\frac{3}{10}(x + 8)$$
$$10y - 6 = -3(x + 8)$$
$$10y - 6 = -3x - 24$$
$$10y = -3x - 18$$
$$y = -\frac{3}{10}x - \frac{9}{5} \quad \text{or} \quad y = -0.3x - 1.8$$



63. $(2, -1), \left(\frac{1}{3}, -1\right)$

$$y + 1 = \frac{-1 - (-1)}{\frac{1}{3} - 2}(x - 2)$$
$$y + 1 = 0$$
$$y = -1$$

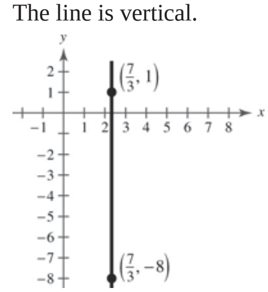
The line is horizontal.



64. $\left(\frac{7}{3}, -8\right), \left(\frac{7}{3}, 1\right)$

$$m = \frac{1 - (-8)}{\frac{7}{3} - \frac{7}{3}} = \frac{9}{0} \text{ and is undefined.}$$
$$x = \frac{7}{3}$$

The line is vertical.



65. $L_1: y = \frac{1}{3}x - 2$

$m_1 = \frac{1}{3}$

$L_2: y = \frac{1}{3}x + 3$

$m_2 = \frac{1}{3}$

The lines are parallel.

66. $L_1: y = 4x - 1$

$m_1 = 4$

$L_2: y = 4x + 7$

$m_2 = 4$

The lines are parallel.

67. $L_1: y = \frac{1}{2}x - 3$

$m_1 = \frac{1}{2}$

$L_2: y = -\frac{1}{2}x + 1$

$m_2 = -\frac{1}{2}$

The lines are neither parallel nor perpendicular.

68. $L_1: y = -\frac{4}{5}x - 5$

$m_1 = -\frac{4}{5}$

$L_2: y = \frac{5}{4}x + 1$

$m_2 = \frac{5}{4}$

The lines are perpendicular.

69. $L_1: (0, -1), (5, 9)$

$$m_1 = \frac{9 + 1}{5 - 0} = 2$$

$L_2: (0, 3), (4, 1)$

$$m_2 = \frac{1 - 3}{4 - 0} = -\frac{1}{2}$$

The lines are perpendicular.

70. $L_1: (-2, -1), (1, 5)$

$$m_1 = \frac{5 - (-1)}{1 - (-2)} = \frac{6}{3} = 2$$

$L_2: (1, 3), (5, -5)$

$$m_2 = \frac{-5 - 3}{5 - 1} = \frac{-8}{4} = -2$$

The lines are neither parallel nor perpendicular.

71. $L_1: (3, 6), (-6, 0)$

$$m_1 = \frac{0 - 6}{-6 - 3} = \frac{2}{3}$$

$L_2: (0, -1), \left(5, \frac{7}{3}\right)$

$$m_2 = \frac{\frac{7}{3} + 1}{5 - 0} = \frac{2}{3}$$

The lines are parallel.

72. $L_1: (4, 8), (-4, 2)$

$$m_1 = \frac{2 - 8}{-4 - 4} = \frac{-6}{-8} = \frac{3}{4}$$

$L_2: (3, -5), \left(-1, \frac{1}{3}\right)$

$$m_2 = \frac{\frac{1}{3} - (-5)}{-1 - 3} = \frac{\frac{16}{3}}{-4} = -\frac{4}{3}$$

The lines are perpendicular.

73. $4x - 2y = 3$

$y = 2x - \frac{3}{2}$

Slope: $m = 2$

(a) $(2, 1), m = 2$

$$y - 1 = 2(x - 2)$$
$$y = 2x - 3$$

(b) $(2, 1), m = -\frac{1}{2}$

$$y - 1 = -\tfrac{1}{2}(x - 2)$$
$$y = -\tfrac{1}{2}x + 2$$

74. $x + y = 7$

$y = -x + 7$

Slope: $m = -1$

(a) $m = -1, (-3, 2)$

$$y - 2 = -1(x + 3)$$
$$y - 2 = -x - 3$$
$$y = -x - 1$$

(b) $m = 1, (-3, 2)$

$$y - 2 = 1(x + 3)$$
$$y = x + 5$$

75. $3x + 4y = 7$

$y = -\frac{3}{4}x + \frac{7}{4}$

Slope: $m = -\frac{3}{4}$

(a) $\left(-\frac{2}{3}, \frac{7}{8}\right), m = -\frac{3}{4}$

$$y - \tfrac{7}{8} = -\tfrac{3}{4}\left(x - \left(-\tfrac{2}{3}\right)\right)$$
$$y = -\tfrac{3}{4}x + \tfrac{3}{8}$$

(b) $\left(-\frac{2}{3}, \frac{7}{8}\right), m = \frac{4}{3}$

$$y - \tfrac{7}{8} = \tfrac{4}{3}\left(x - \left(-\tfrac{2}{3}\right)\right)$$
$$y = \tfrac{4}{3}x + \tfrac{127}{72}$$

76. $5x + 3y = 0$

$$3y = -5x$$

$$y = -\tfrac{5}{3}x$$

Slope: $m = -\tfrac{5}{3}$

(a) $m = -\tfrac{5}{3}, \left(\tfrac{7}{8}, \tfrac{3}{4}\right)$

$$y - \tfrac{3}{4} = -\tfrac{5}{3}\left(x - \tfrac{7}{8}\right)$$

$$24y - 18 = -40\left(x - \tfrac{7}{8}\right)$$

$$24y - 18 = -40x + 35$$

$$24y = -40x + 53$$

$$y = -\tfrac{5}{3}x + \tfrac{53}{24}$$

(b) $m = \tfrac{3}{5}, \left(\tfrac{7}{8}, \tfrac{3}{4}\right)$

$$y - \tfrac{3}{4} = \tfrac{3}{5}\left(x - \tfrac{7}{8}\right)$$

$$40y - 30 = 24\left(x - \tfrac{7}{8}\right)$$

$$40y - 30 = 24x - 21$$

$$40y = 24x + 9$$

$$y = \tfrac{3}{5}x + \tfrac{9}{40}$$

77. $y + 3 = 0$

$$y = -3$$

Slope: $m = 0$

(a) $(-1, 0), m = 0$

$$y = 0$$

(b) $(-1, 0), m$ is undefined.

$$x = -1$$

78. $x - 4 = 0$

$$x = 4$$

Slope: m is undefined.

(a) $(3, -2), m$ is undefined.

$$x = 3$$

(b) $(3, -2), m = 0$

$$y = -2$$

79. $x - y = 4$

$$y = x - 4$$

Slope: $m = 1$

(a) $(2.5, 6.8), m = 1$

$$y - 6.8 = 1(x - 2.5)$$

$$y = x + 4.3$$

(b) $(2.5, 6.8), m = -1$

$$y - 6.8 = (-1)(x - 2.5)$$

$$y = -x + 9.3$$

80. $6x + 2y = 9$

$$2y = -6x + 9$$

$$y = -3x + \tfrac{9}{2}$$

Slope: $m = -3$

(a) $(-3.9, -1.4), m = -3$

$$y - (-1.4) = -3(x - (-3.9))$$

$$y + 1.4 = -3x - 11.7$$

$$y = -3x - 13.1$$

(b) $(-3.9, -1.4), m = \tfrac{1}{3}$

$$y - (-1.4) = \tfrac{1}{3}(x - (-3.9))$$

$$y + 1.4 = \tfrac{1}{3}x + 1.3$$

$$y = \tfrac{1}{3}x - 0.1$$

81. $\dfrac{x}{2} + \dfrac{y}{3} = 1$

$$3x + 2y - 6 = 0$$

82. $(-3, 0), (0, 4)$

$$\frac{x}{-3} + \frac{y}{4} = 1$$

$$(-12)\frac{x}{-3} + (-12)\frac{y}{4} = (-12) \cdot 1$$

$$4x - 3y + 12 = 0$$

83. $\dfrac{x}{-1/6} + \dfrac{y}{-2/3} = 1$

$$6x + \frac{3}{2}y = -1$$

$$12x + 3y + 2 = 0$$

84. $\left(\frac{2}{3}, 0\right), (0, -2)$

$$\frac{x}{2/3} + \frac{y}{-2} = 1$$
$$\frac{3x}{2} - \frac{y}{2} = 1$$
$$3x - y - 2 = 0$$

85. $\frac{x}{c} + \frac{y}{c} = 1, c \neq 0$

$$x + y = c$$
$$1 + 2 = c$$
$$3 = c$$
$$x + y = 3$$
$$x + y - 3 = 0$$

86. $(d, 0), (0, d), (-3, 4)$

$$\frac{x}{d} + \frac{y}{d} = 1$$
$$x + y = d$$
$$-3 + 4 = d$$
$$1 = d$$
$$x + y = 1$$
$$x + y - 1 = 0$$

87. (a) $m = 135$. The sales are increasing 135 units per year.

(b) $m = 0$. There is no change in sales during the year.

(c) $m = -40$. The sales are decreasing 40 units per year.

88. (a) greatest increase = largest slope

$$(9, 36.54), (10, 65.23)$$
$$m_1 = \frac{65.23 - 36.54}{10 - 9} = 28.69$$

So, the sales increased the greatest between the years 2009 and 2010.

least increase = smallest slope

$$(8, 32.48), (9, 36.54)$$
$$m_2 = \frac{36.54 - 32.48}{9 - 8} = 4.06$$

So, the sales increased the least between the years 2008 and 2009.

(b) $(4, 8.28), (10, 65.23)$

$$m = \frac{65.23 - 8.28}{10 - 4} = \frac{56.95}{6} \approx 9.49$$

The slope of the line is about 9.49.

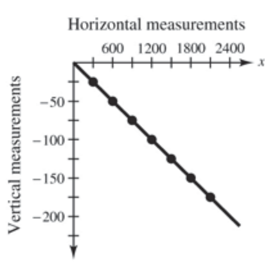
(c) The sales increased \$9.49 billion each year between the years 2004 and 2010.

89. $y = \frac{6}{100}x$

$y = \frac{6}{100}(200) = 12$ feet

90. (a) and (b)

x	300	600	900	1200	1500	1800	2100
y	–25	–50	–75	–100	–125	–150	–175



(c) $m = \dfrac{-50 - (-25)}{600 - 300} = \dfrac{-25}{300} = -\dfrac{1}{12}$

$$y - (-50) = -\frac{1}{12}(x - 600)$$
$$y + 50 = -\frac{1}{12}x + 50$$
$$y = -\frac{1}{12}x$$

(d) Because $m = -\dfrac{1}{12}$, for every change in the horizontal measurement of 12 feet, the vertical measurement decreases by 1 foot.

(e) $\dfrac{1}{12} \approx 0.083 = 8.3\%$ grade

91. $(10, 2540)$, $m = -125$

$$V - 2540 = -125(t - 10)$$
$$V - 2540 = -125t + 1250$$
$$V = -125t + 3790,\ 5 \le t \le 10$$

92. $(10, 156)$, $m = 4.50$

$$V - 156 = 4.50(t - 10)$$
$$V - 156 = 4.50t - 45$$
$$V = 4.5t + 111,\ 5 \le t \le 10$$

93. The C-intercept measures the fixed costs of manufacturing when zero bags are produced.

The slope measures the cost to produce one laptop bag.

94. $W = 0.07S + 2500$

95. Using the points $(0, 875)$ and $(5, 0)$, where the first coordinate represents the year t and the second coordinate represents the value V, you have

$$m = \frac{0 - 875}{5 - 0} = -175$$
$$V = -175t + 875,\ 0 \le t \le 5.$$

96. Using the points $(0, 24{,}000)$ and $(10, 2000)$, where the first coordinate represents the year t and the second coordinate represents the value V, you have

$$m = \frac{2{,}000 - 24{,}000}{10 - 0} = \frac{-22{,}000}{10} = -2200.$$

Since the point $(0, 24{,}000)$ is the V-intercept, $b = 24{,}000$, the equation is $V = -2200t + 24{,}000,\ 0 \le t \le 10.$

97. Using the points $(0, 32)$ and $(100, 212)$, where the first coordinate represents a temperature in degrees Celsius and the second coordinate represents a temperature in degrees Fahrenheit, you have

$$m = \frac{212 - 32}{100 - 0} = \frac{180}{100} = \frac{9}{5}.$$

Since the point $(0, 32)$ is the F- intercept, $b = 32$, the equation is $F = \dfrac{9}{5}C + 32.$

98. (a) Using the points $(1, 970)$ and $(3, 1270)$, you have

$$m = \frac{1270 - 970}{3 - 1} = \frac{300}{2} = 150.$$

Using the point-slope form with $m = 150$ and the point $(1, 970)$, you have

$$y - y_1 = m(t - t_1)$$
$$y - 970 = 150(t - 1)$$
$$y - 970 = 150t - 150$$
$$y = 150t + 820.$$

(b) The slope is $m = 150$. The slope tells you the amount of increase in the weight of average male child's brain each year.

(c) Let $t = 2$:

$$y = 150(2) + 820$$
$$y = 300 + 820$$
$$y = 1120$$

The average brain weight at age 2 is 1120 grams.

(d) Answers will vary.

(e) Answers will vary. *Sample Answer:* No. The brain stops growing after reaching a certain age.

99. (a) Total Cost = cost for fuel and maintainance + cost for operator + purchase cost

$$C = 9.5t + 11.5t + 42{,}000$$
$$C = 21.0t + 42{,}000$$

(b) Revenue = Rate per hour · Hours

$$R = 45t$$

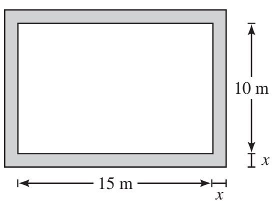
(c)
$$P = R - C$$
$$P = 45t - (21t + 42{,}000)$$
$$P = 24t - 42{,}000$$

(d) Let $P = 0$, and solve for t.

$$0 = 24t - 42{,}000$$
$$42{,}000 = 24t$$
$$1750 = t$$

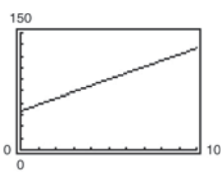
The equipment must be used 1750 hours to yield a profit of 0 dollars.

100. (a)



(b) $y = 2(15 + 2x) + 2(10 + 2x) = 8x + 50$

(c)



(d) Because $m = 8$, each 1-meter increase in x will increase y by 8 meters.

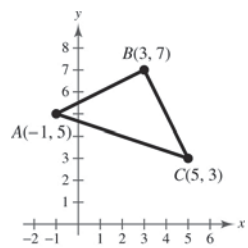
101. False. The slope with the greatest magnitude corresponds to the steepest line.

102. False. The lines are not parallel.

$$(-8, 2) \text{ and } (-1, 4): m_1 = \frac{4 - 2}{-1 - (-8)} = \frac{2}{7}$$

$$(0, -4) \text{ and } (-7, 7): m_2 = \frac{7 - (-4)}{-7 - 0} = \frac{11}{-7}$$

103. Find the slope of the line segments between the points A and B, and B and C.



$$m_{AB} = \frac{7 - 5}{3 - (-1)} = \frac{2}{4} = \frac{1}{2}$$

$$m_{BC} = \frac{3 - 7}{5 - 3} = \frac{-4}{2} = -2$$

Since the slopes are negative reciprocals, the line segments are perpendicular and therefore intersect to form a right angle. So, the triangle is a right triangle.

104. On a vertical line, all the points have the same x-value, so when you evaluate $m = \dfrac{y_2 - y_1}{x_2 - x_1}$, you would have a zero in the denominator, and division by zero is undefined.

105. No. The slope cannot be determined without knowing the scale on the y-axis. The slopes will be the same if the scale on the y-axis of (a) is $2\frac{1}{2}$ and the scale on the y-axis of (b) is 1. Then the slope of both is $\frac{5}{4}$.

106.
$$d_1 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(1 - 0)^2 + (m_1 - 0)^2} = \sqrt{1 + (m_1)^2}$$

$$d_2 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(1 - 0)^2 + (m_2 - 0)^2} = \sqrt{1 + (m_2)^2}$$

Using the Pythagorean Theorem:

$$(d_1)^2 + (d_2)^2 = \left(\text{distance between } (1, m_1), \text{ and } (1, m_2)\right)^2$$

$$\left(\sqrt{1 + (m_1)^2}\right)^2 + \left(\sqrt{1 + (m_2)^2}\right)^2 = \left(\sqrt{(1 - 1)^2 + (m_2 - m_1)^2}\right)^2$$

$$1 + (m_1)^2 + 1 + (m_2)^2 = (m_2 - m_1)^2$$

$$(m_1)^2 + (m_2)^2 + 2 = (m_2)^2 - 2m_1m_2 + (m_1)^2$$

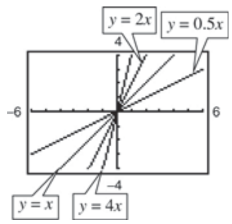
$$2 = -2m_1m_2$$

$$-\frac{1}{m_2} = m_1$$

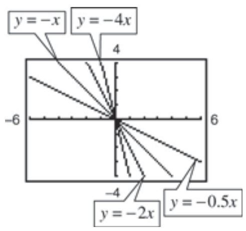
107. No, the slopes of two perpendicular lines have opposite signs. (Assume that neither line is vertical or horizontal.)

108. Because $|-4| > \left|\frac{5}{2}\right|$, the steeper line is the one with a slope of – 4. The slope with the greatest magnitude corresponds to the steepest line.

109. The line $y = 4x$ rises most quickly.



The line $y = -4x$ falls most quickly.



The greater the magnitude of the slope (the absolute value of the slope), the faster the line rises or falls.

110. (a) Matches graph (ii).

The slope is –20, which represents the decrease in the amount of the loan each week. The y-intercept is $(0, 200)$, which represents the original amount of the loan.

(b) Matches graph (iii).

The slope is 2, which represents the increase in the hourly wage for each unit produced. The y-intercept is $(0, 12.5)$, which represents the hourly rate if the employee produces no units.

(c) Matches graph (i).

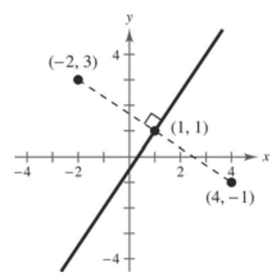
The slope is 0.32, which represents the increase in travel cost for each mile driven. The y-intercept is $(0, 32)$, which represents the fixed cost of \$30 per day for meals. This amount does not depend on the number of miles driven.

(d) Matches graph (iv).

The slope is –100, which represents the amount by which the computer depreciates each year. The y-intercept is $(0, 750)$, which represents the original purchase price.

111. Set the distance between $(4, -1)$ and (x, y) equal to the distance between $(-2, 3)$ and (x, y).

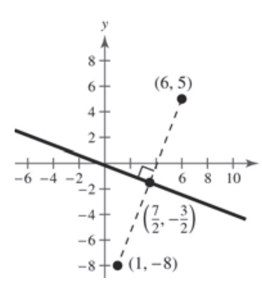
$$
\begin{aligned}
\sqrt{(x-4)^2 + [y-(-1)]^2} &= \sqrt{[x-(-2)]^2 + (y-3)^2} \\
(x-4)^2 + (y+1)^2 &= (x+2)^2 + (y-3)^2 \\
x^2 - 8x + 16 + y^2 + 2y + 1 &= x^2 + 4x + 4 + y^2 - 6y + 9 \\
-8x + 2y + 17 &= 4x - 6y + 13 \\
0 &= 12x - 8y - 4 \\
0 &= 4(3x - 2y - 1) \\
0 &= 3x - 2y - 1
\end{aligned}
$$



This line is the perpendicular bisector of the line segment connecting $(4, -1)$ and $(-2, 3)$.

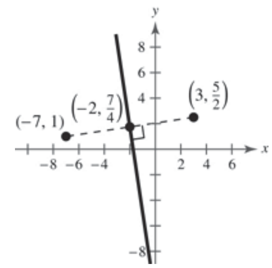
112. Set the distance between $(6, 5)$ and (x, y) equal to the distance between $(1, -8)$ and (x, y).

$$
\begin{aligned}
\sqrt{(x-6)^2 + (y-5)^2} &= \sqrt{(x-1)^2 + (y-(-8))^2} \\
(x-6)^2 + (y-5)^2 &= (x-1)^2 + (y+8)^2 \\
x^2 - 12x + 36 + y^2 - 10y + 25 &= x^2 - 2x + 1 + y^2 + 16y + 64 \\
x^2 + y^2 - 12x - 10y + 61 &= x^2 + y^2 - 2x + 16y + 65 \\
-12x - 10y + 61 &= -2x + 16y + 65 \\
-10x - 26y - 4 &= 0 \\
-2(5x + 13y + 2) &= 0 \\
5x + 13y + 2 &= 0
\end{aligned}
$$



113. Set the distance between $\left(3, \frac{5}{2}\right)$ and (x, y) equal to the distance between $(-7, 1)$ and (x, y).

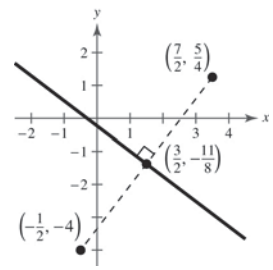
$$\begin{aligned}
\sqrt{(x-3)^2 + \left(y - \tfrac{5}{2}\right)^2} &= \sqrt{\left[x - (-7)\right]^2 + (y-1)^2} \\
(x-3)^2 + \left(y - \tfrac{5}{2}\right)^2 &= (x+7)^2 + (y-1)^2 \\
x^2 - 6x + 9 + y^2 - 5y + \tfrac{25}{4} &= x^2 + 14x + 49 + y^2 - 2y + 1 \\
-6x - 5y + \tfrac{61}{4} &= 14x - 2y + 50 \\
-24x - 20y + 61 &= 56x - 8y + 200 \\
80x + 12y + 139 &= 0
\end{aligned}$$



This line is the perpendicular bisector of the line segment connecting $\left(3, \frac{5}{2}\right)$ and $(-7, 1)$.

114. Set the distance between $\left(-\frac{1}{2}, -4\right)$ and (x, y) equal to the distance between $\left(\frac{7}{2}, \frac{5}{4}\right)$ and (x, y).

$$\begin{aligned}
\sqrt{\left(x - \left(-\tfrac{1}{2}\right)\right)^2 + \left(y - (-4)\right)^2} &= \sqrt{\left(x - \tfrac{7}{2}\right)^2 + \left(y - \tfrac{5}{4}\right)^2} \\
\left(x + \tfrac{1}{2}\right)^2 + (y+4)^2 &= \left(x - \tfrac{7}{2}\right)^2 + \left(y - \tfrac{5}{4}\right)^2 \\
x^2 + x + \tfrac{1}{4} + y^2 + 8y + 16 &= x^2 - 7x + \tfrac{49}{4} + y^2 - \tfrac{5}{2}y + \tfrac{25}{16} \\
x^2 + y^2 + x + 8y + \tfrac{65}{4} &= x^2 + y^2 - 7x - \tfrac{5}{2}y + \tfrac{221}{16} \\
x + 8y + \tfrac{65}{4} &= -7x - \tfrac{5}{2}y + \tfrac{221}{16} \\
8x + \tfrac{21}{2}y + \tfrac{39}{16} &= 0 \\
128x + 168y + 39 &= 0
\end{aligned}$$



Section 1.4 Functions

1. domain; range; function
2. independent; dependent
3. implied domain
4. difference quotient
5. Yes, the relationship is a function. Each domain value is matched with exactly one range value.
6. No, the relationship is not a function. The domain value of –1 is matched with two output values.
7. No, it does not represent a function. The input values of 10 and 7 are each matched with two output values.
8. Yes, the table does represent a function. Each input value is matched with exactly one output value.
9. (a) Each element of A is matched with exactly one element of B, so it does represent a function.
   (b) The element 1 in A is matched with two elements, –2 and 1 of B, so it does not represent a function.
   (c) Each element of A is matched with exactly one element of B, so it does represent a function.
   (d) The element 2 in A is not matched with an element of B, so the relation does not represent a function.
10. (a) The element c in A is matched with two elements, 2 and 3 of B, so it is not a function.
    (b) Each element of A is matched with exactly one element of B, so it does represent a function.
    (c) This is not a function from A to B (it represents a function from B to A instead).
    (d) Each element of A is matched with exactly one element of B, so it does represent a function.
11. $x^2 + y^2 = 4 \Rightarrow y = \pm\sqrt{4 - x^2}$
    No, y *is not* a function of x.
12. $x^2 + y = 4 \Rightarrow y = 4 - x^2$
    Yes, y *is* a function of x.
13. $2x + 3y = 4 \Rightarrow y = \frac{1}{3}(4 - 2x)$
    Yes, y *is* a function of x.
14. $(x-2)^2 + y^2 = 4$
    $$y = \pm\sqrt{4 - (x-2)^2}$$
    No, y *is not* a function of x.

15. $y = \sqrt{16 - x^2}$

Yes, y *is* a function of x.

16. $y = \sqrt{x + 5}$

Yes, y *is* a function of x.

17. $y = |4 - x|$

Yes, y *is* a function of x.

18. $|y| = 4 - x \Rightarrow y = 4 - x$ or $y = -(4 - x)$

No, y *is not* a function of x.

19. $y = -75$ or $y = -75 + 0x$

Yes, y *is* a function of x.

20. $x - 1 = 0$

$x = 1$

No, this *is not* a function of x.

21. $f(x) = 2x - 3$

(a) $f(1) = 2(1) - 3 = -1$

(b) $f(-3) = 2(-3) - 3 = -9$

(c) $f(x - 1) = 2(x - 1) - 3 = 2x - 5$

22. $V(r) = \frac{4}{3}\pi r^3$

(a) $V(3) = \frac{4}{3}\pi(3)^3 = \frac{4}{3}\pi(27) = 36\pi$

(b) $V\left(\frac{3}{2}\right) = \frac{4}{3}\pi\left(\frac{3}{2}\right)^3 = \frac{4}{3}\pi\left(\frac{27}{8}\right) = \frac{9}{2}\pi$

(c) $V(2r) = \frac{4}{3}\pi(2r)^3 = \frac{4}{3}\pi(8r^3) = \frac{32}{3}\pi r^3$

23. $g(t) = 4t^2 - 3t + 5$

(a) $g(2) = 4(2)^2 - 3(2) + 5$

$= 15$

(b) $g(t - 2) = 4(t - 2)^2 - 3(t - 2) + 5$

$= 4t^2 - 19t + 27$

(c) $g(t) - g(2) = 4t^2 - 3t + 5 - 15$

$= 4t^2 - 3t - 10$

24. $h(t) = t^2 - 2t$

(a) $h(2) = 2^2 - 2(2) = 0$

(b) $h(1.5) = (1.5)^2 - 2(1.5) = -0.75$

(c) $h(x + 2) = (x + 2)^2 - 2(x + 2) = x^2 + 2x$

25. $f(y) = 3 - \sqrt{y}$

(a) $f(4) = 3 - \sqrt{4} = 1$

(b) $f(0.25) = 3 - \sqrt{0.25} = 2.5$

(c) $f(4x^2) = 3 - \sqrt{4x^2} = 3 - 2|x|$

26. $f(x) = \sqrt{x + 8} + 2$

(a) $f(-8) = \sqrt{(-8) + 8} + 2 = 2$

(b) $f(1) = \sqrt{(1) + 8} + 2 = 5$

(c) $f(x - 8) = \sqrt{(x - 8) + 8} + 2 = \sqrt{x} + 2$

27. $q(x) = \dfrac{1}{x^2 - 9}$

(a) $q(0) = \dfrac{1}{0^2 - 9} = -\dfrac{1}{9}$

(b) $q(3) = \dfrac{1}{3^2 - 9}$ is undefined.

(c) $q(y + 3) = \dfrac{1}{(y + 3)^2 - 9} = \dfrac{1}{y^2 + 6y}$

28. $q(t) = \dfrac{2t^2 + 3}{t^2}$

(a) $q(2) = \dfrac{2(2)^2 + 3}{(2)^2} = \dfrac{8 + 3}{4} = \dfrac{11}{4}$

(b) $q(0) = \dfrac{2(0)^2 + 3}{(0)^2}$

Division by zero is undefined.

(c) $q(-x) = \dfrac{2(-x)^2 + 3}{(-x)^2} = \dfrac{2x^2 + 3}{x^2}$

29. $f(x) = \dfrac{|x|}{x}$

(a) $f(2) = \dfrac{|2|}{2} = 1$

(b) $f(-2) = \dfrac{|-2|}{-2} = -1$

(c) $f(x - 1) = \dfrac{|x - 1|}{x - 1} = \begin{cases} -1, & \text{if } x < 1 \\ 1, & \text{if } x > 1 \end{cases}$

30. $f(x) = |x| + 4$

(a) $f(2) = |2| + 4 = 6$

(b) $f(-2) = |-2| + 4 = 6$

(c) $f(x^2) = |x^2| + 4 = x^2 + 4$

31. $f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \geq 0 \end{cases}$

(a) $f(-1) = 2(-1) + 1 = -1$

(b) $f(0) = 2(0) + 2 = 2$

(c) $f(2) = 2(2) + 2 = 6$

32. $f(x) = \begin{cases} 4 - 5x, & x \leq -2 \\ 0, & -2 < x \leq 2 \\ x^2 + 1, & x > 2 \end{cases}$

(a) $f(-3) = 4 - 5(-3) = 19$

(b) $f(4) = (4)^2 + 1 = 17$

(c) $f(-1) = 0$

33. $f(x) = x^2 - 3$

$f(-2) = (-2)^2 - 3 = 1$

$f(-1) = (-1)^2 - 3 = -2$

$f(0) = (0)^2 - 3 = -3$

$f(1) = (1)^2 - 3 = -2$

$f(2) = (2)^2 - 3 = 1$

x	-2	-1	0	1	2
$f(x)$	1	-2	-3	-2	1

34. $h(t) = \frac{1}{2}\left|t + 3\right|$

$h(-5) = \frac{1}{2}\left|-5 + 3\right| = 1$

$h(-4) = \frac{1}{2}\left|-4 + 3\right| = \frac{1}{2}$

$h(-3) = \frac{1}{2}\left|-3 + 3\right| = 0$

$h(-2) = \frac{1}{2}\left|-2 + 3\right| = \frac{1}{2}$

$h(-1) = \frac{1}{2}\left|-1 + 3\right| = 1$

t	-5	-4	-3	-2	-1
$h(t)$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	1

35. $f(x) = \begin{cases} -\frac{1}{2}x + 4, & x \leq 0 \\ (x - 2)^2, & x > 0 \end{cases}$

$f(-2) = -\frac{1}{2}(-2) + 4 = 5$

$f(-1) = -\frac{1}{2}(-1) + 4 = 4\frac{1}{2} = \frac{9}{2}$

$f(0) = -\frac{1}{2}(0) + 4 = 4$

$f(1) = (1 - 2)^2 = 1$

$f(2) = (2 - 2)^2 = 0$

x	-2	-1	0	1	2
$f(x)$	5	$\frac{9}{2}$	4	1	0

36. $f(x) = \begin{cases} 9 - x^2, & x < 3 \\ x - 3, & x \geq 3 \end{cases}$

$f(1) = 9 - (1)^2 = 8$

$f(2) = 9 - (2)^2 = 5$

$f(3) = (3) - 3 = 0$

$f(4) = (4) - 3 = 1$

$f(5) = (5) - 3 = 2$

x	1	2	3	4	5
$f(x)$	8	5	0	1	2

37. $15 - 3x = 0$

$3x = 15$

$x = 5$

38. $f(x) = 5x + 1$

$5x + 1 = 0$

$x = -\frac{1}{5}$

39. $\frac{3x - 4}{5} = 0$

$3x - 4 = 0$

$x = \frac{4}{3}$

40. $f(x) = \frac{12 - x^2}{5}$

$\frac{12 - x^2}{5} = 0$

$x^2 = 12$

$x = \pm\sqrt{12} = \pm 2\sqrt{3}$

41. $x^2 - 9 = 0$

$x^2 = 9$

$x = \pm 3$

42. $f(x) = x^2 - 8x + 15$

$x^2 - 8x + 15 = 0$

$(x - 5)(x - 3) = 0$

$x - 5 = 0 \Rightarrow x = 5$

$x - 3 = 0 \Rightarrow x = 3$

43. $x^3 - x = 0$

$x(x^2 - 1) = 0$

$x(x + 1)(x - 1) = 0$

$x = 0$, $x = -1$, or $x = 1$

44. $f(x) = x^3 - x^2 - 4x + 4$

$x^3 - x^2 - 4x + 4 = 0$

$x^2(x - 1) - 4(x - 1) = 0$

$(x - 1)(x^2 - 4) = 0$

$x - 1 = 0 \Rightarrow x = 1$

$x^2 - 4 = 0 \Rightarrow x = \pm 2$

45. $f(x) = g(x)$

$x^2 = x + 2$

$x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0$

$x - 2 = 0 \quad x + 1 = 0$

$x = 2 \quad x = -1$

46. $f(x) = g(x)$

$x^2 + 2x + 1 = 7x - 5$

$x^2 - 5x + 6 = 0$

$(x - 3)(x - 2) = 0$

$x - 3 = 0 \quad x - 2 = 0$

$x = 3 \quad x = 2$

47. $f(x) = g(x)$

$x^4 - 2x^2 = 2x^2$

$x^4 - 4x^2 = 0$

$x^2(x^2 - 4) = 0$

$x^2(x + 2)(x - 2) = 0$

$x^2 = 0 \Rightarrow x = 0$

$x + 2 = 0 \Rightarrow x = -2$

$x - 2 = 0 \Rightarrow x = 2$

48. $f(x) = g(x)$

$\sqrt{x} - 4 = 2 - x$

$x + \sqrt{x} - 6 = 0$

$(\sqrt{x} + 3)(\sqrt{x} - 2) = 0$

$\sqrt{x} + 3 = 0 \Rightarrow \sqrt{x} = -3$, which is a contradiction, since $\sqrt{x}$ represents the principal square root.

$\sqrt{x} - 2 = 0 \Rightarrow \sqrt{x} = 2 \Rightarrow x = 4$

49. $f(x) = 5x^2 + 2x - 1$

Because $f(x)$ is a polynomial, the domain is all real numbers x.

50. $f(x) = 1 - 2x^2$

Because $f(x)$ is a polynomial, the domain is all real numbers x.

51. $h(t) = \dfrac{4}{t}$

The domain is all real numbers t except $t = 0$.

52. $s(y) = \dfrac{3y}{y + 5}$

$y + 5 \neq 0$

$y \neq -5$

The domain is all real numbers y except $y = -5$.

53. $g(y) = \sqrt{y - 10}$

Domain: $y - 10 \geq 0$

$y \geq 10$

The domain is all real numbers y such that $y \geq 10$.

54. $f(t) = \sqrt[3]{t + 4}$

Because $f(t)$ is a cube root, the domain is all real numbers t.

55. $g(x) = \dfrac{1}{x} - \dfrac{3}{x + 2}$

The domain is all real numbers x except $x = 0$, $x = -2$.

56. $h(x) = \dfrac{10}{x^2 - 2x}$

$x^2 - 2x \neq 0$

$x(x - 2) \neq 0$

The domain is all real numbers x except $x = 0, x = 2$.

57. $f(s) = \dfrac{\sqrt{s - 1}}{s - 4}$

Domain: $s - 1 \geq 0 \Rightarrow s \geq 1$ and $s \neq 4$

The domain consists of all real numbers s, such that $s \geq 1$ and $s \neq 4$.

58. $f(x) = \dfrac{\sqrt{x + 6}}{6 + x}$

Domain: $x + 6 \geq 0 \Rightarrow x \geq -6$ and $x \neq -6$

The domain is all real numbers x such that $x > -6$ or $(-6, \infty)$.

59. $f(x) = \dfrac{x - 4}{\sqrt{x}}$

The domain is all real numbers x such that $x > 0$ or $(0, \infty)$.

60. $f(x) = \dfrac{x + 2}{\sqrt{x - 10}}$

$x - 10 > 0$

$x > 10$

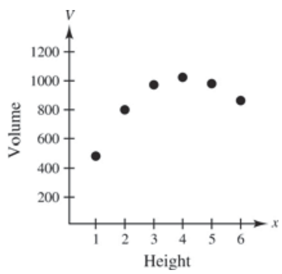
The domain is all real numbers x such that $x > 10$.

61. (a)

Height, x	Volume, V
1	484
2	800
3	972
4	1024
5	980
6	864

The volume is maximum when $x = 4$ and $V = 1024$ cubic centimeters.

(b)



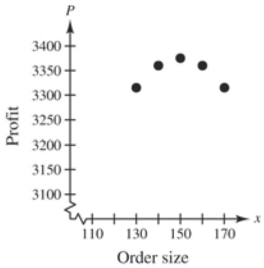
V is a function of x.

(c) $V = x(24 - 2x)^2$

Domain: $0 < x < 12$

62. (a) The maximum profit is \$3375.

(b)



Yes, P is a function of x.

(c) $\text{Profit} = \text{Revenue} - \text{Cost}$

$$= \begin{pmatrix}\text{price}\\ \text{per unit}\end{pmatrix}\begin{pmatrix}\text{number}\\ \text{of units}\end{pmatrix} - (\text{cost})\begin{pmatrix}\text{number}\\ \text{of units}\end{pmatrix}$$

$$= \left[90 - (x - 100)(0.15)\right]x - 60x,\ x > 100$$

$$= (90 - 0.15x + 15)x - 60x$$

$$= (105 - 0.15x)x - 60x$$

$$= 105x - 0.15x^2 - 60x$$

$$= 45x - 0.15x^2,\ x > 100$$

63. $A = s^2$ and $P = 4s \Rightarrow \dfrac{P}{4} = s$

$$A = \left(\frac{P}{4}\right)^2 = \frac{P^2}{16}$$

64. $A = \pi r^2, C = 2\pi r$

$$r = \frac{C}{2\pi}$$

$$A = \pi\left(\frac{C}{2\pi}\right)^2 = \frac{C^2}{4\pi}$$

65. $y = -\frac{1}{10}x^2 + 3x + 6$

$$y(30) = -\tfrac{1}{10}(30)^2 + 3(30) + 6 = 6 \text{ feet}$$

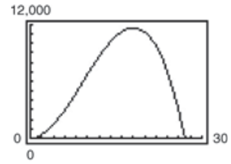
If the child holds a glove at a height of 5 feet, then the ball *will* be over the child's head because it will be at a height of 6 feet.

66. (a) $V = l \cdot w \cdot h = x \cdot y \cdot x = x^2y$ where $4x + y = 108$. So, $y = 108 - 4x$ and

$$V = x^2(108 - 4x) = 108x^2 - 4x^3.$$

Domain: $0 < x < 27$

(b)



(c) The dimensions that will maximize the volume of the package are $18 \times 18 \times 36$. From the graph, the maximum volume occurs when $x = 18$. To find the dimension for y, use the equation $y = 108 - 4x$.

$$y = 108 - 4x = 108 - 4(18) = 108 - 72 = 36$$

67. $A = \dfrac{1}{2}bh = \dfrac{1}{2}xy$

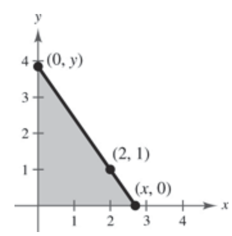
Because $(0, y), (2, 1),$ and $(x, 0)$ all lie on the same line, the slopes between any pair are equal.

$$\frac{1 - y}{2 - 0} = \frac{0 - 1}{x - 2}$$

$$\frac{1 - y}{2} = \frac{-1}{x - 2}$$

$$y = \frac{2}{x - 2} + 1$$

$$y = \frac{x}{x - 2}$$



So, $A = \dfrac{1}{2}x\left(\dfrac{x}{x - 2}\right) = \dfrac{x^2}{2(x - 2)}.$

The domain of A includes x-values such that $x^2/\left[2(x - 2)\right] > 0$. By solving this inequality, the domain is $x > 2$.

68. $A = l \cdot w = (2x)y = 2xy$

But $y = \sqrt{36 - x^2}$, so $A = 2x\sqrt{36 - x^2}$. The domain is $0 < x < 6$.

69. For 2004 through 2007, use

$$p(t) = 4.57t + 27.3.$$

$$2004: p(4) = 4.57(4) + 27.3 = 45.58\%$$

$$2005: p(5) = 4.57(5) + 27.3 = 50.15\%$$

$$2006: p(6) = 4.57(6) + 27.3 = 54.72\%$$

$$2007: p(7) = 4.57(7) + 27.3 = 59.29\%$$

For 2008 through 2010, use

$$p(t) = 3.35t + 37.6.$$

$$2008: p(8) = 3.35(8) + 37.6 = 64.4\%$$

$$2009: p(9) = 3.35(9) + 37.6 = 67.75\%$$

$$2010: p(10) = 3.35(10) + 37.6 = 71.1\%$$

70. For 2000 through 2006, use

$$p(t) = 0.438t^2 + 10.81t + 145.9.$$

$$2000: p(0) = 0.438(0)^2 + 10.81(0) + 145.9 = \$145.9 \text{ thousand}$$

$$2001: p(1) = 0.438(1)^2 + 10.81(1) + 145.9 = \$157.148 \text{ thousand}$$

$$2002: p(2) = 0.438(2)^2 + 10.81(2) + 145.9 = \$169.272 \text{ thousand}$$

$$2003: p(3) = 0.438(3)^2 + 10.81(3) + 145.9 = \$182.272 \text{ thousand}$$

$$2004: p(4) = 0.438(4)^2 + 10.81(4) + 145.9 = \$196.148 \text{ thousand}$$

$$2005: p(5) = 0.438(5)^2 + 10.81(5) + 145.9 = \$210.9 \text{ thousand}$$

$$2006: p(6) = 0.438(6)^2 + 10.81(6) + 145.9 = \$226.528 \text{ thousand}$$

For 2007 though 2010, use

$$p(t) = 5.575t^2 - 110.67t + 720.8.$$

$$2007: p(7) = 5.575(7)^2 - 110.67(7) + 720.8 = \$219.285 \text{ thousand}$$

$$2008: p(8) = 5.575(8)^2 - 110.67(8) + 720.8 = \$192.24 \text{ thousand}$$

$$2009: p(9) = 5.575(9)^2 - 110.67(9) + 720.8 = \$176.345 \text{ thousand}$$

$$2010: p(10) = 5.575(10)^2 - 110.67(10) + 720.8 = \$171.6 \text{ thousand}$$

71. (a) Cost = variable costs + fixed costs

$$C = 12.30x + 98{,}000$$

(b) Revenue = price per unit × number of units

$$R = 17.98x$$

(c) Profit = Revenue − Cost

$$P = 17.98x - (12.30x + 98{,}000)$$

$$P = 5.68x - 98{,}000$$

72. (a) *Model:*

(Total cost) = (Fixed costs) + (Variable costs)

Labels: Total cost $= C$

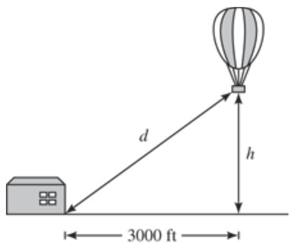
Fixed cost $= 6000$

Variable costs $= 0.95x$

Equation: $C = 6000 + 0.95x$

(b) $$\bar{C} = \frac{C}{x} = \frac{6000 + 0.95x}{x} = \frac{6000}{x} + 0.95$$

73. (a)



(b) $(3000)^2 + h^2 = d^2$

$$h = \sqrt{d^2 - (3000)^2}$$

Domain: $d \geq 3000$ (because both $d \geq 0$ and $d^2 - (3000)^2 \geq 0$)

74. $F(y) = 149.76\sqrt{10}y^{5/2}$

(a)

y	5	10	20	30	40
$F(y)$	26,474.08	149,760.00	847,170.49	2,334,527.36	4,792,320

The force, in tons, of the water against the dam increases with the depth of the water.

(b) It appears that approximately 21 feet of water would produce 1,000,000 tons of force.

(c)
$$1{,}000{,}000 = 149.76\sqrt{10}y^{5/2}$$
$$\frac{1{,}000{,}000}{149.76\sqrt{10}} = y^{5/2}$$
$$2111.56 \approx y^{5/2}$$
$$21.37 \text{ feet} \approx y$$

75. (a) $R = n(\text{rate}) = n[8.00 - 0.05(n - 80)], n \geq 80$

$$R = 12.00n - 0.05n^2 = 12n - \frac{n^2}{20} = \frac{240n - n^2}{20}, n \geq 80$$

(b)

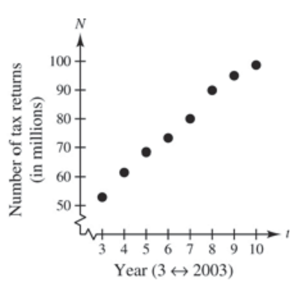
n	90	100	110	120	130	140	150
$R(n)$	\$675	\$700	\$715	\$720	\$715	\$700	\$675

The revenue is maximum when 120 people take the trip.

76. (a)
$$\frac{f(2010) - f(2003)}{2010 - 2003} = \frac{98.7 - 52.9}{7} = \frac{45.8}{7} \approx 6.54$$

Approximately 6.54 million more tax returns were made through e-file each year from 2003 to 2010.

(b)



(c) $N = 6.54t + 33.3$

(d)

t	3	4	5	6
N	52.9	59.5	66.0	72.5

t	7	8	9	10
N	79.1	85.6	92.2	98.7

(e) The algebraic model is a good fit to the actual data.

(f) $y = 6.65x + 34.2$; The models are similar.

77.
$$\begin{aligned}
f(x) &= x^2 - x + 1\\
f(2 + h) &= (2 + h)^2 - (2 + h) + 1\\
&= 4 + 4h + h^2 - 2 - h + 1\\
&= h^2 + 3h + 3\\
f(2) &= (2)^2 - 2 + 1 = 3\\
f(2 + h) - f(2) &= h^2 + 3h\\
\frac{f(2 + h) - f(2)}{h} &= \frac{h^2 + 3h}{h} = h + 3, h \neq 0
\end{aligned}$$

78.
$$\begin{aligned}
f(x) &= 5x - x^2\\
f(5 + h) &= 5(5 + h) - (5 + h)^2\\
&= 25 + 5h - (25 + 10h + h^2)\\
&= 25 + 5h - 25 - 10h - h^2\\
&= -h^2 - 5h\\
f(5) &= 5(5) - (5)^2\\
&= 25 - 25 = 0\\
\frac{f(5 + h) - f(5)}{h} &= \frac{-h^2 - 5h}{h}\\
&= \frac{-h(h + 5)}{h} = -(h + 5), h \neq 0
\end{aligned}$$

79.
$$\begin{aligned}
f(x) &= x^3 + 3x\\
f(x + h) &= (x + h)^3 + 3(x + h)\\
&= x^3 + 3x^2h + 3xh^2 + h^3 + 3x + 3h\\
\frac{f(x + h) - f(x)}{h} &= \frac{(x^3 + 3x^2h + 3xh^2 + h^3 + 3x + 3h) - (x^3 + 3x)}{h}\\
&= \frac{h(3x^2 + 3xh + h^2 + 3)}{h}\\
&= 3x^2 + 3xh + h^2 + 3, h \neq 0
\end{aligned}$$

80.
$$\begin{aligned}
f(x) &= 4x^2 - 2x\\
f(x + h) &= 4(x + h)^2 - 2(x + h)\\
&= 4(x^2 + 2xh + h^2) - 2x - 2h\\
&= 4x^2 + 8xh + 4h^2 - 2x - 2h\\
\frac{f(x + h) - f(x)}{h} &= \frac{4x^2 + 8xh + 4h^2 - 2x - 2h - 4x^2 + 2x}{h}\\
&= \frac{8xh + 4h^2 - 2h}{h}\\
&= \frac{h(8x + 4h - 2)}{h}\\
&= 8x + 4h - 2, h \neq 0
\end{aligned}$$

81.
$$\begin{aligned}
g(x) &= \frac{1}{x^2}\\
\frac{g(x) - g(3)}{x - 3} &= \frac{\frac{1}{x^2} - \frac{1}{9}}{x - 3}\\
&= \frac{9 - x^2}{9x^2(x - 3)}\\
&= \frac{-(x + 3)(x - 3)}{9x^2(x - 3)}\\
&= -\frac{x + 3}{9x^2}, x \neq 3
\end{aligned}$$

82.
$$\begin{aligned}
f(t) &= \frac{1}{t - 2}\\
f(1) &= \frac{1}{1 - 2} = -1\\
\frac{f(t) - f(1)}{t - 1} &= \frac{\frac{1}{t - 2} - (-1)}{t - 1}\\
&= \frac{1 + (t - 2)}{(t - 2)(t - 1)}\\
&= \frac{(t - 1)}{(t - 2)(t - 1)}\\
&= \frac{1}{t - 2}, t \neq 1
\end{aligned}$$

83. $f(x) = \sqrt{5x}$

$$\frac{f(x) - f(5)}{x - 5} = \frac{\sqrt{5x} - 5}{x - 5}, x \neq 5$$

84.
$$f(x) = x^{2/3} + 1$$
$$f(8) = 8^{2/3} + 1 = 5$$
$$\frac{f(x) - f(8)}{x - 8} = \frac{x^{2/3} + 1 - 5}{x - 8} = \frac{x^{2/3} - 4}{x - 8}, x \neq 8$$

85. By plotting the points, we have a parabola, so $g(x) = cx^2$. Because $(-4, -32)$ is on the graph, you have $-32 = c(-4)^2 \Rightarrow c = -2$. So, $g(x) = -2x^2$.

86. By plotting the data, you can see that they represent a line, or $f(x) = cx$. Because $(0, 0)$ and $\left(1, \frac{1}{4}\right)$ are on the line, the slope is $\frac{1}{4}$. So, $f(x) = \frac{1}{4}x$.

87. Because the function is undefined at 0, we have $r(x) = c/x$. Because $(-4, -8)$ is on the graph, you have $-8 = c/-4 \Rightarrow c = 32$. So, $r(x) = 32/x$.

88. By plotting the data, you can see that they represent $h(x) = c\sqrt{|x|}$. Because $\sqrt{|-4|} = 2$ and $\sqrt{|-1|} = 1$, and the corresponding y-values are 6 and 3, $c = 3$ and $h(x) = 3\sqrt{|x|}$.

89. False. The equation $y^2 = x^2 + 4$ is a relation between x and y. However, $y = \pm\sqrt{x^2 + 4}$ does not represent a function.

90. True. A function is a relation by definition.

91. False. The range is $[-1, \infty)$.

92. True. The set represents a function. Each x-value is mapped to exactly one y-value.

93. $f(x) = \sqrt{x - 1}$ Domain: $x \geq 1$

$g(x) = \dfrac{1}{\sqrt{x - 1}}$ Domain: $x > 1$

The value 1 may be included in the domain of $f(x)$ as it is possible to find the square root of 0. However, 1 cannot be included in the domain of $g(x)$ as it causes a zero to occur in the denominator which results in the function being undefined.

94. Because $f(x)$ is a function of an even root, the radicand cannot be negative. $g(x)$ is an odd root, therefore the radicand can be any real number. So, the domain of g is all real numbers x and the domain of f is all real numbers x such that $x \geq 2$.

95. No; x is the independent variable, f is the name of the function.

96. (a) The height h is function of t because for each value of t there is a corresponding value of h for $0 \leq t \leq 2.6$.

(b) Using the graph when $t = 0.5$, $h \approx 20$ feet and when $t = 1.25$, $h \approx 28$ feet.

(c) The domain of h is approximately $0 \leq t \leq 2.6$.

(d) No, the time t is not a function of the height h because some values of h correspond to more than one value of t.

97. (a) Yes. The amount that you pay in sales tax will increase as the price of the item purchased increases.

(b) No. The length of time that you study the night before an exam does not necessarily determine your score on the exam.

98. (a) No. During the course of a year, for example, your salary may remain constant while your savings account balance may vary. That is, there may be two or more outputs (savings account balances) for one input (salary).

(b) Yes. The greater the height from which the ball is dropped, the greater the speed with which the ball will strike the ground.

Section 1.5 Analyzing Graphs of Functions

1. Vertical Line Test

2. zeros

3. decreasing

4. maximum

5. average rate of change; secant

6. odd

7. Domain: $(-\infty, \infty)$; Range: $[-4, \infty)$

(a) $f(-2) = 0$

(b) $f(-1) = -1$

(c) $f\left(\frac{1}{2}\right) = 0$

(d) $f(1) = -2$

8. Domain: $(-\infty, \infty)$; Range: $(-\infty, \infty)$

(a) $f(-1) = 4$

(b) $f(2) = 4$

(c) $f(0) = 2$

(d) $f(1) = 0$

9. Domain: $(-\infty, \infty)$; Range: $(-2, \infty)$

(a) $f(2) = 0$

(b) $f(1) = 1$

(c) $f(3) = 2$

(d) $f(-1) = 3$

10. Domain: $(-\infty, \infty)$; Range: $(-\infty, 1]$

(a) $f(-2) = -3$

(b) $f(1) = 0$

(c) $f(0) = 1$

(d) $f(2) = -3$

11. $y = \frac{1}{4}x^3$

A vertical line intersects the graph at most once, so *y is* a function of *x*.

12. $x - y^2 = 1 \Rightarrow y = \pm\sqrt{x-1}$

y is not a function of *x*. Some vertical lines intersect the graph twice.

13. $x^2 + y^2 = 25$

A vertical line intersects the graph more than once, so *y is not* a function of *x*.

14. $x^2 = 2xy - 1$

A vertical line intersects the graph at most once, so *y is* a function of *x*.

15.
$$\begin{aligned} f(x) &= 2x^2 - 7x - 30 \\ 2x^2 - 7x - 30 &= 0 \\ (2x+5)(x-6) &= 0 \end{aligned}$$
$$2x + 5 = 0 \quad \text{or} \quad x - 6 = 0$$
$$x = -\tfrac{5}{2} \qquad\qquad x = 6$$

16.
$$\begin{aligned} f(x) &= 3x^2 + 22x - 16 \\ 3x^2 + 22x - 16 &= 0 \\ (3x-2)(x+8) &= 0 \\ 3x - 2 = 0 &\Rightarrow x = \tfrac{2}{3} \\ x + 8 = 0 &\Rightarrow x = -8 \end{aligned}$$

17.
$$\begin{aligned} f(x) &= \frac{x}{9x^2 - 4} \\ \frac{x}{9x^2 - 4} &= 0 \\ x &= 0 \end{aligned}$$

18.
$$\begin{aligned} f(x) &= \frac{x^2 - 9x + 14}{4x} \\ \frac{x^2 - 9x + 14}{4x} &= 0 \\ (x-7)(x-2) &= 0 \\ x - 7 = 0 &\Rightarrow x = 7 \\ x - 2 = 0 &\Rightarrow x = 2 \end{aligned}$$

19.
$$\begin{aligned} f(x) &= \tfrac{1}{2}x^3 - x \\ \tfrac{1}{2}x^3 - x &= 0 \\ x^3 - 2x &= 2(0) \\ x(x^2 - 2) &= 0 \end{aligned}$$
$$x = 0 \quad \text{or} \quad x^2 - 2 = 0$$
$$x^2 = 2$$
$$x = \pm\sqrt{2}$$

20.
$$\begin{aligned} f(x) &= x^3 - 4x^2 - 9x + 36 \\ x^3 - 4x^2 - 9x + 36 &= 0 \\ x^2(x-4) - 9(x-4) &= 0 \\ (x-4)(x^2-9) &= 0 \\ x - 4 = 0 &\Rightarrow x = 4 \\ x^2 - 9 = 0 &\Rightarrow x = \pm 3 \end{aligned}$$

21.
$$\begin{aligned} f(x) &= 4x^3 - 24x^2 - x + 6 \\ 4x^3 - 24x^2 - x + 6 &= 0 \\ 4x^2(x-6) - 1(x-6) &= 0 \\ (x-6)(4x^2-1) &= 0 \\ (x-6)(2x+1)(2x-1) &= 0 \end{aligned}$$
$$x - 6 = 0 \quad \text{or} \quad 2x + 1 = 0 \quad \text{or} \quad 2x - 1 = 0$$
$$x = 6 \qquad\qquad x = -\tfrac{1}{2} \qquad\qquad x = \tfrac{1}{2}$$

22. $f(x) = 9x^4 - 25x^2$

$9x^4 - 25x^2 = 0$

$x^2(9x^2 - 25) = 0$

$x^2 = 0 \Rightarrow x = 0$

$9x^2 - 25 = 0 \Rightarrow x = \pm\frac{5}{3}$

23. $f(x) = \sqrt{2x} - 1$

$\sqrt{2x} - 1 = 0$

$\sqrt{2x} = 1$

$2x = 1$

$x = \frac{1}{2}$

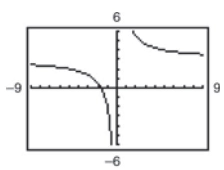
24. $f(x) = \sqrt{3x + 2}$

$\sqrt{3x + 2} = 0$

$3x + 2 = 0$

$-\frac{2}{3} = x$

25. (a)



Zero: $x = -\dfrac{5}{3}$

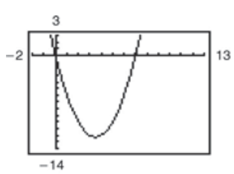
(b) $f(x) = 3 + \frac{5}{x}$

$3 + \frac{5}{x} = 0$

$3x + 5 = 0$

$x = -\frac{5}{3}$

26. (a)



Zeros: $x = 0, x = 7$

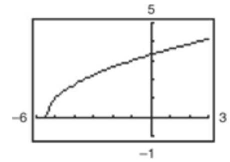
(b) $f(x) = x(x - 7)$

$x(x - 7) = 0$

$x = 0$

$x - 7 = 0 \Rightarrow x = 7$

27. (a)



Zero: $x = -\frac{11}{2}$

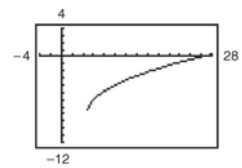
(b) $f(x) = \sqrt{2x + 11}$

$\sqrt{2x + 11} = 0$

$2x + 11 = 0$

$x = -\frac{11}{2}$

28. (a)



Zero: $x = 26$

(b) $f(x) = \sqrt{3x - 14} - 8$

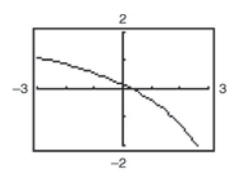
$\sqrt{3x - 14} - 8 = 0$

$\sqrt{3x - 14} = 8$

$3x - 14 = 64$

$x = 26$

29. (a)



Zero: $x = \dfrac{1}{3}$

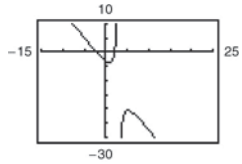
(b) $f(x) = \dfrac{3x - 1}{x - 6}$

$\dfrac{3x - 1}{x - 6} = 0$

$3x - 1 = 0$

$x = \dfrac{1}{3}$

30. (a)



Zeros: $x = \pm 2.1213$

(b) $f(x) = \dfrac{2x^2 - 9}{3 - x}$

$\dfrac{2x^2 - 9}{3 - x} = 0$

$2x^2 - 9 = 0 \Rightarrow x = \pm\dfrac{3\sqrt{2}}{2} = \pm 2.1213$

31. $f(x) = \frac{3}{2}x$

The function is increasing on $(-\infty, \infty)$.

32. $f(x) = x^2 - 4x$

The function is decreasing on $(-\infty, 2)$ and increasing on $(2, \infty)$.

33. $f(x) = x^3 - 3x^2 + 2$

The function is increasing on $(-\infty, 0)$ and $(2, \infty)$ and decreasing on $(0, 2)$.

34. $f(x) = \sqrt{x^2 - 1}$

The function is decreasing on $(-\infty, -1)$ and increasing on $(1, \infty)$.

35. $f(x) = |x + 1| + |x - 1|$

The function is increasing on $(1, \infty)$.

The function is constant on $(-1, 1)$.

The function is decreasing on $(-\infty, -1)$.

36. The function is decreasing on $(-2, -1)$ and $(-1, 0)$ and increasing on $(-\infty, -2)$ and $(0, \infty)$.

37. $f(x) = \begin{cases} x + 3, & x \le 0 \\ 3, & 0 < x \le 2 \\ 2x + 1, & x > 2 \end{cases}$

The function is increasing on $(-\infty, 0)$ and $(2, \infty)$.

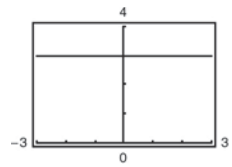
The function is constant on $(0, 2)$.

38. $f(x) = \begin{cases} 2x + 1, & x \le -1 \\ x^2 - 2, & x > -1 \end{cases}$

The function is decreasing on $(-1, 0)$ and increasing on $(-\infty, -1)$ and $(0, \infty)$.

39. $f(x) = 3$

(a)



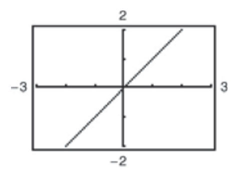
Constant on $(-\infty, \infty)$

(b)

x	−2	−1	0	1	2
$f(x)$	3	3	3	3	3

40. $g(x) = x$

(a)



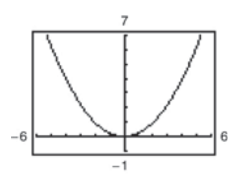
Increasing on $(-\infty, \infty)$

(b)

x	−2	−1	0	1	2
$g(x)$	−2	−1	0	1	2

41. $g(s) = \dfrac{s^2}{4}$

(a)



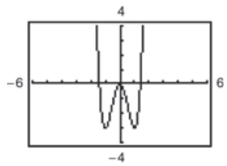
Decreasing on $(-\infty, 0)$; Increasing on $(0, \infty)$

(b)

s	−4	−2	0	2	4
$g(s)$	4	1	0	1	4

42. $f(x) = 3x^4 - 6x^2$

(a)



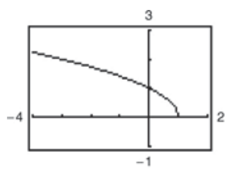
Increasing on $(-1, 0), (1, \infty)$; Decreasing on $(-\infty, -1), (0, 1)$

(b)

x	–2	–1	0	1	2
$f(x)$	24	–3	0	–3	24

43. $f(x) = \sqrt{1 - x}$

(a)



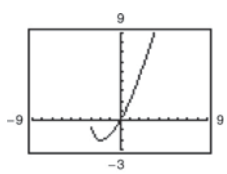
Decreasing on $(-\infty, 1)$

(b)

x	–3	–2	–1	0	1
$f(x)$	2	$\sqrt{3}$	$\sqrt{2}$	1	0

44. $f(x) = x\sqrt{x + 3}$

(a)



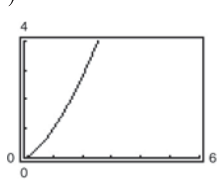
Increasing on $(-2, \infty)$; Decreasing on $(-3, -2)$

(b)

x	–3	–2	–1	0	1
$f(x)$	0	–2	–1.414	0	2

45. $f(x) = x^{3/2}$

(a)



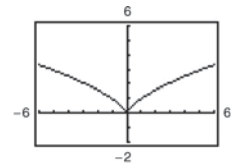
Increasing on $(0, \infty)$

(b)

x	0	1	2	3	4
$f(x)$	0	1	2.8	5.2	8

46. $f(x) = x^{2/3}$

(a)

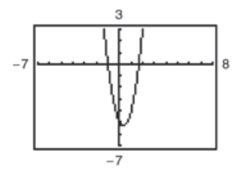


Decreasing on $(-\infty, 0)$; Increasing on $(0, \infty)$

(b)

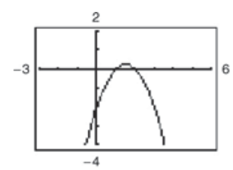
x	–2	–1	0	1	2
$f(x)$	1.59	1	0	1	1.59

47. $f(x) = 3x^2 - 2x - 5$



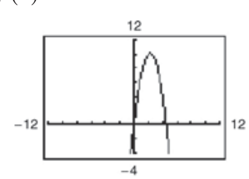
Relative minimum: $\left(\frac{1}{3}, -\frac{16}{3}\right)$ or $(0.33, -5.33)$

48. $f(x) = -x^2 + 3x - 2$



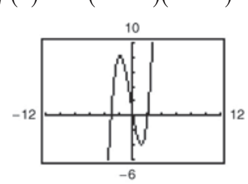
Relative maximum: $(1.5, 0.25)$

49. $f(x) = -2x^2 + 9x$



Relative maximum: $(2.25, 10.125)$

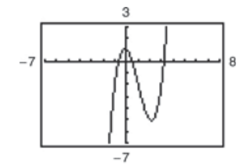
50. $f(x) = x(x - 2)(x + 3)$



Relative minimum: $(1.12, -4.06)$

Relative maximum: $(-1.79, 8.21)$

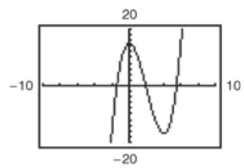
51. $f(x) = x^3 - 3x^2 - x + 1$



Relative maximum: $(-0.15, 1.08)$

Relative minimum: $(2.15, -5.08)$

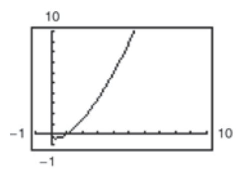
52. $h(x) = x^3 - 6x^2 + 15$



Relative minimum: $(4, -17)$

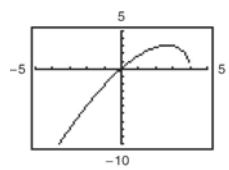
Relative maximum: $(0, 15)$

53. $h(x) = (x - 1)\sqrt{x}$



Relative minimum: $(0.33, -0.38)$

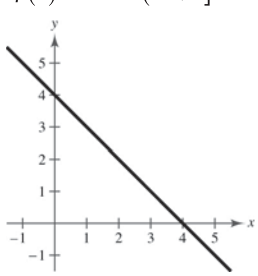
54. $g(x) = x\sqrt{4 - x}$



Relative maximum: $(2.67, 3.08)$

55. $f(x) = 4 - x$

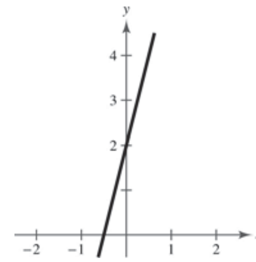
$f(x) \ge 0$ on $(-\infty, 4]$



56. $f(x) = 4x + 2$

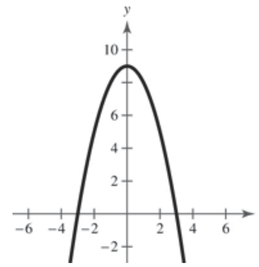
$f(x) \ge 0$ on $\left[-\frac{1}{2}, \infty\right)$

$$4x + 2 \ge 0$$
$$4x \ge -2$$
$$x \ge -\tfrac{1}{2}$$
$$\left[-\tfrac{1}{2}, \infty\right)$$



57. $f(x) = 9 - x^2$

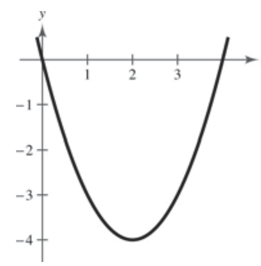
$f(x) \ge 0$ on $[-3, 3]$



58. $f(x) = x^2 - 4x$

$f(x) \ge 0$ on $(-\infty, 0]$ and $[4, \infty)$

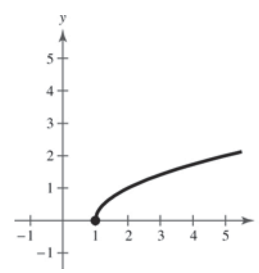
$$x^2 - 4x \ge 0$$
$$x(x - 4) \ge 0$$
$$(-\infty, 0], [4, \infty)$$



59. $f(x) = \sqrt{x - 1}$

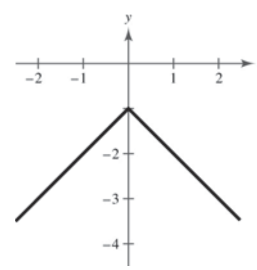
$f(x) \ge 0$ on $[1, \infty)$

$$\sqrt{x - 1} \ge 0$$
$$x - 1 \ge 0$$
$$x \ge 1$$
$$[1, \infty)$$



60. $f(x) = -(1 + |x|)$

$f(x)$ is never greater than 0. $(f(x) < 0$ for all $x.)$



61. $f(x) = -2x + 15$

$$\frac{f(3) - f(0)}{3 - 0} = \frac{9 - 15}{3} = -2$$

The average rate of change from $x_1 = 0$ to $x_2 = 3$ is -2.

62. $f(x) = x^2 - 2x + 8$

$$\frac{f(5) - f(1)}{5 - 1} = \frac{23 - 7}{4} = \frac{16}{4} = 4$$

The average rate of change from $x_1 = 1$ to $x_2 = 5$ is 4.

63. $f(x) = x^3 - 3x^2 - x$

$$\frac{f(3) - f(1)}{3 - 1} = \frac{-3 - (-3)}{2} = 0$$

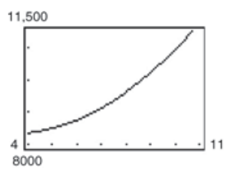
The average rate of change from $x_1 = 1$ to $x_2 = 3$ is 0.

64. $f(x) = -x^3 + 6x^2 + x$

$$\frac{f(6) - f(1)}{6 - 1} = \frac{6 - 6}{5} = \frac{0}{5} = 0$$

The average rate of change from $x_1 = 1$ to $x_2 = 6$ is 0.

65. (a)



(b) To find the average rate of change of the amount the U.S. Department of Energy spent for research and development from 2005 to 2010, find the average rate of change from $(5, f(5))$ to $(10, f(10))$.

$$\frac{f(10) - f(5)}{10 - 5} = \frac{10{,}925 - 8501.25}{5} = 484.75$$

The amount the U.S. Department of Energy spent for research and development increased by about \$484.75 million each year from 2005 to 2010.

66. Average rate of change $= \dfrac{s(t_2) - s(t_1)}{t_2 - t_1}$

$$= \frac{s(9) - s(0)}{9 - 0}$$

$$= \frac{540 - 0}{9 - 0}$$

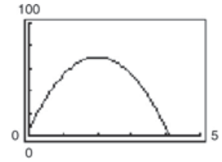
$= 60$ feet per second.

As the time traveled increases, the distance increases rapidly, causing the average speed to increase with each time increment. From $t = 0$ to $t = 4$, the average speed is less than from $t = 4$ to $t = 9$. Therefore, the overall average from $t = 0$ to $t = 9$ falls below the average found in part (b).

67. $s_0 = 6, v_0 = 64$

(a) $s = -16t^2 + 64t + 6$

(b)



(c) $\dfrac{s(3) - s(0)}{3 - 0} = \dfrac{54 - 6}{3} = 16$

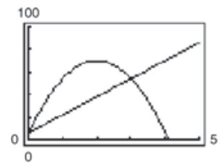
(d) The slope of the secant line is positive.

(e) $s(0) = 6, m = 16$

Secant line: $y - 6 = 16(t - 0)$

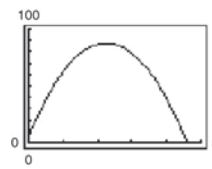
$y = 16t + 6$

(f)



68. (a) $s = -16t^2 + 72t + 6.5$

(b)



(c) The average rate of change from $t = 0$ to $t = 4$:

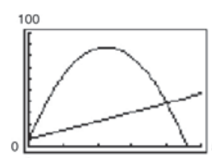
$\dfrac{s(4) - s(0)}{4 - 0} = \dfrac{38.5 - 6.5}{4} = \dfrac{32}{4} = 8$ feet per second

(d) The slope of the secant line through $(0, s(0))$ and $(4, s(4))$ is positive.

(e) The equation of the secant line:

$m = 8, y = 8t + 6.5$

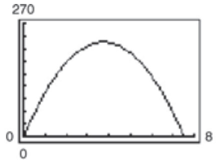
(f)



69. $v_0 = 120,\ s_0 = 0$

(a) $s = -16t^2 + 120t$

(b)



(c) The average rate of change from $t = 3$ to $t = 5$:

$\dfrac{s(5) - s(3)}{5 - 3} = \dfrac{200 - 216}{2} = -\dfrac{16}{2} = -8$ feet per second

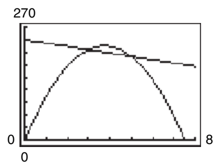
(d) The slope of the secant line through $(3, s(3))$ and $(5, s(5))$ is negative.

(e) The equation of the secant line: $m = -8$

Using $(5, s(5)) = (5, 200)$ we have

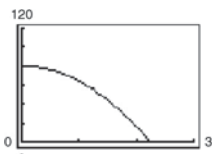
$$y - 200 = -8(t - 5)$$
$$y = -8t + 240.$$

(f)



70. (a) $s = -16t^2 + 80$

(b)



(c) The average rate of change from $t = 1$ to $t = 2$:

$\dfrac{s(2) - s(1)}{2 - 1} = \dfrac{16 - 64}{1} = -\dfrac{48}{1} = -48$ feet per second

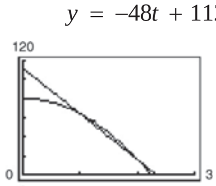
(d) The slope of the secant line through $(1, s(1))$ and $(2, s(2))$ is negative.

(e) The equation of the secant line: $m = -48$

Using $(1, s(1)) = (1, 64)$ we have

$$y - 64 = -48(t - 1)$$
$$y = -48t + 112.$$

(f)



71. $f(x) = x^6 - 2x^2 + 3$

$$\begin{aligned} f(-x) &= (-x)^6 - 2(-x)^2 + 3 \\ &= x^6 - 2x^2 + 3 \\ &= f(x) \end{aligned}$$

The function is even. y-axis symmetry.

72. $g(x) = x^3 - 5x$

$$\begin{aligned} g(-x) &= (-x)^3 - 5(-x) \\ &= -x^3 + 5x \\ &= -g(x) \end{aligned}$$

The function is odd. Origin symmetry.

73. $h(x) = x\sqrt{x + 5}$

$$\begin{aligned} h(-x) &= (-x)\sqrt{-x + 5} \\ &= -x\sqrt{5 - x} \\ &\neq h(x) \\ &\neq -h(x) \end{aligned}$$

The function is neither odd nor even. No symmetry.

74. $f(x) = x\sqrt{1 - x^2}$

$$\begin{aligned} f(-x) &= -x\sqrt{1 - (-x)^2} \\ &= -x\sqrt{1 - x^2} \\ &= -f(x) \end{aligned}$$

The function is odd. Origin symmetry.

75. $f(s) = 4s^{3/2}$

$$\begin{aligned} &= 4(-s)^{3/2} \\ &\neq f(s) \\ &\neq -f(s) \end{aligned}$$

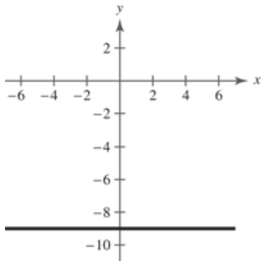
The function is neither odd nor even. No symmetry.

76. $g(s) = 4s^{2/3}$

$$\begin{aligned} g(-s) &= 4(-s)^{2/3} \\ &= 4s^{2/3} \\ &= g(s) \end{aligned}$$

The function is even. y-axis symmetry.

77.

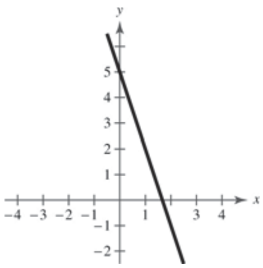


The graph of $f(x) = -9$ is symmetric to the y-axis, which implies $f(x)$ is even.

$$\begin{aligned} f(-x) &= -9 \\ &= f(x) \end{aligned}$$

The function is even.

78. $f(x) = 5 - 3x$

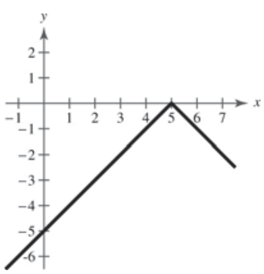


The graph displays no symmetry, which implies $f(x)$ is neither odd nor even.

$$\begin{aligned} f(-x) &= 5 - 3(-x) \\ &= 5 + 3x \\ &\neq f(x) \\ &\neq -f(x) \end{aligned}$$

The function is neither even nor odd.

79. $f(x) = -|x - 5|$

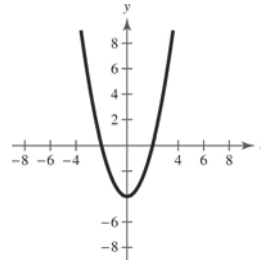


The graph displays no symmetry, which implies $f(x)$ is neither odd nor even.

$$\begin{aligned} f(x) &= -|(-x) - 5| \\ &= -|-x - 5| \\ &\neq f(x) \\ &\neq -f(x) \end{aligned}$$

The function is neither even nor odd.

80. $h(x) = x^2 - 4$

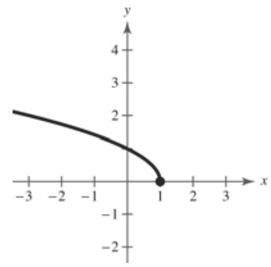


The graph displays y-axis symmetry, which implies $h(x)$ is even.

$$h(-x) = (-x)^2 - 4 = x^2 - 4 = h(x)$$

The function is even.

81. $f(x) = \sqrt{1 - x}$

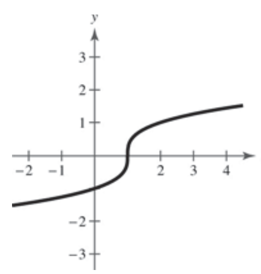


The graph displays no symmetry, which implies $f(x)$ is neither odd nor even.

$$\begin{aligned} f(-x) &= \sqrt{1 - (-x)} \\ &= \sqrt{1 + x} \\ &\neq f(x) \\ &\neq -f(x) \end{aligned}$$

The function is neither even nor odd.

82. $g(t) = \sqrt[3]{t - 1}$



The graph displays no symmetry, which implies $g(t)$ is neither odd nor even.

$$\begin{aligned} g(-t) &= \sqrt[3]{(-t) - 1} \\ &= \sqrt[3]{-t - 1} \\ &\neq g(t) \\ &\neq -g(t) \end{aligned}$$

The function is neither even nor odd.

83. $h = \text{top} - \text{bottom}$

$= 3 - (4x - x^2)$

$= 3 - 4x + x^2$

84. $h = \text{top} - \text{bottom}$

$= (4x - x^2) - 2x$

$= 2x - x^2$

85. $L = \text{right} - \text{left}$

$= 2 - \sqrt[3]{2y}$

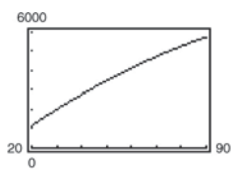
86. $L = \text{right} - \text{left}$

$= \dfrac{2}{y} - 0$

$= \dfrac{2}{y}$

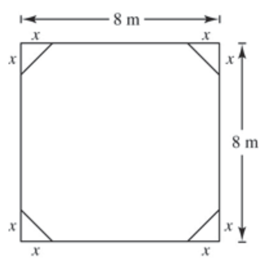
87. $L = -0.294x^2 + 97.744x - 664.875, 20 \le x \le 90$

(a)



(b) $L = 2000$ when $x \approx 29.9645 \approx 30$ watts.

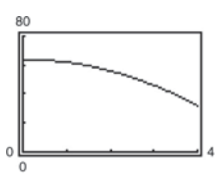
88.



(a) $A = (8)(8) - 4\left(\frac{1}{2}\right)(x)(x) = 64 - 2x^2$

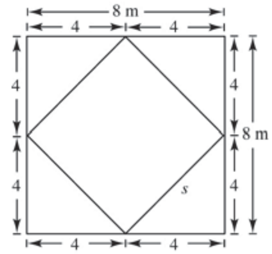
Domain: $0 \le x \le 4$

(b)



Range: $32 \le A \le 64$

(c) When $x = 4$, the resulting figure is a square.



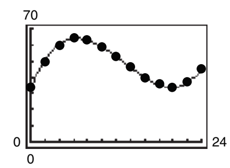
By the Pythagorean Theorem,
$4^2 + 4^2 = s^2 \Rightarrow s = \sqrt{32} = 4\sqrt{2}$ meters.

89. (a) For the average salaries of college professors, a scale of $10,000 would be appropriate.

(b) For the population of the United States, use a scale of 10,000,000.

(c) For the percent of the civilian workforce that is unemployed, use a scale of 1%.

90. (a)



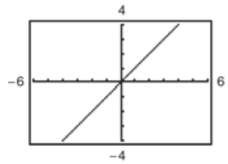
(b) The model is an excellent fit.

(c) The temperature is increasing from 6 A.M. until noon $(x = 0 \text{ to } x = 6)$. Then it decreases until 2 A.M. $(x = 6 \text{ to } x = 20)$. Then the temperature increases until 6 A.M. $(x = 20 \text{ to } x = 24)$.

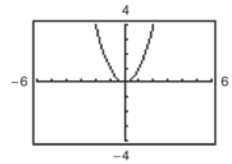
(d) The maximum temperature according to the model is about 63.93°F. According to the data, it is 64°F. The minimum temperature according to the model is about 33.98°F. According to the data, it is 34°F.

(e) Answers may vary. Temperatures will depend upon the weather patterns, which usually change from day to day.

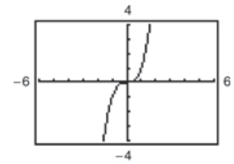
91. (a) $y = x$



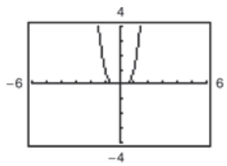
(b) $y = x^2$



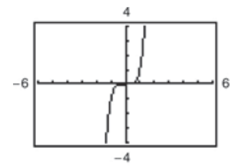
(c) $y = x^3$



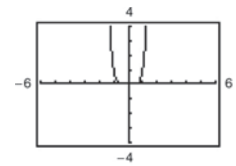
(d) $y = x^4$



(e) $y = x^5$



(f) $y = x^6$



All the graphs pass through the origin. The graphs of the odd powers of x are symmetric with respect to the origin and the graphs of the even powers are symmetric with respect to the y-axis. As the powers increase, the graphs become flatter in the interval $-1 < x < 1$.

92. (a) Domain: $[-4, 5)$; Range: $[0, 9]$

(b) $(3, 0)$

(c) Increasing: $(-4, 0) \cup (3, 5)$; Decreasing: $(0, 3)$

(d) Relative minimum: $(3, 0)$

Relative maximum: $(0, 9)$

(e) Neither

93. False. The function $f(x) = \sqrt{x^2 + 1}$ has a domain of all real numbers.

94. False. An odd function is symmetric with respect to the origin, so its domain must include negative values.

95. $\left(-\frac{5}{3}, -7\right)$

(a) If f is even, another point is $\left(\frac{5}{3}, -7\right)$.

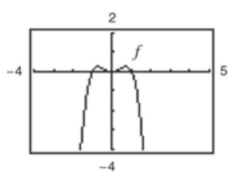
(b) If f is odd, another point is $\left(\frac{5}{3}, 7\right)$.

96. $(2a, 2c)$

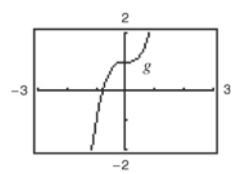
(a) $(-2a, 2c)$

(b) $(-2a, -2c)$

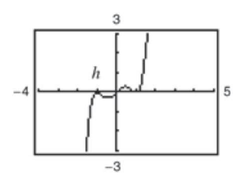
97.



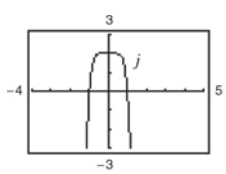
$f(x) = x^2 - x^4$ is even.



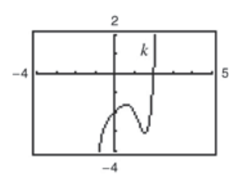
$g(x) = 2x^3 + 1$ is neither.



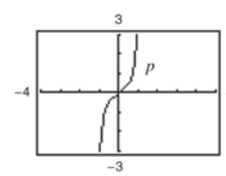
$h(x) = x^5 - 2x^3 + x$ is odd.



$j(x) = 2 - x^6 - x^8$ is even.



$k(x) = x^5 - 2x^4 + x - 2$ is neither.



$p(x) = x^9 + 3x^5 - x^3 + x$ is odd.

Equations of odd functions contain only odd powers of x. Equations of even functions contain only even powers of x. A function that has variables raised to even and odd powers is neither odd nor even.

98. (a) Even. The graph is a reflection in the x-axis.

(b) Even. The graph is a reflection in the y-axis.

(c) Even. The graph is a vertical translation of f.

(d) Neither. The graph is a horizontal translation of f.

Section 1.6 A Library of Parent Functions

1. $f(x) = [\![x]\!]$

(g) greatest integer function

2. $f(x) = x$

(i) identity function

3. $f(x) = \dfrac{1}{x}$

(h) reciprocal function

4. $f(x) = x^2$

(a) squaring function

5. $f(x) = \sqrt{x}$

(b) square root function

6. $f(x) = c$

(e) constant function

7. $f(x) = |x|$

(f) absolute value function

8. $f(x) = x^3$

(c) cubic function

9. $f(x) = ax + b$

(d) linear function

10. linear

11. (a) $f(1) = 4,\ f(0) = 6$

$(1, 4), (0, 6)$

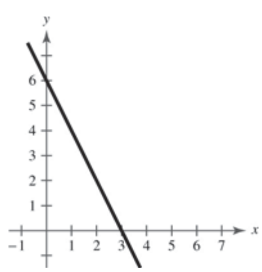
$$m = \frac{6 - 4}{0 - 1} = -2$$

$$y - 6 = -2(x - 0)$$

$$y = -2x + 6$$

$$f(x) = -2x + 6$$

(b)



12. (a) $f(-3) = -8,\ f(1) = 2$

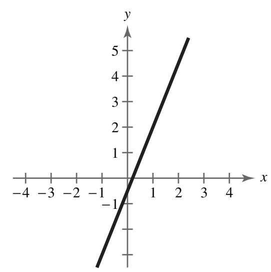
$(-3, -8), (1, 2)$

$$m = \frac{2 - (-8)}{1 - (-3)} = \frac{10}{4} = \frac{5}{2}$$

$$f(x) - 2 = \frac{5}{2}(x - 1)$$

$$f(x) = \frac{5}{2}x - \frac{1}{2}$$

(b)



13. (a) $f(-5) = -1,\ f(5) = -1$

$(-5, -1), (5, -1)$

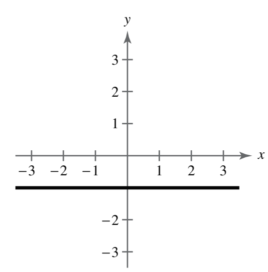
$$m = \frac{-1 - (-1)}{5 - (-5)} = \frac{0}{10} = 0$$

$$y - (-1) = 0(x - (-5))$$

$$y = -1$$

$$f(x) = -1$$

(b)



14. (a) $f\left(\dfrac{2}{3}\right) = -\dfrac{15}{2},\ f(-4) = -11$

$$\left(\frac{2}{3}, -\frac{15}{2}\right), (-4, -11)$$

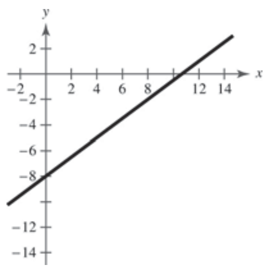
$$m = \frac{-11 - (-15/2)}{-4 - (2/3)}$$

$$= \frac{-7/2}{-14/3} = \left(-\frac{7}{2}\right)\cdot\left(-\frac{3}{14}\right) = \frac{3}{4}$$

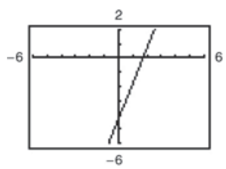
$$f(x) - (-11) = \frac{3}{4}(x - (-4))$$

$$f(x) = \frac{3}{4}x - 8$$

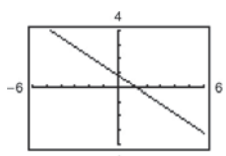
(b)



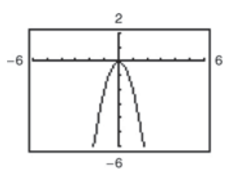
15. $f(x) = 2.5x - 4.25$



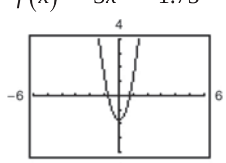
16. $f(x) = \frac{5}{6} - \frac{2}{3}x$



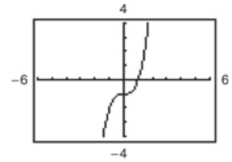
17. $g(x) = -2x^2$



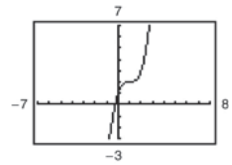
18. $f(x) = 3x^2 - 1.75$



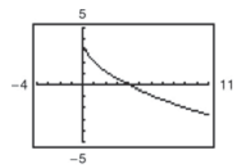
19. $f(x) = x^3 - 1$



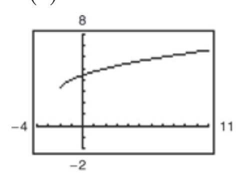
20. $f(x) = (x - 1)^3 + 2$



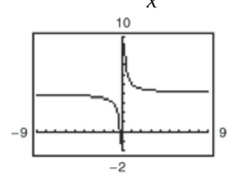
21. $f(x) = 4 - 2\sqrt{x}$



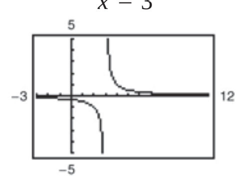
22. $h(x) = \sqrt{x + 2} + 3$



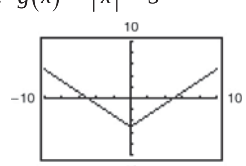
23. $f(x) = 4 + \dfrac{1}{x}$



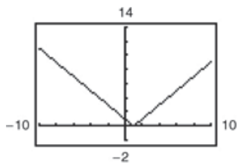
24. $k(x) = \dfrac{1}{x - 3}$



25. $g(x) = |x| - 5$



26. $f(x) = |x - 1|$



27. $f(x) = [\![x]\!]$

(a) $f(2.1) = 2$

(b) $f(2.9) = 2$

(c) $f(-3.1) = -4$

(d) $f\left(\frac{7}{2}\right) = 3$

28. $h(x) = [\![x + 3]\!]$

(a) $h(-2) = [\![1]\!] = 1$

(b) $h\left(\frac{1}{2}\right) = [\![3.5]\!] = 3$

(c) $h(4.2) = [\![7.2]\!] = 7$

(d) $h(-21.6) = [\![-18.6]\!] = -19$

29. $k(x) = \left[\!\!\left[\frac{1}{2}x + 6\right]\!\!\right]$

(a) $k(5) = \left[\!\!\left[\frac{1}{2}(5) + 6\right]\!\!\right] = [\![8.5]\!] = 8$

(b) $k(-6.1) = \left[\!\!\left[\frac{1}{2}(-6.1) + 6\right]\!\!\right] = [\![2.95]\!] = 2$

(c) $k(0.1) = \left[\!\!\left[\frac{1}{2}(0.1) + 6\right]\!\!\right] = [\![6.05]\!] = 6$

(d) $k(15) = \left[\!\!\left[\frac{1}{2}(15) + 6\right]\!\!\right] = [\![13.5]\!] = 13$

30. $g(x) = -7[\![x + 4]\!] + 6$

(a) $g\left(\frac{1}{8}\right) = -7\left[\!\!\left[\frac{1}{8} + 4\right]\!\!\right] + 6$

$= -7\left[\!\!\left[4\frac{1}{8}\right]\!\!\right] + 6 = -7(4) + 6 = -22$

(b) $g(9) = -7[\![9 + 4]\!] + 6$

$= -7[\![13]\!] + 6 = -7(13) + 6 = -85$

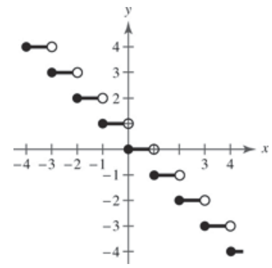
(c) $g(-4) = -7[\![-4 + 4]\!] + 6$

$= -7[\![0]\!] + 6 = -7(0) + 6 = 6$

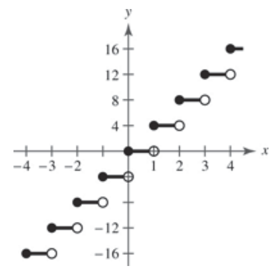
(d) $g\left(\frac{3}{2}\right) = -7\left[\!\!\left[\frac{3}{2} + 4\right]\!\!\right] + 6$

$= -7\left[\!\!\left[5\frac{1}{2}\right]\!\!\right] + 6 = -7(5) + 6 = -29$

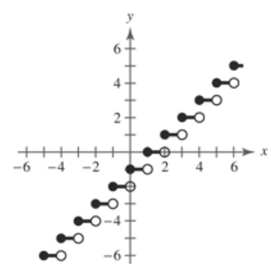
31. $g(x) = -[\![x]\!]$



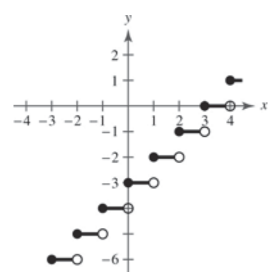
32. $g(x) = 4[\![x]\!]$



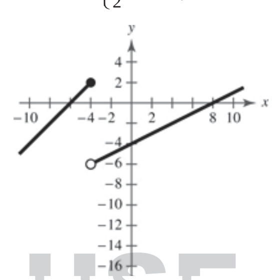
33. $g(x) = [\![x]\!] - 1$



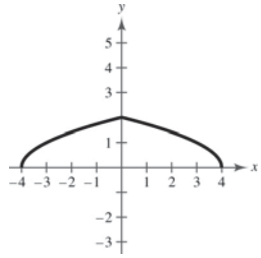
34. $g(x) = [\![x - 3]\!]$



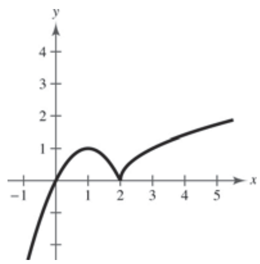
35. $g(x) = \begin{cases} x + 6, & x \le -4 \\ \frac{1}{2}x - 4, & x > -4 \end{cases}$



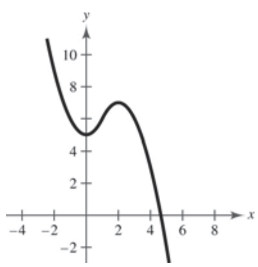
36. $f(x) = \begin{cases} \sqrt{4 + x}, & x < 0 \\ \sqrt{4 - x}, & x \geq 0 \end{cases}$



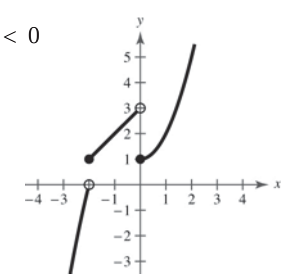
37. $f(x) = \begin{cases} 1 - (x - 1)^2, & x \leq 2 \\ \sqrt{x - 2}, & x > 2 \end{cases}$



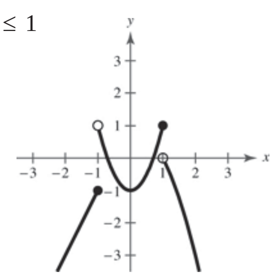
38. $f(x) = \begin{cases} x^2 + 5, & x \leq 1 \\ -x^2 + 4x + 3, & x > 1 \end{cases}$



39. $h(x) = \begin{cases} 4 - x^2, & x < -2 \\ 3 + x, & -2 \leq x < 0 \\ x^2 + 1, & x \geq 0 \end{cases}$

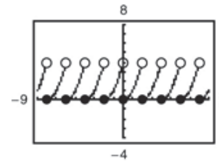


40. $k(x) = \begin{cases} 2x + 1, & x \leq -1 \\ 2x^2 - 1, & -1 < x \leq 1 \\ 1 - x^2, & x > 1 \end{cases}$



41. $s(x) = 2\left(\frac{1}{4}x - [\![\frac{1}{4}x]\!]\right)$

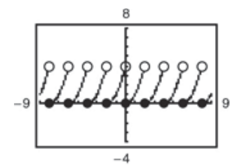
(a)



(b) Domain: $(-\infty, \infty)$; Range: $[0, 2)$

42. $k(x) = 4\left(\frac{1}{2}x - [\![\frac{1}{2}x]\!]\right)^2$

(a)



(b) Domain: $(-\infty, \infty)$; Range: $[0, 4)$

43. (a) $W(30) = 14(30) = 420$

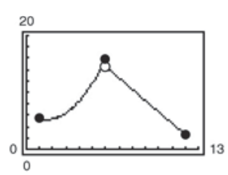
$W(40) = 14(40) = 560$

$W(45) = 21(45 - 40) + 560 = 665$

$W(50) = 21(50 - 40) + 560 = 770$

(b) $W(h) = \begin{cases} 14h, & 0 < h \leq 45 \\ 21(h - 45) + 630, & h > 45 \end{cases}$

44. (a)



The domain of $f(x) = -1.97x + 26.3$ is $6 < x \leq 12$. One way to see this is to notice that this is the equation of a line with negative slope, so the function values are decreasing as x increases, which matches the data for the corresponding part of the table. The domain of $f(x) = 0.505x^2 - 1.47x + 6.3$ is then $1 \leq x \leq 6$.

(b) $f(5) = 0.505(5)^2 - 1.47(5) + 6.3$

$= 0.505(25) - 7.35 + 6.3 = 11.575$

$f(11) = -1.97(11) + 26.3 = 4.63$

These values represent the revenue in thousands of dollars for the months of May and November, respectively.

(c) These values are quite close to the actual data values.

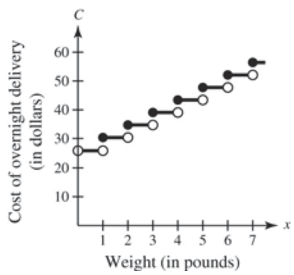
45. Answers will vary. *Sample answer:*

Interval	*Input Pipe*	*Drain Pipe 1*	*Drain Pipe 2*
[0, 5]	Open	Closed	Closed
[5, 10]	Open	Open	Closed
[10, 20]	Closed	Closed	Closed
[20, 30]	Closed	Closed	Open
[30, 40]	Open	Open	Open
[40, 45]	Open	Closed	Open
[45, 50]	Open	Open	Open
[50, 60]	Open	Open	Closed

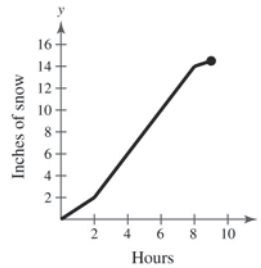
46. (a) Cost = Flat fee + fee per pound

$C(x) = 26.10 + 4.35[\![x]\!]$

(b)



47. For the first two hours the slope is 1. For the next six hours, the slope is 2. For the final hour, the slope is $\frac{1}{2}$.



$$f(t) = \begin{cases} t, & 0 \le t \le 2 \\ 2t - 2, & 2 < t \le 8 \\ \frac{1}{2}t + 10, & 8 < t \le 9 \end{cases}$$

To find $f(t) = 2t - 2$, use $m = 2$ and $(2, 2)$.

$y - 2 = 2(t - 2) \Rightarrow y = 2t - 2$

To find $f(t) = \frac{1}{2}t + 10$, use $m = \frac{1}{2}$ and $(8, 14)$.

$y - 14 = \frac{1}{2}(t - 8) \Rightarrow y = \frac{1}{2}t + 10$

Total accumulation = 14.5 inches

48. $f(x) = x^2$

(a) Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

(b) x-intercept: $(0, 0)$

y-intercept: $(0, 0)$

(c) Increasing: $(0, \infty)$

Decreasing: $(-\infty, 0)$

(d) Even; the graph has y-axis symmetry.

$f(x) = x^3$

(a) Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

(b) x-intercept: $(0, 0)$

y-intercept: $(0, 0)$

(c) Increasing: $(-\infty, \infty)$

(d) Odd; the graph has origin symmetry.

49. False. A piecewise-defined function is a function that is defined by two or more equations over a specified domain. That domain may or may not include x- and y-intercepts.

50. False. The vertical line $x = 2$ has an x-intercept at the point $(2, 0)$ but does not have a y-intercept. The horizontal line $y = 3$ has a y-intercept at the point $(0, 3)$ but does not have an x-intercept.

Section 1.7 Transformations of Functions

1. rigid

2. $-f(x)$; $f(-x)$

3. vertical stretch; vertical shrink

4. (a) iv

(b) ii

(c) iii

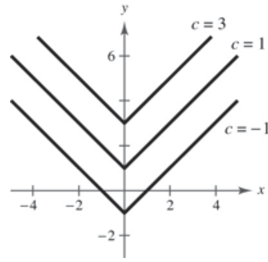
(d) i

5. (a) $f(x) = |x| + c$ — Vertical shifts

$c = -1$: $f(x) = |x| - 1$ — 1 unit down

$c = 1$: $f(x) = |x| + 1$ — 1 unit up

$c = 3$: $f(x) = |x| + 3$ — 3 units up

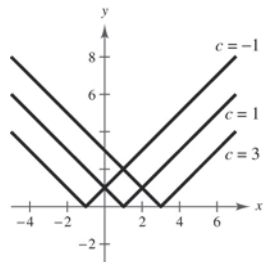


(b) $f(x) = |x - c|$ — Horizontal shifts

$c = -1$: $f(x) = |x + 1|$ — 1 unit left

$c = 1$: $f(x) = |x - 1|$ — 1 unit right

$c = 3$: $f(x) = |x - 3|$ — 3 units right



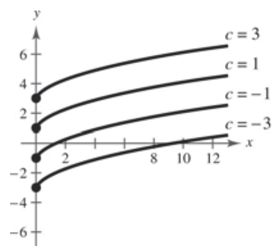
6. (a) $f(x) = \sqrt{x} + c$ — Vertical shifts

$c = -3$: $f(x) = \sqrt{x} - 3$ — 3 units down

$c = -1$: $f(x) = \sqrt{x} - 1$ — 1 unit down

$c = 1$: $f(x) = \sqrt{x} + 1$ — 1 unit up

$c = 3$: $f(x) = \sqrt{x} + 3$ — 3 units up



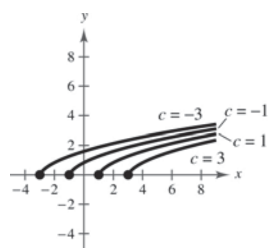
(b) $f(x) = \sqrt{x - c}$ — Horizontal shifts

$c = -3$: $f(x) = \sqrt{x + 3}$ — 3 units left

$c = -1$: $f(x) = \sqrt{x + 1}$ — 1 unit left

$c = 1$: $f(x) = \sqrt{x - 1}$ — 1 unit right

$c = 3$: $f(x) = \sqrt{x - 3}$ — 3 units right

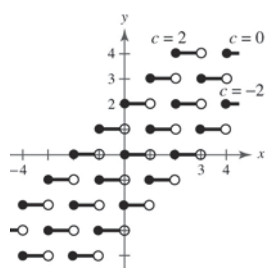


7. (a) $f(x) = [\![x]\!] + c$ — Vertical shifts

$c = -2$: $f(x) = [\![x]\!] - 2$ — 2 units down

$c = 0$: $f(x) = [\![x]\!]$ — Parent function

$c = 2$: $f(x) = [\![x]\!] + 2$ — 2 units up

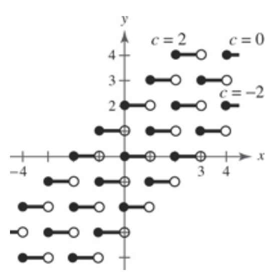


(b) $f(x) = [\![x + c]\!]$ — Horizontal shifts

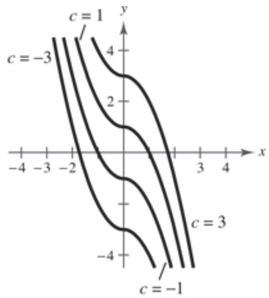
$c = -2$: $f(x) = [\![x - 2]\!]$ — 2 units right

$c = 0$: $f(x) = [\![x]\!]$ — Parent function

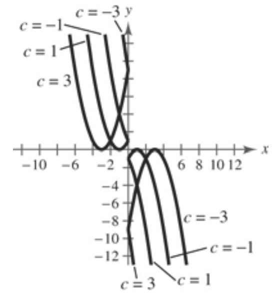
$c = 2$: $f(x) = [\![x + 2]\!]$ — 2 units left



8. (a) $f(x) = \begin{cases} x^2 + c, & x < 0 \\ -x^2 + c, & x \ge 0 \end{cases}$

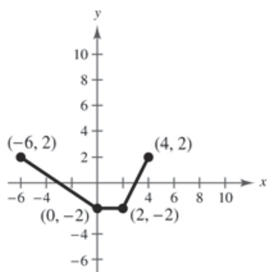


(b) $f(x) = \begin{cases} (x + c)^2, & x < 0 \\ -(x + c)^2, & x \ge 0 \end{cases}$



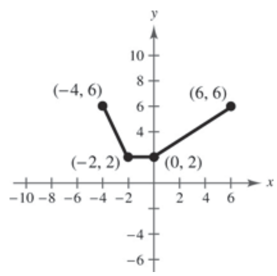
9. (a) $y = f(-x)$

Reflection in the y-axis



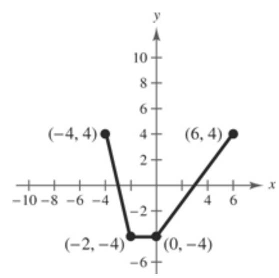
(b) $y = f(x) + 4$

Vertical shift 4 units upward



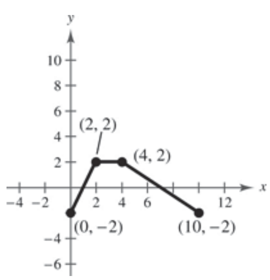
(c) $y = 2f(x)$

Vertical stretch (each y-value is multiplied by 2)



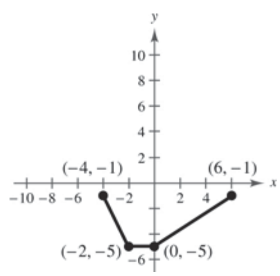
(d) $y = -f(x - 4)$

Reflection in the x-axis and a horizontal shift 4 units to the right



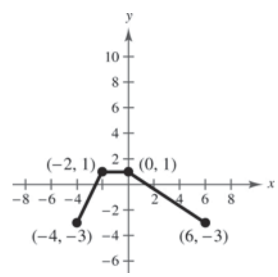
(e) $y = f(x) - 3$

Vertical shift 3 units downward



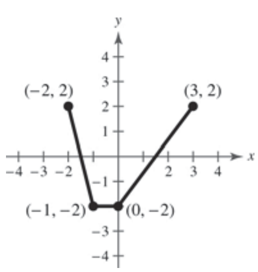
(f) $y = -f(x) - 1$

Reflection in the x-axis and a vertical shift 1 unit downward



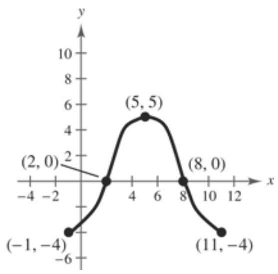
(g) $y = f(2x)$

Horizontal shrink
(each x-value is divided by 2)



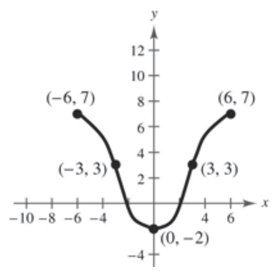
10. (a) $y = f(x - 5)$

Horizontal shift 5 units to the right



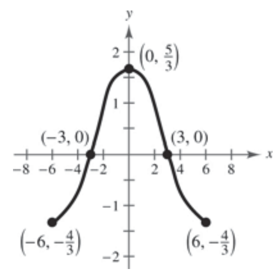
(b) $y = -f(x) + 3$

Reflection in the x-axis and a vertical shift 3 units upward



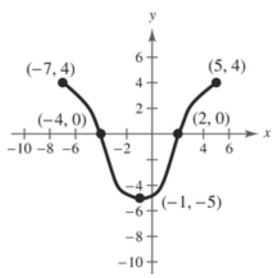
(c) $y = \frac{1}{3}f(x)$

Vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{3}\right)$



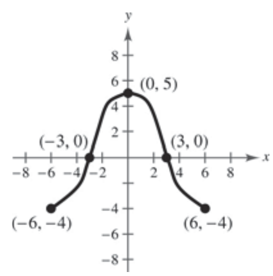
(d) $y = -f(x + 1)$

Reflection in the x-axis and a horizontal shift 1 unit to the left



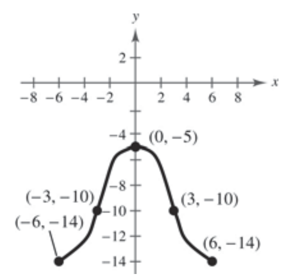
(e) $y = f(-x)$

Reflection in the y-axis



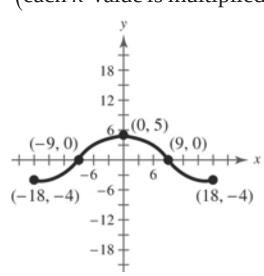
(f) $y = f(x) - 10$

Vertical shift 10 units downward



(g) $y = f\left(\frac{1}{3}x\right)$

Horizontal stretch $(\text{each } x\text{-value is multiplied by } 3)$



11. Parent function: $f(x) = x^2$

(a) Vertical shift 1 unit downward

$g(x) = x^2 - 1$

(b) Reflection in the x-axis, horizontal shift 1 unit to the left, and a vertical shift 1 unit upward

$g(x) = -(x + 1)^2 + 1$

12. Parent function: $f(x) = x^3$

(a) Reflected in the x-axis and shifted upward 1 unit

$g(x) = -x^3 + 1 = 1 - x^3$

(b) Shifted to the left 3 units and down 1 unit

$g(x) = -(x + 3)^3 - 1$

13. Parent function: $f(x) = |x|$

(a) Reflection in the x-axis and a horizontal shift 3 units to the left

$g(x) = -|x + 3|$

(b) Horizontal shift 2 units to the right and a vertical shift 4 units downward

$g(x) = |x - 2| - 4$

14. Parent function: $f(x) = \sqrt{x}$

(a) Shifted downward 7 units and to the left 1 unit

$g(x) = \sqrt{x + 1} - 7$

(d) Reflected about the x- and y-axis and shifted to the right 3 units and downward 4 units

$g(x) = -\sqrt{-x + 3} - 4$

15. Parent function: $f(x) = x^3$

Horizontal shift 2 units to the right

$y = (x - 2)^3$

16. Parent function: $y = x$

Vertical shrink

$y = \frac{1}{2}x$

17. Parent function: $f(x) = x^2$

Reflection in the x-axis

$y = -x^2$

18. Parent function: $y = [\![x]\!]$

Vertical shift

$y = [\![x]\!] + 4$

19. Parent function: $f(x) = \sqrt{x}$

Reflection in the x-axis and a vertical shift 1 unit upward

$y = -\sqrt{x} + 1$

20. Parent function: $y = |x|$

Horizontal shift

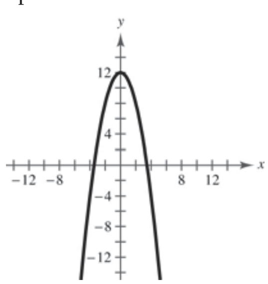
$y = |x + 2|$

21. $g(x) = 12 - x^2$

(a) Parent function: $f(x) = x^2$

(b) Reflection in the x-axis and a vertical shift 12 units upward

(c)



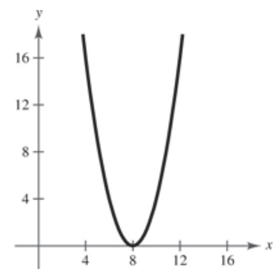
(d) $g(x) = 12 - f(x)$

22. $g(x) = (x - 8)^2$

(a) Parent function: $f(x) = y = x^2$

(b) Horizontal shift of 8 units to the right

(c)



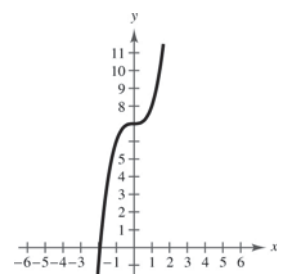
(d) $g(x) = f(x - 8)$

23. $g(x) = x^3 + 7$

(a) Parent function: $f(x) = x^3$

(b) Vertical shift 7 units upward

(c)



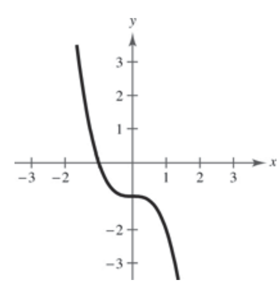
(d) $g(x) = f(x) + 7$

24. $g(x) = -x^3 - 1$

(a) Parent function: $f(x) = x^3$

(b) Reflection in the x-axis, vertical shift of 1 unit downward

(c)



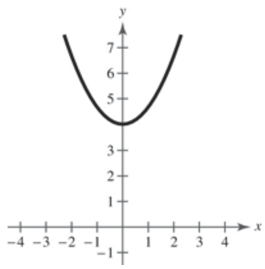
(d) $g(x) = -f(x) - 1$

25. $g(x) = \frac{2}{3}x^2 + 4$

(a) Parent function: $f(x) = x^2$

(b) Vertical shrink of two-thirds, and a vertical shift 4 units upward

(c)



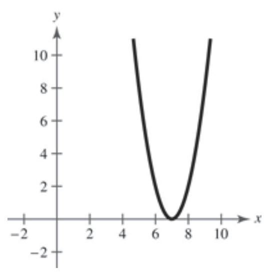
(d) $g(x) = \frac{2}{3}f(x) + 4$

26. $g(x) = 2(x - 7)^2$

(a) Parent function: $f(x) = x^2$

(b) Vertical stretch of 2 and a horizontal shift 7 units to the right of $f(x) = x^2$

(c)



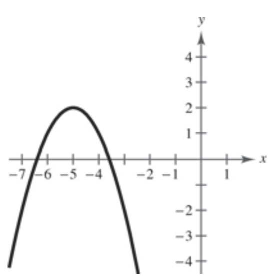
(d) $g(x) = 2f(x - 7)$

27. $g(x) = 2 - (x + 5)^2$

(a) Parent function: $f(x) = x^2$

(b) Reflection in the x-axis, horizontal shift 5 units to the left, and a vertical shift 2 units upward

(c)



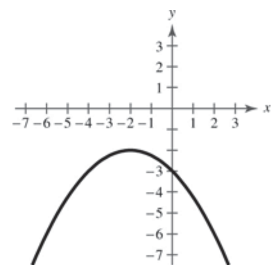
(d) $g(x) = 2 - f(x + 5)$

28. $g(x) = -\frac{1}{4}(x + 2)^2 - 2$

(a) Parent function: $f(x) = x^2$

(b) Horizontal shift 2 units to the left, vertical shrink, reflection in the x-axis, vertical shift 2 units downward

(c)



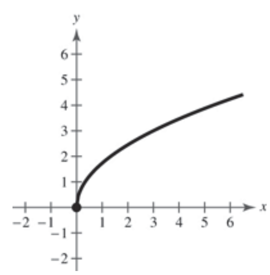
(d) $g(x) = -\frac{1}{4}f(x + 2) - 2$

29. $g(x) = \sqrt{3x}$

(a) Parent function: $f(x) = \sqrt{x}$

(b) Horizontal shrink by $\frac{1}{3}$

(c)



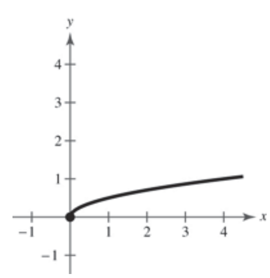
(d) $g(x) = f(3x)$

30. $g(x) = \sqrt{\frac{1}{4}x}$

(a) Parent function: $f(x) = \sqrt{x}$

(b) Horizontal stretch of 4

(c)



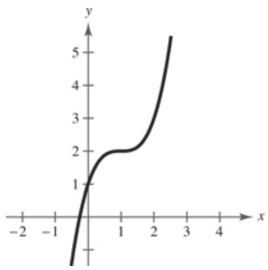
(d) $g(x) = f\left(\frac{1}{4}x\right)$

31. $g(x) = (x-1)^3 + 2$

(a) Parent function: $f(x) = x^3$

(b) Horizontal shift 1 unit to the right and a vertical shift 2 units upward

(c)



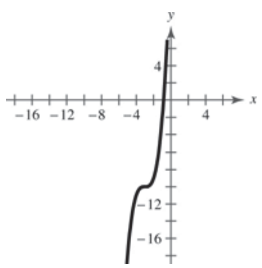
(d) $g(x) = f(x-1) + 2$

32. $g(x) = (x+3)^3 - 10$

(a) Parent function: $f(x) = x^3$

(b) Horizontal shift of 3 units to the left, vertical shift of 10 units downward

(c)



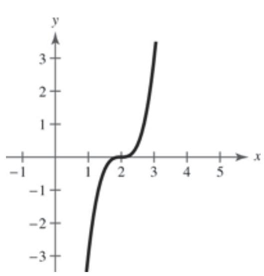
(d) $g(x) = f(x+3) - 10$

33. $g(x) = 3(x-2)^3$

(a) Parent function: $f(x) = x^3$

(b) Horizontal shift 2 units to the right, vertical stretch (each y-value is multiplied by 3)

(c)



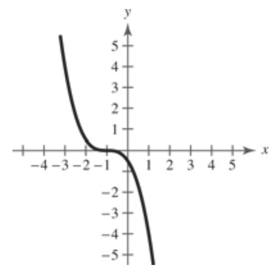
(d) $g(x) = 3f(x-2)$

34. $g(x) = -\frac{1}{2}(x+1)^3$

(a) Parent function: $f(x) = x^3$

(b) Horizontal shift one unit to the right, vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{2}\right)$, reflection in the x-axis.

(c)



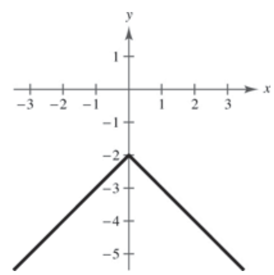
(d) $g(x) = -\frac{1}{2}f(x+1)$

35. $g(x) = -|x| - 2$

(a) Parent function: $f(x) = |x|$

(b) Reflection in the x-axis, vertical shift 2 units downward

(c)



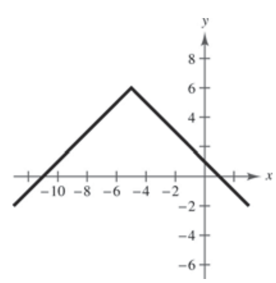
(d) $g(x) = -f(x) - 2$

36. $g(x) = 6 - |x+5|$

(a) Parent function: $f(x) = |x|$

(b) Reflection in the x-axis, horizontal shift of 5 units to the left, vertical shift of 6 units upward

(c)



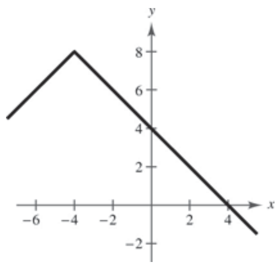
(d) $g(x) = 6 - f(x+5)$

37. $g(x) = -|x + 4| + 8$

(a) Parent function: $f(x) = |x|$

(b) Reflection in the x-axis, horizontal shift 4 units to the left, and a vertical shift 8 units upward

(c)



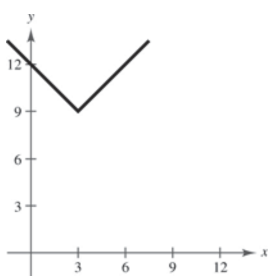
(d) $g(x) = -f(x + 4) + 8$

38. $g(x) = |-x + 3| + 9$

(a) Parent function: $f(x) = |x|$

(b) Reflection in the y-axis, horizontal shift of 3 units to the right, vertical shift of 9 units upward

(c)



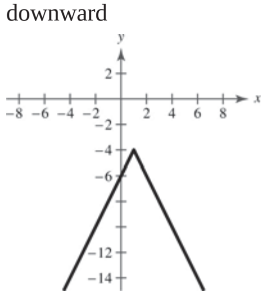
(d) $g(x) = f(-(x - 3)) + 9$

39. $g(x) = -2|x - 1| - 4$

(a) Parent function: $f(x) = |x|$

(b) Horizontal shift one unit to the right, vertical stretch, reflection in the x-axis, vertical shift four units downward

(c)



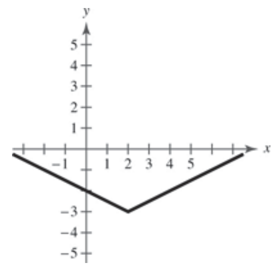
(d) $g(x) = -2f(x - 1) - 4$

40. $g(x) = \frac{1}{2}|x - 2| - 3$

(a) Parent function: $f(x) = |x|$

(b) Horizontal shift 2 units to the right, vertical shrink, vertical shift 3 units downward

(c)



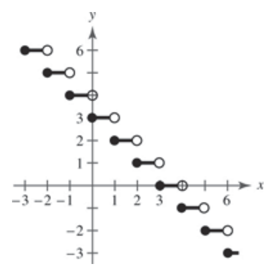
(d) $g(x) = \frac{1}{2}f(x - 2) - 3$

41. $g(x) = 3 - [\![x]\!]$

(a) Parent function: $f(x) = [\![x]\!]$

(b) Reflection in the x-axis and a vertical shift 3 units upward

(c)



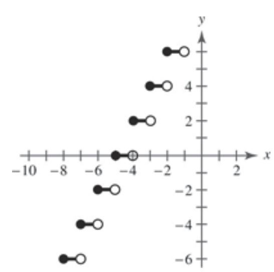
(d) $g(x) = 3 - f(x)$

42. $g(x) = 2[\![x + 5]\!]$

(a) Parent function: $f(x) = [\![x]\!]$

(b) Horizontal shift of 5 units to the left, vertical stretch (each y-value is multiplied by 2)

(c)



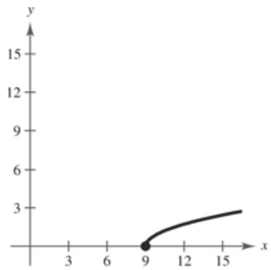
(d) $g(x) = 2f(x + 5)$

43. $g(x) = \sqrt{x - 9}$

(a) Parent function: $f(x) = \sqrt{x}$

(b) Horizontal shift 9 units to the right

(c)



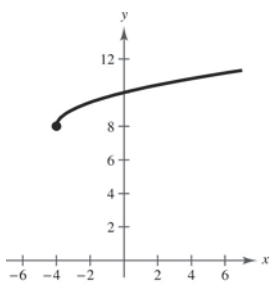
(d) $g(x) = f(x - 9)$

44. $g(x) = \sqrt{x + 4} + 8$

(a) Parent function: $f(x) = \sqrt{x}$

(b) Horizontal shift of 4 units to the left, vertical shift of 8 units upward

(c)



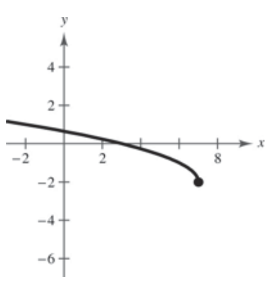
(d) $g(x) = f(x + 4) + 8$

45. $g(x) = \sqrt{7 - x} - 2$ or $g(x) = \sqrt{-(x - 7)} - 2$

(a) Parent function: $f(x) = \sqrt{x}$

(b) Reflection in the y-axis, horizontal shift 7 units to the right, and a vertical shift 2 units downward

(c)



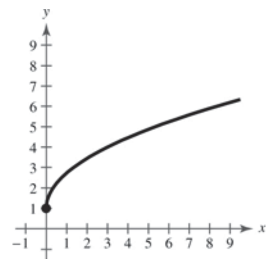
(d) $g(x) = f(7 - x) - 2$

46. $g(x) = \sqrt{3x} + 1$

(a) Parent function: $f(x) = \sqrt{x}$

(b) Horizontal shrink $\left(\text{each } x\text{-value is multiplied by } \frac{1}{3}\right)$, vertical shift of 1 unit upward

(c)



(d) $g(x) = f(3x) + 1$

47. $g(x) = (x - 3)^2 - 7$

48. $g(x) = -(x + 2)^2 + 9$

49. $f(x) = x^3$ moved 13 units to the right

$g(x) = (x - 13)^3$

50. $f(x) = x^3$ moved 6 units to the left, 6 units downward, and reflected in the y-axis (in that order)

$g(x) = (-x + 6)^3 - 6$

51. $g(x) = -|x| + 12$

52. $g(x) = |x + 4| - 8$

53. $f(x) = \sqrt{x}$ moved 6 units to the left and reflected in both the x- and y-axes

$g(x) = -\sqrt{-x + 6}$

54. $f(x) = \sqrt{x}$ moved 9 units downward and reflected in both the x-axis and the y-axis

$g(x) = -\left(\sqrt{-x} - 9\right)$

55. $f(x) = x^2$

(a) Reflection in the x-axis and a vertical stretch (each y-value is multiplied by 3)

$g(x) = -3x^2$

(b) Vertical shift 3 units upward and a vertical stretch (each y-value is multiplied by 4)

$g(x) = 4x^2 + 3$

56. $f(x) = x^3$

(a) Vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{4}\right)$

$g(x) = \frac{1}{4}x^3$

(b) Reflection in the x-axis and a vertical stretch (each y-value is multiplied by 2)

$g(x) = -2x^3$

57. $f(x) = |x|$

(a) Reflection in the x-axis and a vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{2}\right)$

$g(x) = -\frac{1}{2}|x|$

(b) Vertical stretch (each y-value is multiplied by 3) and a vertical shift 3 units downward

$g(x) = 3|x| - 3$

58. $f(x) = \sqrt{x}$

(a) Vertical stretch (each y-value is multiplied by 8)

$g(x) = 8\sqrt{x}$

(b) Reflection in the x-axis and a vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{4}\right)$

$g(x) = -\frac{1}{4}\sqrt{x}$

59. Parent function: $f(x) = x^3$

Vertical stretch (each y-value is multiplied by 2)

$g(x) = 2x^3$

60. Parent function: $f(x) = |x|$

Vertical stretch (each y-value is multiplied by 6)

$g(x) = 6|x|$

61. Parent function: $f(x) = x^2$

Reflection in the x-axis, vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{2}\right)$

$g(x) = -\frac{1}{2}x^2$

62. Parent function: $y = [\![x]\!]$

Horizontal stretch (each x-value is multiplied by 2)

$g(x) = [\![\frac{1}{2}x]\!]$

63. Parent function: $f(x) = \sqrt{x}$

Reflection in the y-axis, vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{2}\right)$

$g(x) = \frac{1}{2}\sqrt{-x}$

64. Parent function: $f(x) = |x|$

Reflection in the x-axis, vertical shift of 2 units downward, vertical stretch (each y-value is multiplied by 2)

$g(x) = -2|x| - 2$

65. Parent function: $f(x) = x^3$

Reflection in the x-axis, horizontal shift 2 units to the right and a vertical shift 2 units upward

$g(x) = -(x - 2)^3 + 2$

66. Parent function: $f(x) = |x|$

Horizontal shift of 4 units to the left and a vertical shift of 2 units downward

$g(x) = |x + 4| - 2$

67. Parent function: $f(x) = \sqrt{x}$

Reflection in the x-axis and a vertical shift 3 units downward

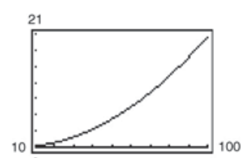
$g(x) = -\sqrt{x} - 3$

68. Parent function: $f(x) = x^2$

Horizontal shift of 2 units to the right and a vertical shift of 4 units upward

$g(x) = (x - 2)^2 + 4$

69. (a)



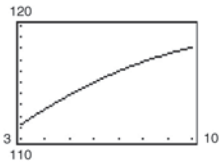
(b)

$$H(x) = 0.002x^2 + 0.005x - 0.029$$

$$\begin{aligned} H\left(\frac{x}{1.6}\right) &= 0.002\left(\frac{x}{1.6}\right)^2 + 0.005\left(\frac{x}{1.6}\right) - 0.029 \\ &= 0.002\left(\frac{x^2}{2.56}\right) + 0.005\left(\frac{x}{1.6}\right) - 0.029 \\ &= 0.00078125x^2 + 0.003125x - 0.029 \end{aligned}$$

The graph of $H\left(\frac{x}{1.6}\right)$ is a horizontal stretch of the graph of $H(x)$.

70. (a) The graph of $N(x) = -0.068(x - 13.68)^2 + 119$ is a reflection in the x-axis, a vertical shrink of a factor of 0.068, a horizontal shift of 13.68 units to the right and a vertical shift of 119 units upward of the graph $f(x) = x^2$.



(b) The average rate of change from $t = 3$ to $t = 10$ is given by the following.

$$\frac{N(10) - N(3)}{10 - 3} \approx \frac{118.079 - 111.244}{7} = \frac{6.835}{7} \approx 0.976$$

Each year, the number of households in the United States increases by an average of 976,000 households.

(c) Let $t = 18$:

$$N(18) = -0.068(18 - 13.68)^2 + 119 \approx 117.7$$

In 2018, the number of households in the United States will be about 117.7 million households.

Answers will vary. *Sample answer:* No, because the number of households has been increasing on average.

71. False. $y = f(-x)$ is a reflection in the y-axis.

72. False. $y = -f(x)$ is a reflection in the x-axis.

73. True. Because $|x| = |-x|$, the graphs of $f(x) = |x| + 6$ and $f(x) = |-x| + 6$ are identical.

74. False. The point $(-2, -61)$ lies on the transformation.

75. $y = f(x + 2) - 1$

Horizontal shift 2 units to the left and a vertical shift 1 unit downward

$(0, 1) \to (0 - 2, 1 - 1) = (-2, 0)$

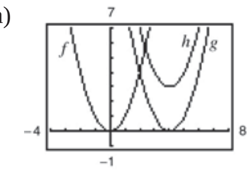
$(1, 2) \to (1 - 2, 2 - 1) = (-1, 1)$

$(2, 3) \to (2 - 2, 3 - 1) = (0, 2)$

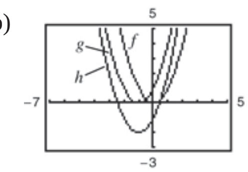
76. (a) Answers will vary. *Sample Answer*: To graph $f(x) = 3x^2 - 4x + 1$ use the point-plotting method since it is not written in a form that is easily identified by a sequence of translations of the parent function $y = x^2$.

(b) Answers will vary. *Sample Answer*: To graph $f(x) = 2(x - 1)^2 - 6$ use the method of translating the parent function $y = x^2$ since it is written in a form such that a sequence of translations is easily identified.

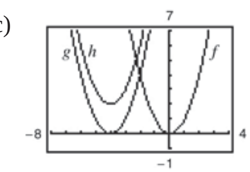
77. (a)



(b)



(c)



78. (a) Increasing on the interval $(-2, 1)$ and decreasing on the intervals $(-\infty, -2)$ and $(1, \infty)$

(b) Increasing on the interval $(-1, 2)$ and decreasing on the intervals $(-\infty, -1)$ and $(2, \infty)$

(c) Increasing on the intervals $(-\infty, -1)$ and $(2, \infty)$ and decreasing on the interval $(-1, 2)$

(d) Increasing on the interval $(0, 3)$ and decreasing on the intervals $(-\infty, 0)$ and $(3, \infty)$

(e) Increasing on the intervals $(-\infty, 1)$ and $(4, \infty)$ and decreasing on the interval $(1, 4)$

79. (a) The profits were only $\frac{3}{4}$ as large as expected:

$g(t) = \frac{3}{4}f(t)$

(b) The profits were \$10,000 greater than predicted:

$g(t) = f(t) + 10{,}000$

(c) There was a two-year delay: $g(t) = f(t - 2)$

80. No. $g(x) = -x^4 - 2$. Yes. $h(x) = -(x - 3)^4$

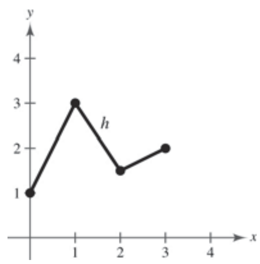
Section 1.8 Combinations of Functions: Composite Functions

1. addition; subtraction; multiplication; division

2. composition

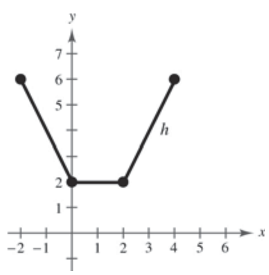
3.

x	0	1	2	3
f	2	3	1	2
g	–1	0	$\frac{1}{2}$	0
$f + g$	1	3	$\frac{3}{2}$	2



4.

x	–2	0	1	2	4
f	2	0	1	2	4
g	4	2	1	0	2
$f + g$	6	2	2	2	6



5. $f(x) = x + 2, g(x) = x - 2$

(a) $(f + g)(x) = f(x) + g(x)$
$= (x + 2) + (x - 2)$
$= 2x$

(b) $(f - g)(x) = f(x) - g(x)$
$= (x + 2) - (x - 2)$
$= 4$

(c) $(fg)(x) = f(x) \cdot g(x)$
$= (x + 2)(x - 2)$
$= x^2 - 4$

(d) $\left(\dfrac{f}{x}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x + 2}{x - 2}$

Domain: all real numbers x except $x = 2$

6. $f(x) = 2x - 5, g(x) = 2 - x$

(a) $(f + g)(x) = 2x - 5 + 2 - x = x - 3$

(b) $(f - g)(x) = 2x - 5 - (2 - x)$
$= 2x - 5 - 2 + x$
$= 3x - 7$

(c) $(fg)(x) = (2x - 5)(2 - x)$
$= 4x - 2x^2 - 10 + 5x$
$= -2x^2 + 9x - 10$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{2x - 5}{2 - x}$

Domain: all real numbers x except $x = 2$

7. $f(x) = x^2, g(x) = 4x - 5$

(a) $(f + g)(x) = f(x) + g(x)$
$= x^2 + (4x - 5)$
$= x^2 + 4x - 5$

(b) $(f - g)(x) = f(x) - g(x)$
$= x^2 - (4x - 5)$
$= x^2 - 4x + 5$

(c) $(fg)(x) = f(x) \cdot g(x)$
$= x^2(4x - 5)$
$= 4x^3 - 5x^2$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}$
$= \dfrac{x^2}{4x - 5}$

Domain: all real numbers x except $x = \dfrac{5}{4}$

8. $f(x) = 3x + 1, g(x) = 5x - 4$

(a) $(f + g)(x) = f(x) + g(x)$
$= 3x + 1 + 5x - 4$
$= 8x - 3$

(b) $(f - g)(x) = f(x) - g(x)$
$= 3x + 1 - (5x - 4)$
$= -2x + 5$

(c) $(fg)(x) = f(x) \cdot g(x)$
$= (3x + 1)(5x - 4)$
$= 15x^2 - 7x - 4$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{3x + 1}{5x - 4}$

Domain: all real numbers x except $x = \dfrac{4}{5}$

9. $f(x) = x^2 + 6, g(x) = \sqrt{1 - x}$

(a) $(f + g)(x) = f(x) + g(x) = x^2 + 6 + \sqrt{1 - x}$

(b) $(f - g)(x) = f(x) - g(x) = x^2 + 6 - \sqrt{1 - x}$

(c) $(fg)(x) = f(x) \cdot g(x) = (x^2 + 6)\sqrt{1 - x}$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x^2 + 6}{\sqrt{1 - x}} = \dfrac{(x^2 + 6)\sqrt{1 - x}}{1 - x}$

Domain: $x < 1$

10. $f(x) = \sqrt{x^2 - 4}, g(x) = \dfrac{x^2}{x^2 + 1}$

(a) $(f + g)(x) = \sqrt{x^2 - 4} + \dfrac{x^2}{x^2 + 1}$

(b) $(f - g)(x) = \sqrt{x^2 - 4} - \dfrac{x^2}{x^2 + 1}$

(c) $(fg)(x) = \sqrt{x^2 - 4}\left(\dfrac{x^2}{x^2 + 1}\right) = \dfrac{x^2\sqrt{x^2 - 4}}{x^2 + 1}$

(d) $\left(\dfrac{f}{g}\right)(x) = \sqrt{x^2 - 4} \div \dfrac{x^2}{x^2 + 1}$
$= \dfrac{(x^2 + 1)\sqrt{x^2 - 4}}{x^2}$

Domain: $x^2 - 4 \geq 0$
$x^2 \geq 4 \Rightarrow x \geq 2$ or $x \leq -2$

$|x| \geq 2$

11. $f(x) = \dfrac{1}{x}, g(x) = \dfrac{1}{x^2}$

(a) $(f + g)(x) = f(x) + g(x) = \dfrac{1}{x} + \dfrac{1}{x^2} = \dfrac{x + 1}{x^2}$

(b) $(f - g)(x) = f(x) - g(x) = \dfrac{1}{x} - \dfrac{1}{x^2} = \dfrac{x - 1}{x^2}$

(c) $(fg)(x) = f(x) \cdot g(x) = \dfrac{1}{x}\left(\dfrac{1}{x^2}\right) = \dfrac{1}{x^3}$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{1/x}{1/x^2} = \dfrac{x^2}{x} = x$

Domain: all real numbers x except $x = 0$

12. $f(x) = \dfrac{x}{x + 1}, g(x) = x^3$

(a) $(f + g)(x) = \dfrac{x}{x + 1} + x^3 = \dfrac{x + x^4 + x^3}{x + 1}$

(b) $(f - g)(x) = \dfrac{x}{x + 1} - x^3 = \dfrac{x - x^4 - x^3}{x + 1}$

(c) $(fg)(x) = \dfrac{x}{x + 1} \cdot x^3 = \dfrac{x^4}{x + 1}$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{x}{x + 1} \div x^3 = \dfrac{x}{x + 1} \cdot \dfrac{1}{x^3} = \dfrac{1}{x^2(x + 1)}$

Domain: all real numbers x except $x = 0$ and $x = -1$

For Exercises 13–24, $f(x) = x^2 + 1$ and $g(x) = x - 4$.

13. $(f + g)(2) = f(2) + g(2) = (2^2 + 1) + (2 - 4) = 3$

14. $(f - g)(-1) = f(-1) - g(-1)$
$= (-1)^2 + 1 - (-1 - 4)$
$= 1 + 1 - (-5)$
$= 7$

15. $(f - g)(0) = f(0) - g(0)$
$= (0^2 + 1) - (0 - 4)$
$= 5$

16. $(f + g)(1) = f(1) + g(1)$
$= (1)^2 + 1 + (1) - 4$
$= -1$

17. $(f - g)(3t) = f(3t) - g(3t)$
$= \left[(3t)^2 + 1\right] - (3t - 4)$
$= 9t^2 - 3t + 5$

18. $(f+g)(t-2) = f(t-2) + g(t-2)$

$= (t-2)^2 + 1 + (t-2) - 4$

$= t^2 - 4t + 4 + 1 + t - 2 - 4$

$= t^2 - 3t - 1$

19. $(fg)(6) = f(6)g(6)$

$= (6^2+1)(6-4)$

$= 74$

20. $(fg)(-6) = f(-6) \cdot g(-6)$

$= \left[(-6)^2 + 1\right]\left[(-6) - 4\right]$

$= (37)(-10)$

$= -370$

21. $\left(\dfrac{f}{g}\right)(5) = \dfrac{f(5)}{g(5)} = \dfrac{5^2+1}{5-4} = 26$

22. $\left(\dfrac{f}{g}\right)(0) = \dfrac{f(0)}{g(0)} = \dfrac{0^2+1}{0-4} = -\dfrac{1}{4}$

23. $\left(\dfrac{f}{g}\right)(-1) - g(3) = \dfrac{f(-1)}{g(-1)} - g(3)$

$= \dfrac{(-1)^2+1}{-1-4} - (3-4)$

$= -\dfrac{2}{5} + 1 = \dfrac{3}{5}$

24. $(fg)(5) + f(4) = f(5)g(5) + f(4)$

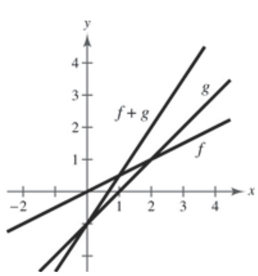
$= (5^2+1)(5-4) + (4^2+1)$

$= 26 \cdot 1 + 17$

$= 43$

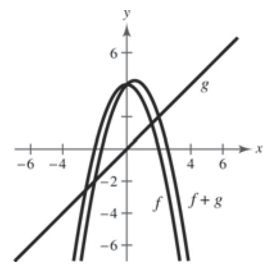
25. $f(x) = \frac{1}{2}x,\ g(x) = x - 1$

$(f+g)(x) = \frac{3}{2}x - 1$



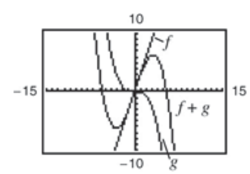
26. $f(x) = 4 - x^2,\ g(x) = x$

$(f+g)(x) = 4 - x^2 + x = 4 + x - x^2$



27. $f(x) = 3x,\ g(x) = -\dfrac{x^3}{10}$

$(f+g)(x) = 3x - \dfrac{x^3}{10}$

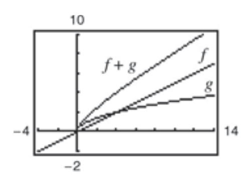


For $0 \le x \le 2$, $f(x)$ contributes most to the magnitude.

For $x > 6$, $g(x)$ contributes most to the magnitude.

28. $f(x) = \dfrac{x}{2},\ g(x) = \sqrt{x}$

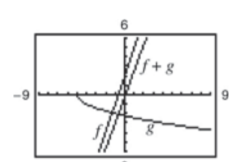
$(f+g)(x) = \dfrac{x}{2} + \sqrt{x}$



$g(x)$ contributes most to the magnitude of the sum for $0 \le x \le 2$. $f(x)$ contributes most to the magnitude of the sum for $x > 6$.

29. $f(x) = 3x + 2,\ g(x) = -\sqrt{x+5}$

$(f+g)x = 3x - \sqrt{x+5} + 2$

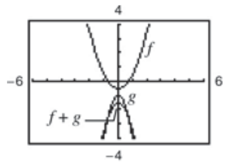


For $0 \le x \le 2$, $f(x)$ contributes most to the magnitude.

For $x > 6$, $f(x)$ contributes most to the magnitude.

30. $f(x) = x^2 - \frac{1}{2}, g(x) = -3x^2 - 1$

$(f + g)(x) = -2x^2 - \frac{3}{2}$



For $0 \le x \le 2$, $g(x)$ contributes most to the magnitude.

For $x > 6$, $g(x)$ contributes most to the magnitude.

31. $f(x) = x^2, g(x) = x - 1$

(a) $(f \circ g)(x) = f(g(x)) = f(x - 1) = (x - 1)^2$

(b) $(g \circ f)(x) = g(f(x)) = g(x^2) = x^2 - 1$

(c) $(g \circ g)(x) = g(g(x)) = g(x - 1) = x - 2$

32. $f(x) = 3x + 5, g(x) = 5 - x$

(a) $(f \circ g)(x) = f(g(x))$
$= f(5 - x) = 3(5 - x) + 5$
$= 20 - 3x$

(b) $(g \circ f)(x) = g(f(x))$
$= g(3x + 5) = 5 - (3x + 5)$
$= -3x$

(c) $(g \circ g)(x) = g(g(x)) = g(5 - x) = x$

33. $f(x) = \sqrt[3]{x - 1}, g(x) = x^3 + 1$

(a) $(f \circ g)(x) = f(g(x))$
$= f(x^3 + 1)$
$= \sqrt[3]{(x^3 + 1) - 1}$
$= \sqrt[3]{x^3} = x$

(b) $(g \circ f)(x) = g(f(x))$
$= g(\sqrt[3]{x - 1})$
$= (\sqrt[3]{x - 1})^3 + 1$
$= (x - 1) + 1 = x$

(c) $(g \circ g)(x) = g(g(x))$
$= g(x^3 + 1)$
$= (x^3 + 1)^3 + 1$
$= x^9 + 3x^6 + 3x^3 + 2$

34. $f(x) = x^3, g(x) = \dfrac{1}{x}$

(a) $(f \circ g)(x) = f(g(x)) = f\left(\dfrac{1}{x}\right) = \left(\dfrac{1}{x}\right)^3 = \dfrac{1}{x^3}$

(b) $(g \circ f)(x) = g(f(x)) = g(x^3) = \dfrac{1}{x^3}$

(c) $(g \circ g)(x) = g(g(x)) = g\left(\dfrac{1}{x}\right) = x$

35. $f(x) = \sqrt{x + 4}$ Domain: $x \ge -4$

$g(x) = x^2$ Domain: all real numbers x

(a) $(f \circ g)(x) = f(g(x)) = f(x^2) = \sqrt{x^2 + 4}$

Domain: all real numbers x

(b) $(g \circ f)(x) = g(f(x))$
$= g(\sqrt{x + 4}) = (\sqrt{x + 4})^2 = x + 4$

Domain: $x \ge -4$

36. $f(x) = \sqrt[3]{x - 5}$ Domain: all real numbers x

$g(x) = x^3 + 1$ Domain: all real numbers x

(a) $(f \circ g)(x) = f(g(x))$
$= f(x^3 + 1)$
$= \sqrt[3]{x^3 + 1 - 5}$
$= \sqrt[3]{x^3 - 4}$

Domain: all real numbers x

(b) $(g \circ f)(x) = g(f(x))$
$= g(\sqrt[3]{x - 5})$
$= (\sqrt[3]{x - 5})^3 + 1$
$= x - 5 + 1 = x - 4$

Domain: all real numbers x

37. $f(x) = x^2 + 1$ Domain: all real numbers x

$g(x) = \sqrt{x}$ Domain: $x \ge 0$

(a) $(f \circ g)(x) = f(g(x))$
$= f(\sqrt{x})$
$= (\sqrt{x})^2 + 1$
$= x + 1$

Domain: $x \ge 0$

(b) $(g \circ f)(x) = g(f(x)) = g(x^2 + 1) = \sqrt{x^2 + 1}$

Domain: all real numbers x

38. $f(x) = x^{2/3}$ Domain: all real numbers x

$g(x) = x^6$ Domain: all real numbers x

(a) $(f \circ g)(x) = f(g(x)) = f(x^6) = (x^6)^{2/3} = x^4$

Domain: all real numbers x

(b) $(g \circ f)(x) = g(f(x)) = g(x^{2/3}) = (x^{2/3})^6 = x^4$

Domain: all real numbers x

39. $f(x) = |x|$ Domain: all real numbers x

$g(x) = x + 6$ Domain: all real numbers x

(a) $(f \circ g)(x) = f(g(x)) = f(x + 6) = |x + 6|$

Domain: all real numbers x

(b) $(g \circ f)(x) = g(f(x)) = g(|x|) = |x| + 6$

Domain: all real numbers x

40. $f(x) = |x - 4|$ Domain: all real numbers x

$g(x) = 3 - x$ Domain: all real numbers x

(a) $(f \circ g)(x) = f(g(x)) = f(3 - x) = |(3 - x) - 4| = |-x - 1|$

Domain: all real numbers x

(b) $(g \circ f)(x) = g(f(x)) = g(|x - 4|) = 3 - (|x - 4|) = 3 - |x - 4|$

Domain: all real numbers x

41. $f(x) = \frac{1}{x}$ Domain: all real numbers x except $x = 0$

$g(x) = x + 3$ Domain: all real numbers x

(a) $(f \circ g)(x) = f(g(x)) = f(x + 3) = \frac{1}{x + 3}$

Domain: all real numbers x except $x = -3$

(b) $(g \circ f)(x) = g(f(x)) = g\left(\frac{1}{x}\right) = \frac{1}{x} + 3$

Domain: all real numbers x except $x = 0$

42. $f(x) = \frac{3}{x^2 - 1}$ Domain: all real numbers x except $x = \pm 1$

$g(x) = x + 1$ Domain: all real numbers x

(a) $(f \circ g)(x) = f(g(x)) = f(x + 1) = \frac{3}{(x + 1)^2 - 1} = \frac{3}{x^2 + 2x + 1 - 1} = \frac{3}{x^2 + 2x}$

Domain: all real numbers x except $x = 0$ and $x = -2$

(b) $(g \circ f)(x) = g(f(x)) = g\left(\frac{3}{x^2 - 1}\right) = \frac{3}{x^2 - 1} + 1 = \frac{3 + x^2 - 1}{x^2 - 1} = \frac{x^2 + 2}{x^2 - 1}$

Domain: all real numbers x except $x = \pm 1$

43. (a) $(f + g)(3) = f(3) + g(3) = 2 + 1 = 3$

(b) $\left(\frac{f}{g}\right)(2) = \frac{f(2)}{g(2)} = \frac{0}{2} = 0$

44. (a) $(f - g)(1) = f(1) - g(1) = 2 - 3 = -1$

(b) $(fg)(4) = f(4) \cdot g(4) = 4 \cdot 0 = 0$

45. (a) $(f \circ g)(2) = f(g(2)) = f(2) = 0$

(b) $(g \circ f)(2) = g(f(2)) = g(0) = 4$

46. (a) $(f \circ g)(1) = f(g(1)) = f(3) = 2$

(b) $(g \circ f)(3) = g(f(3)) = g(2) = 2$

47. $h(x) = (2x^2 + 1)^2$

One possibility: Let $f(x) = x^2$ and $g(x) = 2x + 1$, then $(f \circ g)(x) = h(x)$.

48. $h(x) = (1 - x)^3$

One possibility: Let $g(x) = 1 - x$ and $f(x) = x^3$, then $(f \circ g)(x) = h(x)$.

49. $h(x) = \sqrt[3]{x^2 - 4}$

One possibility: Let $f(x) = \sqrt[3]{x}$ and $g(x) = x^2 - 4$, then $(f \circ g)(x) = h(x)$.

50. $h(x) = \sqrt{9 - x}$

One possibility: Let $g(x) = 9 - x$ and $f(x) = \sqrt{x}$, then $(f \circ g)(x) = h(x)$.

51. $h(x) = \dfrac{1}{x + 2}$

One possibility: Let $f(x) = 1/x$ and $g(x) = x + 2$, then $(f \circ g)(x) = h(x)$.

52. $h(x) = \dfrac{4}{(5x + 2)^2}$

One possibility: Let $g(x) = 5x + 2$ and $f(x) = \dfrac{4}{x^2}$, then $(f \circ g)(x) = h(x)$.

53. $h(x) = \dfrac{-x^2 + 3}{4 - x^2}$

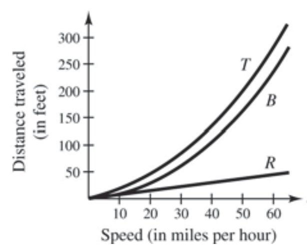
One possibility: Let $f(x) = \dfrac{x + 3}{4 + x}$ and $g(x) = -x^2$, then $(f \circ g)(x) = h(x)$.

54. $h(x) = \dfrac{27x^3 + 6x}{10 - 27x^3}$

One possibility: Let $g(x) = x^3$ and $f(x) = \dfrac{27x + 6\sqrt[3]{x}}{10 - 27x}$, then $(f \circ g)(x) = h(x)$.

55. (a) $T(x) = R(x) + B(x) = \frac{3}{4}x + \frac{1}{15}x^2$

(b)



(c) $B(x)$; As x increases, $B(x)$ increases at a faster rate.

56. (a) $c(t) = \dfrac{b(t) - d(t)}{p(t)} \times 100$

(b) $c(5)$ represents the percent change in the population due to births and deaths in the year 2005.

57. (a) $p(t) = d(t) + c(t)$

(b) $p(5)$ represents the number of dogs and cats in 2005.

(c) $h(t) = \dfrac{p(t)}{n(t)} = \dfrac{d(t) + c(t)}{n(t)}$

$h(t)$ represents the number of dogs and cats at time t compared to the population at time t or the number of dogs and cats per capita.

58. (a) T is a function of t since for each time t there corresponds one and only one temperature T.

(b) $T(4) \approx 60°$; $T(15) \approx 72°$

(c) $H(t) = T(t - 1)$; All the temperature changes would be one hour later.

(d) $H(t) = T(t) - 1$; The temperature would be decreased by one degree.

(e) The points at the endpoints of the individual functions that form each "piece" appear to be $(0, 60), (6, 60), (7, 72), (20, 72), (21, 60),$ and $(24, 60)$. Note that the value $t = 24$ is chosen for the last ordered pair because that is when the day ends and the cycle starts over.

From $t = 0$ to $t = 6$: This is the constant function $T(t) = 60$.

From $t = 6$ to $t = 7$: Use the points $(6, 60)$ and $(7, 72)$.

$$m = \frac{72 - 60}{7 - 6} = 12$$

$$y - 60 = 12(x - 6) \Rightarrow y = 12x - 12, \quad \text{or} \quad T(t) = 12t - 12$$

From $t = 7$ to $t = 20$: This is the constant function $T(t) = 72$.

From $t = 20$ to $t = 21$: Use the points $(20, 72)$ and $(21, 60)$.

$$m = \frac{72 - 60}{20 - 21} = -12$$

$$y - 60 = -12(x - 21) \Rightarrow y = -12x + 312, \text{ or } T(t) = -12t + 312$$

From $t = 21$ to $t = 24$: This is the constant function $T(t) = 60$.

A piecewise-defined function is $T(t) = \begin{cases} 60, & 0 \le t \le 6 \\ 12t - 12, & 6 < t < 7 \\ 72, & 7 \le t \le 20 \\ -12t + 312, & 20 < t < 21 \\ 60, & 21 \le t \le 24 \end{cases}$

59. (a) $r(x) = \dfrac{x}{2}$

(b) $A(r) = \pi r^2$

(c) $(A \circ r)(x) = A(r(x)) = A\left(\dfrac{x}{2}\right) = \pi\left(\dfrac{x}{2}\right)^2$

$(A \circ r)(x)$ represents the area of the circular base of the tank on the square foundation with side length x.

60. (a)
$$\begin{aligned} N(T(t)) &= N(3t + 2) \\ &= 10(3t + 2)^2 - 20(3t + 2) + 600 \\ &= 10(9t^2 + 12t + 4) - 60t - 40 + 600 \\ &= 90t^2 + 60t + 600 \\ &= 30(3t^2 + 2t + 20), \quad 0 \le t \le 6 \end{aligned}$$

This represents the number of bacteria in the food as a function of time.

(b) Use $t = 0.5$.

$$N(T(0.5)) = 30\left(3(0.5)^2 + 2(0.5) + 20\right) = 652.5$$

After half an hour, there will be about 653 bacteria.

(c)
$$\begin{aligned} 30(3t^2 + 2t + 20) &= 1500 \\ 3t^2 + 2t + 20 &= 50 \\ 3t^2 + 2t - 30 &= 0 \end{aligned}$$

By the Quadratic Formula, $t \approx -3.513$ or 2.846.

Choosing the positive value for t, you have $t \approx 2.846$ hours.

61. (a) $f(g(x)) = f(0.03x) = 0.03x - 500{,}000$

(b) $g(f(x)) = g(x - 500{,}000) = 0.03(x - 500{,}000)$

$g(f(x))$ represents your bonus of 3% of an amount over \$500,000.

62. (a) $R(p) = p - 2000$ the cost of the car after the factory rebate.

(b) $S(p) = 0.9p$ the cost of the car with the dealership discount.

(c) $(R \circ S)(p) = R(0.9p) = 0.9p - 2000$

$$\begin{aligned} (S \circ R)(p) &= S(p - 2000) \\ &= 0.9(p - 2000) = 0.9p - 1800 \end{aligned}$$

$(R \circ S)(p)$ represents the factory rebate *after* the dealership discount.

$(S \circ R)(p)$ represents the dealership discount after the factory rebate.

(d)
$$\begin{aligned} (R \circ S)(p) &= (R \circ S)(20{,}500) \\ &= 0.9(20{,}500) - 2000 = \$16{,}450 \end{aligned}$$

$$\begin{aligned} (S \circ R)(p) &= (S \circ R)(20{,}500) \\ &= 0.9(20{,}500) - 1800 = \$16{,}650 \end{aligned}$$

$(R \circ S)(20{,}500)$ yields the lower cost because 10% of the price of the car is more than \$2000

63. Let O = oldest sibling, M = middle sibling, Y = youngest sibling.
Then the ages of each sibling can be found using the equations:

$$O = 2M$$
$$M = \tfrac{1}{2}Y + 6$$

(a) $O(M(Y)) = 2\left(\frac{1}{2}(Y) + 6\right) = 12 + Y$; Answers will vary.

(b) Oldest sibling is 16: $O = 16$

Middle sibling: $$\begin{aligned} O &= 2M \\ 16 &= 2M \\ M &= 8 \text{ years old} \end{aligned}$$

Youngest sibling: $$\begin{aligned} M &= \tfrac{1}{2}Y + 6 \\ 8 &= \tfrac{1}{2}Y + 6 \\ 2 &= \tfrac{1}{2}Y \\ Y &= 4 \text{ years old} \end{aligned}$$

64. (a) $Y(M(O)) = 2\left(\frac{1}{2}O\right) - 12 = O - 12$; Answers will vary.

(b) Youngest sibling is $2 \to Y = 2$

Middle sibling: $$\begin{aligned} M &= \tfrac{1}{2}Y + 6 \\ M &= \tfrac{1}{2}(2) + 6 \\ M &= 7 \text{ years old} \end{aligned}$$

Oldest sibling: $$\begin{aligned} O &= 2M \\ O &= 2(7) \\ O &= 14 \text{ years old} \end{aligned}$$

65. False. $(f \circ g)(x) = 6x + 1$ and $(g \circ f)(x) = 6x + 6$

66. True. The range of g must be a subset of the domain of f for $(f \circ g)(x)$ to be defined.

67. Let $f(x)$ and $g(x)$ be two odd functions and define $h(x) = f(x)g(x)$. Then

$$\begin{aligned} h(-x) &= f(-x)g(-x) \\ &= [-f(x)][-g(x)] \quad \text{because } f \text{ and } g \text{ are odd} \\ &= f(x)g(x) \\ &= h(x). \end{aligned}$$

So, $h(x)$ is even.

Let $f(x)$ and $g(x)$ be two even functions and define $h(x) = f(x)g(x)$. Then

$$\begin{aligned} h(-x) &= f(-x)g(-x) \\ &= f(x)g(x) \quad \text{because } f \text{ and } g \text{ are even} \\ &= h(x). \end{aligned}$$

So, $h(x)$ is even.

68. (a) $f(p)$: matches L_2; For example, an original price of $p = \$15.00$ corresponds to a sale price of $S = \$7.50$.

(b) $g(p)$: matches L_1; For example an original price of $p = \$20.00$ corresponds to a sale price of $S = \$15.00$.

(c) $(g \circ f)(p)$: matches L_4; This function represents applying a 50% discount to the original price p, then subtracting a \$5 discount.

(d) $(f \circ g)(p)$ matches L_3; This function represents subtracting a \$5 discount from the original price p, then applying a 50% discount.

69. Let $f(x)$ be an odd function, $g(x)$ be an even function, and define $h(x) = f(x)g(x)$. Then

$$\begin{aligned} h(-x) &= f(-x)g(-x) \\ &= [-f(x)]g(x) \quad \text{because } f \text{ is odd and } g \text{ is even} \\ &= -f(x)g(x) \\ &= -h(x). \end{aligned}$$

So, h is odd and the product of an odd function and an even function is odd.

70. (a) $g(x) = \frac{1}{2}\left[f(x) + f(-x)\right]$

To determine if $g(x)$ is even, show $g(-x) = g(x)$.

$$\begin{aligned} g(-x) &= \frac{1}{2}\left[f(-x) + f(-(-x))\right] \\ &= \frac{1}{2}\left[f(-x) + f(x)\right] \\ &= \frac{1}{2}\left[f(x) + f(-x)\right] \\ &= g(x) \checkmark \end{aligned}$$

$h(x) = \frac{1}{2}\left[f(x) - f(-x)\right]$

To determine if $h(x)$ is odd show $h(-x) = -h(x)$.

$$\begin{aligned} h(-x) &= \frac{1}{2}\left[f(-x) - f(-(-x))\right] \\ &= \frac{1}{2}\left[f(-x) - f(x)\right] \\ &= -\frac{1}{2}\left[f(x) - f(-x)\right] \\ &= -h(x) \checkmark \end{aligned}$$

(b) Let $f(x) = a$ function

$f(x) =$ even function $+$ odd function.

Using the result from part (a) $g(x)$ is an even function and $h(x)$ is an odd function.

$$\begin{aligned} f(x) &= g(x) + h(x) \\ &= \frac{1}{2}\left[f(x) + f(-x)\right] + \frac{1}{2}\left[f(x) - f(-x)\right] \\ &= \frac{1}{2}f(x) + \frac{1}{2}f(-x) + \frac{1}{2}f(x) - \frac{1}{2}f(-x) \\ &= f(x) \checkmark \end{aligned}$$

(c) $f(x) = x^2 - 2x + 1$

$f(x) = g(x) + h(x)$

$$\begin{aligned} g(x) &= \frac{1}{2}\left[f(x) + f(-x)\right] \\ &= \frac{1}{2}\left[x^2 - 2x + 1 + (-x)^2 - 2(-x) + 1\right] \\ &= \frac{1}{2}\left[x^2 - 2x + 1 + x^2 + 2x + 1\right] \\ &= \frac{1}{2}\left[2x^2 + 2\right] = x^2 + 1 \end{aligned}$$

$$\begin{aligned} h(x) &= \frac{1}{2}\left[f(x) - f(-x)\right] \\ &= \frac{1}{2}\left[x^2 - 2x + 1 - \left((-x)^2 - 2(-x) + 1\right)\right] \\ &= \frac{1}{2}\left[x^2 - 2x + 1 - x^2 - 2x - 1\right] \\ &= \frac{1}{2}\left[-4x\right] = -2x \end{aligned}$$

$f(x) = \left(x^2 + 1\right) + (-2x)$

$k(x) = \dfrac{1}{x + 1}$

$k(x) = g(x) + h(x)$

$$\begin{aligned} g(x) &= \frac{1}{2}\left[k(x) + k(-x)\right] \\ &= \frac{1}{2}\left[\frac{1}{x + 1} + \frac{1}{-x + 1}\right] \\ &= \frac{1}{2}\left[\frac{1 - x + x + 1}{(x + 1)(1 - x)}\right] \\ &= \frac{1}{2}\left[\frac{2}{(x + 1)(1 - x)}\right] \\ &= \frac{1}{(x + 1)(1 - x)} \\ &= \frac{-1}{(x + 1)(x - 1)} \end{aligned}$$

$$\begin{aligned} h(x) &= \frac{1}{2}\left[k(x) - k(-x)\right] \\ &= \frac{1}{2}\left[\frac{1}{x + 1} - \frac{1}{1 - x}\right] \\ &= \frac{1}{2}\left[\frac{1 - x - (x + 1)}{(x + 1)(1 - x)}\right] \\ &= \frac{1}{2}\left[\frac{-2x}{(x + 1)(1 - x)}\right] \\ &= \frac{-x}{(x + 1)(1 - x)} \\ &= \frac{x}{(x + 1)(x - 1)} \end{aligned}$$

$$k(x) = \left(\frac{-1}{(x + 1)(x - 1)}\right) + \left(\frac{x}{(x + 1)(x - 1)}\right)$$

Section 1.9 Inverse Functions

1. inverse

2. f^{-1}

3. range; domain

4. $y = x$

5. one-to-one

6. Horizontal

7. $f(x) = 6x$

$$f^{-1}(x) = \frac{x}{6} = \frac{1}{6}x$$

$$f(f^{-1}(x)) = f\left(\frac{x}{6}\right) = 6\left(\frac{x}{6}\right) = x$$

$$f^{-1}(f(x)) = f^{-1}(6x) = \frac{6x}{6} = x$$

8. $f(x) = \frac{1}{3}x$

$$f^{-1}(x) = 3x$$

$$f(f^{-1}(x)) = f(3x) = \tfrac{1}{3}(3x) = x$$

$$f^{-1}(f(x)) = f^{-1}\left(\tfrac{1}{3}x\right) = 3\left(\tfrac{1}{3}x\right) = x$$

9. $f(x) = 3x + 1$

$$f^{-1}(x) = \frac{x-1}{3}$$

$$f(f^{-1}(x)) = f\left(\frac{x-1}{3}\right) = 3\left(\frac{x-1}{3}\right) + 1 = x$$

$$f^{-1}(f(x)) = f^{-1}(3x+1) = \frac{(3x+1)-1}{3} = x$$

10. $f(x) = \dfrac{x-1}{5}$

$$f^{-1}(x) = 5x + 1$$

$$f(f^{-1}(x)) = f(5x+1) = \frac{5x+1-1}{5} = \frac{5x}{5} = x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{x-1}{5}\right) = 5\left(\frac{x-1}{5}\right) + 1$$

$$= x - 1 + 1 = x$$

11. $f(x) = \sqrt[3]{x}$

$$f^{-1}(x) = x^3$$

$$f(f^{-1}(x)) = f(x^3) = \sqrt[3]{x^3} = x$$

$$f^{-1}(f(x)) = f^{-1}(\sqrt[3]{x}) = (\sqrt[3]{x})^3 = x$$

12. $f(x) = x^5$

$$f^{-1}(x) = \sqrt[5]{x}$$

$$f(f^{-1}(x)) = f(\sqrt[5]{x}) = (\sqrt[5]{x})^5 = x$$

$$f^{-1}(f(x)) = f^{-1}(x^5) = \sqrt[5]{x^5} = x$$

13. $(f \circ g)(x) = f(g(x)) = f\left(-\dfrac{2x+6}{7}\right) = -\dfrac{7}{2}\left(-\dfrac{2x+6}{7}\right) - 3 = x + 3 - 3 = x$

$$(g \circ f)(x) = g(f(x)) = g\left(-\frac{7}{2}x - 3\right) = -\frac{2\left(-\frac{7}{2}x - 3\right) + 6}{7} = \frac{-(-7x)}{7} = x$$

14. $(f \circ g)(x) = f(g(x)) = f(4x + 9) = \dfrac{4x + 9 - 9}{4} = \dfrac{4x}{4} = x$

$$(g \circ f)(x) = g(f(x)) = g\left(\frac{x-9}{4}\right) = 4\left(\frac{x-9}{4}\right) + 9 = x - 9 + 9 = x$$

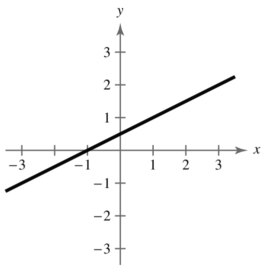
15. $(f \circ g)(x) = f(g(x)) = f(\sqrt[3]{x-5}) = (\sqrt[3]{x-5})^3 + 5 = x - 5 + 5 = x$

$$(g \circ f)(x) = g(f(x)) = g(x^3 + 5) = \sqrt[3]{x^3 + 5 - 5} = \sqrt[3]{x^3} = x$$

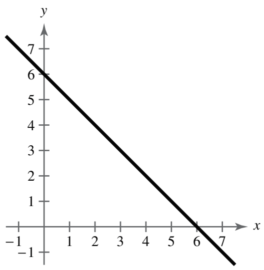
16. $(f \circ g)(x) = f(g(x)) = f(\sqrt[3]{2x}) = \dfrac{(\sqrt[3]{2x})^3}{2} = \dfrac{2x}{2} = x$

$$(g \circ f)(x) = g(f(x)) = g\left(\frac{x^3}{2}\right) = \sqrt[3]{2\left(\frac{x^3}{2}\right)} = \sqrt[3]{x^3} = x$$

17.



18.



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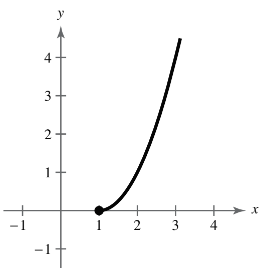

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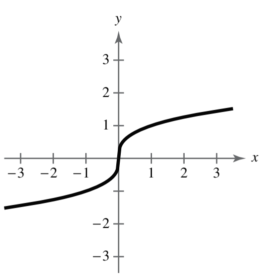
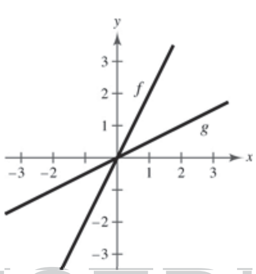


21. $f(x) = 2x,\ g(x) = \dfrac{x}{2}$

(a) $f(g(x)) = f\left(\dfrac{x}{2}\right) = 2\left(\dfrac{x}{2}\right) = x$

$g(f(x)) = g(2x) = \dfrac{2x}{2} = x$

(b)

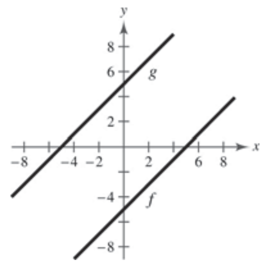


22. $f(x) = x - 5,\ g(x) = x + 5$

(a) $f(g(x)) = f(x + 5) = (x + 5) - 5 = x$

$g(f(x)) = g(x - 5) = (x - 5) + 5 = x$

(b)

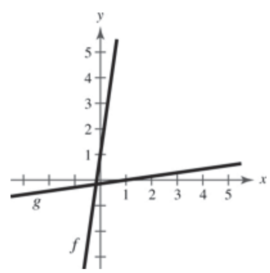


23. $f(x) = 7x + 1,\ g(x) = \dfrac{x - 1}{7}$

(a) $f(g(x)) = f\left(\dfrac{x - 1}{7}\right) = 7\left(\dfrac{x - 1}{7}\right) + 1 = x$

$g(f(x)) = g(7x + 1) = \dfrac{(7x + 1) - 1}{7} = x$

(b)



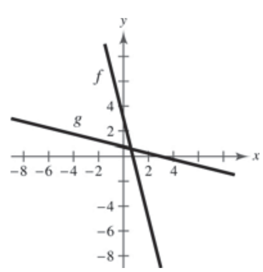
24. $f(x) = 3 - 4x,\ g(x) = \dfrac{3 - x}{4}$

(a) $f(g(x)) = f\left(\dfrac{3 - x}{4}\right) = 3 - 4\left(\dfrac{3 - x}{4}\right)$

$= 3 - (3 - x) = x$

$g(f(x)) = g(3 - 4x) = \dfrac{3 - (3 - 4x)}{4} = \dfrac{4x}{4} = x$

(b)

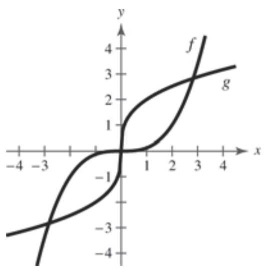


25. $f(x) = \dfrac{x^3}{8},\ g(x) = \sqrt[3]{8x}$

(a) $f(g(x)) = f\left(\sqrt[3]{8x}\right) = \dfrac{\left(\sqrt[3]{8x}\right)^3}{8} = \dfrac{8x}{8} = x$

$g(f(x)) = g\left(\dfrac{x^3}{8}\right) = \sqrt[3]{8\left(\dfrac{x^3}{8}\right)} = \sqrt[3]{x^3} = x$

(b)

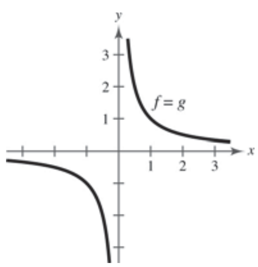


26. $f(x) = \dfrac{1}{x},\ g(x) = \dfrac{1}{x}$

(a) $f(g(x)) = f\left(\dfrac{1}{x}\right) = \dfrac{1}{1/x} = 1 \div \dfrac{1}{x} = 1 \cdot \dfrac{x}{1} = x$

$g(f(x)) = g\left(\dfrac{1}{x}\right) = \dfrac{1}{1/x} = 1 \div \dfrac{1}{x} = 1 \cdot \dfrac{x}{1} = x$

(b)



27. $f(x) = \sqrt{x-4},\ g(x) = x^2 + 4,\ x \ge 0$

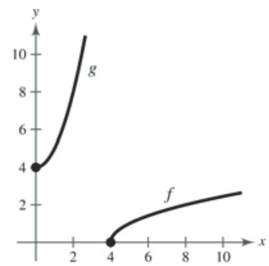
(a) $f(g(x)) = f\left(x^2 + 4\right),\ x \ge 0$

$= \sqrt{\left(x^2 + 4\right) - 4} = x$

$g(f(x)) = g\left(\sqrt{x-4}\right)$

$= \left(\sqrt{x-4}\right)^2 + 4 = x$

(b)



28. $f(x) = 1 - x^3,\ g(x) = \sqrt[3]{1-x}$

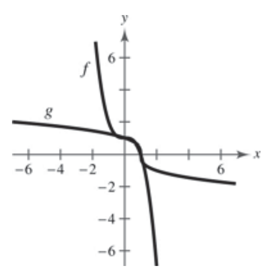
(a) $f(g(x)) = f\left(\sqrt[3]{1-x}\right) = 1 - \left(\sqrt[3]{1-x}\right)^3$

$= 1 - (1 - x) = x$

$g(f(x)) = g\left(1 - x^3\right) = \sqrt[3]{1 - \left(1 - x^3\right)}$

$= \sqrt[3]{x^3} = x$

(b)

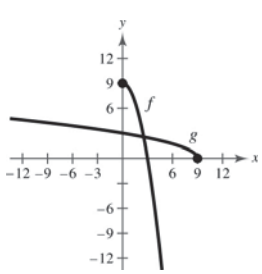


29. $f(x) = 9 - x^2,\ x \ge 0;\ g(x) = \sqrt{9-x},\ x \le 9$

(a) $f(g(x)) = f\left(\sqrt{9-x}\right),\ x \le 9 = 9 - \left(\sqrt{9-x}\right)^2 = x$

$g(f(x)) = g\left(9 - x^2\right),\ x \ge 0 = \sqrt{9 - \left(9 - x^2\right)} = x$

(b)

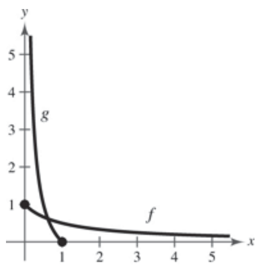


30. $f(x) = \dfrac{1}{1+x}, x \geq 0; g(x) = \dfrac{1-x}{x}, 0 < x \leq 1$

(a) $f(g(x)) = f\left(\dfrac{1-x}{x}\right) = \dfrac{1}{1+\left(\dfrac{1-x}{x}\right)} = \dfrac{1}{\dfrac{x}{x}+\dfrac{1-x}{x}} = \dfrac{1}{\dfrac{1}{x}} = x$

$g(f(x)) = g\left(\dfrac{1}{1+x}\right) = \dfrac{1-\left(\dfrac{1}{1+x}\right)}{\left(\dfrac{1}{1+x}\right)} = \dfrac{\dfrac{1+x}{1+x}-\dfrac{1}{1+x}}{\dfrac{1}{1+x}} = \dfrac{\dfrac{x}{1+x}}{\dfrac{1}{1+x}} = \dfrac{x}{1+x}\cdot\dfrac{x+1}{1} = x$

(b)

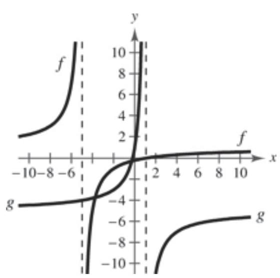


31. $f(x) = \dfrac{x-1}{x+5}, g(x) = -\dfrac{5x+1}{x-1}$

(a) $f(g(x)) = f\left(-\dfrac{5x+1}{x-1}\right) = \dfrac{\left(-\dfrac{5x+1}{x-1}-1\right)}{\left(-\dfrac{5x+1}{x-1}+5\right)}\cdot\dfrac{x-1}{x-1} = \dfrac{-(5x+1)-(x-1)}{-(5x+1)+5(x-1)} = \dfrac{-6x}{-6} = x$

$g(f(x)) = g\left(\dfrac{x-1}{x+5}\right) = -\dfrac{\left[5\left(\dfrac{x-1}{x+5}\right)+1\right]}{\left[\dfrac{x-1}{x+5}-1\right]}\cdot\dfrac{x+5}{x+5} = -\dfrac{5(x-1)+(x+5)}{(x-1)-(x+5)} = -\dfrac{6x}{-6} = x$

(b)

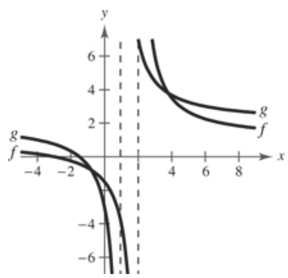


32. $f(x) = \dfrac{x+3}{x-2},\ g(x) = \dfrac{2x+3}{x-1}$

(a) $$f(g(x)) = f\left(\frac{2x+3}{x-1}\right) = \frac{\dfrac{2x+3}{x-1}+3}{\dfrac{2x+3}{x-1}-2} = \frac{\dfrac{2x+3+3x-3}{x-1}}{\dfrac{2x+3-2x+2}{x-1}} = \frac{5x}{5} = x$$

$$g(f(x)) = g\left(\frac{x+3}{x-2}\right) = \frac{2\left(\dfrac{x+3}{x-2}\right)+3}{\dfrac{x+3}{x-2}-1} = \frac{\dfrac{2x+6+3x-6}{x-2}}{\dfrac{x+3-x+2}{x-2}} = \frac{5x}{5} = x$$

(b)



33. No, $\{(-2,-1),(1,0),(2,1),(1,2),(-2,3),(-6,4)\}$ does not represent a function. -2 and 1 are paired with two different values.

34. Yes, $\{(10,-3),(6,-2),(4,-1),(1,0),(-3,2),(10,2)\}$ does represent a function.

35.

x	-2	0	2	4	6	8
$f^{-1}(x)$	-2	-1	0	1	2	3

36.

x	-10	-7	-4	-1	2	5
$f^{-1}(x)$	-3	-2	-1	0	1	2

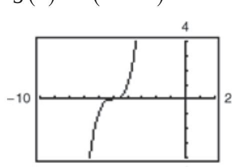
37. Yes, because no horizontal line crosses the graph of *f* at more than one point, *f has* an inverse.

38. No, because some horizontal lines intersect the graph of *f* twice, *f does not* have an inverse.

39. No, because some horizontal lines cross the graph of *f* twice, *f does not* have an inverse.

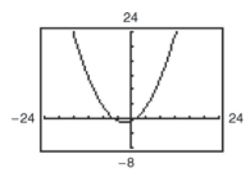
40. Yes, because no horizontal lines intersect the graph, of *f* at more than one point, *f has* an inverse.

41. $g(x) = (x+5)^3$



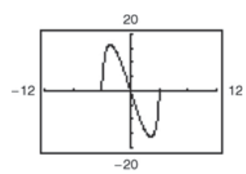
g passes the Horizontal Line Test, so *g has* an inverse.

42. $f(x) = \frac{1}{8}(x+2)^2 - 1$



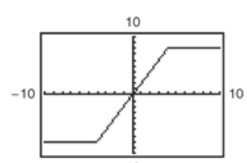
f does not pass the Horizontal Line Test, so *f does not* have an inverse.

43. $f(x) = -2x\sqrt{16-x^2}$



f does not pass the Horizontal Line Test, so *f does not* have an inverse.

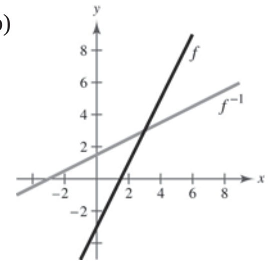
44. $h(x) = |x+4| - |x-4|$



h does not pass the Horizontal Line Test, so *h does not* have an inverse.

45. (a)
$$\begin{aligned} f(x) &= 2x - 3 \\ y &= 2x - 3 \\ x &= 2y - 3 \\ y &= \frac{x+3}{2} \\ f^{-1}(x) &= \frac{x+3}{2} \end{aligned}$$

(b)

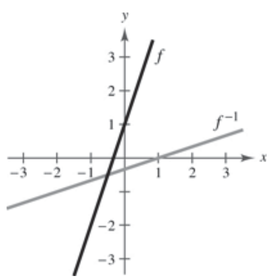


(c) The graph of f^{-1} is the reflection of the graph of f in the line $y = x$.

(d) The domains and ranges of f and f^{-1} are all real numbers.

46. (a)
$$\begin{aligned} f(x) &= 3x + 1 \\ y &= 3x + 1 \\ x &= 3y + 1 \\ \frac{x-1}{3} &= y \\ f^{-1}(x) &= \frac{x-1}{3} \end{aligned}$$

(b)

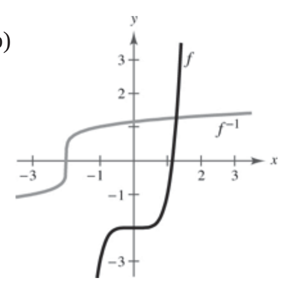


(c) The graph of f^{-1} is the reflection of f in the line $y = x$.

(d) The domains and ranges of f and f^{-1} are all real numbers.

47. (a)
$$\begin{aligned} f(x) &= x^5 - 2 \\ y &= x^5 - 2 \\ x &= y^5 - 2 \\ y &= \sqrt[5]{x+2} \\ f^{-1}(x) &= \sqrt[5]{x+2} \end{aligned}$$

(b)

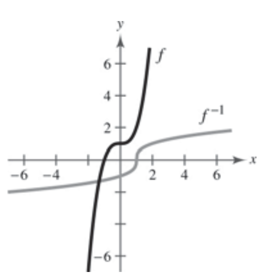


(c) The graph of f^{-1} is the reflection of the graph of f in the line $y = x$.

(d) The domains and ranges of f and f^{-1} are all real numbers.

48. (a)
$$\begin{aligned} f(x) &= x^3 + 1 \\ y &= x^3 + 1 \\ x &= y^3 + 1 \\ x - 1 &= y^3 \\ \sqrt[3]{x-1} &= y \\ f^{-1}(x) &= \sqrt[3]{x-1} \end{aligned}$$

(b)

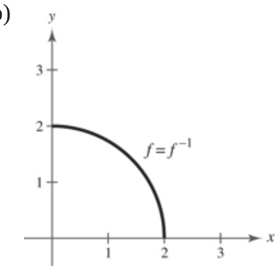


(c) The graph of f^{-1} is the reflection of f in the line $y = x$.

(d) The domains and ranges of f and f^{-1} are all real numbers.

49. (a)
$$\begin{aligned} f(x) &= \sqrt{4 - x^2},\ 0 \le x \le 2 \\ y &= \sqrt{4 - x^2} \\ x &= \sqrt{4 - y^2} \\ x^2 &= 4 - y^2 \\ y^2 &= 4 - x^2 \\ y &= \sqrt{4 - x^2} \\ f^{-1}(x) &= \sqrt{4 - x^2},\ 0 \le x \le 2 \end{aligned}$$

(b)

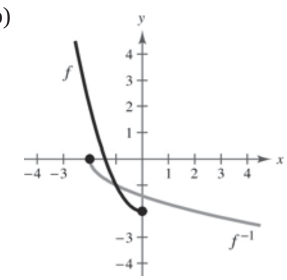


(c) The graph of f^{-1} is the same as the graph of f.

(d) The domains and ranges of f and f^{-1} are all real numbers x such that $0 \le x \le 2$.

50. (a)
$$\begin{aligned} f(x) &= x^2 - 2,\ x \le 0 \\ y &= x^2 - 2 \\ x &= y^2 - 2 \\ \pm\sqrt{x+2} &= y \\ f^{-1}(x) &= -\sqrt{x+2} \end{aligned}$$

(b)



(c) The graph of f^{-1} is the reflection of f in the line $y = x$.

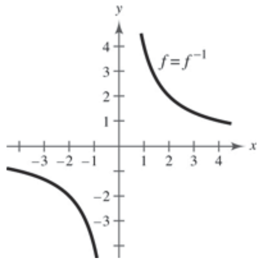
(d) $[-2, \infty)$ is the range of f and domain of f^{-1}.

$(-\infty, 0]$ is the domain of f and the range of f^{-1}.

51. (a)

$$\begin{aligned} f(x) &= \frac{4}{x} \\ y &= \frac{4}{x} \\ x &= \frac{4}{y} \\ xy &= 4 \\ y &= \frac{4}{x} \\ f^{-1}(x) &= \frac{4}{x} \end{aligned}$$

(b)



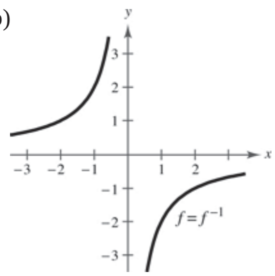
(c) The graph of f^{-1} is the same as the graph of f.

(d) The domains and ranges of f and f^{-1} are all real numbers except for 0.

52. (a)

$$\begin{aligned} f(x) &= -\frac{2}{x} \\ y &= -\frac{2}{x} \\ x &= -\frac{2}{y} \\ y &= -\frac{2}{x} \\ f^{-1}(x) &= -\frac{2}{x} \end{aligned}$$

(b)



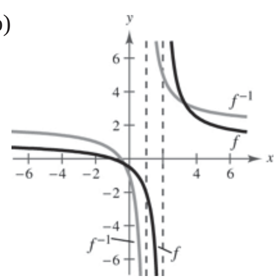
(c) The graphs are the same.

(d) The domains and ranges of f and f^{-1} are all real numbers except for 0.

53. (a)

$$\begin{aligned} f(x) &= \frac{x+1}{x-2} \\ y &= \frac{x+1}{x-2} \\ x &= \frac{y+1}{y-2} \\ x(y-2) &= y+1 \\ xy - 2x &= y+1 \\ xy - y &= 2x+1 \\ y(x-1) &= 2x+1 \\ y &= \frac{2x+1}{x-1} \\ f^{-1}(x) &= \frac{2x+1}{x-1} \end{aligned}$$

(b)



(c) The graph of f^{-1} is the reflection of graph of f in the line $y = x$.

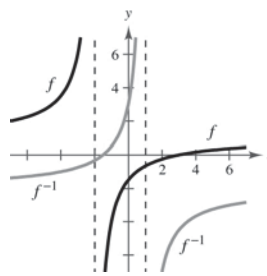
(d) The domain of f and the range of f^{-1} is all real numbers except 2.

The range of f and the domain of f^{-1} is all real numbers except 1.

54. (a)

$$\begin{aligned} f(x) &= \frac{x-3}{x+2} \\ y &= \frac{x-3}{x+2} \\ x &= \frac{y-3}{y+2} \\ xy + 2x - y + 3 &= 0 \\ y(x-1) &= -2x-3 \\ y &= \frac{-2x-3}{x-1} \\ f^{-1}(x) &= \frac{-2x-3}{x-1} \end{aligned}$$

(b)



(c) The graph of f^{-1} is the reflection of the graph of f in the line $y = x$.

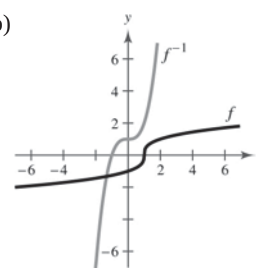
(d) The domain of f and the range of f^{-1} is all real numbers except $x = -2$.

The range of f and the domain of f^{-1} is all real numbers x except $x = 1$.

55. (a)

$$\begin{aligned} f(x) &= \sqrt[3]{x-1} \\ y &= \sqrt[3]{x-1} \\ x &= \sqrt[3]{y-1} \\ x^3 &= y-1 \\ y &= x^3+1 \\ f^{-1}(x) &= x^3+1 \end{aligned}$$

(b)

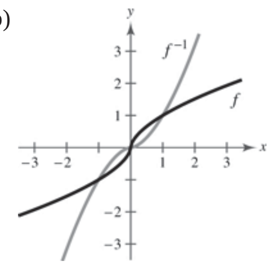


(c) The graph of f^{-1} is the reflection of the graph of f in the line $y = x$.

(d) The domains and ranges of f and f^{-1} are all real numbers.

56. (a)
$$f(x) = x^{3/5}$$
$$y = x^{3/5}$$
$$x = y^{3/5}$$
$$x^{5/3} = \left(y^{3/5}\right)^{5/3}$$
$$x^{5/3} = y$$
$$f^{-1}(x) = x^{5/3}$$

(b)



(c) The graph of f^{-1} is the reflection of the graph of f in the line $y = x$.

(d) The domains and ranges of f and f^{-1} are all real numbers.

57.
$$f(x) = x^4$$
$$y = x^4$$
$$x = y^4$$
$$y = \pm\sqrt[4]{x}$$

This does not represent y as a function of x. f does not have an inverse.

58.
$$f(x) = \frac{1}{x^2}$$
$$y = \frac{1}{x^2}$$
$$x = \frac{1}{y^2}$$
$$y^2 = \frac{1}{x}$$
$$y = \pm\sqrt{\frac{1}{x}}$$

This does not represent y as a function of x. f does not have an inverse.

59.
$$g(x) = \frac{x}{8}$$
$$y = \frac{x}{8}$$
$$x = \frac{y}{8}$$
$$y = 8x$$

This is a function of x, so g has an inverse.

$$g^{-1}(x) = 8x$$

60.
$$f(x) = 3x + 5$$
$$y = 3x + 5$$
$$x = 3y + 5$$
$$x - 5 = 3y$$
$$\frac{x - 5}{3} = y$$

This is a function of x, so f has an inverse.

$$f^{-1}(x) = \frac{x - 5}{3}$$

61.
$$p(x) = -4$$
$$y = -4$$

Because $y = -4$ for all x, the graph is a horizontal line and fails the Horizontal Line Test. p does not have an inverse.

62.
$$f(x) = \frac{3x + 4}{5}$$
$$y = \frac{3x + 4}{5}$$
$$x = \frac{3y + 4}{5}$$
$$5x = 3y + 4$$
$$5x - 4 = 3y$$
$$\frac{5x - 4}{3} = y$$

This is a function of x, so f has an inverse.

$$f^{-1}(x) = \frac{5x - 4}{3}$$

63.
$$f(x) = (x + 3)^2,\ x \ge -3 \Rightarrow y \ge 0$$
$$y = (x + 3)^2,\ x \ge -3,\ y \ge 0$$
$$x = (y + 3)^2,\ y \ge -3,\ x \ge 0$$
$$\sqrt{x} = y + 3,\ y \ge -3,\ x \ge 0$$
$$y = \sqrt{x} - 3,\ x \ge 0,\ y \ge -3$$

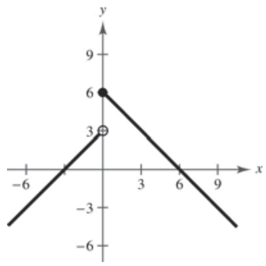
This is a function of x, so f has an inverse.

$$f^{-1}(x) = \sqrt{x} - 3,\ x \ge 0$$

64.
$$q(x) = (x - 5)^2$$
$$y = (x - 5)^2$$
$$x = (y - 5)^2$$
$$\pm\sqrt{x} = y - 5$$
$$5 \pm \sqrt{x} = y$$

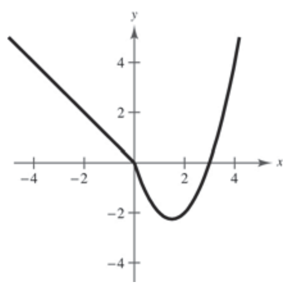
This does not represent y as a function of x, so q does not have an inverse.

65. $f(x) = \begin{cases} x + 3, & x < 0 \\ 6 - x, & x \geq 0 \end{cases}$



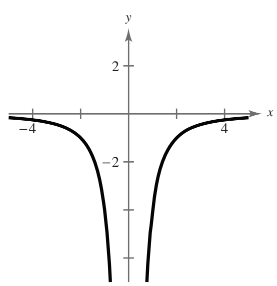
This graph fails the Horizontal Line Test, so f does not have an inverse.

66. $f(x) = \begin{cases} -x, & x \leq 0 \\ x^2 - 3x, & x > 0 \end{cases}$



The graph fails the Horizontal Line Test, so f does not have an inverse.

67. $h(x) = -\dfrac{4}{x^2}$



The graph fails the Horizontal Line Test so h does not have an inverse.

68. $f(x) = |x - 2|, x \leq 2 \Rightarrow y \geq 0$

$$y = |x - 2|, x \leq 2, y \geq 0$$

$$x = |y - 2|, y \leq 2, x \geq 0$$

$$x = y - 2 \quad \text{or} \quad -x = y - 2$$

$$2 + x = y \quad \text{or} \quad 2 - x = y$$

The portion that satisfies the conditions $y \leq 2$ and $x \geq 0$ is $2 - x = y$. This is a function of x, so f has an inverse.

$f^{-1}(x) = 2 - x, x \geq 0$

69. $f(x) = \sqrt{2x + 3} \Rightarrow x \geq -\dfrac{3}{2}, y \geq 0$

$$y = \sqrt{2x + 3}, x \geq -\frac{3}{2}, y \geq 0$$

$$x = \sqrt{2y + 3}, y \geq -\frac{3}{2}, x \geq 0$$

$$x^2 = 2y + 3, x \geq 0, y \geq -\frac{3}{2}$$

$$y = \frac{x^2 - 3}{2}, x \geq 0, y \geq -\frac{3}{2}$$

This is a function of x, so f has an inverse.

$f^{-1}(x) = \dfrac{x^2 - 3}{2}, x \geq 0$

70. $f(x) = \sqrt{x - 2} \Rightarrow x \geq 2, y \geq 0$

$$y = \sqrt{x - 2}, x \geq 2, y \geq 0$$

$$x = \sqrt{y - 2}, y \geq 2, x \geq 0$$

$$x^2 = y - 2, x \geq 0, y \geq 2$$

$$x^2 + 2 = y, x \geq 0, y \geq 2$$

This is a function of x, so f has an inverse.

$f^{-1}(x) = x^2 + 2, x \geq 0$

71.

$$f(x) = \frac{6x + 4}{4x + 5}$$

$$y = \frac{6x + 4}{4x + 5}$$

$$x = \frac{6y + 4}{4y + 5}$$

$$x(4y + 5) = 6y + 4$$

$$4xy + 5x = 6y + 4$$

$$4xy - 6y = -5x + 4$$

$$y(4x - 6) = -5x + 4$$

$$y = \frac{-5x + 4}{4x - 6} = \frac{5x - 4}{6 - 4x}$$

This is a function of x, so f has an inverse.

$f^{-1}(x) = \dfrac{5x - 4}{6 - 4x}$

72. The graph of f passes the Horizontal Line Test. So, you know f is one-to-one and has an inverse function.

$$f(x) = \frac{5x - 3}{2x + 5}$$
$$y = \frac{5x - 3}{2x + 5}$$
$$x = \frac{5y - 3}{2y + 5}$$
$$x(2y + 5) = 5y - 3$$
$$2xy + 5x = 5y - 3$$
$$2xy - 5y = -5x - 3$$
$$y(2x - 5) = -(5x + 3)$$
$$y = -\frac{5x + 3}{2x - 5}$$
$$f^{-1}(x) = -\frac{5x + 3}{2x - 5}$$

73. $f(x) = (x - 2)^2$

domain of $f: x \ge 2$, range of $f: y \ge 0$

$$f(x) = (x - 2)^2$$
$$y = (x - 2)^2$$
$$x = (y - 2)^2$$
$$\sqrt{x} = y - 2$$
$$\sqrt{x} + 2 = y$$

So, $f^{-1}(x) = \sqrt{x} + 2.$

domain of $f^{-1}: x \ge 0$, range of $f^{-1}: x \ge 2$

74. $f(x) = 1 - x^4$

domain of $f: x \ge 0$, range of $f: y \le 1$

$$f(x) = 1 - x^4$$
$$y = 1 - x^4$$
$$x = 1 - y^4$$
$$x - 1 = -y^4$$
$$\sqrt[4]{1 - x} = y$$

So, $f^{-1}(x) = \sqrt[4]{1 - x}.$

domain of $f^{-1}: x \le 1$, range of $f^{-1}: y \ge 0$

75. $f(x) = |x + 2|$

domain of $f: x \ge -2$, range of $f: y \ge 0$

$$f(x) = |x + 2|$$
$$y = |x + 2|$$
$$x = y + 2$$
$$x - 2 = y$$

So, $f^{-1}(x) = x - 2.$

domain of $f^{-1}: x \ge 0$, range of $f^{-1}: y \ge -2$

76. $f(x) = |x - 5|$

domain of $f: x \ge 5$, range of $f: y \ge 0$

$$f(x) = |x - 5|$$
$$y = x - 5$$
$$x = y - 5$$
$$x + 5 = y$$

So, $f^{-1}(x) = x + 5.$

domain $f^{-1}: x \ge 0$, range of $f^{-1}: y \ge 5$

77. $f(x) = (x + 6)^2$

domain of $f: x \ge -6$, range of $f: y \ge 0$

$$f(x) = (x + 6)^2$$
$$y = (x + 6)^2$$
$$x = (y + 6)^2$$
$$\sqrt{x} = y + 6$$
$$\sqrt{x} - 6 = y$$

So, $f^{-1}(x) = \sqrt{x} - 6.$

domain of $f^{-1}: x \ge 0$, range of $f^{-1}: y \ge -6$

78. $f(x) = (x - 4)^2$

domain of $f: x \ge 4$, range of $f: y \ge 0$

$$f(x) = (x - 4)^2$$
$$y = (x - 4)^2$$
$$x = (y - 4)^2$$
$$\sqrt{x} = y - 4$$
$$\sqrt{x} + 4 = y$$

So, $f^{-1}(x) = \sqrt{x} + 4.$

domain of $f^{-1}: x \ge 0$, range of $f^{-1}: y \ge 4$

79. $f(x) = -2x^2 + 5$

domain of $f: x \geq 0$, range of $f: y \leq 5$

$$\begin{aligned} f(x) &= -2x^2 + 5 \\ y &= -2x^2 + 5 \\ x &= -2y^2 + 5 \\ x - 5 &= -2y^2 \\ 5 - x &= 2y^2 \\ \sqrt{\frac{5-x}{2}} &= y \\ \frac{\sqrt{5-x}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} &= y \\ \frac{\sqrt{2(5-x)}}{2} &= y \end{aligned}$$

So, $f^{-1}(x) = \dfrac{\sqrt{-2(x-5)}}{2}$.

domain of $f^{-1}(x): x \leq 5$, range of $f^{-1}(x): y \geq 0$

80. $f(x) = \frac{1}{2}x^2 - 1$

domain of $f: x \geq 0$, range of $f: y \geq -1$

$$\begin{aligned} f(x) &= \tfrac{1}{2}x^2 - 1 \\ y &= \tfrac{1}{2}x^2 - 1 \\ x &= \tfrac{1}{2}y^2 - 1 \\ x + 1 &= \tfrac{1}{2}y^2 \\ 2x + 2 &= y^2 \\ \sqrt{2x+2} &= y \end{aligned}$$

So, $f^{-1}(x) = \sqrt{2x+2}$.

domain of $f^{-1}: x \geq -1$, range of $f^{-1}: y \geq 0$

81. $f(x) = |x - 4| + 1$

domain of $f: x \geq 4$, range of $f: y \geq 1$

$$\begin{aligned} f(x) &= |x - 4| + 1 \\ y &= x - 3 \\ x &= y - 3 \\ x + 3 &= y \end{aligned}$$

So, $f^{-1}(x) = x + 3$.

domain of $f^{-1}: x \geq 1$, range of $f^{-1}: y \geq 4$

82. $f(x) = -|x - 1| - 2$

domain of $f: x \geq 1$, range of $f: y \leq -2$

$$\begin{aligned} f(x) &= -|x - 1| - 2 \\ y &= -|x - 1| - 2 \\ x &= -(y - 1) - 2 \\ x &= -y - 1 \\ -x - 1 &= y \end{aligned}$$

So, $f^{-1}(x) = -x - 1$.

domain of $f^{-1}: x \leq -2$, range of $f^{-1}: y \geq 1$

In Exercises 83–88, $f(x) = \frac{1}{8}x - 3$, $f^{-1}(x) = 8(x + 3)$, $g(x) = x^3$, $g^{-1}(x) = \sqrt[3]{x}$.

83.
$$\begin{aligned} (f^{-1} \circ g^{-1})(1) &= f^{-1}(g^{-1}(1)) \\ &= f^{-1}(\sqrt[3]{1}) \\ &= 8(\sqrt[3]{1} + 3) = 32 \end{aligned}$$

84.
$$\begin{aligned} (g^{-1} \circ f^{-1})(-3) &= g^{-1}(f^{-1}(-3)) \\ &= g^{-1}(8(-3 + 3)) \\ &= g^{-1}(0) = \sqrt[3]{0} = 0 \end{aligned}$$

85.
$$\begin{aligned} (f^{-1} \circ f^{-1})(6) &= f^{-1}(f^{-1}(6)) \\ &= f^{-1}(8[6 + 3]) \\ &= 8[8(6 + 3) + 3] = 600 \end{aligned}$$

86.
$$\begin{aligned} (g^{-1} \circ g^{-1})(-4) &= g^{-1}(g^{-1}(-4)) \\ &= g^{-1}(\sqrt[3]{-4}) \\ &= \sqrt[3]{\sqrt[3]{-4}} = \sqrt[9]{-4} \end{aligned}$$

87.
$$\begin{aligned} (f \circ g)(x) &= f(g(x)) = f(x^3) = \tfrac{1}{8}x^3 - 3 \\ y &= \tfrac{1}{8}x^3 - 3 \\ x &= \tfrac{1}{8}y^3 - 3 \\ x + 3 &= \tfrac{1}{8}y^3 \\ 8(x + 3) &= y^3 \\ \sqrt[3]{8(x+3)} &= y \\ (f \circ g)^{-1}(x) &= 2\sqrt[3]{x + 3} \end{aligned}$$

88. $g^{-1} \circ f^{-1} = g^{-1}\left(f^{-1}(x)\right)$
$= g^{-1}\left(8(x + 3)\right)$
$= \sqrt[3]{8(x + 3)}$
$= 2\sqrt[3]{x + 3}$

In Exercises 89–92, $f(x) = x + 4$, $f^{-1}(x) = x - 4$, $g(x) = 2x - 5$, $g^{-1}(x) = \frac{x + 5}{2}$.

89. $\left(g^{-1} \circ f^{-1}\right)(x) = g^{-1}\left(f^{-1}(x)\right)$
$= g^{-1}(x - 4)$
$= \frac{(x - 4) + 5}{2}$
$= \frac{x + 1}{2}$

90. $\left(f^{-1} \circ g^{-1}\right)(x) = f^{-1}\left(g^{-1}(x)\right)$
$= f^{-1}\left(\frac{x + 5}{2}\right)$
$= \frac{x + 5}{2} - 4$
$= \frac{x + 5 - 8}{2}$
$= \frac{x - 3}{2}$

91. $(f \circ g)(x) = f\left(g(x)\right)$
$= f(2x - 5)$
$= (2x - 5) + 4$
$= 2x - 1$

$(f \circ g)^{-1}(x) = \frac{x + 1}{2}$

Note: Comparing Exercises 89 and 91,
$(f \circ g)^{-1}(x) = \left(g^{-1} \circ f^{-1}\right)(x).$

92. $(g \circ f)(x) = g\left(f(x)\right)$
$= g(x + 4)$
$= 2(x + 4) - 5$
$= 2x + 8 - 5$
$= 2x + 3$
$y = 2x + 3$
$x = 2y + 3$
$x - 3 = 2y$
$\frac{x - 3}{2} = y$
$(g \circ f)^{-1}(x) = \frac{x - 3}{2}$

93. (a) $y = 10 + 0.75x$
$x = 10 + 0.75y$
$x - 10 = 0.75y$
$\frac{x - 10}{0.75} = y$

So, $f^{-1}(x) = \frac{x - 10}{0.75}.$

$x =$ hourly wage, $y =$ number of units produced

(b) $y = \frac{24.25 - 10}{0.75} = 19$

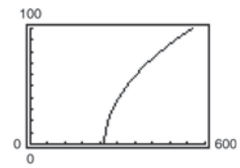
So, 19 units are produced.

94. (a) $y = 0.03x^2 + 245.50,\ 0 < x < 100$
$\Rightarrow 245.50 < y < 545.50$

$x = 0.03y^2 + 245.50$
$x - 245.50 = 0.03y^2$
$\frac{x - 245.50}{0.03} = y^2$
$\sqrt{\frac{x - 245.50}{0.03}} = y,\ 245.50 < x < 545.50$
$f^{-1}(x) = \sqrt{\frac{x - 245.50}{0.03}}$

$x =$ temperature in degrees Fahrenheit
$y =$ percent load for a diesel engine

(b)



(c) $0.03x^2 + 245.50 \le 500$
$0.03x^2 \le 254.50$
$x^2 \le 8483.33$
$x \le 92.10$

Thus, $0 < x \le 92.10$.

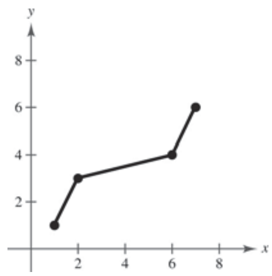
95. False. $f(x) = x^2$ is even and does not have an inverse.

96. True. If $f(x)$ has an inverse and it has a y-intercept at $(0, b)$, then the point $(b, 0)$, must be a point on the graph of $f^{-1}(x)$.

97.

x	1	3	4	6
f	1	2	6	7

x	1	2	6	7
$f^{-1}(x)$	1	3	4	6



98.

x	-4	-2	0	3
f	3	4	0	-1

The graph does not pass the Horizontal Line Test, so $f^{-1}(x)$ does not exist.

99. Let $(f \circ g)(x) = y$. Then $x = (f \circ g)^{-1}(y)$. Also,

$$\begin{aligned}(f \circ g)(x) = y \Rightarrow f(g(x)) &= y\\ g(x) &= f^{-1}(y)\\ x &= g^{-1}(f^{-1}(y))\\ x &= (g^{-1} \circ f^{-1})(y).\end{aligned}$$

Because f and g are both one-to-one functions, $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

100. Let $f(x)$ be a one-to-one odd function. Then $f^{-1}(x)$ exists and $f(-x) = -f(x)$. Letting (x, y) be any point on the graph of $f(x) \Rightarrow (-x, -y)$ is also on the graph of $f(x)$ and $f^{-1}(-y) = -x = -f^{-1}(y)$. So, $f^{-1}(x)$ is also an odd function.

101. If $f(x) = k(2 - x - x^3)$ has an inverse and $f^{-1}(3) = -2$, then $f(-2) = 3$. So,

$$f(-2) = k\left(2 - (-2) - (-2)^3\right) = 3$$

$$\begin{aligned}k(2 + 2 + 8) &= 3\\ 12k &= 3\\ k &= \tfrac{3}{12} = \tfrac{1}{4}.\end{aligned}$$

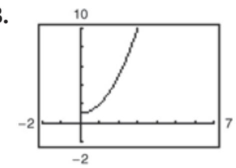
So, $k = \frac{1}{4}$.

102.

x	-10	0	7	45
$f(f^{-1}(x))$	-10	0	7	45
$f^{-1}(f(x))$	-10	0	7	45

$f(x)$ and $f^{-1}(x)$ are inverses of each other.

103.



There is an inverse function $f^{-1}(x) = \sqrt{x - 1}$ because the domain of f is equal to the range of f^{-1} and the range of f is equal to the domain of f^{-1}.

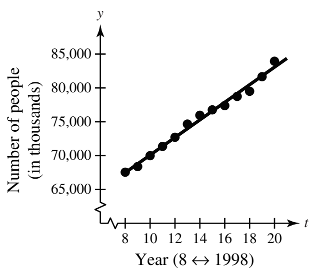
104. (a) $C(x)$ is represented by graph m and $C^{-1}(x)$ is represented by graph n.

(b) $C(x)$ represents the cost of making x units of personalized T-shirts. $C^{-1}(x)$ represents the number of personalized T-shirts that can be made for a given cost.

105. This situation could be represented by a one-to-one function if the runner does not stop to rest. The inverse function would represent the time in hours for a given number of miles completed.

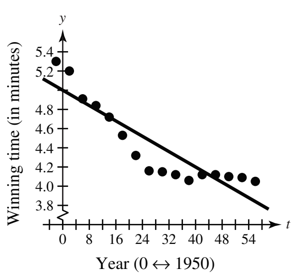
106. This situation could be represented by a one-to-one function if the population continues to increase. The inverse function would represent the year for a given population.

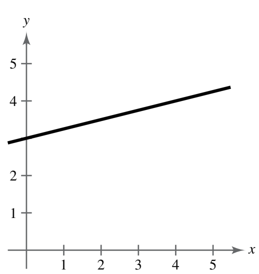
Section 1.10 Mathematical Modeling and Variation

1. variation; regression
2. sum of square differences
3. least squares regression
4. correlation coefficient
5. directly proportional
6. constant of variation
7. directly proportional
8. inverse
9. combined
10. jointly proportional
11. 

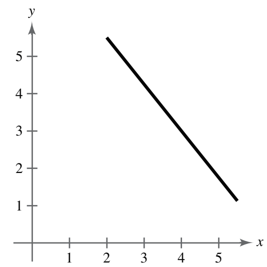
The model fits the data well.

12. The model is not a good fit for the actual data.

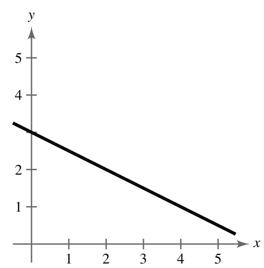


13. 

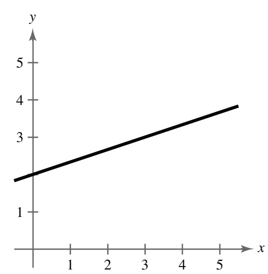
Using the point $(0, 3)$ and $(4, 4)$, $y = \frac{1}{4}x + 3$.

14. 

The line appears to pass through $(2, 5.5)$ and $(6, 0.5)$, so its equation is $y = -\frac{5}{4}x + 8$.

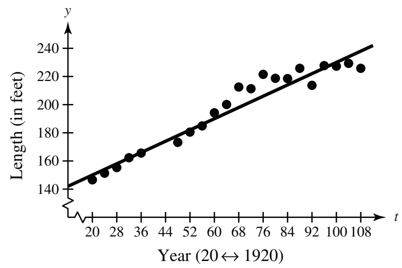
15. 

Using the points $(2, 2)$ and $(4, 1)$, $y = -\frac{1}{2}x + 3$.

16. 

The line appears to pass through $(0, 2)$ and $(3, 3)$ so its equation is $y = \frac{1}{3}x + 2$.

17. (a)



(b) Using the points $(32, 162.3)$ and $(96, 227.7)$:

$$m = \frac{227.7 - 162.3}{96 - 32}$$
$$\approx 1.02$$
$$y - 162.3 = 1.02(t - 32)$$
$$y = 1.02t + 129.66$$

(c) $y \approx 1.01t + 130.82$

(d) The models are similar.

$2012 \rightarrow$ use $t = 112$

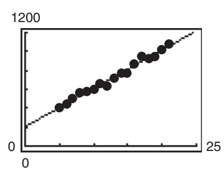
Model from part (b):

$y = 1.02(112) + 129.66 = 243.9$ feet

Model from part (c):

$y = 1.01(112) + 130.82 = 243.94$ feet

18. (a) and (c)



The model fits the data well.

(b) $S = 40.6t + 204$

(d) $2017 \rightarrow$ use $t = 27$

Model from part (b).

$S = 40.6(27) + 204 = 1300$

In 2017, the annual gross ticket sales will be about $1300 million.

(e) Each year, the gross ticket sales for Broadway shows in New York City increase by about $40.6 million.

19.
$$y = kx$$
$$14 = k(2)$$
$$7 = k$$
$$y = 7x$$

20.
$$y = kx$$
$$12 = k(5)$$
$$\tfrac{12}{5} = k$$
$$y = \tfrac{12}{5}x$$

21.
$$y = kx$$
$$2050 = k(10)$$
$$205 = k$$
$$y = 205x$$

22.
$$y = kx$$
$$580 = k(6)$$
$$\tfrac{290}{3} = k$$
$$y = \tfrac{290}{3}x$$

23.
$$y = kx$$
$$1 = k(5)$$
$$\tfrac{1}{5} = k$$
$$y = \tfrac{1}{5}x$$

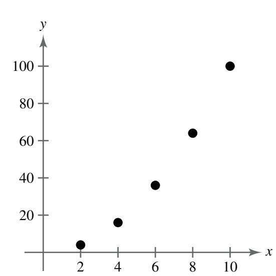
24.
$$y = kx$$
$$3 = k(-24)$$
$$-\tfrac{1}{8} = k$$
$$y = -\tfrac{1}{8}x$$

25.
$$y = kx$$
$$8\pi = k(4)$$
$$\pi = k$$
$$y = \frac{\pi}{2}x$$

26.
$$y = kx$$
$$-1 = k(\pi)$$
$$-\frac{1}{\pi} = k$$
$$y = -\frac{1}{\pi}x$$

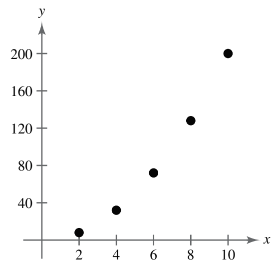
27. $k = 1$

x	2	4	6	8	10
$y = kx^2$	4	16	36	64	100



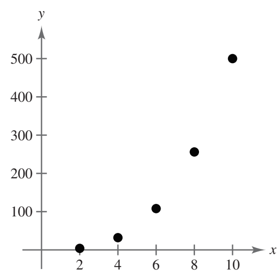
28. $k = 2$

x	2	4	6	8	10
$y = kx^2$	8	32	72	128	200



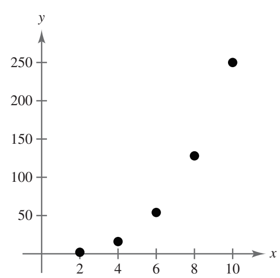
29. $k = \frac{1}{2}$

x	2	4	6	8	10
$y = \frac{1}{2}x^3$	4	32	108	256	500



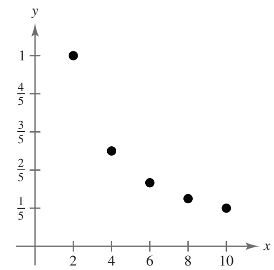
30. $k = \frac{1}{4}, n = 3$

x	2	4	6	8	10
$y = \frac{1}{4}x^3$	2	16	54	128	250



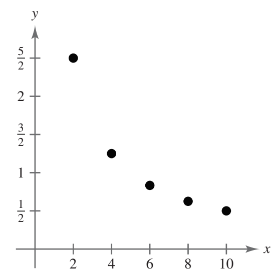
31. $k = 2, n = 1$

x	2	4	6	8	10
$y = \frac{2}{x}$	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$



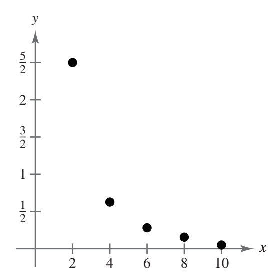
32. $k = 5, n = 1$

x	2	4	6	8	10
$y = \frac{5}{x}$	$\frac{5}{2}$	$\frac{5}{4}$	$\frac{5}{6}$	$\frac{5}{8}$	$\frac{1}{2}$



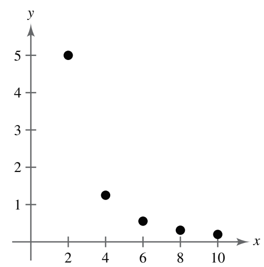
33. $k = 10$

x	2	4	6	8	10
$y = \frac{k}{x^2}$	$\frac{5}{2}$	$\frac{5}{8}$	$\frac{5}{18}$	$\frac{5}{32}$	$\frac{1}{10}$



34. $k = 20$

x	2	4	6	8	10
$y = \dfrac{k}{x^2}$	5	$\dfrac{5}{4}$	$\dfrac{5}{9}$	$\dfrac{5}{16}$	$\dfrac{1}{5}$



35. The graph appears to represent $y = 4/x$, so y varies inversely as x.

36. The graph appears to represent $y = \frac{3}{2}x$, so y varies directly with x.

37.
$$y = \frac{k}{x}$$
$$1 = \frac{k}{5}$$
$$5 = k$$
$$y = \frac{5}{x}$$

This equation checks with the other points given in the table.

38.
$$y = kx$$
$$2 = k5$$
$$\tfrac{2}{5} = k$$
$$y = \tfrac{2}{5}x$$

This equation checks with the other points given in the table.

39.
$$y = kx$$
$$-7 = k(10)$$
$$-\tfrac{7}{10} = k$$
$$y = -\tfrac{7}{10}x$$

This equation checks with the other points given in the table.

40.
$$y = \frac{k}{x}$$
$$24 = \frac{k}{5}$$
$$120 = k$$
$$y = \frac{120}{x}$$

This equation checks with the other points given in the table.

41. $A = kr^2$

42. $V = ke^3$

43. $y = \dfrac{k}{x^2}$

44. $h = \dfrac{k}{\sqrt{s}}$

45. $F = \dfrac{kg}{r^2}$

46. $z = kx^2y^3$

47. $R = k(T - T_e)$

48. $P = \dfrac{k}{V}$

49. $R = kS(S - L)$

50. $F = \dfrac{km_1m_2}{r^2}$

51. $S = 4\pi r^2$

The surface area of a sphere varies directly as the square of the radius r.

52. $r = \dfrac{d}{t}$

Average speed is directly proportional to the distance and inversely proportional to the time.

53. $A = \frac{1}{2}bh$

The area of a triangle is jointly proportional to its base and height.

54. $V = \pi r^2h$

The volume of a right circular cylinder is jointly proportional to the height and the square of the radius.

55.
$$A = kr^2$$
$$9\pi = k(3)^2$$
$$\pi = k$$
$$A = \pi r^2$$

56.
$$y = \frac{k}{x}$$
$$3 = \frac{k}{25}$$
$$75 = k$$
$$y = \frac{75}{x}$$

57.
$$y = \frac{k}{x}$$
$$7 = \frac{k}{4}$$
$$28 = k$$
$$y = \frac{28}{x}$$

58.
$$z = kxy$$
$$64 = k(4)(8)$$
$$2 = k$$
$$z = 2xy$$

59.
$$F = krs^3$$
$$4158 = k(11)(3)^3$$
$$k = 14$$
$$F = 14rs^3$$

60.
$$P = \frac{kx}{y^2}$$
$$\frac{28}{3} = \frac{k(42)}{9^2}$$
$$\frac{28}{3} \cdot \frac{81}{42} = k$$
$$\frac{2 \cdot 27}{3} = k$$
$$18 = k$$
$$P = \frac{18x}{y^2}$$

61.
$$z = \frac{kx^2}{y}$$
$$6 = \frac{k(6)^2}{4}$$
$$\frac{24}{36} = k$$
$$\frac{2}{3} = k$$
$$z = \frac{2/3x^2}{y} = \frac{2x^2}{3y}$$

62.
$$v = \frac{kpq}{s^2}$$
$$1.5 = \frac{k(4.1)(6.3)}{(1.2)^2}$$
$$\frac{(1.5)(1.44)}{(4.1)(6.3)} = k$$
$$\frac{2.16}{25.83} = k$$
$$k = \frac{24}{287}$$
$$v = \frac{24pq}{287s^2}$$

63.
$$I = kP$$
$$113.75 = k(3250)$$
$$0.035 = k$$
$$I = 0.035P$$

64.
$$I = kP$$
$$211.25 = k(6500)$$
$$0.0325 = k$$
$$I = 0.0325P$$

65.
$$y = kx$$
$$33 = k(13)$$
$$\tfrac{33}{13} = k$$
$$y = \tfrac{33}{13}x$$

When $x = 10$ inches, $y \approx 25.4$ centimeters.

When $x = 20$ inches, $y \approx 50.8$ centimeters.

66.
$$y = kx$$
$$53 = k(14)$$
$$\tfrac{53}{14} = k$$
$$y = \tfrac{53}{14}x$$

5 gallons: $y = \tfrac{53}{14}(5) \approx 18.9$ liters

25 gallons: $y = \tfrac{53}{14}(5) \approx 94.6$ liters

67.
$$\begin{aligned} d &= kF \\ 0.12 &= k(220) \\ \tfrac{3}{5500} &= k \\ d &= \tfrac{3}{5500}F \\ 0.16 &= \tfrac{3}{5500}F \\ \tfrac{880}{3} &= F \end{aligned}$$

The required force is $293\frac{1}{3}$ newtons.

68.
$$\begin{aligned} d &= kF \\ 0.15 &= k(265) \\ \tfrac{3}{5300} &= k \\ d &= \tfrac{3}{5300}F \end{aligned}$$

(a) $d = \frac{3}{5300}(90) \approx 0.05$ meter

(b) $0.1 = \frac{3}{5300}F$

$\frac{530}{3} = F$

$F = 176\frac{2}{3}$ newtons

69.
$$\begin{aligned} d &= kF \\ 1.9 &= k(25) \Rightarrow k = 0.076 \\ d &= 0.076F \end{aligned}$$

When the distance compressed is 3 inches, we have

$$\begin{aligned} 3 &= 0.076F \\ F &\approx 39.47. \end{aligned}$$

No child over 39.47 pounds should use the toy.

70.
$$\begin{aligned} d &= kF \\ 1 &= k(15) \\ k &= \tfrac{1}{15} \\ d &= \tfrac{1}{15}F \\ \tfrac{8}{2} &= \tfrac{1}{15}F \\ F &= 60 \text{ lb per spring} \end{aligned}$$

Combined lifting force $= 2F = 120$ lb

71.
$$\begin{aligned} d &= kv^2 \\ 0.02 &= k\left(\frac{1}{4}\right)^2 \\ k &= 0.32 \\ d &= 0.32v^2 \\ 0.12 &= 0.32v^2 \\ v^2 &= \frac{0.12}{0.32} = \frac{3}{8} \\ v &= \frac{\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{6}}{4} \approx 0.61 \text{ mi/hr} \end{aligned}$$

72. $f = k\dfrac{\sqrt{T}}{l}$ where $f =$ frequency, $T =$ tension, and $l =$ length of string

$$\begin{aligned} 440 &= k\frac{\sqrt{T}}{l} \\ f &= k\frac{\sqrt{1.25T}}{1.2l} \end{aligned}$$

$$\frac{440l}{\sqrt{T}} = k \quad \text{and} \quad \frac{1.2\ fl}{\sqrt{1.25T}} = k$$

$$\begin{aligned} \frac{440l}{\sqrt{T}} &= \frac{1.2\,fl}{\sqrt{1.25T}} \\ 440l\sqrt{1.25T} &= 1.2\,fl\sqrt{T} \qquad (l > 0) \\ 440\sqrt{1.25T} &= 1.2\,f\sqrt{T} \\ 242{,}000T &= 1.44\,f^2T \qquad (T > 0) \\ 242{,}000 &= 1.44\,f^2 \\ 168{,}055.56 &= f^2 \\ f &\approx 409.95 \text{ vibrations per second} \end{aligned}$$

73. $W = kmh$

$2116.8 = k(120)(1.8)$

$k = \dfrac{2116.8}{(120)(1.8)} = 9.8$

$W = 9.8mh$

When $m = 100$ kilograms and $h = 1.5$ meters, we have $W = 9.8(100)(1.5) = 1470$ joules.

74. Load $= \dfrac{kwd^2}{l}$

(a) load $= \dfrac{k(2w)d^2}{2l} = \dfrac{kwd^2}{l}$

The safe load is unchanged.

(b) load $= \dfrac{k(2w)(2d)^2}{l} = \dfrac{8kwd^2}{l}$

The safe load is eight times as great.

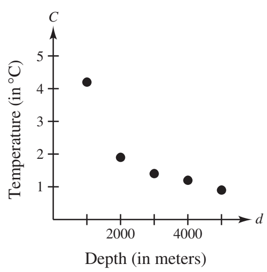
(c) load $= \dfrac{k(2w)(2d)^2}{2l} = \dfrac{4kwd^2}{l}$

The safe load is four times as great.

(d) load $= \dfrac{kw(d/2)^2}{l} = \dfrac{(1/4)kwd^2}{l}$

The safe load is one-fourth as great.

75. (a)



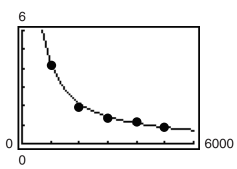
(b) Yes, the data appears to be modeled (approximately) by the inverse proportion model.

$4.2 = \dfrac{k_1}{1000}$ $\quad 1.9 = \dfrac{k_2}{2000}$ $\quad 1.4 = \dfrac{k_3}{3000}$ $\quad 1.2 = \dfrac{k_4}{4000}$ $\quad 0.9 = \dfrac{k_5}{5000}$

$4200 = k_1$ $\quad 3800 = k_2$ $\quad 4200 = k_3$ $\quad 4800 = k_4$ $\quad 4500 = k_5$

(c) Mean: $k = \dfrac{4200 + 3800 + 4200 + 4800 + 4500}{5} = 4300$, Model: $C = \dfrac{4300}{d}$

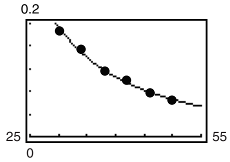
(d)



(e) $3 = \dfrac{4300}{d}$

$d = \dfrac{4300}{3} = 1433\frac{1}{3}$ meters

76. $y = \dfrac{262.76}{x^{2.12}}$

(a) 

(b) $y = \dfrac{262.76}{(25)^{2.12}}$

≈ 0.2857 microwatts per sq. cm.

77. False. π is a constant, not a variable. So, the area A varies directly as the square of the radius, r.

78. False. The closer the value of $|r|$ is to 1, the better the fit.

79. (a) y will change by a factor of one-fourth.

(b) y will change by a factor of four.

80. (a) The data shown could be represented by a linear model which would be a good approximation.

(b) The points do not follow a linear pattern. A linear model would be a poor approximation. A quadratic model would be better.

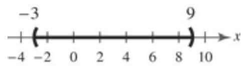
(c) The points do not follow a linear pattern. A linear model would be a poor approximation.

(d) The data shown could be represented by a linear model which would be a good approximation.

Review Exercises for Chapter 1

1. $x^2 - 6x - 27 < 0$

$(x + 3)(x - 9) < 0$



Key numbers: $x = -3, x = 9$

Test intervals: $(-\infty, -3), (-3, 9), (9, \infty)$

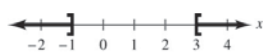
Test: Is $(x + 3)(x - 9) < 0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is $(-3, 9)$.

2. $x^2 - 2x \geq 3$

$x2 - 2x - 3 \geq 0$

$(x - 3)(x + 1) \geq 0$



Key numbers: $x = -1, x = 3$

Test intervals: $(-\infty, -1), (-1, 3), (3, \infty)$

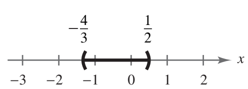
Test: Is $(x - 3)(x + 1) \geq 0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is $(-\infty, -1] \cup [3, \infty)$.

3. $6x^2 + 5x < 4$

$6x^2 + 5x - 4 < 0$

$(3x + 4)(2x - 1) < 0$



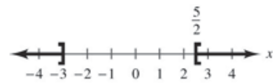
Key numbers: $x = -\frac{4}{3}, x = \frac{1}{2}$

Test intervals: $\left(-\infty, -\frac{4}{3}\right), \left(-\frac{4}{3}, \frac{1}{2}\right), \left(\frac{1}{2}, \infty\right)$

Test: Is $(3x + 4)(2x - 1) < 0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is $\left(-\frac{4}{3}, \frac{1}{2}\right)$.

4. $$2x^2 + x \geq 15$$
$$2x^2 + x - 15 \geq 0$$
$$(2x - 5)(x + 3) \geq 0$$



Key numbers: $x = \frac{5}{2}, x = -3$

Test intervals: $(-\infty, -3), \left(-3, \frac{5}{2}\right), \left(\frac{5}{2}, \infty\right)$

Test: Is $(2x - 5)(x + 3) \geq 0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is $(-\infty, -3] \cup \left[\frac{5}{2}, \infty\right)$.

5. $\dfrac{x - 5}{3 - x} < 0$

Key numbers: $x = 5, x = 3$

Test intervals: $(-\infty, 3), (3, 5), (5, \infty)$

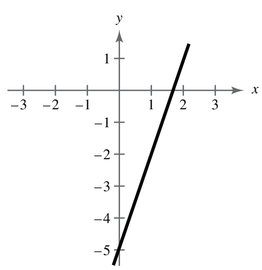
Test: Is $\dfrac{x - 5}{3 - x} < 0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is $(-\infty, 3) \cup (5, \infty)$.

6. Rational equations, equations involving radicals, and absolute value equations, may have "solutions" that are extraneous. So checking solutions, in the original equations, is crucial to eliminate these extraneous values.

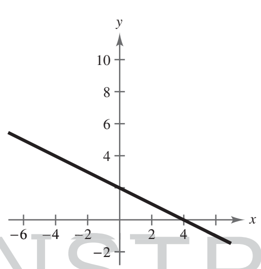
7. $y = 3x - 5$

x	-2	-1	0	1	2
y	-11	-8	-5	-2	1



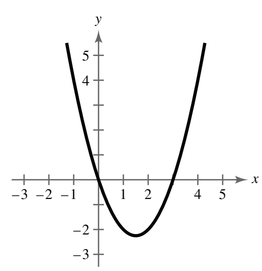
8. $y = -\frac{1}{2}x + 2$

x	-4	-2	0	2	4
y	4	3	2	1	0



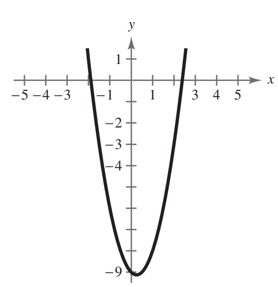
9. $y = x^2 - 3x$

x	-1	0	1	2	3	4
y	4	0	-2	-2	0	4



10. $y = 2x^2 - x - 9$

x	-2	-1	0	1	2	3
y	1	-6	-9	-8	-3	6



11. $y = 2x + 7$

x-intercept: Let $y = 0$.

$$0 = 2x + 7$$
$$x = -\frac{7}{2}$$
$$\left(-\frac{7}{2}, 0\right)$$

y-intercept: Let $x = 0$.

$$y = 2(0) + 7$$
$$y = 7$$
$$(0, 7)$$

12. x-intercept: Let $y = 0$.

$$y = |x + 1| - 3$$
$$0 = |x + 1| - 3$$

For $x + 1 > 0, 0 = x + 1 - 3,$ or $2 = x$.

For $x + 1 < 0, 0 = -(x + 1) - 3,$ or $-4 = x$.

$(2, 0), (4, 0)$

y-intercept: Let $x = 0$.

$$y = |x + 1| - 3$$
$$y = |0 + 1| - 3 \text{ or } y = -2$$

$(0, -2)$

13. $y = (x - 3)^2 - 4$

x-intercepts: $0 = (x - 3)^2 - 4 \Rightarrow (x - 3)^2 = 4$

$$\Rightarrow x - 3 = \pm 2$$
$$\Rightarrow x = 3 \pm 2$$
$$\Rightarrow x = 5 \text{ or } x = 1$$
$$(5, 0), (1, 0)$$

y-intercept: $y = (0 - 3)^2 - 4$

$$y = 9 - 4$$
$$y = 5$$
$$(0, 5)$$

14. $y = x\sqrt{4 - x^2}$

x-intercepts: $0 = x\sqrt{4 - x^2}$

$$x = 0 \qquad \sqrt{4 - x^2} = 0$$
$$4 - x^2 = 0$$
$$x = \pm 2$$
$$(0, 0), (-2, 0), (2, 0)$$

y-intercept: $y = 0 \cdot \sqrt{4 - 0} = 0$

$(0, 0)$

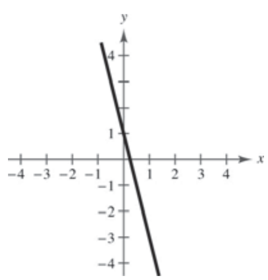
15. $y = -4x + 1$

Intercepts: $\left(\frac{1}{4}, 0\right), (0, 1)$

$y = -4(-x) + 1 \Rightarrow y = 4x + 1 \Rightarrow$ No y-axis symmetry

$-y = -4x + 1 \Rightarrow y = 4x - 1 \Rightarrow$ No x-axis symmetry

$-y = -4(-x) + 1 \Rightarrow y = -4x - 1 \Rightarrow$ No origin symmetry



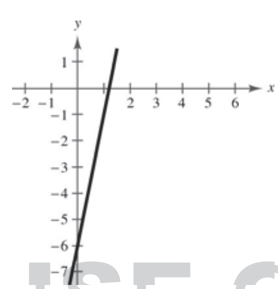
16. $y = 5x - 6$

Intercepts: $\left(\frac{6}{5}, 0\right), (0, -6)$

$y = 5(-x) - 6 \Rightarrow y = -5x - 6 \Rightarrow$ No y-axis symmetry

$-y = 5x - 6 \Rightarrow y = -5x + 6 \Rightarrow$ No x-axis symmetry

$-y = 5(-x) - 6 \Rightarrow y = 5x + 6 \Rightarrow$ No origin symmetry



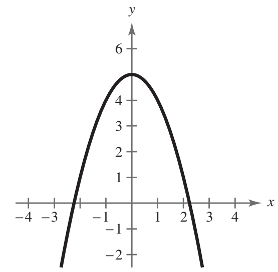
17. $y = 5 - x^2$

Intercepts: $\left(\pm\sqrt{5}, 0\right), (0, 5)$

$y = 5 - (-x)^2 \Rightarrow y = 5 - x^2 \Rightarrow$ y-axis symmetry

$-y = 5 - x^2 \Rightarrow y = -5 + x^2 \Rightarrow$ No x-axis symmetry

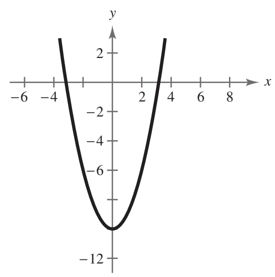
$-y = 5 - (-x)^2 \Rightarrow y = -5 + x^2 \Rightarrow$ No origin symmetry



18. $y = x^2 - 10$

Intercepts: $\left(\pm\sqrt{10}, 0\right), (0, -10)$

y-axis symmetry



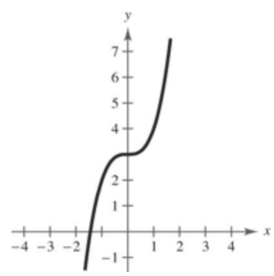
19. $y = x^3 + 3$

Intercepts: $\left(-\sqrt[3]{3}, 0\right), (0, 3)$

$y = (-x)^3 + 3 \Rightarrow y = -x^3 + 3 \Rightarrow$ No y-axis symmetry

$-y = x^3 + 3 \Rightarrow y = -x^3 - 3 \Rightarrow$ No x-axis symmetry

$-y = (-x)^3 + 3 \Rightarrow y = x^3 - 3 \Rightarrow$ No origin symmetry



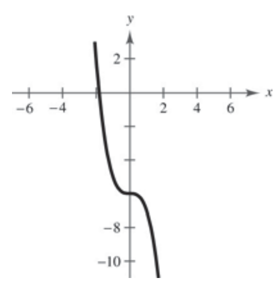
20. $y = -6 - x^3$

Intercepts: $\left(\sqrt[3]{-6}, 0\right), (0, -6)$

$y = -6 - (-x)^3 \Rightarrow y = -6 + x \Rightarrow$ No y-axis symmetry

$-y = -6 - x^3 \Rightarrow y = 6 + x^3 \Rightarrow$ No x-axis symmetry

$-y = -6 - (-x)^3 \Rightarrow y = 6 - x^3 \Rightarrow$ No origin symmetry



21. $y = \sqrt{x + 5}$

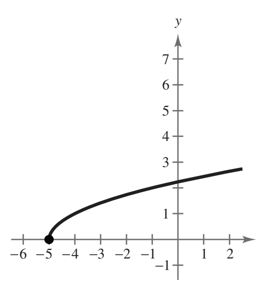
Domain: $[-5, \infty)$

Intercepts: $(-5, 0), \left(0, \sqrt{5}\right)$

$y = \sqrt{-x + 5} \Rightarrow$ No y-axis symmetry

$-y = \sqrt{x + 5} \Rightarrow y = -\sqrt{x + 5} \Rightarrow$ No x-axis symmetry

$-y = \sqrt{-x + 5} \Rightarrow y = -\sqrt{-x + 5} \Rightarrow$ No origin symmetry



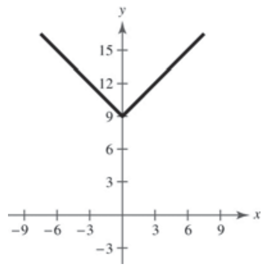
22. $y = |x| + 9$

Intercept: $(0, 9)$

$y = |-x| + 9 \Rightarrow y = |x| + 9 \Rightarrow$ y-axis symmetry

$-y = |x| + 9 \Rightarrow y = -|x| - 9 \Rightarrow$ No x-axis symmetry

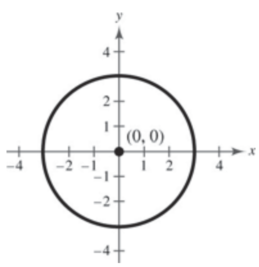
$-y = |-x| + 9 \Rightarrow y = -|x| - 9 \Rightarrow$ No origin symmetry



23. $x^2 + y^2 = 9$

Center: $(0, 0)$

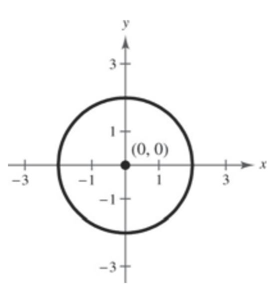
Radius: 3



24. $x^2 + y^2 = 4$

Center: $(0, 0)$

Radius: 2

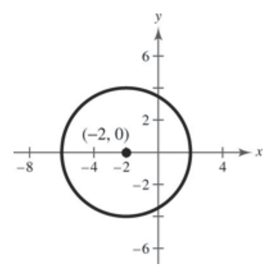


25. $(x + 2)^2 + y^2 = 16$

$(x - (-2))^2 + (y - 0)^2 = 4^2$

Center: $(-2, 0)$

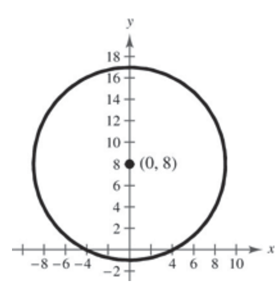
Radius: 4



26. $x^2 + (y - 8)^2 = 81$

Center: $(0, 8)$

Radius: 9



27. Endpoints of a diameter: $(0, 0)$ and $(4, -6)$

Center: $\left(\dfrac{0+4}{2}, \dfrac{0+(-6)}{2}\right) = (2, -3)$

Radius: $r = \sqrt{(2-0)^2 + (-3-0)^2} = \sqrt{4+9} = \sqrt{13}$

Standard form: $(x-2)^2 + (y-(-3))^2 = \left(\sqrt{13}\right)^2$

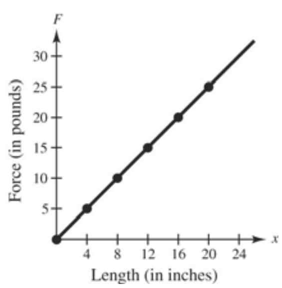
$$(x-2)^2 + (y+3)^2 = 13$$

28. $F = \frac{5}{4}x, 0 \le x \le 20$

(a)

x	0	4	8	12	16	20
F	0	5	10	15	20	25

(b)

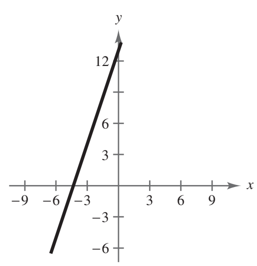


(c) When $x = 10$, $F = \frac{50}{4} = 12.5$ pounds.

29. $y = 3x + 13$

Slope: $m = 3$

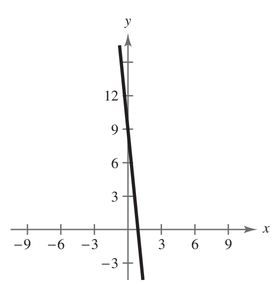
y-intercept: $(0, 13)$



30. $y = -10x + 9$

Slope: $m = -10$

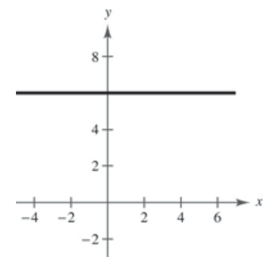
y-intercept: $(0, 9)$



31. $y = 6$

Slope: $m = 0$

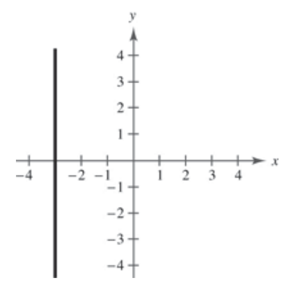
y-intercept: $(0, 6)$



32. $x = -3$

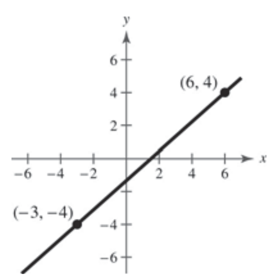
Slope: m is undefined.

y-intercept: none



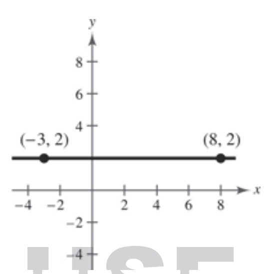
33. $(6, 4), (-3, -4)$

$$m = \frac{4-(-4)}{6-(-3)} = \frac{4+4}{6+3} = \frac{8}{9}$$



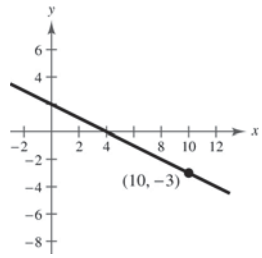
34. $(-3, 2), (8, 2)$

$$m = \frac{2-2}{-3-8} = \frac{0}{-11} = 0$$



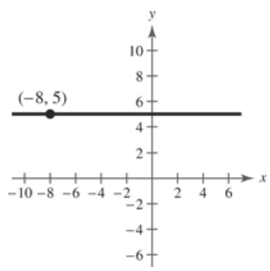
35. $(10, -3)$, $m = -\frac{1}{2}$

$$y - (-3) = -\tfrac{1}{2}(x - 10)$$
$$y + 3 = -\tfrac{1}{2}x + 5$$
$$y = -\tfrac{1}{2}x + 2$$



36. $(-8, 5)$, $m = 0$

$$y - 5 = 0(x - (-8))$$
$$y = 5$$



37. $(-1, 0), (6, 2)$

$$m = \frac{2 - (0)}{6 - (-1)} = \frac{2}{7}$$
$$y - 0 = \frac{2}{7}(x - (-1))$$
$$y = \frac{2}{7}(x + 1)$$
$$y = \frac{2}{7}x + \frac{2}{7}$$

38. $(11, -2), (6, -1)$

$$m = \frac{-1 - (-2)}{6 - 11} = -\frac{1}{5}$$
$$y - (-2) = -\frac{1}{5}(x - 11)$$
$$5y + 10 = -x + 11$$
$$5y = -x + 1$$
$$y = -\frac{1}{5}x + \frac{1}{5}$$

39. Point: $(3, -2)$

$$5x - 4y = 8$$
$$y = \tfrac{5}{4}x - 2$$

(a) Parallel slope: $m = \frac{5}{4}$

$$y - (-2) = \tfrac{5}{4}(x - 3)$$
$$y + 2 = \tfrac{5}{4}x - \tfrac{15}{4}$$
$$y = \tfrac{5}{4}x - \tfrac{23}{4}$$

(b) Perpendicular slope: $m = -\frac{4}{5}$

$$y - (-2) = -\tfrac{4}{5}(x - 3)$$
$$y + 2 = -\tfrac{4}{5}x + \tfrac{12}{5}$$
$$y = -\tfrac{4}{5}x + \tfrac{2}{5}$$

40. Point: $(-8, 3)$, $2x + 3y = 5$

$$3y = 5 - 2x$$
$$y = \tfrac{5}{3} - \tfrac{2}{3}x$$

(a) Parallel slope: $m = -\frac{2}{3}$

$$y - 3 = -\tfrac{2}{3}(x + 8)$$
$$3y - 9 = -2x - 16$$
$$3y = -2x - 7$$
$$y = -\tfrac{2}{3}x - \tfrac{7}{3}$$

(b) Perpendicular slope: $m = \frac{3}{2}$

$$y - 3 = \tfrac{3}{2}(x + 8)$$
$$2y - 6 = 3x + 24$$
$$2y = 3x + 30$$
$$y = \tfrac{3}{2}x + 15$$

41. *Verbal Model*: Sale price = (List price) − (Discount)

Labels: Sale price = S
List price = L
Discount = 20% of $L = 0.2L$

Equation: $S = L - 0.2L$
$S = 0.8L$

42. *Verbal Model*: Hourly wage = (Base wage per hour) + (Piecework rate) $\cdot$ (Number of units)

Labels: Hourly wage = W

Base wage = 12.25

Piecework rate = 0.75

Number of units = x

Equation: $W = 12.25 + 0.75x$

43. $16x - y^4 = 0$

$y^4 = 16x$

$y = \pm 2\sqrt[4]{x}$

No, y is not a function of x. Some x-values correspond to two y-values.

44. $2x - y - 3 = 0$

$2x - 3 = y$

Yes, the equation represents y as a function of x.

45. $y = \sqrt{1 - x}$

Yes, the equation represents y as a function of x. Each x-value, $x \le 1$, corresponds to only one y-value.

46. $|y| = x + 2$ corresponds to $y = x + 2$ or $-y = x + 2$.

No, y is not a function of x. Some x-values correspond to two y-values.

47. $f(x) = x^2 + 1$

(a) $f(2) = (2)^2 + 1 = 5$

(b) $f(-4) = (-4)^2 + 1 = 17$

(c) $f(t^2) = (t^2)^2 + 1 = t^4 + 1$

(d) $f(t + 1) = (t + 1)^2 + 1$

$= t^2 + 2t + 2$

48. $h(x) = \begin{cases} 2x + 1, & x \le -1 \\ x^2 + 2, & x > -1 \end{cases}$

(a) $h(-2) = 2(-2) + 1 = -3$

(b) $h(-1) = 2(-1) + 1 = -1$

(c) $h(0) = 0^2 + 2 = 2$

(d) $h(2) = 2^2 + 2 = 6$

49. $f(x) = \sqrt{25 - x^2}$

Domain: $25 - x^2 \ge 0$

$(5 + x)(5 - x) \ge 0$

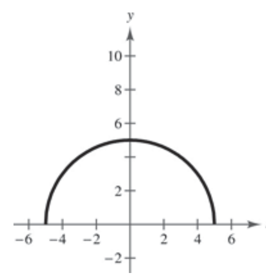
Critical numbers: $x = \pm 5$

Test intervals: $(-\infty, -5), (-5, 5), (5, \infty)$

Test: Is $25 - x^2 \ge 0$?

Solution set: $-5 \le x \le 5$

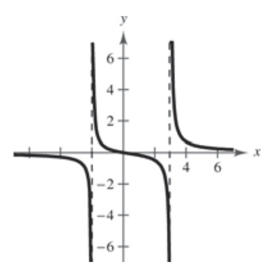
Domain: all real numbers x such that $-5 \le x \le 5$, or $[-5, 5]$



50. $h(x) = \dfrac{x}{x^2 - x - 6}$

$= \dfrac{x}{(x + 2)(x - 3)}$

Domain: All real numbers x except $x = -2, 3$



51. $v(t) = -32t + 48$

$v(1) = 16$ feet per second

52. $0 = -32t + 48$

$t = \frac{48}{32} = 1.5$ seconds

53. $f(x) = 2x^2 + 3x - 1$

$$\frac{f(x+h) - f(x)}{h} = \frac{\left[2(x+h)^2 + 3(x+h) - 1\right] - \left(2x^2 + 3x - 1\right)}{h}$$
$$= \frac{2x^2 + 4xh + 2h^2 + 3x + 3h - 1 - 2x^2 - 3x + 1}{h}$$
$$= \frac{h(4x + 2h + 3)}{h}$$
$$= 4x + 2h + 3, \quad h \neq 0$$

54.
$$f(x) = x^3 - 5x^2 + x$$
$$f(x+h) = (x+h)^3 - 5(x+h)^2 + (x+h)$$
$$= x^3 + 3x^2h + 3xh^2 + h^3 - 5x^2 - 10xh - 5h^2 + x + h$$
$$\frac{f(x+h) - f(x)}{h} = \frac{x^3 + 3x^2h + 3xh^2 + h^3 - 5x^2 - 10xh - 5h^2 + x + h - x^3 + 5x^2 - x}{h}$$
$$= \frac{3x^2h + 3xh^2 + h^3 - 10xh - 5h^2 + h}{h}$$
$$= \frac{h\left(3x^2 + 3xh + h^2 - 10x - 5h + 1\right)}{h}$$
$$= 3x^2 + 3xh + h^2 - 10x - 5h + 1, \quad h \neq 0$$

55. $y = (x - 3)^2$

A vertical line intersects the graph no more than once, so *y is* a function of *x*.

56. $x = -|4 - y|$

A vertical line intersects the graph more than once, so *y is not* a function of *x*.

57. $f(x) = 3x^2 - 16x + 21$

$$3x^2 - 16x + 21 = 0$$
$$(3x - 7)(x - 3) = 0$$
$$3x - 7 = 0 \quad \text{or} \quad x - 3 = 0$$
$$x = \tfrac{7}{3} \quad \text{or} \quad x = 3$$

58. $f(x) = 5x^2 + 4x - 1$

$$5x^2 + 4x - 1 = 0$$
$$(5x - 1)(x + 1) = 0$$
$$5x - 1 = 0 \Rightarrow x = \tfrac{1}{5}$$
$$x + 1 = 0 \Rightarrow x = -1$$

59. $f(x) = \dfrac{8x + 3}{11 - x}$

$$\frac{8x + 3}{11 - x} = 0$$
$$8x + 3 = 0$$
$$x = -\tfrac{3}{8}$$

60. $f(x) = x^3 - x^2$

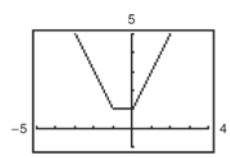
$$x^3 - x^2 = 0$$
$$x^2(x - 1) = 0$$
$$x^2 = 0 \quad \text{or} \quad x - 1 = 0$$
$$x = 0 \qquad\qquad x = 1$$

61. $f(x) = |x| + |x + 1|$

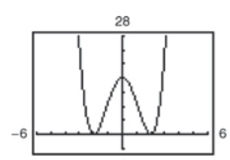
f is increasing on $(0, \infty)$.

f is decreasing on $(-\infty, -1)$.

f is constant on $(-1, 0)$.



62. $f(x) = \left(x^2 - 4\right)^2$

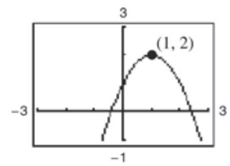


f is increasing on $(-2, 0)$ and $(2, \infty)$.

f is decreasing on $(-\infty, -2)$ and $(0, 2)$.

63. $f(x) = -x^2 + 2x + 1$

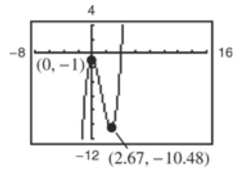
Relative maximum: $(1, 2)$



64. $f(x) = x^3 - 4x^2 - 1$

Relative minimum: $(2.67, -10.48)$

Relative maximum: $(0, -1)$



65. $f(x) = -x^2 + 8x - 4$

$$\frac{f(4) - f(0)}{4 - 0} = \frac{12 - (-4)}{4} = 4$$

The average rate of change of f from $x_1 = 0$ to $x_2 = 4$ is 4.

66. $f(x) = 2 - \sqrt{x + 1}$

$$\frac{f(7) - f(3)}{7 - 3} = \frac{(2 - \sqrt{8}) - (2 - 2)}{4}$$
$$= \frac{2 - 2\sqrt{2}}{4} = \frac{1 - \sqrt{2}}{2}$$

The average rate of change of f from $x_1 = 3$ to $x_2 = 7$ is $(1 - \sqrt{2})/2$.

67. $f(x) = x^4 - 20x^2$

$f(-x) = (-x)^4 - 20(-x)^2 = x^4 - 20x^2 = f(x)$

The function is even.

68. $f(x) = 2x\sqrt{x^2 + 3}$

$$f(-x) = 2(-x)\sqrt{(-x)^2 + 3}$$
$$= -2x\sqrt{x^2 + 3}$$
$$= -f(x)$$

The function is odd.

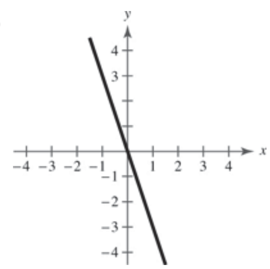
69. (a) $f(2) = -6,\ f(-1) = 3$

Points: $(2, -6), (-1, 3)$

$$m = \frac{3 - (-6)}{-1 - 2} = \frac{9}{-3} = -3$$

$$y - (-6) = -3(x - 2)$$
$$y + 6 = -3x + 6$$
$$y = -3x$$
$$f(x) = -3x$$

(b)



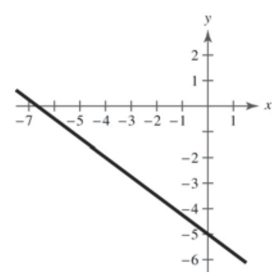
70. (a) $f(0) = -5,\ f(4) = -8$

$(0, -5), (4, -8)$

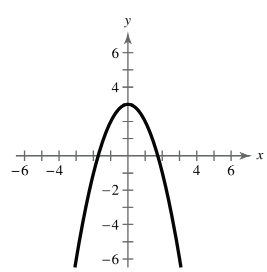
$$m = \frac{-8 - (-5)}{4 - 0} = -\frac{3}{4}$$

$$y - (-5) = -\frac{3}{4}(x - 0)$$
$$y = -\frac{3}{4}x - 5$$
$$f(x) = -\frac{3}{4}x - 5$$

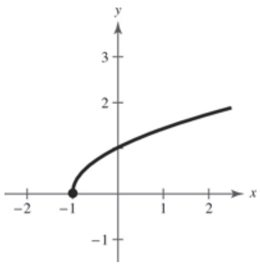
(b)



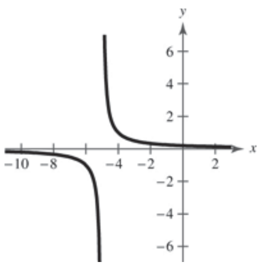
71. $f(x) = 3 - x^2$



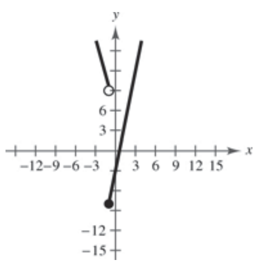
72. $f(x) = \sqrt{x + 1}$



73. $g(x) = \dfrac{1}{x + 5}$



74. $f(x) = \begin{cases} 5x - 3, & x \geq -1 \\ -4x + 5, & x < -1 \end{cases}$

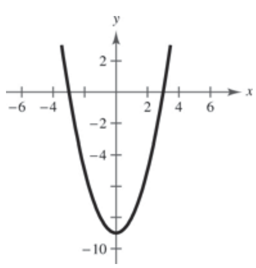


75. (a) $f(x) = x^2$

(b) $h(x) = x^2 - 9$

Vertical shift 9 units downward

(c)



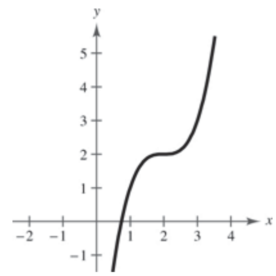
(d) $h(x) = f(x) - 9$

76. (a) $f(x) = x^3$

(b) $h(x) = (x - 2)^3 + 2$

Horizontal shift 2 units to the right; vertical shift 2 units upward

(c)



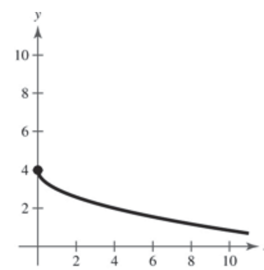
(d) $h(x) = f(x - 2) + 2$

77. (a) $f(x) = \sqrt{x}$

(b) $h(x) = -\sqrt{x} + 4$

Vertical shift 4 units upward, reflection in the *x*-axis

(c)



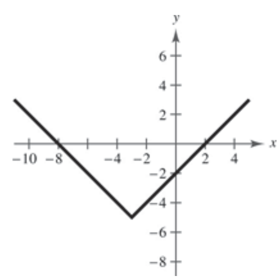
(d) $h(x) = -f(x) + 4$

78. (a) $f(x) = |x|$

(b) $h(x) = |x + 3| - 5$

Horizontal shift 3 units to the left; vertical shift 5 units downward

(c)



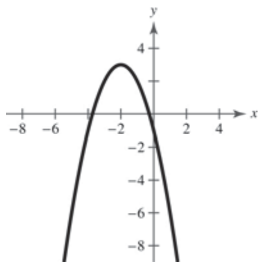
(d) $h(x) = f(x + 3) - 5$

79. (a) $f(x) = x^2$

(b) $h(x) = -(x + 2)^2 + 3$

Horizontal shift two units to the left, vertical shift 3 units upward, reflection in the x-axis.

(c)



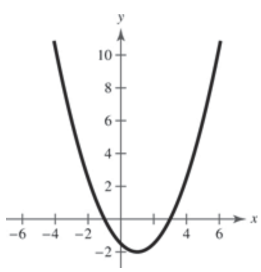
(d) $h(x) = -f(x + 2) + 3$

80. (a) $f(x) = x^2$

(b) $h(x) = \frac{1}{2}(x - 1)^2 - 2$

Horizontal shift one unit to the right, vertical shrink, vertical shift 2 units downward

(c)



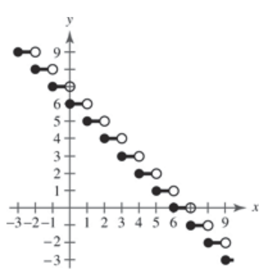
(d) $h(x) = \frac{1}{2}f(x - 1) - 2$

81. (a) $f(x) = [\![x]\!]$

(b) $h(x) = -[\![x]\!] + 6$

Reflection in the x-axis and a vertical shift 6 units upward

(c)



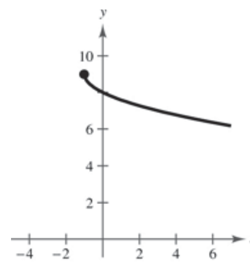
(d) $h(x) = -f(x) + 6$

82. (a) $f(x) = \sqrt{x}$

(b) $h(x) = -\sqrt{x + 1} + 9$

Reflection in the x-axis, a horizontal shift 1 unit to the left, and a vertical shift 9 units upward

(c)



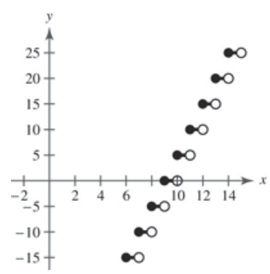
(d) $h(x) = -f(x + 1) + 9$

83. (a) $f(x) = [\![x]\!]$

(b) $h(x) = 5[\![x - 9]\!]$

Horizontal shift 9 units to the right and a vertical stretch (each y-value is multiplied by 5)

(c)



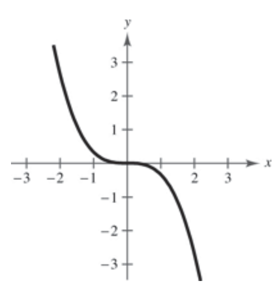
(d) $h(x) = 5f(x - 9)$

84. (a) $f(x) = x^3$

(b) $h(x) = -\frac{1}{3}x^3$

Reflection in the x-axis; vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{3}\right)$

(c)



(d) $h(x) = -\frac{1}{3}f(x)$

85. $f(x) = x^2 + 3, g(x) = 2x - 1$

(a) $(f + g)(x) = (x^2 + 3) + (2x - 1) = x^2 + 2x + 2$

(b) $(f - g)(x) = (x^2 + 3) - (2x - 1) = x^2 - 2x + 4$

(c) $(fg)(x) = (x^2 + 3)(2x - 1) = 2x^3 - x^2 + 6x - 3$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{x^2 + 3}{2x - 1}$, Domain: $x \neq \dfrac{1}{2}$

86. $f(x) = x^2 - 4, g(x) = \sqrt{3 - x}$

(a) $(f + g)(x) = f(x) + g(x) = x^2 - 4 + \sqrt{3 - x}$

(b) $(f - g)(x) = f(x) - g(x) = x^2 - 4 - \sqrt{3 - x}$

(c) $(fg)(x) = f(x)g(x) = (x^2 - 4)(\sqrt{3 - x})$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x^2 - 4}{\sqrt{3 - x}}$, Domain: $x < 3$

87. $f(x) = \frac{1}{3}x - 3, g(x) = 3x + 1$

The domains of f and g are all real numbers.

(a)
$$\begin{aligned}(f \circ g)(x) &= f(g(x))\\ &= f(3x + 1)\\ &= \tfrac{1}{3}(3x + 1) - 3\\ &= x + \tfrac{1}{3} - 3\\ &= x - \tfrac{8}{3}\end{aligned}$$

Domain: all real numbers

(b)
$$\begin{aligned}(g \circ f)(x) &= g(f(x))\\ &= g\left(\tfrac{1}{3}x - 3\right)\\ &= 3\left(\tfrac{1}{3}x - 3\right) + 1\\ &= x - 9 + 1\\ &= x - 8\end{aligned}$$

Domain: all real numbers

88. $f(x) = x^3 - 4, g(x) = \sqrt[3]{x + 7}$

The domains of $f(x)$ and $g(x)$ are all real numbers.

(a)
$$\begin{aligned}(f \circ g)(x) &= f(g(x))\\ &= \left(\sqrt[3]{x + 7}\right)^3 - 4\\ &= x + 7 - 4\\ &= x + 3\end{aligned}$$

Domain: all real numbers

(b)
$$\begin{aligned}(g \circ f)(x) &= g(f(x))\\ &= \sqrt[3]{(x^3 - 4) + 7}\\ &= \sqrt[3]{x^3 + 3}\end{aligned}$$

Domain: all real numbers

89.
$$\begin{aligned}N(T(t)) &= 25(2t + 1)^2 - 50(2t + 1) + 300,\ 2 \le t \le 20\\ &= 25(4t^2 + 4t + 1) - 100t - 50 + 300\\ &= 100t^2 + 100t + 25 - 100t + 250\\ &= 100t^2 + 275\end{aligned}$$

The composition $N(T(t))$ represents the number of bacteria in the food as a function of time.

90. When $N = 750$,

$$\begin{aligned}750 &= 100t^2 + 275\\ 100t^2 &= 475\\ t^2 &= 4.75\\ t &= 2.18 \text{ hours.}\end{aligned}$$

After about 2.18 hours, the bacterial count will reach 750.

91. $f(x) = 3x + 8$

$$\begin{aligned} y &= 3x + 8 \\ x &= 3y + 8 \\ x - 8 &= 3y \\ y &= \frac{x - 8}{3} \\ y &= \frac{1}{3}(x - 8) \end{aligned}$$

So, $f^{-1}(x) = \frac{1}{3}(x - 8)$

$$f\left(f^{-1}(x)\right) = f\left(\frac{1}{3}(x - 8)\right) = 3\left(\frac{1}{3}(x - 8)\right) + 8 = x - 8 + 8 = x$$

$$f^{-1}\left(f(x)\right) = f^{-1}(3x + 8) = \frac{1}{3}(3x + 8 - 8) = \frac{1}{3}(3x) = x$$

92. $f(x) = \dfrac{x - 4}{5}$

$$\begin{aligned} y &= \frac{x - 4}{5} \\ x &= \frac{y - 4}{5} \\ 5x &= y - 4 \\ y &= 5x + 4 \end{aligned}$$

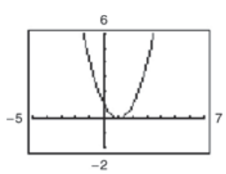
So, $f^{-1}(x) = 5x + 4$

$$f\left(f^{-1}(x)\right) = f(5x + 4) = \frac{5x + 4 - 4}{5} = \frac{5x}{5} = x$$

$$f^{-1}\left(f(x)\right) = f^{-1}\left(\frac{x - 4}{5}\right) = 5\left(\frac{x - 4}{5}\right) + 4 = x - 4 + 4 = x$$

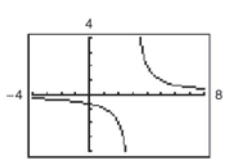
93. $f(x) = (x - 1)^2$

No, the function does not have an inverse because some horizontal lines intersect the graph twice.



94. $h(t) = \dfrac{2}{t - 3}$

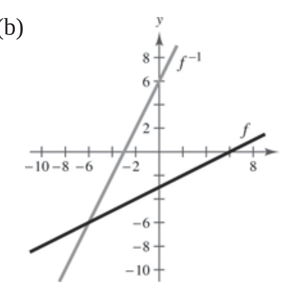
Yes, the function has an inverse because no horizontal lines intersect the graph at more than one point. The function has an inverse.



95. (a)

$$\begin{aligned} f(x) &= \tfrac{1}{2}x - 3 \\ y &= \tfrac{1}{2}x - 3 \\ x &= \tfrac{1}{2}y - 3 \\ x + 3 &= \tfrac{1}{2}y \\ 2(x + 3) &= y \\ f^{-1}(x) &= 2x + 6 \end{aligned}$$

(b)



(c) The graph of f^{-1} is the reflection of the graph of f in the line $y = x$.

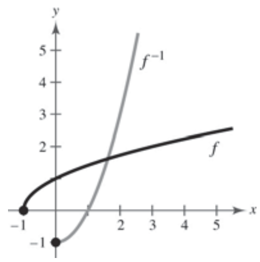
(d) The domains and ranges of f and f^{-1} are the set of all real numbers.

96. (a)

$$\begin{aligned} f(x) &= \sqrt{x+1} \\ y &= \sqrt{x+1} \\ x &= \sqrt{y+1} \\ x^2 &= y+1 \\ x^2 - 1 &= y \\ f^{-1}(x) &= x^2 - 1, \quad x \ge 0 \end{aligned}$$

Note: The inverse must have a restricted domain.

(b)



(c) The graph of f^{-1} is the reflection of the graph of f in the line $y = x$.

(d) The domain of f and the range of f^{-1} is $[-1, \infty)$.

The range of f and the domain of f^{-1} is $[0, \infty)$.

97. $f(x) = 2(x-4)^2$ is increasing on $(4, \infty)$.

Let $f(x) = 2(x-4)^2, x > 4$ and $y > 0$.

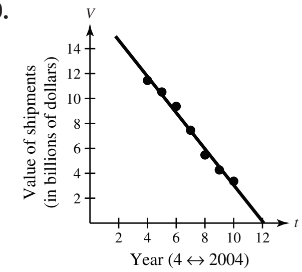
$$\begin{aligned} y &= 2(x-4)^2 \\ x &= 2(y-4)^2, x > 0, y > 4 \\ \frac{x}{2} &= (y-4)^2 \\ \sqrt{\frac{x}{2}} &= y - 4 \\ \sqrt{\frac{x}{2}} + 4 &= y \\ f^{-1}(x) &= \sqrt{\frac{x}{2}} + 4, x > 0 \end{aligned}$$

98. $f(x) = |x-2|$ is increasing on $(2, \infty)$.

Let $f(x) = x - 2, x > 2, y > 0$.

$$\begin{aligned} y &= x - 2 \\ x &= y - 2, x > 0, y > 2 \\ x + 2 &= y, x > 0, y > 2 \\ f^{-1}(x) &= x + 2, x > 0 \end{aligned}$$

99.



The model fits the data well.

100.

$$\begin{aligned} T &= \frac{k}{r} \\ 3 &= \frac{k}{65} \\ k &= 3(65) = 195 \\ T &= \frac{195}{r} \end{aligned}$$

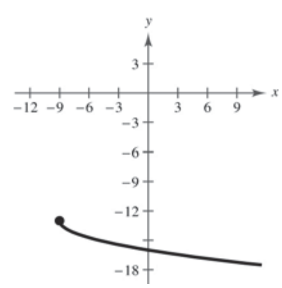
When $r = 80$ mph,

$$\begin{aligned} T &= \frac{195}{80} = 2.4375 \text{ hours} \\ &\approx 2 \text{ hours, 26 minutes.} \end{aligned}$$

101.

$$\begin{aligned} C &= khw^2 \\ 28.80 &= k(16)(6)^2 \\ k &= 0.05 \\ C &= (0.05)(14)(8)^2 = \$44.80 \end{aligned}$$

102. False. The graph is reflected in the x-axis, shifted 9 units to the left, then shifted 13 units downward.

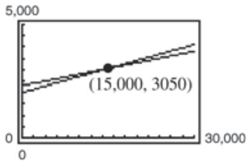


103. True. If $f(x) = x^3$ and $g(x) = \sqrt[3]{x}$, then the domain of g is all real numbers, which is equal to the range of f and vice versa.

Problem Solving for Chapter 1

1. (a) $W_1 = 0.07S + 2000$

(b) $W_2 = 0.05S + 2300$

(c) 

Point of intersection: $(15{,}000, 3050)$

Both jobs pay the same, \$3050, if you sell \$15,000 per month.

(d) No. If you think you can sell \$20,000 per month, keep your current job with the higher commission rate. For sales over \$15,000 it pays more than the other job.

2. Mapping numbers onto letters is *not* a function. Each number between 2 and 9 is mapped to more than one letter.

$$\{(2, A), (2, B), (2, C), (3, D), (3, E), (3, F), (4, G), (4, H), (4, I), (5, J), (5, K), (5, L),$$
$$(6, M), (6, N), (6, O), (7, P), (7, Q), (7, R), (7, S), (8, T), (8, U), (8, V), (9, W), (9, X), (9, Y), (9, Z)\}$$

Mapping letters onto numbers *is* a function. Each letter is only mapped to one number.

$$\{(A, 2), (B, 2), (C, 2), (D, 3), (E, 3), (F, 3), (G, 4), (H, 4), (I, 4), (J, 5), (K, 5), (L, 5),$$
$$(M, 6), (N, 6), (O, 6), (P, 7), (Q, 7), (R, 7), (S, 7), (T, 8), (U, 8), (V, 8), (W, 9), (X, 9), (Y, 9), (Z, 9)\}$$

3. (a) Let $f(x)$ and $g(x)$ be two even functions.

Then define $h(x) = f(x) \pm g(x)$.

$$\begin{aligned} h(-x) &= f(-x) \pm g(-x) \\ &= f(x) \pm g(x) \text{ because } f \text{ and } g \text{ are even} \\ &= h(x) \end{aligned}$$

So, $h(x)$ is also even.

(b) Let $f(x)$ and $g(x)$ be two odd functions.

Then define $h(x) = f(x) \pm g(x)$.

$$\begin{aligned} h(-x) &= f(-x) \pm g(-x) \\ &= -f(x) \pm g(x) \text{ because } f \text{ and } g \text{ are odd} \\ &= -h(x) \end{aligned}$$

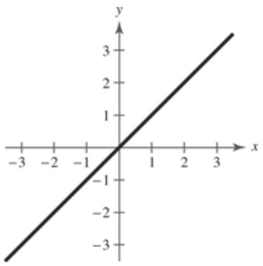
So, $h(x)$ is also odd. $\left(\text{If } f(x) \neq g(x)\right)$

(c) Let $f(x)$ be odd and $g(x)$ be even. Then define $h(x) = f(x) \pm g(x)$.

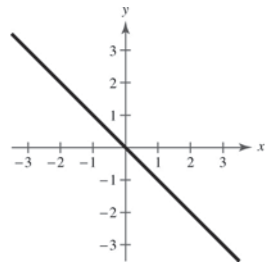
$$\begin{aligned} h(-x) &= f(-x) \pm g(-x) \\ &= -f(x) \pm g(x) \text{ because } f \text{ is odd and } g \text{ is even} \\ &\neq h(x) \\ &\neq -h(x) \end{aligned}$$

So, $h(x)$ is neither odd nor even.

4. $f(x) = x$



$g(x) = -x$



$(f \circ f)(x) = x$ and $(g \circ g)(x) = x$

These are the only two linear functions that are their own inverse functions since m has to equal $1/m$ for this to be true.

General formula: $y = -x + c$

5. $f(x) = a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0$

$f(-x) = a_{2n}(-x)^{2n} + a_{2n-2}(-x)^{2n-2} + \cdots + a_2(-x)^2 + a_0 = a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0 = f(x)$

So, $f(x)$ is even.

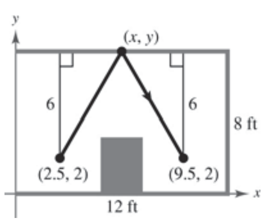
6. It appears, from the drawing, that the triangles are equal; thus $(x, y) = (6, 8)$.

The line between $(2.5, 2)$ and $(6, 8)$ is $y = \frac{12}{7}x - \frac{16}{7}$.

The line between $(9.5, 2)$ and $(6, 8)$ is $y = -\frac{12}{7}x + \frac{128}{7}$.

The path of the ball is:

$$f(x) = \begin{cases} \frac{12}{7}x - \frac{16}{7}, & 2.5 \le x \le 6 \\ -\frac{12}{7}x + \frac{128}{7}, & 6 < x \le 9.5 \end{cases}$$



7. (a) April 11: 10 hours

April 12: 24 hours

April 13: 24 hours

April 14: $23\frac{2}{3}$ hours

Total: $81\frac{2}{3}$ hours

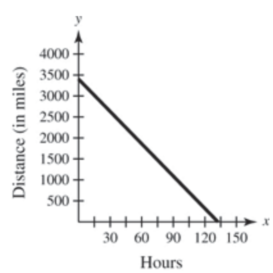
(b) $\text{Speed} = \dfrac{\text{distance}}{\text{time}} = \dfrac{2100}{81\frac{2}{3}} = \dfrac{180}{7} = 25\frac{5}{7}$ mph

(c) $D = -\dfrac{180}{7}t + 3400$

Domain: $0 \le t \le \dfrac{1190}{9}$

Range: $0 \le D \le 3400$

(d)



8. (a) $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{f(2) - f(1)}{2 - 1} = \dfrac{1 - 0}{1} = 1$

(b) $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{f(1.5) - f(1)}{1.5 - 1} = \dfrac{0.75 - 0}{0.5} = 1.5$

(c) $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{f(1.25) - f(1)}{1.25 - 1} = \dfrac{0.4375 - 0}{0.25} = 1.75$

(d) $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{f(1.125) - f(1)}{1.125 - 1} = \dfrac{0.234375 - 0}{0.125} = 1.875$

(e) $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{f(1.0625) - f(1)}{1.0625 - 1} = \dfrac{0.12109375 - 0}{0.625} = 1.9375$

(f) Yes, the average rate of change appears to be approaching 2.

(g) a. $(1, 0), (2, 1), m = 1, y = x - 1$

b. $(1, 0), (1.5, 0.75), m = \dfrac{0.75}{0.5} = 1.5, y = 1.5x - 1.5$

c. $(1, 0), (1.25, 0.4375), m = \dfrac{0.4375}{0.25} = 1.75, y = 1.75x - 1.75$

d. $(1, 0), (1.125, 0.234375), m = \dfrac{0.234375}{0.125} = 1.875, y = 1.875x - 1.875$

e. $(1, 0), (1.0625, 0.12109375), m = \dfrac{0.12109375}{0.0625} = 1.9375, y = 1.9375x - 1.9375$

(h) $(1, f(1)) = (1, 0), m \to 2, y = 2(x - 1), y = 2x - 2$

9. (a)–(d) Use $f(x) = 4x$ and $g(x) = x + 6$.

(a) $(f \circ g)(x) = f(x + 6) = 4(x + 6) = 4x + 24$

(b) $(f \circ g)^{-1}(x) = \dfrac{x - 24}{4} = \dfrac{1}{4}x - 6$

(c) $f^{-1}(x) = \dfrac{1}{4}x$

$g^{-1}(x) = x - 6$

(d) $(g^{-1} \circ f^{-1})(x) = g^{-1}\left(\dfrac{1}{4}x\right) = \dfrac{1}{4}x - 6$

(e) $f(x) = x^3 + 1$ and $g(x) = 2x$

$(f \circ g)(x) = f(2x) = (2x)^3 + 1 = 8x^3 + 1$

$(f \circ g)^{-1}(x) = \sqrt[3]{\dfrac{x - 1}{8}} = \dfrac{1}{2}\sqrt[3]{x - 1}$

$f^{-1}(x) = \sqrt[3]{x - 1}$

$g^{-1}(x) = \dfrac{1}{2}x$

$(g^{-1} \circ f^{-1})(x) = g^{-1}\left(\sqrt[3]{x - 1}\right) = \dfrac{1}{2}\sqrt[3]{x - 1}$

(f) Answers will vary.

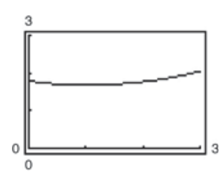
(g) Conjecture: $(f \circ g)^{-1}(x) = (g^{-1} \circ f^{-1})(x)$

10. (a) The length of the trip in the water is $\sqrt{2^2 + x^2}$, and the length of the trip over land is $\sqrt{1 + (3 - x)^2}$. The total time is

$$T(x) = \frac{\sqrt{4 + x^2}}{2} + \frac{\sqrt{1 + (3 - x)^2}}{4}$$
$$= \frac{1}{2}\sqrt{4 + x^2} + \frac{1}{4}\sqrt{x^2 - 6x + 10}.$$

(b) Domain of $T(x)$: $0 \le x \le 3$

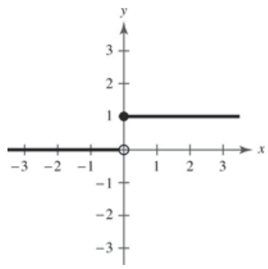
(c)



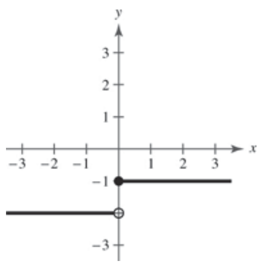
(d) $T(x)$ is a minimum when $x = 1$.

(e) Answers will vary. *Sample answer:* To reach point Q in the shortest amount of time, you should row to a point one mile down the coast, and then walk the rest of the way.

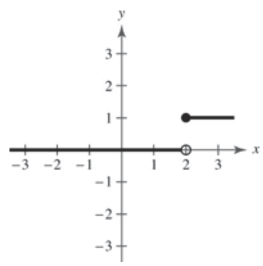
11. $H(x) = \begin{cases} 1, & x \ge 0 \\ 0, & x < 0 \end{cases}$



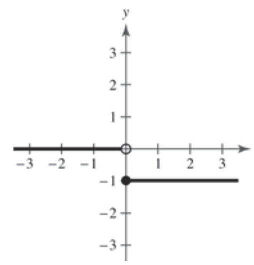
(a) $H(x) - 2$



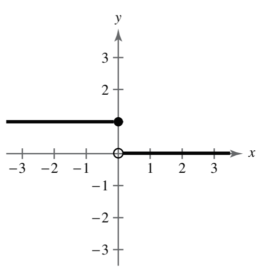
(b) $H(x - 2)$



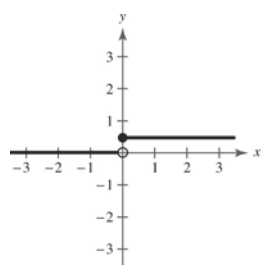
(c) $-H(x)$



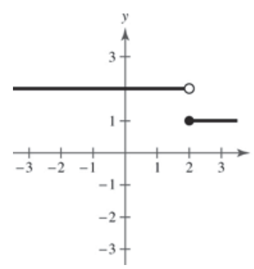
(d) $H(-x)$



(e) $\frac{1}{2}H(x)$



(f) $-H(x - 2) + 2$



12. $f(x) = y = \dfrac{1}{1 - x}$

(a) Domain: all real numbers x except $x = 1$

Range: all real numbers y except $y = 0$

(b) $f(f(x)) = f\left(\dfrac{1}{1 - x}\right)$

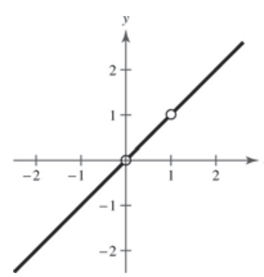
$$= \frac{1}{1 - \left(\dfrac{1}{1 - x}\right)} = \frac{1}{\dfrac{1 - x - 1}{1 - x}}$$

$$= \frac{1 - x}{-x} = \frac{x - 1}{x}$$

Domain: all real numbers x except $x = 0$ and $x = 1$

(c) $f(f(f(x))) = f\left(\dfrac{x - 1}{x}\right) = \dfrac{1}{1 - \left(\dfrac{x - 1}{x}\right)} = \dfrac{1}{\dfrac{1}{x}} = x$

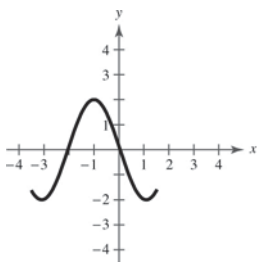
The graph is not a line. It has holes at $(0, 0)$ and $(1, 1)$.



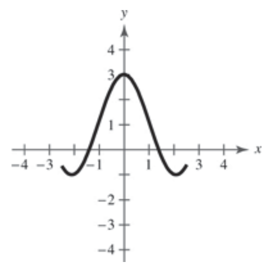
13. $(f \circ (g \circ h))(x) = f((g \circ h)(x)) = f(g(h(x))) = (f \circ g \circ h)(x)$

$((f \circ g) \circ h)(x) = (f \circ g)(h(x)) = f(g(h(x))) = (f \circ g \circ h)(x)$

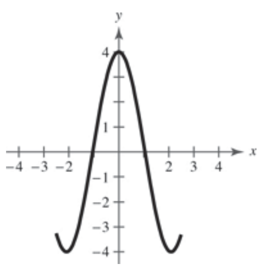
14. (a) $f(x + 1)$



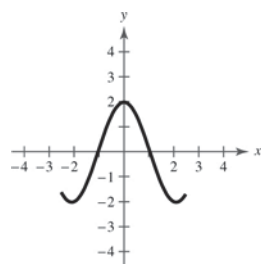
(b) $f(x) + 1$



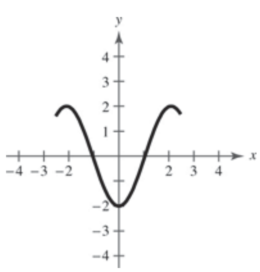
(c) $2f(x)$



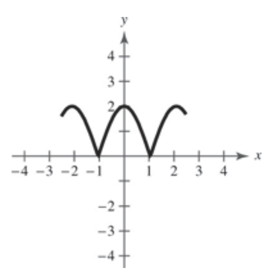
(d) $f(-x)$



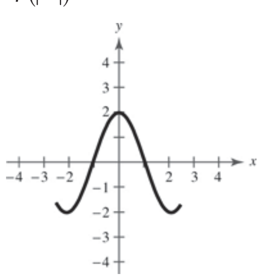
(e) $-f(x)$



(f) $|f(x)|$



(g) $f(|x|)$



15.

x	$f(x)$	$f^{-1}(x)$
-4	—	2
-3	4	1
-2	1	0
-1	0	—
0	-2	-1
1	-3	-2
2	-4	—
3	—	—
4	—	-3

(a)

x	$f(f^{-1}(x))$
-4	$f(f^{-1}(-4)) = f(2) = -4$
-2	$f(f^{-1}(-2)) = f(0) = -2$
0	$f(f^{-1}(0)) = f(-1) = 0$
4	$f(f^{-1}(4)) = f(-3) = 4$

(b)

x	$(f + f^{-1})(x)$
-3	$f(-3) + f^{-1}(-3) = 4 + 1 = 5$
-2	$f(-2) + f^{-1}(-2) = 1 + 0 = 1$
0	$f(0) + f^{-1}(0) = -2 + (-1) = -3$
1	$f(1) + f^{-1}(1) = -3 + (-2) = -5$

(c)

x	$(f \cdot f^{-1})(x)$
-3	$f(-3)f^{-1}(-3) = (4)(1) = 4$
-2	$f(-2)f^{-1}(-2) = (1)(0) = 0$
0	$f(0)f^{-1}(0) = (-2)(-1) = 2$
1	$f(1)f^{-1}(1) = (-3)(-2) = 6$

(d)

x	$\lvert f^{-1}(x)\rvert$
-4	$\lvert f^{-1}(-4)\rvert = \lvert 2\rvert = 2$
-3	$\lvert f^{-1}(-3)\rvert = \lvert 1\rvert = 1$
0	$\lvert f^{-1}(0)\rvert = \lvert -1\rvert = 1$
4	$\lvert f^{-1}(4)\rvert = \lvert -3\rvert = 3$

Practice Test for Chapter 1

1. Given the points $(-3, 4)$ and $(5, -6)$, find (a) the midpoint of the line segment joining the points, and (b) the distance between the points.

2. Graph $y = \sqrt{7 - x}$.

3. Write the standard equation of the circle with center $(-3, 5)$ and radius 6.

4. Find the equation of the line through $(2, 4)$ and $(3, -1)$.

5. Find the equation of the line with slope $m = 4/3$ and y-intercept $b = -3$.

6. Find the equation of the line through $(4, 1)$ perpendicular to the line $2x + 3y = 0$.

7. If it costs a company \$32 to produce 5 units of a product and \$44 to produce 9 units, how much does it cost to produce 20 units? (Assume that the cost function is linear.)

8. Given $f(x) = x^2 - 2x + 1$, find $f(x - 3)$.

9. Given $f(x) = 4x - 11$, find $\dfrac{f(x) - f(3)}{x - 3}$

10. Find the domain and range of $f(x) = \sqrt{36 - x^2}$.

11. Which equations determine y as a function of x?

(a) $6x - 5y + 4 = 0$

(b) $x^2 + y^2 = 9$

(c) $y^3 = x^2 + 6$

12. Sketch the graph of $f(x) = x^2 - 5$.

13. Sketch the graph of $f(x) = |x + 3|$.

14. Sketch the graph of $f(x) = \begin{cases} 2x + 1, & \text{if } x \geq 0, \\ x^2 - x, & \text{if } x < 0. \end{cases}$

15. Use the graph of $f(x) = |x|$ to graph the following:

(a) $f(x + 2)$

(b) $-f(x) + 2$

16. Given $f(x) = 3x + 7$ and $g(x) = 2x^2 - 5$, find the following:

(a) $(g - f)(x)$

(b) $(fg)(x)$

17. Given $f(x) = x^2 - 2x + 16$ and $g(x) = 2x + 3$, find $f(g(x))$.

18. Given $f(x) = x^3 + 7$, find $f^{-1}(x)$.

19. Which of the following functions have inverses?

(a) $f(x) = |x - 6|$

(b) $f(x) = ax + b, a \neq 0$

(c) $f(x) = x^3 - 19$

20. Given $f(x) = \sqrt{\dfrac{3 - x}{x}}, 0 < x \leq 3,$ find $f^{-1}(x)$.

Exercises 21–23, true or false?

21. $y = 3x + 7$ and $y = \frac{1}{3}x - 4$ are perpendicular.

22. $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$

23. If a function has an inverse, then it must pass both the Vertical Line Test and the Horizontal Line Test.

24. If z varies directly as the cube of x and inversely as the square root of y, and $z = -1$ when $x = -1$ and $y = 25$, find z in terms of x and y.

25. Use your calculator to find the least square regression line for the data.

x	–2	–1	0	1	2	3
y	1	2.4	3	3.1	4	4.7

CHAPTER 2
Polynomial and Rational Functions

CHAPTER 2
Polynomial and Rational Functions

Section 2.1 Quadratic Functions and Models

1. polynomial

2. nonnegative integer; real

3. quadratic; parabola

4. axis

5. positive; minimum

6. negative; maximum

7. $f(x) = (x - 2)^2$ opens upward and has vertex $(2, 0)$. Matches graph (e).

8. $f(x) = (x + 4)^2$ opens upward and has vertex $(-4, 0)$. Matches graph (c).

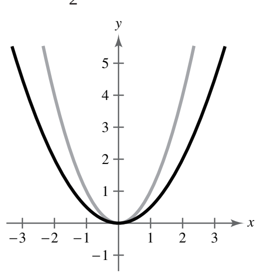
9. $f(x) = x^2 - 2$ opens upward and has vertex $(0, -2)$. Matches graph (b).

10. $f(x) = (x + 1)^2 - 2$ opens upward and has vertex $(-1, -2)$. Matches graph (a).

11. $f(x) = 4 - (x - 2)^2 = -(x - 2)^2 + 4$ opens downward and has vertex $(2, 4)$. Matches graph (f).

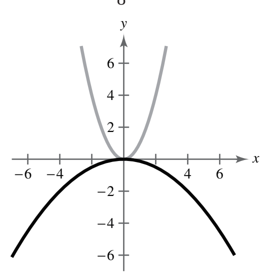
12. $f(x) = -(x - 4)^2$ opens downward and has vertex $(4, 0)$. Matches graph (d).

13. (a) $y = \frac{1}{2}x^2$



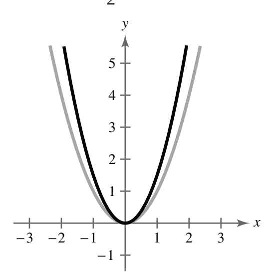
Vertical shrink

(b) $y = -\frac{1}{8}x^2$



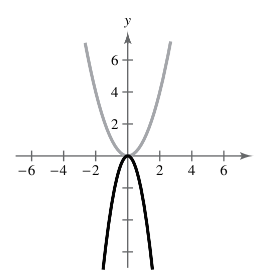
Vertical shrink and reflection in the x-axis

(c) $y = \frac{3}{2}x^2$



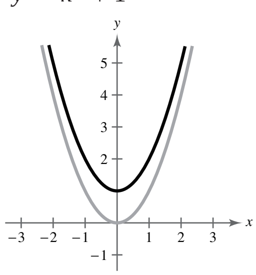
Vertical stretch

(d) $y = -3x^2$



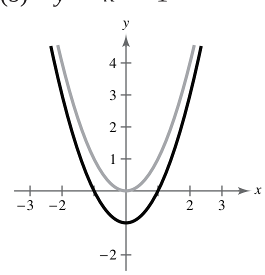
Vertical stretch and reflection in the x-axis

14. (a) $y = x^2 + 1$



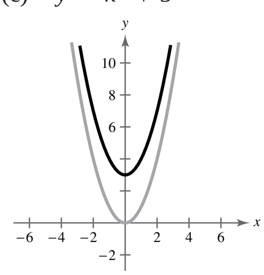
Vertical shift one unit upward

(b) $y = x^2 - 1$



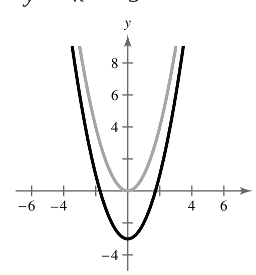
Vertical shift one unit downward

(c) $y = x^2 + 3$



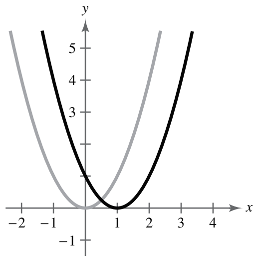
Vertical shift three units upward

(d) $y = x^2 - 3$



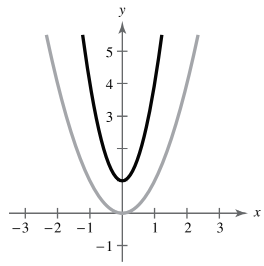
Vertical shift three units downward

15. (a) $y = (x - 1)^2$



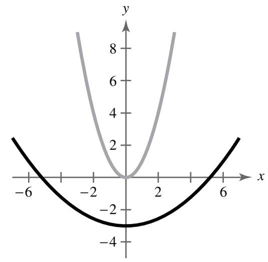
Horizontal shift one unit to the right

(b) $y = (3x)^2 + 1$



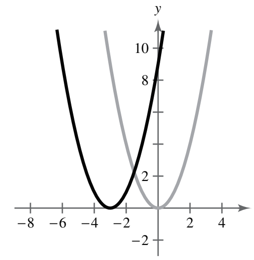
Horizontal shrink and a vertical shift one unit upward

(c) $y = \left(\frac{1}{3}x\right)^2 - 3$



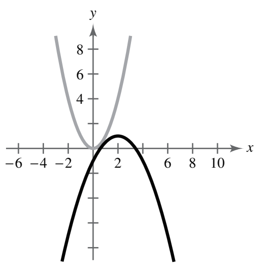
Horizontal stretch and a vertical shift three units downward

(d) $y = (x + 3)^2$



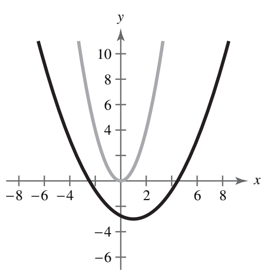
Horizontal shift three units to the left

16. (a) $y = -\frac{1}{2}(x - 2)^2 + 1$



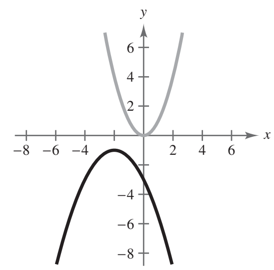
Horizontal shift two units to the right, vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{2}\right)$, reflection in the x-axis, and vertical shift one unit upward

(b) $y = \left[\frac{1}{2}(x - 1)\right]^2 - 3$



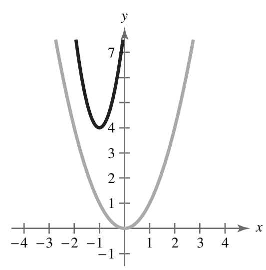
Horizontal shift one unit to the right, horizontal stretch $\left(\text{each } x\text{-value is multiplied by } 2\right)$, and vertical shift three units downward

(c) $y = -\frac{1}{2}(x + 2)^2 - 1$



Horizontal shift two units to the left, vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{2}\right)$, reflection in x-axis, and vertical shift one unit downward

(d) $y = \left[2(x + 1)\right]^2 + 4$



Horizontal shift one unit to the left, horizontal shrink $\left(\text{each } x\text{-value is multiplied by } \frac{1}{2}\right)$, and vertical shift four units upward

17. $f(x) = x^2 - 6x$
$= (x^2 - 6x + 9) - 9$
$= (x - 3)^2 - 9$

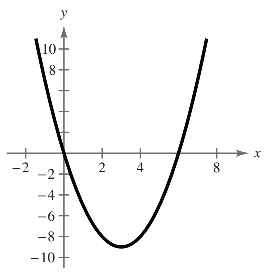
Vertex: $(3, -9)$

Axis of symmetry: $x = 3$

Find x-intercepts:

$x^2 - 6x = 0$
$x(x - 6) = 0$
$x = 0$
$x - 6 = 0 \Rightarrow x = 6$

x-intercepts: $(0, 0), (6, 0)$



18. $g(x) = x^2 - 8x$
$= (x^2 - 8x + 16) - 16$
$= (x - 4)^2 - 16$

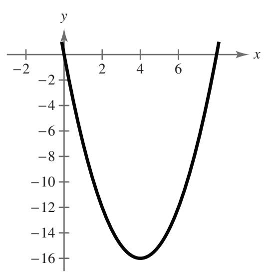
Vertex: $(4, -16)$

Axis of symmetry: $x = 4$

Find x-intercepts:

$x^2 - 8x = 0$
$x(x - 8) = 0$
$x = 0$
$x - 8 = 0 \Rightarrow x = 8$

x-intercepts: $(0, 0), (8, 0)$



19. $h(x) = x^2 - 8x + 16 = (x - 4)^2$

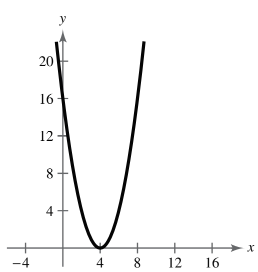
Vertex: $(4, 0)$

Axis of symmetry: $x = 4$

Find x-intercepts:

$(x - 4)^2 = 0$
$x - 4 = 0$
$x = 4$

x-intercept: $(4, 0)$



20. $g(x) = x^2 + 2x + 1 = (x + 1)^2$

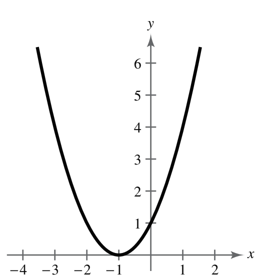
Vertex: $(-1, 0)$

Axis of symmetry: $x = -1$

Find x-intercepts:

$(x + 1)^2 = 0$
$x + 1 = 0$
$x = -1$

x-intercept: $(-1, 0)$



21. $f(x) = x^2 + 8x + 13$
$= (x^2 + 8x + 16) - 16 + 13$
$= (x + 4)^2 - 3$

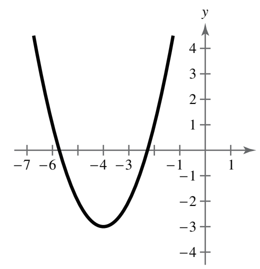
Vertex: $(-4, -3)$

Axis of symmetry: $x = -4$

Find x-intercepts:

$x^2 + 8x + 13 = 0$
$x^2 + 8x = -13$
$x^2 + 8x + 16 = 16 - 13$
$(x + 4)^2 = 3$
$x + 4 = \pm\sqrt{3}$
$x = -4 \pm\sqrt{3}$

x-intercepts: $(-4 \pm\sqrt{3}, 0)$



22. $f(x) = x^2 - 12x + 44$
$= (x^2 - 12x + 36) - 36 + 44$
$= (x - 6)^2 + 12$

Vertex: $(6, 12)$

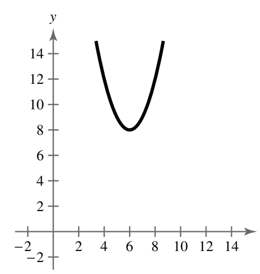
Axis of symmetry: $x = 6$

Find x-intercepts:

$x^2 - 12x + 44 = 0$
$x^2 - 12x = -44$
$x^2 - 12x + 36 = -44 + 36$
$(x - 6)^2 = -8$
$x - 6 = \pm\sqrt{-8}$
$x = 6 \pm 2\sqrt{2}i$

Not a real number

No x-intercepts



23. $f(x) = x^2 - 14x + 54$
$= (x^2 - 14x + 49) - 49 + 54$
$= (x - 7)^2 + 5$

Vertex: $(7, 5)$

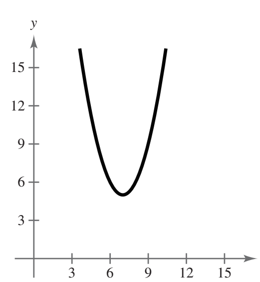
Axis of symmetry: $x = 7$

Find x-intercepts:

$x^2 - 14x + 54 = 0$
$x^2 - 14x = -54$
$x^2 - 14x + 49 = -54 + 49$
$(x - 7)^2 = -5$
$x - 7 = \pm\sqrt{-5}$
$x = 7 \pm\sqrt{5}i$

Not a real number

No x-intercepts



24. $h(x) = x^2 + 16x - 17$

$= (x^2 + 16x + 64) - 64 - 17$

$= (x + 8)^2 - 81$

Vertex: $(-8, -81)$

Axis of symmetry: $x = -8$

Find x-intercepts:

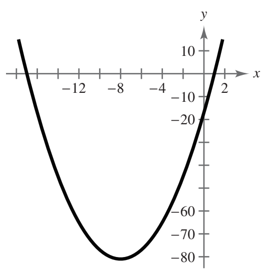
$x^2 + 16x - 17 = 0$

$(x + 17)(x - 1) = 0$

$x + 17 = 0 \Rightarrow x = -17$

$x - 1 = 0 \Rightarrow x = 1$

x-intercepts: $(-17, 0), (1, 0)$



25. $f(x) = x^2 + 34x + 289$

$= (x + 17)^2$

Vertex: $(-17, 0)$

Axis of symmetry: $x = -17$

Find x-intercepts:

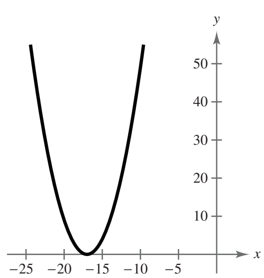
$x^2 + 34x + 289 = 0$

$(x + 17)^2 = 0$

$x + 17 = 0$

$x = -17$

x-intercept: $(-17, 0)$



26. $f(x) = x^2 - 30x + 225$

$= (x - 15)^2$

Vertex: $(15, 0)$

Axis of symmetry: $x = 15$

Find x-intercepts:

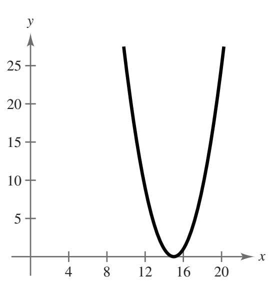
$x^2 - 30x + 225 = 0$

$(x - 15)^2 = 0$

$x - 15 = 0$

$x = 15$

x-intercept: $(15, 0)$



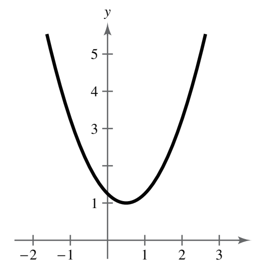
27. $f(x) = x^2 - x + \frac{5}{4}$

$= \left(x^2 - x + \frac{1}{4}\right) - \frac{1}{4} + \frac{5}{4}$

$= \left(x - \frac{1}{2}\right)^2 + 1$

Vertex: $\left(\frac{1}{2}, 1\right)$

Axis of symmetry: $x = \frac{1}{2}$



Find x-intercepts:

$x^2 - x + \frac{5}{4} = 0$

$x = \frac{1 \pm \sqrt{1 - 5}}{2}$

Not a real number

No x-intercepts

28. $f(x) = x^2 + 3x + \frac{1}{4}$

$= \left(x^2 + 3x + \frac{9}{4}\right) - \frac{9}{4} + \frac{1}{4}$

$= \left(x + \frac{3}{2}\right)^2 - 2$

Vertex: $\left(-\frac{3}{2}, -2\right)$

Axis of symmetry: $x = -\frac{3}{2}$

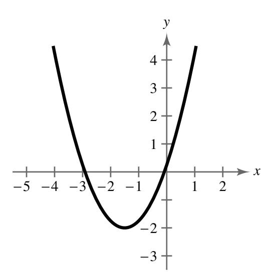
Find x-intercepts:

$x^2 + 3x + \frac{1}{4} = 0$

$x = \frac{-3 \pm \sqrt{9 - 1}}{2}$

$= -\frac{3}{2} \pm \sqrt{2}$

x-intercepts: $\left(-\frac{3}{2} - \sqrt{2}, 0\right), \left(-\frac{3}{2} + \sqrt{2}, 0\right)$



29. $f(x) = -x^2 + 2x + 5$

$= -(x^2 - 2x + 1) - (-1) + 5$

$= -(x - 1)^2 + 6$

Vertex: $(1, 6)$

Axis of symmetry: $x = 1$

Find x-intercepts:

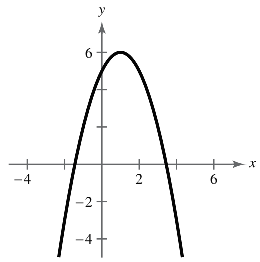
$-x^2 + 2x + 5 = 0$

$x^2 - 2x - 5 = 0$

$$x = \frac{2 \pm \sqrt{4 + 20}}{2}$$

$$= 1 \pm \sqrt{6}$$

x-intercepts: $(1 - \sqrt{6}, 0), (1 + \sqrt{6}, 0)$



30. $f(x) = -x^2 - 4x + 1 = -(x^2 + 4x) + 1$

$= -(x^2 + 4x + 4) - (-4) + 1$

$= -(x + 2)^2 + 5$

Vertex: $(-2, 5)$

Axis of symmetry: $x = -2$

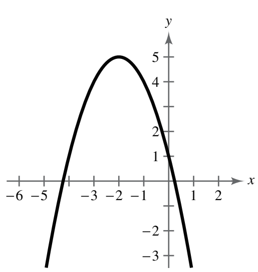
Find x-intercepts: $-x^2 - 4x + 1 = 0$

$x^2 + 4x - 1 = 0$

$$x = \frac{-4 \pm \sqrt{16 + 4}}{2}$$

$$= -2 \pm \sqrt{5}$$

x-intercepts: $(-2 - \sqrt{5}, 0), (-2 + \sqrt{5}, 0)$



31. $h(x) = 4x^2 - 4x + 21$

$= 4\left(x^2 - x + \frac{1}{4}\right) - 4\left(\frac{1}{4}\right) + 21$

$= 4\left(x - \frac{1}{2}\right)^2 + 20$

Vertex: $\left(\frac{1}{2}, 20\right)$

Axis of symmetry: $x = \frac{1}{2}$

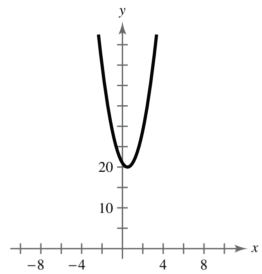
Find x-intercepts:

$4x^2 - 4x + 21 = 0$

$$x = \frac{4 \pm \sqrt{16 - 336}}{2(4)}$$

Not a real number

No x-intercepts



32. $f(x) = 2x^2 - x + 1$

$= 2\left(x^2 - \frac{1}{2}x\right) + 1$

$= 2\left(x - \frac{1}{4}\right)^2 - 2\left(\frac{1}{16}\right) + 1$

$= 2\left(x - \frac{1}{4}\right)^2 + \frac{7}{8}$

Vertex: $\left(\frac{1}{4}, \frac{7}{8}\right)$

Axis of symmetry: $x = \frac{1}{4}$

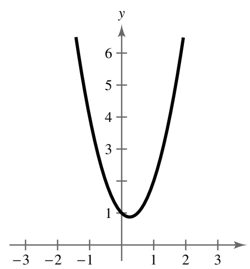
Find x-intercepts:

$2x^2 - x + 1 = 0$

$$x = \frac{1 \pm \sqrt{1 - 8}}{2(2)}$$

Not a real number

No x-intercepts



33. $$\begin{aligned} f(x) &= \tfrac{1}{4}x^2 - 2x - 12 \\ &= \tfrac{1}{4}(x^2 - 8x + 16) - \tfrac{1}{4}(16) - 12 \\ &= \tfrac{1}{4}(x - 4)^2 - 16 \end{aligned}$$

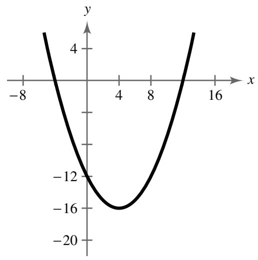
Vertex: $(4, -16)$

Axis of symmetry: $x = 4$

Find x-intercepts:

$$\begin{aligned} \tfrac{1}{4}x^2 - 2x - 12 &= 0 \\ x^2 - 8x - 48 &= 0 \\ (x + 4)(x - 12) &= 0 \\ x = -4 \text{ or } x &= 12 \end{aligned}$$

x-intercepts: $(-4, 0), (12, 0)$



34. $$\begin{aligned} f(x) &= -\tfrac{1}{3}x^2 + 3x - 6 \\ &= -\tfrac{1}{3}(x^2 - 9x) - 6 \\ &= -\tfrac{1}{3}\left(x^2 - 9x + \tfrac{81}{4}\right) + \tfrac{1}{3}\left(\tfrac{81}{4}\right) - 6 \\ &= -\tfrac{1}{3}\left(x - \tfrac{9}{2}\right)^2 + \tfrac{3}{4} \end{aligned}$$

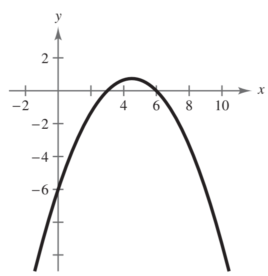
Vertex: $\left(\frac{9}{2}, \frac{3}{4}\right)$

Axis of symmetry: $x = \frac{9}{2}$

Find x-intercepts:

$$\begin{aligned} -\tfrac{1}{3}x^2 + 3x - 6 &= 0 \\ x^2 - 9x + 18 &= 0 \\ (x - 3)(x - 6) &= 0 \end{aligned}$$

x-intercepts: $(3, 0), (6, 0)$

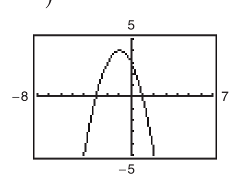


35. $f(x) = -(x^2 + 2x - 3) = -(x + 1)^2 + 4$

Vertex: $(-1, 4)$

Axis of symmetry: $x = -1$

x-intercepts: $(-3, 0), (1, 0)$

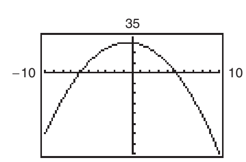


36. $$\begin{aligned} f(x) &= -(x^2 + x - 30) \\ &= -(x^2 + x) + 30 \\ &= -\left(x^2 + x + \tfrac{1}{4}\right) + \tfrac{1}{4} + 30 \\ &= -\left(x + \tfrac{1}{2}\right)^2 + \tfrac{121}{4} \end{aligned}$$

Vertex: $\left(-\frac{1}{2}, \frac{121}{4}\right)$

Axis of symmetry: $x = -\frac{1}{2}$

x-intercepts: $(-6, 0), (5, 0)$

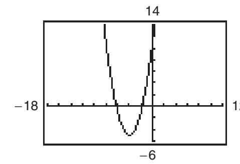


37. $g(x) = x^2 + 8x + 11 = (x + 4)^2 - 5$

Vertex: $(-4, -5)$

Axis of symmetry: $x = -4$

x-intercepts: $\left(-4 \pm \sqrt{5}, 0\right)$

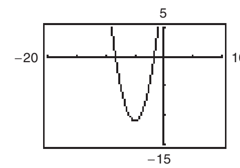


38. $$\begin{aligned} f(x) &= x^2 + 10x + 14 \\ &= (x^2 + 10x + 25) - 25 + 14 \\ &= (x + 5)^2 - 11 \end{aligned}$$

Vertex: $(-5, -11)$

Axis of symmetry: $x = -5$

x-intercepts: $\left(-5 \pm \sqrt{11}, 0\right)$

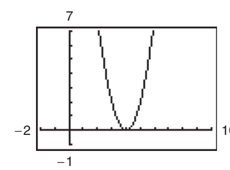


39. $$\begin{aligned} f(x) &= 2x^2 - 16x + 32 \\ &= 2(x^2 - 8x + 16) \\ &= 2(x - 4)^2 \end{aligned}$$

Vertex: $(4, 0)$

Axis of symmetry: $x = 4$

x-intercepts: $(4, 0)$

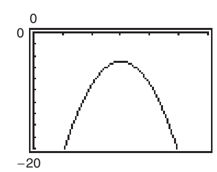


40. $$\begin{aligned} f(x) &= -4x^2 + 24x - 41 \\ &= -4(x^2 - 6x) - 41 \\ &= -4(x^2 - 6x + 9) + 36 - 41 \\ &= -4(x - 3)^2 - 5 \end{aligned}$$

Vertex: $(3, -5)$

Axis of symmetry: $x = 3$

No x-intercepts

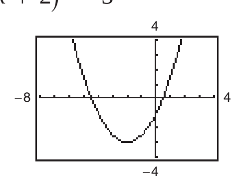


41. $g(x) = \frac{1}{2}(x^2 + 4x - 2) = \frac{1}{2}(x + 2)^2 - 3$

Vertex: $(-2, -3)$

Axis of symmetry: $x = -2$

x-intercepts: $\left(-2 \pm \sqrt{6}, 0\right)$

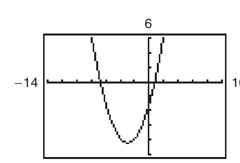


42. $$\begin{aligned} f(x) &= \tfrac{3}{5}(x^2 + 6x - 5) \\ &= \tfrac{3}{5}(x^2 + 6x + 9) - \tfrac{27}{5} - 3 \\ &= \tfrac{3}{5}(x + 3)^2 - \tfrac{42}{5} \end{aligned}$$

Vertex: $\left(-3, -\frac{42}{5}\right)$

Axis of symmetry: $x = -3$

x-intercepts: $\left(-3 \pm \sqrt{14}, 0\right)$



43. $(-1, 4)$ is the vertex.

$$y = a(x + 1)^2 + 4$$

Because the graph passes through $(1, 0)$,

$$\begin{aligned} 0 &= a(1 + 1)^2 + 4 \\ -4 &= 4a \\ -1 &= a. \end{aligned}$$

So, $y = -1(x + 1)^2 + 4 = -(x + 1)^2 + 4.$

44. $(-2, -1)$ is the vertex.

$$f(x) = a(x + 2)^2 - 1$$

Because the graph passes through $(0, 3)$,

$$\begin{aligned} 3 &= a(0 + 2)^2 - 1 \\ 3 &= 4a - 1 \\ 4 &= 4a \\ 1 &= a. \end{aligned}$$

So, $y = (x + 2)^2 - 1.$

45. $(-2, 2)$ is the vertex.

$$y = a(x + 2)^2 + 2$$

Because the graph passes through $(-1, 0)$,

$$\begin{aligned} 0 &= a(-1 + 2)^2 + 2 \\ -2 &= a. \end{aligned}$$

So, $y = -2(x + 2)^2 + 2.$

46. $(2, 0)$ is the vertex.

$$f(x) = a(x - 2)^2 + 0 = a(x - 2)^2$$

Because the graph passes through $(3, 2)$,

$$\begin{aligned} 2 &= a(3 - 2)^2 \\ 2 &= a. \end{aligned}$$

So, $y = 2(x - 2)^2.$

47. $(-2, 5)$ is the vertex.

$$f(x) = a(x + 2)^2 + 5$$

Because the graph passes through $(0, 9)$,

$$\begin{aligned} 9 &= a(0 + 2)^2 + 5 \\ 4 &= 4a \\ 1 &= a. \end{aligned}$$

So, $f(x) = 1(x + 2)^2 + 5 = (x + 2)^2 + 5.$

48. $(4, -1)$ is the vertex.

$$f(x) = a(x - 4)^2 - 1$$

Because the graph passes through $(2, 3)$,

$$\begin{aligned} 3 &= a(2 - 4)^2 - 1 \\ 3 &= 4a - 1 \\ 4 &= 4a \\ 1 &= a. \end{aligned}$$

So, $f(x) = (x - 4)^2 - 1.$

49. $(1, -2)$ is the vertex.

$$f(x) = a(x - 1)^2 - 2$$

Because the graph passes through $(-1, 14)$,

$$\begin{aligned} 14 &= a(-1 - 1)^2 - 2 \\ 14 &= 4a - 2 \\ 16 &= 4a \\ 4 &= a. \end{aligned}$$

So, $f(x) = 4(x - 1)^2 - 2.$

50. $(2, 3)$ is the vertex.

$$f(x) = a(x - 2)^2 + 3$$

Because the graph passes through $(0, 2)$,

$$\begin{aligned} 2 &= a(0 - 2)^2 + 3 \\ 2 &= 4a + 3 \\ -1 &= 4a \\ -\tfrac{1}{4} &= a. \end{aligned}$$

So, $f(x) = -\frac{1}{4}(x - 2)^2 + 3.$

51. $(5, 12)$ is the vertex.

$$f(x) = a(x - 5)^2 + 12$$

Because the graph passes through $(7, 15)$,

$$\begin{aligned} 15 &= a(7 - 5)^2 + 12 \\ 3 &= 4a \Rightarrow a = \tfrac{3}{4}. \end{aligned}$$

So, $f(x) = \frac{3}{4}(x - 5)^2 + 12.$

52. $(-2, -2)$ is the vertex.

$f(x) = a(x + 2)^2 - 2$

Because the graph passes through $(-1, 0)$,

$0 = a(-1 + 2)^2 - 2$
$0 = a - 2$
$2 = a.$

So, $f(x) = 2(x + 2)^2 - 2.$

53. $\left(-\frac{1}{4}, \frac{3}{2}\right)$ is the vertex.

$f(x) = a\left(x + \frac{1}{4}\right)^2 + \frac{3}{2}$

Because the graph passes through $(-2, 0)$,

$0 = a\left(-2 + \frac{1}{4}\right)^2 + \frac{3}{2}$
$-\frac{3}{2} = \frac{49}{16}a \Rightarrow a = -\frac{24}{49}.$

So, $f(x) = -\frac{24}{49}\left(x + \frac{1}{4}\right)^2 + \frac{3}{2}.$

54. $\left(\frac{5}{2}, -\frac{3}{4}\right)$ is the vertex.

$f(x) = a\left(x - \frac{5}{2}\right)^2 - \frac{3}{4}$

Because the graph passes through $(-2, 4)$,

$4 = a\left(-2 - \frac{5}{2}\right)^2 - \frac{3}{4}$
$4 = \frac{81}{4}a - \frac{3}{4}$
$\frac{19}{4} = \frac{81}{4}a$
$\frac{19}{81} = a.$

So, $f(x) = \frac{19}{81}\left(x - \frac{5}{2}\right)^2 - \frac{3}{4}.$

55. $\left(-\frac{5}{2}, 0\right)$ is the vertex.

$f(x) = a\left(x + \frac{5}{2}\right)^2$

Because the graph passes through $\left(-\frac{7}{2}, -\frac{16}{3}\right)$,

$-\frac{16}{3} = a\left(-\frac{7}{2} + \frac{5}{2}\right)^2$
$-\frac{16}{3} = a.$

So, $f(x) = -\frac{16}{3}\left(x + \frac{5}{2}\right)^2.$

56. $(6, 6)$ is the vertex.

$f(x) = a(x - 6)^2 + 6$

Because the graph passes through $\left(\frac{61}{10}, \frac{3}{2}\right)$,

$\frac{3}{2} = a\left(\frac{61}{10} - 6\right)^2 + 6$
$\frac{3}{2} = \frac{1}{100}a + 6$
$-\frac{9}{2} = \frac{1}{100}a$
$-450 = a.$

So, $f(x) = -450(x - 6)^2 + 6.$

57. $y = x^2 - 4x - 5$

x-intercepts: $(5, 0), (-1, 0)$

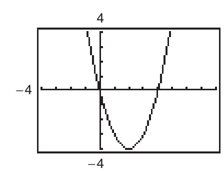
$0 = x^2 - 4x - 5$
$0 = (x - 5)(x + 1)$
$x = 5$ or $x = -1$

58. $y = 2x^2 + 5x - 3$

x-intercepts: $\left(\frac{1}{2}, 0\right), (-3, 0)$

$0 = 2x^2 + 5x - 3$
$0 = (2x - 1)(x + 3)$
$2x - 1 = 0 \Rightarrow x = \frac{1}{2}$
$x + 3 = 0 \Rightarrow x = -3$

59. $f(x) = x^2 - 4x$

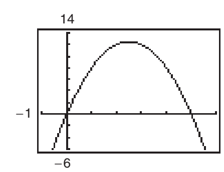


x-intercepts: $(0, 0), (4, 0)$

$0 = x^2 - 4x$
$0 = x(x - 4)$
$x = 0$ or $x = 4$

The x-intercepts and the solutions of $f(x) = 0$ are the same.

60. $f(x) = -2x^2 + 10x$



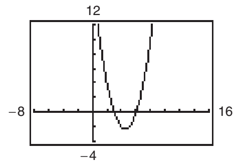
x-intercepts: $(0, 0), (5, 0)$

$0 = -2x^2 + 10x$
$0 = -2x(x - 5)$
$-2x = 0 \Rightarrow x = 0$
$x - 5 = 0 \Rightarrow x = 5$

The x-intercepts and the solutions of $f(x) = 0$ are the same.

61. $f(x) = x^2 - 9x + 18$

x-intercepts: $(3, 0), (6, 0)$



$0 = x^2 - 9x + 18$

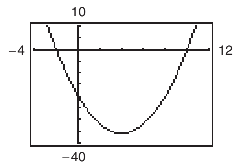
$0 = (x - 3)(x - 6)$

$x = 3 \quad \text{or} \quad x = 6$

The x-intercepts and the solutions of $f(x) = 0$ are the same.

62. $f(x) = x^2 - 8x - 20$

x-intercepts: $(-2, 0), (10, 0)$



$0 = x^2 - 8x - 20$

$0 = (x + 2)(x - 10)$

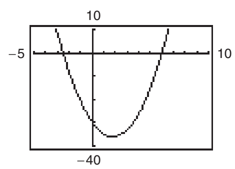
$x + 2 = 0 \Rightarrow x = -2$

$x - 10 = 0 \Rightarrow x = 10$

The x-intercepts and the solutions of $f(x) = 0$ are the same.

63. $f(x) = 2x^2 - 7x - 30$

x-intercepts: $\left(-\frac{5}{2}, 0\right), (6, 0)$



$0 = 2x^2 - 7x - 30$

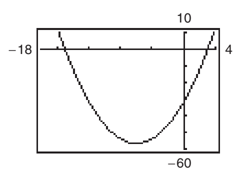
$0 = (2x + 5)(x - 6)$

$x = -\frac{5}{2} \quad \text{or} \quad x = 6$

The x-intercepts and the solutions of $f(x) = 0$ are the same.

64. $f(x) = \frac{7}{10}\left(x^2 + 12x - 45\right)$

x-intercepts: $(-15, 0), (3, 0)$



$0 = \frac{7}{10}\left(x^2 + 12x - 45\right)$

$0 = (x + 15)(x - 3)$

$x + 15 = 0 \Rightarrow x = -15$

$x - 3 = 0 \Rightarrow x = 3$

The x-intercepts and the solutions of $f(x) = 0$ are the same.

65. $f(x) = [x - (-1)](x - 3)$ opens upward

$= (x + 1)(x - 3)$

$= x^2 - 2x - 3$

$g(x) = -[x - (-1)](x - 3)$ opens downward

$= -(x + 1)(x - 3)$

$= -\left(x^2 - 2x - 3\right)$

$= -x^2 + 2x + 3$

Note: $f(x) = a(x + 1)(x - 3)$ has x-intercepts $(-1, 0)$ and $(3, 0)$ for all real numbers $a \neq 0$.

66. $f(x) = [x - (-5)](x - 5)$

$= (x + 5)(x - 5)$

$= x^2 - 25$, opens upward

$g(x) = -f(x)$, opens downward

$g(x) = -x^2 + 25$

Note: $f(x) = a\left(x^2 - 25\right)$ has x-intercepts $(-5, 0)$ and $(5, 0)$ for all real numbers $a \neq 0$.

67. $f(x) = (x - 0)(x - 10)$ opens upward

$= x^2 - 10x$

$g(x) = -(x - 0)(x - 10)$ opens downward

$= -x^2 + 10x$

Note: $f(x) = a(x - 0)(x - 10) = ax(x - 10)$ has x-intercepts $(0, 0)$ and $(10, 0)$ for all real numbers $a \neq 0$.

68. $f(x) = (x - 4)(x - 8)$

$= x^2 - 12x + 32$, opens upward

$g(x) = -f(x)$, opens downward

$g(x) = -x^2 + 12x - 32$

Note: $f(x) = a(x - 4)(x - 8)$ has x-intercepts $(4, 0)$ and $(8,0)$ for all real numbers $a \neq 0$.

69. $f(x) = [x - (-3)]\left[x - \left(-\frac{1}{2}\right)\right](2)$ opens upward

$= (x + 3)\left(x + \frac{1}{2}\right)(2)$

$= (x + 3)(2x + 1)$

$= 2x^2 + 7x + 3$

$g(x) = -\left(2x^2 + 7x + 3\right)$ opens downward

$= -2x^2 - 7x - 3$

Note: $f(x) = a(x + 3)(2x + 1)$ has x-intercepts $(-3, 0)$ and $\left(-\frac{1}{2}, 0\right)$ for all real numbers $a \neq 0$.

70. $f(x) = 2\left[x - \left(-\frac{5}{2}\right)\right](x - 2)$

$= 2\left(x + \frac{5}{2}\right)(x - 2)$

$= 2\left(x^2 + \frac{1}{2}x - 5\right)$

$= 2x^2 + x - 10$, opens upward

$g(x) = -f(x)$, opens downward

$g(x) = -2x^2 - x + 10$

Note: $f(x) = a\left(x + \frac{5}{2}\right)(x - 2)$ has x-intercepts $\left(-\frac{5}{2}, 0\right)$ and $(2, 0)$ for all real numbers $a \neq 0$.

71. Let $x =$ the first number and $y =$ the second number. Then the sum is

$x + y = 110 \Rightarrow y = 110 - x.$

The product is $P(x) = xy = x(110 - x) = 110x - x^2$.

$$\begin{aligned} P(x) &= -x^2 + 110x \\ &= -\left(x^2 - 110x + 3025 - 3025\right) \\ &= -\left[(x - 55)^2 - 3025\right] \\ &= -(x - 55)^2 + 3025 \end{aligned}$$

The maximum value of the product occurs at the vertex of $P(x)$ and is 3025. This happens when $x = y = 55$.

72. Let $x =$ the first number and $y =$ the second number. Then the sum is

$x + y = S \Rightarrow y = S - x.$

The product is $P(x) = xy = x(S - x) = Sx - x^2$.

$$\begin{aligned} P(x) &= Sx - x^2 \\ &= -x^2 + Sx \\ &= -\left(x^2 - Sx + \frac{S^2}{4} - \frac{S^2}{4}\right) \\ &= -\left(x - \frac{S}{2}\right)^2 + \frac{S^2}{4} \end{aligned}$$

The maximum value of the product occurs at the vertex of $P(x)$ and is $S^2/4$. This happens when $x = y = S/2$.

73. Let $x =$ the first number and $y =$ the second number. Then the sum is

$x + 2y = 24 \Rightarrow y = \frac{24 - x}{2}.$

The product is $P(x) = xy = x\left(\frac{24 - x}{2}\right)$.

$$\begin{aligned} P(x) &= \frac{1}{2}\left(-x^2 + 24x\right) \\ &= -\frac{1}{2}\left(x^2 - 24x + 144 - 144\right) \\ &= -\frac{1}{2}\left[(x - 12)^2 - 144\right] = -\frac{1}{2}(x - 12)^2 + 72 \end{aligned}$$

The maximum value of the product occurs at the vertex of $P(x)$ and is 72. This happens when $x = 12$ and $y = (24 - 12)/2 = 6$. So, the numbers are 12 and 6.

74. Let $x =$ the first number and $y =$ the second number.

Then the sum is $x + 3y = 42 \Rightarrow y = \frac{42 - x}{3}.$

The product is $P(x) = xy = x\left(\frac{42 - x}{3}\right)$.

$$\begin{aligned} P(x) &= \frac{1}{3}\left(-x^2 + 42x\right) \\ &= -\frac{1}{3}\left(x^2 - 42x + 441 - 441\right) \\ &= -\frac{1}{3}\left[(x - 21)^2 - 441\right] = -\frac{1}{3}(x - 21)^2 + 147 \end{aligned}$$

The maximum value of the product occurs at the vertex of $P(x)$ and is 147. This happens when $x = 21$ and $y = \frac{42 - 21}{3} = 7$. So, the numbers are 21 and 7.

75. $y = -\frac{4}{9}x^2 + \frac{24}{9}x + 12$

The vertex occurs at $-\frac{b}{2a} = \frac{-24/9}{2(-4/9)} = 3$. The maximum height is

$y(3) = -\frac{4}{9}(3)^2 + \frac{24}{9}(3) + 12 = 16$ feet.

76. $y = -\frac{16}{2025}x^2 + \frac{9}{5}x + 1.5$

(a) The ball height when it is punted is the y-intercept.

$$y = -\frac{16}{2025}(0)^2 + \frac{9}{5}(0) + 1.5 = 1.5 \text{ feet}$$

(b) The vertex occurs at $x = -\frac{b}{2a} = -\frac{9/5}{2(-16/2025)} = \frac{3645}{32}$.

The maximum height is $f\left(\frac{3645}{32}\right) = -\frac{16}{2025}\left(\frac{3645}{32}\right)^2 + \frac{9}{5}\left(\frac{3645}{32}\right) + 1.5$

$$= -\frac{6561}{64} + \frac{6561}{32} + 1.5 = -\frac{6561}{64} + \frac{13{,}122}{64} + \frac{96}{64} = \frac{6657}{64} \text{ feet} \approx 104.02 \text{ feet.}$$

(c) The length of the punt is the positive x-intercept.

$$0 = -\frac{16}{2025}x^2 + \frac{9}{5}x + 1.5$$

$$x = \frac{-(9/5) \pm \sqrt{(9/5)^2 - (4)(1.5)(-16/2025)}}{-32/2025} \approx \frac{1.8 \pm 1.81312}{-0.01580247}$$

$x \approx -0.83031$ or $x \approx 228.64$

The punt is about 228.64 feet.

77. $C = 800 - 10x + 0.25x^2 = 0.25x^2 - 10x + 800$

The vertex occurs at $x = -\frac{b}{2a} = -\frac{-10}{2(0.25)} = 20$.

The cost is minimum when $x = 20$ fixtures.

78. $P = 230 + 20x - 0.5x^2$

The vertex occurs at $x = -\frac{b}{2a} = -\frac{20}{2(-0.5)} = 20$.

Because x is in hundreds of dollars, $20 \times 100 = 2000$ dollars is the amount spent on advertising that gives maximum profit.

79. $R(p) = -25p^2 + 1200p$

(a) $R(20) = \$14{,}000$ thousand $= \$14{,}000{,}000$

$R(25) = \$14{,}375$ thousand $= \$14{,}375{,}000$

$R(30) = \$13{,}500$ thousand $= \$13{,}500{,}000$

(b) The revenue is a maximum at the vertex.

$$-\frac{b}{2a} = \frac{-1200}{2(-25)} = 24$$

$R(24) = 14{,}400$

The unit price that will yield a maximum revenue of \$14,400 thousand is \$24.

80. $R(p) = -12p^2 + 150p$

(a) $R(\$4) = -12(\$4)^2 + 150(\$4) = \$408$

$R(\$6) = -12(\$6)^2 + 150(\$6) = \$468$

$R(\$8) = -12(\$8)^2 + 150(\$8) = \$432$

(b) The vertex occurs at

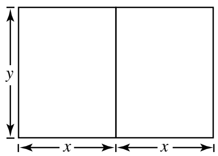
$$p = -\frac{b}{2a} = -\frac{150}{2(-12)} = \$6.25.$$

Revenue is maximum when price $= \$6.25$ per pet.

The maximum revenue is

$$R(\$6.25) = -12(\$6.25)^2 + 150(\$6.25) = \$468.75.$$

81. (a)



$$4x + 3y = 200 \Rightarrow y = \frac{1}{3}(200 - 4x) = \frac{4}{3}(50 - x)$$

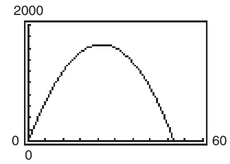
$$A = 2xy = 2x\left[\frac{4}{3}(50 - x)\right] = \frac{8}{3}x(50 - x) = \frac{8x(50 - x)}{3}$$

(b)

x	A
5	600
10	$1066\frac{2}{3}$
15	1400
20	1600
25	$1666\frac{2}{3}$
30	1600

This area is maximum when $x = 25$ feet and $y = \frac{100}{3} = 33\frac{1}{3}$ feet.

(c)



This area is maximum when $x = 25$ feet and $y = \frac{100}{3} = 33\frac{1}{3}$ feet.

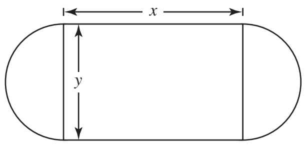
(d)
$$\begin{aligned} A &= \frac{8}{3}x(50 - x) \\ &= -\frac{8}{3}(x^2 - 50x) \\ &= -\frac{8}{3}(x^2 - 50x + 625 - 625) \\ &= -\frac{8}{3}\left[(x - 25)^2 - 625\right] \\ &= -\frac{8}{3}(x - 25)^2 + \frac{5000}{3} \end{aligned}$$

The maximum area occurs at the vertex and is $5000/3$ square feet. This happens when $x = 25$ feet and $y = (200 - 4(25))/3 = 100/3$ feet. The dimensions are $2x = 50$ feet by $33\frac{1}{3}$ feet.

(e) They are all identical.

$x = 25$ feet and $y = 33\frac{1}{3}$ feet

82. (a)



(b) Radius of semicircular ends of track: $r = \frac{1}{2}y$

Distance around two semicircular parts of track: $d = 2\pi r = 2\pi\left(\frac{1}{2}y\right) = \pi y$

(c) Distance traveled around track in one lap: $d = \pi y + 2x = 200$

$$\pi y = 200 - 2x$$

$$y = \frac{200 - 2x}{\pi}$$

(d) Area of rectangular region:

$$\begin{aligned} A = xy &= x\left(\frac{200 - 2x}{\pi}\right) \\ &= \frac{1}{\pi}\left(200x - 2x^2\right) \\ &= -\frac{2}{\pi}\left(x^2 - 100x\right) \\ &= -\frac{2}{\pi}\left(x^2 - 100x + 2500 - 2500\right) \\ &= -\frac{2}{\pi}(x - 50)^2 + \frac{5000}{\pi} \end{aligned}$$

The area is maximum when $x = 50$ and $y = \frac{200 - 2(50)}{\pi} = \frac{100}{\pi}$.

83. (a) Revenue $=$ (number of tickets sold)(price per ticket)

Let $y =$ attendance, or the number of tickets sold.

$$m = -100, (20, 1500)$$

$$\begin{aligned} y - 1500 &= -100(x - 20) \\ y - 1500 &= -100x + 2000 \\ y &= -100x + 3500 \end{aligned}$$

$$\begin{aligned} R(x) &= (y)(x) \\ R(x) &= (-100x + 3500)(x) \\ R(x) &= -100x^2 + 3500x \end{aligned}$$

(b) The revenue is at a maximum at the vertex.

$$-\frac{b}{2a} = \frac{-3500}{2(-100)} = 17.5$$

$$R(17.5) = -100(17.5)^2 + 3500(17.5) = \$30{,}625$$

A ticket price of $17.50 will yield a maximum revenue of $30,625.

84. (a) Area of window $=$ Area of rectangle $+$ Area of semicircle $= xy + \frac{1}{2}\pi(\text{radius})^2 = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2 = xy + \frac{\pi x^2}{8}$

To eliminate the y in the equation for area, introduce a secondary equation.

Perimeter $=$ perimeter of rectangle $+$ perimeter of semicircle

$$16 = 2y + x + \frac{1}{2}(\text{circumference})$$

$$16 = 2y + x + \frac{1}{2}(2\pi \cdot \text{radius})$$

$$16 = 2y + x + \pi\left(\frac{x}{2}\right)$$

$$y = 8 - \frac{1}{2}x - \frac{\pi x}{4}$$

Substitute the secondary equation into the area equation.

$$\text{Area} = xy + \frac{\pi x^2}{8} = x\left(8 - \frac{1}{2}x - \frac{\pi x}{4}\right) + \frac{\pi x^2}{8} = 8x - \frac{1}{2}x^2 - \frac{\pi x^2}{4} + \frac{\pi x^2}{8} = 8x - \frac{1}{2}x^2 - \frac{\pi x^2}{8} = \frac{1}{8}\left(64x - 4x^2 - \pi x^2\right)$$

(b) The area is maximum at the vertex.

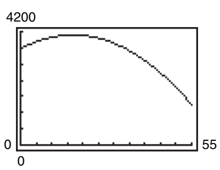
$$\text{Area} = 8x - \frac{1}{2}x^2 - \frac{\pi x^2}{8} = \left(-\frac{1}{2} - \frac{\pi}{8}\right)x^2 + 8x$$

$$x = -\frac{b}{2a} = \frac{-8}{2\left(-\frac{1}{2} - \frac{\pi}{8}\right)} \approx 4.48$$

$$y = 8 - \frac{1}{2}(4.48) - \frac{\pi(4.48)}{4} \approx 2.24$$

The area will be at a maximum when the width is about 4.48 feet and the length is about 2.24 feet.

85. (a)



(b) The maximum annual consumption occurs at the point $(16.9, 4074.813)$.

4075 cigarettes

$1966 \rightarrow t = 16$

The maximum consumption occurred in 1966. After that year, the consumption decreases.
It is likely that the warning was responsible for the decrease in consumption.

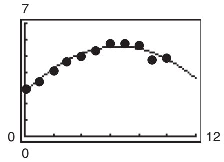
(c) $$\text{Annual consumption per smoker} = \frac{\text{Annual consumption in 2005} \cdot \text{total population}}{\text{total number of smokers in 2005}} = \frac{1487.9(296{,}329{,}000)}{59{,}858{,}458} = 7365.8$$

About 7366 cigarettes per smoker annually

$$\text{Daily consumption per smoker} = \frac{\text{Number of cigarettes per year}}{\text{Number of days per year}} = \frac{7366}{365} \approx 20.2$$

About 20 cigarettes per day

86. (a) and (c)



The model fits the data well.

(b) $y = -0.0666x^2 + 0.875x + 2.69$

(d) 2006

(e) Answers will vary.

(f) Let $t = 13$, $y = -0.0666(13)^2 + 0.875(13) + 2.69$

≈ 2.81

In the year 2013, the sales will be about $2.81 billion.

87. True. The equation $-12x^2 - 1 = 0$ has no real solution, so the graph has no x-intercepts.

88. True. The vertex of $f(x)$ is $\left(-\frac{5}{4}, \frac{53}{4}\right)$ and the vertex of $g(x)$ is $\left(-\frac{5}{4}, -\frac{71}{4}\right)$.

89. $f(x) = -x^2 + bx - 75$, maximum value: 25

The maximum value, 25, is the y-coordinate of the vertex.

Find the x-coordinate of the vertex:

$$x = -\frac{b}{2a} = -\frac{b}{2(-1)} = \frac{b}{2}$$

$$\begin{aligned} f(x) &= -x^2 + bx - 75 \\ f\left(\frac{b}{2}\right) &= -\left(\frac{b}{2}\right)^2 + b\left(\frac{b}{2}\right) - 75 \\ 25 &= -\frac{b^2}{4} + \frac{b^2}{2} - 75 \\ 100 &= \frac{b^2}{4} \\ 400 &= b^2 \\ \pm 20 &= b \end{aligned}$$

90. $f(x) = -x^2 + bx - 16$, maximum value: 48

The maximum value, 48, is the y-coordinate of the vertex.

Find the x-coordinate of the vertex:

$$x = -\frac{b}{2a} = -\frac{b}{2(-1)} = \frac{b}{2}$$

$$\begin{aligned} f(x) &= -x^2 + bx - 16 \\ f\left(\frac{b}{2}\right) &= -\left(\frac{b}{2}\right)^2 + b\left(\frac{b}{2}\right) - 16 \\ 48 &= -\frac{b^2}{4} + \frac{b^2}{2} - 16 \\ 64 &= \frac{b^2}{4} \\ 256 &= b^2 \\ \pm 16 &= b \end{aligned}$$

91. $f(x) = x^2 + bx + 26$, minimum value: 10

The minimum value, 10, is the y-coordinate of the vertex.

Find the x-coordinate of the vertex:

$$x = -\frac{b}{2a} = -\frac{b}{2(1)} = -\frac{b}{2}$$

$$\begin{aligned} f(x) &= x^2 + bx + 26 \\ f\left(-\frac{b}{2}\right) &= \left(-\frac{b}{2}\right)^2 + b\left(-\frac{b}{2}\right) + 26 \\ 10 &= \frac{b^2}{4} - \frac{b^2}{2} + 26 \\ -16 &= -\frac{b^2}{4} \\ 64 &= b^2 \\ \pm 8 &= b \end{aligned}$$

92. $f(x) = x^2 + bx - 25$, minimum value: –50

The minimum value, –50, is the y-coordinate of the vertex.

Find the x-coordinate:

$$x = -\frac{b}{2a} = -\frac{b}{2(1)} = -\frac{b}{2}$$

$$\begin{aligned} f(x) &= x^2 + bx - 25 \\ f\left(-\frac{b}{2}\right) &= \left(-\frac{b}{2}\right)^2 + b\left(-\frac{b}{2}\right) - 25 \\ -50 &= \frac{b^2}{4} - \frac{b^2}{2} - 25 \\ -25 &= \frac{-b^2}{4} \\ 100 &= b^2 \\ \pm 10 &= b \end{aligned}$$

93. $$\begin{aligned} f(x) &= ax^2 + bx + c \\ &= a\left(x^2 + \frac{b}{a}x\right) + c \\ &= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2}\right) + c \\ &= a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c \\ &= a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a} \end{aligned}$$

$$\begin{aligned} f\left(-\frac{b}{2a}\right) &= a\left(\frac{b^2}{4a^2}\right) + b\left(-\frac{b}{2a}\right) + c \\ &= \frac{b^2}{4a} - \frac{b^2}{2a} + c \\ &= \frac{b^2 - 2b^2 + 4ac}{4a} = \frac{4ac - b^2}{4a} \end{aligned}$$

So, the vertex occurs at

$$\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right) = \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right).$$

94. (a) Since the graph of P opens upward, the value of a is positive.

(b) Since the graph of P opens upward, the vertex of the parabola is a relative minimum at $t = \frac{-b}{2a}$.

(c) Because of the symmetrical property of the graph of a parabola, and since the company made the same yearly profit in 2004 and 2012, the midpoint of the interval $4 \le t \le 12$ or $t = 8$ corresponds to the year 2008,when the company made the least profit.

(d) Since the year 2008 is when the company made the least profit, profit has been increasing since 2008, and is also currently increasing.

95. If $f(x) = ax^2 + bx + c$ has two real zeros, then by the Quadratic Formula they are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The average of the zeros of f is

$$\frac{\frac{-b - \sqrt{b^2 - 4ac}}{2a} + \frac{-b + \sqrt{b^2 - 4ac}}{2a}}{2} = \frac{\frac{-2b}{2a}}{2} = -\frac{b}{2a}.$$

This is the x-coordinate of the vertex of the graph.

Section 2.2 Polynomial Functions of Higher Degree

1. continuous

2. Leading Coefficient Test

3. n; $n - 1$

4. (a) solution; (b) $(x - a)$; (c) x-intercept

5. touches; crosses

6. multiplicity

7. standard

8. Intermediate Value

9. $f(x) = -2x^2 - 5x$ is a parabola with x-intercepts $(0, 0)$ and $\left(-\frac{5}{2}, 0\right)$ and opens downward. Matches graph (h).

10. $f(x) = 2x^3 - 3x + 1$ has intercepts $(0, 1), (1, 0), \left(-\frac{1}{2} - \frac{1}{2}\sqrt{3}, 0\right)$ and $\left(-\frac{1}{2} + \frac{1}{2}\sqrt{3}, 0\right)$. Matches graph (f).

11. $f(x) = -\frac{1}{4}x^4 + 3x^2$ has intercepts $(0, 0)$ and $\left(\pm 2\sqrt{3}, 0\right)$. Matches graph (a).

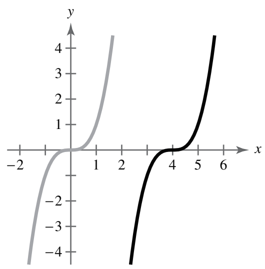
12. $f(x) = -\frac{1}{3}x^3 + x^2 - \frac{4}{3}$ has y-intercept $\left(0, -\frac{4}{3}\right)$. Matches graph (e).

13. $f(x) = x^4 + 2x^3$ has intercepts $(0, 0)$ and $(-2, 0)$. Matches graph (d).

14. $f(x) = \frac{1}{5}x^5 - 2x^3 + \frac{9}{5}x$ has intercepts $(0, 0), (1, 0), (-1, 0), (3, 0), (-3, 0)$. Matches graph (b).

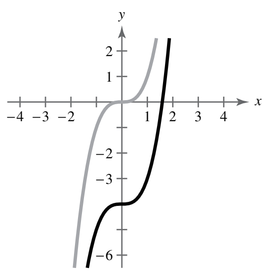
15. $y = x^3$

(a) $f(x) = (x - 4)^3$



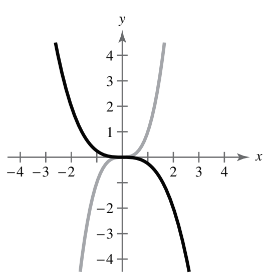
Horizontal shift four units to the right

(b) $f(x) = x^3 - 4$



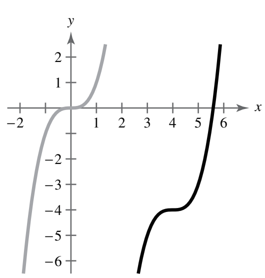
Vertical shift four units downward

(c) $f(x) = -\dfrac{1}{4}x^3$



Reflection in the x-axis and a vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{4}\right)$

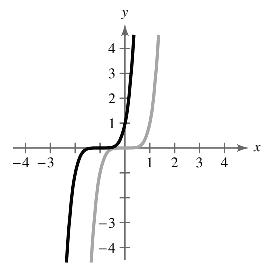
(d) $f(x) = (x - 4)^3 - 4$



Horizontal shift four units to the right and vertical shift four units downward

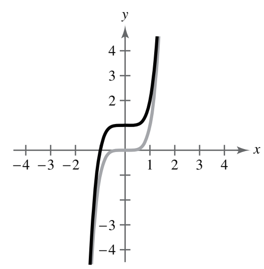
16. $y = x^5$

(a) $f(x) = (x + 1)^5$



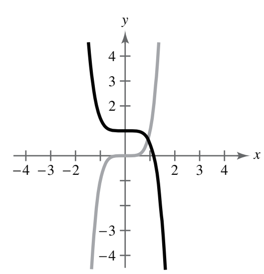
Horizontal shift one unit to the left

(b) $f(x) = x^5 + 1$



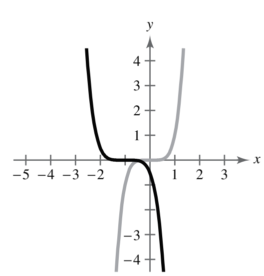
Vertical shift one unit upward

(c) $f(x) = 1 - \frac{1}{2}x^5$



Reflection in the x-axis, vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{2}\right)$, and vertical shift one unit upward

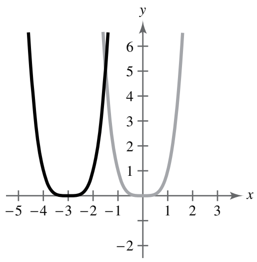
(d) $f(x) = -\frac{1}{2}(x + 1)^5$



Reflection in the x-axis, vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{2}\right)$, and horizontal shift one unit to the left

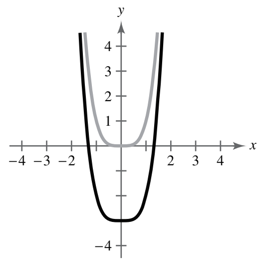
17. $y = x^4$

(a) $f(x) = (x + 3)^4$



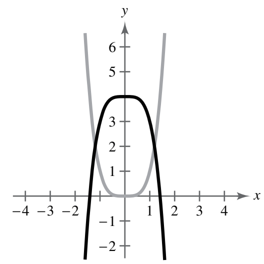
Horizontal shift three units to the left

(b) $f(x) = x^4 - 3$



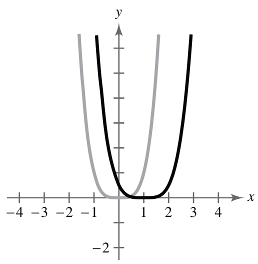
Vertical shift three units downward

(c) $f(x) = 4 - x^4$



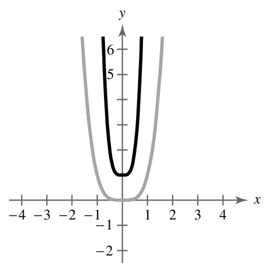
Reflection in the x-axis and then a vertical shift four units upward

(d) $f(x) = \frac{1}{2}(x - 1)^4$



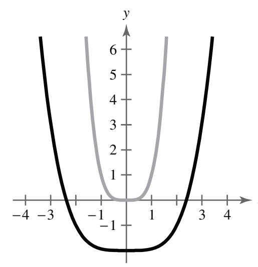
Horizontal shift one unit to the right and a vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{2}\right)$

(e) $f(x) = (2x)^4 + 1$



Vertical shift one unit upward and a horizontal shrink (each y-value is multiplied by 16)

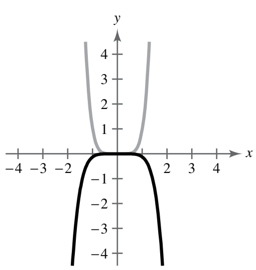
(f) $f(x) = \left(\frac{1}{2}x\right)^4 - 2$



Vertical shift two units downward and a horizontal stretch $\left(\text{each } y\text{-value is multipied by } \frac{1}{16}\right)$

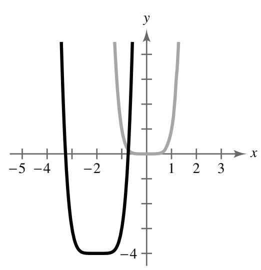
18. $y = x^6$

(a) $f(x) = -\frac{1}{8}x^6$



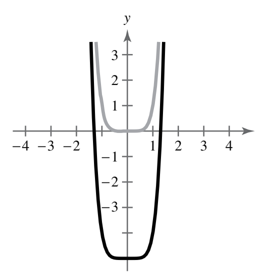
Vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{8}\right)$ and reflection in the x-axis

(b) $f(x) = (x + 2)^6 - 4$



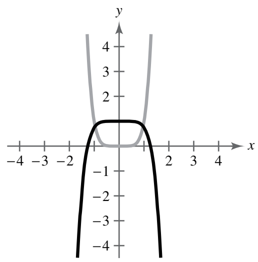
Horizontal shift two units to the left and a vertical shift four units downward

(c) $f(x) = x^6 - 5$



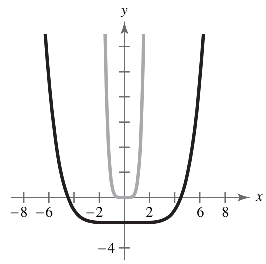
Vertical shift five units downward

(d) $f(x) = -\frac{1}{4}x^6 + 1$



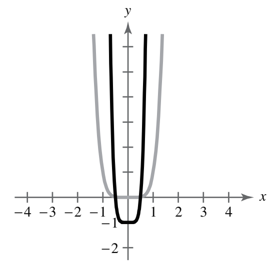
Reflection in the x-axis, vertical shrink (each y-value is multiplied by $\frac{1}{4}$), and vertical shift one unit upward

(e) $f(x) = \left(\frac{1}{4}x\right)^6 - 2$



Horizontal stretch (each x-value is multiplied by 4), and vertical shift two units downward

(f) $f(x) = (2x)^6 - 1$



Horizontal shrink (each x-value is multiplied by $\frac{1}{2}$), and vertical shift one unit downward

19. $f(x) = \frac{1}{5}x^3 + 4x$

Degree: 3

Leading coefficient: $\frac{1}{5}$

The degree is odd and the leading coefficient is positive. The graph falls to the left and rises to the right.

20. $f(x) = 2x^2 - 3x + 1$

Degree: 2

Leading coefficient: 2

The degree is even and the leading coefficient is positive.

The graph rises to the left and rises to the right.

21. $g(x) = 5 - \frac{7}{2}x - 3x^2$

Degree: 2

Leading coefficient: -3

The degree is even and the leading coefficient is negative. The graph falls to the left and falls to the right.

22. $h(x) = 1 - x^6$

Degree: 6

Leading coefficient: -1

The degree is even and the leading coefficient is negative. The graph falls to the left and falls to the right.

23. $g(x) = -x^3 + 3x^2$

Degree: 3

Leading coefficient: -1

The degree is odd and the leading coefficient is negative. The graph rises to the left and falls to the right.

24. $g(x) = -x^4 + 4x - 6$

Degree: 4

Leading coefficient: -1

The degree is even and the leading coefficient is negative. The graph falls to the left and falls to the right.

25. $f(x) = -2.1x^5 + 4x^3 - 2$

Degree: 5

Leading coefficient: -2.1

The degree is odd and the leading coefficient is negative.

The graph rises to the left and falls to the right.

26. $f(x) = 4x^5 - 7x + 6.5$

Degree: 5

Leading coefficient: 4

The degree is odd and the leading coefficient is positive. The graph falls to the left and rises to the right.

27. $f(x) = 6 - 2x + 4x^2 - 5x^3$

Degree: 3

Leading coefficient: -5

The degree is odd and the leading coefficient is negative.

The graph rises to the left and falls to the right.

28. $f(x) = \dfrac{3x^4 - 2x + 5}{4}$

Degree: 4

Leading coefficient: $\dfrac{3}{4}$

The degree is even and the leading coefficient is positive.

The graph rises to the left and rises to the right.

29. $h(x) = -\frac{3}{4}(t^2 - 3t + 6)$

Degree: 2

Leading coefficient: $-\frac{3}{4}$

The degree is even and the leading coefficient is negative. The graph falls to the left and falls to the right.

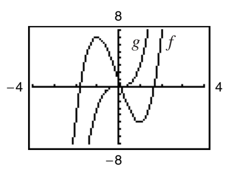
30. $f(s) = -\frac{7}{8}(s^3 + 5s^2 - 7s + 1)$

Degree: 3

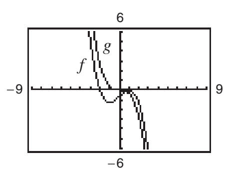
Leading coefficient: $-\frac{7}{8}$

The degree is odd and the leading coefficient is negative. The graph rises to the left and falls to the right.

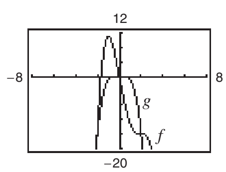
31. $f(x) = 3x^3 - 9x + 1$; $g(x) = 3x^3$



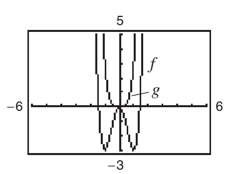
32. $f(x) = -\frac{1}{3}(x^3 - 3x + 2)$, $g(x) = -\frac{1}{3}x^3$



33. $f(x) = -(x^4 - 4x^3 + 16x)$; $g(x) = -x^4$



34. $f(x) = 3x^4 - 6x^2$, $g(x) = 3x^4$



35. $f(x) = x^2 - 36$

(a) $0 = x^2 - 36$

$0 = (x + 6)(x - 6)$

$x + 6 = 0 \qquad x - 6 = 0$

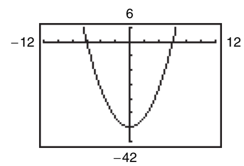
$x = -6 \qquad x = 6$

Zeros: ± 6

(b) Each zero has a multiplicity of one (odd multiplicity).

(c) Turning points: 1 (the vertex of the parabola)

(d)



36. $f(x) = 81 - x^2$

(a) $0 = 81 - x^2$

$0 = (9 - x)(9 + x)$

$9 - x = 0 \qquad 9 + x = 0$

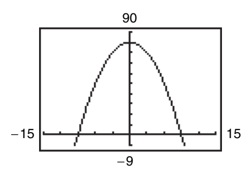
$9 = x \qquad x = -9$

Zeros: ± 9

(b) Each zero has a multiplicity of one (odd multiplicity).

(c) Turning points: 1 (the vertex of the parabola)

(d)



37. $h(t) = t^2 - 6t + 9$

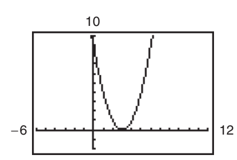
(a) $0 = t^2 - 6t + 9 = (t - 3)^2$

Zero: $t = 3$

(b) $t = 3$ has a multiplicity of 2 (even multiplicity).

(c) Turning points: 1 (the vertex of the parabola)

(d)



38. $f(x) = x^2 + 10x + 25$

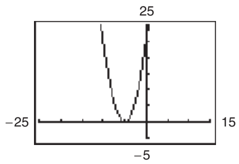
(a) $0 = x^2 + 10x + 25 = (x + 5)^2$

Zero: $x = -5$

(b) $x = -5$ has a multiplicity of 2 (even multiplicity).

(c) Turning points: 1 (the vertex of the parabola)

(d)



39. $f(x) = \frac{1}{3}x^2 + \frac{1}{3}x - \frac{2}{3}$

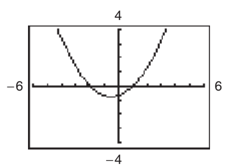
(a) $$\begin{aligned} 0 &= \tfrac{1}{3}x^2 + \tfrac{1}{3}x - \tfrac{2}{3} \\ &= \tfrac{1}{3}(x^2 + x - 2) \\ &= \tfrac{1}{3}(x + 2)(x - 1) \end{aligned}$$

Zeros: $x = -2, x = 1$

(b) Each zero has a multiplicity of 1 (odd multiplicity).

(c) Turning points: 1 (the vertex of the parabola)

(d)



40. $f(x) = \dfrac{1}{2}x^2 + \dfrac{5}{2}x - \dfrac{3}{2}$

(a) For $\dfrac{1}{2}x^2 + \dfrac{5}{2}x - \dfrac{3}{2} = 0, a = \dfrac{1}{2}, b = \dfrac{5}{2}, c = -\dfrac{3}{2}.$

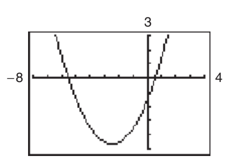
$$\begin{aligned} x &= \frac{-\frac{5}{2} \pm \sqrt{\left(\frac{5}{2}\right)^2 - 4\left(\frac{1}{2}\right)\left(-\frac{3}{2}\right)}}{1} \\ &= -\frac{5}{2} \pm \sqrt{\frac{37}{4}} \end{aligned}$$

Zeros: $x = \dfrac{-5 \pm \sqrt{37}}{2}$

(b) Each zero has a multiplicity of 1 (odd multiplicity).

(c) Turning points: 1 (the vertex of the parabola)

(d)



41. $f(x) = 3x^3 - 12x^2 + 3x$

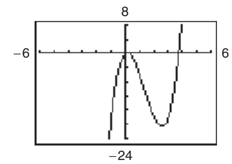
(a) $0 = 3x^3 - 12x^2 + 3x = 3x(x^2 - 4x + 1)$

Zeros: $x = 0, x = 2 \pm \sqrt{3}$ (by the Quadratic Formula)

(b) Each zero has a multiplicity of 1 (odd multiplicity).

(c) Turning points: 2

(d)



42. $g(x) = 5x(x^2 - 2x - 1)$

(a) $0 = 5x(x^2 - 2x - 1)$

$0 = x(x^2 - 2x - 1)$

For $x^2 - 2x - 1 = 0, a = 1, b = -2, c = -1.$

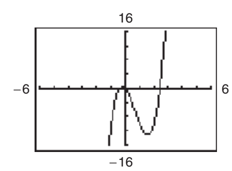
$$\begin{aligned} x &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)} \\ &= \frac{2 \pm \sqrt{8}}{2} \\ &= 1 \pm \sqrt{2} \end{aligned}$$

Zeros: $x = 0, x = 1 \pm \sqrt{2}$

(b) Each zero has a multiplicity of 1 (odd multiplicity).

(c) Turning points: 2

(d)



43. $f(t) = t^3 - 8t^2 + 16t$

(a) $0 = t^3 - 8t^2 + 16t$

$0 = t(t^2 - 8t + 16)$

$0 = t(t - 4)(t - 4)$

$t = 0 \quad t - 4 = 0 \quad t - 4 = 0$

$t = 0 \quad\quad t = 4 \quad\quad t = 4$

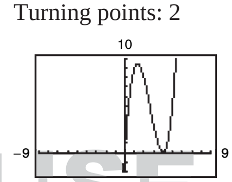
Zeros: $t = 0, t = 4$

(b) The multiplicity of $t = 0$ is 1 (odd multiplicity).

The multiplicity of $t = 4$ is 2 (even multiplicity).

(c) Turning points: 2

(d)



44. $f(x) = x^4 - x^3 - 30x^2$

(a) $0 = x^4 - x^3 - 30x^2$

$0 = x^2(x^2 - x - 30)$

$0 = x^2(x - 6)(x + 5)$

$x^2 = 0 \qquad x - 6 = 0 \qquad x + 5 = 0$

$x = 0 \qquad\quad x = 6 \qquad\quad x = -5$

Zeros: $x = 0, x = 6, x = -5$

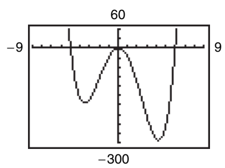
(b) The multiplicity of $x = 0$ is 2 (even multiplicity).

The multiplicity of $x = 6$ is 1 (odd multiplicity).

The multiplicity of $x = -5$ is 1 (odd multiplicity).

(c) Turning points: 3

(d)



45. $g(t) = t^5 - 6t^3 + 9t$

(a) $0 = t^5 - 6t^3 + 9t = t(t^4 - 6t^2 + 9) = t(t^2 - 3)^2$

$= t(t + \sqrt{3})^2(t - \sqrt{3})^2$

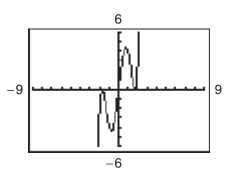
Zeros: $t = 0, t = \pm\sqrt{3}$

(b) $t = 0$ has a multiplicity of 1 (odd multiplicity).

$t = \pm\sqrt{3}$ each have a multiplicity of 2 (even multiplicity).

(c) Turning points: 4

(d)



46. (a) $f(x) = x^5 + x^3 - 6x$

$0 = x(x^4 + x^2 - 6)$

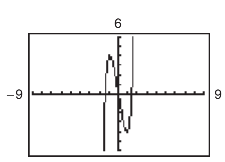
$0 = x(x^2 + 3)(x^2 - 2)$

Zeros: $x = 0, \pm\sqrt{2}$

(b) Each zero has a multiplicity of 1 (odd multiplicity).

(c) Turning points: 2

(d)



47. $f(x) = 3x^4 + 9x^2 + 6$

(a) $0 = 3x^4 + 9x^2 + 6$

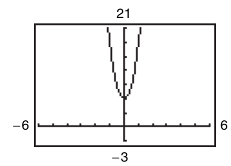
$0 = 3(x^4 + 3x^2 + 2)$

$0 = 3(x^2 + 1)(x^2 + 2)$

(b) No real zeros

(c) Turning points: 1

(d)



48. $f(x) = 2x^4 - 2x^2 - 40$

(a) $0 = 2x^4 - 2x^2 - 40$

$0 = 2(x^4 - x^2 - 20)$

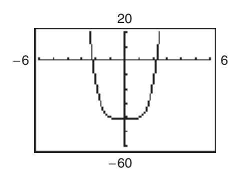
$0 = 2(x^2 + 4)(x^2 - 5)$

Zeros: $x = \pm\sqrt{5}$

(b) Each zero has a multiplicity of 1 (odd multiplicity).

(c) Turning points: 3

(d)



49. $g(x) = x^3 + 3x^2 - 4x - 12$

(a) $0 = x^3 + 3x^2 - 4x - 12 = x^2(x + 3) - 4(x + 3)$

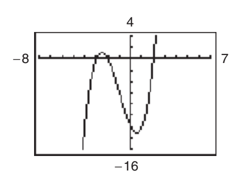
$= (x^2 - 4)(x + 3) = (x - 2)(x + 2)(x + 3)$

Zeros: $x = \pm 2, x = -3$

(b) Each zero has a multiplicity of 1 (odd multiplicity).

(c) Turning points: 2

(d)



50. $f(x) = x^3 - 4x^2 - 25x + 100$

(a) $0 = x^2(x - 4) - 25(x - 4)$

$0 = (x^2 - 25)(x - 4)$

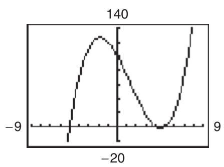
$0 = (x + 5)(x - 5)(x - 4)$

Zeros: $x = \pm 5, 4$

(b) Each zero has a multiplicity of 1 (odd multiplicity).

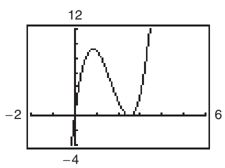
(c) Turning points: 2

(d)



51. $y = 4x^3 - 20x^2 + 25x$

(a)



(b) x-intercepts: $(0, 0), \left(\frac{5}{2}, 0\right)$

(c) $0 = 4x^3 - 20x^2 + 25x$

$0 = x(4x^2 - 20x + 25)$

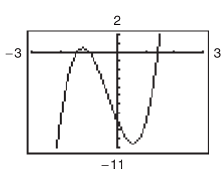
$0 = x(2x - 5)^2$

$x = 0, \frac{5}{2}$

(d) The solutions are the same as the x-coordinates of the x-intercepts.

52. $y = 4x^3 + 4x^2 - 8x - 8$

(a)



(b) $(-1, 0), \left(-\sqrt{2}, 0\right), \left(\sqrt{2}, 0\right)$

(c) $0 = 4x^3 + 4x^2 - 8x - 8$

$0 = 4x^2(x + 1) - 8(x + 1)$

$0 = (4x^2 - 8)(x + 1)$

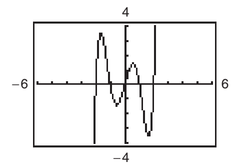
$0 = 4(x^2 - 2)(x + 1)$

$x = \pm\sqrt{2}, -1$

(d) The solutions are the same as the x-coordinates of the x-intercepts.

53. $y = x^5 - 5x^3 + 4x$

(a)



(b) x-intercepts: $(0, 0), (\pm 1, 0), (\pm 2, 0)$

(c) $0 = x^5 - 5x^3 + 4x$

$0 = x(x^2 - 1)(x^2 - 4)$

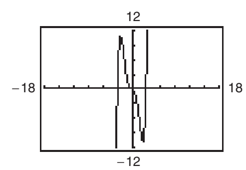
$0 = x(x + 1)(x - 1)(x + 2)(x - 2)$

$x = 0, \pm 1, \pm 2$

(d) The solutions are the same as the x-coordinates of the x-intercepts.

54. $y = \frac{1}{4}x^3(x^2 - 9)$

(a)



(b) x-intercepts: $(0, 0), (3, 0), (-3, 0)$

(c) $0 = \frac{1}{4}x^3(x^2 - 9)$

$x = 0, \pm 3$

(d) The solutions are the same as the x-coordinates of the x-intercepts.

55. $f(x) = (x - 0)(x - 8)$

$= x^2 - 8x$

Note: $f(x) = ax(x - 8)$ has zeros 0 and 8 for all real numbers $a \neq 0$.

56. $f(x) = (x - 0)(x + 7)$

$= x^2 + 7x$

Note: $f(x) = ax(x + 7)$ has zeros 0 and -7 for all real numbers $a \neq 0$.

57. $f(x) = (x - 2)(x + 6)$

$= x^2 + 4x - 12$

Note: $f(x) = a(x - 2)(x + 6)$ has zeros 2 and -6 for all real numbers $a \neq 0$.

58. $f(x) = (x + 4)(x - 5)$

$= x^2 - x - 20$

Note: $f(x) = a(x + 4)(x - 5)$ has zeros -4 and 5 for all real numbers $a \neq 0$.

59. $f(x) = (x - 0)(x + 4)(x + 5)$
$= x(x^2 + 9x + 20)$
$= x^3 + 9x^2 + 20x$

Note: $f(x) = ax(x + 4)(x + 5)$ has zeros $0, -4,$ and -5 for all real numbers $a \neq 0$.

60. $f(x) = (x - 0)(x - 1)(x - 10)$
$= x(x^2 - 11x + 10)$
$= x^3 - 11x^2 + 10x$

Note: $f(x) = ax(x - 1)(x - 10)$ has zeros $0, 1,$ and 10 for all real numbers $a \neq 0$.

61. $f(x) = (x - 4)(x + 3)(x - 3)(x - 0)$
$= (x - 4)(x^2 - 9)x$
$= x^4 - 4x^3 - 9x^2 + 36x$

Note: $f(x) = a(x^4 - 4x^3 - 9x^2 + 36x)$ has zeros $4, -3, 3,$ and 0 for all real numbers $a \neq 0$.

62. $f(x) = (x - (-2))(x - (-1))(x - 0)(x - 1)(x - 2)$
$= x(x + 2)(x + 1)(x - 1)(x - 2)$
$= x(x^2 - 4)(x^2 - 1)$
$= x(x^4 - 5x^2 + 4)$
$= x^5 - 5x^3 + 4x$

Note: $f(x) = ax(x + 2)(x + 1)(x - 1)(x - 2)$ has zeros $-2, -1, 0, 1,$ and 2 for all real numbers $a \neq 0$.

63. $f(x) = \left[x - \left(1 + \sqrt{3}\right)\right]\left[x - \left(1 - \sqrt{3}\right)\right]$
$= \left[(x - 1) - \sqrt{3}\right]\left[(x - 1) + \sqrt{3}\right]$
$= (x - 1)^2 - \left(\sqrt{3}\right)^2$
$= x^2 - 2x + 1 - 3$
$= x^2 - 2x - 2$

Note: $f(x) = a(x^2 - 2x - 2)$ has zeros $1 + \sqrt{3}$ and $1 - \sqrt{3}$ for all real numbers

64. $f(x) = (x - 2)\left[x - \left(4 + \sqrt{5}\right)\right]\left[x - \left(4 - \sqrt{5}\right)\right]$
$= (x - 2)\left[(x - 4) - \sqrt{5}\right]\left[(x - 4) + \sqrt{5}\right]$
$= (x - 2)\left[(x - 4)^2 - 5\right]$
$= x(x - 4)^2 - 5x - 2(x - 4)^2 + 10$
$= x^3 - 8x^2 + 16x - 5x - 2x^2 + 16x - 32 + 10$
$= x^3 - 10x^2 + 27x - 22$

Note: $f(x) = a(x^3 - 10x^2 + 27x - 22)$ has zeros $2, 4 + \sqrt{5},$ and $4 - \sqrt{5}$ for all real numbers $a \neq 0$.

65. $f(x) = (x + 3)(x + 3) = x^2 + 6x + 9$

Note: $f(x) = a(x^2 + 6x + 9), a \neq 0,$ has degree 2 and zero $x = -3$.

66. $f(x) = (x + 12)(x + 6) = x^2 + 18x + 72$

Note: $f(x) = a(x^2 + 18x + 72), a \neq 0,$ has degree 2 and zeros $x = -12$ and -6.

67. $f(x) = (x - 0)(x + 5)(x - 1)$
$= x(x^2 + 4x - 5)$
$= x^3 + 4x^2 - 5x$

Note: $f(x) = ax(x^2 + 4x - 5), a \neq 0,$ has degree 3 and zeros $x = 0, -5,$ and 1.

68. $f(x) = (x + 2)(x - 4)(x - 7)$
$= (x + 2)(x^2 - 11x + 28) = x^3 - 9x^2 + 6x + 56$

Note: $f(x) = a(x^3 - 9x^2 + 6x + 56), a \neq 0,$ has degree 3 and zeros $x = -2, 4,$ and 7.

69. $f(x) = (x - 0)\left(x - \sqrt{3}\right)\left(x - \left(-\sqrt{3}\right)\right)$
$= x\left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right) = x^3 - 3x$

Note: $f(x) = a(x^3 - 3x), a \neq 0,$ has degree 3 and zeros $x = 0, \sqrt{3},$ and $-\sqrt{3}$.

70. $f(x) = (x - 9)^3 = x^3 - 27x^2 + 243x - 729$

Note:
$f(x) = a(x^3 - 27x^2 + 243x - 729), a \neq 0,$ has degree 3 and zero $x = 9$.

71. $f(x) = (x - (-5))^2(x - 1)(x - 2) = x^4 + 7x^3 - 3x^2 - 55x + 50$
or $f(x) = (x - (-5))(x - 1)^2(x - 2) = x^4 + x^3 - 15x^2 + 23x - 10$
or $f(x) = (x - (-5))(x - 1)(x - 2)^2 = x^4 - 17x^2 + 36x - 20$

Note: Any nonzero scalar multiple of these functions would also have degree 4 and zeros $x = -5, 1,$ and 2.

72. $f(x) = (x + 4)(x + 1)(x - 3)(x - 6) = x^4 - 4x^3 - 23x^2 + 54x + 72$

Note: $f(x) = a(x^4 - 4x^3 - 23x^2 + 54x + 72)$, $a \neq 0$, has degree 4 and zeros $x = -4, -1, 3,$ and 6.

73. $f(x) = x^4(x + 4) = x^5 + 4x^4$

or $f(x) = x^3(x + 4)^2 = x^5 + 8x^4 + 16x^3$

or $f(x) = x^2(x + 4)^3 = x^5 + 12x^4 + 48x^3 + 64x^2$

or $f(x) = x(x + 4)^4 = x^5 + 16x^4 + 96x^3 + 256x^2 + 256x$

Note: Any nonzero scalar multiple of these functions would also have degree 5 and zeros $x = 0$ and -4.

74. $f(x) = (x + 1)^2(x - 4)(x - 7)(x - 8) = x^5 - 17x^4 + 79x^3 - 11x^2 - 332x - 224$

or $f(x) = (x + 1)(x - 4)^2(x - 7)(x - 8) = x^5 - 22x^4 + 169x^3 - 496x^2 + 208x + 896$

or $f(x) = (x + 1)(x - 4)(x - 7)^2(x - 8) = x^5 - 25x^4 + 223x^3 - 787x^2 + 532x + 1568$

or $f(x) = (x + 1)(x - 4)(x - 7)(x - 8)^2 = x^5 - 26x^4 + 241x^3 - 884x^2 + 640x + 1792$

Note: Any nonzero scalar multiple of these functions would also have degree 5 and zeros $x = -1, 4, 7,$ and 8.

75. $f(x) = x^3 - 25x = x(x + 5)(x - 5)$

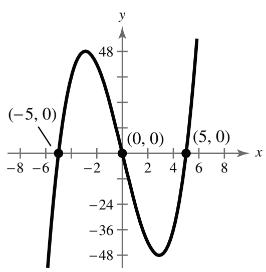
(a) Falls to the left; rises to the right

(b) Zeros: $0, -5, 5$

(c)

x	-2	-1	0	1	2
$f(x)$	42	24	0	-24	-42

(d)



76. $g(x) = x^4 - 9x^2 = x^2(x + 3)(x - 3)$

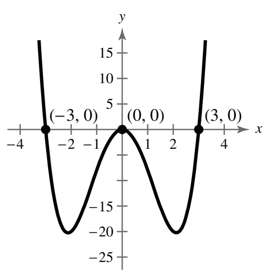
(a) Rises to the left; rises to the right

(b) Zeros: $-3, 0, 3$

(c)

x	-2	-1	0	1	2
$f(x)$	-24	-8	0	-8	-24

(d)



77. $f(t) = \frac{1}{4}(t^2 - 2t + 15) = \frac{1}{4}(t - 1)^2 + \frac{7}{2}$

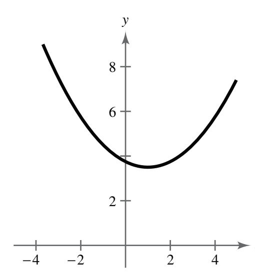
(a) Rises to the left; rises to the right

(b) No real zeros (no x-intercepts)

(c)

t	-1	0	1	2	3
$f(t)$	4.5	3.75	3.5	3.75	4.5

(d) The graph is a parabola with vertex $\left(1, \frac{7}{2}\right)$.



78. $g(x) = -x^2 + 10x - 16 = -(x - 2)(x - 8)$

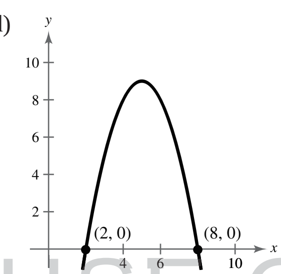
(a) Falls to the left; falls to the right

(b) Zeros: 2, 8

(c)

x	1	3	5	7	9
$g(x)$	-7	5	9	5	-7

(d)



79. $f(x) = x^3 - 2x^2 = x^2(x - 2)$

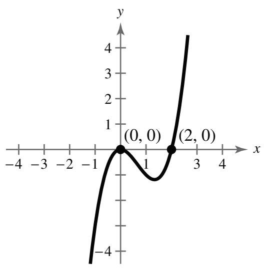
(a) Falls to the left; rises to the right

(b) Zeros: 0, 2

(c)

x	-1	0	$\frac{1}{2}$	1	2	3
$f(x)$	-3	0	$-\frac{3}{8}$	-1	0	9

(d)



80. $f(x) = 8 - x^3 = (2 - x)(4 + 2x + x^2)$

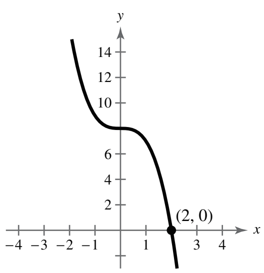
(a) Rises to the left; falls to the right

(b) Zero: 2

(c)

x	-2	-1	0	1	2
$f(x)$	16	9	8	7	0

(d)



81. $f(x) = 3x^3 - 15x^2 + 18x = 3x(x - 2)(x - 3)$

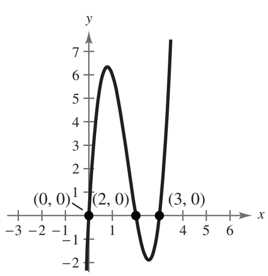
(a) Falls to the left; rises to the right

(b) Zeros: 0, 2, 3

(c)

x	0	1	2	2.5	3	3.5
$f(x)$	0	6	0	-1.875	0	7.875

(d)



82. $f(x) = -4x^3 + 4x^2 + 15x$

$= -x(4x^2 - 4x - 15)$

$= -x(2x - 5)(2x + 3)$

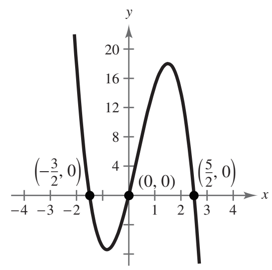
(a) Rises to the left; falls to the right

(b) Zeros: $-\frac{3}{2}, 0, \frac{5}{2}$

(c)

x	-3	-2	-1	0	1	2	3
$f(x)$	99	18	-7	0	15	14	-27

(d)



83. $f(x) = -5x^2 - x^3 = -x^2(5 + x)$

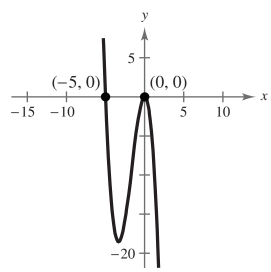
(a) Rises to the left; falls to the right

(b) Zeros: 0, -5

(c)

x	-5	-4	-3	-2	-1	0	1
$f(x)$	0	-16	-18	-12	-4	0	-6

(d)



84. $f(x) = -48x^2 + 3x^4$
$= 3x^2(x^2 - 16)$

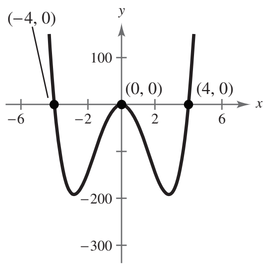
(a) Rises to the left; rises to the right

(b) Zeros; $0, \pm 4$

(c)

x	–5	–4	–3	–2	–1	0	1	2	3	4	5
$f(x)$	675	0	–189	–144	–45	0	–45	–144	–189	0	675

(d)



85. $f(x) = x^2(x - 4)$

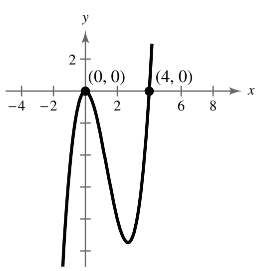
(a) Falls to the left; rises to the right

(b) Zeros: 0, 4

(c)

x	–1	0	1	2	3	4	5
$f(x)$	–5	0	–3	–8	–9	0	25

(d)



86. $h(x) = \frac{1}{3}x^3(x - 4)^2$

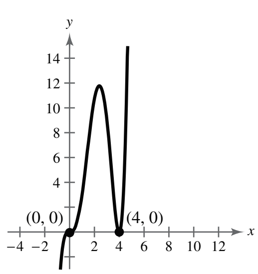
(a) Falls to the left; rises to the right

(b) Zeros: 0, 4

(c)

x	–1	0	1	2	3	4	5
$h(x)$	$-\frac{25}{3}$	0	3	$\frac{32}{3}$	9	0	$\frac{125}{3}$

(d)



87. $g(t) = -\frac{1}{4}(t - 2)^2(t + 2)^2$

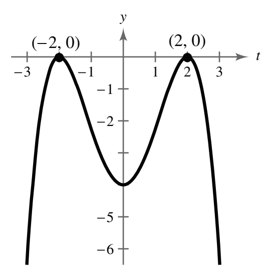
(a) Falls to the left; falls to the right

(b) Zeros: 2, – 2

(c)

t	–3	–2	–1	0	1	2	3
$g(t)$	$-\frac{25}{4}$	0	$-\frac{9}{4}$	–4	$-\frac{9}{4}$	0	$-\frac{25}{4}$

(d)



88. $g(x) = \frac{1}{10}(x + 1)^2(x - 3)^3$

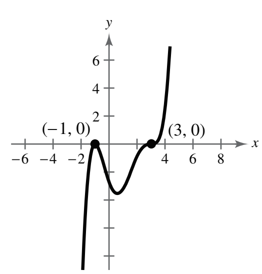
(a) Falls to the left; rises to the right

(b) Zeros: –1, 3

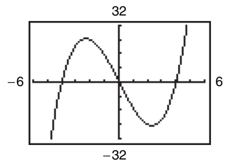
(c)

x	–2	–1	0	1	2	4
$g(x)$	–12.5	0	–2.7	–3.2	–0.9	2.5

(d)

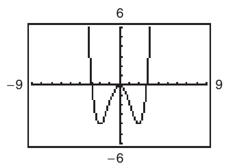


89. $f(x) = x^3 - 16x = x(x - 4)(x + 4)$



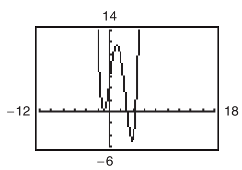
Zeros: 0 of multiplicity 1; 4 of multiplicity 1; and −4 of multiplicity 1

90. $f(x) = \frac{1}{4}x^4 - 2x^2$



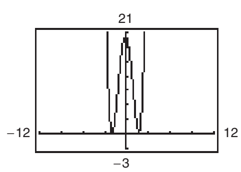
Zeros: −2.828 and 2.828 of multiplicity 1; 0 of multiplicity 2

91. $g(x) = \frac{1}{5}(x + 1)^2(x - 3)(2x - 9)$



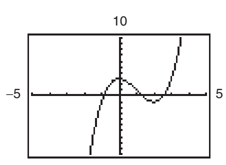
Zeros: −1 of multiplicity 2; 3 of multiplicity 1; $\frac{9}{2}$ of multiplicity 1

92. $h(x) = \frac{1}{5}(x + 2)^2(3x - 5)^2$



Zeros: $-2, \frac{5}{3}$, both with multiplicity 2

93. $f(x) = x^3 - 3x^2 + 3$

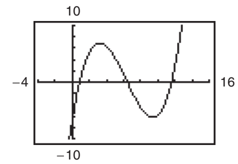


The function has three zeros. They are in the intervals $[-1, 0], [1, 2]$, and $[2, 3]$. They are $x \approx -0.879, 1.347, 2.532$.

x	y
−3	−51
−2	−17
−1	−1
0	3
1	1
2	−1
3	3
4	19

94. $f(x) = 0.11x^3 - 2.07x^2 + 9.81x - 6.88$

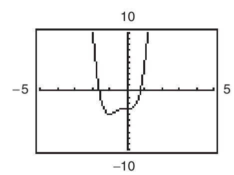
The function has three zeros. They are in the intervals $[0, 1], [6, 7]$, and $[11, 12]$. They are approximately 0.845, 6.385, and 11.588.



x	y
0	−6.88
1	0.97
2	5.34
3	6.89
4	6.28
5	4.17
6	1.12

x	y
7	−1.91
8	−4.56
9	−6.07
10	−5.78
11	3.03
12	2.84

95. $g(x) = 3x^4 + 4x^3 - 3$

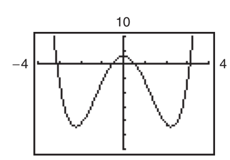


The function has two zeros. They are in the intervals $[-2, -1]$ and $[0, 1]$. They are $x \approx -1.585, 0.779$.

x	y
−4	509
−3	132
−2	13
−1	−4
0	−3
1	4
2	77
3	348

96. $h(x) = x^4 - 10x^2 + 3$

The function has four zeros. They are in the intervals $[-4, -3], [-1, 0], [0, 1]$, and $[3, 4]$. They are approximately ± 3.113 and ± 0.556.



x	y
−4	99
−3	−6
−2	−21
−1	−6
0	3
1	−6
2	−21
3	−6
4	99

97. (a) Volume $= l \cdot w \cdot h$

height $= x$

length $=$ width $= 36 - 2x$

So, $V(x) = (36 - 2x)(36 - 2x)(x) = x(36 - 2x)^2$.

(b) Domain: $0 < x < 18$

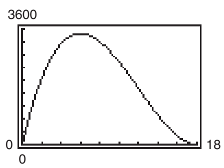
The length and width must be positive.

(c)

Box Height	Box Width	Box Volume, V
1	$36 - 2(1)$	$1[36 - 2(1)]^2 = 1156$
2	$36 - 2(2)$	$2[36 - 2(2)]^2 = 2048$
3	$36 - 2(3)$	$3[36 - 2(3)]^2 = 2700$
4	$36 - 2(4)$	$4[36 - 2(4)]^2 = 3136$
5	$36 - 2(5)$	$5[36 - 2(5)]^2 = 3380$
6	$36 - 2(6)$	$6[36 - 2(6)]^2 = 3456$
7	$36 - 2(7)$	$7[36 - 2(7)]^2 = 3388$

The volume is a maximum of 3456 cubic inches when the height is 6 inches and the length and width are each 24 inches. So the dimensions are $6 \times 24 \times 24$ inches.

(d)



The maximum point on the graph occurs at $x = 6$. This agrees with the maximum found in part (c).

98. (a) Volume $= l \cdot w \cdot h = (24 - 2x)(24 - 4x)x$

$= 2(12 - x) \cdot 4(6 - x)x$

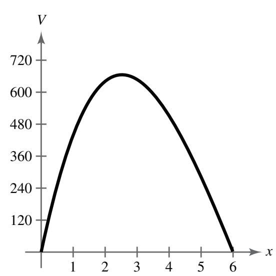
$= 8x(12 - x)(6 - x)$

(b) $x > 0,\quad 12 - x > 0,\quad 6 - x > 0$

$x < 12 \qquad x < 6$

Domain: $0 < x < 6$

(c)



$x \approx 2.5$ corresponds to a maximum of 665 cubic inches.

99. (a) $A = l \cdot w = (12 - 2x)(x) = -2x^2 + 12x$

(b) 16 feet = 192 inches

$V = l \cdot w \cdot h$

$= (12 - 2x)(x)(192)$

$= -384x^2 + 2304x$

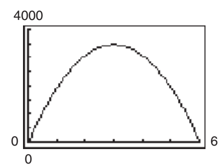
(c) Because x and $12 - 2x$ cannot be negative, we have $0 < x < 6$ inches for the domain.

(d)

x	V
0	0
1	1920
2	3072
3	3456
4	3072
5	1920
6	0

When $x = 3$, the volume is a maximum with $V = 3456$ in.^{3}. The dimensions of the gutter cross-section are 3 inches $\times$ 6 inches $\times$ 3 inches.

(e)



Maximum: (3, 3456)

The maximum value is the same.

(f) No. The volume is a product of the constant length and the cross-sectional area. The value of x would remain the same; only the value of V would change if the length was changed.

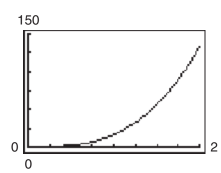
100. (a) $V = \frac{4}{3}\pi r^3 + \pi r^2(4r)$

$V = \frac{4}{3}\pi r^3 + 4\pi r^3$

$= \frac{16}{3}\pi r^3$

(b) $r \geq 0$

(c)

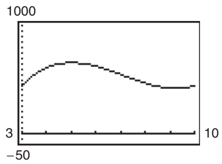


(d) $V = 120 \text{ ft}^3 = \frac{16}{3}\pi r^3$

$r \approx 1.93$ ft

length $= 4r \approx 7.72$ ft

101. (a)



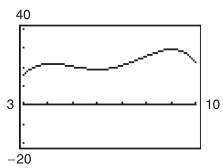
Relative maximum: $(5.01, 655.75)$

Relative minimum: $(9.25, 417.42)$

(b) The revenue is increasing over $(3, 5.01)$ and decreasing over $(5.01, 9.25)$, and then increasing over $(9.25, 10)$.

(c) The revenue for this company is increasing from 2003 to 2005, when it reached a (relative) maximum of \$655.75 million. From 2005 to 2009, revenue was decreasing when it dropped to \$417.42 million. From 2009 to 2010, revenue began to increase again.

102. (a)



Relative maxima: $(4.11, 21.87), (9.02, 29.25)$

Relative minimum: $(6.01, 18.81)$

(b) The revenue was increasing over $(3, 4.11)$ and $(6.01, 9.02)$ and was decreasing over $(4.11, 6.01)$ and $(9.02, 10)$.

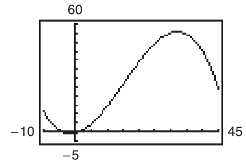
(c) The revenue for this company was increasing from 2003 to 2004 when it reached a (relative) maximum of \$21.87 million. From 2004 to 2006 when revenue dropped to a (relative) minimum of \$18.81 million. From 2006 to 2009, revenue again was increasing to a (relative) maxima of \$29.25 million. From 2009 to 2010, revenue again began to decrease.

103. $R = \dfrac{1}{100{,}000}\left(-x^3 + 600x^2\right)$

The point of diminishing returns (where the graph changes from curving upward to curving downward) occurs when $x = 200$. The point is $(200, 160)$ which corresponds to spending \$2,000,000 on advertising to obtain a revenue of \$160 million.

104. $G = -0.003t^3 + 0.137t^2 + 0.458t - 0.839,\ 2 \le t \le 34$

(a)



(b) The tree is growing most rapidly at $t \approx 15$.

(c) $y = -0.009t^2 + 0.274t + 0.458$

$-\dfrac{b}{2a} = \dfrac{-0.274}{2(-0.009)} \approx 15.222$

$y(15.222) \approx 2.543$

Vertex $\approx (15.22, 2.54)$

(d) The x-value of the vertex in part (c) is approximately equal to the value found in part (b).

105. False. A fifth-degree polynomial can have at most four turning points.

106. False. f has at least one real zero between $x = 2$ and $x = 6$.

107. False. The function $f(x) = (x - 2)^2$ has one turning point and two real (repeated) zeros.

108. True. A polynomial function only falls to the right when the leading coefficient is negative.

109. False. $f(x) = -x^3$ rises to the left.

110. False. The graph rises to the left and to the right.

111. True. A polynomial of degree 7 with a negative leading coefficient rises to the left and falls to the right.

112. (a) Degree: 3
Leading coefficient: Positive

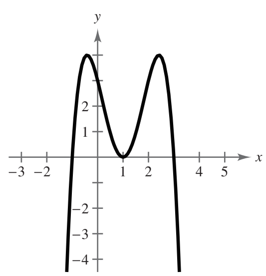
(b) Degree: 2
Leading coefficient: Positive

(c) Degree: 4
Leading coefficient: Positive

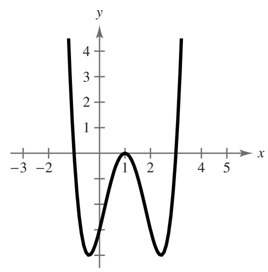
(d) Degree: 5
Leading coefficient: Positive

113. Answers will vary. *Sample answers:*

$a_4 < 0$

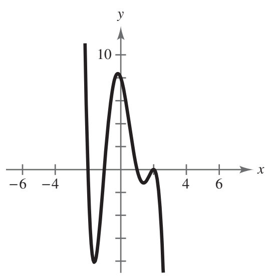


$a_4 > 0$

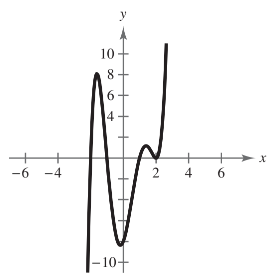


114. Answers will vary. *Sample answers:*

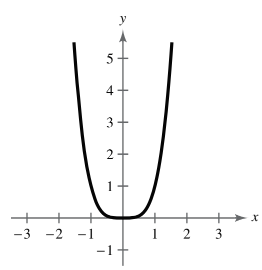
$a_5 < 0$



$a_5 > 0$



115. $f(x) = x^4$; $f(x)$ is even.



(a) $g(x) = f(x) + 2$

Vertical shift two units upward

$$\begin{aligned} g(-x) &= f(-x) + 2 \\ &= f(x) + 2 \\ &= g(x) \end{aligned}$$

Even

(b) $g(x) = f(x + 2)$

Horizontal shift two units to the left

Neither odd nor even

(c) $g(x) = f(-x) = (-x)^4 = x^4$

Reflection in the y-axis. The graph looks the same.

Even

(d) $g(x) = -f(x) = -x^4$

Reflection in the x-axis

Even

(e) $g(x) = f\left(\frac{1}{2}x\right) = \frac{1}{16}x^4$

Horizontal stretch

Even

(f) $g(x) = \frac{1}{2}f(x) = \frac{1}{2}x^4$

Vertical shrink

Even

(g) $g(x) = f\left(x^{3/4}\right) = \left(x^{3/4}\right)^4 = x^3, x \ge 0$

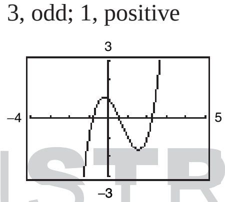
Neither odd nor even

(h) $g(x) = (f \circ f)(x) = f(f(x)) = f\left(x^4\right) = \left(x^4\right)^4 = x^{16}$

Even

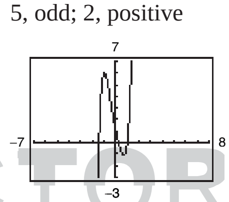
116. (a) $f(x) = x^3 - 2x^2 - x + 1$

3, odd; 1, positive



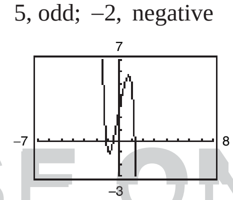
(b) $f(x) = 2x^5 + 2x^2 - 5x + 1$

5, odd; 2, positive



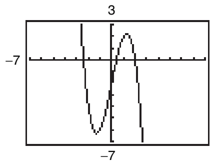
(c) $f(x) = -2x^5 - x^2 + 5x + 3$

5, odd; -2, negative



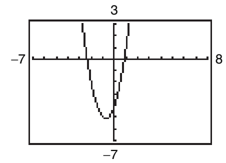
(d) $f(x) = -x^3 + 5x - 2$

3, odd; -1, negative



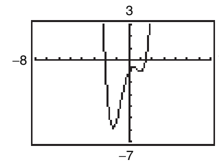
(e) $f(x) = 2x^2 + 3x - 4$

2, even; 2, positive



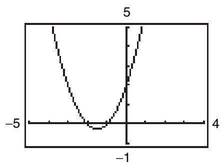
(f) $f(x) = x^4 - 3x^2 + 2x - 1$

4, even; 1, positive



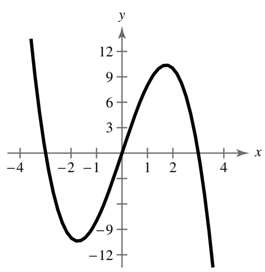
(g) $f(x) = x^2 + 3x + 2$

2, even; 1, positive



When the degree of the function is odd and the leading coefficient is positive, the graph falls to the left and rises to the right, but if the leading coefficient is negative, the graph falls to the right and rises to the left. When the degree of the function is even and the leading coefficient is positive, the graph rises to the left and right, but if the leading coefficient is negative, the graph falls to the left and right.

117. (a)



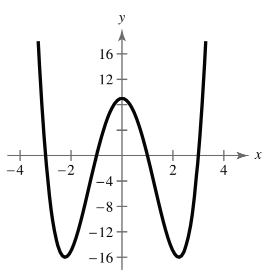
Zeros: 3

Relative minimum: 1

Relative maximum: 1

The number of zeros is the same as the degree and the number of extrema is one less than the degree.

(b)



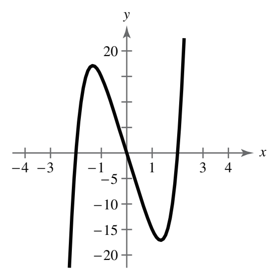
Zeros: 4

Relative minima: 2

Relative maximum: 1

The number of zeros is the same as the degree and the number of extrema is one less than the degree.

(c)



Zeros: 3

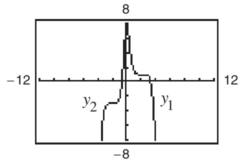
Relative minimum: 1

Relative maximum: 1

The number of zeros and the number of extrema are both less than the degree.

118. (a) $y_1 = -\frac{1}{3}(x - 2)^5 + 1$ is decreasing.

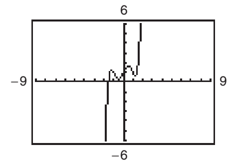
$y_2 = \frac{3}{5}(x + 2)^5 - 3$ is increasing.



(b) The graph is either always increasing or always decreasing. The behavior is determined by a. If $a > 0$, $g(x)$ will always be increasing. If $a < 0$, $g(x)$ will always be decreasing.

(c) $H(x) = x^5 - 3x^3 + 2x + 1$

Because $H(x)$ is not always increasing or always decreasing, $H(x)$ cannot be written in the form $a(x - h)^5 + k$.



Section 2.3 Polynomial and Synthetic Division

1. $f(x)$ is the dividend; $d(x)$ is the divisor: $q(x)$ is the quotient: $r(x)$ is the remainder

2. proper

3. improper

4. synthetic division

5. Factor

6. Remainder

7. $y_1 = \dfrac{x^2}{x + 2}$ and $y_2 = x - 2 + \dfrac{4}{x + 2}$

$$\begin{array}{r|l} & x - 2 \\ x + 2 & x^2 + 0x + 0 \\ & \underline{x^2 + 2x} \\ & -2x + 0 \\ & \underline{-2x - 4} \\ & 4 \end{array}$$

So, $\dfrac{x^2}{x + 2} = x - 2 + \dfrac{4}{x + 2}$ and $y_1 = y_2$.

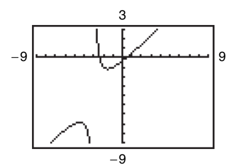
8. $y_1 = \dfrac{x^4 - 3x^2 - 1}{x^2 + 5}$ and $y_2 = x^2 - 8 + \dfrac{39}{x^2 + 5}$

$$\begin{array}{r|l} & x^2 - 8 \\ x^2 + 5 & x^4 - 3x^2 - 1 \\ & \underline{x^4 + 5x^2} \\ & -8x^2 - 1 \\ & \underline{-8x^2 - 40} \\ & 39 \end{array}$$

So, $\dfrac{x^4 - 3x^2 - 1}{x^2 + 5} = x^2 - 8 + \dfrac{39}{x^2 + 5}$ and $y_1 = y_2$.

9. $y_1 = \dfrac{x^2 + 2x - 1}{x + 3}$, $y_2 = x - 1 + \dfrac{2}{x + 3}$

(a) and (b)

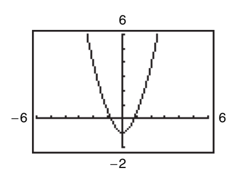


(c)
$$\begin{array}{r|l} & x - 1 \\ x + 3 & x^2 + 2x - 1 \\ & \underline{x^2 + 3x} \\ & -x - 1 \\ & \underline{-x - 3} \\ & 2 \end{array}$$

So, $\dfrac{x^2 + 2x - 1}{x + 3} = x - 1 + \dfrac{2}{x + 3}$ and $y_1 = y_2$.

10. $y_1 = \dfrac{x^4 + x^2 - 1}{x^2 + 1}$, $y_2 = x^2 - \dfrac{1}{x^2 + 1}$

(a) and (b)



(c)
$$\begin{array}{r|l} & x^2 \\ x^2 + 0x + 1 & x^4 + 0x^3 + x^2 + 0x - 1 \\ & \underline{x^4 + 0x^3 + x^2} \\ & -1 \end{array}$$

So, $\dfrac{x^4 + x^2 - 1}{x^2 + 1} = x^2 - \dfrac{1}{x^2 + 1}$ and $y_1 = y_2$.

11.
$$\begin{array}{r}
2x + 4 \\
x + 3\overline{)2x^2 + 10x + 12} \\
\underline{2x^2 + 6x} \\
4x + 12 \\
\underline{4x + 12} \\
0
\end{array}$$

$$\frac{2x^2 + 10x + 12}{x + 3} = 2x + 4,\ x \neq 3$$

12.
$$\begin{array}{r}
5x + 3 \\
x - 4\overline{)5x^2 - 17x - 12} \\
\underline{5x^2 - 20x} \\
3x - 12 \\
\underline{3x - 12} \\
0
\end{array}$$

$$\frac{5x^2 - 17x - 12}{x - 4} = 5x + 3,\ x \neq 4$$

13.
$$\begin{array}{r}
x^2 - 3x + 1 \\
4x + 5\overline{)4x^3 - 7x^2 - 11x + 5} \\
\underline{4x^3 + 5x^2} \\
-12x^2 - 11x \\
\underline{-12x^2 - 15x} \\
4x + 5 \\
\underline{4x + 5} \\
0
\end{array}$$

$$\frac{4x^3 - 7x^2 - 11x + 5}{4x + 5} = x^2 - 3x + 1,\ x \neq -\frac{5}{4}$$

14.
$$\begin{array}{r}
2x^2 - 4x + 3 \\
3x - 2\overline{)6x^3 - 16x^2 + 17x - 6} \\
\underline{6x^3 - 4x^2} \\
-12x^2 + 17x \\
\underline{-12x^2 + 8x} \\
9x - 6 \\
\underline{9x - 6} \\
0
\end{array}$$

$$\frac{6x^3 - 16x^2 + 17x - 6}{3x - 2} = 2x^2 - 4x + 3,\ x \neq \frac{2}{3}$$

15.
$$\begin{array}{r}
x^3 + 3x^2 \quad -1 \\
x + 2\overline{)x^4 + 5x^3 + 6x^2 - x - 2} \\
\underline{x^4 + 2x^3} \\
3x^3 + 6x^2 \\
\underline{3x^3 + 6x^2} \\
-x - 2 \\
\underline{-x - 2} \\
0
\end{array}$$

$$\frac{x^4 + 5x^3 + 6x^2 - x - 2}{x + 2} = x^3 + 3x^2 - 1,\ x \neq -2$$

16.
$$\begin{array}{r}
x^2 + 7x + 18 \\
x - 3\overline{)x^3 + 4x^2 - 3x - 12} \\
\underline{x^3 - 3x^2} \\
7x^2 - 3x \\
\underline{7x^2 - 21x} \\
18x - 12 \\
\underline{18x - 54} \\
42
\end{array}$$

$$\frac{x^3 + 4x^2 - 3x - 12}{x - 3} = x^2 + 7x + 18 + \frac{42}{x - 3}$$

17.
$$\begin{array}{r}
x^2 + 3x + 9 \\
x - 3\overline{)x^3 + 0x^2 + 0x - 27} \\
\underline{x^3 - 3x^2} \\
3x^2 + 0x \\
\underline{3x^2 - 9x} \\
9x - 27 \\
\underline{9x - 27} \\
0
\end{array}$$

$$\frac{x^3 - 27}{x - 3} = x^2 + 3x + 9,\ x \neq 3$$

18.
$$\begin{array}{r}
x^2 - 5x + 25 \\
x + 5\overline{)x^3 + 0x^2 + 0x + 125} \\
\underline{x^3 + 5x^2} \\
-5x^2 + 0x \\
\underline{-5x^2 - 25x} \\
25x + 125 \\
\underline{25x + 125} \\
0
\end{array}$$

$$\frac{x^3 + 125}{x + 5} = x^2 - 5x + 25,\ x \neq -5$$

19.
$$\begin{array}{r}
7 \\
x + 2\overline{)7x + 3} \\
\underline{7x + 14} \\
-11
\end{array}$$

$$\frac{7x + 3}{x + 2} = 7 - \frac{11}{x + 2}$$

20.
$$\begin{array}{r}
4 \\
2x + 1\overline{)8x - 5} \\
\underline{8x + 4} \\
-9
\end{array}$$

$$\frac{8x - 5}{2x + 1} = 4 - \frac{9}{2x + 1}$$

21.
$$\begin{array}{r}
x \\
x^2 + 0x + 1\overline{)x^3 + 0x^2 + 0x - 9} \\
\underline{x^3 + 0x^2 + x} \\
-x - 9
\end{array}$$

$$\frac{x^3 - 9}{x^2 + 1} = x - \frac{x + 9}{x^2 + 1}$$

22.
$$\begin{array}{r} x^2 \\ x^3+0x^2+0x-1\overline{\big)x^5+0x^4+0x^3+0x^2+0x+7} \\ \underline{x^5+0x^4+0x^3-x^2} \\ x^2+7 \end{array}$$

$$\frac{x^5+7}{x^3-1} = x^2 + \frac{x^2+7}{x^3-1}$$

23.
$$\begin{array}{r} 2x-8 \\ x^2+0x+1\overline{\big)2x^3-8x^2+3x-9} \\ \underline{2x^3+0x^2+2x} \\ -8x^2+x-9 \\ \underline{-8x^2-0x-8} \\ x-1 \end{array}$$

$$\frac{2x^3-8x^2+3x-9}{x^2+1} = 2x-8+\frac{x-1}{x^2+1}$$

24.
$$\begin{array}{r} x^2+6x+9 \\ x^2-x-3\overline{\big)x^4+5x^3+0x^2-20x-16} \\ \underline{x^4-x^3-3x^2} \\ 6x^3+3x^2-20x \\ \underline{6x^3-6x^2-18x} \\ 9x^2-2x-16 \\ \underline{9x^2-9x-27} \\ 7x+11 \end{array}$$

$$\frac{x^4+5x^3-20x-16}{x^2-x-3} = x^2+6x+9+\frac{7x+11}{x^2-x-3}$$

25.
$$\begin{array}{r} x+3 \\ x^3-3x^2+3x-1\overline{\big)x^4+0x^3+0x^2+0x+0} \\ \underline{x^4-3x^3+3x^2-x} \\ 3x^3-3x^2+x+0 \\ \underline{3x^3-9x^2+9x-3} \\ 6x^2-8x+3 \end{array}$$

$$\frac{x^4}{(x-1)^3} = x+3+\frac{6x^2-8x+3}{(x-1)^3}$$

26.
$$\begin{array}{r} 2x \\ x^2-2x+1\overline{\big)2x^3-4x^2-15x+5} \\ \underline{2x^3-4x^2+2x} \\ -17x+5 \end{array}$$

$$\frac{2x^3-4x^2-15x+5}{(x-1)^2} = 2x-\frac{17x-5}{x^2-2x+1}$$

27.
$$\begin{array}{r|rrrr} 5 & 3 & -17 & 15 & -25 \\ & & 15 & -10 & 25 \\ \hline & 3 & -2 & 5 & 0 \end{array}$$

$$\frac{3x^3-17x^2+15x-25}{x-5} = 3x^2-2x+5,\ x \neq 5$$

28.
$$\begin{array}{r|rrrr} -3 & 5 & 18 & 7 & -6 \\ & & -15 & -9 & 6 \\ \hline & 5 & 3 & -2 & 0 \end{array}$$

$$\frac{5x^3+18x^2+7x-6}{x+3} = 5x^2+3x-2,\ x \neq -3$$

29.
$$\begin{array}{r|rrrr} 3 & 6 & 7 & -1 & 26 \\ & & 18 & 75 & 222 \\ \hline & 6 & 25 & 74 & 248 \end{array}$$

$$\frac{6x^3+7x^2-x+26}{x-3} = 6x^2+25x+74+\frac{248}{x-3}$$

30.
$$\begin{array}{r|rrrr} -6 & 2 & 14 & -20 & 7 \\ & & -12 & -12 & 192 \\ \hline & 2 & 2 & -32 & 199 \end{array}$$

$$\frac{2x^3+14x^2-20x+7}{x+6} = 2x^2+2x-32+\frac{199}{x+6}$$

31.
$$\begin{array}{r|rrrr} -2 & 4 & 8 & -9 & -18 \\ & & -8 & 0 & 18 \\ \hline & 4 & 0 & -9 & 0 \end{array}$$

$$\frac{4x^3+8x^2-9x-18}{x+2} = 4x^2-9,\ x \neq -2$$

32.
$$\begin{array}{r|rrrr} 2 & 9 & -18 & -16 & 32 \\ & & 18 & 0 & -32 \\ \hline & 9 & 0 & -16 & 0 \end{array}$$

$$\frac{9x^3-18x^2-16x+32}{x-2} = 9x^2-16,\ x \neq 2$$

33.
$$\begin{array}{r|rrrr} -10 & -1 & 0 & 75 & -250 \\ & & 10 & -100 & 250 \\ \hline & -1 & 10 & -25 & 0 \end{array}$$

$$\frac{-x^3+75x-250}{x+10} = -x^2+10x-25,\ x \neq -10$$

34.
$$\begin{array}{r|rrrr} 6 & 3 & -16 & 0 & -72 \\ & & 18 & 12 & 72 \\ \hline & 3 & 2 & 12 & 0 \end{array}$$

$$\frac{3x^3-16x^2-72}{x-6} = 3x^2+2x+12,\ x \neq 6$$

35.
$$\begin{array}{r|rrrr} 4 & 5 & -6 & 0 & 8 \\ & & 20 & 56 & 224 \\ \hline & 5 & 14 & 56 & 232 \end{array}$$

$$\frac{5x^3-6x^2+8}{x-4} = 5x^2+14x+56+\frac{232}{x-4}$$

36.
$$\begin{array}{r|rrrr} -2 & 5 & 0 & 6 & 8 \\ & & -10 & 20 & -52 \\ \hline & 5 & -10 & 26 & -44 \end{array}$$

$$\frac{5x^3 + 6x + 8}{x + 2} = 5x^2 - 10x + 26 - \frac{44}{x + 2}$$

37.
$$\begin{array}{r|rrrrr} 6 & 10 & -50 & 0 & 0 & -800 \\ & & 60 & 60 & 360 & 2160 \\ \hline & 10 & 10 & 60 & 360 & 1360 \end{array}$$

$$\frac{10x^4 - 50x^3 - 800}{x - 6} = 10x^3 + 10x^2 + 60x + 360 + \frac{1360}{x - 6}$$

38.
$$\begin{array}{r|rrrrrr} -3 & 1 & -13 & 0 & 0 & -120 & 80 \\ & & -3 & 48 & -144 & 432 & -936 \\ \hline & 1 & -16 & 48 & -144 & 312 & -856 \end{array}$$

$$\frac{x^5 - 13x^4 - 120x + 80}{x + 3} = x^4 - 16x^3 + 48x^2 - 144x + 312 - \frac{856}{x + 3}$$

39.
$$\begin{array}{r|rrrr} -8 & 1 & 0 & 0 & 512 \\ & & -8 & 64 & -512 \\ \hline & 1 & -8 & 64 & 0 \end{array}$$

$$\frac{x^3 + 512}{x + 8} = x^2 - 8x + 64,\ x \neq -8$$

40.
$$\begin{array}{r|rrrr} 9 & 1 & 0 & 0 & -729 \\ & & 9 & 81 & 729 \\ \hline & 1 & 9 & 81 & 0 \end{array}$$

$$\frac{x^3 - 729}{x - 9} = x^2 + 9x + 81,\ x \neq 9$$

41.
$$\begin{array}{r|rrrrr} 2 & -3 & 0 & 0 & 0 & 0 \\ & & -6 & -12 & -24 & -48 \\ \hline & -3 & -6 & -12 & -24 & -48 \end{array}$$

$$\frac{-3x^4}{x - 2} = -3x^3 - 6x^2 - 12x - 24 - \frac{48}{x - 2}$$

42.
$$\begin{array}{r|rrrrr} -2 & -3 & 0 & 0 & 0 & 0 \\ & & 6 & -12 & 24 & -48 \\ \hline & -3 & 6 & -12 & 24 & -48 \end{array}$$

$$\frac{-3x^4}{x + 2} = -3x^3 + 6x^2 - 12x + 24 - \frac{48}{x + 2}$$

43.
$$\begin{array}{r|rrrrr} 6 & -1 & 0 & 0 & 180 & 0 \\ & & -6 & -36 & -216 & -216 \\ \hline & -1 & -6 & -36 & -36 & -216 \end{array}$$

$$\frac{180x - x^4}{x - 6} = -x^3 - 6x^2 - 36x - 36 - \frac{216}{x - 6}$$

44.
$$\begin{array}{r|rrrr} -1 & -1 & 2 & -3 & 5 \\ & & 1 & -3 & 6 \\ \hline & -1 & 3 & -6 & 11 \end{array}$$

$$\frac{5 - 3x + 2x^2 - x^3}{x + 1} = -x^2 + 3x - 6 + \frac{11}{x + 1}$$

45.
$$\begin{array}{r|rrrr} -\frac{1}{2} & 4 & 16 & -23 & -15 \\ & & -2 & -7 & 15 \\ \hline & 4 & 14 & -30 & 0 \end{array}$$

$$\frac{4x^3 + 16x^2 - 23x - 15}{x + \frac{1}{2}} = 4x^2 + 14x - 30,\ x \neq -\frac{1}{2}$$

46.
$$\begin{array}{r|rrrr} \frac{3}{2} & 3 & -4 & 0 & 5 \\ & & \frac{9}{2} & \frac{3}{4} & \frac{9}{8} \\ \hline & 3 & \frac{1}{2} & \frac{3}{4} & \frac{49}{8} \end{array}$$

$$\frac{3x^3 - 4x^2 + 5}{x - \frac{3}{2}} = 3x^2 + \frac{1}{2}x + \frac{3}{4} + \frac{49}{8x - 12}$$

47. $f(x) = x^3 - x^2 - 14x + 11, k = 4$

$$\begin{array}{r|rrrr} 4 & 1 & -1 & -14 & 11 \\ & & 4 & 12 & -8 \\ \hline & 1 & 3 & -2 & 3 \end{array}$$

$f(x) = (x - 4)(x^2 + 3x - 2) + 3$

$f(4) = 4^3 - 4^2 - 14(4) + 11 = 3$

48. $f(x) = x^3 - 5x^2 - 11x + 8, k = -2$

$$\begin{array}{r|rrrr} -2 & 1 & -5 & -11 & 8 \\ & & -2 & 14 & -6 \\ \hline & 1 & -7 & 3 & 2 \end{array}$$

$f(x) = (x + 2)(x^2 - 7x + 3) + 2$

$f(-2) = (-2)^3 - 5(-2)^2 - 11(-2) + 8 = 2$

49. $f(x) = 15x^4 + 10x^3 - 6x^2 + 14, k = -\frac{2}{3}$

$$\begin{array}{r|rrrrr} -\frac{2}{3} & 15 & 10 & -6 & 0 & 14 \\ & & -10 & 0 & 4 & -\frac{8}{3} \\ \hline & 15 & 0 & -6 & 4 & \frac{34}{3} \end{array}$$

$f(x) = \left(x + \frac{2}{3}\right)(15x^3 - 6x + 4) + \frac{34}{3}$

$f\left(-\frac{2}{3}\right) = 15\left(-\frac{2}{3}\right)^4 + 10\left(-\frac{2}{3}\right)^3 - 6\left(-\frac{2}{3}\right)^2 + 14 = \frac{34}{3}$

50. $f(x) = 10x^3 - 22x^2 - 3x + 4, k = \frac{1}{5}$

$$\begin{array}{r|rrrr} \frac{1}{5} & 10 & -22 & -3 & 4 \\ & & 2 & -4 & -\frac{7}{5} \\ \hline & 10 & -20 & -7 & \frac{13}{5} \end{array}$$

$f(x) = \left(x - \frac{1}{5}\right)(10x^2 - 20x - 7) + \frac{13}{5}$

$f\left(\frac{1}{5}\right) = 10\left(\frac{1}{5}\right)^3 - 22\left(\frac{1}{5}\right)^2 - 3\left(\frac{1}{5}\right) + 4 = \frac{13}{5}$

51. $f(x) = x^3 + 3x^2 - 2x - 14, k = \sqrt{2}$

$$\begin{array}{r|rrrr} \sqrt{2} & 1 & 3 & -2 & -14 \\ & & \sqrt{2} & 2 + 3\sqrt{2} & 6 \\ \hline & 1 & 3 + \sqrt{2} & 3\sqrt{2} & -8 \end{array}$$

$f(x) = (x - \sqrt{2})\left[x^2 + (3 + \sqrt{2})x + 3\sqrt{2}\right] - 8$

$f(\sqrt{2}) = (\sqrt{2})^3 + 3(\sqrt{2})^2 - 2\sqrt{2} - 14 = -8$

52. $f(x) = x^3 + 2x^2 - 5x - 4, k = -\sqrt{5}$

$$\begin{array}{r|rrrr} -\sqrt{5} & 1 & 2 & -5 & -4 \\ & & -\sqrt{5} & -2\sqrt{5} + 5 & 10 \\ \hline & 1 & 2 - \sqrt{5} & -2\sqrt{5} & 6 \end{array}$$

$f(x) = (x + \sqrt{5})\left[x^2 + (2 - \sqrt{5})x - 2\sqrt{5}\right] + 6$

$f(-\sqrt{5}) = (-\sqrt{5})^3 + 2(-\sqrt{5})^2 - 5(-\sqrt{5}) - 4 = 6$

53. $f(x) = -4x^3 + 6x^2 + 12x + 4, k = 1 - \sqrt{3}$

$$\begin{array}{r|rrrr} 1 - \sqrt{3} & -4 & 6 & 12 & 4 \\ & & -4 + 4\sqrt{3} & -10 + 2\sqrt{3} & -4 \\ \hline & -4 & 2 + 4\sqrt{3} & 2 + 2\sqrt{3} & 0 \end{array}$$

$f(x) = (x - 1 + \sqrt{3})\left[-4x^2 + (2 + 4\sqrt{3})x + (2 + 2\sqrt{3})\right]$

$f(1 - \sqrt{3}) = -4(1 - \sqrt{3})^3 + 6(1 - \sqrt{3})^2 + 12(1 - \sqrt{3}) + 4 = 0$

54. $f(x) = -3x^3 + 8x^2 + 10x - 8, k = 2 + \sqrt{2}$

$$\begin{array}{r|rrrr} 2+\sqrt{2} & -3 & 8 & 10 & -8 \\ & & -6-3\sqrt{2} & -2-4\sqrt{2} & 8 \\ \hline & -3 & 2-3\sqrt{2} & 8-4\sqrt{2} & 0 \end{array}$$

$$f(x) = \left(x - 2 - \sqrt{2}\right)\left[-3x^2 + \left(2 - 3\sqrt{2}\right)x + 8 - 4\sqrt{2}\right]$$

$$f\left(2 + \sqrt{2}\right) = -3\left(2 + \sqrt{2}\right)^3 + 8\left(2 + \sqrt{2}\right)^2 + 10\left(2 + \sqrt{2}\right) - 8 = 0$$

55. $f(x) = 2x^3 - 7x + 3$

(a) Using the Remainder Theorem:

$$f(1) = 2(1)^3 - 7(1) + 3 = -2$$

Using synthetic division:

$$\begin{array}{r|rrrr} 1 & 2 & 0 & -7 & 3 \\ & & 2 & 2 & -5 \\ \hline & 2 & 2 & -5 & -2 \end{array}$$

Verify using long division:

$$\begin{array}{r l} & 2x^2 + 2x - 5 \\ x - 1 \big) & 2x^3 + 0x^2 - 7x + 3 \\ & \underline{2x^3 - 2x^2} \\ & 2x^2 - 7x \\ & \underline{2x^2 - 2x} \\ & -5x + 3 \\ & \underline{-5x + 5} \\ & -2 \end{array}$$

(b) Using the Remainder Theorem:

$$f(-2) = 2(-2)^3 - 7(-2) + 3 = 1$$

Using synthetic division:

$$\begin{array}{r|rrrr} -2 & 2 & 0 & -7 & 3 \\ & & -4 & 8 & -2 \\ \hline & 2 & -4 & 1 & 1 \end{array}$$

Verify using long division:

$$\begin{array}{r l} & 2x^2 - 4x + 1 \\ x + 2 \big) & 2x^3 + 0x^2 - 7x + 3 \\ & \underline{2x^3 + 4x^2} \\ & -4x^2 - 7x \\ & \underline{-4x^2 - 8x} \\ & x + 3 \\ & \underline{x + 2} \\ & 1 \end{array}$$

(c) Using the Remainder Theorem:

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 - 7\left(\frac{1}{2}\right) + 3 = -\frac{1}{4}$$

Using synthetic division:

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & 0 & -7 & 3 \\ & & 1 & \frac{1}{2} & -\frac{13}{4} \\ \hline & 2 & 1 & -\frac{13}{2} & -\frac{1}{4} \end{array}$$

Verify using long division:

$$\begin{array}{r l} & 2x^2 + x - \frac{13}{2} \\ x - \frac{1}{2} \big) & 2x^3 + 0x^2 - 7x + 3 \\ & \underline{2x^3 - x^2} \\ & x^2 - 7x \\ & \underline{x^2 - \frac{1}{2}x} \\ & -\frac{13}{2}x + 3 \\ & \underline{-\frac{13}{2}x + \frac{13}{4}} \\ & -\frac{1}{4} \end{array}$$

(d) Using the Remainder Theorem:

$$f(2) = 2(2)^3 - 7(2) + 3 = 5$$

Using synthetic division:

$$\begin{array}{r|rrrr} 2 & 2 & 0 & -7 & 3 \\ & & 4 & 8 & 2 \\ \hline & 2 & 4 & 1 & 5 \end{array}$$

Verify using long division:

$$\begin{array}{r l} & 2x^2 + 4x + 1 \\ x - 2 \big) & 2x^3 + 0x^2 - 7x + 3 \\ & \underline{2x^3 - 4x^2} \\ & 4x^2 - 7x \\ & \underline{4x^2 - 8x} \\ & x + 3 \\ & \underline{x - 2} \\ & 5 \end{array}$$

56. $g(x) = 2x^6 + 3x^4 - x^2 + 3$

(a) Using the Remainder Theorem:

$$g(2) = 2(2)^6 + 3(2)^4 - (2)^2 + 3 = 175$$

Using synthetic division:

$$\begin{array}{r|rrrrrrr} 2 & 2 & 0 & 3 & 0 & -1 & 0 & 3 \\ & & 4 & 8 & 22 & 44 & 86 & 172 \\ \hline & 2 & 4 & 11 & 22 & 43 & 86 & 175 \end{array}$$

Verify using long division:

$$\begin{array}{l}
\qquad\quad 2x^5 + 4x^4 + 11x^3 + 22x^2 + 43x + 86 \\
x - 2\overline{)2x^6 + 0x^5 + 3x^4 + 0x^3 - x^2 + 0x + 3} \\
\qquad\ \underline{2x^6 - 4x^5} \\
\qquad\qquad 4x^5 + 3x^4 \\
\qquad\qquad \underline{4x^5 - 8x^4} \\
\qquad\qquad\qquad 11x^4 + 0x^3 \\
\qquad\qquad\qquad \underline{11x^4 - 22x^3} \\
\qquad\qquad\qquad\qquad 22x^3 - x^2 \\
\qquad\qquad\qquad\qquad \underline{22x^3 - 44x^2} \\
\qquad\qquad\qquad\qquad\qquad 43x^2 + 0x \\
\qquad\qquad\qquad\qquad\qquad \underline{43x^2 - 86x} \\
\qquad\qquad\qquad\qquad\qquad\qquad 86x + 3 \\
\qquad\qquad\qquad\qquad\qquad\qquad \underline{86x - 172} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad 175
\end{array}$$

(b) Using the Remainder Theorem:

$$g(1) = 2(1)^6 + 3(1)^4 - (1)^2 + 3 = 7$$

Using synthetic division:

$$\begin{array}{r|rrrrrrr} 1 & 2 & 0 & 3 & 0 & -1 & 0 & 3 \\ & & 2 & 2 & 5 & 5 & 4 & 4 \\ \hline & 2 & 2 & 5 & 5 & 4 & 4 & 7 \end{array}$$

Verify using long division:

$$\begin{array}{l}
\qquad\quad 2x^5 + 2x^4 + 5x^3 + 5x^2 + 4x + 4 \\
x - 1\overline{)2x^6 + 0x^5 + 3x^4 + 0x^3 - x^2 + 0x + 3} \\
\qquad\ \underline{2x^6 - 2x^5} \\
\qquad\qquad 2x^5 + 3x^4 \\
\qquad\qquad \underline{2x^5 - 2x^4} \\
\qquad\qquad\qquad 5x^4 + 0x^3 \\
\qquad\qquad\qquad \underline{5x^4 - 5x^3} \\
\qquad\qquad\qquad\qquad 5x^3 - x^2 \\
\qquad\qquad\qquad\qquad \underline{5x^3 - 5x^2} \\
\qquad\qquad\qquad\qquad\qquad 4x^2 + 0x \\
\qquad\qquad\qquad\qquad\qquad \underline{4x^2 - 4x} \\
\qquad\qquad\qquad\qquad\qquad\qquad 4x + 3 \\
\qquad\qquad\qquad\qquad\qquad\qquad \underline{4x - 4} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad 7
\end{array}$$

(c) Using the Remainder Theorem:

$$g(3) = 2(3)^6 + 3(3)^4 - (3)^2 + 3 = 1695$$

Using synthetic division:

$$\begin{array}{r|rrrrrrr} 3 & 2 & 0 & 3 & 0 & -1 & 0 & 3 \\ & & 6 & 18 & 63 & 189 & 564 & 1692 \\ \hline & 2 & 6 & 21 & 63 & 188 & 564 & 1695 \end{array}$$

Verify using long division:

$$\begin{array}{l}
\qquad\quad 2x^5 + 6x^4 + 21x^3 + 63x^2 + 188x + 564 \\
x - 3\overline{)2x^6 + 0x^5 + 3x^4 + 0x^3 - x^2 + 0x + 3} \\
\qquad\ \underline{2x^6 - 6x^5} \\
\qquad\qquad 6x^5 + 3x^4 \\
\qquad\qquad \underline{6x^4 - 18x^4} \\
\qquad\qquad\qquad 21x^4 + 0x^3 \\
\qquad\qquad\qquad \underline{21x^4 - 63x^3} \\
\qquad\qquad\qquad\qquad 63x^3 - x^2 \\
\qquad\qquad\qquad\qquad \underline{63x^3 - 189x^2} \\
\qquad\qquad\qquad\qquad\qquad 188x^2 + 0x \\
\qquad\qquad\qquad\qquad\qquad \underline{188x^2 - 564x} \\
\qquad\qquad\qquad\qquad\qquad\qquad 564x + 3 \\
\qquad\qquad\qquad\qquad\qquad\qquad \underline{564x - 1692} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad 1695
\end{array}$$

(d) Using the Remainder Theorem:

$$g(-1) = 2(-1)^6 + 3(-1)^4 - (-1)^2 + 3 = 7$$

Using synthetic division:

$$\begin{array}{r|rrrrrrr} -1 & 2 & 0 & 3 & 0 & -1 & 0 & 3 \\ & & -2 & 2 & -5 & 5 & -4 & 4 \\ \hline & 2 & -2 & 5 & -5 & 4 & -4 & 7 \end{array}$$

Verify using long division:

$$\begin{array}{l}
\qquad\quad 2x^5 - 2x^4 + 5x^3 - 5x^2 + 4x - 4 \\
x + 1\overline{)2x^6 + 0x^5 + 3x^4 + 0x^3 - x^2 + 0x + 3} \\
\qquad\ \underline{2x^6 + 2x^5} \\
\qquad\qquad -2x^5 + 3x^4 \\
\qquad\qquad \underline{-2x^5 - 2x^4} \\
\qquad\qquad\qquad 5x^4 + 0x^3 \\
\qquad\qquad\qquad \underline{5x^4 + 5x^3} \\
\qquad\qquad\qquad\qquad -5x^3 - x^2 \\
\qquad\qquad\qquad\qquad \underline{-5x^3 - 5x^2} \\
\qquad\qquad\qquad\qquad\qquad 4x^2 + 0x \\
\qquad\qquad\qquad\qquad\qquad \underline{4x^2 + 4x} \\
\qquad\qquad\qquad\qquad\qquad\qquad -4x + 3 \\
\qquad\qquad\qquad\qquad\qquad\qquad \underline{-4x - 4} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad 7
\end{array}$$

57. $h(x) = x^3 - 5x^2 - 7x + 4$

(a) Using the Remainder Theorem:

$h(3) = (3)^3 - 5(3)^2 - 7(3) + 4 = -35$

Using synthetic division:

$$\begin{array}{r|rrrr} 3 & 1 & -5 & -7 & 4 \\ & & 3 & -6 & -39 \\ \hline & 1 & -2 & -13 & -35 \end{array}$$

Verify using long division:

$$\begin{array}{r} x^2 - 2x - 13 \\ x - 3\overline{)x^3 - 5x^2 - 7x + 4} \\ \underline{x^3 - 3x^2} \qquad\qquad \\ -2x^2 - 7x \qquad \\ \underline{-2x^2 + 6x} \qquad \\ -13x + 4 \\ \underline{-13x + 39} \\ -35 \end{array}$$

(b) Using the Remainder Theorem:

$h(2) = (2)^3 - 5(2)^2 - 7(2) + 4 = -22$

Using synthetic division:

$$\begin{array}{r|rrrr} 2 & 1 & -5 & -7 & 4 \\ & & 2 & -6 & -26 \\ \hline & 1 & -3 & -13 & -22 \end{array}$$

Verify using long division:

$$\begin{array}{r} x^2 - 3x - 13 \\ x - 2\overline{)x^3 - 5x^2 - 7x + 4} \\ \underline{x^3 - 2x^2} \qquad\qquad \\ -3x^2 - 7x \qquad \\ \underline{-3x^2 + 6x} \qquad \\ -13x + 4 \\ \underline{-13x + 26} \\ -22 \end{array}$$

(c) Using the Remainder Theorem:

$h(-2) = (-2)^3 - 5(-2)^2 - 7(-2) + 4 = -10$

Using synthetic division:

$$\begin{array}{r|rrrr} -2 & 1 & -5 & -7 & 4 \\ & & -2 & 14 & -14 \\ \hline & 1 & -7 & 7 & -10 \end{array}$$

Verify using long division:

$$\begin{array}{r} x^2 - 7x + 7 \\ x + 2\overline{)x^3 - 5x^2 - 7x + 4} \\ \underline{x^3 + 2x^2} \qquad\qquad \\ -7x^2 - 7x \qquad \\ \underline{-7x^2 - 14x} \qquad \\ 7x + 4 \\ \underline{7x + 14} \\ -10 \end{array}$$

(d) Using the Remainder Theorem:

$h(-5) = (-5)^3 - 5(-5)^2 - 7(-5) + 4 = -211$

Using synthetic division:

$$\begin{array}{r|rrrr} -5 & 1 & -5 & -7 & 4 \\ & & -5 & 50 & -215 \\ \hline & 1 & -10 & 43 & -211 \end{array}$$

Verify using long division:

$$\begin{array}{r} x^2 - 10x + 43 \\ x + 5\overline{)x^3 - 5x^2 - 7x + 4} \\ \underline{x^3 + 5x^2} \qquad\qquad \\ -10x^2 - 7x \qquad \\ \underline{-10x^2 - 50x} \qquad \\ 43x + 4 \\ \underline{43x + 215} \\ -211 \end{array}$$

58. $f(x) = 4x^4 - 16x^3 + 7x^2 + 20$

(a) Using the Remainder Theorem:

$f(1) = 4(1)^4 - 16(1)^3 + 7(1) + 20 = 15$

Using synthetic division:

$$\begin{array}{r|rrrrr} 1 & 4 & -16 & 7 & 0 & 20 \\ & & 4 & -12 & -5 & -5 \\ \hline & 4 & -12 & -5 & -5 & 15 \end{array}$$

Verify using long division:

$$\begin{array}{r} 4x^3 - 12x^2 - 5x - 5 \\ x - 1\overline{)4x^4 - 16x^3 + 7x^2 + 0x + 20} \\ \underline{4x^4 - 4x^3} \qquad\qquad\qquad\quad \\ -12x^3 + 7x^2 \qquad\qquad \\ \underline{-12x^3 + 12x^2} \qquad\qquad \\ -5x^2 + 0x \qquad \\ \underline{-5x^2 + 5x} \qquad \\ -5x + 20 \\ \underline{-5x + 5} \\ 15 \end{array}$$

(b) Using the Remainder Theorem:

$f(-2) = 4(-2)^4 - 16(-2)^3 + 7(-2)^2 + 20 = 240$

Using synthetic division:

$$\begin{array}{r|rrrrr} -2 & 4 & -16 & 7 & 0 & 20 \\ & & -8 & 48 & -110 & 220 \\ \hline & 4 & -24 & 55 & -110 & 240 \end{array}$$

Verify using long division:

$$\begin{array}{r} 4x^3 - 24x^2 + 55x - 110 \\ x + 2\overline{)4x^4 - 16x^3 + 7x^2 + 0x + 20} \\ \underline{4x^3 + 8x^3} \qquad\qquad\qquad\quad \\ -24x^3 + 7x^2 \qquad\qquad \\ \underline{-24x^3 - 48x^2} \qquad\qquad \\ 55x^2 + 0x \qquad \\ \underline{55x^2 + 110x} \qquad \\ -110x + 20 \\ \underline{-110x - 220} \\ 240 \end{array}$$

(c) Using the Remainder Theorem:

$$f(5) = 4(5)^4 - 16(5)^3 + 7(5)^2 + 20 = 695$$

Using synthetic division:

$$\begin{array}{r|rrrrr} 5 & 4 & -16 & 7 & 0 & 20 \\ & & 20 & 20 & 135 & 675 \\ \hline & 4 & 4 & 27 & 135 & 695 \end{array}$$

Verify using long division:

$$\begin{array}{r} 4x^3 + 4x^2 + 27x + 135 \\ x - 5\overline{)4x^4 - 16x^3 + 7x^2 + 0x + 20} \\ \underline{4x^4 - 20x^3} \qquad\qquad\qquad\quad \\ 4x^3 + 7x^2 \qquad\qquad\quad \\ \underline{4x^3 - 20x^2} \qquad\qquad\quad \\ 27x^2 + 0x \qquad\quad \\ \underline{27x^2 - 135x} \qquad\quad \\ 135x + 20 \\ \underline{135x - 675} \\ 695 \end{array}$$

(d) Using the Remainder Theorem:

$$f(-10) = 4(-10)^4 - 16(-10)^3 + 7(-10)^2 + 20 = 56{,}720$$

Using synthetic division:

$$\begin{array}{r|rrrrr} -10 & 4 & -16 & 7 & 0 & 20 \\ & & -40 & 560 & -5670 & 56{,}700 \\ \hline & 4 & -56 & 567 & -5670 & 56{,}720 \end{array}$$

Verify using long division:

$$\begin{array}{r} 4x^3 - 56x^2 + 567x - 5670 \\ x + 10\overline{)4x^4 - 16x^3 + 7x^2 + 0x + 20} \\ \underline{4x^4 + 40x^3} \qquad\qquad\qquad\qquad \\ -56x^3 + 7x^2 \qquad\qquad\qquad \\ \underline{-56x^3 - 560x^2} \qquad\qquad\qquad \\ 567x^2 + 0x \qquad\qquad \\ \underline{567x^2 + 5670x} \qquad\qquad \\ -5670x + 20 \\ \underline{-5670x - 56{,}700} \\ 56{,}720 \end{array}$$

59.

$$\begin{array}{r|rrrr} 2 & 1 & 0 & -7 & 6 \\ & & 2 & 4 & -6 \\ \hline & 1 & 2 & -3 & 0 \end{array}$$

$$\begin{aligned} x^3 - 7x + 6 &= (x - 2)(x^2 + 2x - 3) \\ &= (x - 2)(x + 3)(x - 1) \end{aligned}$$

Zeros: $2, -3, 1$

60.

$$\begin{array}{r|rrrr} -4 & 1 & 0 & -28 & -48 \\ & & -4 & 16 & 48 \\ \hline & 1 & -4 & -12 & 0 \end{array}$$

$$\begin{aligned} x^3 - 28x - 48 &= (x + 4)(x^2 - 4x - 12) \\ &= (x + 4)(x - 6)(x + 2) \end{aligned}$$

Zeros: $-4, -2, 6$

61.

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -15 & 27 & -10 \\ & & 1 & -7 & 10 \\ \hline & 2 & -14 & 20 & 0 \end{array}$$

$$\begin{aligned} 2x^3 - 15x^2 + 27x - 10 &= \left(x - \tfrac{1}{2}\right)(2x^2 - 14x + 20) \\ &= (2x - 1)(x - 2)(x - 5) \end{aligned}$$

Zeros: $\frac{1}{2}, 2, 5$

62.

$$\begin{array}{r|rrrr} \frac{2}{3} & 48 & -80 & 41 & -6 \\ & & 32 & -32 & 6 \\ \hline & 48 & -48 & 9 & 0 \end{array}$$

$$\begin{aligned} 48x^3 - 80x^2 + 41x - 6 &= \left(x - \tfrac{2}{3}\right)(48x^2 - 48x + 9) \\ &= \left(x - \tfrac{2}{3}\right)(4x - 3)(12x - 3) \\ &= (3x - 2)(4x - 3)(4x - 1) \end{aligned}$$

Zeros: $\frac{2}{3}, \frac{3}{4}, \frac{1}{4}$

63.

$$\begin{array}{r|rrrr} \sqrt{3} & 1 & 2 & -3 & -6 \\ & & \sqrt{3} & 3 + 2\sqrt{3} & 6 \\ \hline & 1 & 2 + \sqrt{3} & 2\sqrt{3} & 0 \end{array}$$

$$\begin{array}{r|rrr} -\sqrt{3} & 1 & 2 + \sqrt{3} & 2\sqrt{3} \\ & & -\sqrt{3} & -2\sqrt{3} \\ \hline & 1 & 2 & 0 \end{array}$$

$$x^3 + 2x^2 - 3x - 6 = (x - \sqrt{3})(x + \sqrt{3})(x + 2)$$

Zeros: $-\sqrt{3}, \sqrt{3}, -2$

64.

$$\begin{array}{r|rrrr} \sqrt{2} & 1 & 2 & -2 & -4 \\ & & \sqrt{2} & 2\sqrt{2} + 2 & 4 \\ \hline & 1 & 2 + \sqrt{2} & 2\sqrt{2} & 0 \end{array}$$

$$\begin{array}{r|rrr} -\sqrt{2} & 1 & 2 + \sqrt{2} & 2\sqrt{2} \\ & & -\sqrt{2} & -2\sqrt{2} \\ \hline & 1 & 2 & 0 \end{array}$$

$$x^3 + 2x^2 - 2x - 4 = (x - \sqrt{2})(x + 2)(x + \sqrt{2})$$

Zeros: $-2, -\sqrt{2}, \sqrt{2}$

65.

$$\begin{array}{r|rrrr} 1+\sqrt{3} & 1 & -3 & 0 & 2 \\ & & 1+\sqrt{3} & 1-\sqrt{3} & -2 \\ \hline & 1 & -2+\sqrt{3} & 1-\sqrt{3} & 0 \end{array}$$

$$\begin{array}{r|rrr} 1-\sqrt{3} & 1 & -2+\sqrt{3} & 1-\sqrt{3} \\ & & 1-\sqrt{3} & -1+\sqrt{3} \\ \hline & 1 & -1 & 0 \end{array}$$

$$\begin{aligned} x^3 - 3x^2 + 2 &= \left[x - \left(1+\sqrt{3}\right)\right]\left[x - \left(1-\sqrt{3}\right)\right](x-1) \\ &= (x-1)\left(x-1-\sqrt{3}\right)\left(x-1+\sqrt{3}\right) \end{aligned}$$

Zeros: $1, 1-\sqrt{3}, 1+\sqrt{3}$

66.

$$\begin{array}{r|rrrr} 2-\sqrt{5} & 1 & -1 & -13 & -3 \\ & & 2-\sqrt{5} & 7-3\sqrt{5} & 3 \\ \hline & 1 & 1-\sqrt{5} & -6-3\sqrt{5} & 0 \end{array}$$

$$\begin{array}{r|rrr} 2+\sqrt{5} & 1 & 1-\sqrt{5} & -6-3\sqrt{5} \\ & & 2+\sqrt{5} & 6+3\sqrt{5} \\ \hline & 1 & 3 & 0 \end{array}$$

$$x^3 - x^2 - 13x - 3 = \left(x-2+\sqrt{5}\right)\left(x-2-\sqrt{5}\right)(x+3)$$

Zeros: $2-\sqrt{5}, 2+\sqrt{5}, -3$

67. $f(x) = 2x^3 + x^2 - 5x + 2$; Factors: $(x+2), (x-1)$

(a)
$$\begin{array}{r|rrrr} -2 & 2 & 1 & -5 & 2 \\ & & -4 & 6 & -2 \\ \hline & 2 & -3 & 1 & 0 \end{array}$$

$$\begin{array}{r|rrr} 1 & 2 & -3 & 1 \\ & & 2 & -1 \\ \hline & 2 & -1 & 0 \end{array}$$

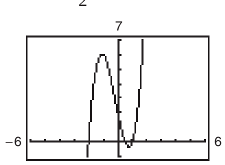
Both are factors of $f(x)$ because the remainders are zero.

(b) The remaining factor of $f(x)$ is $(2x-1)$.

(c) $f(x) = (2x-1)(x+2)(x-1)$

(d) Zeros: $\frac{1}{2}, -2, 1$

(e)



68. $f(x) = 3x^3 + 2x^2 - 19x + 6$;
Factors: $(x+3), (x-2)$

(a)
$$\begin{array}{r|rrrr} -3 & 3 & 2 & -19 & 6 \\ & & -9 & 21 & -6 \\ \hline & 3 & -7 & 2 & 0 \end{array}$$

$$\begin{array}{r|rrr} 2 & 3 & -7 & 2 \\ & & 6 & -2 \\ \hline & 3 & -1 & 0 \end{array}$$

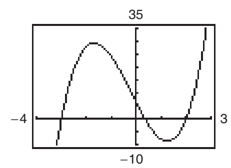
Both are factors of $f(x)$ because the remainders are zero.

(b) The remaining factor is $(3x-1)$.

(c)
$$\begin{aligned} f(x) &= 3x^3 + 2x^2 - 19x + 6 \\ &= (3x-1)(x+3)(x-2) \end{aligned}$$

(d) Zeros: $\frac{1}{3}, -3, 2$

(e)



69. $f(x) = x^4 - 4x^3 - 15x^2 + 58x - 40$;
Factors: $(x-5), (x+4)$

(a)
$$\begin{array}{r|rrrrr} 5 & 1 & -4 & -15 & 58 & -40 \\ & & 5 & 5 & -50 & 40 \\ \hline & 1 & 1 & -10 & 8 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -4 & 1 & 1 & -10 & 8 \\ & & -4 & 12 & -8 \\ \hline & 1 & -3 & 2 & 0 \end{array}$$

Both are factors of $f(x)$ because the remainders are zero.

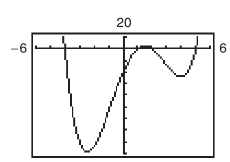
(b) $x^2 - 3x + 2 = (x-1)(x-2)$

The remaining factors are $(x-1)$ and $(x-2)$.

(c) $f(x) = (x-1)(x-2)(x-5)(x+4)$

(d) Zeros: $1, 2, 5, -4$

(e)



70. $f(x) = 8x^4 - 14x^3 - 71x^2 - 10x + 24$;

Factors: $(x + 2), (x - 4)$

(a)
$$\begin{array}{r|rrrrr} -2 & 8 & -14 & -71 & -10 & 24 \\ & & -16 & 60 & 22 & -24 \\ \hline & 8 & -30 & -11 & 12 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 4 & 8 & -30 & -11 & 12 \\ & & 32 & 8 & -12 \\ \hline & 8 & 2 & -3 & 0 \end{array}$$

Both are factors of $f(x)$ because the remainders are zero.

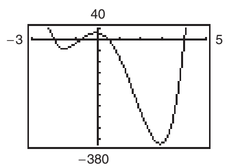
(b) $8x^2 + 2x - 3 = (4x + 3)(2x - 1)$

The remaining factors are $(4x + 3)$ and $(2x - 1)$.

(c) $f(x) = (4x + 3)(2x - 1)(x + 2)(x - 4)$

(d) Zeros: $-\frac{3}{4}, \frac{1}{2}, -2, 4$

(e)



71. $f(x) = 6x^3 + 41x^2 - 9x - 14$;

Factors: $(2x + 1), (3x - 2)$

(a)
$$\begin{array}{r|rrrr} -\frac{1}{2} & 6 & 41 & -9 & -14 \\ & & -3 & -19 & 14 \\ \hline & 6 & 38 & -28 & 0 \end{array}$$

$$\begin{array}{r|rrr} \frac{2}{3} & 6 & 38 & -28 \\ & & 4 & 28 \\ \hline & 6 & 42 & 0 \end{array}$$

Both are factors of $f(x)$ because the remainders are zero.

(b) $6x + 42 = 6(x + 7)$

This shows that $\dfrac{f(x)}{\left(x + \frac{1}{2}\right)\left(x - \frac{2}{3}\right)} = 6(x + 7)$,

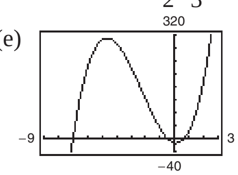
so $\dfrac{f(x)}{(2x + 1)(3x - 2)} = x + 7$.

The remaining factor is $(x + 7)$.

(c) $f(x) = (x + 7)(2x + 1)(3x - 2)$

(d) Zeros: $-7, -\dfrac{1}{2}, \dfrac{2}{3}$

(e)



72. $f(x) = 10x^3 - 11x^2 - 72x + 45$;

Factors: $(2x + 5), (5x - 3)$

(a)
$$\begin{array}{r|rrrr} -\frac{5}{2} & 10 & -11 & -72 & 45 \\ & & -25 & 90 & -45 \\ \hline & 10 & -36 & 18 & 0 \end{array}$$

$$\begin{array}{r|rrr} \frac{3}{5} & 10 & -36 & 18 \\ & & 6 & -18 \\ \hline & 10 & -30 & 0 \end{array}$$

Both are factors of $f(x)$ because the remainders are zero.

(b) $10x - 30 = 10(x - 3)$

This shows that $\dfrac{f(x)}{\left(x + \frac{5}{2}\right)\left(x - \frac{3}{5}\right)} = 10(x - 3)$,

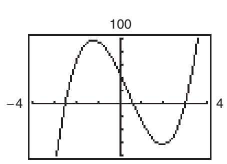
so $\dfrac{f(x)}{(2x + 5)(5x - 3)} = x - 3$.

The remaining factor is $(x - 3)$.

(c) $f(x) = (x - 3)(2x + 5)(5x - 3)$

(d) Zeros: $3, -\dfrac{5}{2}, \dfrac{3}{5}$

(e)



73. $f(x) = 2x^3 - x^2 - 10x + 5;$

Factors: $(2x - 1), \left(x + \sqrt{5}\right)$

(a)
$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -1 & -10 & 5 \\ & & 1 & 0 & -5 \\ \hline & 2 & 0 & -10 & 0 \end{array}$$

$$\begin{array}{r|rrr} -\sqrt{5} & 2 & 0 & -10 \\ & & -2\sqrt{5} & 10 \\ \hline & 2 & -2\sqrt{5} & 0 \end{array}$$

Both are factors of $f(x)$ because the remainders are zero.

(b) $2x - 2\sqrt{5} = 2\left(x - \sqrt{5}\right)$

This shows that $\dfrac{f(x)}{\left(x - \frac{1}{2}\right)\left(x + \sqrt{5}\right)} = 2\left(x - \sqrt{5}\right),$

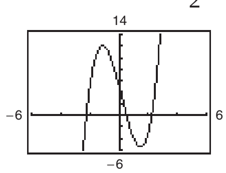
so $\dfrac{f(x)}{(2x - 1)\left(x + \sqrt{5}\right)} = x - \sqrt{5}.$

The remaining factor is $\left(x - \sqrt{5}\right)$.

(c) $f(x) = \left(x + \sqrt{5}\right)\left(x - \sqrt{5}\right)(2x - 1)$

(d) Zeros: $-\sqrt{5}, \sqrt{5}, \dfrac{1}{2}$

(e)



74. $f(x) = x^3 + 3x^2 - 48x - 144;$

Factors: $\left(x + 4\sqrt{3}\right), (x + 3)$

(a)
$$\begin{array}{r|rrrr} -3 & 1 & 3 & -48 & -144 \\ & & -3 & 0 & 144 \\ \hline & 1 & 0 & -48 & 0 \end{array}$$

$$\begin{array}{r|rrr} -4\sqrt{3} & 1 & 0 & -48 \\ & & -4\sqrt{3} & 48 \\ \hline & 1 & -4\sqrt{3} & 0 \end{array}$$

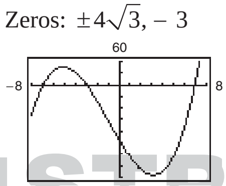
Both are factors of $f(x)$ because the remainders are zero.

(b) The remaining factor is $\left(x - 4\sqrt{3}\right)$.

(c) $f(x) = \left(x - 4\sqrt{3}\right)\left(x + 4\sqrt{3}\right)(x + 3)$

(d) Zeros: $\pm 4\sqrt{3}, -3$

(e)



75. $f(x) = x^3 - 2x^2 - 5x + 10$

(a) The zeros of f are $x = 2$ and $x \approx \pm 2.236$.

(b) An exact zero is $x = 2$.

(c)
$$\begin{array}{r|rrrr} 2 & 1 & -2 & -5 & 10 \\ & & 2 & 0 & -10 \\ \hline & 1 & 0 & -5 & 0 \end{array}$$

$$\begin{aligned} f(x) &= (x - 2)\left(x^2 - 5\right) \\ &= (x - 2)\left(x - \sqrt{5}\right)\left(x + \sqrt{5}\right) \end{aligned}$$

76. $g(x) = x^3 - 4x^2 - 2x + 8$

(a) The zeros of g are $x = 4, x \approx -1.414, x \approx 1.414$.

(b) $x = 4$ is an exact zero.

(c)
$$\begin{array}{r|rrrr} 4 & 1 & -4 & -2 & 8 \\ & & 4 & 0 & -8 \\ \hline & 1 & 0 & -2 & 0 \end{array}$$

$$\begin{aligned} g(x) &= (x - 4)\left(x^2 - 2\right) \\ &= (x - 4)\left(x - \sqrt{2}\right)\left(x + \sqrt{2}\right) \end{aligned}$$

77. $h(t) = t^3 - 2t^2 - 7t + 2$

(a) The zeros of h are $t = -2, t \approx 3.732, t \approx 0.268$.

(b) An exact zero is $t = -2$.

(c)
$$\begin{array}{r|rrrr} -2 & 1 & -2 & -7 & 2 \\ & & -2 & 8 & -2 \\ \hline & 1 & -4 & 1 & 0 \end{array}$$

$h(t) = (t + 2)\left(t^2 - 4t + 1\right)$

By the Quadratic Formula, the zeros of $t^2 - 4t + 1$ are $2 \pm \sqrt{3}$. Thus,

$h(t) = (t + 2)\left[t - \left(2 + \sqrt{3}\right)\right]\left[t - \left(2 - \sqrt{3}\right)\right].$

78. $f(s) = s^3 - 12s^2 + 40s - 24$

(a) The zeros of f are $s = 6, s \approx 0.764, s \approx 5.236$

(b) $s = 6$ is an exact zero.

(c)
$$\begin{array}{r|rrrr} 6 & 1 & -12 & 40 & -24 \\ & & 6 & -36 & 24 \\ \hline & 1 & -6 & 4 & 0 \end{array}$$

$$\begin{aligned} f(s) &= (s-6)(s^2 - 6s + 4) \\ &= (s-6)\left[s - \left(3 + \sqrt{5}\right)\right]\left[s - \left(3 - \sqrt{5}\right)\right] \end{aligned}$$

79. $h(x) = x^5 - 7x^4 + 10x^3 + 14x^2 - 24x$

(a) The zeros of h are $x = 0, x = 3, x = 4,$ $x \approx 1.414, x \approx -1.414.$

(b) An exact zero is $x = 4$.

(c)
$$\begin{array}{r|rrrrr} 4 & 1 & -7 & 10 & 14 & -24 \\ & & 4 & -12 & -8 & 24 \\ \hline & 1 & -3 & -2 & 6 & 0 \end{array}$$

$$\begin{aligned} h(x) &= (x-4)(x^4 - 3x^3 - 2x^2 + 6x) \\ &= x(x-4)(x-3)\left(x + \sqrt{2}\right)\left(x - \sqrt{2}\right) \end{aligned}$$

80. $g(x) = 6x^4 - 11x^3 - 51x^2 + 99x - 27$

(a) The zeros of a are $x = 3, x = -3, x = 1.5,$ $x \approx 0.333.$

(b) An exact zero is $x = -3$.

(c)
$$\begin{array}{r|rrrrr} -3 & 6 & -11 & -51 & 99 & -27 \\ & & -18 & 87 & -108 & 27 \\ \hline & 6 & -29 & 36 & -9 & 0 \end{array}$$

$$\begin{aligned} a(x) &= (x+3)(6x^3 - 29x^2 + 36x - 9) \\ &= (x+3)(x-3)(2x-3)(3x-1) \end{aligned}$$

81. $\dfrac{4x^3 - 8x^2 + x + 3}{2x - 3}$

$$\begin{array}{r|rrrr} \frac{3}{2} & 4 & -8 & 1 & 3 \\ & & 6 & -3 & -3 \\ \hline & 4 & -2 & -2 & 0 \end{array}$$

$$\frac{4x^3 - 8x^2 + x + 3}{x - \frac{3}{2}} = 4x^2 - 2x - 2 = 2(2x^2 - x - 1)$$

So, $\dfrac{4x^3 - 8x^2 + x + 3}{2x - 3} = 2x^2 - x - 1, x \neq \dfrac{3}{2}.$

82. $\dfrac{x^3 + x^2 - 64x - 64}{x + 8}$

$$\begin{array}{r|rrrr} -8 & 1 & 1 & -64 & -64 \\ & & -8 & 56 & 64 \\ \hline & 1 & -7 & -8 & 0 \end{array}$$

$$\frac{x^3 + x^2 - 64x - 64}{x + 8} = x^2 - 7x - 8, x \neq -8$$

83. $\dfrac{x^4 + 6x^3 + 11x^2 + 6x}{x^2 + 3x + 2} = \dfrac{x^4 + 6x^3 + 11x^2 + 6x}{(x+1)(x+2)}$

$$\begin{array}{r|rrrrr} -1 & 1 & 6 & 11 & 6 & 0 \\ & & -1 & -5 & -6 & 0 \\ \hline & 1 & 5 & 6 & 0 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 1 & 5 & 6 & 0 \\ & & -2 & -6 & 0 \\ \hline & 1 & 3 & 0 & 0 \end{array}$$

$$\frac{x^4 + 6x^3 + 11x^2 + 6x}{(x+1)(x+2)} = x^2 + 3x, x \neq -2, -1$$

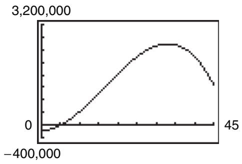
84. $\dfrac{x^4 + 9x^3 - 5x^2 - 36x + 4}{x^2 - 4} = \dfrac{x^4 + 9x^3 - 5x^2 - 36x + 4}{(x+2)(x-2)}$

$$\begin{array}{r|rrrrr} 2 & 1 & 9 & -5 & -36 & 4 \\ & & 2 & 22 & 34 & -4 \\ \hline & 1 & 11 & 17 & -2 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 1 & 11 & 17 & -2 \\ & & -2 & -18 & 2 \\ \hline & 1 & 9 & -1 & 0 \end{array}$$

$$\frac{x^4 + 9x^3 - 5x^2 - 36x + 4}{x^2 - 4} = x^2 + 9x - 1, x \neq \pm 2$$

85. (a)



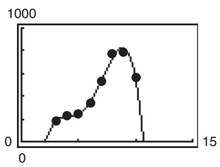
(b) Using the trace and zoom features, when $x = 25$, an advertising expense of about \$250,000 would produce the same profit of \$2,174,375.

(c) $x = 25$

25	−152	7545	0	−169,625
		−3800	93,625	2,340,625
	−152	3745	93,625	2,171,000

So, an advertising expense of \$250,000 yields a profit of \$2,171,000, which is close to \$2,174,375.

86. (a) and (b)



$N = -3.1705t^4 + 71.205t^3 - 551.75t^2 + 1821.2t - 1985$

(c)

t	3	4	5	6	7	8	9	10
N	179	217	246	351	539	743	821	552

The estimated values are close to the original data values.

(d)

10	−3.1705	71.205	−551.75	1821.2	−1985
		−31.705	395	−1567.5	2537
		39.5	−156.75	253.7	552

Because the remainder is $r = 552$, you can conclude that $N(10) = 552$. This confirms the estimated value.

87. False. If $(7x + 4)$ is a factor of f, then $-\frac{4}{7}$ is a zero of f.

88. True.

$\frac{1}{2}$	6	1	−92	45	184	4	−48
		3	2	−45	0	92	48
	6	4	−90	0	184	96	0

$f(x) = (2x - 1)(x + 1)(x - 2)(x - 3)(3x + 2)(x + 4)$

89. True. The degree of the numerator is greater than the degree of the denominator.

90. False. The equation $\dfrac{x^3 - 3x^2 + 4}{x + 1} = x^2 - 4x + 4$ it is not true for $x = -1$ since this value would result in division by zero in the original equation. So, the equation should be written as $\dfrac{x^3 - 3x^2 + 4}{x + 1} = x^2 - 4x + 4$, $x \neq -1$.

91.
$$
\begin{array}{r}
x^{2n} + 6x^n + 9 \\
x^n + 3 \overline{\big) x^{3n} + 9x^{2n} + 27x^n + 27} \\
\underline{x^{3n} + 3x^{2n}} \qquad\qquad\quad \\
6x^{2n} + 27x^n \qquad \\
\underline{6x^{2n} + 18x^n} \qquad \\
9x^n + 27 \\
\underline{9x^n + 27} \\
0
\end{array}
$$

$$\frac{x^{3n} + 9x^{2n} + 27x^n + 27}{x^n + 3} = x^{2n} + 6x^n + 9,\ x^n \neq -3$$

92.
$$
\begin{array}{r}
x^{2n} - x^n + 3 \\
x^n - 2 \overline{\big) x^{3n} - 3x^{2n} + 5x^n - 6} \\
\underline{x^{3n} - 2x^{2n}} \qquad\qquad\quad \\
-x^{2n} + 5x^n \qquad \\
\underline{-x^{2n} + 2x^n} \qquad \\
3x^n - 6 \\
\underline{3x^n - 6} \\
0
\end{array}
$$

$$\frac{x^{3n} - 3x^{2n} + 5x^n - 6}{x^n - 2} = x^{2n} - x^n + 3,\ x^n \neq 2$$

Section 2.4 Complex Numbers

1. real

2. imaginary

3. pure imaginary

4. $\sqrt{-1}$; -1

5. principal square

6. complex conjugates

7. $a + bi = -12 + 7i$
$$a = -12$$
$$b = 7$$

8. $a + bi = 13 + 4i$
$$a = 13$$
$$b = 4$$

9. $(a - 1) + (b + 3)i = 5 + 8i$
$$a - 1 = 5 \Rightarrow a = 6$$
$$b + 3 = 8 \Rightarrow b = 5$$

10. $(a + 6) + 2bi = 6 - 5i$
$$a + 6 = 6 \Rightarrow a = 0$$
$$2b = -5 \Rightarrow b = -\tfrac{5}{2}$$

11. $8 + \sqrt{-25} = 8 + 5i$

12. $5 + \sqrt{-36} = 5 + 6i$

13. $2 - \sqrt{-27} = 2 - \sqrt{27}i$
$$= 2 - 3\sqrt{3}i$$

14. $1 + \sqrt{-8} = 1 + 2\sqrt{2}i$

15. $\sqrt{-80} = 4\sqrt{5}i$

16. $\sqrt{-4} = 2i$

17. $14 = 14 + 0i = 14$

18. $75 = 75 + 0i = 75$

19. $-10i + i^2 = -10i - 1 = -1 - 10i$

20. $-4i^2 + 2i = -4(-1) + 2i$
$$= 4 + 2i$$

21. $\sqrt{-0.09} = \sqrt{0.09}i$
$$= 0.3i$$

22. $\sqrt{-0.0049} = \sqrt{0.0049}i$
$$= 0.07i$$

23. $(7 + i) + (3 - 4i) = 10 - 3i$

24. $(13 - 2i) + (-5 + 6i) = 8 + 4i$

25. $(9 - i) - (8 - i) = 1$

26. $(3 + 2i) - (6 + 13i) = 3 + 2i - 6 - 13i$
$$= -3 - 11i$$

27. $\left(-2 + \sqrt{-8}\right) + \left(5 - \sqrt{-50}\right) = -2 + 2\sqrt{2}i + 5 - 5\sqrt{2}i$
$$= 3 - 3\sqrt{2}i$$

28. $\left(8 + \sqrt{-18}\right) - \left(4 + 3\sqrt{2}i\right) = 8 + 3\sqrt{2}i - 4 - 3\sqrt{2}i$
$$= 4$$

29. $13i - (14 - 7i) = 13i - 14 + 7i$
$$= -14 + 20i$$

30. $25 + (-10 + 11i) + 15i = 15 + 26i$

31. $-\left(\frac{3}{2} + \frac{5}{2}i\right) + \left(\frac{5}{3} + \frac{11}{3}i\right) = -\frac{3}{2} - \frac{5}{2}i + \frac{5}{3} + \frac{11}{3}i$
$$= -\frac{9}{6} - \frac{15}{6}i + \frac{10}{6} + \frac{22}{6}i$$
$$= \frac{1}{6} + \frac{7}{6}i$$

32. $(1.6 + 3.2i) + (-5.8 + 4.3i) = -4.2 + 7.5i$

33. $(1+i)(3-2i) = 3 - 2i + 3i - 2i^2$
$= 3 + i + 2 = 5 + i$

34. $(7-2i)(3-5i) = 21 - 35i - 6i + 10i^2$
$= 21 - 41i - 10$
$= 11 - 41i$

35. $12i(1-9i) = 12i - 108i^2$
$= 12i + 108$
$= 108 + 12i$

36. $-8i(9+4i) = -72i - 32i^2$
$= 32 - 72i$

37. $\left(\sqrt{14} + \sqrt{10}i\right)\left(\sqrt{14} - \sqrt{10}i\right) = 14 - 10i^2$
$= 14 + 10 = 24$

38. $\left(\sqrt{3} + \sqrt{15}i\right)\left(\sqrt{3} - \sqrt{15}i\right) = 3 - 15i^2$
$= 3 + 15 = 18$

39. $(6+7i)^2 = 36 + 84i + 49i^2$
$= 36 + 84i - 49$
$= -13 + 84i$

40. $(5-4i)^2 = 25 - 40i + 16i^2$
$= 25 - 40i - 16$
$= 9 - 40i$

41. $(2+3i)^2 + (2-3i)^2 = 4 + 12i + 9i^2 + 4 - 12i + 9i^2$
$= 4 + 12i - 9 + 4 - 12i - 9$
$= -10$

42. $(1-2i)^2 - (1+2i)^2 = 1 - 4i + 4i^2 - \left(1 + 4i + 4i^2\right)$
$= 1 - 4i + 4i^2 - 1 - 4i - 4i^2$
$= -8i$

43. The complex conjugate of $9 + 2i$ is $9 - 2i$.
$(9+2i)(9-2i) = 81 - 4i^2$
$= 81 + 4$
$= 85$

44. The complex conjugate of $8 - 10i$ is $8 + 10i$.
$(8-10i)(8+10i) = 64 - 100i^2$
$= 64 + 100$
$= 164$

45. The complex conjugate of $-1 - \sqrt{5}i$ is $-1 + \sqrt{5}i$.
$\left(-1 - \sqrt{5}i\right)\left(-1 + \sqrt{5}i\right) = 1 - 5i^2$
$= 1 + 5 = 6$

46. The complex conjugate of $-3 + \sqrt{2}i$ is $-3 - \sqrt{2}i$.
$\left(-3 + \sqrt{2}i\right)\left(-3 - \sqrt{2}i\right) = 9 - 2i^2$
$= 9 + 2$
$= 11$

47. The complex conjugate of $\sqrt{-20} = 2\sqrt{5}i$ is $-2\sqrt{5}i$.
$\left(2\sqrt{5}i\right)\left(-2\sqrt{5}i\right) = -20i^2 = 20$

48. The complex conjugate of $\sqrt{-15} = \sqrt{15}i$ is $-\sqrt{15}i$.
$\left(\sqrt{15}i\right)\left(-\sqrt{15}i\right) = -15i^2 = 15$

49. The complex conjugate of $\sqrt{6}$ is $\sqrt{6}$.
$\left(\sqrt{6}\right)\left(\sqrt{6}\right) = 6$

50. The complex conjugate of $1 + \sqrt{8}$ is $1 + \sqrt{8}$.
$\left(1 + \sqrt{8}\right)\left(1 + \sqrt{8}\right) = 1 + 2\sqrt{8} + 8$
$= 9 + 4\sqrt{2}$

51. $\dfrac{3}{i} \cdot \dfrac{-i}{-i} = \dfrac{-3i}{-i^2} = -3i$

52. $-\dfrac{14}{2i} \cdot \dfrac{-2i}{-2i} = \dfrac{28i}{-4i^2} = \dfrac{28i}{4} = 7i$

53. $\dfrac{2}{4-5i} = \dfrac{2}{4-5i} \cdot \dfrac{4+5i}{4+5i}$
$= \dfrac{2(4+5i)}{16+25} = \dfrac{8+10i}{41} = \dfrac{8}{41} + \dfrac{10}{41}i$

54. $\dfrac{13}{1-i} \cdot \dfrac{(1+i)}{(1+i)} = \dfrac{13+13i}{1-i^2} = \dfrac{13+13i}{2} = \dfrac{13}{2} + \dfrac{13}{2}i$

55. $\dfrac{5+i}{5-i} \cdot \dfrac{(5+i)}{(5+i)} = \dfrac{25+10i+i^2}{25-i^2}$
$= \dfrac{24+10i}{26} = \dfrac{12}{13} + \dfrac{5}{13}i$

56. $\dfrac{6-7i}{1-2i} \cdot \dfrac{1+2i}{1+2i} = \dfrac{6+12i-7i-14i^2}{1-4i^2}$
$= \dfrac{20+5i}{5} = 4 + i$

57. $\dfrac{9-4i}{i}\cdot\dfrac{-i}{-i}=\dfrac{-9i+4i^2}{-i^2}=-4-9i$

58. $\dfrac{8+16i}{2i}\cdot\dfrac{-2i}{-2i}=\dfrac{-16i-32i^2}{-4i^2}=8-4i$

59. $$\begin{aligned}\frac{3i}{(4-5i)^2}&=\frac{3i}{16-40i+25i^2}=\frac{3i}{-9-40i}\cdot\frac{-9+40i}{-9+40i}\\&=\frac{-27i+120i^2}{81+1600}=\frac{-120-27i}{1681}\\&=-\frac{120}{1681}-\frac{27}{1681}i\end{aligned}$$

60. $$\begin{aligned}\frac{5i}{(2+3i)^2}&=\frac{5i}{4+12i+9i^2}\\&=\frac{5i}{-5+12i}\cdot\frac{-5-12i}{-5-12i}\\&=\frac{-25i-60i^2}{25-144i^2}\\&=\frac{60-25i}{169}=\frac{60}{169}-\frac{25}{169}i\end{aligned}$$

61. $$\begin{aligned}\frac{2}{1+i}-\frac{3}{1-i}&=\frac{2(1-i)-3(1+i)}{(1+i)(1-i)}\\&=\frac{2-2i-3-3i}{1+1}\\&=\frac{-1-5i}{2}\\&=-\frac{1}{2}-\frac{5}{2}i\end{aligned}$$

62. $$\begin{aligned}\frac{2i}{2+i}+\frac{5}{2-i}&=\frac{2i(2-i)+5(2+i)}{(2+i)(2-i)}\\&=\frac{4i-2i^2+10+5i}{4-i^2}\\&=\frac{12+9i}{5}\\&=\frac{12}{5}+\frac{9}{5}i\end{aligned}$$

63. $$\begin{aligned}\frac{i}{3-2i}+\frac{2i}{3+8i}&=\frac{i(3+8i)+2i(3-2i)}{(3-2i)(3+8i)}\\&=\frac{3i+8i^2+6i-4i^2}{9+24i-6i-16i^2}\\&=\frac{4i^2+9i}{9+18i+16}\\&=\frac{-4+9i}{25+18i}\cdot\frac{25-18i}{25-18i}\\&=\frac{-100+72i+225i-162i^2}{625+324}\\&=\frac{62+297i}{949}=\frac{62}{949}+\frac{297}{949}i\end{aligned}$$

64. $$\begin{aligned}\frac{1+i}{i}-\frac{3}{4-i}&=\frac{(1+i)(4-i)-3i}{i(4-i)}\\&=\frac{4-i+4i-i^2-3i}{4i-i^2}\\&=\frac{5}{1+4i}\cdot\frac{1-4i}{1-4i}\\&=\frac{5-20i}{1-16i^2}\\&=\frac{5}{17}-\frac{20}{17}i\end{aligned}$$

65. $$\begin{aligned}\sqrt{-6}\cdot\sqrt{-2}&=\left(\sqrt{6}i\right)\left(\sqrt{2}i\right)=\sqrt{12}i^2=\left(2\sqrt{3}\right)(-1)\\&=-2\sqrt{3}\end{aligned}$$

66. $$\begin{aligned}\sqrt{-5}\cdot\sqrt{-10}&=\left(\sqrt{5}i\right)\left(\sqrt{10}i\right)\\&=\sqrt{50}i^2=5\sqrt{2}(-1)=-5\sqrt{2}\end{aligned}$$

67. $\left(\sqrt{-15}\right)^2=\left(\sqrt{15}i\right)^2=15i^2=-15$

68. $\left(\sqrt{-75}\right)^2=\left(\sqrt{75}i\right)^2=75i^2=-75$

69. $$\begin{aligned}\left(3+\sqrt{-5}\right)\left(7-\sqrt{-10}\right)&=\left(3+\sqrt{5}i\right)\left(7-\sqrt{10}i\right)\\&=21-3\sqrt{10}i+7\sqrt{5}i-\sqrt{50}i^2\\&=\left(21+\sqrt{50}\right)+\left(7\sqrt{5}-3\sqrt{10}\right)i\\&=\left(21+5\sqrt{2}\right)+\left(7\sqrt{5}-3\sqrt{10}\right)i\end{aligned}$$

70. $\left(2-\sqrt{-6}\right)^2 = \left(2-\sqrt{6}i\right)\left(2-\sqrt{6}i\right)$

$= 4 - 2\sqrt{6}i - 2\sqrt{6}i + 6i^2$

$= 4 - 2\sqrt{6}i - 2\sqrt{6}i + 6(-1)$

$= 4 - 6 - 4\sqrt{6}i$

$= -2 - 4\sqrt{6}i$

71. $x^2 - 2x + 2 = 0;\ a = 1, b = -2, c = 2$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)} = \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm 2i}{2} = 1 \pm i$$

72. $x^2 + 6x + 10 = 0;\ a = 1, b = 6, c = 10$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(10)}}{2(1)} = \frac{-6 \pm \sqrt{-4}}{2} = \frac{-6 + 2i}{2} = -3 \pm i$$

73. $4x^2 + 16x + 17 = 0;\ a = 4, b = 16, c = 17$

$$x = \frac{-16 \pm \sqrt{(16)^2 - 4(4)(17)}}{2(4)} = \frac{-16 \pm \sqrt{-16}}{8} = \frac{-16 \pm 4i}{8} = -2 \pm \frac{1}{2}i$$

74. $9x^2 - 6x + 37 = 0;\ a = 9, b = -6, c = 37$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(37)}}{2(9)} = \frac{6 \pm \sqrt{-1296}}{18} = \frac{6 \pm 36i}{18} = \frac{1}{3} \pm 2i$$

75. $4x^2 + 16x + 15 = 0;\ a = 4, b = 16, c = 15$

$$x = \frac{-16 \pm \sqrt{(16)^2 - 4(4)(15)}}{2(4)} = \frac{-16 \pm \sqrt{16}}{8} = \frac{-16 \pm 4}{8}$$

$x = -\frac{12}{8} = -\frac{3}{2}$ or $x = -\frac{20}{8} = -\frac{5}{2}$

76. $16t^2 - 4t + 3 = 0;\ a = 16, b = -4, c = 3$

$$t = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(16)(3)}}{2(16)} = \frac{4 \pm \sqrt{-176}}{32} = \frac{4 \pm 4\sqrt{11}i}{32} = \frac{1}{8} \pm \frac{\sqrt{11}}{8}i$$

77. $\frac{3}{2}x^2 - 6x + 9 = 0$ Multiply both sides by 2.

$3x^2 - 12x + 18 = 0;\ a = 3, b = -12, c = 18$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(18)}}{2(3)} = \frac{12 \pm \sqrt{-72}}{6} = \frac{12 \pm 6\sqrt{2}i}{6} = 2 \pm \sqrt{2}i$$

78. $\frac{7}{8}x^2 - \frac{3}{4}x + \frac{5}{16} = 0$ Multiply both sides by 16.

$14x^2 - 12x + 5 = 0;\ a = 14, b = -12, c = 5$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(14)(5)}}{2(14)} = \frac{12 \pm \sqrt{-136}}{28} = \frac{12 \pm 2\sqrt{34}i}{28} = \frac{3}{7} \pm \frac{\sqrt{34}}{14}i$$

79. $1.4x^2 - 2x - 10 = 0$ Multiply both sides by 5.

$7x^2 - 10x - 50 = 0; a = 7, b = -10, c = -50$

$$\begin{aligned} x &= \frac{-(-10) \pm \sqrt{(-10)^2 - 4(7)(-50)}}{2(7)} \\ &= \frac{10 \pm \sqrt{1500}}{14} \\ &= \frac{10 \pm 10\sqrt{15}}{14} \\ &= \frac{5}{7} \pm \frac{5\sqrt{15}}{7} \end{aligned}$$

80. $4.5x^2 - 3x + 12 = 0; a = 4.5, b = -3, c = 12$

$$\begin{aligned} x &= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4.5)(12)}}{2(4.5)} \\ &= \frac{3 \pm \sqrt{-207}}{9} \\ &= \frac{3 \pm 3\sqrt{23}i}{9} \\ &= \frac{1}{3} \pm \frac{\sqrt{23}}{3}i \end{aligned}$$

81. $$\begin{aligned} -6i^3 + i^2 &= -6i^2 i + i^2 \\ &= -6(-1)i + (-1) \\ &= 6i - 1 \\ &= -1 + 6i \end{aligned}$$

82. $4i^2 - 2i^3 = 4i^2 - 2i^2 i = 4(-1) - 2(-1)i = -4 + 2i$

83. $-14i^5 = -14i^2 i^2 i = -14(-1)(-1)(i) = -14i$

84. $(-i)^3 = (-1)(i^3) = (-1)i^2 i = (-1)(-1)i = i$

85. $$\begin{aligned} \left(\sqrt{-72}\right)^3 &= \left(6\sqrt{2}i\right)^3 \\ &= 6^3\left(\sqrt{2}\right)^3 i^3 \\ &= 216\left(2\sqrt{2}\right)i^2 i \\ &= 432\sqrt{2}(-1)i \\ &= -432\sqrt{2}i \end{aligned}$$

86. $$\begin{aligned} \left(\sqrt{-2}\right)^6 &= \left(\sqrt{2}i\right)^6 \\ &= 8i^6 = 8i^2 i^2 i^2 \\ &= 8(-1)(-1)(-1) \\ &= -8 \end{aligned}$$

87. $\frac{1}{i^3} = \frac{1}{i^2 i} = \frac{1}{-i} = \frac{1}{-i} \cdot \frac{i}{i} = \frac{i}{-i^2} = i$

88. $\frac{1}{(2i)^3} = \frac{1}{8i^3} = \frac{1}{8i^2 i} = \frac{1}{-8i} = \frac{1}{-8i} \cdot \frac{8i}{8i} = \frac{8i}{-64i^2} = \frac{1}{8}i$

89. $(3i)^4 = 81i^4 = 81i^2 i^2 = 81(-1)(-1) = 81$

90. $(-i)^6 = i^6 = i^2 i^2 i^2 = (-1)(-1)(-1) = -1$

91. (a) $z_1 = 9 + 16i, z_2 = 20 - 10i$

(b) $\frac{1}{z} = \frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{9 + 16i} + \frac{1}{20 - 10i} = \frac{20 - 10i + 9 + 16i}{(9 + 16i)(20 - 10i)} = \frac{29 + 6i}{340 + 230i}$

$z = \left(\frac{340 + 230i}{29 + 6i}\right)\left(\frac{29 - 6i}{29 - 6i}\right) = \frac{11{,}240 + 4630i}{877} = \frac{11{,}240}{877} + \frac{4630}{877}i$

92. (a) $$\begin{aligned} \left(-1 + \sqrt{3}i\right)^3 &= (-1)^3 + 3(-1)^2\left(\sqrt{3}i\right) + 3(-1)\left(\sqrt{3}i\right)^2 + \left(\sqrt{3}i\right)^3 \\ &= -1 + 3\sqrt{3}i - 9i^2 + 3\sqrt{3}i^3 \\ &= -1 + 3\sqrt{3}i - 9i^2 + 3\sqrt{3}i^2 i \\ &= -1 + 3\sqrt{3}i + 9 - 3\sqrt{3}i \\ &= 8 \end{aligned}$$

(b) $$\begin{aligned} \left(-1 - \sqrt{3}i\right)^3 &= (-1)^3 + 3(-1)^2\left(-\sqrt{3}i\right) + 3(-1)\left(-\sqrt{3}i\right)^2 + \left(-\sqrt{3}i\right)^3 \\ &= -1 - 3\sqrt{3}i - 9i^2 - 3\sqrt{3}i^3 \\ &= -1 - 3\sqrt{3}i - 9i^2 - 3\sqrt{3}i^2 i \\ &= -1 - 3\sqrt{3}i + 9 + 3\sqrt{3}i \\ &= 8 \end{aligned}$$

93. False.

If $b = 0$ then $a + bi = a - bi = a$.

That is, if the complex number is real, the number equals its conjugate.

94. True.

$$\begin{aligned} x^4 - x^2 + 14 &= 56 \\ \left(-i\sqrt{6}\right)^4 - \left(-i\sqrt{6}\right)^2 + 14 &\stackrel{?}{=} 56 \\ 36 + 6 + 14 &\stackrel{?}{=} 56 \\ 56 &= 56 \end{aligned}$$

95. False.

$$\begin{aligned} i^{44} + i^{150} - i^{74} - i^{109} + i^{61} &= \left(i^2\right)^{22} + \left(i^2\right)^{75} - \left(i^2\right)^{37} - \left(i^2\right)^{54} i + \left(i^2\right)^{30} i \\ &= (-1)^{22} + (-1)^{75} - (-1)^{37} - (-1)^{54} i + (-1)^{30} i \\ &= 1 - 1 + 1 - i + i = 1 \end{aligned}$$

96. False.

Sample answer: $4i + (3 + 2i) = 3 + 6i$, which is not a real number.

97.

$$\begin{aligned} i &= i \\ i^2 &= -1 \\ i^3 &= -i \\ i^4 &= 1 \\ i^5 &= i^4 i = i \\ i^6 &= i^4 i^2 = -1 \\ i^7 &= i^4 i^3 = -i \\ i^8 &= i^4 i^4 = 1 \\ i^9 &= i^4 i^4 i = i \\ i^{10} &= i^4 i^4 i^2 = -1 \\ i^{11} &= i^4 i^4 i^3 = -i \\ i^{12} &= i^4 i^4 i^4 = 1 \end{aligned}$$

The pattern $i, -1, -i, 1$ repeats. Divide the exponent by 4.

If the remainder is 1, the result is i.

If the remainder is 2, the result is -1.

If the remainder is 3, the result is $-i$.

If the remainder is 0, the result is 1.

98. (i) D

(ii) F

(iii) B

(iv) E

(v) A

(vi) C

99. $\sqrt{-6}\sqrt{-6} = \sqrt{6}i\sqrt{6}i = 6i^2 = -6$

100.

$$\begin{aligned} (a_1 + b_1 i)(a_2 + b_2 i) &= a_1 a_2 + a_1 b_2 i + a_2 b_1 i + b_1 b_2 i^2 \\ &= (a_1 a_2 - b_1 b_2) + (a_1 b_2 + a_2 b_1) i \end{aligned}$$

The complex conjugate of this product is $(a_1 a_2 - b_1 b_2) - (a_1 b_2 + a_2 b_1) i$.

The product of the complex conjugates is

$$\begin{aligned} (a_1 - b_1 i)(a_2 - b_2 i) &= a_1 a_2 - a_1 b_2 i - a_2 b_1 i - b_1 b_2 i^2 \\ &= (a_1 a_2 - b_1 b_2) - (a_1 b_2 + a_2 b_1) i. \end{aligned}$$

So, the complex conjugate of the product of two complex numbers is the product of their complex conjugates.

101. $(a_1 + b_1 i) + (a_2 + b_2 i) = (a_1 + a_2) + (b_1 + b_2) i$

The complex conjugate of this sum is $(a_1 + a_2) - (b_1 + b_2) i$.

The sum of the complex conjugates is $(a_1 - b_1 i) + (a_2 - b_2 i) = (a_1 + a_2) - (b_1 + b_2) i$.

So, the complex conjugate of the sum of two complex numbers is the sum of their complex conjugates.

Section 2.5 Zeros of Polynomial Functions

1. Fundamental Theorem of Algebra

2. Linear Factorization Theorem

3. Rational Zero

4. conjugate

5. linear; quadratic; quadratic

6. irreducible; reals

7. Descartes's Rule of Signs

8. lower; upper

9. Since f is a 1st degree polynomial function, there is one zero.

10. Since f is a 2nd degree polynomial function, there are two zeros.

11. Since f is a 3rd degree polynomial function, there are three zeros.

12. Since f is a 7th degree polynomial function, there are seven zeros.

13. Since f is a 2nd degree polynomial function, there are two zeros.

14. $$\begin{aligned} h(t) &= (t-1)^2 - (t+1)^2 \\ &= (t^2 - 2t + 1) - (t^2 + 2t + 1) \\ &= -4t \end{aligned}$$

Since h is a 1st degree polynomial function, there is one real zero.

15. $f(x) = x^3 + 2x^2 - x - 2$

Possible rational zeros: $\pm 1, \pm 2$

Zeros shown on graph: $-2, -1, 1, 2$

16. $f(x) = x^3 - 4x^2 - 4x + 16$

Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

Zeros shown on graph: $-2, 2, 4$

17. $f(x) = 2x^4 - 17x^3 + 35x^2 + 9x - 45$

Possible rational zeros: $\pm 1, \pm 3, \pm 5, \pm 9, \pm 15, \pm 45, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \pm \frac{9}{2}, \pm \frac{15}{2}, \pm \frac{45}{2}$

Zeros shown on graph: $-1, \frac{3}{2}, 3, 5$

18. $f(x) = 4x^5 - 8x^4 - 5x^3 + 10x^2 + x - 2$

Possible rational zeros: $\pm 1, \pm 2, \pm \frac{1}{2}, \pm \frac{1}{4}$

Zeros shown on graph: $-1, -\frac{1}{2}, \frac{1}{2}, 1, 2$

19. $f(x) = x^3 - 7x - 6$

Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 6$

$$\begin{array}{r|rrrr} 3 & 1 & 0 & -7 & -6 \\ & & 3 & 9 & 6 \\ \hline & 1 & 3 & 2 & 0 \end{array}$$

$$\begin{aligned} f(x) &= (x-3)(x^2 + 3x + 2) \\ &= (x-3)(x+2)(x+1) \end{aligned}$$

So, the rational zeros are $-2, -1$, and 3.

20. $f(x) = x^3 - 13x + 12$

Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$$\begin{array}{r|rrrr} 3 & 1 & 0 & -13 & -12 \\ & & 3 & 9 & -12 \\ \hline & & 3 & -4 & 0 \end{array}$$

$$\begin{aligned} f(x) &= (x-3)(x^2 + 3x - 4) \\ &= (x-3)(x+4)(x-1) \end{aligned}$$

So, the rational zeros are $3, -4$, and 1.

21. $$\begin{aligned} g(x) &= x^3 - 4x^2 - x + 4 \\ &= x^2(x-4) - 1(x-4) \\ &= (x-4)(x^2 - 1) \\ &= (x-4)(x-1)(x+1) \end{aligned}$$

So, the rational zeros are $4, 1$, and -1.

22. $h(x) = x^3 - 9x^2 + 20x - 12$

Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$$\begin{array}{r|rrrr} 1 & 1 & -9 & 20 & -12 \\ & & 1 & -8 & 12 \\ \hline & 1 & -8 & 12 & 0 \end{array}$$

$$\begin{aligned} h(x) &= (x-1)(x^2 - 8x + 12) \\ &= (x-1)(x-2)(x-6) \end{aligned}$$

So, the rational zeros are 1, 2, and 6.

23. $h(t) = t^3 + 8t^2 + 13t + 6$

Possible rational zeros: $\pm1, \pm2, \pm3, \pm6$

$$\begin{array}{r|rrrr} -6 & 1 & 8 & 13 & 6 \\ & & -6 & -12 & -6 \\ \hline & 1 & 2 & 1 & 0 \end{array}$$

$$\begin{aligned} t^3 + 8t^2 + 13t + 6 &= (t + 6)(t^2 + 2t + 1) \\ &= (t + 6)(t + 1)(t + 1) \end{aligned}$$

So, the rational zeros are -1 and -6.

24. $p(x) = x^3 - 9x^2 + 27x - 27$

Possible rational zeros: $\pm1, \pm3, \pm9, \pm27$

$$\begin{array}{r|rrrr} 3 & 1 & -9 & 27 & -27 \\ & & 3 & -18 & 27 \\ \hline & 1 & -6 & 9 & 0 \end{array}$$

$$\begin{aligned} p(x) &= (x - 3)(x^2 - 6x + 9) \\ &= (x - 3)(x - 3)(x - 3) \end{aligned}$$

So, the rational zero is 3.

25. $C(x) = 2x^3 + 3x^2 - 1$

Possible rational zeros: $\pm1, \pm\frac{1}{2}$

$$\begin{array}{r|rrrr} -1 & 2 & 3 & 0 & -1 \\ & & -2 & -1 & 1 \\ \hline & 2 & 1 & -1 & 0 \end{array}$$

$$\begin{aligned} 2x^3 + 3x^2 - 1 &= (x + 1)(2x^2 + x - 1) \\ &= (x + 1)(x + 1)(2x - 1) \\ &= (x + 1)^2(2x - 1) \end{aligned}$$

So, the rational zeros are -1 and $\frac{1}{2}$.

26. $f(x) = 3x^3 - 19x^2 + 33x - 9$

Possible rational zeros: $\pm1, \pm3, \pm9, \pm\frac{1}{3}$

$$\begin{array}{r|rrrr} 3 & 3 & -19 & 33 & -9 \\ & & 9 & -30 & 9 \\ \hline & 3 & -10 & 3 & 0 \end{array}$$

$$\begin{aligned} f(x) &= (x - 3)(3x^2 - 10x + 3) \\ &= (x - 3)(3x - 1)(x - 3) \end{aligned}$$

So, the rational zeros are 3 and $\frac{1}{3}$.

27. $f(x) = 9x^4 - 9x^3 - 58x^2 + 4x + 24$

Possible rational zeros:
$\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24,$
$\pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}, \pm\frac{1}{9}, \pm\frac{2}{9}, \pm\frac{4}{9}, \pm\frac{8}{9}$

$$\begin{array}{r|rrrrr} -2 & 9 & -9 & -58 & 4 & 24 \\ & & -18 & 54 & 8 & -24 \\ \hline & 9 & -27 & -4 & 12 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 3 & 9 & -27 & -4 & 12 \\ & & 27 & 0 & -12 \\ \hline & 9 & 0 & -4 & 0 \end{array}$$

$$\begin{aligned} f(x) &= (x + 2)(x - 3)(9x^2 - 4) \\ &= (x + 2)(x - 3)(3x - 2)(3x + 2) \end{aligned}$$

So, the rational zeros are $-2, 3, \frac{2}{3},$ and $-\frac{2}{3}$.

28. $f(x) = 2x^4 - 15x^3 + 23x^2 + 15x - 25$

Possible rational zeros: $\pm1, \pm5, \pm25, \pm\frac{1}{2}, \pm\frac{5}{2}, \pm\frac{25}{2}$

$$\begin{array}{r|rrrrr} 5 & 2 & -15 & 23 & 15 & -25 \\ & & 10 & -25 & -10 & 25 \\ \hline & 2 & -5 & -2 & 5 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 1 & 2 & -5 & -2 & 5 \\ & & 2 & -3 & -5 \\ \hline & 2 & -3 & -5 & 0 \end{array}$$

$$\begin{array}{r|rrr} -1 & 2 & -3 & -5 \\ & & -2 & 5 \\ \hline & 2 & -5 & 0 \end{array}$$

$$f(x) = (x - 5)(x - 1)(x + 1)(2x - 5)$$

So, the rational zeros are $5, 1, -1$ and $\frac{5}{2}$.

29. $z^4 + z^3 + z^2 + 3z - 6 = 0$

Possible rational zeros: $\pm1, \pm2, \pm3, \pm6$

$$\begin{array}{r|rrrrr} 1 & 1 & 1 & 1 & 3 & -6 \\ & & 1 & 2 & 3 & 6 \\ \hline & 1 & 2 & 3 & 6 & 0 \end{array}$$

$$\begin{aligned} (z - 1)(z^3 + 2z^2 + 3z + 6) &= 0 \\ (z - 1)(z^2 + 3)(z + 2) &= 0 \end{aligned}$$

So, the real zeros are -2 and 1.

30. $x^4 - 13x^2 - 12x = 0$

$x\left(x^3 - 13x - 12\right) = 0$

Possible rational zeros of $x^3 - 13x - 12$:

$\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -13 & -12 \\ & & -1 & 1 & 12 \\ \hline & 1 & -1 & -12 & 0 \end{array}$$

$x(x + 1)\left(x^2 - x - 12\right) = 0$

$x(x + 1)(x - 4)(x + 3) = 0$

The real zeros are $0, -1, 4,$ and -3.

31. $2y^4 + 3y^3 - 16y^2 + 15y - 4 = 0$

Possible rational zeros: $\pm\frac{1}{2}, \pm1, \pm2, \pm4$

$$\begin{array}{r|rrrrr} 1 & 2 & 3 & -16 & 15 & -4 \\ & & 2 & 5 & -11 & 4 \\ \hline & 2 & 5 & -11 & 4 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 1 & 2 & 5 & -11 & 4 \\ & & 2 & 7 & -4 \\ \hline & 2 & 7 & -4 & 0 \end{array}$$

$(y - 1)(y - 1)\left(2y^2 + 7y - 4\right) = 0$

$(y - 1)(y - 1)(2y - 1)(y + 4) = 0$

So, the real zeros are $-4, \frac{1}{2}$ and 1.

32. $x^5 - x^4 - 3x^3 + 5x^2 - 2x = 0$

$x\left(x^4 - x^3 - 3x^2 + 5x - 2\right) = 0$

Possible rational zeros of $x^4 - x^3 - 3x^2 + 5x - 2$:

$\pm1, \pm2$

$$\begin{array}{r|rrrrr} 1 & 1 & -1 & -3 & 5 & -2 \\ & & 1 & 0 & -3 & 2 \\ \hline & 1 & 0 & -3 & 2 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -3 & 2 \\ & & -2 & 4 & -2 \\ \hline & 1 & -2 & 1 & 0 \end{array}$$

$x(x - 1)(x + 2)\left(x^2 - 2x + 1\right) = 0$

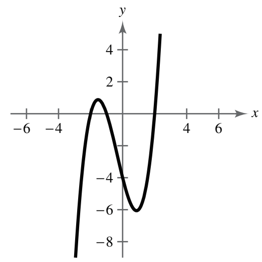
$x(x - 1)(x + 2)(x - 1)(x - 1) = 0$

The real zeros are $-2, 0,$ and 1.

33. $f(x) = x^3 + x^2 - 4x - 4$

(a) Possible rational zeros: $\pm1, \pm2, \pm4$

(b)

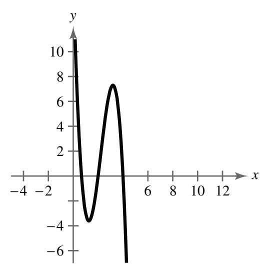


(c) Real zeros: $-2, -1, 2$

34. $f(x) = -3x^3 + 20x^2 - 36x + 16$

(a) Possible rational zeros: $\pm1, \pm2, \pm4, \pm8, \pm16, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}, \pm\frac{16}{3}$

(b)

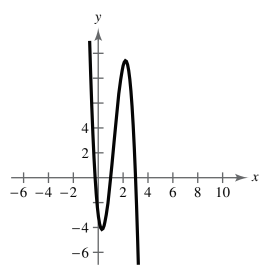


(c) Real zeros: $\frac{2}{3}, 2, 4$

35. $f(x) = -4x^3 + 15x^2 - 8x - 3$

(a) Possible rational zeros: $\pm1, \pm3, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}$

(b)

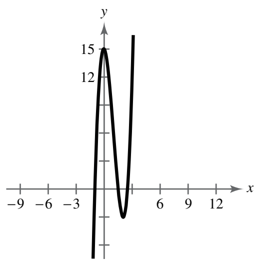


(c) Real zeros: $-\frac{1}{4}, 1, 3$

36. $f(x) = 4x^3 - 12x^2 - x + 15$

(a) Possible rational zeros: $\pm1, \pm3, \pm5, \pm15, \pm\frac{1}{2}, \pm\frac{3}{2},$
$\pm\frac{5}{2}, \pm\frac{15}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{5}{4}, \pm\frac{15}{4}$

(b)

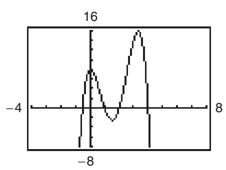


(c) Real zeros: $-1, \frac{3}{2}, \frac{5}{2}$

37. $f(x) = -2x^4 + 13x^3 - 21x^2 + 2x + 8$

(a) Possible rational zeros: $\pm1, \pm2, \pm4, \pm8, \pm\frac{1}{2}$

(b)

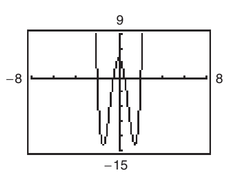


(c) Real zeros: $-\frac{1}{2}, 1, 2, 4$

38. $f(x) = 4x^4 - 17x^2 + 4$

(a) Possible rational zeros: $\pm1, \pm2, \pm4, \pm\frac{1}{2}, \pm\frac{1}{4}$

(b)

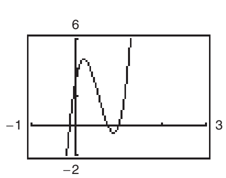


(c) Real zeros: $-2, -\frac{1}{2}, \frac{1}{2}, 2$

39. $f(x) = 32x^3 - 52x^2 + 17x + 3$

(a) Possible rational zeros: $\pm1, \pm3, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4},$
$\pm\frac{1}{8}, \pm\frac{3}{8}, \pm\frac{1}{16}, \pm\frac{3}{16}, \pm\frac{1}{32}, \pm\frac{3}{32}$

(b)

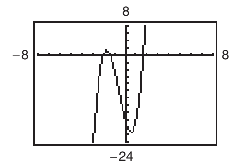


(c) Real zeros: $-\frac{1}{8}, \frac{3}{4}, 1$

40. $f(x) = 4x^3 + 7x^2 - 11x - 18$

(a) Possible rational zeros: $\pm1, \pm2, \pm3, \pm6, \pm9, \pm18,$
$\pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{9}{4}$

(b)



(c) Real zeros: $-2, \frac{1}{8} \pm \frac{\sqrt{145}}{8}$

41. $f(x) = x^4 - 3x^2 + 2$

(a) $x = \pm1$, about ±1.414

(b) An exact zero is $x = 1$.

$$\begin{array}{r|rrrrr} 1 & 1 & 0 & -3 & 0 & 2 \\ & & 1 & 1 & -2 & -2 \\ \hline & 1 & 1 & -2 & -2 & 0 \end{array}$$

(c)
$$\begin{array}{r|rrrr} -1 & 1 & 1 & -2 & -2 \\ & & -1 & 0 & 2 \\ \hline & 1 & 0 & -2 & 0 \end{array}$$

$$\begin{aligned} f(x) &= (x-1)(x+1)(x^2-2) \\ &= (x-1)(x+1)(x-\sqrt{2})(x+\sqrt{2}) \end{aligned}$$

42. $P(t) = t^4 - 7t^2 + 12$

(a) $t = \pm2$, about ±1.732

(b) An exact zero is $t = 2$.

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & -7 & 0 & 12 \\ & & 2 & 4 & -6 & -12 \\ \hline & 1 & 2 & -3 & -6 & 0 \end{array}$$

An exact zero is $t = -2$.

$$\begin{array}{r|rrrr} -2 & 1 & 2 & -3 & -6 \\ & & -2 & 0 & 6 \\ \hline & 1 & 0 & -3 & 0 \end{array}$$

(c) $$\begin{aligned} P(t) &= (t-2)(t+2)(t^2-3) \\ &= (t-2)(t+2)(t-\sqrt{3})(t+\sqrt{3}) \end{aligned}$$

43. $h(x) = x^5 - 7x^4 + 10x^3 + 14x^2 - 24x$

(a) $h(x) = x(x^4 - 7x^3 + 10x^2 + 14x - 24)$

$x = 0, 3, 4,$ about ± 1.414

(b) An exact zero is $x = 3$.

$$\begin{array}{r|rrrrr} 3 & 1 & -7 & 10 & 14 & -24 \\ & & 3 & -12 & -6 & 24 \\ \hline & 1 & -4 & -2 & 8 & 0 \end{array}$$

(c)
$$\begin{array}{r|rrrr} 4 & 1 & -4 & -2 & 8 \\ & & 4 & 0 & -8 \\ \hline & 1 & 0 & -2 & 0 \end{array}$$

$$\begin{aligned} h(x) &= x(x-3)(x-4)(x^2-2) \\ &= x(x-3)(x-4)(x-\sqrt{2})(x+\sqrt{2}) \end{aligned}$$

44. $g(x) = 6x^4 - 11x^3 - 51x^2 + 99x - 27$

(a) $x = \pm 3, 1.5,$ about 0.333

(b) An exact zero is $x = 3$.

$$\begin{array}{r|rrrrr} 3 & 6 & -11 & -51 & 99 & -27 \\ & & 18 & 21 & -90 & 27 \\ \hline & 6 & 7 & -30 & 9 & 0 \end{array}$$

An exact zero is $x = -3$.

$$\begin{array}{r|rrrr} -3 & 6 & 7 & -30 & 9 \\ & & -18 & 33 & -9 \\ \hline & 6 & -11 & 3 & 0 \end{array}$$

(c) $$\begin{aligned} g(x) &= (x-3)(x+3)(6x^2 - 11x + 3) \\ &= (x-3)(x+3)(3x-1)(2x-3) \end{aligned}$$

45. $$\begin{aligned} f(x) &= (x-1)(x-5i)(x+5i) \\ &= (x-1)(x^2+25) \\ &= x^3 - x^2 + 25x - 25 \end{aligned}$$

Note: $f(x) = a(x^3 - x^2 + 25x - 25)$, where a is any nonzero real number, has the zeros 1 and $\pm 5i$.

46. $$\begin{aligned} f(x) &= (x-4)(x-3i)(x+3i) \\ &= (x-4)(x^2+9) \\ &= x^3 - 4x^2 + 9x - 36 \end{aligned}$$

Note: $f(x) = a(x^3 - 4x^2 + 9x - 36)$, where a is any real number, has the zeros $4, 3i,$ and $-3i$.

47. If $5 + i$ is a zero, so is its conjugate, $5 - i$.

$$\begin{aligned} f(x) &= (x-2)(x-(5+i))(x-(5-i)) \\ &= (x-2)(x^2 - 10x + 26) \\ &= x^3 - 12x^2 + 46x - 52 \end{aligned}$$

Note: $f(x) = a(x^3 - 12x^2 + 46x - 52)$, where a is any nonzero real number, has the zeros 2 and $5 \pm i$.

48. If $3 - 2i$ is a zero, so is its conjugate, $3 + 2i$.

$$\begin{aligned} f(x) &= (x-5)(x-(3-2i))(x-(3+2i)) \\ &= (x-5)(x^2 - 6x + 13) \\ &= x^3 - 11x^2 + 43x - 65 \end{aligned}$$

Note: $f(x) = a(x^3 - 11x^2 + 43x - 65)$, where a is any nonzero real number, has the zeros 5 and $3 \pm 2i$.

49. If $3 + \sqrt{2}i$ is a zero, so is its conjugate, $3 - \sqrt{2}i$.

$$\begin{aligned} f(x) &= (3x-2)(x+1)\left[x - (3+\sqrt{2}i)\right]\left[x - (3-\sqrt{2}i)\right] \\ &= (3x-2)(x+1)\left[(x-3) - \sqrt{2}i\right]\left[(x-3) + \sqrt{2}i\right] \\ &= (3x^2 + x - 2)\left[(x-3)^2 - (\sqrt{2}i)^2\right] \\ &= (3x^2 + x - 2)(x^2 - 6x + 9 + 2) \\ &= (3x^2 + x - 2)(x^2 - 6x + 11) \\ &= 3x^4 - 17x^3 + 25x^2 + 23x - 22 \end{aligned}$$

Note: $f(x) = a(3x^4 - 17x^3 + 25x^2 + 23x - 22)$, where a is any nonzero real number, has the zeros $\frac{2}{3}, -1,$ and $3 \pm \sqrt{2}i$.

50. If $1 + \sqrt{3}i$ is a zero, so is its conjugate, $1 - \sqrt{3}i$.

$$\begin{aligned} f(x) &= (x+5)^2\left(x - 1 - \sqrt{3}i\right)\left(x - 1 + \sqrt{3}i\right) \\ &= \left(x^2 + 10x + 25\right)\left(x^2 - 2x + 4\right) \\ &= x^4 + 8x^3 + 9x^2 - 10x + 100 \end{aligned}$$

Note: $f(x) = a\left(x^4 + 8x^3 + 9x^2 - 10x + 100\right)$, where a is any real number, has the zeros $-5, -5$, and $1 \pm \sqrt{3}i$.

51. $f(x) = x^4 + 6x^2 - 27$

(a) $f(x) = \left(x^2 + 9\right)\left(x^2 - 3\right)$

(b) $f(x) = \left(x^2 + 9\right)\left(x + \sqrt{3}\right)\left(x - \sqrt{3}\right)$

(c) $f(x) = (x + 3i)(x - 3i)\left(x + \sqrt{3}\right)\left(x - \sqrt{3}\right)$

52. $f(x) = x^4 - 2x^3 - 3x^2 + 12x - 18$

$$\begin{array}{r} x^2 - 2x + 3 \\ x^2 - 6 \overline{\big) x^4 - 2x^3 - 3x^2 + 12x - 18} \\ \underline{x^4 \qquad - 6x^2} \qquad\qquad\quad \\ -2x^3 + 3x^2 + 12x \qquad \\ \underline{-2x^3 \qquad\quad + 12x} \qquad \\ 3x^2 \qquad\quad -18 \\ \underline{3x^2 \qquad\quad -18} \\ 0 \end{array}$$

(a) $f(x) = \left(x^2 - 6\right)\left(x^2 - 2x + 3\right)$

(b) $f(x) = \left(x + \sqrt{6}\right)\left(x - \sqrt{6}\right)\left(x^2 - 2x + 3\right)$

(c) $f(x) = \left(x + \sqrt{6}\right)\left(x - \sqrt{6}\right)\left(x - 1 - \sqrt{2}i\right)\left(x - 1 + \sqrt{2}i\right)$

Note: Use the Quadratic Formula for (c).

53. $f(x) = x^4 - 4x^3 + 5x^2 - 2x - 6$

$$\begin{array}{r} x^2 - 2x + 3 \\ x^2 - 2x - 2 \overline{\big) x^4 - 4x^3 + 5x^2 - 2x - 6} \\ \underline{x^4 - 2x^3 - 2x^2} \qquad\qquad \\ -2x^3 + 7x^2 - 2x \qquad \\ \underline{-2x^3 + 4x^2 + 4x} \qquad \\ 3x^2 - 6x - 6 \\ \underline{3x^2 - 6x - 6} \\ 0 \end{array}$$

(a) $f(x) = \left(x^2 - 2x - 2\right)\left(x^2 - 2x + 3\right)$

(b) $f(x) = \left(x - 1 + \sqrt{3}\right)\left(x - 1 - \sqrt{3}\right)\left(x^2 - 2x + 3\right)$

(c) $f(x) = \left(x - 1 + \sqrt{3}\right)\left(x - 1 - \sqrt{3}\right)\left(x - 1 + \sqrt{2}i\right)\left(x - 1 - \sqrt{2}i\right)$

Note: Use the Quadratic Formula for (b) and (c).

54. $f(x) = x^4 - 3x^3 - x^2 - 12x - 20$

$$\begin{array}{r|l}
 & x^2 - 3x - 5 \\ \hline
x^2 + 4 & x^4 - 3x^3 - x^2 - 12x - 20 \\
 & \underline{x^4 \quad\quad + 4x^2} \\
 & -3x^3 - 5x^2 - 12x \\
 & \underline{-3x^3 \quad\quad - 12x} \\
 & -5x^2 \quad\quad - 20 \\
 & \underline{-5x^2 \quad\quad - 20} \\
 & 0
\end{array}$$

(a) $f(x) = (x^2 + 4)(x^2 - 3x - 5)$

(b) $f(x) = (x^2 + 4)\left(x - \dfrac{3 + \sqrt{29}}{2}\right)\left(x - \dfrac{3 - \sqrt{29}}{2}\right)$

(c) $f(x) = (x + 2i)(x - 2i)\left(x - \dfrac{3 + \sqrt{29}}{2}\right)\left(x - \dfrac{3 - \sqrt{29}}{2}\right)$

Note: Use the Quadratic Formula for (b).

55. $f(x) = x^3 - x^2 + 4x - 4$

Because $2i$ is a zero, so is $-2i$.

$$\begin{array}{r|rrrr}
2i & 1 & -1 & 4 & -4 \\
 & & 2i & -4 - 2i & 4 \\ \hline
 & 1 & 2i - 1 & -2i & 0
\end{array}$$

$$\begin{array}{r|rrr}
-2i & 1 & 2i - 1 & -2i \\
 & & -2i & 2i \\ \hline
 & 1 & -1 & 0
\end{array}$$

$f(x) = (x - 2i)(x + 2i)(x - 1)$

The zeros of $f(x)$ are $x = 1, \pm 2i$.

Alternate Solution:

Because $x = \pm 2i$ are zeros of $f(x)$,

$(x + 2i)(x - 2i) = x^2 + 4$ is a factor of $f(x)$.

By long division, you have:

$$\begin{array}{r|l}
 & x - 1 \\ \hline
x^2 + 0x + 4 & x^3 - x^2 + 4x - 4 \\
 & \underline{x^3 + 0x^2 + 4x} \\
 & -x^2 + 0x - 4 \\
 & \underline{-x^2 + 0x - 4} \\
 & 0
\end{array}$$

$f(x) = (x^2 + 4)(x - 1)$

The zeros of $f(x)$ are $x = 1, \pm 2i$.

56. $f(x) = 2x^3 + 3x^2 + 18x + 27$

Because $3i$ is a zero, so is $-3i$.

$$\begin{array}{r|rrrr}
3i & 2 & 3 & 18 & 27 \\
 & & 6i & 9i - 18 & -27 \\ \hline
 & 2 & 3 + 6i & 9i & 0
\end{array}$$

$$\begin{array}{r|rrr}
-3i & 2 & 3 + 6i & 9i \\
 & & -6i & -9i \\ \hline
 & 2 & 3 & 0
\end{array}$$

$f(x) = (x - 3i)(x + 3i)(2x + 3)$

The zeros of $f(x)$ are $x = \pm 3i, -\frac{3}{2}$.

Alternate Solution:

Because $x = \pm 3i$ are zeros of $f(x)$,

$(x - 3i)(x + 3i) = x^2 + 9$ is a factor of $f(x)$.

By long division, you have:

$$\begin{array}{r|l}
 & 2x + 3 \\ \hline
x^2 + 0x + 9 & 2x^3 + 3x^2 + 18x + 27 \\
 & \underline{2x^3 + 0x^2 + 18x} \\
 & 3x^2 + 0x + 27 \\
 & \underline{3x^2 + 0x + 27} \\
 & 0
\end{array}$$

$f(x) = (x^2 + 9)(2x + 3)$

The zeros of $f(x)$ are $x = \pm 3i, -\frac{3}{2}$.

57. $f(x) = 2x^4 - x^3 + 49x^2 - 25x - 25$

Because $5i$ is a zero, so is $-5i$.

$$\begin{array}{r|rrrrr} 5i & 2 & -1 & 49 & -25 & -25 \\ & & 10i & -5i - 50 & -5i + 25 & 25 \\ \hline & 2 & -1 + 10i & -1 - 5i & -5i & 0 \end{array}$$

$$\begin{array}{r|rrrr} -5i & 2 & -1 + 10i & -1 - 5i & -5i \\ & & -10i & 5i & 5i \\ \hline & 2 & -1 & -1 & 0 \end{array}$$

$$\begin{aligned} f(x) &= (x - 5i)(x + 5i)(2x^2 - x - 1) \\ &= (x - 5i)(x + 5i)(2x + 1)(x - 1) \end{aligned}$$

The zeros of $f(x)$ are $x = \pm 5i, -\frac{1}{2}, 1$.

Alternate Solution:

Because $x = \pm 5i$ are zeros of $f(x)$, $(x - 5i)(x + 5i) = x^2 + 25$ is a factor of $f(x)$.

By long division, you have:

$$\begin{array}{r} 2x^2 - x - 1 \\ x^2 + 0x + 25 \overline{\smash{)}\, 2x^4 - x^3 + 49x^2 - 25x - 25} \\ \underline{2x^4 + 0x^3 + 50x^2} \qquad\qquad\quad \\ -x^3 - x^2 - 25x \qquad \\ \underline{-x^3 + 0x^2 - 25x} \qquad \\ -x^2 + 0x - 25 \\ \underline{-x^2 + 0x - 25} \\ 0 \end{array}$$

$$f(x) = (x^2 + 25)(2x^2 - x - 1)$$

The zeros of $f(x)$ are $x = \pm 5i, -\frac{1}{2}, 1$.

58. $g(x) = x^3 - 7x^2 - x + 87$

Because $5 + 2i$ is a zero, so is $5 - 2i$.

$$\begin{array}{r|rrrr} 5 + 2i & 1 & -7 & -1 & 87 \\ & & 5 + 2i & -14 + 6i & -87 \\ \hline & 1 & -2 + 2i & -15 + 6i & 0 \end{array}$$

$$\begin{array}{r|rrr} 5 - 2i & 1 & -2 + 2i & -15 + 6i \\ & & 5 - 2i & 15 - 6i \\ \hline & 1 & 3 & 0 \end{array}$$

The zero of $x + 3$ is $x = -3$. The zeros of $f(x)$ are $x = -3, 5 \pm 2i$.

59. $g(x) = 4x^3 + 23x^2 + 34x - 10$

Because $-3 + i$ is a zero, so is $-3 - i$.

$$\begin{array}{r|rrrr} -3+i & 4 & 23 & 34 & -10 \\ & & -12+4i & -37-i & 10 \\ \hline & 4 & 11+4i & -3-i & 0 \end{array}$$

$$\begin{array}{r|rrr} -3-i & 4 & 11+4i & -3-i \\ & & -12-4i & 3+i \\ \hline & 4 & -1 & 0 \end{array}$$

The zero of $4x - 1$ is $x = \frac{1}{4}$. The zeros of $g(x)$ are $x = -3 \pm i, \frac{1}{4}$.

Alternate Solution

Because $-3 \pm i$ are zeros of $g(x)$,

$$\begin{aligned} [x - (-3 + i)][x - (-3 - i)] &= [(x + 3) - i][(x + 3) + i] \\ &= (x + 3)^3 - i^2 \\ &= x^2 + 6x + 10 \end{aligned}$$

is a factor of $g(x)$. By long division, you have:

$$\begin{array}{r} 4x - 1 \\ x^2 + 6x + 10 \overline{\smash{)}\, 4x^3 + 23x^2 + 34x - 10} \\ \underline{4x^3 + 24x^2 + 40x \phantom{{}-10}} \\ -x^2 - 6x - 10 \\ \underline{-x^2 - 6x - 10} \\ 0 \end{array}$$

$g(x) = (x^2 + 6x + 10)(4x - 1)$

The zeros of $g(x)$ are $x = -3 \pm i, \frac{1}{4}$.

60. $h(x) = 3x^3 - 4x^2 + 8x + 8$

Because $1 - \sqrt{3}i$ is a zero, so is $1 + \sqrt{3}i$.

$$\begin{array}{r|rrrr} 1-\sqrt{3}i & 3 & -4 & 8 & 8 \\ & & 3-3\sqrt{3}i & -10-2\sqrt{3}i & -8 \\ \hline & 3 & -1-3\sqrt{3}i & -2-2\sqrt{3}i & 0 \end{array}$$

$$\begin{array}{r|rrr} 1+\sqrt{3}i & 3 & -1-3\sqrt{3}i & -2-2\sqrt{3}i \\ & & 3+3\sqrt{3}i & 2+2\sqrt{3}i \\ \hline & 3 & 2 & 0 \end{array}$$

The zero of $3x + 2$ is $x = -\frac{2}{3}$. The zeros of $f(x)$ are $x = -\frac{2}{3}, 1 \pm \sqrt{3}i$.

61. $f(x) = x^4 + 3x^3 - 5x^2 - 21x + 22$

Because $-3 + \sqrt{2}i$ is a zero, so is $-3 - \sqrt{2}i$, and

$$\begin{aligned}\left[x - \left(-3 + \sqrt{2}i\right)\right]\left[x - \left(-3 - \sqrt{2}i\right)\right] &= \left[(x+3) - \sqrt{2}i\right]\left[(x+3) + \sqrt{2}i\right]\\ &= (x+3)^2 - \left(\sqrt{2}i\right)^2\\ &= x^2 + 6x + 11\end{aligned}$$

is a factor of $f(x)$. By long division, you have:

$$\begin{array}{r} x^2 - 3x + 2 \\ x^2 + 6x + 11\overline{\big)\, x^4 + 3x^3 - 5x^2 - 21x + 22} \\ \underline{x^4 + 6x^3 + 11x^2} \qquad\qquad \\ -3x^3 - 16x^2 - 21x \qquad \\ \underline{-3x^3 - 18x^2 - 33x} \qquad \\ 2x^2 + 12x + 22 \\ \underline{2x^2 + 12x + 22} \\ 0 \end{array}$$

$$\begin{aligned}f(x) &= \left(x^2 + 6x + 11\right)\left(x^2 - 3x + 2\right)\\ &= \left(x^2 + 6x + 11\right)(x-1)(x-2)\end{aligned}$$

The zeros of $f(x)$ are $x = -3 \pm \sqrt{2}i, 1, 2$.

62. $f(x) = x^3 + 4x^2 + 14x + 20$

Because $-1 - 3i$ is zero, so is $-1 + 3i$.

$$\begin{array}{r|rrrr} -1 - 3i & 1 & 4 & 14 & 20 \\ & & -1 - 3i & -12 - 6i & -20 \\ \hline & 1 & 3 - 3i & 2 - 6i & 0 \end{array}$$

$$\begin{array}{r|rrr} -1 + 3i & 1 & 3 - 3i & 2 - 6i \\ & & -1 + 3i & -2 + 6i \\ \hline & 1 & 2 & 0 \end{array}$$

The zero of $x + 2$ is $x = -2$.

The zeros of $f(x)$ are $x = -2, -1 \pm 3i$.

63. $f(x) = x^2 + 36$

$= (x + 6i)(x - 6i)$

The zeros of $f(x)$ are $x = \pm 6i$.

64. $f(x) = x^2 - x + 56$

By the Quadratic Formula, the zeros of $f(x)$ are

$$x = \frac{1 \pm \sqrt{1 - 224}}{2} = \frac{1 \pm \sqrt{223}i}{2}.$$

$$f(x) = \left(x - \frac{1 - \sqrt{223}i}{2}\right)\left(x - \frac{1 + \sqrt{223}i}{2}\right)$$

65. $h(x) = x^2 - 2x + 17$

By the Quadratic Formula, the zeros of $f(x)$ are

$$x = \frac{2 \pm \sqrt{4 - 68}}{2} = \frac{2 \pm \sqrt{-64}}{2} = 1 \pm 4i.$$

$$\begin{aligned}f(x) &= \left(x - (1 + 4i)\right)\left(x - (1 - 4i)\right)\\ &= (x - 1 - 4i)(x - 1 + 4i)\end{aligned}$$

66. $g(x) = x^2 + 10x + 17$

By the Quadratic Formula, the zeros of $f(x)$ are:

$$x = \frac{-10 \pm \sqrt{100 - 68}}{2} = \frac{-10 \pm \sqrt{32}}{2} = -5 \pm 2\sqrt{2}.$$

$$\begin{aligned}f(x) &= \left(x - \left(-5 + 2\sqrt{2}\right)\right)\left(x - \left(-5 - 2\sqrt{2}\right)\right)\\ &= \left(x + 5 - 2\sqrt{2}\right)\left(x + 5 + 2\sqrt{2}\right)\end{aligned}$$

67. $f(x) = x^4 - 16$

$$\begin{aligned}&= \left(x^2 - 4\right)\left(x^2 + 4\right)\\ &= (x - 2)(x + 2)(x - 2i)(x + 2i)\end{aligned}$$

Zeros: $\pm 2, \pm 2i$

68. $f(y) = y^4 - 256$

$= (y^2 - 16)(y^2 + 16)$

$= (y - 4)(y + 4)(y - 4i)(y + 4i)$

Zeros: $\pm 4, \pm 4i$

69. $f(z) = z^2 - 2z + 2$

By the Quadratic Formula, the zeros of $f(z)$ are

$$z = \frac{2 \pm \sqrt{4 - 8}}{2} = 1 \pm i.$$

$$f(z) = [z - (1 + i)][z - (1 - i)]$$
$$= (z - 1 - i)(z - 1 + i)$$

70. $h(x) = x^3 - 3x^2 + 4x - 2$

Possible rational zeros: $\pm 1, \pm 2$

$$\begin{array}{r|rrrr} 1 & 1 & -3 & 4 & -2 \\ & & 1 & -2 & 2 \\ \hline & 1 & -2 & 2 & 0 \end{array}$$

By the Quadratic Formula, the zeros of $x^2 - 2x + 2$ are $x = \dfrac{2 \pm \sqrt{4 - 8}}{2} = 1 \pm i.$

Zeros: $1, 1 \pm i$

$h(x) = (x - 1)(x - 1 - i)(x - 1 + i)$

71. $g(x) = x^3 - 3x^2 + x + 5$

Possible rational zeros: $\pm 1, \pm 5$

$$\begin{array}{r|rrrr} -1 & 1 & -3 & 1 & 5 \\ & & -1 & 4 & -5 \\ \hline & 1 & -4 & 5 & 0 \end{array}$$

By the Quadratic Formula, the zeros of $x^2 - 4x + 5$ are: $x = \dfrac{4 \pm \sqrt{16 - 20}}{2} = 2 \pm i$

Zeros: $-1, 2 \pm i$

$g(x) = (x + 1)(x - 2 - i)(x - 2 + i)$

72. $f(x) = x^3 - x^2 + x + 39$

Possible rational zeros: $\pm 1, \pm 3, \pm 13, \pm 39$

$$\begin{array}{r|rrrr} -3 & 1 & -1 & 1 & 39 \\ & & -3 & 12 & -39 \\ \hline & 1 & -4 & 13 & 0 \end{array}$$

By the Quadratic Formula, the zeros of $x^2 - 4x + 13$ are: $x = \dfrac{4 \pm \sqrt{16 - 52}}{2} = 2 \pm 3i$

Zeros: $-3, 2 \pm 3i$

$f(x) = (x + 3)(x - 2 - 3i)(x - 2 + 3i)$

73. $h(x) = x^3 - x + 6$

Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 6$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -1 & 6 \\ & & -2 & 4 & -6 \\ \hline & 1 & -2 & 3 & 0 \end{array}$$

By the Quadratic Formula, the zeros of $x^2 - 2x + 3$ are

$$x = \frac{2 \pm \sqrt{4 - 12}}{2} = 1 \pm \sqrt{2}i.$$

Zeros: $-2, 1 \pm \sqrt{2}i$

$$h(x) = (x + 2)\left[x - \left(1 + \sqrt{2}i\right)\right]\left[x - \left(1 - \sqrt{2}i\right)\right]$$
$$= (x + 2)\left(x - 1 - \sqrt{2}i\right)\left(x - 1 + \sqrt{2}i\right)$$

74. $h(x) = x^3 + 9x^2 + 27x + 35$

Possible rational zeros: $\pm 1, \pm 5, \pm 7, \pm 35$

$$\begin{array}{r|rrrr} -5 & 1 & 9 & 27 & 35 \\ & & -5 & -20 & -35 \\ \hline & 1 & 4 & 7 & 0 \end{array}$$

By the Quadratic Formula, the zeros of $x^2 + 4x + 7$ are $x = \dfrac{-4 \pm \sqrt{16 - 28}}{2} = -2 \pm \sqrt{3}i.$

Zeros: $-5, -2 \pm \sqrt{3}i$

$h(x) = (x + 5)\left(x + 2 + \sqrt{3}i\right)\left(x + 2 - \sqrt{3}i\right)$

75. $f(x) = 5x^3 - 9x^2 + 28x + 6$

Possible rational zeros:

$\pm1, \pm2, \pm3, \pm6, \pm\frac{1}{5}, \pm\frac{2}{5}, \pm\frac{3}{5}, \pm\frac{6}{5}$

$$\begin{array}{r|rrrr} -\frac{1}{5} & 5 & -9 & 28 & 6 \\ & & -1 & 2 & -6 \\ \hline & 5 & -10 & 30 & 0 \end{array}$$

By the Quadratic Formula, the zeros of $5x^2 - 10x + 30 = 5(x^2 - 2x + 6)$ are

$$x = \frac{2 \pm \sqrt{4 - 24}}{2} = 1 \pm \sqrt{5}i.$$

Zeros: $-\frac{1}{5}, 1 \pm \sqrt{5}i$

$$f(x) = \left[x - \left(-\frac{1}{5}\right)\right](5)\left[x - \left(1 + \sqrt{5}i\right)\right]\left[x - \left(1 - \sqrt{5}i\right)\right]$$
$$= (5x + 1)\left(x - 1 - \sqrt{5}i\right)\left(x - 1 + \sqrt{5}i\right)$$

76. $g(x) = 2x^3 - x^2 + 8x + 21$

Possible rational roots:

$\pm\frac{1}{2}, \pm1, \pm\frac{3}{2}, \pm3, \pm\frac{7}{2}, \pm7, \pm\frac{21}{2}, \pm21$

$$\begin{array}{r|rrrr} -\frac{3}{2} & 2 & -1 & 8 & 21 \\ & & -3 & 6 & -21 \\ \hline & 2 & -4 & 14 & 0 \end{array}$$

By the Quadratic Formula, the zeros of $2x^2 - 4x + 14$ are $x = \frac{4 \pm \sqrt{16 - 112}}{4} = \frac{4 \pm \sqrt{-96}}{4} = 1 \pm \sqrt{6}i.$

Zeros: $-\frac{3}{2}, 1 \pm \sqrt{6}i$

$$f(x) = \left(x + \frac{3}{2}\right)\left(x - 1 - \sqrt{6}i\right)\left(x - 1 + \sqrt{6}i\right)$$

77. $g(x) = x^4 - 4x^3 + 8x^2 - 16x + 16$

Possible rational zeros: $\pm1, \pm2, \pm4, \pm8, \pm16$

$$\begin{array}{r|rrrrr} 2 & 1 & -4 & 8 & -16 & 16 \\ & & 2 & -4 & 8 & -16 \\ \hline & 1 & -2 & 4 & -8 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & 4 & -8 \\ & & 2 & 0 & 8 \\ \hline & 1 & 0 & 4 & 0 \end{array}$$

$$g(x) = (x - 2)(x - 2)(x^2 + 4)$$
$$= (x - 2)^2(x + 2i)(x - 2i)$$

Zeros: $2, \pm2i$

78. $h(x) = x^4 + 6x^3 + 10x^2 + 6x + 9$

Possible rational zeros: $\pm1, \pm3, \pm9$

$$\begin{array}{r|rrrrr} -3 & 1 & 6 & 10 & 6 & 9 \\ & & -3 & -9 & -3 & -9 \\ \hline & 1 & 3 & 1 & 3 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -3 & 1 & 3 & 1 & 3 \\ & & -3 & 0 & -3 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

The zeros of $x^2 + 1$ are $x = \pm i$.

Zeros: $-3, \pm i$

$$h(x) = (x + 3)^2(x + i)(x - i)$$

79. $f(x) = x^4 + 10x^2 + 9$
$$= (x^2 + 1)(x^2 + 9)$$
$$= (x + i)(x - i)(x + 3i)(x - 3i)$$

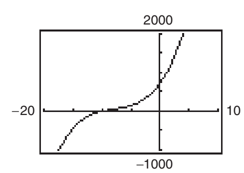
Zeros: $\pm i, \pm 3i$

80. $f(x) = x^4 + 29x^2 + 100$
$$= (x^2 + 25)(x^2 + 4)$$
$$= (x + 2i)(x - 2i)(x + 5i)(x - 5i)$$

Zeros: $\pm 2i, \pm 5i$

81. $f(x) = x^3 + 24x^2 + 214x + 740$

Possible rational zeros: $\pm1, \pm2, \pm4, \pm5, \pm10, \pm20, \pm37, \pm74, \pm148, \pm185, \pm370, \pm740$



Based on the graph, try $x = -10$.

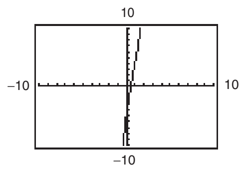
$$\begin{array}{r|rrrr} -10 & 1 & 24 & 214 & 740 \\ & & -10 & -140 & -740 \\ \hline & 1 & 14 & 74 & 0 \end{array}$$

By the Quadratic Formula, the zeros of $x^2 + 14x + 74$ are $x = \frac{-14 \pm \sqrt{196 - 296}}{2} = -7 \pm 5i.$

The zeros of $f(x)$ are $x = -10$ and $x = -7 \pm 5i$.

82. $f(s) = 2s^3 - 5s^2 + 12s - 5$

Possible rational zeros: $\pm 1, \pm 5, \pm\frac{1}{2}, \pm\frac{5}{2}$



Based on the graph, try $s = \frac{1}{2}$.

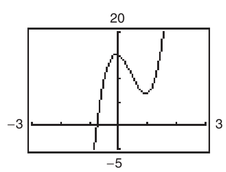
$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -5 & 12 & -5 \\ & & 1 & -2 & 5 \\ \hline & 2 & -4 & 10 & 0 \end{array}$$

By the Quadratic Formula, the zeros of $2(s^2 - 2s + 5)$ are $s = \dfrac{2 \pm \sqrt{4 - 20}}{2} = 1 \pm 2i$.

The zeros of $f(s)$ are $s = \frac{1}{2}$ and $s = 1 \pm 2i$.

83. $f(x) = 16x^3 - 20x^2 - 4x + 15$

Possible rational zeros:
$\pm 1, \pm 3, \pm 5, \pm 15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}, \pm\frac{1}{4}, \pm\frac{3}{4},$
$\pm\frac{5}{4}, \pm\frac{15}{4}, \pm\frac{1}{8}, \pm\frac{3}{8}, \pm\frac{5}{8}, \pm\frac{15}{8}, \pm\frac{1}{16}, \pm\frac{3}{16}, \pm\frac{5}{16}, \pm\frac{15}{16}$



Based on the graph, try $x = -\frac{3}{4}$.

$$\begin{array}{r|rrrr} -\frac{3}{4} & 16 & -20 & -4 & 15 \\ & & -12 & 24 & -15 \\ \hline & 16 & -32 & 20 & 0 \end{array}$$

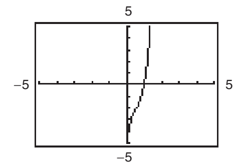
By the Quadratic Formula, the zeros of $16x^2 - 32x + 20 = 4(4x^2 - 8x + 5)$ are

$x = \dfrac{8 \pm \sqrt{64 - 80}}{8} = 1 \pm \frac{1}{2}i$.

The zeros of $f(x)$ are $x = -\frac{3}{4}$ and $x = 1 \pm \frac{1}{2}i$.

84. $f(x) = 9x^3 - 15x^2 + 11x - 5$

Possible rational zeros: $\pm 1, \pm 5, \pm\frac{1}{3}, \pm\frac{5}{3}, \pm\frac{1}{9}, \pm\frac{5}{9}$



Based on the graph, try $x = 1$.

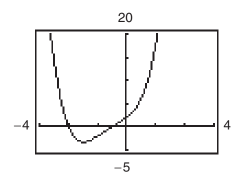
$$\begin{array}{r|rrrr} 1 & 9 & -15 & 11 & -5 \\ & & 9 & -6 & 5 \\ \hline & 9 & -6 & 5 & 0 \end{array}$$

By the Quadratic Formula, the zeros of $9x^2 - 6x + 5$ are $x = \dfrac{6 \pm \sqrt{36 - 180}}{18} = \frac{1}{3} \pm \frac{2}{3}i$.

The zeros of $f(x)$ are $x = 1$ and $x = \frac{1}{3} \pm \frac{2}{3}i$.

85. $f(x) = 2x^4 + 5x^3 + 4x^2 + 5x + 2$

Possible rational zeros: $\pm 1, \pm 2, \pm\frac{1}{2}$



Based on the graph, try $x = -2$ and $x = -\frac{1}{2}$.

$$\begin{array}{r|rrrrr} -2 & 2 & 5 & 4 & 5 & 2 \\ & & -4 & -2 & -4 & -2 \\ \hline & 2 & 1 & 2 & 1 & 0 \end{array}$$

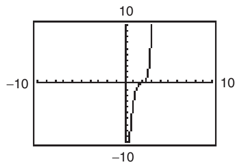
$$\begin{array}{r|rrrr} -\frac{1}{2} & 2 & 1 & 2 & 1 \\ & & -1 & 0 & -1 \\ \hline & 2 & 0 & 2 & 0 \end{array}$$

The zeros of $2x^2 + 2 = 2(x^2 + 1)$ are $x = \pm i$.

The zeros of $f(x)$ are $x = -2, x = -\frac{1}{2}$, and $x = \pm i$.

86. $g(x) = x^5 - 8x^4 + 28x^3 - 56x^2 + 64x - 32$

Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32$



Based on the graph, try $x = 2$.

$$\begin{array}{r|rrrrrr} 2 & 1 & -8 & 28 & -56 & 64 & -32 \\ & & 2 & -12 & 32 & -48 & 32 \\ \hline & 1 & -6 & 16 & -24 & 16 & 0 \end{array}$$

$$\begin{array}{r|rrrrr} 2 & 1 & -6 & 16 & -24 & 16 \\ & & 2 & -8 & 16 & -16 \\ \hline & 1 & -4 & 8 & -8 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -4 & 8 & -8 \\ & & 2 & -4 & 8 \\ \hline & 1 & -2 & 4 & 0 \end{array}$$

By the Quadratic Formula, the zeros of $x^2 - 2x + 4$ are $x = \dfrac{2 \pm \sqrt{4 - 16}}{2} = 1 \pm \sqrt{3}i$.

The zeros of $g(x)$ are $x = 2$ and $x = 1 \pm \sqrt{3}i$.

87. $g(x) = 2x^3 - 3x^2 - 3$

Sign variations: 1, positive zeros: 1

$g(-x) = -2x^3 - 3x^2 - 3$

Sign variations: 0, negative zeros: 0

88. $h(x) = 4x^2 - 8x + 3$

Sign variations: 2, positive zeros: 2 or 0

$h(-x) = 4x^2 + 8x + 3$

Sign variations: 0, negative zeros: 0

89. $h(x) = 2x^3 + 3x^2 + 1$

Sign variations: 0, positive zeros: 0

$h(-x) = -2x^3 + 3x^2 + 1$

Sign variations: 1, negative zeros: 1

90. $h(x) = 2x^4 - 3x + 2$

Sign variations: 2, positive zeros: 2 or 0

$h(-x) = 2x^4 + 3x + 2$

Sign variations: 0, negative zeros: 0

91. $g(x) = 5x^5 - 10x = 5x\left(x^4 - 2\right)$

Let $g(x) = x^4 - 2$.

Sign variations: 1, positive zeros: 1

$g(-x) = x^4 - 2$

Sign variations: 1, negative zeros: 1

92. $f(x) = 4x^3 - 3x^2 + 2x - 1$

Sign variations: 3, positive zeros: 3 or 1

$f(-x) = -4x^3 - 3x^2 - 2x - 1$

Sign variations: 0, negative zeros: 0

93. $f(x) = -5x^3 + x^2 - x + 5$

Sign variations: 3, positive zeros: 3 or 1

$f(-x) = 5x^3 + x^2 + x + 5$

Sign variations: 0, negative zeros: 0

94. $f(x) = 3x^3 + 2x^2 + x + 3$

Sign variations: 0, positive zeros: 0

$f(-x) = -3x^3 + 2x^2 - x + 3$

Sign variations: 3, negative zeros: 3 or 1

95. $f(x) = x^3 + 3x^2 - 2x + 1$

(a) $$\begin{array}{r|rrrr} 1 & 1 & 3 & -2 & 1 \\ & & 1 & 4 & 2 \\ \hline & 1 & 4 & 2 & 3 \end{array}$$

1 is an upper bound.

(b) $$\begin{array}{r|rrrr} -4 & 1 & 3 & -2 & 1 \\ & & -4 & 4 & -8 \\ \hline & 1 & -1 & 2 & -7 \end{array}$$

-4 is a lower bound.

96. $f(x) = x^3 - 4x^2 + 1$

(a) $$\begin{array}{r|rrrr} 4 & 1 & -4 & 0 & 1 \\ & & 4 & 0 & 0 \\ \hline & 1 & 0 & 0 & 1 \end{array}$$

4 is an upper bound.

(b) $$\begin{array}{r|rrrr} -1 & 1 & -4 & 0 & 1 \\ & & -1 & 5 & -5 \\ \hline & 1 & -5 & 5 & -4 \end{array}$$

-1 is a lower bound.

97. $f(x) = x^4 - 4x^3 + 16x - 16$

(a)
$$\begin{array}{r|rrrrr} 5 & 1 & -4 & 0 & 16 & -16 \\ & & 5 & 5 & 25 & 205 \\ \hline & 1 & 1 & 5 & 41 & 189 \end{array}$$

5 is an upper bound.

(b)
$$\begin{array}{r|rrrrr} -3 & 1 & -4 & 0 & 16 & -16 \\ & & -3 & 21 & -63 & 141 \\ \hline & 1 & -7 & 21 & -47 & 125 \end{array}$$

-3 is a lower bound.

98. $f(x) = 2x^4 - 8x + 3$

(a)
$$\begin{array}{r|rrrrr} 3 & 2 & 0 & 0 & -8 & 3 \\ & & 6 & 18 & 54 & 138 \\ \hline & 2 & 6 & 18 & 46 & 141 \end{array}$$

3 is an upper bound.

(b)
$$\begin{array}{r|rrrrr} -4 & 2 & 0 & 0 & -8 & 3 \\ & & -8 & 32 & -128 & 544 \\ \hline & 2 & -8 & 32 & -136 & 547 \end{array}$$

-3 is a lower bound.

99. $f(x) = 4x^3 - 3x - 1$

Possible rational zeros: $\pm1, \pm\frac{1}{2}, \pm\frac{1}{4}$

$$\begin{array}{r|rrrr} 1 & 4 & 0 & -3 & -1 \\ & & 4 & 4 & 1 \\ \hline & 4 & 4 & 1 & 0 \end{array}$$

$$4x^3 - 3x - 1 = (x - 1)(4x^2 + 4x + 1)$$
$$= (x - 1)(2x + 1)^2$$

So, the zeros are 1 and $-\dfrac{1}{2}$.

100. $f(z) = 12z^3 - 4z^2 - 27z + 9$

Possible rational zeros: $\pm1, \pm3, \pm9, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}, \pm\frac{1}{3}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{9}{4}, \pm\frac{1}{6}, \pm\frac{1}{12}$

$$\begin{array}{r|rrrr} \frac{3}{2} & 12 & -4 & -27 & 9 \\ & & 18 & 21 & -9 \\ \hline & 12 & 14 & -6 & 0 \end{array}$$

$$f(z) = 2\left(z - \tfrac{3}{2}\right)(6z^2 + 7z - 3)$$
$$= (2z - 3)(3z - 1)(2z + 3)$$

So, the real zeros are $-\frac{3}{2}, \frac{1}{3},$ and $\frac{3}{2}$.

101. $f(y) = 4y^3 + 3y^2 + 8y + 6$

Possible rational zeros: $\pm1, \pm2, \pm3, \pm6, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}$

$$\begin{array}{r|rrrr} -\frac{3}{4} & 4 & 3 & 8 & 6 \\ & & -3 & 0 & -6 \\ \hline & 4 & 0 & 8 & 0 \end{array}$$

$$4y^3 + 3y^2 + 8y + 6 = \left(y + \tfrac{3}{4}\right)(4y^2 + 8)$$
$$= \left(y + \tfrac{3}{4}\right)4(y^2 + 2)$$
$$= (4y + 3)(y^2 + 2)$$

So, the only real zero is $-\frac{3}{4}$.

102. $g(x) = 3x^3 - 2x^2 + 15x - 10$

Possible rational zeros:
$\pm1, \pm2, \pm5, \pm10, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{5}{3}, \pm\frac{10}{3}$

$$\begin{array}{r|rrrr} \frac{2}{3} & 3 & -2 & 15 & -10 \\ & & 2 & 0 & 10 \\ \hline & 3 & 0 & 15 & 0 \end{array}$$

$$g(x) = \left(x - \tfrac{2}{3}\right)(3x^2 + 15) = (3x - 2)(x^2 + 5)$$

So, the only real zero is $\frac{2}{3}$.

103. $P(x) = x^4 - \frac{25}{4}x^2 + 9$
$$= \tfrac{1}{4}(4x^4 - 25x^2 + 36)$$
$$= \tfrac{1}{4}(4x^2 - 9)(x^2 - 4)$$
$$= \tfrac{1}{4}(2x + 3)(2x - 3)(x + 2)(x - 2)$$

The rational zeros are $\pm\frac{3}{2}$ and ±2.

104. $f(x) = \frac{1}{2}(2x^3 - 3x^2 - 23x + 12)$

Possible rational zeros: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12, \pm\frac{1}{2}, \frac{3}{2}$

$$\begin{array}{r|rrrr} 4 & 2 & -3 & -23 & 12 \\ & & 8 & 20 & -12 \\ \hline & 2 & 5 & -3 & 0 \end{array}$$

$$f(x) = \tfrac{1}{2}(x - 4)(2x^2 + 5x - 3)$$
$$= \tfrac{1}{2}(x - 4)(2x - 1)(x + 3)$$

The rational zeros are $-3, \frac{1}{2},$ and 4.

105. $f(x) = x^3 - \frac{1}{4}x^2 - x + \frac{1}{4}$

$$= \tfrac{1}{4}(4x^3 - x^2 - 4x + 1)$$
$$= \tfrac{1}{4}[x^2(4x - 1) - 1(4x - 1)]$$
$$= \tfrac{1}{4}(4x - 1)(x^2 - 1)$$
$$= \tfrac{1}{4}(4x - 1)(x + 1)(x - 1)$$

The rational zeros are $\frac{1}{4}$ and ± 1.

106. $f(z) = \frac{1}{6}(6z^3 + 11z^2 - 3z - 2)$

Possible rational zeros: $\pm 1, \pm 2, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{1}{6}$

$$\begin{array}{r|rrrr} -2 & 6 & 11 & -3 & -2 \\ & & -12 & 2 & 2 \\ \hline & 6 & -1 & -1 & 0 \end{array}$$

$$f(x) = \tfrac{1}{6}(x + 2)(6x^2 - x - 1)$$
$$= \tfrac{1}{6}(x + 2)(3x + 1)(2x - 1)$$

The rational zeros are $-2, -\frac{1}{3},$ and $\frac{1}{2}$.

107. $f(x) = x^3 - 1 = (x - 1)(x^2 + x + 1)$

Rational zeros: $1\ (x = 1)$

Irrational zeros: 0

Matches (d).

108. $f(x) = x^3 - 2$

$$= \left(x - \sqrt[3]{2}\right)\left(x^2 + \sqrt[3]{2}x + \sqrt[3]{4}\right)$$

Rational zeros: 0

Irrational zeros: $1\left(x = \sqrt[3]{2}\right)$

Matches (a).

109. $f(x) = x^3 - x = x(x + 1)(x - 1)$

Rational zeros: $3\ (x = 0, \pm 1)$

Irrational zeros: 0

Matches (b).

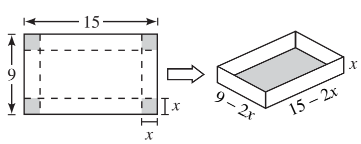
110. $f(x) = x^3 - 2x$

$$= x(x^2 - 2)$$
$$= x\left(x + \sqrt{2}\right)\left(x - \sqrt{2}\right)$$

Rational zeros: $1\ (x = 0)$

Irrational zeros: $2\left(x = \pm\sqrt{2}\right)$

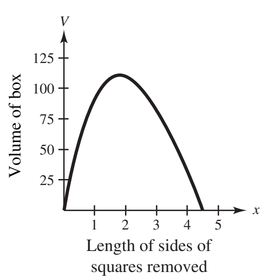
Matches (c).

111. (a) 

(b) $V = l \cdot w \cdot h = (15 - 2x)(9 - 2x)x$

$$= x(9 - 2x)(15 - 2x)$$

Because length, width, and height must be positive, you have $0 < x < \frac{9}{2}$ for the domain.

(c) 

The volume is maximum when $x \approx 1.82$.

The dimensions are: length $\approx 15 - 2(1.82) = 11.36$

width $\approx 9 - 2(1.82) = 5.36$

height $= x \approx 1.82$

1.82 cm × 5.36 cm × 11.36 cm

(d) $56 = x(9 - 2x)(15 - 2x)$

$$56 = 135x - 48x^2 + 4x^3$$
$$0 = 4x^3 - 48x^2 + 135x - 56$$

The zeros of this polynomial are $\frac{1}{2}, \frac{7}{2},$ and 8.

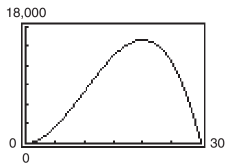
x cannot equal 8 because it is not in the domain of V. [The length cannot equal -1 and the width cannot equal -7. The product of $(8)(-1)(-7) = 56$ so it showed up as an extraneous solution.]

So, the volume is 56 cubic centimeters when $x = \frac{1}{2}$ centimeter or $x = \frac{7}{2}$ centimeters.

112. (a) Combined length and width:

$4x + y = 120 \Rightarrow y = 120 - 4x$

$$\begin{aligned}\text{Volume} &= l \cdot w \cdot h = x^2 y\\ &= x^2(120 - 4x)\\ &= 4x^2(30 - x)\end{aligned}$$

(b) 

Dimensions with maximum volume:
20 in. × 20 in. × 40 in.

(c) $$\begin{aligned}13{,}500 &= 4x^2(30 - x)\\ 4x^3 - 120x^2 + 13{,}500 &= 0\\ x^3 - 30x^2 + 3375 &= 0\end{aligned}$$

$$\begin{array}{r|rrrr} 15 & 1 & -30 & 0 & 3375\\ & & 15 & -225 & -3375\\ \hline & 1 & -15 & -225 & 0\end{array}$$

$(x - 15)(x^2 - 15x - 225) = 0$

Using the Quadratic Formula, $x = 15, \dfrac{15 \pm 15\sqrt{5}}{2}$.

The value of $\dfrac{15 - 15\sqrt{5}}{2}$ is not possible because it is negative.

113. (a) Current bin: $V = 2 \times 3 \times 4 = 24$ cubic feet
New bin: $V = 5(24) = 120$ cubic feet

$V(x) = (2 + x)(3 + x)(4 + x) = 120$

(b) $x^3 + 9x^2 + 26x + 24 = 120$

$x^3 + 9x^2 + 26x - 96 = 0$

The only real zero of this polynomial is $x = 2$. All the dimensions should be increased by 2 feet, so the new bin will have dimensions of 4 feet by 5 feet by 6 feet.

114. $C = 100\left(\dfrac{200}{x^2} + \dfrac{x}{x + 30}\right), x \geq 1$

C is minimum when

$3x^3 - 40x^2 - 2400x - 36000 = 0.$

The only real zero is $x \approx 40$ or 4000 units.

115. False. The most complex zeros it can have is two, and the Linear Factorization Theorem guarantees that there are three linear factors, so one zero must be real.

116. False. f does not have real coefficients.

117. $g(x) = -f(x)$. This function would have the same zeros as $f(x)$, so $r_1, r_2,$ and r_3 are also zeros of $g(x)$.

118. $g(x) = 3f(x)$. This function has the same zeros as f because it is a vertical stretch of f. The zeros of g are $r_1, r_2,$ and r_3.

119. $g(x) = f(x - 5)$. The graph of $g(x)$ is a horizontal shift of the graph of $f(x)$ five units of the right, so the zeros of $g(x)$ are $5 + r_1, 5 + r_2,$ and $5 + r_3$.

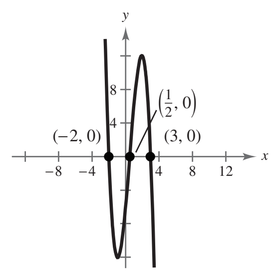
120. $g(x) = f(2x)$. Note that x is a zero of g if and only if $2x$ is a zero of f. The zeros of g are $\dfrac{r_1}{2}, \dfrac{r_2}{2},$ and $\dfrac{r_3}{2}$.

121. $g(x) = 3 + f(x)$. Because $g(x)$ is a vertical shift of the graph of $f(x)$, the zeros of $g(x)$ cannot be determined.

122. $g(x) = f(-x)$. Note that x is a zero of g if and only if $-x$ is a zero of f. The zeros of g are $-r_1, -r_2,$ and $-r_3$.

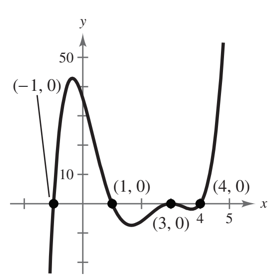
123. Zeros: $-2, \frac{1}{2}, 3$

$$\begin{aligned}f(x) &= -(x + 2)(2x - 1)(x - 3)\\ &= -2x^3 + 3x^2 + 11x - 6\end{aligned}$$



Any nonzero scalar multiple of f would have the same three zeros. Let $g(x) = af(x), a > 0$. There are infinitely many possible functions for f.

124.



Section 2.6 Rational Functions

1. rational functions

2. vertical asymptote

3. horizontal asymptote

4. slant asymptote

5. Because the denominator is zero when $x - 1 = 0$, the domain of f is all real numbers except $x = 1$.

x	0	0.5	0.9	0.99	$\to 1$
$f(x)$	–1	–2	–10	–100	$\to -\infty$

x	$1 \leftarrow$	1.01	1.1	1.5	2
$f(x)$	$\infty \leftarrow$	100	10	2	1

As x approaches 1 from the left, $f(x)$ decreases without bound. As x approaches 1 from the right, $f(x)$ increases without bound.

6. Because the denominator is zero when $x + 2 = 0$, the domain of f is all real numbers except $x = -2$.

x	–3	–2.5	–2.1	–2.01	$\to -2$
$f(x)$	15	25	105	1005	$\to \infty$

x	$-2 \leftarrow$	–1.99	–1.9	–1.5	–1
$f(x)$	$-\infty \leftarrow$	–955	–95	–15	–5

As x approaches -2 from the left, $f(x)$ increases without bound. As x approaches -2 from the right, $f(x)$ increases without bound.

7. Because the denominator is zero when $x^2 - 1 = 0$, the domain of f is all real numbers except $x = -1$ and $x = 1$.

x	–2	–1.5	–1.1	–1.01	$\to -1$
$f(x)$	4	5.4	17.3	152.3	$-\infty$

x	$-1 \leftarrow$	–0.99	–0.9	–0.5	0
$f(x)$	$-\infty \leftarrow$	–147.8	–12.8	–1	0

As x approaches -1 from the left, $f(x)$ increases without bound. As x approaches -1 from the right, $f(x)$ decreases without bound.

x	0	0.5	0.9	0.99	$\to 1$
$f(x)$	0	–1	–12.8	–147.8	$\to -\infty$

x	$1 \leftarrow$	1.01	1.1	1.5	2
$f(x)$	$\infty \leftarrow$	152.3	17.3	5.4	4

As x approaches 1 from the left, $f(x)$ increases without bound. As x approaches 1 from the right, $f(x)$ decreases without bound.

8. Because the denominator is zero when $x^2 - 4 = 0$, the domain of f is all real numbers except $x = -2$ and $x = 2$.

x	–3	–2.5	–2.1	–2.01
$f(x)$	–1.2	–2.2	–10.2	–100.2

x	–2	–1.99	–1.9	–1.5	–1
$f(x)$	Undef.	99.7	9.7	1.7	0.7

As x approaches -2 from the left, $f(x)$ decreases without bound. As x approaches -2 from the right, $f(x)$ increases without bound.

x	1	1.5	1.9	1.99
$f(x)$	–0.7	–1.7	–9.7	–99.7

x	2	2.01	2.1	2.5	3
$f(x)$	Undef.	100.2	10.2	2.2	1.2

As x approaches 2 from the left, $f(x)$ decreases without bound. As x approaches 2 from the right, $f(x)$ increases without bound.

9. $f(x) = \dfrac{4}{x^2}$

Domain: all real numbers except $x = 0$

Vertical asymptote: $x = 0$

Horizontal asymptote: $y = 0$

$\left[\text{Degree of } N(x) < \text{degree of } D(x)\right]$

10. $f(x) = \dfrac{1}{(x-2)^3}$

Domain: all real numbers except $x = 2$

Vertical asymptote: $x = 2$

Horizontal asymptote: $y = 0$

$\left[\text{Degree of } N(x) < \text{degree of } D(x)\right]$

11. $f(x) = \dfrac{5+x}{5-x} = \dfrac{x+5}{-x+5}$

Domain: all real numbers except $x = 5$

Vertical asymptote: $x = 5$

Horizontal asymptote: $y = -1$

$\left[\text{Degree of } N(x) = \text{degree of } D(x)\right]$

12. $f(x) = \dfrac{3-7x}{3+2x} = \dfrac{-7x+3}{2x+3}$

Domain: all real numbers except $x = -\dfrac{3}{2}$

Vertical asymptote: $x = -\dfrac{3}{2}$

Horizontal asymptote: $y = -\dfrac{7}{2}$

$\left[\text{Degree of } N(x) = \text{degree of } D(x)\right]$

13. $f(x) = \dfrac{x^3}{x^2-1}$

Domain: all real numbers except $x = \pm 1$

Vertical asymptotes: $x = \pm 1$

Horizontal asymptote: None

$\left[\text{Degree of } N(x) > \text{degree of } D(x)\right]$

14. $f(x) = \dfrac{4x^2}{x+2}$

Vertical asymptote: $x = -2$

Horizontal asymptote: None

$\left[\text{Degree of } N(x) = \text{degree of } D(x)\right]$

15. $f(x) = \dfrac{3x^2+1}{x^2+x+9}$

Domain: All real numbers. The denominator has no real zeros. [Try the Quadratic Formula on the denominator.]

Vertical asymptote: None

Horizontal asymptote: $y = 3$

$\left[\text{Degree of } N(x) = \text{degree of } D(x)\right]$

16. $f(x) = \dfrac{3x^2+x-5}{x^2+1}$

Domain: All real numbers. The denominator has no real zeros. [Try the Quadratic Formula on the denominator.]

Vertical asymptote: None

Horizontal asymptote: $y = 3$

$\left[\text{Degree of } N(x) = \text{degree of } D(x)\right]$

17. $f(x) = \dfrac{1}{x+2}$

(a) Domain: all real numbers x except $x = -2$

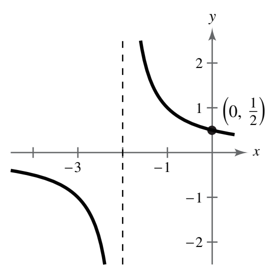
(b) y-intercept: $\left(0, \dfrac{1}{2}\right)$

(c) Vertical asymptote: $x = -2$

Horizontal asymptote: $y = 0$

(d)

x	-4	-3	-1	0	1
$f(x)$	$-\frac{1}{2}$	-1	1	$\frac{1}{2}$	$\frac{1}{3}$



18. $f(x) = \dfrac{1}{x - 3}$

(a) Domain: all real numbers x except $x = 3$

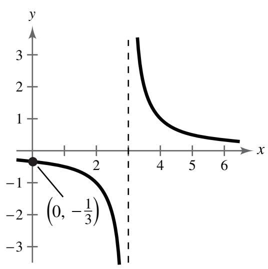
(b) y-intercept: $\left(0, -\dfrac{1}{3}\right)$

(c) Vertical asymptote: $x = 3$

Horizontal asymptote: $y = 0$

(d)

x	0	1	2	4	5	6
$f(x)$	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	1	$\frac{1}{2}$	$\frac{1}{3}$



19. $h(x) = \dfrac{-1}{x + 4}$

(a) Domain: all real numbers x except $x = -4$

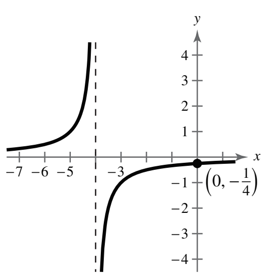
(b) y-intercept: $\left(0, -\dfrac{1}{4}\right)$

(c) Vertical asymptote: $x = -4$

Horizontal asymptote: $y = 0$

(d)

x	-6	-5	-3	-2	-1	0
$h(x)$	$\frac{1}{2}$	1	-1	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{4}$



20. $g(x) = \dfrac{1}{6 - x} = -\dfrac{1}{x - 6}$

(a) Domain: all real numbers x except $x = 6$

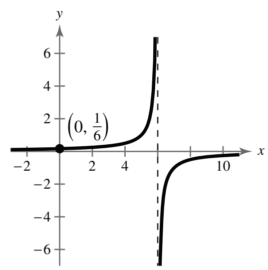
(b) y-intercept: $\left(0, \dfrac{1}{6}\right)$

(c) Vertical asymptote: $x = 6$

Horizontal asymptote: $y = 0$

(d)

x	-2	0	2	4	8
$g(x)$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{2}$	$-\frac{1}{2}$



21. $C(x) = \dfrac{7 + 2x}{2 + x}$

(a) Domain: all real numbers x except $x = -2$

(b) x-intercept: $\left(-\dfrac{7}{2}, 0\right)$

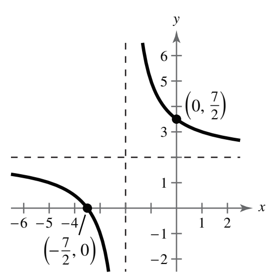
y-intercept: $\left(0, \dfrac{7}{2}\right)$

(c) Vertical asymptote: $x = -2$

Horizontal asymptote: $y = 2$

(d)

x	-4	-3	-1	0	1
$C(x)$	$\frac{1}{2}$	-1	5	$\frac{7}{2}$	3



22. $P(x) = \dfrac{1 - 3x}{1 - x} = \dfrac{3x - 1}{x - 1}$

(a) Domain: all real numbers x except $x = 1$

(b) x-intercept: $\left(\dfrac{1}{3}, 0\right)$

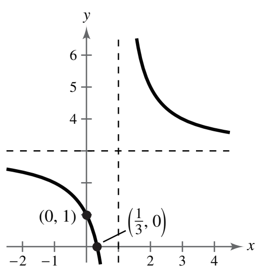
y-intercept: $(0, 1)$

(c) Vertical asymptote: $x = 1$

Horizontal asymptote: $y = 3$

(d)

x	-1	0	2	3
$P(x)$	2	1	5	4



23. $f(x) = \dfrac{x^2}{x^2 + 9}$

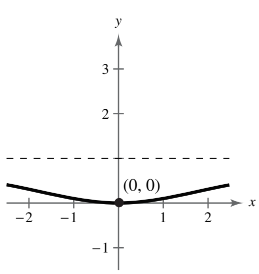
(a) Domain: all real numbers x

(b) Intercept: $(0, 0)$

(c) Horizontal asymptote: $y = 1$

(d)

x	± 1	± 2	± 3
$f(x)$	$\dfrac{1}{10}$	$\dfrac{4}{13}$	$\dfrac{1}{2}$



24. $g(s) = \dfrac{4s}{s^2 + 4}$

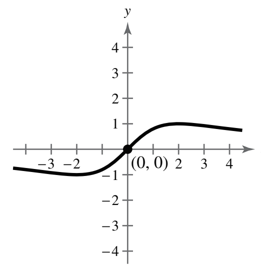
(a) Domain: all real numbers s

(b) Intercept: $(0, 0)$

(c) Horizontal asymptote: $y = 0$

(d)

s	-2	-1	0	1	2
$g(s)$	-1	$-\dfrac{4}{5}$	0	$\dfrac{4}{5}$	1



25. $f(t) = \dfrac{1 - 2t}{t} = -\dfrac{2t - 1}{t}$

(a) Domain: all real numbers t except $t = 0$

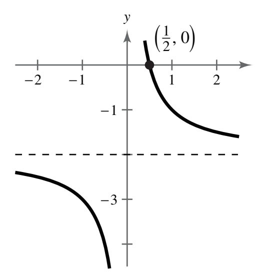
(b) t-intercept: $\left(\dfrac{1}{2}, 0\right)$

(c) Vertical asymptote: $t = 0$

Horizontal asymptote: $y = -2$

(d)

t	-2	-1	$\dfrac{1}{2}$	1	2
$f(t)$	$-\dfrac{5}{2}$	-3	0	-1	$-\dfrac{3}{2}$



26. $f(x) = -\dfrac{1}{(x-2)^2}$

(a) Domain: all real numbers x except $x = 2$

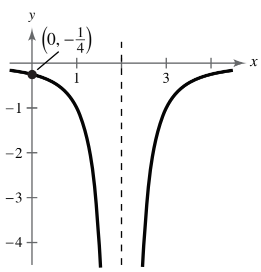
(b) y-intercept: $\left(0, -\dfrac{1}{4}\right)$

(c) Vertical asymptote: $x = 2$

Horizontal asymptote: $y = 0$

(d)

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	$\frac{5}{2}$	3	$\frac{7}{2}$	4
$f(x)$	$-\frac{1}{4}$	$-\frac{4}{9}$	-1	-4	-4	-1	$-\frac{4}{9}$	$-\frac{1}{4}$



27. $h(x) = \dfrac{x^2 - 5x + 4}{x^2 - 4} = \dfrac{(x-1)(x-4)}{(x+2)(x-2)}$

(a) Domain: all real numbers x except $x = \pm 2$

(b) x-intercepts: $(1, 0), (4, 0)$

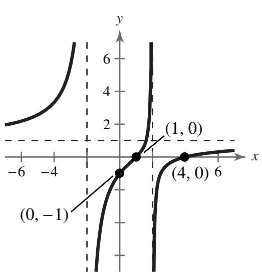
y-intercept: $(0, -1)$

(c) Vertical asymptotes: $x = -2, x = 2$

Horizontal asymptote: $y = 1$

(d)

x	-4	-3	-1	0	1	3	4
$h(x)$	$\frac{10}{3}$	$\frac{28}{5}$	$-\frac{10}{3}$	-1	0	$-\frac{2}{5}$	0



28. $g(x) = \dfrac{x^2 - 2x - 8}{x^2 - 9} = \dfrac{(x-4)(x+2)}{(x-3)(x+3)}$

(a) Domain: all real numbers x except $x = \pm 3$

(b) x-intercepts: $(4, 0), (-2, 0)$

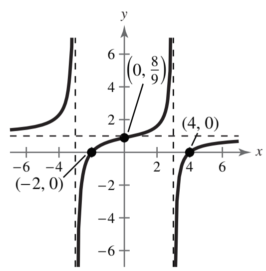
y-intercept: $\left(0, \dfrac{8}{9}\right)$

(c) Vertical asymptotes: $x = \pm 3$

Horizontal asymptote: $y = 1$

(d)

x	-5	-4	-2	0	2	4	5
$g(x)$	$\frac{27}{16}$	$\frac{16}{7}$	0	$\frac{8}{9}$	$\frac{8}{5}$	0	$\frac{7}{16}$



29. $f(x) = \dfrac{x-4}{x^2-16} = \dfrac{x-4}{(x-4)(x+4)} = \dfrac{1}{x+4}, x \neq 4$

Domain: all real numbers x except $x = \pm 4$

Vertical asymptote: $x = -4$ (Because $x - 4$ is a common factor of $N(x)$ and $D(x)$, $x = 4$ is not a vertical asymptote of $f(x)$.)

Horizontal asymptote: $y = 0$

[Degree of $N(x)$ < degree of $D(x)$]

30. $f(x) = \dfrac{x+1}{x^2-1} = \dfrac{x+1}{(x+1)(x-1)} = \dfrac{1}{x-1}, x \neq -1$

Domain: all real numbers x except $x = \pm 1$

Vertical asymptote: $x = 1$ (Because $x + 1$ is a common factor of $N(x)$ and $D(x)$, $x = -1$ is not a vertical asymptote of $f(x)$.)

Horizontal asymptote: $y = 0$

[Degree of $N(x)$ < degree of $D(x)$]

31. $f(t) = \dfrac{t^2 - 1}{t - 1} = \dfrac{(t + 1)(t - 1)}{t - 1} = t + 1, \quad t \neq 1$

(a) Domain: all real numbers t except $t = 1$

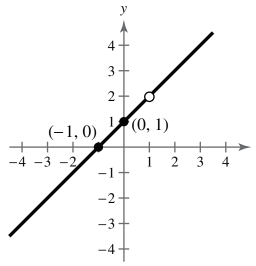
(b) t-intercept: $(-1, 0)$

y-intercept: $(0, 1)$

(c) No asymptotes

(d)

t	-3	-2	-1	0	1	2
$f(t)$	-2	-1	0	1	Undef.	3



32. $f(x) = \dfrac{x^2 - 36}{x + 6} = \dfrac{(x + 6)(x - 6)}{x + 6} = x - 6, \quad x \neq -6$

(a) Domain: all real numbers x except $x = -6$

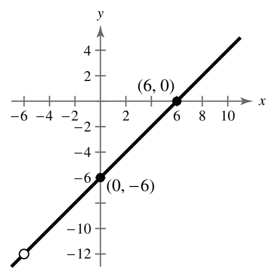
(b) x-intercept: $(6, 0)$

y-intercept: $(0, -6)$

(c) No asymptotes

(d)

x	-6	-4	-2	0	2	4	6	8
$f(x)$	Undef.	-10	-8	-6	-4	-2	0	2



33. $f(x) = \dfrac{x^2 - 25}{x^2 - 4x - 5} = \dfrac{(x + 5)(x - 5)}{(x - 5)(x + 1)} = \dfrac{x + 5}{x + 1}, \; x \neq 5$

Domain: all real numbers x except $x = 5$ and $x = -1$

Vertical asymptote: $x = -1$ (Because $x - 5$ is a common factor of $N(x)$ and $D(x)$, $x = 5$ is not a vertical asymptote of $f(x)$.)

Horizontal asymptote: $y = 1$

[Degree of $N(x)$ = degree of $D(x)$]

34. $f(x) = \dfrac{x^2 - 4}{x^2 - 3x + 2}$

$= \dfrac{(x - 2)(x + 2)}{(x - 2)(x - 1)} = \dfrac{x + 2}{x - 1}, \; x \neq 2$

Domain: all real numbers x except $x = 1$ and $x = 2$

Vertical asymptote: $x = 1$ (Because $x - 2$ is a common factor of $N(x)$ and $D(x)$, $x = 2$ is not a vertical asymptote of $f(x)$.)

Horizontal asymptote: $y = 1$

[Degree of $N(x)$ = degree of $D(x)$]

35. $f(x) = \dfrac{x^2 + 3x}{x^2 + x - 6} = \dfrac{x(x + 3)}{(x + 3)(x - 2)} = \dfrac{x}{x - 2}, \quad x \neq -3$

(a) Domain: all real numbers x except $x = -3$ and $x = 2$

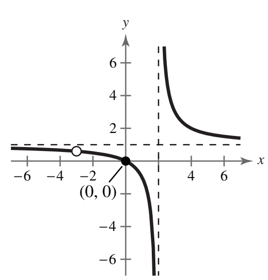
(b) Intercept: $(0, 0)$

(c) Vertical asymptote: $x = 2$

Horizontal asymptote: $y = 1$

(d)

x	-1	0	1	3	4
$f(x)$	$\frac{1}{3}$	0	-1	3	2



36. $f(x) = \dfrac{5(x+4)}{x^2+x-12} = \dfrac{5(x+4)}{(x+4)(x-3)} = \dfrac{5}{x-3}, \quad x \neq -4$

(a) Domain: all real numbers x except $x = -4$ or $x = 3$

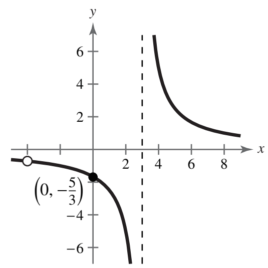
(b) y-intercept: $\left(0, -\dfrac{5}{3}\right)$

(c) Vertical asymptote: $x = 3$

Horizontal asymptote: $y = 0$

(d)

x	-2	0	2	5	7
$f(x)$	-1	$-\frac{5}{3}$	-5	$\frac{5}{2}$	$\frac{5}{4}$



37. $f(x) = \dfrac{x^2-3x-4}{2x^2+x-1}$

$= \dfrac{(x+1)(x-4)}{(2x-1)(x+1)} = \dfrac{x-4}{2x-1}, x \neq -1$

Domain: all real numbers x except $x = \dfrac{1}{2}$ and $x = -1$

Vertical asymptote: $x = \dfrac{1}{2}$ (Because $x + 1$ is a common factor of $N(x)$ and $D(x)$, $x = -1$ is not a vertical asymptote of $f(x)$.)

Horizontal asymptote: $y = \dfrac{1}{2}$

[Degree of $N(x)$ = degree of $D(x)$]

38. $f(x) = \dfrac{6x^2-11x+3}{6x^2-7x-3}$

$= \dfrac{(2x-3)(3x-1)}{(2x-3)(3x+1)} = \dfrac{3x-1}{3x+1}, x \neq \dfrac{3}{2}$

Domain: all real numbers x except

$x = \dfrac{3}{2}$ or $x = -\dfrac{1}{3}$

Vertical asymptote: $x = -\dfrac{1}{3}$ (Because $2x - 3$ is a common factor of $N(x)$ and $D(x)$, $x = \dfrac{3}{2}$ is not a vertical asymptote of $f(x)$.)

Horizontal asymptote: $y = 1$

[Degree of $N(x)$ = degree of $D(x)$]

39. $f(x) = \dfrac{2x^2-5x-3}{x^3-2x^2-x+2} = \dfrac{(2x+1)(x-3)}{(x-2)(x+1)(x-1)}$

(a) Domain: all real numbers x except $x = 2, x = \pm 1$

(b) x-intercepts: $\left(-\dfrac{1}{2}, 0\right), (3, 0)$

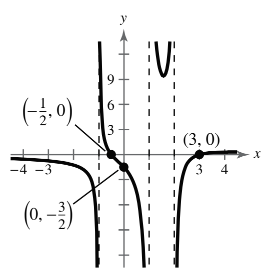
y-intercept: $\left(0, -\dfrac{3}{2}\right)$

(c) Vertical asymptotes: $x = 2, x = -1,$ and $x = 1$

Horizontal asymptote: $y = 0$

(d)

x	-3	-2	0	$\frac{3}{2}$	3	4
$f(x)$	$-\frac{3}{4}$	$-\frac{5}{4}$	$-\frac{3}{2}$	$\frac{48}{5}$	0	$\frac{3}{10}$



40. $f(x) = \dfrac{x^2 - x - 2}{x^3 - 2x^2 - 5x + 6} = \dfrac{(x+1)(x-2)}{(x-1)(x+2)(x-3)}$

(a) Domain: all real numbers x except $x = 1$, $x = -2$, and $x = 3$

(b) x-intercepts: $(-1, 0)$, $(2, 0)$

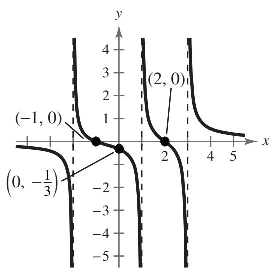
y-intercept: $\left(0, -\dfrac{1}{3}\right)$

(c) Vertical asymptotes: $x = -2$, $x = 1$, $x = 3$

Horizontal asymptote: $y = 0$

(d)

x	-4	-3	-1	0	2	4
$f(x)$	$-\dfrac{9}{35}$	$-\dfrac{5}{12}$	0	$-\dfrac{1}{3}$	0	$\dfrac{5}{9}$



41. g

42. e

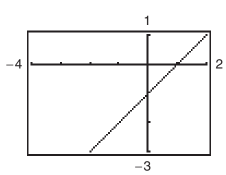
43. a

44. f

45. (a) Domain of f: all real numbers x except $x = -1$

Domain of g: all real numbers x

(b)



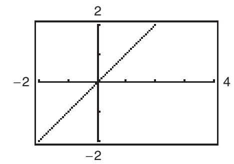
(c) Because there are only finitely many pixels, the graphing utility may not attempt to evaluate the function where it does not exist.

46. $f(x) = \dfrac{x^2(x-2)}{x^2 - 2x}$, $g(x) = x$

(a) Domain of f: All real numbers x except $x = 0$ and $x = 2$

Domain of g: All real numbers x

(b)

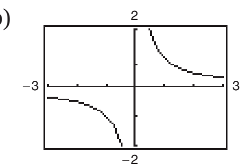


(c) Because there are only a finite number of pixels, the utility may not attempt to evaluate the function where it does not exist.

47. (a) Domain of f: all real numbers x except $x = 0, 2$

Domain of g: all real numbers $x = 0$

(b)



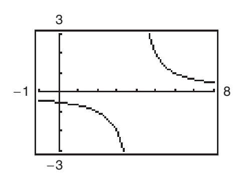
(c) Because there are only finitely many pixels, the graphing utility may not attempt to evaluate the function where it does not exist.

48. $f(x) = \dfrac{2x - 6}{x^2 - 7x + 12}$, $g(x) = \dfrac{2}{x - 4}$

(a) Domain of f: All real numbers x except $x = 3$ and $x = 4$

Domain of g: All real numbers x except $x = 4$

(b)



(c) Because there are only a finite number of pixels, the utility may not attempt to evaluate the function where it does not exist.

49. $h(x) = \dfrac{x^2 - 9}{x} = x - \dfrac{9}{x}$

(a) Domain: all real numbers x except $x = 0$

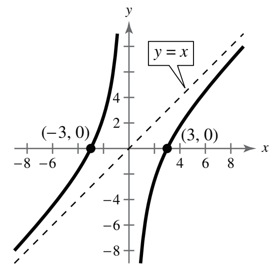
(b) x-intercepts: $(-3, 0), (3, 0)$

(c) Vertical asymptote: $x = 0$

Slant asymptote: $y = x$

(d)

x	-6	-4	-3	-2	2	3	4	6
$h(x)$	$-\frac{9}{2}$	$-\frac{7}{4}$	0	$\frac{5}{2}$	$-\frac{5}{2}$	0	$\frac{7}{4}$	$\frac{9}{2}$



50. $g(x) = \dfrac{x^2 + 5}{x} = x + \dfrac{5}{x}$

(a) Domain: all real numbers x except $x = 0$

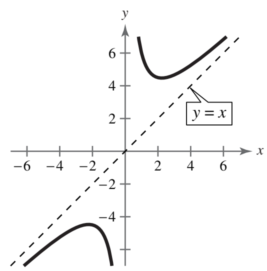
(b) No intercepts

(c) Vertical asymptote: $x = 0$

Slant asymptote: $y = x$

(d)

x	-3	-2	-1	1	2	3
$g(x)$	$-\frac{14}{3}$	$-\frac{9}{2}$	-6	6	$\frac{9}{2}$	$\frac{14}{3}$



51. $f(x) = \dfrac{2x^2 + 1}{x} = 2x + \dfrac{1}{x}$

(a) Domain: all real numbers x except $x = 0$

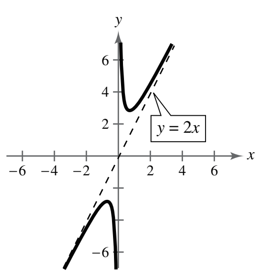
(b) No intercepts

(c) Vertical asymptote: $x = 0$

Slant asymptote: $y = 2x$

(d)

x	-4	-2	2	4	6
$f(x)$	$-\frac{33}{4}$	$-\frac{9}{2}$	$\frac{9}{2}$	$\frac{33}{4}$	$\frac{73}{6}$



52. $f(x) = \dfrac{1 - x^2}{x} = -x + \dfrac{1}{x}$

(a) Domain: all real numbers x except $x = 0$

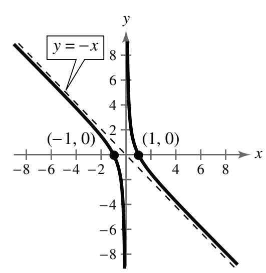
(b) x-intercepts: $(-1, 0), (1, 0)$

(c) Vertical asymptote: $x = 0$

Slant asymptote: $y = -x$

(d)

x	-6	-4	-2	2	4	6
$f(x)$	$\frac{35}{6}$	$\frac{15}{4}$	$\frac{3}{2}$	$-\frac{3}{2}$	$-\frac{15}{4}$	$-\frac{35}{6}$



53. $g(x) = \dfrac{x^2 + 1}{x} = x + \dfrac{1}{x}$

(a) Domain: all real numbers x except $x = 0$

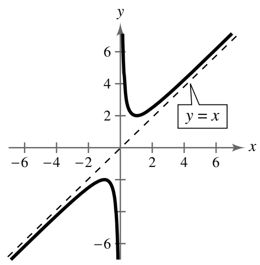
(b) No intercepts

(c) Vertical asymptote: $x = 0$

Slant asymptote: $y = x$

(d)

x	-4	-2	2	4	6
$g(x)$	$-\frac{17}{4}$	$-\frac{5}{2}$	$\frac{5}{2}$	$\frac{17}{4}$	$\frac{37}{6}$



54. $h(x) = \dfrac{x^2}{x - 1} = x + 1 + \dfrac{1}{x - 1}$

(a) Domain: all real numbers x except $x = 1$

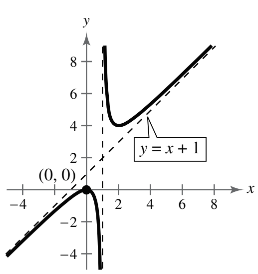
(b) Intercept: $(0, 0)$

(c) Vertical asymptote: $x = 1$

Slant asymptote: $y = x + 1$

(d)

x	-4	-2	2	4	6
$h(x)$	$-\frac{16}{5}$	$-\frac{4}{3}$	4	$\frac{16}{3}$	$\frac{36}{5}$



55. $f(t) = \dfrac{t^2 + 1}{t + 5} = -t + 5 - \dfrac{26}{t + 5}$

(a) Domain: all real numbers t except $t = -5$

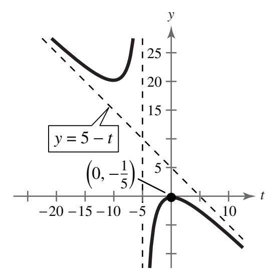
(b) Intercept: $\left(0, -\dfrac{1}{5}\right)$

(c) Vertical asymptote: $t = -5$

Slant asymptote: $y = -t + 5$

(d)

t	-7	-6	-4	-3	0
$f(t)$	25	37	-17	-5	$-\frac{1}{5}$



56. $f(x) = \dfrac{x^2}{3x + 1} = \dfrac{1}{3}x - \dfrac{1}{9} + \dfrac{1}{9(3x + 1)}$

(a) Domain: all real numbers x except $x = -\dfrac{1}{3}$

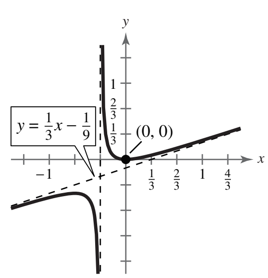
(b) Intercept: $(0, 0)$

(c) Vertical asymptote: $x = -\dfrac{1}{3}$

Slant asymptote: $y = \dfrac{1}{3}x - \dfrac{1}{9}$

(d)

x	-3	-2	-1	$-\frac{1}{2}$	-1	0	2
$f(x)$	$-\frac{9}{8}$	$-\frac{4}{5}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{4}{7}$



57. $f(x) = \dfrac{x^3}{x^2 - 4} = x + \dfrac{4x}{x^2 - 4}$

(a) Domain: all real numbers x except $x = \pm 2$

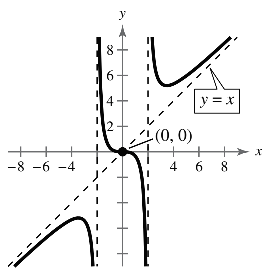
(b) Intercept: $(0, 0)$

(c) Vertical asymptotes: $x = \pm 2$

Slant asymptote: $y = x$

(d)

x	-6	-4	-1	0	1	4	6
$f(x)$	$-\frac{27}{4}$	$-\frac{16}{3}$	$\frac{1}{3}$	0	$-\frac{1}{3}$	$\frac{16}{3}$	$\frac{27}{4}$



58. $g(x) = \dfrac{x^3}{2x^2 - 8} = \dfrac{1}{2}x + \dfrac{4x}{2x^2 - 8}$

(a) Domain: all real numbers x except $x = \pm 2$

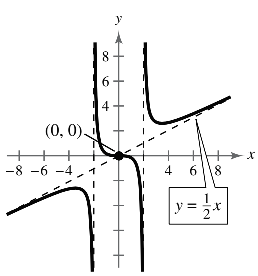
(b) Intercept: $(0, 0)$

(c) Vertical asymptotes: $x = \pm 2$

Slant asymptote: $y = \dfrac{1}{2}x$

(d)

x	-6	-4	-1	1	4	6
$g(x)$	$-\frac{27}{8}$	$-\frac{8}{3}$	$\frac{1}{6}$	$-\frac{1}{6}$	$\frac{8}{3}$	$\frac{27}{8}$



59. $f(x) = \dfrac{x^2 - x + 1}{x - 1} = x + \dfrac{1}{x - 1}$

(a) Domain: all real numbers x except $x = 1$

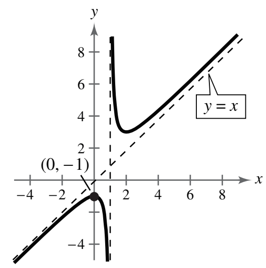
(b) y-intercept: $(0, -1)$

(c) Vertical asymptotes: $x = 1$

Slant asymptote: $y = x$

(d)

x	-4	-2	0	2	4
$f(x)$	$-\frac{21}{5}$	$-\frac{7}{3}$	-1	3	$\frac{13}{3}$



60. $f(x) = \dfrac{2x^2 - 5x + 5}{x - 2} = 2x - 1 + \dfrac{3}{x - 2}$

(a) Domain: all real numbers x except $x = 2$

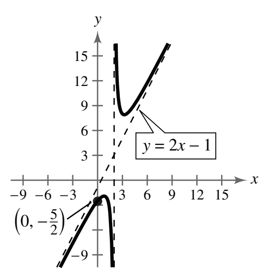
(b) y-intercept: $\left(0, -\dfrac{5}{2}\right)$

(c) Vertical asymptote: $x = 2$

Slant asymptote: $y = 2x - 1$

(d)

x	-6	-3	1	3	6	7
$f(x)$	$-\frac{107}{8}$	$-\frac{38}{5}$	-2	8	$\frac{47}{4}$	$\frac{68}{5}$



61. $f(x) = \dfrac{2x^3 - x^2 - 2x + 1}{x^2 + 3x + 2} = \dfrac{(2x-1)(x+1)(x-1)}{(x+1)(x+2)} = \dfrac{(2x-1)(x-1)}{x+2}, \quad x \neq -1$

$= \dfrac{2x^2 - 3x + 1}{x + 2} = 2x - 7 + \dfrac{15}{x+2}, \quad x \neq -1$

(a) Domain: all real numbers x except $x = -1$ and $x = -2$

(b) y-intercept: $\left(0, \dfrac{1}{2}\right)$

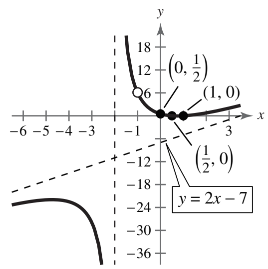
x-intercepts: $\left(\dfrac{1}{2}, 0\right), (1, 0)$

(c) Vertical asymptote: $x = -2$

Slant asymptote: $y = 2x - 7$

(d)

x	-4	-3	$-\frac{3}{2}$	0	1
$f(x)$	$-\frac{45}{2}$	-28	20	$\frac{1}{2}$	0



62. $f(x) = \dfrac{2x^3 + x^2 - 8x - 4}{x^2 - 3x + 2} = \dfrac{(x-2)(x+2)(2x+1)}{(x-2)(x-1)}$

$= 2x + 7 + \dfrac{9}{x-1}, \quad x \neq 2$

(a) Domain: all real numbers x except $x = 1$ and $x = 2$

(b) y-intercept: $(0, -2)$

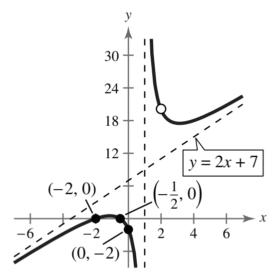
x-intercepts: $(-2, 0), \left(-\dfrac{1}{2}, 0\right)$

(c) Vertical asymptote: $x = 1$

Slant asymptote: $y = 2x + 7$

(d)

x	-3	-2	-1	0	$\frac{1}{2}$	$\frac{3}{2}$	3	4
$f(x)$	$-\frac{5}{4}$	0	$\frac{1}{2}$	-2	-10	28	$\frac{35}{2}$	18



63. $f(x) = \dfrac{x^2 + 5x + 8}{x + 3} = x + 2 + \dfrac{2}{x+3}$

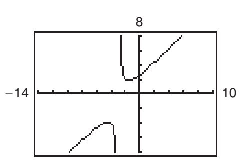
Domain: all real numbers x except $x = -3$

y-intercept: $\left(0, \dfrac{8}{3}\right)$

Vertical asymptote: $x = -3$

Slant asymptote: $y = x + 2$

Line: $y = x + 2$



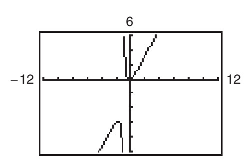
64. $f(x) = \dfrac{2x^2 + x}{x + 1} = 2x - 1 + \dfrac{1}{x+1}$

Domain: all real numbers x except $x = -1$

Vertical asymptote: $x = -1$

Slant asymptote: $y = 2x - 1$

Line: $y = 2x - 1$



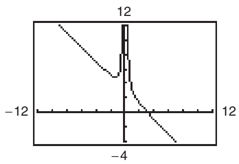
65. $g(x) = \dfrac{1 + 3x^2 - x^3}{x^2} = \dfrac{1}{x^2} + 3 - x = -x + 3 + \dfrac{1}{x^2}$

Domain: all real numbers x except $x = 0$

Vertical asymptote: $x = 0$

Slant asymptote: $y = -x + 3$

Line: $y = -x + 3$



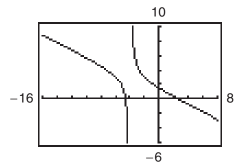
66. $h(x) = \dfrac{12 - 2x - x^2}{2(4 + x)} = -\dfrac{1}{2}x + 1 + \dfrac{2}{4 + x}$

Domain: all real numbers x except $x = -4$

Vertical asymptote: $x = -4$

Slant asymptote: $y = -\dfrac{1}{2}x + 1$

Line: $y = -\dfrac{1}{2}x + 1$



67. $y = \dfrac{x + 1}{x - 3}$

(a) x-intercept: $(-1, 0)$

(b) $0 = \dfrac{x + 1}{x - 3}$

$0 = x + 1$

$-1 = x$

68. $y = \dfrac{2x}{x - 3}$

(a) x-intercept: $(0, 0)$

(b) $0 = \dfrac{2x}{x - 3}$

$0 = 2x$

$0 = x$

69. $y = \dfrac{1}{x} - x$

(a) x-intercepts: $(-1, 0), (1, 0)$

(b) $0 = \dfrac{1}{x} - x$

$x = \dfrac{1}{x}$

$x^2 = 1$

$x = \pm 1$

70. $y = x - 3 + \dfrac{2}{x}$

(a) x-intercepts: $(1, 0), (2, 0)$

(b) $0 = x - 3 + \dfrac{2}{x}$

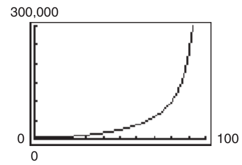
$0 = x^2 - 3x + 2$

$0 = (x - 1)(x - 2)$

$x = 1, x = 2$

71. $C = \dfrac{25{,}000p}{100 - p}, \ 0 \le p < 100$

(a)



(b) $C = \dfrac{25{,}000(15)}{100 - 15} \approx \$4411.76$

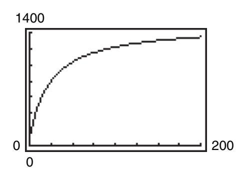
$C = \dfrac{25{,}000(50)}{100 - 50} = \$25{,}000$

$C = \dfrac{25{,}000(90)}{100 - 90} = \$225{,}000$

(c) $C \to \infty$ as $x \to 100$. No, it would not be possible to supply bins to 100% of the residents because the model is undefined for $p = 100$.

72. $N = \dfrac{20(5 + 3t)}{1 + 0.04t}, t \ge 0$

(a)



(b) $N(5) \approx 333$ deer

$N(10) = 500$ deer

$N(25) = 800$ deer

(c) The herd is limited by the horizontal asymptote:

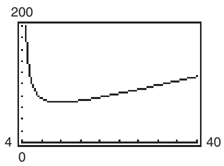
$N = \dfrac{60}{0.04} = 1500$ deer

73. (a) $A = xy$ and

$$(x-4)(y-2) = 30$$
$$y - 2 = \frac{30}{x-4}$$
$$y = 2 + \frac{30}{x-4} = \frac{2x+22}{x-4}$$

Thus, $A = xy = x\left(\frac{2x+22}{x-4}\right) = \frac{2x(x+11)}{x-4}.$

(b) Domain: Since the margins on the left and right are each 2 inches, $x > 4$. In interval notation, the domain is $(4, \infty)$.

(c) 

The area is minimum when $x \approx 11.75$ inches and $y \approx 5.87$ inches.

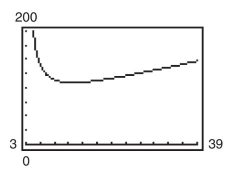
x	5	6	7	8	9	10	11	12	13	14	15
y_1 (Area)	160	102	84	76	72	70	69.143	69	69.333	70	70.909

The area is minimum when x is approximately 12.

74. $A = xy$ and

$$(x-3)(y-2) = 64$$
$$y - 2 = \frac{64}{x-3}$$
$$y = 2 + \frac{64}{x-3} = \frac{2x+58}{x-3}$$

Thus, $A = xy = x\left(\frac{2x+58}{x-3}\right) = \frac{2x(x+29)}{x-3}, x > 3.$



By graphing the area function, we see that A is minimum when $x \approx 12.8$ inches and $y \approx 8.5$ inches.

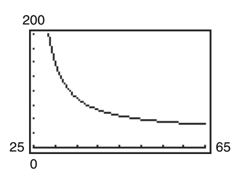
75. (a) Let t_1 = time from Akron to Columbus and t_2 = time from Columbus back to Akron.

$$xt_1 = 100 \Rightarrow t_1 = \frac{100}{x}$$
$$yt_2 = 100 \Rightarrow t_2 = \frac{100}{y}$$
$$50(t_1 + t_2) = 200$$
$$t_1 + t_2 = 4$$
$$\frac{100}{x} + \frac{100}{y} = 4$$
$$100y + 100x = 4xy$$
$$25y + 25x = xy$$
$$25x = xy - 25y$$
$$25x = y(x - 25)$$

Thus, $y = \frac{25x}{x-25}.$

(b) Vertical asymptote: $x = 25$

Horizontal asymptote: $y = 25$

(c) 

(d)

x	30	35	40	45	50	55	60
y	150	87.5	66.7	56.3	50	45.8	42.9

(e) Sample answer: No. You might expect the average speed for the round trip to be the average of the average speeds for the two parts of the trip.

(f) No. At 20 miles per hour you would use more time in one direction than is required for the round trip at an average speed of 50 miles per hour.

76. Yes; No; Every rational function is the ratio of two polynomial functions of the form $f(x) = \dfrac{N(x)}{D(x)}$.

77. False. Polynomial functions do not have vertical asymptotes.

78. False. The graph of $f(x) = \dfrac{x}{x^2 + 1}$ crosses $y = 0$, which is a horizontal asymptote.

79. False. A graph can have a vertical asymptote and a horizontal asymptote or a vertical asymptote and a slant asymptote, but a graph cannot have both a horizontal asymptote and a slant asymptote.

A horizontal asymptote occurs when the degree of $N(x)$ is equal to the degree of $D(x)$ or when the degree of $N(x)$ is less than the degree of $D(x)$. A slant asymptote occurs when the degree of $N(x)$ is greater than the degree of $D(x)$ by one. Because the degree of a polynomial is constant, it is impossible to have both relationships at the same time.

80. (a) True. When $x = 1$ the graph of f has a vertical asymptote, therefore $D(1) = D$.

(b) True. Since the graph of f has a horizontal asymptote at $y = 2$, the degrees of $N(x)$ and $D(x)$ are equal.

(c) False. Since the horizontal asymtptote is at $y = 2$, which shows that the ratio of the leading coefficients of $N(x)$ and $D(x)$ is 2, not 1.

81. b

82. c

Section 2.7 Nonlinear Inequalities

1. positive; negative

2. key; test intervals

3. zeros; undefined values

4. $P = R - C$

5. $x^2 - 3 < 0$

(a) $x = 3$

$$(3)^2 - 3 \overset{?}{<} 0$$
$$6 \not< 0$$

No, $x = 3$ *is not* a solution.

(b) $x = 0$

$$(0)^2 - 3 \overset{?}{<} 0$$
$$-3 < 0$$

Yes, $x = 0$ *is* a solution.

(c) $x = \frac{3}{2}$

$$\left(\tfrac{3}{2}\right)^2 - 3 \overset{?}{<} 0$$
$$-\tfrac{3}{4} < 0$$

Yes, $x = \frac{3}{2}$ *is* a solution.

(d) $x = -5$

$$(-5)^2 - 3 \overset{?}{<} 0$$
$$22 \not< 0$$

No, $x = -5$ *is not* a solution.

6. $x^2 - x - 12 \ge 0$

(a) $x = 5$

$$(5)^2 - (5) - 12 \overset{?}{\ge} 0$$
$$8 \ge 0$$

Yes, $x = 5$ *is* a solution.

(b) $x = 0$

$$(0)^2 - 0 - 12 \overset{?}{\ge} 0$$
$$-12 \not\ge 0$$

No, $x = 0$ *is not* a solution.

(c) $x = -4$

$$(-4)^2 - (-4) - 12 \overset{?}{\ge} 0$$
$$16 + 4 - 12 \overset{?}{\ge} 0$$
$$8 \ge 0$$

Yes, $x = -4$ *is* a solution.

(d) $x = -3$

$$(-3)^2 - (-3) - 12 \overset{?}{\ge} 0$$
$$9 + 3 - 12 \overset{?}{\ge} 0$$
$$0 \ge 0$$

Yes, $x = -3$ *is* a solution.

7. $\dfrac{x+2}{x-4} \ge 3$

(a) $x = 5$

$\dfrac{5+2}{5-4} \overset{?}{\ge} 3$

$7 \ge 3$

Yes, $x = 5$ *is* a solution.

(b) $x = 4$

$\dfrac{4+2}{4-4} \overset{?}{\ge} 3$

$\dfrac{6}{0}$ is undefined.

No, $x = 4$ *is not* a solution.

(c) $x = -\dfrac{9}{2}$

$\dfrac{-\frac{9}{2}+2}{-\frac{9}{2}-4} \overset{?}{\ge} 3$

$\dfrac{5}{17} \not\ge 3$

No, $x = -\dfrac{9}{2}$ *is not* a solution.

(d) $x = \dfrac{9}{2}$

$\dfrac{\frac{9}{2}+2}{\frac{9}{2}-4} \overset{?}{\ge} 3$

$13 \ge 3$

Yes, $x = \dfrac{9}{2}$ *is* a solution.

8. $\dfrac{3x^2}{x^2+4} < 1$

(a) $x = -2$

$\dfrac{3(-2)^2}{(-2)^2+4} \overset{?}{<} 1$

$\dfrac{12}{8} \not< 1$

No, $x = -2$ *is not* a solution.

(b) $x = -1$

$\dfrac{3(-1)^2}{(-1)^2+4} \overset{?}{<} 1$

$\dfrac{3}{5} < 1$

Yes, $x = -1$ *is* a solution.

(c) $x = 0$

$\dfrac{3(0)^2}{(0)^2+4} \overset{?}{<} 1$

$0 < 1$

Yes, $x = 0$ *is* a solution.

(d) $x = 3$

$\dfrac{3(3)^2}{(3)^2+4} \overset{?}{<} 1$

$\dfrac{27}{13} \not< 1$

No, $x = 3$ *is not* a solution.

9. $3x^2 - x - 2 = (3x+2)(x-1)$

$3x + 2 = 0 \Rightarrow x = -\frac{2}{3}$

$x - 1 = 0 \Rightarrow x = 1$

The key numbers are $-\frac{2}{3}$ and 1.

10. $9x^3 - 25x^2 = 0$

$x^2(9x - 25) = 0$

$x^2 = 0 \Rightarrow x = 0$

$9x - 25 = 0 \Rightarrow x = \frac{25}{9}$

The key numbers are 0 and $\frac{25}{9}$.

11. $\dfrac{1}{x-5} + 1 = \dfrac{1 + 1(x-5)}{x-5}$

$= \dfrac{x-4}{x-5}$

$x - 4 = 0 \Rightarrow x = 4$

$x - 5 = 0 \Rightarrow x = 5$

The key numbers are 4 and 5.

12. $\dfrac{x}{x+2} - \dfrac{2}{x-1} = \dfrac{x(x-1) - 2(x+2)}{(x+2)(x-1)}$

$= \dfrac{x^2 - x - 2x - 4}{(x+2)(x-1)}$

$= \dfrac{(x-4)(x+1)}{(x+2)(x-1)}$

$(x-4)(x+1) = 0$

$x - 4 = 0 \Rightarrow x = 4$

$x + 1 = 0 \Rightarrow x = -1$

$(x+2)(x-1) = 0$

$x + 2 = 0 \Rightarrow x = -2$

$x - 1 = 0 \Rightarrow x = 1$

The key numbers are $-2, -1, 1$, and 4.

13.
$$x^2 < 9$$
$$x^2 - 9 < 0$$
$$(x+3)(x-3) < 0$$

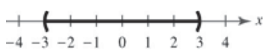
Key numbers: $x = \pm 3$

Test intervals: $(-\infty, -3), (-3, 3), (3, \infty)$

Test: Is $(x+3)(x-3) < 0$?

Interval	x-Value	Value of $x^2 - 9$	Conclusion
$(-\infty, -3)$	–4	7	Positive
$(-3, 3)$	0	–9	Negative
$(3, \infty)$	4	7	Positive

Solution set: $(-3, 3)$



14.
$$x^2 \le 16$$
$$x^2 - 16 \le 0$$
$$(x+4)(x-4) \le 0$$

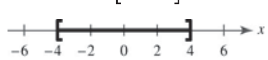
Key numbers: $x = \pm 4$

Test intervals: $(-\infty, -4), (-4, 4), (4, \infty)$

Test: Is $(x+4)(x-4) \le 0$?

Interval	x-Value	Value of $x^2 - 16$	Conclusion
$(-\infty, -4)$	–5	9	Positive
$(-4, 4)$	0	–16	Negative
$(4, \infty)$	5	9	Positive

Solution set: $[-4, 4]$



15.
$$(x+2)^2 \le 25$$
$$x^2 + 4x + 4 \le 25$$
$$x^2 + 4x - 21 \le 0$$
$$(x+7)(x-3) \le 0$$

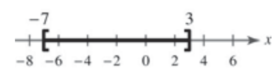
Key numbers: $x = -7, x = 3$

Test intervals: $(-\infty, -7), (-7, 3), (3, \infty)$

Test: Is $(x+7)(x-3) \le 0$?

Interval	x-Value	Value of $(x+7)(x-3)$	Conclusion
$(-\infty, -7)$	–8	$(-1)(-11) = 11$	Positive
$(-7, 3)$	0	$(7)(-3) = -21$	Negative
$(3, \infty)$	4	$(11)(1) = 11$	Positive

Solution set: $[-7, 3]$



16.
$$(x-3)^2 \ge 1$$
$$x^2 - 6x + 8 \ge 0$$
$$(x-2)(x-4) \ge 0$$

Key numbers: $x = 2, x = 4$

Test intervals: $(-\infty, 2) \Rightarrow (x-2)(x-4) > 0$

$(2, 4) \Rightarrow (x-2)(x-4) < 0$

$(4, \infty) \Rightarrow (x-2)(x-4) > 0$

Solution set: $(-\infty, 2] \cup [4, \infty)$

17.
$$x^2 + 4x + 4 \ge 9$$
$$x^2 + 4x - 5 \ge 0$$
$$(x+5)(x-1) \ge 0$$

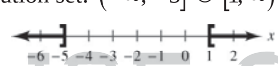
Key numbers: $x = -5, x = 1$

Test intervals: $(-\infty, -5), (-5, 1), (1, \infty)$

Test: Is $(x+5)(x-1) \ge 0$?

Interval	x-Value	Value of $(x+5)(x-1)$	Conclusion
$(-\infty, -5)$	−6	$(-1)(-7) = 7$	Positive
$(-5, 1)$	0	$(5)(-1) = -5$	Negative
$(1, \infty)$	2	$(7)(1) = 7$	Positive

Solution set: $(-\infty, -5] \cup [1, \infty)$



18. $x^2 - 6x + 9 < 16$

$x^2 - 6x - 7 < 0$

$(x + 1)(x - 7) < 0$

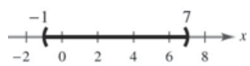
Key numbers: $x = -1, x = 7$

Test intervals: $(-\infty, -1) \Rightarrow (x + 1)(x - 7) > 0$

$(-1, 7) \Rightarrow (x + 1)(x - 7) < 0$

$(7, \infty) \Rightarrow (x + 1)(x - 7) > 0$

Solution set: $(-1, 7)$



19. $x^2 + x < 6$

$x^2 + x - 6 < 0$

$(x + 3)(x - 2) < 0$

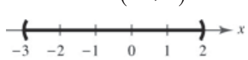
Key numbers: $x = -3, x = 2$

Test intervals: $(-\infty, -3), (-3, 2), (2, \infty)$

Test: Is $(x + 3)(x - 2) < 0$?

Interval	x-Value	Value of $(x + 3)(x - 2)$	Conclusion
$(-\infty, -3)$	-4	$(-1)(-6) = 6$	Positive
$(-3, 2)$	0	$(3)(-2) = -6$	Negative
$(2, \infty)$	3	$(6)(1) = 6$	Positive

Solution set: $(-3, 2)$



20. $x^2 + 2x > 3$

$x^2 + 2x - 3 > 0$

$(x + 3)(x - 1) > 0$

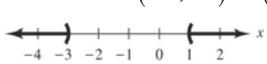
Key numbers: $x = -3, x = 1$

Test intervals: $(-\infty, -3) \Rightarrow (x + 3)(x - 1) > 0$

$(-3, 1) \Rightarrow (x + 3)(x - 1) < 0$

$(1, \infty) \Rightarrow (x + 3)(x - 1) > 0$

Solution set: $(-\infty, -3) \cup (1, \infty)$



21. $x^2 + 2x - 3 < 0$

$(x + 3)(x - 1) < 0$

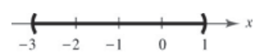
Key numbers: $x = -3, x = 1$

Test intervals: $(-\infty, -3), (-3, 1), (1, \infty)$

Test: Is $(x + 3)(x - 1) < 0$?

Interval	x-Value	Value of $(x + 3)(x - 1)$	Conclusion
$(-\infty, -3)$	-4	$(-1)(-5) = 5$	Positive
$(-3, 1)$	0	$(3)(-1) = -3$	Negative
$(1, \infty)$	2	$(5)(1) = 5$	Positive

Solution set: $(-3, 1)$



22. $x^2 > 2x + 8$

$x^2 - 2x - 8 > 0$

$(x - 4)(x + 2) > 0$

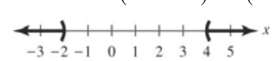
Key numbers: $x = -2, x = 4$

Test intervals: $(-\infty, -2), (-2, 4), (4, \infty)$

Test: Is $(x - 4)(x + 2) > 0$?

Interval	x-Value	Value of $(x - 4)(x + 2)$	Conclusion
$(-\infty, -2)$	-3	$(-7)(-1) = 7$	Positive
$(-2, 4)$	0	$(-4)(2) = -8$	Negative
$(4, \infty)$	5	$(1)(7) = 7$	Positive

Solution set: $(-\infty, -2) \cup (4, \infty)$



23. $3x^2 - 11x > 20$

$3x^2 - 11x - 20 > 0$

$(3x + 4)(x - 5) > 0$

Key numbers: $x = 5, x = -\frac{4}{3}$

Test intervals: $\left(-\infty, -\frac{4}{5}\right), \left(-\frac{4}{3}, 5\right), (5, \infty)$

Test: Is $(3x + 4)(x - 5) > 0$?

Interval	x-Value	Value of $(3x + 4)(x - 5)$	Conclusion
$\left(-\infty, -\frac{4}{3}\right)$	-3	$(-5)(-8) = 40$	Positive
$\left(-\frac{4}{3}, 5\right)$	0	$(4)(-5) = -20$	Negative
$(5, \infty)$	6	$(22)(1) = 22$	Positive

Solution set: $\left(-\infty, -\frac{4}{3}\right) \cup (5, \infty)$

24. $-2x^2 + 6x + 15 \le 0$

$2x^2 - 6x - 15 \ge 0$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(-15)}}{2(2)}$$

$$= \frac{6 \pm \sqrt{156}}{4}$$

$$= \frac{6 \pm 2\sqrt{39}}{4}$$

$$= \frac{3}{2} \pm \frac{\sqrt{39}}{2}$$

Key numbers: $x = \frac{3}{2} - \frac{\sqrt{39}}{2}, x = \frac{3}{2} + \frac{\sqrt{39}}{2}$

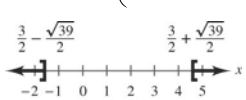
Test intervals:

$$\left(-\infty, \frac{3}{2} - \frac{\sqrt{39}}{2}\right) \Rightarrow -2x^2 + 6x + 15 < 0$$

$$\left(\frac{3}{2} - \frac{\sqrt{39}}{2}, \frac{3}{2} + \frac{\sqrt{39}}{2}\right) \Rightarrow -2x^2 + 6x + 15 > 0$$

$$\left(\frac{3}{2} + \frac{\sqrt{39}}{2}, \infty\right) \Rightarrow -2x^2 + 6x + 15 < 0$$

Solution set: $\left(-\infty, \frac{3}{2} - \frac{\sqrt{39}}{2}\right] \cup \left[\frac{3}{2} + \frac{\sqrt{39}}{2}, \infty\right)$



25. $x^2 - 3x - 18 > 0$

$(x + 3)(x - 6) > 0$

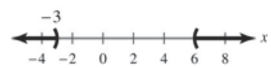
Key numbers: $x = -3, x = 6$

Test intervals: $(-\infty, -3), (-3, 6), (6, \infty)$

Test: Is $(x + 3)(x - 6) > 0$?

Interval	x-Value	Value of $(x + 3)(x - 6)$	Conclusion
$(-\infty, -3)$	-4	$(-1)(-10) = 10$	Positive
$(-3, 6)$	0	$(3)(-6) = -18$	Negative
$(6, \infty)$	7	$(10)(1) = 10$	Positive

Solution set: $(-\infty, -3) \cup (6, \infty)$



26. $x^3 + 2x^2 - 4x - 8 \le 0$

$x^2(x + 2) - 4(x + 2) \le 0$

$(x + 2)(x^2 - 4) \le 0$

$(x + 2)^2(x - 2) \le 0$

Key numbers: $x = -2, x = 2$

Test intervals: $(-\infty, -2) \Rightarrow x^3 + 2x^2 - 4x - 8 < 0$

$(-2, 2) \Rightarrow x^3 + 2x^2 - 4x - 8 < 0$

$(2, \infty) \Rightarrow x^3 + 2x^2 - 4x - 8 > 0$

Solution set: $(-\infty, 2]$

27.
$$x^3 - 3x^2 - x > -3$$
$$x^3 - 3x^2 - x + 3 > 0$$
$$x^2(x - 3) - (x - 3) > 0$$
$$(x - 3)(x^2 - 1) > 0$$
$$(x - 3)(x + 1)(x - 1) > 0$$

Key numbers: $x = -1, x = 1, x = 3$

Test intervals: $(-\infty, -1), (-1, 1), (1, 3), (3, \infty)$

Test: Is $(x - 3)(x + 1)(x - 1) > 0$?

Interval	x-Value	Value of $(x - 3)(x + 1)(x - 1)$	Conclusion
$(-\infty, -1)$	-2	$(-5)(-1)(-3) = -15$	Negative
$(-1, 1)$	0	$(-3)(1)(-1) = 3$	Positive
$(1, 3)$	2	$(-1)(3)(1) = -3$	Negative
$(3, \infty)$	4	$(1)(5)(3) = 15$	Positive

Solution set: $(-1, 1) \cup (3, \infty)$

28.
$$2x^3 + 13x^2 - 8x - 46 \ge 6$$
$$2x^3 + 13x^2 - 8x - 52 \ge 0$$
$$x^2(2x + 13) - 4(2x + 13) \ge 0$$
$$(2x + 13)(x^2 - 4) \ge 0$$
$$(2x + 13)(x + 2)(x - 2) \ge 0$$

Key numbers: $x = -\frac{13}{2}, x = -2, x = 2$

Test intervals:

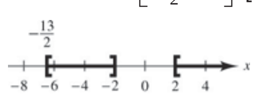
$\left(-\infty, -\frac{13}{2}\right) \Rightarrow 2x^3 + 13x^2 - 8x - 52 < 0$

$\left(-\frac{13}{2}, -2\right) \Rightarrow 2x^3 + 13x^2 - 8x - 52 > 0$

$(-2, 2) \Rightarrow 2x^3 + 13x^2 - 8x - 52 < 0$

$(2, \infty) \Rightarrow 2x^3 + 13x^2 - 8x - 52 > 0$

Solution set: $\left[-\frac{13}{2}, -2\right], [2, \infty)$



29.
$$4x^3 - 6x^2 < 0$$
$$2x^2(2x - 3) < 0$$

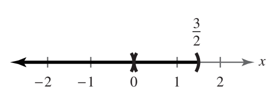
Key numbers: $x = 0, x = \frac{3}{2}$

Test intervals: $(-\infty, 0) \Rightarrow 2x^2(2x - 3) < 0$

$\left(0, \frac{3}{2}\right) \Rightarrow 2 \Rightarrow 2x^2(2x - 3) < 0$

$\left(\frac{3}{2}, \infty\right) \Rightarrow 2x^2(2x - 3) > 0$

Solution set: $(-\infty, 0) \cup \left(0, \frac{3}{2}\right)$



30.
$$4x^3 - 12x^2 > 0$$
$$4x^2(x - 3) > 0$$

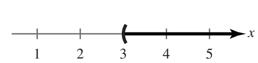
Key numbers: $x = 0, x = 3$

Test intervals: $(-\infty, 0) \Rightarrow 4x^2(x - 3) < 0$

$(0, 3) \Rightarrow 4x^2(x - 3) < 0$

$(3, \infty) \Rightarrow 4x^2(x - 3) > 0$

Solution set: $(3, \infty)$



31. $x^3 - 4x \ge 0$

$x(x+2)(x-2) \ge 0$

Key numbers: $x = 0, x = \pm 2$

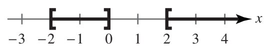
Test intervals: $(-\infty, -2) \Rightarrow x(x+2)(x-2) < 0$

$(-2, 0) \Rightarrow x(x+2)(x-2) > 0$

$(0, 2) \Rightarrow x(x+2)(x-2) < 0$

$(2, \infty) \Rightarrow x(x+2)(x-2) > 0$

Solution set: $[-2, 0] \cup [2, \infty)$



32. $2x^3 - x^4 \le 0$

$x^3(2 - x) \le 0$

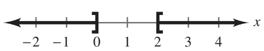
Key numbers: $x = 0, x = 2$

Test intervals: $(-\infty, 0) \Rightarrow x^3(2-x) < 0$

$(0, 2) \Rightarrow x^3(2-x) > 0$

$(2, \infty) \Rightarrow x^3(2-x) < 0$

Solution set: $(-\infty, 0] \cup [2, \infty)$



33. $(x-1)^2(x+2)^3 \ge 0$

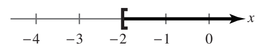
Key numbers: $x = 1, x = -2$

Test intervals: $(-\infty, -2) \Rightarrow (x-1)^2(x+2)^3 < 0$

$(-2, 1) \Rightarrow (x-1)^2(x+2)^3 > 0$

$(1, \infty) \Rightarrow (x-1)^2(x+2)^3 > 0$

Solution set: $[-2, \infty)$



34. $x^4(x-3) \le 0$

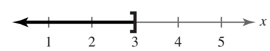
Key numbers: $x = 0, x = 3$

Test intervals: $(-\infty, 0) \Rightarrow x^4(x-3) < 0$

$(0, 3) \Rightarrow x^4(x-3) < 0$

$(3, \infty) \Rightarrow x^4(x-3) > 0$

Solution set: $(-\infty, 3]$



35. $4x^2 - 4x + 1 \le 0$

$(2x-1)^2 \le 0$

Key number: $x = \dfrac{1}{2}$

Test Interval	x-Value	Polynomial Value	Conclusion
$\left(-\infty, \dfrac{1}{2}\right]$	$x = 0$	$[2(0) - 1]^2 = 1$	Positive
$\left(\dfrac{1}{2}, \infty\right)$	$x = 1$	$[2(1) - 1]^2 = 1$	Positive

The solution set consists of the single real number $\dfrac{1}{2}$.

36. $x^2 + 3x + 8 > 0$

Using the Quadratic Formula you can determine the key numbers are $x = -\dfrac{3}{2} \pm \dfrac{\sqrt{23}}{2}i$.

Test Interval	x-Value	Polynomial Value	Conclusion
$(-\infty, \infty)$	$x = 0$	$(0)^2 + 3(0) + 8 = 8$	Positive

The solution set is the set of all real numbers.

37. $x^2 - 6x + 12 \le 0$

Using the Quadratic Formula, you can determine that the key numbers are $x = 3 \pm \sqrt{3}i$.

Test Interval	x-Value	Polynomial Value	Conclusion
$(-\infty, \infty)$	$x = 0$	$(0)^2 - 6(0) + 12 = 12$	Positive

The solution set is empty, that is there are no real solutions.

38. $x^2 - 8x + 16 > 0$

$(x - 4)^2 > 0$

Key number: $x = 4$

Test Interval	x-Value	Polynomial Value	Conclusion
$(-\infty, 4)$	$x = 0$	$(0 - 4)^2 = 16$	Positive
$(4, \infty)$	$x = 5$	$(5 - 4)^2 = 1$	Positive

The solution set consists of all real numbers except $x = 4$, or $(-\infty, 4) \cup (4, \infty)$.

39. $\dfrac{4x - 1}{x} > 0$

Key numbers: $x = 0, x = \dfrac{1}{4}$

Test intervals: $(-\infty, 0), \left(0, \frac{1}{4}\right), \left(\frac{1}{4}, \infty\right)$

Test: Is $\dfrac{4x - 1}{x} > 0$?

Interval	x-Value	Value of $\dfrac{4x - 1}{x}$	Conclusion
$(-\infty, 0)$	-1	$\dfrac{-5}{-1} = 5$	Positive
$\left(0, \dfrac{1}{4}\right)$	$\dfrac{1}{8}$	$\dfrac{-\frac{1}{2}}{\frac{1}{8}} = -4$	Negative
$\left(\dfrac{1}{4}, \infty\right)$	1	$\dfrac{3}{1} = 3$	Positive

Solution set: $(-\infty, 0) \cup \left(\dfrac{1}{4}, \infty\right)$

40. $\dfrac{x^2 - 1}{x} < 0$

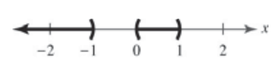
$\dfrac{(x - 1)(x + 1)}{x} < 0$

Key numbers: $x = -1, x = 0, x = 1$

Test intervals: $(-\infty, -1), (-1, 0), (0, 1), (1, \infty)$

Interval	x-Value	Value of $\dfrac{(x - 1)(x + 1)}{x}$	Conclusion
$(-\infty, -1)$	-2	$\dfrac{(-3)(-1)}{-2} = -\dfrac{3}{2}$	Negative
$(-1, 0)$	$-\dfrac{1}{2}$	$\dfrac{\left(-\frac{3}{2}\right)\left(\frac{1}{2}\right)}{-\frac{1}{2}} = \dfrac{3}{2}$	Positive
$(0, 1)$	$\dfrac{1}{2}$	$\dfrac{\left(-\frac{1}{2}\right)\left(\frac{3}{2}\right)}{\frac{1}{2}} = -\dfrac{3}{2}$	Negative
$(1, \infty)$	2	$\dfrac{(1)(3)}{2} = \dfrac{3}{2}$	Positive

Solution set: $(-\infty, -1) \cup (0, 1)$



41. $\dfrac{3x - 5}{x - 5} \geq 0$

Key numbers: $x = \dfrac{5}{3}, x = 5$

Test intervals: $\left(-\infty, \dfrac{5}{3}\right), \left(\dfrac{5}{3}, 5\right), (5, \infty)$

Test: Is $\dfrac{3x - 5}{x - 5} \geq 0$?

Interval	x-Value	Value of $\dfrac{3x-5}{x-5}$	Conclusion
$\left(-\infty, \dfrac{5}{3}\right)$	0	$\dfrac{-5}{-5} = 1$	Positive
$\left(\dfrac{5}{3}, 5\right)$	2	$\dfrac{6-5}{2-5} = -\dfrac{1}{3}$	Negative
$(5, \infty)$	6	$\dfrac{18-5}{6-5} = 13$	Positive

Solution set: $\left(-\infty, \dfrac{5}{3}\right] \cup (5, \infty)$

42. $\dfrac{5 + 7x}{1 + 2x} \leq 4$

$\dfrac{5 + 7x - 4(1 + 2x)}{1 + 2x} \leq 0$

$\dfrac{1 - x}{1 + 2x} \leq 0$

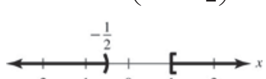
Key numbers: $x = -\dfrac{1}{2}, x = 1$

Test intervals: $\left(-\infty, -\dfrac{1}{2}\right), \left(-\dfrac{1}{2}, 1\right), (1, \infty)$

Test: Is $\dfrac{1 - x}{1 + 2x} \leq 0$?

Interval	x-Value	Value of $\dfrac{1-x}{1+2x}$	Conclusion
$\left(-\infty, -\dfrac{1}{2}\right)$	-1	$\dfrac{2}{-1} = -2$	Negative
$\left(-\dfrac{1}{2}, 1\right)$	0	$\dfrac{1}{1} = 1$	Positive
$(1, \infty)$	2	$\dfrac{-1}{5} = -\dfrac{1}{5}$	Negative

Solution set: $\left(-\infty, -\dfrac{1}{2}\right) \cup [1, \infty)$



43.
$$\frac{x + 6}{x + 1} - 2 < 0$$
$$\frac{x + 6 - 2(x + 1)}{x + 1} < 0$$
$$\frac{4 - x}{x + 1} < 0$$

Key numbers: $x = -1, x = 4$

Test intervals: $(-\infty, -1) \Rightarrow \dfrac{4 - x}{x + 1} < 0$

$(-1, 4) \Rightarrow \dfrac{4 - x}{x + 1} > 0$

$(4, \infty) \Rightarrow \dfrac{4 - x}{x + 1} < 0$

Solution set: $(-\infty, -1) \cup (4, \infty)$

44.
$$\frac{x + 12}{x + 2} - 3 \geq 0$$
$$\frac{x + 12 - 3(x + 2)}{x + 2} \geq 0$$
$$\frac{6 - 2x}{x + 2} \geq 0$$

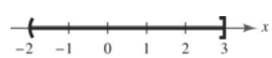
Key numbers: $x = -2, x = 3$

Test intervals: $(-\infty, -2) \Rightarrow \dfrac{6 - 2x}{x + 2} < 0$

$(-2, 3) \Rightarrow \dfrac{6 - 2x}{x + 2} > 0$

$(3, \infty) \Rightarrow \dfrac{6 - 2x}{x + 2} < 0$

Solution interval: $(-2, 3]$



45.
$$\frac{2}{x+5} > \frac{1}{x-3}$$
$$\frac{2}{x+5} - \frac{1}{x-3} > 0$$
$$\frac{2(x-3) - 1(x+5)}{(x+5)(x-3)} > 0$$
$$\frac{x-11}{(x+5)(x-3)} > 0$$

Key numbers: $x = -5, x = 3, x = 11$

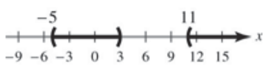
Test intervals: $(-\infty, -5) \Rightarrow \frac{x-11}{(x+5)(x-3)} < 0$

$(-5, 3) \Rightarrow \frac{x-11}{(x+5)(x-3)} > 0$

$(3, 11) \Rightarrow \frac{x-11}{(x+5)(x-3)} < 0$

$(11, \infty) \Rightarrow \frac{x-11}{(x+5)(x-3)} > 0$

Solution set: $(-5, 3) \cup (11, \infty)$



46.
$$\frac{5}{x-6} > \frac{3}{x+2}$$
$$\frac{5(x+2) - 3(x-6)}{(x-6)(x+2)} > 0$$
$$\frac{2x+28}{(x-6)(x+2)} > 0$$

Key numbers: $x = -14, x = -2, x = 6$

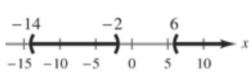
Test intervals: $(-\infty, -14) \Rightarrow \frac{2x+28}{(x-6)(x+2)} < 0$

$(-14, -2) \Rightarrow \frac{2x+28}{(x-6)(x+2)} > 0$

$(-2, 6) \Rightarrow \frac{2x+28}{(x-6)(x+2)} < 0$

$(6, \infty) \Rightarrow \frac{2x+28}{(x-6)(x+2)} > 0$

Solution intervals: $(-14, -2) \cup (6, \infty)$



47.
$$\frac{1}{x-3} \le \frac{9}{4x+3}$$
$$\frac{1}{x-3} - \frac{9}{4x+3} \le 0$$
$$\frac{4x+3-9(x-3)}{(x-3)(4x+3)} \le 0$$
$$\frac{30-5x}{(x-3)(4x+3)} \le 0$$

Key numbers: $x = 3, x = -\frac{3}{4}, x = 6$

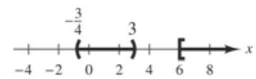
Test intervals: $\left(-\infty, -\frac{3}{4}\right) \Rightarrow \frac{30-5x}{(x-3)(4x+3)} > 0$

$\left(-\frac{3}{4}, 3\right) \Rightarrow \frac{30-5x}{(x-3)(4x+3)} < 0$

$(3, 6) \Rightarrow \frac{30-5x}{(x-3)(4x+3)} > 0$

$(6, \infty) \Rightarrow \frac{30-5x}{(x-3)(4x+3)} < 0$

Solution set: $\left(-\frac{3}{4}, 3\right) \cup [6, \infty)$



48.
$$\frac{1}{x} \ge \frac{1}{x+3}$$
$$\frac{1(x+3) - 1(x)}{x(x+3)} \ge 0$$
$$\frac{3}{x(x+3)} \ge 0$$

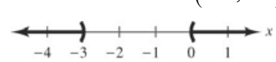
Key numbers: $x = -3, x = 0$

Test intervals: $(-\infty, -3) \Rightarrow \frac{3}{x(x+3)} > 0$

$(-3, 0) \Rightarrow \frac{3}{x(x+3)} < 0$

$(0, \infty) \Rightarrow \frac{3}{x(x+3)} > 0$

Solution intervals: $(-\infty, -3) \cup (0, \infty)$



49. $$\frac{x^2 + 2x}{x^2 - 9} \le 0$$

$$\frac{x(x+2)}{(x+3)(x-3)} \le 0$$

Key numbers: $x = 0, x = -2, x = \pm 3$

Test intervals: $(-\infty, -3) \Rightarrow \frac{x(x+2)}{(x+3)(x-3)} > 0$

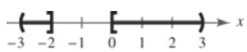
$(-3, -2) \Rightarrow \frac{x(x+2)}{(x+3)(x-3)} < 0$

$(-2, 0) \Rightarrow \frac{x(x+2)}{(x+3)(x-3)} > 0$

$(0, 3) \Rightarrow \frac{x(x+2)}{(x+3)(x-3)} < 0$

$(3, \infty) \Rightarrow \frac{x(x+2)}{(x+3)(x-3)} > 0$

Solution set: $(-3, -2] \cup [0, 3)$



50. $$\frac{x^2 + x - 6}{x} \ge 0$$

$$\frac{(x+3)(x-2)}{x} \ge 0$$

Key numbers: $x = -3, x = 0, x = 2$

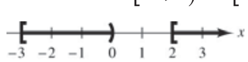
Test intervals: $(-\infty, -3) \Rightarrow \frac{(x+3)(x-2)}{x} < 0$

$(-3, 0) \Rightarrow \frac{(x+3)(x-2)}{x} > 0$

$(0, 2) \Rightarrow \frac{(x+3)(x-2)}{x} < 0$

$(2, \infty) \Rightarrow \frac{(x+3)(x-2)}{x} > 0$

Solution set: $[-3, 0) \cup [2, \infty)$



51. $$\frac{3}{x-1} + \frac{2x}{x+1} > -1$$

$$\frac{3(x+1) + 2x(x-1) + 1(x+1)(x-1)}{(x-1)(x+1)} > 0$$

$$\frac{3x^2 + x + 2}{(x-1)(x+1)} > 0$$

Key numbers: $x = -1, x = 1$

Test intervals: $(-\infty, -1) \Rightarrow \frac{3x^2 + x + 2}{(x-1)(x+1)} > 0$

$(-1, 1) \Rightarrow \frac{3x^2 + x + 2}{(x-1)(x+1)} < 0$

$(1, \infty) \Rightarrow \frac{3x^2 + x + 2}{(x-1)(x+1)} > 0$

Solution set: $(-\infty, -1) \cup (1, \infty)$

52. $$\frac{3x}{x-1} \le \frac{x}{x+4} + 3$$

$$\frac{3x(x+4) - x(x-1) - 3(x+4)(x-1)}{(x-1)(x+4)} \le 0$$

$$\frac{-x^2 + 4x + 12}{(x-1)(x+4)} \le 0$$

$$\frac{-(x-6)(x+2)}{(x-1)(x+4)} \le 0$$

Key numbers: $x = -4, x = -2, x = 1, x = 6$

Test intervals: $(-\infty, -4) \Rightarrow \frac{-(x-6)(x+2)}{(x-1)(x+4)} < 0$

$(-4, -2) \Rightarrow \frac{-(x-6)(x+2)}{(x-1)(x+4)} > 0$

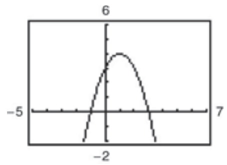
$(-2, 1) \Rightarrow \frac{-(x-6)(x+2)}{(x-1)(x+4)} < 0$

$(1, 6) \Rightarrow \frac{-(x-6)(x+2)}{(x-1)(x+4)} > 0$

$(6, \infty) \Rightarrow \frac{-(x-6)(x+2)}{(x-1)(x+4)} < 0$

Solution set: $(-\infty, -4) \cup [-2, 1) \cup [6, \infty)$

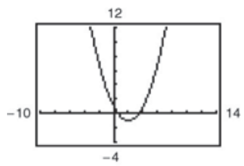
53. $y = -x^2 + 2x + 3$



(a) $y \le 0$ when $x \le -1$ or $x \ge 3$.

(b) $y \ge 3$ when $0 \le x \le 2$.

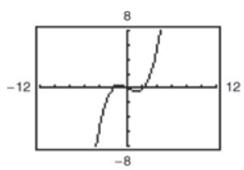
54. $y = \frac{1}{2}x^2 - 2x + 1$



(a) $y \le 0$ when $2 - \sqrt{2} \le x \le 2 + \sqrt{2}$.

(b) $y \ge 7$ when $x \le -2$ or $x \ge 6$.

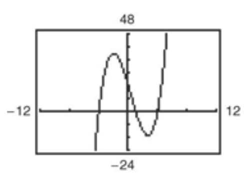
55. $y = \frac{1}{8}x^3 - \frac{1}{2}x$



(a) $y \ge 0$ when $-2 \le x \le 0$ or $2 \le x < \infty$.

(b) $y \le 6$ when $x \le 4$.

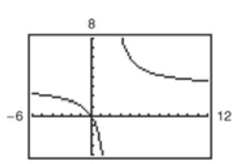
56. $y = x^3 - x^2 - 16x + 16$



(a) $y \le 0$ when $-\infty < x \le -4$ or $1 \le x \le 4$.

(b) $y \ge 36$ when $x = -2$ or $5 \le x < \infty$.

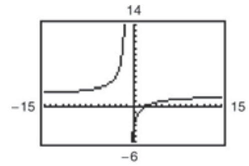
57. $y = \dfrac{3x}{x - 2}$



(a) $y \le 0$ when $0 \le x < 2$.

(b) $y \ge 6$ when $2 < x \le 4$.

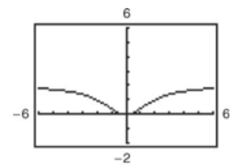
58. $y = \dfrac{2(x - 2)}{x + 1}$



(a) $y \le 0$ when $-1 < x \le 2$.

(b) $y \ge 8$ when $-2 \le x < -1$.

59. $y = \dfrac{2x^2}{x^2 + 4}$



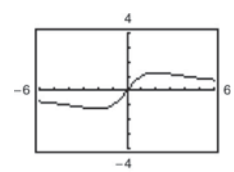
(a) $y \ge 1$ when $x \le -2$ or $x \ge 2$.

This can also be expressed as $|x| \ge 2$.

(b) $y \le 2$ for all real numbers x.

This can also be expressed as $-\infty < x < \infty$.

60. $y = \dfrac{5x}{x^2 + 4}$



(a) $y \ge 1$ when $1 \le x \le 4$.

(b) $y \le 0$ when $-\infty < x \le 0$.

61.

$$4 - x^2 \ge 0$$
$$(2 + x)(2 - x) \ge 0$$

Key numbers: $x = \pm 2$

Test intervals: $(-\infty, -2) \Rightarrow 4 - x^2 < 0$

$(-2, 2) \Rightarrow 4 - x^2 > 0$

$(2, \infty) \Rightarrow 4 - x^2 < 0$

Domain: $[-2, 2]$

62.

$$x^2 - 4 \ge 0$$
$$(x + 2)(x - 2) \ge 0$$

Key numbers: $x = -2, x = 2$

Test intervals: $(-\infty, -2) \Rightarrow (x + 2)(x - 2) > 0$

$(-2, 2) \Rightarrow (x + 2)(x - 2) < 0$

$(2, \infty) \Rightarrow (x + 2)(x - 2) > 0$

Domain: $(-\infty, -2] \cup [2, \infty)$

63. $x^2 - 9x + 20 \geq 0$

$(x - 4)(x - 5) \geq 0$

Key numbers: $x = 4, x = 5$

Test intervals: $(-\infty, 4), (4, 5), (5, \infty)$

Interval	x-Value	Value of $(x - 4)(x - 5)$	Conclusion
$(-\infty, 4)$	0	$(-4)(-5) = 20$	Positive
$(4, 5)$	$\frac{9}{2}$	$\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right) = -\frac{1}{4}$	Negative
$(5, \infty)$	6	$(2)(1) = 2$	Positive

Domain: $(-\infty, 4] \cup [5, \infty)$

64. $81 - 4x^2 \geq 0$

$(9 - 2x)(9 + 2x) \geq 0$

Key numbers: $x = \pm\frac{9}{2}$

Test intervals: $\left(-\infty, -\frac{9}{2}\right), \left(-\frac{9}{2}, \frac{9}{2}\right), \left(\frac{9}{2}, \infty\right)$

Interval	x-Value	Value of $(9 - 2x)(9 + 2x)$	Conclusion
$\left(-\infty, -\frac{9}{2}\right)$	-5	$(19)(-1) = -19$	Negative
$\left(-\frac{9}{2}, \frac{9}{2}\right)$	0	$(9)(9) = 81$	Positive
$\left(\frac{9}{2}, \infty\right)$	5	$(-1)(19) = -19$	Negative

Domain: $\left[-\frac{9}{2}, \frac{9}{2}\right]$

65. $\dfrac{x}{x^2 - 2x - 35} \geq 0$

$\dfrac{x}{(x + 5)(x - 7)} \geq 0$

Key numbers: $x = 0, x = -5, x = 7$

Test intervals: $(-\infty, -5) \Rightarrow \dfrac{x}{(x + 5)(x - 7)} < 0$

$(-5, 0) \Rightarrow \dfrac{x}{(x + 5)(x - 7)} > 0$

$(0, 7) \Rightarrow \dfrac{x}{(x + 5)(x - 7)} < 0$

$(7, \infty) \Rightarrow \dfrac{x}{(x + 5)(x - 7)} > 0$

Domain: $(-5, 0] \cup (7, \infty)$

66. $\dfrac{x}{x^2 - 9} \geq 0$

$\dfrac{x}{(x + 3)(x - 3)} \geq 0$

Key numbers: $x = -3, x = 0, x = 3$

Test intervals: $(-\infty, -3) \Rightarrow \dfrac{x}{(x + 3)(x - 3)} < 0$

$(-3, 0) \Rightarrow \dfrac{x}{(x + 3)(x - 3)} > 0$

$(0, 3) \Rightarrow \dfrac{x}{(x + 3)(x - 3)} < 0$

$(3, \infty) \Rightarrow \dfrac{x}{(x + 3)(x - 3)} > 0$

Domain: $(-3, 0] \cup (3, \infty)$

67. $0.4x^2 + 5.26 < 10.2$

$0.4x^2 - 4.94 < 0$

$0.4(x^2 - 12.35) < 0$

Key numbers: $x \approx \pm 3.51$

Test intervals: $(-\infty, -3.51), (-3.51, 3.51), (3.51, \infty)$

Solution set: $(-3.51, 3.51)$

68. $-1.3x^2 + 3.78 > 2.12$

$-1.3x^2 + 1.66 > 0$

Key numbers: $x \approx \pm 1.13$

Test intervals: $(-\infty, -1.13), (-1.13, 1.13), (1.13, \infty)$

Solution set: $(-1.13, 1.13)$

69. $-0.5x^2 + 12.5x + 1.6 > 0$

Key numbers: $x \approx -0.13, x \approx 25.13$

Test intervals: $(-\infty, -0.13), (-0.13, 25.13), (25.13, \infty)$

Solution set: $(-0.13, 25.13)$

70. $1.2x^2 + 4.8x + 3.1 < 5.3$

$1.2x^2 + 4.8x - 2.2 < 0$

Key numbers: $x \approx -4.42, x \approx 0.42$

Test intervals: $(-\infty, -4.42), (-4.42, 0.42), (0.42, \infty)$

Solution set: $(-4.42, 0.42)$

71. $$\frac{1}{2.3x - 5.2} > 3.4$$
$$\frac{1}{2.3x - 5.2} - 3.4 > 0$$
$$\frac{1 - 3.4(2.3x - 5.2)}{2.3x - 5.2} > 0$$
$$\frac{-7.82x + 18.68}{2.3x - 5.2} > 0$$

Key numbers: $x \approx 2.39$, $x \approx 2.26$

Test intervals: $(-\infty, 2.26), (2.26, 2.39), (2.39, \infty)$

Solution set: $(2.26, 2.39)$

72. $$\frac{2}{3.1x - 3.7} > 5.8$$
$$\frac{2 - 5.8(3.1x - 3.7)}{3.1x - 3.7} > 0$$
$$\frac{23.46 - 17.98x}{3.1x - 3.7} > 0$$

Key numbers: $x \approx 1.19$, $x \approx 1.30$

Test intervals: $(-\infty, 1.19) \Rightarrow \dfrac{23.46 - 17.98x}{3.1x - 3.7} < 0$

$(1.19, 1.30) \Rightarrow \dfrac{23.46 - 17.98x}{3.1x - 3.7} > 0$

$(1.30, \infty) \Rightarrow \dfrac{23.46 - 17.98x}{3.1x - 3.7} < 0$

Solution set: $(1.19, 1.30)$

73. $s = -16t^2 + v_0t + s_0 = -16t^2 + 160t$

(a) $-16t^2 + 160t = 0$

$-16t(t - 10) = 0$

$t = 0, t = 10$

It will be back on the ground in 10 seconds.

(b) $-16t^2 + 160t > 384$

$-16t^2 + 160t - 384 > 0$

$-16(t^2 - 10t + 24) > 0$

$t^2 - 10t + 24 < 0$

$(t - 4)(t - 6) < 0$

Key numbers: $t = 4, t = 6$

Test intervals: $(-\infty, 4), (4, 6), (6, \infty)$

Solution set: 4 seconds $< t <$ 6 seconds

74. $s = -16t^2 + v_0t + s_0 = -16t^2 + 128t$

(a) $-16t^2 + 128t = 0$

$-16t(t - 8) = 0$

$-16t = 0 \Rightarrow t = 0$

$t - 8 = 0 \Rightarrow t = 8$

It will be back on the ground in 8 seconds.

(b) $-16t^2 + 128t < 128$

$-16t^2 + 128t - 128 < 0$

$-16(t^2 - 8t + 8) < 0$

$t^2 - 8t + 8 > 0$

Key numbers: $t = 4 - 2\sqrt{2}, t = 4 + 2\sqrt{2}$

Test intervals:
$(-\infty, 4 - 2\sqrt{2}), (4 - 2\sqrt{2}, 4 + 2\sqrt{2}),$
$(4 + 2\sqrt{2}, \infty)$

Solution set: 0 seconds $\le t < 4 - 2\sqrt{2}$ seconds
and
$4 + 2\sqrt{2}$ seconds $< t \le$ 8 seconds

75. $2L + 2W = 100 \Rightarrow W = 50 - L$

$LW \ge 500$

$L(50 - L) \ge 500$

$-L^2 + 50L - 500 \ge 0$

By the Quadratic Formula you have:

Key numbers: $L = 25 \pm 5\sqrt{5}$

Test: Is $-L^2 + 50L - 500 \ge 0$?

Solution set: $25 - 5\sqrt{5} \le L \le 25 + 5\sqrt{5}$
13.8 meters $\le L \le$ 36.2 meters

76. $2L + 2W = 440 \Rightarrow W = 220 - L$

$LW \ge 8000$

$L(220 - L) \ge 8000$

$-L^2 + 220L - 8000 \ge 0$

By the Quadratic Formula we have:

Key numbers: $L = 110 \pm 10\sqrt{41}$

Test: Is $-L^2 + 220L - 8000 \ge 0$?

Solution set: $110 - 10\sqrt{41} \le L \le 110 + 10\sqrt{41}$
45.97 feet $\le L \le$ 174.03 feet

77. $R = x(75 - 0.0005x)$ and $C = 30x + 250{,}000$

$$\begin{aligned} P &= R - C \\ &= (75x - 0.0005x^2) - (30x + 250{,}000) \\ &= -0.0005x^2 + 45x - 250{,}000 \end{aligned}$$

$$P \geq 750{,}000$$

$$-0.0005x^2 + 45x - 250{,}000 \geq 750{,}000$$

$$-0.0005x^2 + 45x - 1{,}000{,}000 \geq 0$$

Key numbers: $x = 40{,}000$, $x = 50{,}000$

(These were obtained by using the Quadratic Formula.)

Test intervals:

$(0, 40{,}000)$, $(40{,}000, 50{,}000)$, $(50{,}000, \infty)$

The solution set is $[40{,}000, 50{,}000]$ or $40{,}000 \leq x \leq 50{,}000$. The price per unit is

$$p = \frac{R}{x} = 75 - 0.0005x.$$

For $x = 40{,}000$, $p = \$55$. For $x = 50{,}000$, $p = \$50$. So, for $40{,}000 \leq x \leq 50{,}000$, $\$50.00 \leq p \leq \$55.00$.

78. $R = x(50 - 0.0002x)$ and $C = 12x + 150{,}000$

$$\begin{aligned} P &= R - C \\ &= (50x - 0.0002x^2) - (12x + 150{,}000) \\ &= -0.0002x^2 + 38x - 150{,}000 \end{aligned}$$

$$P \geq 1{,}650{,}000$$

$$-0.0002x^2 + 38x - 150{,}000 \geq 1{,}650{,}000$$

$$-0.0002x^2 + 38x - 1{,}800{,}000 \geq 0$$

Key numbers: $x = 90{,}000$ and $x = 100{,}000$

Test intervals:

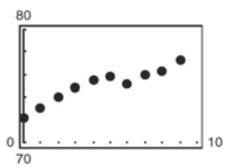
$(0, 90{,}000)$, $(90{,}000, 100{,}000)$, $(100{,}000, \infty)$

The solution set is $[90{,}000, 100{,}000]$ or $90{,}000 \leq x \leq 100{,}000$. The price per unit is

$$p = \frac{R}{x} = 50 - 0.0002x.$$

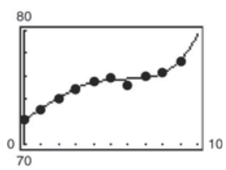
For $x = 90{,}000$, $p = \$32$. For $x = 100{,}000$, $p = \$30$. So, for $90{,}000 \leq x \leq 100{,}000$, $\$30 \leq p \leq \$32$.

79. (a)



(b) $N = 0.00406t^4 - 0.0564t^3 + 0.147t^2 + 0.86t + 72.2$

(c)



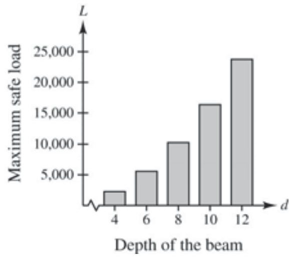
The model fits the data well.

(d) Using the zoom and trace features, the number of students enrolled in schools exceeded 74 million in the year 2001.

(e) No. The model can be used to predict enrollments for years close to those in its domain but when you project too far into the future, the numbers predicted by the model increase too rapidly to be considered reasonable.

80. (a)

d	4	6	8	10	12
Load	2223.9	5593.9	10,312	16,378	23,792



(b)
$$\begin{aligned} 2000 &\le 168.5d^2 - 472.1 \\ 2472.1 &\le 168.5d^2 \\ 14.67 &\le d^2 \\ 3.83 &\le d \end{aligned}$$

The minimum depth is 3.83 inches.

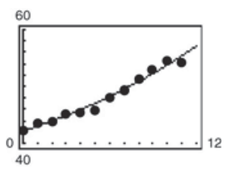
81.
$$\begin{aligned} \frac{1}{R} &= \frac{1}{R_1} + \frac{1}{2} \\ 2R_1 &= 2R + RR_1 \\ 2R_1 &= R(2 + R_1) \\ \frac{2R_1}{2 + R_1} &= R \end{aligned}$$

Because $R \ge 1$,

$$\begin{aligned} \frac{2R_1}{2 + R_1} &\ge 1 \\ \frac{2R_1}{2 + R_1} - 1 &\ge 0 \\ \frac{R_1 - 2}{2 + R_1} &\ge 0. \end{aligned}$$

Because $R_1 > 0$, the only key number is $R_1 = 2$. The inequality is satisfied when $R_1 \ge 2$ ohms.

82. (a)



(b) The model fits the data well.

(c)
$$\begin{aligned} S &= \frac{42.16 - 0.236t}{1 - 0.026t} \\ 60 &\le \frac{42.16 - 0.236t}{1 - 0.026t} \\ 0 &\le \frac{42.16 - 0.236t}{1 - 0.026t} - 60 \\ 0 &\le \frac{-17.84 + 1.324t}{1 - 0.026t} \end{aligned}$$

Key numbers: $t \approx 38.5$ and $t \approx 13.5$

Test Intervals	t-Value	Expression Value	Conclusion
$(0, 13.5)$	$t = 1$	-17.0	Negative
$(13.5, 38.5)$	$t = 20$	18.0	Positive
$(38.5, \infty)$	$t = 40$	-878.0	Negative

So, the mean salary for classroom teachers will exceed $60,000 during the year 2013.

(d) No. The model yields negative values for values of $t \ge 38.5$. The graph also has a vertical asymptote at

$$t = \frac{500}{13} \approx 38.5.$$

After testing the intervals, you can see that the inequality is satisfied on the open interval (13.5, 38.5).

83. True.

$$x^3 - 2x^2 - 11x + 12 = (x + 3)(x - 1)(x - 4)$$

The test intervals are $(-\infty, -3), (-3, 1), (1, 4)$, and $(4, \infty)$.

84. True.

The y-values are greater than zero for all values of x.

85. $x^2 + bx + 4 = 0$

(a) To have at least one real solution, $b^2 - 4ac \geq 0$.

$b^2 - 4(1)(4) \geq 0$

$b^2 - 16 \geq 0$

Key numbers: $b = -4, b = 4$

Test intervals: $(-\infty, -4) \Rightarrow b^2 - 16 > 0$

$(-4, 4) \Rightarrow b^2 - 16 < 0$

$(4, \infty) \Rightarrow b^2 - 16 > 0$

Solution set: $(-\infty, -4] \cup [4, \infty]$

(b) $b^2 - 4ac \geq 0$

Key numbers: $b = -2\sqrt{ac}, b = 2\sqrt{ac}$

Similar to part (a), if $a > 0$ and $c > 0$,

$b \leq -2\sqrt{ac}$ or $b \geq 2ac$.

86. $x^2 + bx - 4 = 0$

(a) To have at least one real solution, $b^2 - 4ac \geq 0$.

$b^2 - 4(1)(-4) \geq 0$

$b^2 + 16 \geq 0$

Key numbers: none

Test intervals: $(-\infty, \infty) \Rightarrow b^2 + 16 > 0$

Solution set: $(-\infty, \infty)$

(b) $b^2 - 4ac \geq 0$

Similar to part (a), if $a > 0$ and $c < 0$, b can be any real number.

87. $3x^2 + bx + 10 = 0$

(a) To have at least one real solution, $b^2 - 4ac \geq 0$.

$b^2 - 4(3)(10) \geq 0$

$b^2 - 120 \geq 0$

Key numbers: $b = -2\sqrt{30}, b = 2\sqrt{30}$

Test intervals: $(-\infty, -2\sqrt{30}) \Rightarrow b^2 - 120 > 0$

$(-2\sqrt{30}, 2\sqrt{30}) \Rightarrow b^2 - 120 < 0$

$(2\sqrt{30}, \infty) \Rightarrow b^2 - 120 > 0$

Solution set: $(-\infty, -2\sqrt{30}] \cup [2\sqrt{30}, \infty]$

(b) $b^2 - 4ac \geq 0$

Similar to part (a), if $a > 0$ and $c > 0$,

$b \leq -2\sqrt{ac}$ or $b \geq 2ac$.

88. $2x^2 + bx + 5 = 0$

(a) To have at least one real solution, $b^2 - 4ac \geq 0$.

$b^2 - 4(2)(5) \geq 0$

$b^2 - 40 \geq 0$

Key numbers: $b = -2\sqrt{10}, b = 2\sqrt{10}$

Test intervals: $(-\infty, -2\sqrt{10}) \Rightarrow b^2 - 40 > 0$

$(-2\sqrt{10}, 2\sqrt{10}) \Rightarrow b^2 - 40 < 0$

$(2\sqrt{10}, \infty) \Rightarrow b^2 - 40 > 0$

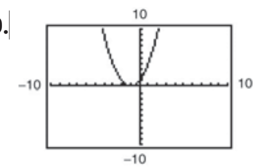
Solution set: $(-\infty, -2\sqrt{10}] \cup [2\sqrt{10}, \infty)$

(b) $b^2 - 4ac \geq 0$

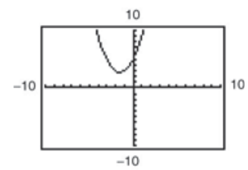
Similar to part (a), if $a > 0$ and $c > 0$,

$b \leq -2\sqrt{ac}$ or $b \geq 2ac$.

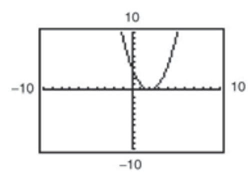
89.



For part (b), the y-values that are less than or equal to 0 occur only at $x = -1$.



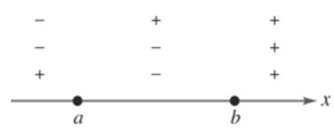
For part (c), there are no y-values that are less than 0.



For part (d), the y-values that are greater than 0 occur for all values of x except 2

90. (a) $x = a, x = b$

(b)

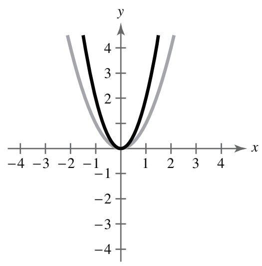


(c) The real zeros of the polynomial

Review Exercises for Chapter 2

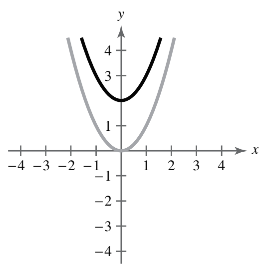
1. (a) $y = 2x^2$

Vertical stretch



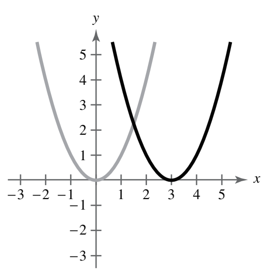
(b) $y = x^2 + 2$

Vertical shift two units upward



2. (a) $y = (x - 3)^2$

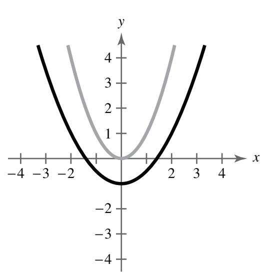
Horizontal shift three units to the right



(b) $y = \frac{1}{2}x^2 - 1$

Vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{2}\right)$,

and a vertical shift one unit downward



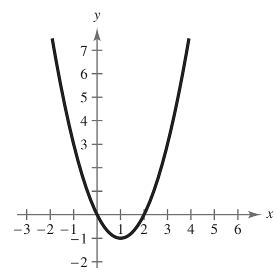
3.
$$\begin{aligned} g(x) &= x^2 - 2x \\ &= x^2 - 2x + 1 - 1 \\ &= (x - 1)^2 - 1 \end{aligned}$$

Vertex: $(1, -1)$

Axis of symmetry: $x = 1$

$0 = x^2 - 2x = x(x - 2)$

x-intercepts: $(0, 0), (2, 0)$

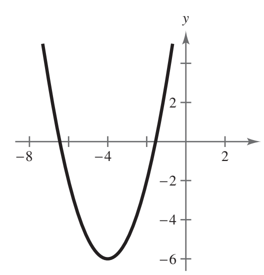


4.
$$\begin{aligned} f(x) &= x^2 + 8x + 10 \\ &= x^2 + 8x + 16 - 16 + 10 \\ &= (x + 4)^2 - 6 \end{aligned}$$

Vertex: $(-4, -6)$

Axis of symmetry: $x = -4$

$$\begin{aligned} 0 &= (x + 4)^2 - 6 \\ (x + 4)^2 &= 6 \\ x + 4 &= \pm\sqrt{6} \\ x &= -4 \pm \sqrt{6} \end{aligned}$$



x-intercepts: $\left(-4 \pm \sqrt{6}, 0\right)$

5. $h(x) = 3 + 4x - x^2$

$= -(x^2 - 4x - 3)$

$= -(x^2 - 4x + 4 - 4 - 3)$

$= -[(x-2)^2 - 7]$

$= -(x-2)^2 + 7$

Vertex: $(2, 7)$

Axis of symmetry: $x = 2$

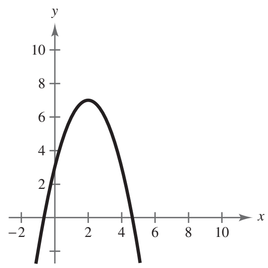
$0 = 3 + 4x - x^2$

$0 = x^2 - 4x - 3$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{28}}{2} = 2 \pm \sqrt{7}$$

x-intercepts: $\left(2 \pm \sqrt{7}, 0\right)$



6. $f(t) = -2t^2 + 4t + 1$

$= -2(t^2 - 2t + 1 - 1) + 1$

$= -2[(t-1)^2 - 1] + 1$

$= -2(t-1)^2 + 3$

Vertex: $(1, 3)$

Axis of symmetry: $t = 1$

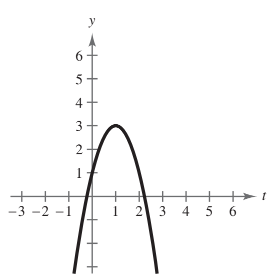
$0 = -2(t-1)^2 + 3$

$2(t-1)^2 = 3$

$t - 1 = \pm\sqrt{\frac{3}{2}}$

$t = 1 \pm \frac{\sqrt{6}}{2}$

t-intercepts: $\left(1 \pm \frac{\sqrt{6}}{2}, 0\right)$



7. $h(x) = 4x^2 + 4x + 13$

$= 4(x^2 + x) + 13$

$= 4\left(x^2 + x + \frac{1}{4} - \frac{1}{4}\right) + 13$

$= 4\left(x^2 + x + \frac{1}{4}\right) - 1 + 13$

$= 4\left(x + \frac{1}{2}\right)^2 + 12$

Vertex: $\left(-\frac{1}{2}, 12\right)$

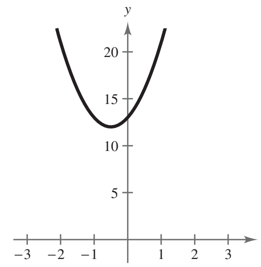
Axis of symmetry: $x = -\frac{1}{2}$

$0 = 4\left(x + \frac{1}{2}\right)^2 + 12$

$\left(x + \frac{1}{2}\right)^2 = -3$

No real zeros

x-intercepts: none



8. $f(x) = \frac{1}{3}(x^2 + 5x - 4)$

$= \frac{1}{3}\left(x^2 + 5x + \frac{25}{4} - \frac{25}{4} - 4\right)$

$= \frac{1}{3}\left[\left(x + \frac{5}{2}\right)^2 - \frac{41}{4}\right]$

$= \frac{1}{3}\left(x + \frac{5}{2}\right)^2 - \frac{41}{12}$

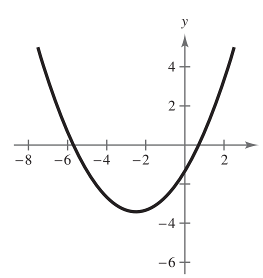
Vertex: $\left(-\frac{5}{2}, -\frac{41}{12}\right)$

Axis of symmetry: $x = -\frac{5}{2}$

$0 = x^2 + 5x - 4$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(-4)}}{2(1)} = \frac{-5 \pm \sqrt{41}}{2}$$

x-intercepts: $\left(\frac{-5 \pm \sqrt{41}}{2}, 0\right)$



9. (a) $x + x + y + y = P$

$$2x + 2y = 1000$$
$$y = 500 - x$$

$$A = xy$$
$$= x(500 - x)$$
$$= 500x - x^2$$

(b) $A = 500x - x^2$

$$= -(x^2 - 500x + 62{,}500) + 62{,}500$$
$$= -(x - 250)^2 + 62{,}500$$

The maximum area occurs at the vertex when $x = 250$ and $y = 500 - 250 = 250$.

The dimensions with the maximum area are $x = 250$ meters and $y = 250$ meters.

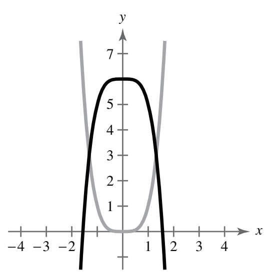
10. $C = 70{,}000 - 120x + 0.055x^2$

The minimum cost occurs at the vertex of the parabola.

Vertex: $-\dfrac{b}{2a} = -\dfrac{-120}{2(0.055)} \approx 1091$ units

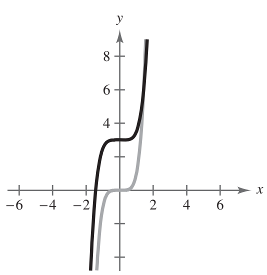
About 1091 units should be produced each day to yield a minimum cost.

11. $y = x^4,\ f(x) = 6 - x^4$



Transformation: Reflection in the x-axis and a vertical shift six units upward

12. $y = x^5,\ f(x) = \frac{1}{2}x^5 + 3$



Transformation: Vertical shrink and a vertical shift three units upward

13. $f(x) = -2x^2 - 5x + 12$

The degree is even and the leading coefficient is negative. The graph falls to the left and falls to the right.

14. $f(x) = \frac{1}{2}x^3 + 2x$

The degree is odd and the leading coefficient is positive. The graph falls to the left and rises to the right.

15. $g(x) = \frac{3}{4}(x^4 + 3x^2 + 2)$

The degree is even and the leading coefficient is positive. The graph rises to the left and rises to the right.

16. $f(x) = -x^7 + 8x^2 - 8x$

The degree is odd and the leading coefficient is negative. The graph rises to the left and falls to the right.

17. $g(x) = 2x^3 + 4x^2$

(a) The degree is odd and the leading coefficient, 2, is positive. The graph falls to the left and rises to the right.

(b) $g(x) = 2x^3 + 4x^2$

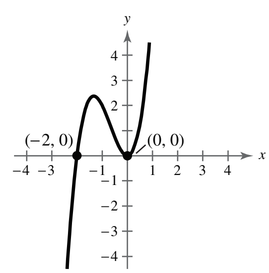
$$0 = 2x^3 + 4x^2$$
$$0 = 2x^2(x + 2)$$
$$0 = x^2(x + 2)$$

Zeros: $x = -2, 0$

(c)

x	-3	-2	-1	0	1
$g(x)$	-18	0	2	0	6

(d)



18. $h(x) = 3x^2 - x^4$

(a) The degree is even and the leading coefficient, -1, is negative. The graph falls to the left and falls to the right.

(b) $g(x) = 3x^2 - x^4$

$$0 = 3x^2 - x^4$$

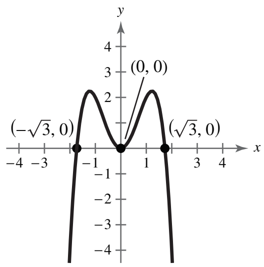
$$0 = x^2(3 - x^2)$$

Zeros: $x = 0, \pm\sqrt{3}$

(c)

x	-2	-1	0	1	2
$h(x)$	-4	2	0	2	-4

(d)



19. $f(x) = -x^3 + x^2 - 2$

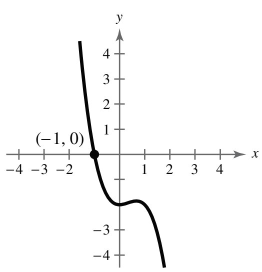
(a) The degree is odd and the leading coefficient is negative. The graph rises to the left and falls to the right.

(b) Zero: $x = -1$

(c)

x	-3	-2	-1	0	1	2
$f(x)$	34	10	0	-2	-2	-6

(d)



20. $f(x) = x(x^3 + x^2 - 5x + 3)$

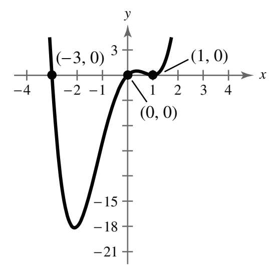
(a) The degree is even and the leading coefficient is positive. The graph rises to the left and rises to the right.

(b) Zeros: $x = 0, 1, -3$

(c)

x	-4	-3	-2	-1	0	1	2	3
$f(x)$	100	0	-18	-8	0	0	10	72

(d)



21. (a) $f(x) = 3x^3 - x^2 + 3$

x	-3	-2	-1	0	1	2	3
$f(x)$	-87	-25	-1	3	5	23	75

The zero is in the interval $[-1, 0]$.

(b) Zero: $x \approx -0.900$

22. (a) $f(x) = x^4 - 5x - 1$

x	-3	-2	-1	0	1	2	3
$f(x)$	95	25	5	-1	-5	5	65

There are zeros in the intervals $[-1, 0]$ and $[1, 2]$.

(b) Zeros: $x \approx -0.200$, $x \approx 1.772$

23.

$$\begin{array}{r}
6x + 3 \\
5x - 3\overline{\smash{)}30x^2 - 3x + 8} \\
\underline{30x^2 - 18x} \\
15x + 8 \\
\underline{15x - 9} \\
17
\end{array}$$

$$\frac{30x^2 - 3x + 8}{5x - 3} = 6x + 3 + \frac{17}{5x - 3}$$

24.
$$\begin{array}{r}
5x + 4 \\
x^2 - 5x - 1\overline{\smash{)}\,5x^3 - 21x^2 - 25x - 4} \\
\underline{5x^3 - 25x^2 - 5x} \\
4x^2 - 20x - 4 \\
\underline{4x^2 - 20x - 4} \\
0
\end{array}$$

$$\frac{5x^3 - 21x^2 - 25x - 4}{x^2 - 5x - 1} = 5x + 4,\ x \neq \frac{5}{2} \pm \frac{\sqrt{29}}{2}$$

25. $f(x) = 20x^4 + 9x^3 - 14x^2 - 3x$

(a)
$$\begin{array}{r|rrrrr}
-1 & 20 & 9 & -14 & -3 & 0 \\
 & & -20 & 11 & 3 & 0 \\
\hline
 & 20 & -11 & -3 & 0 & 0
\end{array}$$

Yes, $x = -1$ is a zero of f.

(b)
$$\begin{array}{r|rrrrr}
\frac{3}{4} & 20 & 9 & -14 & -3 & 0 \\
 & & 15 & 18 & 3 & 0 \\
\hline
 & 20 & 24 & 4 & 0 & 0
\end{array}$$

Yes, $x = \frac{3}{4}$ is a zero of f.

(c)
$$\begin{array}{r|rrrrr}
0 & 20 & 9 & -14 & -3 & 0 \\
 & & 0 & 0 & 0 & 0 \\
\hline
 & 20 & 9 & -14 & -3 & 0
\end{array}$$

Yes, $x = 0$ is a zero of f.

(d)
$$\begin{array}{r|rrrrr}
1 & 20 & 9 & -14 & -3 & 0 \\
 & & 20 & 29 & 15 & 12 \\
\hline
 & 20 & 29 & 15 & 12 & 12
\end{array}$$

No, $x = 1$ is not a zero of f.

26. $f(x) = 3x^3 - 8x^2 - 20x + 16$

(a)
$$\begin{array}{r|rrrr}
4 & 3 & -8 & -20 & 16 \\
 & & 12 & 16 & -16 \\
\hline
 & 3 & 4 & -4 & 0
\end{array}$$

Yes, $x = 4$ is a zero of f.

(b)
$$\begin{array}{r|rrrr}
-4 & 3 & -8 & -20 & 16 \\
 & & -12 & 80 & -240 \\
\hline
 & 3 & -20 & 60 & -224
\end{array}$$

No, $x = -4$ is not a zero of f.

(c)
$$\begin{array}{r|rrrr}
\frac{2}{3} & 3 & -8 & -20 & 16 \\
 & & 2 & -4 & -16 \\
\hline
 & 3 & -6 & -24 & 0
\end{array}$$

Yes, $x = \frac{2}{3}$ is a zero of f.

(d)
$$\begin{array}{r|rrrr}
-1 & 3 & -8 & -20 & 16 \\
 & & -3 & 11 & 9 \\
\hline
 & 3 & -11 & -9 & 25
\end{array}$$

No, $x = -1$ is not a zero of f.

27. $f(x) = 2x^3 + 11x^2 - 21x - 90$; Factor: $(x + 6)$

(a)
$$\begin{array}{r|rrrr}
-6 & 2 & 11 & -21 & -90 \\
 & & -12 & 6 & 90 \\
\hline
 & 2 & -1 & -15 & 0
\end{array}$$

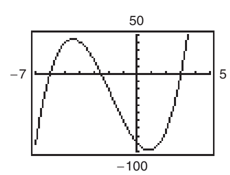
Yes, $(x + 6)$ is a factor of $f(x)$.

(b) $2x^2 - x - 15 = (2x + 5)(x - 3)$

The remaining factors are $(2x + 5)$ and $(x - 3)$.

(c) $f(x) = (2x + 5)(x - 3)(x + 6)$

(d) Zeros: $x = -\frac{5}{2}, 3, -6$

(e)


28. $f(x) = x^4 - 4x^3 - 7x^2 + 22x + 24$

Factors: $(x + 2), (x - 3)$

(a)
$$\begin{array}{r|rrrrr} -2 & 1 & -4 & -7 & 22 & 24 \\ & & -2 & 12 & -10 & -24 \\ \hline & 1 & -6 & 5 & 12 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 3 & 1 & -6 & 5 & 12 \\ & & 3 & -9 & -12 \\ \hline & 1 & -3 & -4 & 0 \end{array}$$

Yes, $(x + 2)$ and $(x - 3)$ are both factors of $f(x)$.

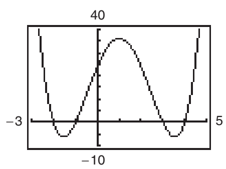
(b) $x^2 - 3x - 4 = (x + 1)(x - 4)$

The remaining factors are $(x + 1)$ and $(x - 4)$.

(c) $f(x) = (x + 1)(x - 4)(x + 2)(x - 3)$

(d) Zeros: $-2, -1, 3, 4$

(e)



29. $8 + \sqrt{-100} = 8 + 10i$

30. $-5i + i^2 = -1 - 5i$

31. $(7 + 5i) + (-4 + 2i) = (7 - 4) + (5i + 2i) = 3 + 7i$

32.
$$\begin{aligned} \left(\sqrt{2} - \sqrt{2}i\right) - \left(\sqrt{2} + \sqrt{2}i\right) &= \left(\sqrt{2} - \sqrt{2}\right) + \left(-\sqrt{2} - \sqrt{2}i\right) \\ &= 0 + (-2)\sqrt{2}i \\ &= -2\sqrt{2}i \end{aligned}$$

33. $7i(11 - 9i) = 77i - 63i^2 = 63 + 77i$

34.
$$\begin{aligned} (1 + 6i)(5 - 2i) &= 5 - 2i + 30i - 12i^2 \\ &= 5 + 28i + 12 \\ &= 17 + 28i \end{aligned}$$

35.
$$\begin{aligned} \frac{6 + i}{4 - i} &= \frac{6 + i}{4 - i} \cdot \frac{4 + i}{4 + i} \\ &= \frac{24 + 10i + i^2}{16 + 1} \\ &= \frac{23 + 10i}{17} \\ &= \frac{23}{17} + \frac{10}{17}i \end{aligned}$$

36.
$$\begin{aligned} \frac{4}{2 - 3i} + \frac{2}{1 + i} &= \frac{4}{2 - 3i} \cdot \frac{2 + 3i}{2 + 3i} + \frac{2}{1 + i} \cdot \frac{1 - i}{1 - i} \\ &= \frac{8 + 12i}{4 + 9} + \frac{2 - 2i}{1 + 1} \\ &= \frac{8}{13} + \frac{12}{13}i + 1 - i \\ &= \left(\frac{8}{13} + 1\right) + \left(\frac{12}{13}i - i\right) \\ &= \frac{21}{13} - \frac{1}{13}i \end{aligned}$$

37. $x^2 - 2x + 10 = 0$
$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(10)}}{2(1)} \\ &= \frac{2 \pm \sqrt{-36}}{2} \\ &= \frac{2 \pm 6i}{2} \\ &= 1 \pm 3i \end{aligned}$$

38. $4x^2 + 4x + 7 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-4 \pm \sqrt{(4)^2 - 4(4)(7)}}{2(4)}$$
$$= \frac{-4 \pm \sqrt{-96}}{8}$$
$$= \frac{-4 \pm 4\sqrt{6}i}{8}$$
$$= -\frac{1}{2} \pm \frac{\sqrt{6}}{2}i$$

39. Since $g(x) = x^2 - 2x - 8$ is a 2nd degree polynomial function, it has two zeros.

40. Since $h(t) = t^2 - t^5$ is a 5th degree polynomial function, it has five zeros.

41. $f(x) = x^3 + 3x^2 - 28x - 60$

Possible rational zeros:
$\pm1, \pm2, \pm3, \pm4, \pm5, \pm6, \pm10, \pm12, \pm15, \pm20, \pm30, \pm60$

$$\begin{array}{r|rrrr} -2 & 1 & 3 & -28 & -60 \\ & & -2 & -2 & 60 \\ \hline & 1 & 1 & -30 & 0 \end{array}$$

$$x^3 + 3x^2 - 28x - 60 = (x + 2)(x^2 + x - 30)$$
$$= (x + 2)(x + 6)(x - 5)$$

The zeros of $f(x)$ are $x = -2$, $x = -6$, and $x = 5$.

42. $f(x) = 4x^4 - 11x^3 + 14x^2 - 6x$

$$= x(4x^3 - 11x^2 + 14x - 6)$$

One zero is $x = 0$. Because $1 - i$ is a zero, so is $1 + i$.

$$\begin{array}{r|rrrr} 1 - i & 4 & -11 & 14 & -6 \\ & & 4 - 4i & -11 + 3i & 6 \\ \hline & 4 & -7 - 4i & 3 + 3i & 0 \end{array}$$

$$\begin{array}{r|rrr} 1 + i & 4 & -7 - 4i & 3 + 3i \\ & & 4 + 4i & -3 - 3i \\ \hline & 4 & -3 & 0 \end{array}$$

$$f(x) = x[x - (1 - i)][x - (1 + i)](4x - 3)$$
$$= x(x - 1 + i)(x - 1 - i)(4x - 3)$$

Zeros: $0, \frac{3}{4}, 1 + i, 1 - i$

43. $g(x) = x^3 - 7x^2 + 36$

$$\begin{array}{r|rrrr} -2 & 1 & -7 & 0 & 36 \\ & & -2 & 18 & -36 \\ \hline & 1 & -9 & 18 & 0 \end{array}$$

The zeros of $x^2 - 9x + 18 = (x - 3)(x - 6)$ are $x = 3, 6$. The zeros of $g(x)$ are $x = -2, 3, 6$.

$$g(x) = (x + 2)(x - 3)(x - 6)$$

44. $f(x) = x^4 + 8x^3 + 8x^2 - 72x - 153$

$$\begin{array}{r|rrrrr} 3 & 1 & 8 & 8 & -72 & -153 \\ & & 3 & 33 & 123 & 153 \\ \hline & 1 & 11 & 41 & 51 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -3 & 1 & 11 & 41 & 51 \\ & & -3 & -24 & -51 \\ \hline & 1 & 8 & 17 & \end{array}$$

By the Quadratic Formula, the zeros of
$x^2 + 8x + 17$ are

$$x = \frac{-8 \pm \sqrt{(8)^2 - 4(1)(17)}}{2(1)} = \frac{-8 \pm \sqrt{-4}}{2} = -4 \pm i.$$

The zeros of $f(x)$ are $-3, 3, -4 - i, -4 + i$.

$$f(x) = (x + 3)(x - 3)(x + 4 - i)(x + 4 + i)$$

45. $h(x) = -2x^5 + 4x^3 - 2x^2 + 5$

$h(x)$ has three variations in sign, so h has either three or one positive real zeros.

$$h(-x) = -2(-x)^5 + 4(-x)^3 - 2(-x)^2 + 5$$
$$= 2x^5 - 4x^3 - 2x^2 + 5$$

$h(-x)$ has two variations in sign, so h has either two or no negative real zeros.

46. $f(x) = 4x^3 - 3x^2 + 4x - 3$

(a)
$$\begin{array}{r|rrrr} 1 & 4 & -3 & 4 & -3 \\ & & 4 & 1 & 5 \\ \hline & 4 & 1 & 5 & 2 \end{array}$$

Because the last row has all positive entries, $x = 1$ is an upper bound.

47. Because the denominator is zero when $x + 10 = 0$, the domain of f is all real numbers except $x = -10$.

x	-11	-10.5	-10.1	-10.01	-10.001	$\to -10$
$f(x)$	33	63	303	3003	30,003	$\to \infty$

x	$-10 \leftarrow$	-9.999	-9.99	-9.9	-9.5	-9
$f(x)$	$-\infty$	$-29{,}997$	-2997	-297	-57	-27

As x approaches -10 from the left, $f(x)$ increases without bound.

As x approaches -10 from the right, $f(x)$ decreases without bound.

48. Because the denominator is zero when $x^2 - 10x + 24 = (x - 4)(x - 6) = 0$, the domain of f is all real numbers except $x = 4$ and $x = 6$.

x	3	3.5	3.9	3.99	3.999	$\to 4$
$f(x)$	2.667	6.4	38.095	398.010	3998.001	$\to \infty$

x	$4 \leftarrow$	4.001	4.01	4.1	4.5	5
$f(x)$	$-\infty \leftarrow$	-4002.001	-402.010	-42.105	-10.67	-8

As x approaches 4 from the left, $f(x)$ increases without bound.

As x approaches 4 from the right, $f(x)$ decreases without bound.

x	5	5.5	5.9	5.99	5.999	$\to 6$
$f(x)$	-8	-10.67	-42.015	-402.010	-4002.001	$\to -\infty$

x	$6 \leftarrow$	6.001	6.01	6.1	6.5	7
$f(x)$	$\infty \leftarrow$	3998.001	398.010	38.095	6.4	2.667

As x approaches 6 from the left, $f(x)$ decreases without bound.

As x approaches 6 from the right, $f(x)$ increases without bound.

49. $f(x) = \dfrac{4}{x}$

(a) Domain: all real numbers x except $x = 0$

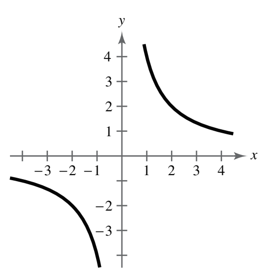
(b) No intercepts

(c) Vertical asymptote: $x = 0$

Horizontal asymptote: $y = 0$

(d)

x	-3	-2	-1	1	2	3
$f(x)$	$-\frac{4}{3}$	-2	-4	4	2	$\frac{4}{3}$



50. $h(x) = \dfrac{x-4}{x-7}$

(a) Domain: all real numbers x except $x = 7$

(b) x-intercept: $(4, 0)$

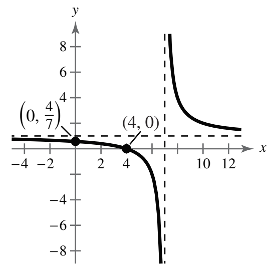
y-intercept: $\left(0, \dfrac{4}{7}\right)$

(c) Vertical asymptote: $x = 7$

Horizontal asymptote: $y = 1$

(d)

x	-2	-1	0	1	2	3	4	5	6	8
$h(x)$	$\frac{2}{3}$	$\frac{5}{8}$	$\frac{4}{7}$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{1}{4}$	0	$-\frac{1}{2}$	-2	4



51. $f(x) = \dfrac{x}{x^2+1}$

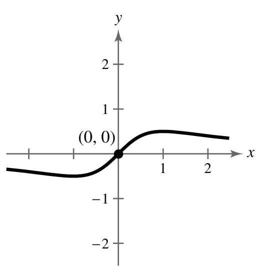
(a) Domain: all real numbers x

(b) Intercept: $(0, 0)$

(c) Horizontal asymptote: $y = 0$

(d)

x	-2	-1	0	1	2
$f(x)$	$-\frac{2}{5}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{2}{5}$



52. $y = \dfrac{2x^2}{x^2-4}$

(a) Domain: all real numbers x except $x = \pm 2$

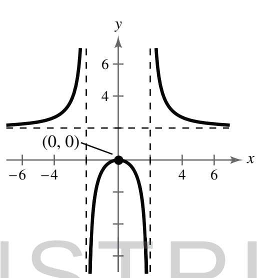
(b) Intercept: $(0, 0)$

(c) Vertical asymptotes: $x = 2, x = -2$

Horizontal asymptote: $y = 2$

(d)

x	± 5	± 4	± 3	± 1	0
y	$\frac{50}{21}$	$\frac{8}{3}$	$\frac{18}{5}$	$-\frac{2}{3}$	0



53. $f(x) = \dfrac{6x^2 - 11x + 3}{3x^2 - x}$

$$= \frac{(3x-1)(2x-3)}{x(3x-1)} = \frac{2x-3}{x},\ x \neq \frac{1}{3}$$

(a) Domain: all real numbers x except $x = 0$ and $x = \dfrac{1}{3}$

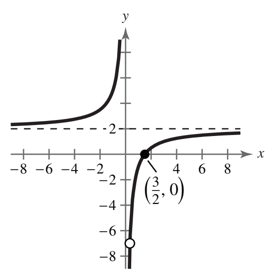
(b) x-intercept: $\left(\dfrac{3}{2}, 0\right)$

(c) Vertical asymptote: $x = 0$

Horizontal asymptote: $y = 2$

(d)

x	-2	-1	1	2	3	4
$f(x)$	$\frac{7}{2}$	5	-1	$\frac{1}{2}$	1	$\frac{5}{4}$



54. $f(x) = \dfrac{6x^2 - 7x + 2}{4x^2 - 1}$

$= \dfrac{(2x-1)(3x-2)}{(2x-1)(2x+1)} = \dfrac{3x-2}{2x+1},\ x \neq \dfrac{1}{2}$

(a) Domain: all real numbers x except $x = \pm\dfrac{1}{2}$

(b) y-intercept: $(0, -2)$

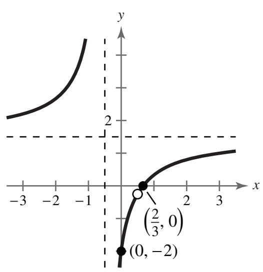
x-intercept: $\left(\dfrac{2}{3}, 0\right)$

(c) Vertical asymptote: $x = -\dfrac{1}{2}$

Horizontal asymptote: $y = \dfrac{3}{2}$

(d)

x	-3	-2	-1	0	$\frac{2}{3}$	1	2
$f(x)$	$\frac{11}{5}$	$\frac{8}{3}$	5	-2	0	$\frac{1}{3}$	$\frac{4}{5}$



55. $f(x) = \dfrac{2x^3}{x^2 + 1} = 2x - \dfrac{2x}{x^2 + 1}$

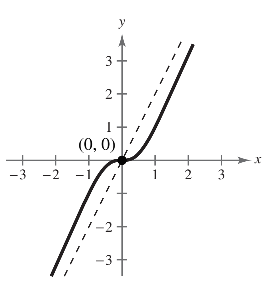
(a) Domain: all real numbers x

(b) Intercept: $(0, 0)$

(c) Slant asymptote: $y = 2x$

(d)

x	-2	-1	0	1	2
$f(x)$	$-\frac{16}{5}$	-1	0	1	$\frac{16}{5}$



56. $f(x) = \dfrac{x^2 + 1}{x + 1} = x - 1 + \dfrac{2}{x + 1}$

(a) Domain: all real numbers x except $x = -1$

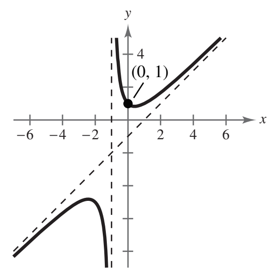
(b) y-intercept: $(0, 1)$

(c) Vertical asymptote: $x = -1$

Slant asymptote: $y = x - 1$

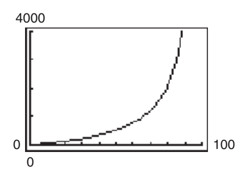
(d)

x	-6	-2	$-\frac{3}{2}$	$-\frac{1}{2}$	0	4
$f(x)$	$-\frac{37}{5}$	-5	$-\frac{13}{2}$	$\frac{5}{2}$	1	$\frac{17}{5}$



57. $C = \dfrac{528p}{100 - p},\ 0 \le p < 100$

(a)



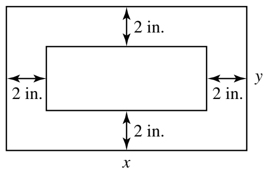
(b) When $p = 25$, $C = \dfrac{528(25)}{100 - 25} = \$176$ million.

When $p = 50$, $C = \dfrac{528(50)}{100 - 50} = \$528$ million.

When $p = 75$, $C = \dfrac{528(75)}{100 - 75} = \$1584$ million.

(c) As $p \to 100$, $C \to \infty$. No, it is not possible.

58. (a)



(b) The area of print is $(x - 4)(y - 4)$, which is 30 square inches.

$$(x - 4)(y - 4) = 30$$
$$y - 4 = \frac{30}{x - 4}$$
$$y = \frac{30}{x - 4} + 4$$
$$y = \frac{30 + 4(x - 4)}{x - 4}$$
$$y = \frac{4x + 14}{x - 4}$$
$$y = \frac{2(2x + 7)}{x - 4}$$
$$\text{Total area} = xy = x\left[\frac{2(2x + 7)}{x - 4}\right] = \frac{2x(2x + 7)}{x - 4}$$

(c) Because the horizontal margins total 4 inches, x must be greater than 4 inches. The domain is $x > 4$.

59.
$$12x^2 + 5x < 2$$
$$12x^2 + 5x - 2 < 0$$
$$(4x - 1)(3x + 2) < 0$$

Key numbers: $x = -\frac{2}{3}, x = \frac{1}{4}$

Test intervals: $\left(-\infty, -\frac{2}{3}\right), \left(-\frac{2}{3}, \frac{1}{4}\right), \left(\frac{1}{4}, \infty\right)$

Test: Is $(4x - 1)(3x + 2) < 0$?

By testing an x-value in each test interval in the inequality, you see that the solution set is $\left(-\frac{2}{3}, \frac{1}{4}\right)$.

60.
$$x^3 - 16x \geq 0$$
$$x(x + 4)(x - 4) \geq 0$$

Key numbers: $x = 0, x = \pm 4$

Test intervals: $(-\infty, -4), (-4, 0), (0, 4), (4, \infty)$

Test: Is $x(x + 4)(x - 4) \geq 0$?

By testing an x-value in each test interval in the inequality, you see that the solution set is $[-4, 0] \cup [4, \infty)$.

61.
$$\frac{2}{x + 1} \leq \frac{3}{x - 1}$$
$$\frac{2(x - 1) - 3(x + 1)}{(x + 1)(x - 1)} \leq 0$$
$$\frac{2x - 2 - 3x - 3}{(x + 1)(x - 1)} \leq 0$$
$$\frac{-(x + 5)}{(x + 1)(x - 1)} \leq 0$$

Key numbers: $x = -5, x = \pm 1$

Test intervals: $(-\infty, -5), (-5, -1), (-1, 1), (1, \infty)$

Test: Is $\frac{-(x + 5)}{(x + 1)(x - 1)} \leq 0$?

By testing an x-value in each test interval in the inequality, you see that the solution set is $[-5, -1) \cup (1, \infty)$.

62.
$$\frac{x^2 - 9x + 20}{x} < 0$$
$$\frac{(x - 4)(x - 5)}{x} < 0$$

Key numbers: $x = 0, x = 4, x = 5$

Test intervals: $(-\infty, 0), (0, 4), (4, 5), (5, \infty)$

Test: Is $\frac{(x - 4)(x - 5)}{x} < 0$?

By testing an x-value in each test interval in the inequality, you see that the solution set is $(-\infty, 0) \cup (4, 5)$.

63.
$$P = \frac{1000(1 + 3t)}{5 + t}$$
$$2000 \leq \frac{1000(1 + 3t)}{5 + t}$$
$$2000(5 + t) \leq 1000(1 + 3t)$$
$$10{,}000 + 2000t \leq 1000 + 3000t$$
$$-1000t \leq -9000$$
$$t \geq 9 \text{ days}$$

64. An asymptote of a graph is a line to which the graph becomes arbitrarily close as x increases or decreases without bound.

65. False. A fourth-degree polynomial can have at most four zeros, and complex zeros occur in conjugate pairs.

66. False.

The domain of

$$f(x) = \frac{1}{x^2 + 1}$$

is the set of all real numbers x.

Problem Solving for Chapter 2

1. $f(x) = ax^3 + bx^2 + cx + d$

$$\begin{array}{r} ax^2 + (ak+b)x + (ak^2+bk+c) \\ x - k\overline{\big)ax^3 + bx^2 \qquad + cx \qquad + d} \\ \underline{ax^3 - akx^2} \qquad\qquad\qquad\qquad \\ (ak+b)x^2 + cx \qquad\qquad \\ \underline{(ak+b)x^2 - (ak^2+bk)x} \qquad\qquad \\ (ak^2+bk+c)x + d \\ \underline{(ak^2+bk+c)x - (ak^3+bk^2+ck)} \\ (ak^3+bk^2+ck+d) \end{array}$$

So, $f(x) = ax^3 + bx^2 + cx + d = (x-k)\left[ax^2 + (ak+b)x + (ak^2+bx+c)\right] + ak^3 + bk^2 + ck + d$ and $f(x) = ak^3 + bk^2 + ck + d$. Because the remainder is $r = ak^3 + bk^2 + ck + d$, $f(k) = r$.

2. (a)

y	$y^3 + y^2$
1	2
2	12
3	36
4	80
5	150
6	252
7	392
8	576
9	810
10	1100

(b) (i) $x^3 + x^2 = 252 \Rightarrow x = 6$

(ii) $x^3 + 2x^2 = 288; a = 1, b = 2 \Rightarrow \dfrac{a^2}{b^3} = \dfrac{1}{8}$

$\dfrac{1}{8}x^3 + \dfrac{1}{8}(2x^2) = \dfrac{1}{8}(288)$

$\left(\dfrac{x}{2}\right)^3 + \left(\dfrac{x}{2}\right)^2 = 36 \Rightarrow \dfrac{x}{2} = 3 \Rightarrow x = 6$

(iii) $3x^3 + x^2 = 90; a = 3, b = 1 \Rightarrow \dfrac{a^2}{b^3} = 9$

$9(3x^3) + 9x^2 = 9(90)$

$(3x)^3 + (3x)^2 = 810 \Rightarrow 3x = 9 \Rightarrow x = 3$

(iv) $2x^3 + 5x^2 = 2500; a = 2, b = 5 \Rightarrow \dfrac{a^2}{b^3} = \dfrac{4}{125}$

$\dfrac{4}{125}(2x^3) + \dfrac{4}{125}(5x^2) = \dfrac{4}{125}(2500)$

$\left(\dfrac{2x}{5}\right)^3 + \left(\dfrac{2x}{5}\right)^2 = 80 \Rightarrow \dfrac{2x}{5} = 4 \Rightarrow x = 10$

(v) $7x^3 + 6x^2 = 1728;$

$a = 7, b = 6 \Rightarrow \dfrac{a^2}{b^3} = \dfrac{49}{216}$

$\dfrac{49}{216}(7x^3) + \dfrac{49}{216}(6x^2) = \dfrac{49}{216}(1728)$

$\left(\dfrac{7x}{6}\right)^3 + \left(\dfrac{7x}{6}\right)^2 = 392 \Rightarrow \dfrac{7x}{6} = 7 \Rightarrow x = 6$

(vi) $10x^3 + 3x^2 = 297;$

$a = 10, b = 3 \Rightarrow \dfrac{a^2}{b^3} = \dfrac{100}{27}$

$\dfrac{100}{27}(10x^3) + \dfrac{100}{27}(3x^2) = \dfrac{100}{27}(297)$

$\left(\dfrac{10x}{3}\right)^3 + \left(\dfrac{10x}{3}\right)^2 = 1100 \Rightarrow \dfrac{10x}{3}$

$= 10 \Rightarrow x = 3$

(c) Answers will vary.

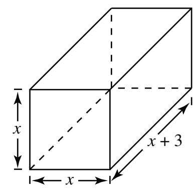
3. $V = l \cdot w \cdot h = x^2(x + 3)$

$x^2(x + 3) = 20$

$x^3 + 3x^2 - 20 = 0$

Possible rational zeros: $\pm1, \pm2, \pm4, \pm5, \pm10, \pm20$

$$\begin{array}{r|rrrr} 2 & 1 & 3 & 0 & -20 \\ & & 2 & 10 & 20 \\ \hline & 1 & 5 & 10 & 0 \end{array}$$



$(x - 2)(x^2 + 5x + 10) = 0$

$x = 2$ or $x = \dfrac{-5 \pm \sqrt{15}i}{2}$

Choosing the real positive value for x we have:
$x = 2$ and $x + 3 = 5$.

The dimensions of the mold are
2 inches × 2 inches × 5 inches.

4. False. Since $f(x) = d(x)q(x) + r(x)$, we have

$$\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}.$$

The statement should be corrected to read $f(-1) = 2$

since $\dfrac{f(x)}{x + 1} = q(x) + \dfrac{f(-1)}{x + 1}$.

5. (a) $y = ax^2 + bx + c$

$(0, -4)$: $-4 = a(0)^2 + b(0) + c$

$-4 = c$

$(4, 0)$: $0 = a(4)^2 + b(4) - 4$

$0 = 16a + 4b - 4 = 4(4a + b - 1)$

$0 = 4a + b - 1$ or $b = 1 - 4a$

$(1, 0)$: $0 = a(1)^2 + b(1) - 4$

$4 = a + b$

$4 = a + (1 - 4a)$

$4 = 1 - 3a$

$3 = -3a$

$a = -1$

$b = 1 - 4(-1) = 5$

$y = -x^2 + 5x - 4$

(b) Enter the data points $(0, -4)$, $(1, 0)$, $(2, 2)$, $(4, 0)$, $(6, -10)$ and use the regression feature to obtain $y = -x^2 + 5x - 4$.

6. (a) Slope $= \dfrac{9 - 4}{3 - 2} = 5$

Slope of tangent line is less than 5.

(b) Slope $= \dfrac{4 - 1}{2 - 1} = 3$

Slope of tangent line is greater than 3.

(c) Slope $= \dfrac{4.41 - 4}{2.1 - 2} = 4.1$

Slope of tangent line is less than 4.1.

(d) Slope $= \dfrac{f(2 + h) - f(2)}{(2 + h) - 2}$

$= \dfrac{(2 + h)^2 - 4}{h}$

$= \dfrac{4h + h^2}{h}$

$= 4 + h, h \neq 0$

(e) Slope $= 4 + h, \quad h \neq 0$

$4 + (-1) = 3$

$4 + 1 = 5$

$4 + 0.1 = 4.1$

The results are the same as in (a)–(c).

(f) Letting h get closer and closer to 0, the slope approaches 4. So, the slope at $(2, 4)$ is 4.

7. $f(x) = (x - k)q(x) + r$

(a) Cubic, passes through $(2, 5)$, rises to the right

One possibility:

$f(x) = (x - 2)x^2 + 5$

$= x^3 - 2x^2 + 5$

(b) Cubic, passes through $(-3, 1)$, falls to the right

One possibility:

$f(x) = -(x + 3)x^2 + 1$

$= -x^3 - 3x^2 + 1$

8. (a) $z_m = \dfrac{1}{z}$

$= \dfrac{1}{1+i} = \dfrac{1}{1+i} \cdot \dfrac{1-i}{1-i}$

$= \dfrac{1-i}{2} = \dfrac{1}{2} - \dfrac{1}{2}i$

(b) $z_m = \dfrac{1}{z}$

$= \dfrac{1}{3-i} = \dfrac{1}{3-i} \cdot \dfrac{3+i}{3+i}$

$= \dfrac{3+i}{10} = \dfrac{3}{10} + \dfrac{1}{10}i$

(c) $z_m = \dfrac{1}{z}$

$= \dfrac{1}{-2+8i}$

$= \dfrac{1}{-2+8i} \cdot \dfrac{-2-8i}{-2-8i}$

$= \dfrac{-2-8i}{68} = -\dfrac{1}{34} - \dfrac{2}{17}i$

9. $(a+bi)(a-bi) = a^2 - abi + abi - b^2i^2 = a^2 + b^2$

Since a and b are real numbers, $a^2 + b^2$ is also a real number.

10. $f(x) = \dfrac{ax+b}{cx+d}$

Vertical asymptote: $x = -\dfrac{d}{c}$

Horizontal asymptote: $y = \dfrac{a}{c}$

(i) $a > 0, b < 0, c > 0, d < 0$

Both the vertical asymptote and the horizontal asymptote are positive. Matches graph (d).

(ii) $a > 0, b > 0, c < 0, d < 0$

Both the vertical asymptote and the horizontal asymptote are negative. Matches graph (b).

(iii) $a < 0, b > 0, c > 0, d < 0$

The vertical asymptote is positive and the horizontal asymptote is negative. Matches graph (a).

(iv) $a > 0, b < 0, c > 0, d > 0$

The vertical asymptote is negative and the horizontal asymptote is positive. Matches graph (c).

11. $f(x) = \dfrac{ax}{(x-b)^2}$

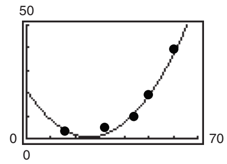
(a) $b \neq 0 \Rightarrow x = b$ is a vertical asymptote.

a causes a vertical stretch if $|a| > 1$ and a vertical shrink if $0 < |a| < 1$. For $|a| > 1$, the graph becomes wider as $|a|$ increases. When a is negative, the graph is reflected about the x-axis.

(b) $a \neq 0$. Varying the value of b varies the vertical asymptote of the graph of f. For $b > 0$, the graph is translated to the right. For $b < 0$, the graph is reflected in the x-axis and is translated to the left.

12. (a)

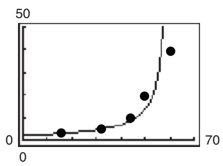
Age, x	Near point, y
16	3.0
32	4.7
44	9.8
50	19.7
60	39.4



$y \approx 0.031x^2 - 1.59x + 21.0$

(b) $\dfrac{1}{y} \approx -0.007x + 0.44$

$$y \approx \frac{1}{-0.007x + 0.44}$$



(c)

Age, x	Near point, y	Quadratic Model	Rational Model
16	3.0	3.66	3.05
32	4.7	2.32	4.63
44	9.8	11.83	7.58
50	19.7	19.97	11.11
60	39.4	38.54	50.00

The models are fairly good fits to the data. The quadratic model seems to be a better fit for older ages and the rational model a better fit for younger ages.

(d) For $x = 25$, the quadratic model yields $y \approx 0.625$ inch and the rational model yields $y \approx 3.774$ inches.

(e) The reciprocal model cannot be used to predict the near point for a person who is 70 years old because it results in a negative value $(y \approx -20)$. The quadratic model yields $y \approx 63.37$ inches.

13. Because complex zeros always occur in conjugate pairs, and a cubic equation has three zeros and not four, a cubic equation with real coefficients can not have two real zeros and one complex zero.

Practice Test for Chapter 2

1. Sketch the graph of $f(x) = x^2 - 6x + 5$ and identify the vertex and the intercepts.

2. Find the number of units x that produce a minimum cost C if $C = 0.01x^2 - 90x + 15{,}000$.

3. Find the quadratic function that has a maximum at $(1, 7)$ and passes through the point $(2, 5)$.

4. Find two quadratic functions that have x-intercepts $(2, 0)$ and $\left(\frac{4}{3}, 0\right)$.

5. Use the leading coefficient test to determine the right and left end behavior of the graph of the polynomial function $f(x) = -3x^5 + 2x^3 - 17$.

6. Find all the real zeros of $f(x) = x^5 - 5x^3 + 4x$.

7. Find a polynomial function with 0, 3, and -2 as zeros.

8. Sketch $f(x) = x^3 - 12x$.

9. Divide $3x^4 - 7x^2 + 2x - 10$ by $x - 3$ using long division.

10. Divide $x^3 - 11$ by $x^2 + 2x - 1$.

11. Use synthetic division to divide $3x^5 + 13x^4 + 12x - 1$ by $x + 5$.

12. Use synthetic division to find $f(-6)$ given $f(x) = 7x^3 + 40x^2 - 12x + 15$.

13. Find the real zeros of $f(x) = x^3 - 19x - 30$.

14. Find the real zeros of $f(x) = x^4 + x^3 - 8x^2 - 9x - 9$.

15. List all possible rational zeros of the function $f(x) = 6x^3 - 5x^2 + 4x - 15$.

16. Find the rational zeros of the polynomial $f(x) = x^3 - \frac{20}{3}x^2 + 9x - \frac{10}{3}$.

17. Write $f(x) = x^4 + x^3 + 5x - 10$ as a product of linear factors.

18. Find a polynomial with real coefficients that has $2, 3 + i,$ and $3 - 2i$ as zeros.

19. Use synthetic division to show that $3i$ is a zero of $f(x) = x^3 + 4x^2 + 9x + 36$.

20. Sketch the graph of $f(x) = \dfrac{x - 1}{2x}$ and label all intercepts and asymptotes.

21. Find all the asymptotes of $f(x) = \dfrac{8x^2 - 9}{x^2 + 1}$.

22. Find all the asymptotes of $f(x) = \dfrac{4x^2 - 2x + 7}{x - 1}$.

23. Given $z_1 = 4 - 3i$ and $z_2 = -2 + i$, find the following:

(a) $z_1 - z_2$

(b) $z_1 z_2$

(c) z_1/z_2

24. Solve the inequality: $x^2 - 49 \leq 0$

25. Solve the inequality: $\dfrac{x+3}{x-7} \geq 0$