Chapter 1: Introduction to Functions and Graphs

1.1: Numbers, Data, and Problem Solving

1. $\frac{21}{24}$ is a real and rational number.
2. 695,00 is a real number, rational number, integer, and natural number.
3. 7.5 is a real and rational number.
4. 8.4 is a real and rational number.
5. $90\sqrt{2}$ is a real number.
6. -71 is an integer, real number, and rational number.
7. Natural number: $\sqrt{9} = 3$; integers: –3, $\sqrt{9}$; rational numbers: $-3, \frac{2}{9}, \sqrt{9}, 1.\overline{3}$; irrational numbers: $\pi, -\sqrt{2}$
8. Natural numbers: $\frac{3}{1} = 3, 5.6 \times 10^3 = 5600$; integers: $\frac{3}{1}, 0, 5.6 \times 10^3$;

   rational numbers: $\frac{3}{1}, -\frac{5}{8}, 0.\overline{45}, 0, 5.6 \times 10^3$; irrational number: $\sqrt{7}$
9. Natural number: None; integer: $-\sqrt{4} = -2$; rational numbers: $\frac{1}{3}, 5.1 \times 10^{-6}, -2.33, 0.\overline{7}, -\sqrt{4}$;

   irrational number: $\sqrt{13}$
10. Natural numbers: $\sqrt{100} = 10$; integers:

    $-103, \sqrt{100}$; rational numbers: $-103, \frac{21}{25}, \sqrt{100}, -\frac{5.7}{10}, \frac{2}{9}, -1.457$;

    irrational number: $\sqrt{3}$
11. Shoe sizes are normally measured to within half sizes. Rational numbers are most appropriate.
12. Population is measured using natural numbers.
13. Speed limit is measured using natural numbers.
14. Gasoline is usually measured to a fraction of a gallon using rational numbers.
15. Temperature is typically measured to the nearest degree in a weather forecast. Since temperature can include negative numbers, the integers would be most appropriate.
16. Compact disc sales could be measured in natural numbers, since fractions of a disc are not allowed.
17. $|5 - 8 \cdot 7| = |5 - 56| = |-51| = 51$
18. $-2(16 - 3 \cdot 5) \div 2 = -2(16 - 15) \div 2 = -2(1) \div 2 = -2 \div 2 = -1$
19. $-6^2 - 3(2 - 4)^4 = -6^2 - 3(-2)^4 = -36 - 3(16) = -36 - 48 = -84$
20. $(4 - 5)^2 - 3^2 - 3\sqrt{9} = (-1)^2 - 9 - 3(3) = 1 - 9 - 9 = -17$
21. $\sqrt{9 - 5} - \frac{8 - 4}{4 - 2} = \sqrt{4} - \frac{4}{2} = 2 - 2 = 0$
22. $\frac{6 - 4^2 \div 2^3}{3 - 4} = \frac{6 - 16 \div 8}{-1} = \frac{6 - 2}{-1} = -4$
23. $\sqrt{13^2 - 12^2} = \sqrt{169 - 144} = \sqrt{25} = 5$

24. $\dfrac{13 - \sqrt{9 + 16}}{|5 - 7|^2} = \dfrac{13 - \sqrt{25}}{|-2|^2} = \dfrac{13 - 5}{4} = \dfrac{8}{4} = 2$

25. $\dfrac{4 + 9}{2 + 3} - \dfrac{-3^2 \cdot 3}{5} = \dfrac{13}{5} - \dfrac{-27}{5} = \dfrac{40}{5} = 8$

26. $10 \div 2 \div \dfrac{5 + 10}{5} = 10 \div 2 \div \dfrac{15}{5} = 10 \div 2 \div 3 = 5 \div 3 = \dfrac{5}{3}$

27. $-5^2 - 20 \div 4 - 2 = -25 - 5 - 2 = -32$

28. $5 - (-4)^3 - 4^3 = 5 - (-64) - 64 = 5 + 64 - 64 = 5$

29. 4×10^1

30. 1.17×10^7

31. 3.65×10^{-3}

32. $0.62 = 6.2 \times 10^{-1}$

33. $2450 = 2.45 \times 10^3$

34. $105.6 = 1.056 \times 10^2$

35. $0.56 = 5.6 \times 10^{-1}$

36. $-0.00456 = -4.56 \times 10^{-3}$

37. $-0.0087 = -8.7 \times 10^{-3}$

38. $1{,}250{,}000 = 1.25 \times 10^6$

39. $206.8 = 2.068 \times 10^2$

40. $0.00007 = 7 \times 10^{-5}$

41. $10^{-6} = 0.000001$

42. $9.11 \times 10^{-31} = 0.000000000000000000000000000000911$

    There are 30 zeros between the decimal point and the digits 911.

43. $2 \times 10^8 = 200{,}000{,}000$

44. $8{,}000{,}000{,}000$

45. $1.567 \times 10^2 = 156.7$

46. $-5.68 \times 10^{-1} = -0.568$

47. $5 \times 10^5 = 500{,}000$

48. $3.5 \times 10^3 = 3500$

49. $0.045 \times 10^5 = 4500$

50. $-5.4 \times 10^{-5} = -0.000054$

51. $67 \times 10^3 = 67{,}000$

52. $0.0032 \times 10^{-1} = 0.00032$

53. $(4 \times 10^3)(2 \times 10^5) = 4 \cdot 2 \times 10^{3+5} = 8 \times 10^8$; $800{,}000{,}000$

54. $(3 \times 10^1)(3 \times 10^4) = 3 \cdot 3 \times 10^{1+4} = 9 \times 10^5$; $900{,}000$

55. $(5 \times 10^2)(7 \times 10^{-4}) = 5 \cdot 7 \times 10^{2-4} = 35 \times 10^{-2} = 3.5 \times 10^{-1}$; 0.35

56. $(8 \times 10^{-3})(7 \times 10^{1}) = 8 \cdot 7 \times 10^{-3+1} = 56 \times 10^{-2} = 5.6 \times 10^{-1};\ 0.56$

57. $\dfrac{6.3 \times 10^{-2}}{3 \times 10^{1}} = \dfrac{6.3}{3} \times 10^{-2-1} = 2.1 \times 10^{-3};\ 0.0021$

58. $\dfrac{8.2 \times 10^{2}}{2 \times 10^{-2}} = \dfrac{8.2}{2} \times 10^{2-(-2)} = 4.1 \times 10^{4};\ 41{,}000$

59. $\dfrac{4 \times 10^{-3}}{8 \times 10^{-1}} = \dfrac{4}{8} \times 10^{-3-(-1)} = 0.5 \times 10^{-2} = 5 \times 10^{-3};\ 0.005$

60. $\dfrac{2.4 \times 10^{-5}}{4.8 \times 10^{-7}} = \dfrac{2.4}{4.8} \times 10^{-5-(-7)} = 0.5 \times 10^{2} = 5 \times 10^{1};\ 50$

61. $\dfrac{8.947 \times 10^{7}}{0.00095}(4.5 \times 10^{8}) \approx 42381 \times 10^{15} = 4.2381 \times 10^{19} \approx 4.24 \times 10^{19}$

62. $(9.87 \times 10^{6})(34 \times 10^{11}) = 335.58 \times 10^{17} = 3.3558 \times 10^{19} \approx 3.36 \times 10^{19}$

63. $\left(\dfrac{101 + 23}{0.42}\right)^2 + \sqrt{3.4 \times 10^{-2}} \approx 87166 + 0.2 \approx 87166.2 \approx 8.72 \times 10^{4}$

64. $\sqrt[n]{(2.5 \times 10^{-8})} + 10^{-7} \approx 0.005 = 5.0 \times 10^{-3}$

65. $(8.5 \times 10^{-5})(-9.5 \times 10^{7})^2 = (8.5 \times 10^{-5})(9.025 \times 10^{15}) \approx 76.7 \times 10^{10} = 7.67 \times 10^{11}$

66. $\sqrt{\pi(4.56 \times 10^{4}) + (3.1 \times 10^{-2})} \approx 378.5 \approx 3.78 \times 10^{2}$

67. $\sqrt[m]{192} \approx 5.769$

68. $\sqrt{(32 + \pi^3)} \approx 7.938$

69. $|\pi - 3.2| \approx 0.058$

70. $\dfrac{1.72 - 5.98}{35.6 + 1.02} \approx -0.116$

71. $\dfrac{0.3 + 1.5}{5.5 - 1.2} \approx 0.419$

72. $3.2(1.1)^2 - 4(1.1) + 2 = 1.472$

73. $\dfrac{1.5^3}{\sqrt{2 + \pi} - 5} \approx \dfrac{3.375}{-2.732} \approx -1.235$

74. $4.3^2 - \dfrac{5}{17} \approx 18.49 - 0.294 = 18.196$

75. $15 + \dfrac{4 + \sqrt{3}}{7} \approx 15.819$

76. $\dfrac{5 + \sqrt{5}}{2} \approx 3.618$

77. $0.14 = 1.4 \times 10^{-1}$ watt

78. The movement of the Pacific plate in one year is 7.1×10^{-5} kilometers per year. In one million years, the Pacific plate will move $(7.1 \times 10^{-5}) \cdot (1 \times 10^{6}) = 7.1 \times 10^{1} = 71$ km.

79. The distance Mars travels around the sun is $2\pi r = 2\pi(141{,}000{,}000) \approx 885{,}929{,}128$ miles. The number of hours in 1.88 years is $365 \times 1.88 \times 24 \approx 16{,}469$ hours. So Mars' speed is $\dfrac{885{,}929{,}128}{16{,}469} \approx 53{,}794$ miles per hour.

80. In one year light travels $186{,}000 \times 60 \times 60 \times 24 \times 365 \approx 5.87 \times 10^{12}$ miles. It takes $\dfrac{6 \times 10^{17}}{5.87 \times 10^{12}} \approx 102{,}000$ years for light to cross the Milky Way.

81. (a) First we will write both numbers in scientific notation. 208 million $= 2.08 \times 10^8$, $3{,}000{,}000 = 3 \times 10^6$. Then, divide the numbers to find the percentage. $\dfrac{3 \times 10^6}{2.08 \times 10^8} \approx 0.144 = 1.44\%$

(b) First we will write both numbers in scientific notation. 300 million $= 3 \times 10^8$, $11{,}700{,}000 = 1.17 \times 10^7$.

Then, divide the numbers to find the percentage. $\dfrac{1.17 \times 10^7}{3 \times 10^8} = 0.39 = 3.9\%$

82. One cubic mile is equal to $5280 \times 5280 \times 5280 \approx 1.47 \times 10^{11}$ cubic feet. In one day, the Amazon River discharges $(4.2 \times 10^6) \times 60 \times 60 \times 24 \approx 3.6 \times 10^{11}$ cubic feet, which is approximately 2.5 cubic miles.

83. (a) It would take $\dfrac{5.54 \times 10^{12}}{100} \approx 5.54 \times 10^{10}$ or 55.4 billion \$100-dollar bills to equal the federal debt. The height of the stacked bills would be $\dfrac{5.54 \times 10^{10}}{250} \approx 2.216 \times 10^8$ inches or $\dfrac{2.216 \times 10^8}{12} \approx 18{,}466{,}667$ feet.

(b) There are 5280 feet in one mile, so the stacked bills would span $\dfrac{18{,}466{,}667}{5280} \approx 3497$ miles. It would reach farther than the distance between Los Angles and New York.

84. $V = \dfrac{1}{3}\pi r^2 h \Rightarrow V = \dfrac{1}{3}\pi(4)^2(12) \Rightarrow V = \dfrac{1}{3}(\pi)(16)(12) \Rightarrow V = 201.06 \text{ in}^3$

85. (a) $V = \pi r^2 h \Rightarrow V = \pi(1.3)^2(4.4) \Rightarrow V = 7.436\pi \approx 23.4 \text{ in}^3$

(b) $1 \text{ in}^3 = 0.55$ fluid ounce $\Rightarrow 23.4 \cdot 0.55 = 12.87$ fluid ounces; Yes, it can hold 12 fluid ounces.

86. $V = \dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\pi(3)^3 = 36\pi \text{ ft}^3 \approx 113.1 \text{ ft}^3$

1.2: Visualizing and Graphing Data

1. (a) 

(b) Maximum: 6; minimum: −2

(c) $\dfrac{3 + (-2) + 5 + 0 + 6 + (-1)}{6} = \dfrac{11}{6} = 1.8\overline{3}$

2. (a) 

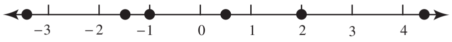
(b) Maximum: 6; minimum: −3

(c) $\dfrac{5 + (-3) + 4 + (-2) + 1 + 6}{6} = \dfrac{11}{6} = 1.8\overline{3}$

3. (a) 

(b) Maximum: 30; minimum: −20

(c) $\dfrac{-10 + 20 + 30 + (-20) + 0 + 10}{6} = 5$

4. (a) 

(b) Maximum: 4.5; minimum: -3.5

(c) $\dfrac{0.5 + (-1.5) + 2.0 + 4.5 + (-3.5) + (-1.0)}{6} = \dfrac{1}{6} = 0.1\overline{6}$

5.

-30	-30	-10	5	15	25	45	55	61

(a) The maximum is 61 and the minimum is –30.

(b) The mean is $\dfrac{-30 - 30 - 10 + 5 + 15 + 25 + 45 + 55 + 61}{9} \approx 15.11$ and the median is 15.

6.

-3.5	-1.25	1.5	1.5	2.5	4.75	4.75

(a) The maximum is 4.75 and the minimum is –3.5.

(b) The mean is $\dfrac{-3.5 - 1.25 + 1.5 + 1.5 + 2.5 + 4.75 + 4.75}{7} \approx 1.46$ and the median is 1.5.

7. $\sqrt{15} \approx 3.87$, $2^{2.3} \approx 4.92$, $\sqrt[3]{69} \approx 4.102$, $\pi^2 \approx 9.87$, $2^{\pi} \approx 8.82$, 4.1

$\sqrt{15}$	4.1	$\sqrt[3]{69}$	$2^{2.3}$	2^{π}	π^2

(a) The maximum is π^2 and the minimum is $\sqrt{15}$.

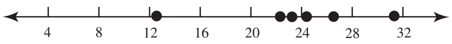
(b) The mean is $\dfrac{\sqrt{15} + 4.1 + \sqrt[3]{69} + 2^{2.3} + 2^{\pi} + \pi^2}{6} \approx 5.95$ and the median is $\dfrac{\sqrt[3]{69} + 2^{2.3}}{2} \approx 4.51$.

8. $\dfrac{22}{7} \approx 3.1429$, 3.14, $\sqrt[3]{28} \approx 3.04$, $\sqrt{9.4} \approx 3.07$, $4^{0.9} \approx 3.48$, $3^{1.2} \approx 3.74$

$\sqrt[3]{28}$	$\sqrt{9.4}$	3.14	$\frac{22}{7}$	$4^{0.9}$	$3^{1.2}$

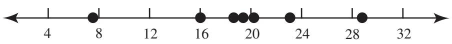
(a) The maximum is $3^{1.2}$ and the minimum is $\sqrt[3]{28}$.

(b) The mean is $\dfrac{\sqrt[3]{28} + \sqrt{9.4} + 3.14 + \frac{22}{7} + 4^{0.9} + 3^{1.2}}{6} \approx 3.27$ and the median is $\dfrac{3.14 + \frac{22}{7}}{2} \approx 3.14$.

9. (a) 

(b) Mean $= \dfrac{31.7 + 22.3 + 12.3 + 26.8 + 24.9 + 23.0}{6} = 23.5$; Median $= \dfrac{23.0 + 24.9}{2} = 23.95$. The average area of the six largest freshwater lakes is 23,500 square miles. Half of the lakes have areas larger than 23,950 square miles and half have less. The largest difference in area between any two lakes is 19,400 square miles.

(c) The freshwater lake with the largest area is Lake Superior.

10. (a) 

(b) Mean $= \dfrac{19.3 + 18.5 + 29.0 + 7.31 + 16.1 + 22.8 + 20.3}{7} \approx 19.0$; Median $= 19.3$.

The average maximum elevation of the seven continents is about 19,000 feet. About half of these continents have maximum elevations below 19,300 feet and about half are above. The largest difference between these elevations is about 21,700 feet.

(c) The mountain with the highest elevation is Mount Everest in Asia.

11. *Answers may vary.* 16, 18, 26; No

12. *Answers may vary.* 3, 5, 9, 15, 18; No

13. (a) $S = \{(-1, 5), (2, 2), (3, -1), (5, -4), (9, -5)\}$

    (b) $D = \{-1, 2, 3, 5, 9\}$

    $R = \{-5, -4, -1, 2, 5\}$

14. (a) $S = \{(-2, -4), (0, -2), (2, -1), (4, 0), (6, 4)\}$

    (b) $D = \{-2, 0, 2, 4, 6\}$

    $R = \{-4, -2, -1, 0, 4\}$

15. (a) $S = \{(1, 5), (4, 5), (5, 6), (4, 6), (1, 5)\}$

    (b) $D = \{1, 4, 5\}$

    $R = \{5, 6\}$

16. (a) $S = \left\{\left(-1, \frac{1}{2}\right), (0, 1), \left(3, \frac{3}{4}\right), (-1, 3), \left(-2, -\frac{5}{6}\right)\right\}$

    (b) $D = \{-2, -1, 0, 3\}$

    $R = \left\{-\frac{5}{6}, \frac{1}{2}, \frac{3}{4}, 1, 3\right\}$

17. (a) The domain is $D = \{1, 3, -5, 8, 0\}$ and the range is $R = \{1, 0, -5, -2, 3\}$.

    (b) The minimum x-value is –5, and the maximum x-value is 8. The minimum y-value is –5, and the maximum y-value is 3.

    (c) The axes must include at least $-5 \le x \le 8$ and $-5 \le y \le 3$. Extend each axis slightly beyond these intervals and let each tick mark represent 2 units. Plot the points $(1, 1)$, $(3, 0)$, $(-5, -5)$, $(8, -2)$, and $(0, 3)$.

    (d) See Figure 17.

18. (a) The domain is $D = \{1, 3, -5, 8, 0\}$ and the range is $R = \{1, 0, -5, -2, 3\}$.

    (b) The minimum x-value is –5, and the maximum x-value is 8. The minimum y-value is –5, and the maximum y-value is 3.

    (c) The axes must include at least $-5 \le x \le 8$ and $-5 \le y \le 3$. Extend each axis slightly beyond these intervals and let each tick mark represent 2 units. Plot the points $(1, 1)$, $(3, 0)$, $(-5, -5)$, $(8, -2)$, and $(0, 3)$.

    (d) See Figure 18.

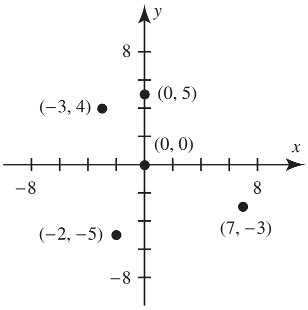


Figure 17

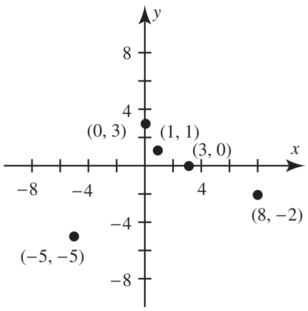


Figure 18

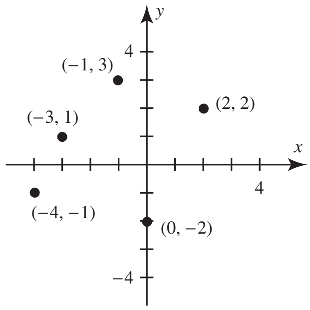


Figure 19

19. (a) The domain is $D = \{2, -3, -4, -1, 0\}$ and the range is $R = \{2, 1, -1, 3, -2\}$.

(b) The minimum x-value is -4, and the maximum x-value is 2. The minimum y-value is -2, and the maximum y-value is 3.

(c) The axes must include at least $-4 \le x \le 2$ and $-2 \le y \le 3$. Extend each axis slightly beyond these intervals and let each tick mark represent 1 unit. Plot the points $(2, 2)$, $(-3, 1)$, $(-4, -1)$, $(-1, 3)$, and $(0, -2)$.

(d) See Figure 19.

20. (a) The domain is $D = \{1, 2, -1\}$ and the range is $R = \{1, -3, -1, 2, 0\}$.

(b) The minimum x-value is -1, and the maximum x-value is 2. The minimum y-value is -3, and the maximum y-value is 2.

(c) The axes must include at least $-1 \le x \le 2$ and $-3 \le y \le 2$. Extend each axis slightly beyond these intervals and let each tick mark represent 1 unit. Plot the points $(1, 1)$, $(2, -3)$, $(-1, -1)$, $(-1, 2)$, and $(-1, 0)$.

(d) See Figure 20.

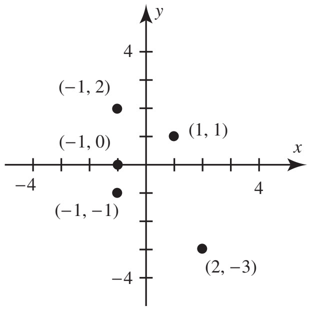


Figure 20

21. (a) The domain is $D = \{10, -35, 0, 75, -25\}$ and the range is $R = \{50, 45, -55, 25, -25\}$

(b) The minimum x-value is -35, and the maximum x-value is 75. The minimum y-value is -55, and the maximum y-value is 50.

(c) The axes must include at least $-35 \le x \le 75$ and $-55 \le y \le 50$, and let each tick mark represent 25 units. It would be appropriate to extend the axes slightly beyond these intervals. Plot the points $(10, 50)$, $(-35, 45)$, $(0, -55)$, $(75, 25)$, and $(-25, -25)$.

(d) See Figure 21.

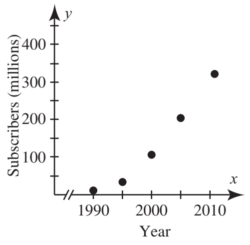


Figure 21

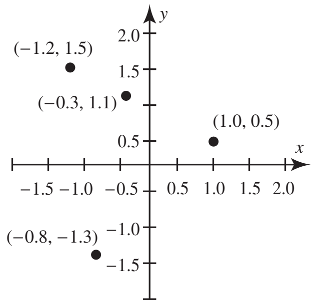


Figure 22

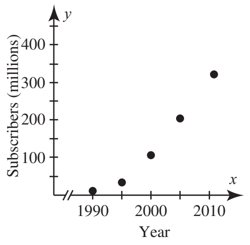


Figure 23a

22. (a) The domain is $D = \{-1.2, 1.0, -0.3, -0.8\}$ and the range is $R = \{1.5, 0.5, 1.1, -1.3\}$

(b) The minimum x-value is -1.2, and the maximum x-value is 1.0. The minimum y-value is -1.3, and the maximum y-value is 1.5.

(c) The axes must include at least $-1.2 \le x \le 1.0$ and $-1.3 \le y \le 1.5$beyond these intervals and let each tick mark represent 0.5 unit. It would be appropriate to extend the axes slightly beyond these intervals. Plot the points $(-1.2, 1.5)$, $(1.0, 0.5)$, $(-0.3, 1.1)$, and $(-0.8, -1.3)$.

(d) See Figure 22.

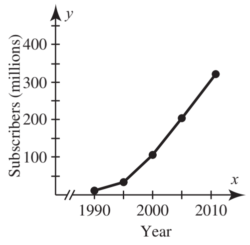


Figure 23b

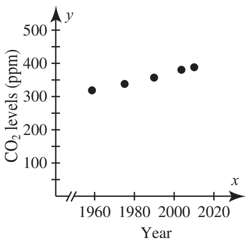


Figure 24a

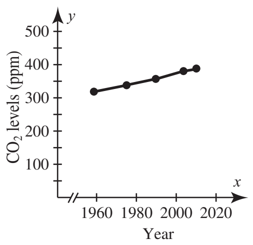


Figure 24b

23. See Figures 23a and 23b

24. See Figures 24a and 24b

25. $d = \sqrt{(5-2)^2 + (2-(-2))^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

26. $d = \sqrt{(12-0)^2 + (-8-(-3))^2} = \sqrt{12^2 + (-5)^2} = \sqrt{169} = 13$

27. $d = \sqrt{(9-7)^2 + (1-(-4))^2} = \sqrt{2^2 + 5^2} = \sqrt{29} \approx 5.39$

28. $d = \sqrt{(-8-(-1))^2 + (-5-(-6))^2} = \sqrt{(-7)^2 + 1^2} = \sqrt{50} \approx 7.07$

29. $d = \sqrt{(-2.1-3.6)^2 + (8.7-5.7)^2} = \sqrt{(-5.7)^2 + 3^2} = \sqrt{41.49} \approx 6.44$

30. $d = \sqrt{(3.6-(-6.5))^2 + (-2.9-2.7)^2} = \sqrt{10.1^2 + (-5.6)^2} = \sqrt{133.37} \approx 11.55$

31. $d = \sqrt{(-3-(-3))^2 + (10-2)^2} = \sqrt{0^2 + 8^2} = \sqrt{64} = 8$

32. $d = \sqrt{(-1-7)^2 + (9-9)^2} = \sqrt{(-8)^2 + 0^2} = \sqrt{64} = 8$

33. $d = \sqrt{\left(\frac{3}{4}-\frac{1}{2}\right)^2 + \left(\frac{1}{2}-\left(-\frac{1}{2}\right)\right)^2} = \sqrt{\left(\frac{1}{4}\right)^2 + 1^2} = \sqrt{\frac{1}{16}+1} = \sqrt{\frac{17}{16}} = \frac{\sqrt{17}}{4} \approx 1.03$

34. $d = \sqrt{\left(\frac{1}{3}-\left(-\frac{1}{3}\right)\right)^2 + \left(-\frac{4}{3}-\frac{2}{3}\right)^2} = \sqrt{\left(\frac{2}{3}\right)^2 + (-2)^2} = \sqrt{\frac{4}{9}+4} = \sqrt{\frac{40}{9}} = \frac{\sqrt{40}}{3} \approx 2.11$

35. $d = \sqrt{\left(-\frac{1}{10}-\frac{2}{5}\right)^2 + \left(\frac{4}{5}-\frac{3}{10}\right)^2} = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{1}{4}+\frac{1}{4}} = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2} = 0.71$

36. $d = \sqrt{\left(\frac{1}{3}-\left(-\frac{1}{2}\right)\right)^2 + \left(-\frac{5}{2}-\frac{2}{3}\right)^2} = \sqrt{\left(\frac{5}{6}\right)^2 + \left(-\frac{19}{6}\right)^2} = \sqrt{\frac{25}{36}+\frac{361}{36}} = \sqrt{\frac{386}{36}} = \frac{\sqrt{386}}{6} = 3.27$

37. $d = \sqrt{(-30-20)^2 + (-90-30)^2} = \sqrt{(-50)^2 + (-120)^2} = \sqrt{2500 + 14{,}400} = \sqrt{16{,}900} = 130$

38 $d = \sqrt{(-20-40)^2 + (17-6)^2} = \sqrt{(-60)^2 + (11)^2} = \sqrt{3600 + 121} = \sqrt{3721} = 61$

39. $d = \sqrt{(0-a)^2 + (-b-0)^2} = \sqrt{(-a)^2 + (-b)^2} = \sqrt{a^2 + b^2}$

40. $d = \sqrt{(x-1)^2 + (y-2)^2}$

41. $d = \sqrt{(3-0)^2 + (4-0)^2} = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$

$d = \sqrt{(7-3)^2 + (1-4)^2} = \sqrt{4^2 + (-3)^2} = \sqrt{16+9} = \sqrt{25} = 5$

The side between (0,0) and (3,4) and the side between (3,4) and (7,1) have equal length, so the triangle is isosceles.

42. $d = \sqrt{(2-(-1))^2 + (3-(-1))^2} = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$

$d = \sqrt{(-4-2)^2 + (3-3)^2} = \sqrt{(-6)^2 + 0^2} = \sqrt{36} = 6$

Since two of the sides have different lengths, the triangle cannot be an equilateral triangle.

43. (a) See Figure 43.

(b) $d = \sqrt{(0-(-40))^2 + (50-0)^2} = \sqrt{40^2 + 50^2} = \sqrt{1600 + 2500} = \sqrt{4100} \approx 64.0$ miles.

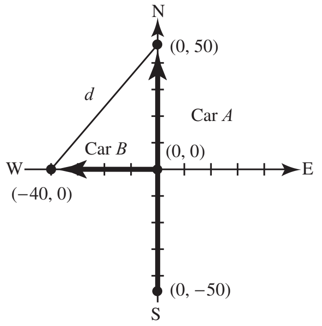


Figure 43

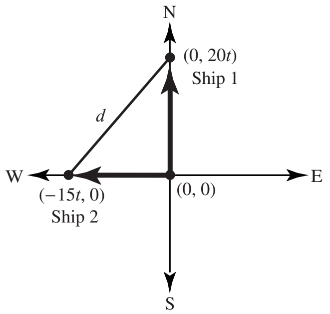


Figure 44

44. $d = \sqrt{(0 - (-15t))^2 + (20t - 0)^2} = \sqrt{(15t)^2 + (20t)^2} = \sqrt{225t^2 + 400t^2} = \sqrt{625t^2} = 25t$

See Figure 44.

45. Use the midpoint formula $M = \left(\frac{24 + 6}{2}, \frac{44 + 10}{2}\right) = (15{,}27)$. According to the midpoint estimate the number of Nintendo Wii units sold after 15 months was 27 million units.

46. Use the midpoint formula $M = \left(\frac{2050 + 1874}{2}, \frac{9 + 1.24}{2}\right) = (1962, 5.12)$. According to the midpoint estimate the world population in 1962 was 5.12 billion.

47. Assuming the distance of 0 meters requires 0 seconds to run, the midpoint between the data points (0, 0) and (200, 20) is equal to $M = \left(\frac{0 + 200}{2}, \frac{0 + 20}{2}\right) = (100, 10)$. According to the midpoint estimate the time required to run 100 meters is 10 seconds or half the time required to run 200 meters.

48. Use the midpoint formula to compute the value of $\frac{a + b}{2}$.

49. $M = \left(\frac{1 + 5}{2}, \frac{2 + (-3)}{2}\right) = (3, -0.5)$

50. $M = \left(\frac{-6 + 9}{2}, \frac{7 + (-4)}{2}\right) = (1.5, 1.5)$

51. $M = \left(\frac{-30 + 50}{2}, \frac{50 + (-30)}{2}\right) = (10, 10)$

52. $M = \left(\frac{28 + 52}{2}, \frac{-33 + 38}{2}\right) = (40, 2.5)$

53. $M = \left(\frac{1.5 + (-5.7)}{2}, \frac{2.9 + (-3.6)}{2}\right) = (-2.1, -0.35)$

54. $M = \left(\frac{9.4 + (-7.7)}{2}, \frac{-4.5 + 9.5}{2}\right) = (0.85, 2.5)$

55. $M = \left(\frac{\sqrt{2} + \sqrt{2}}{2}, \frac{\sqrt{5} + (-\sqrt{5})}{2}\right) = (\sqrt{2}, 0)$

56. $M = \left(\frac{\sqrt{7} + (-\sqrt{7}\,)}{2}, \frac{3\sqrt{3} + (-\sqrt{3})}{2}\right) = (0, \sqrt{3})$

57. $M = \left(\frac{a + (-a)}{2}, \frac{b + 3b}{2}\right) = (0, 2b)$

58 $M = \left(\frac{-a + 3a}{2}, \frac{b + b}{2}\right) = (a, b)$

59. $x^2 + y^2 = 25 \Rightarrow (x - 0)^2 + (y - 0)^2 = 5^2 \Rightarrow$ Center: $(0, 0)$; Radius: 5

60. $x^2 + y^2 = 100 \Rightarrow (x - 0)^2 + (y - 0)^2 = 10^2 \Rightarrow$ Center: $(0, 0)$; Radius: 10

61. $x^2 + y^2 = 7 \Rightarrow (x - 0)^2 + (y - 0)^2 = (\sqrt{7}\,)^2 \Rightarrow$ Center: $(0, 0)$; Radius: $\sqrt{7}$

62. $x^2 + y^2 = 20 \Rightarrow (x - 0)^2 + (y - 0)^2 = (\sqrt{20})^2 \Rightarrow$ Center: $(0, 0)$; Radius: $\sqrt{20} = 2\sqrt{5}$

63. $(x - 2)^2 + (y + 3)^2 = 9 \Rightarrow (x - 2)^2 + (y - (-3))^2 = 3^2 \Rightarrow$ Center: $(2, -3)$; Radius: 3

64. $(x + 1)^2 + (y - 1)^2 = 16 \Rightarrow (x - (-1))^2 + (y - 1)^2 = 4^2 \Rightarrow$ Center: $(-1, 1)$; Radius: 4

65. $x^2 + (y + 1)^2 = 100 \Rightarrow (x - 0)^2 + (y - (-1))^2 = 10^2 \Rightarrow$ Center: $(0, -1)$; Radius: 10

66. $(x-5)^2+y^2=19 \Rightarrow (x-5)^2+(y-0)^2=(\sqrt{19})^2 \Rightarrow$ Center: $(5,0)$; Radius: $\sqrt{19}$

67. Since the center is (1, –2) and the radius is 1, the equation is $(x-1)^2+(y+2)^2=1$.

68. Since the center is (0, 0) and the radius is 3, the equation is $x^2+y^2=9$.

69. Since the center is (–2, 1) and the radius is 2, the equation is $(x+2)^2+(y-1)^2=4$.

70. Since the center is (–2, 0) and the radius is 4, the equation is .

71. $(x-3)^2+(y-(-5))^2=8^2 \Rightarrow (x-3)^2+(y+5)^2=64$

72. $(x-(-1))^2+(y-4)^2=5^2 \Rightarrow (x+1)^2+(y-4)^2=25$

73. $(x-3)^2+(y-0)^2=7^2 \Rightarrow (x-3)^2+y^2=49$

74. $(x-0)^2+(y-0)^2=1^2 \Rightarrow x^2+y^2=1$

75. First find the radius using the distance formula: $r=\sqrt{(4-3)^2+(2-(-5))^2}=\sqrt{50}$.

    $(x-3)^2+(y-(-5))^2=(\sqrt{50})^2 \Rightarrow (x-3)^2+(y+5)^2=50$

76. First find the radius using the distance formula: $r=\sqrt{(-3-0)^2+(-1-0)^2}=\sqrt{10}$.

    $(x-0)^2+(y-0)^2=(\sqrt{10})^2 \Rightarrow x^2+y^2=10$

77. First find the center using the midpoint formula: $C=\left(\frac{-5+1}{2},\frac{-7+1}{2}\right)=(-2,-3)$

    Then find the radius using the distance formula: $r=\sqrt{(-2-1)^2+(-3-1)^2}=\sqrt{25}=5$.

    $(x-(-2))^2+(y-(-3))^2=5^2 \Rightarrow (x+2)^2+(y+3)^2=25$

78. First find the center using the midpoint formula: $C=\left(\frac{-3+1}{2},\frac{-2+(-4)}{2}\right)=(-1,-3)$

    Then find the radius using the distance formula: $r=\sqrt{(-1-1)^2+(-3-(-4))^2}=\sqrt{5}$.

    $(x-(-1))^2+(y-(-3))^2=(\sqrt{5})^2 \Rightarrow (x+1)^2+(y+3)^2=5$

79. Find the center using the midpoint formula: $C=\left(\frac{5+2}{2},\frac{5+1}{2}\right)=\left(\frac{7}{2},3\right)$. Then find the radius using the

    distance formula: $r=\sqrt{\left(\frac{7}{2}-2\right)^2+(3-1)^2}=\sqrt{\frac{25}{4}}=\frac{5}{2}$.

    $\left(x-\frac{7}{2}\right)^2+(y-3)^2=\left(\frac{5}{2}\right)^2 \Rightarrow \left(x-\frac{7}{2}\right)^2+(y-3)^2=\frac{25}{4}$

80. Find the center using the midpoint formula: $C=\left(\frac{-2+6}{2},\frac{1+(-3)}{2}\right)=(2,-1)$. Then find the radius using

    the distance formula: $r=\sqrt{(2-(-2))^2+(-1-1)^2}=\sqrt{20}$.

    $(x-2)^2+\left(1-(-1)\right)^2=\left(\sqrt{20}\right)^2 \Rightarrow (x-2)^2+(y+1)^2=20$

81. See Figure 81.

82. See Figure 82.

83. See Figure 83.

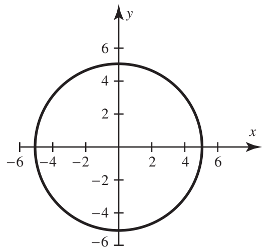


Figure 81

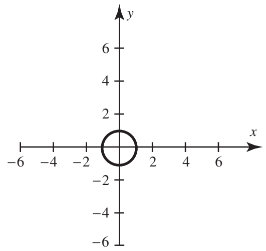


Figure 82

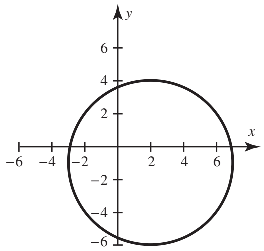


Figure 83

84. See Figure 84.
85. See Figure 85.
86. See Figure 86.

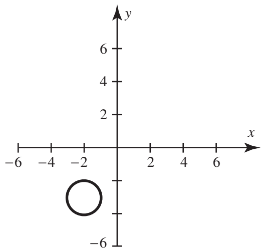


Figure 84

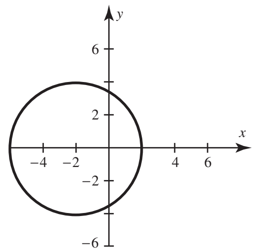


Figure 85

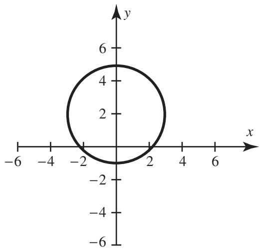


Figure 86

87. See Figure 87.
88. See Figure 88.
89. See Figure 89.

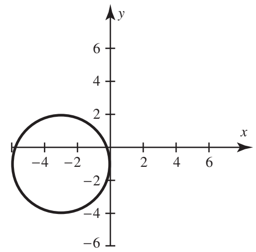


Figure 87

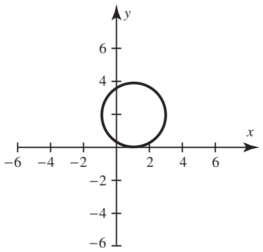


Figure 88

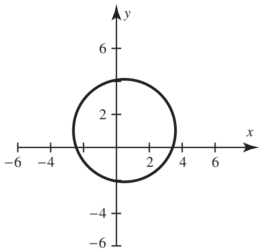


Figure 89

90. See Figure 90.

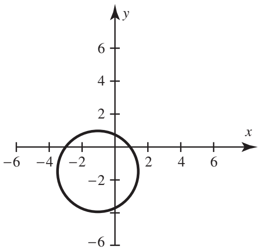


Figure 90

91. x-axis: 10 tick marks; y-axis: 10 tick marks. See Figure 91.

92. x-axis: 4 tick marks; y-axis: 3 tick marks. See Figure 92.

[–10, 10, 1] by [–10, 10, 1]

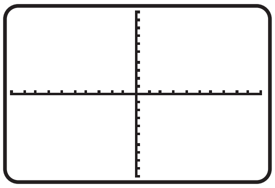


Figure 91

[–4.7, 4.7, 1] by [–3.1, 3.1, 1]

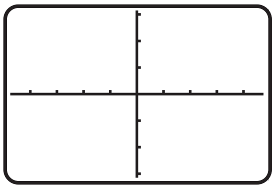


Figure 92

[0, 100, 10] by [–50, 50, 10]

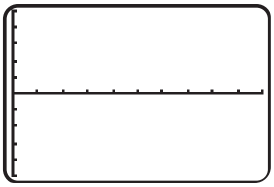


Figure 93

[–30, 30, 5] by [–20, 20, 5]

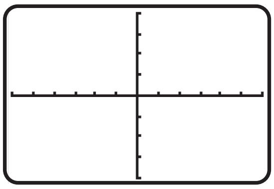


Figure 94

93. x-axis: 10 tick marks; y-axis: 5 tick marks. See Figure 93.

94. x-axis: 6 tick marks; y-axis: 4 tick marks. See Figure 94.

95. x-axis: 16 tick marks; y-axis: 5 tick marks. See Figure 95.

96. x-axis: 11 tick marks; y-axis: 4 tick marks. See Figure 96.

[1980, 1995, 1] by [12000, 16000, 1000]

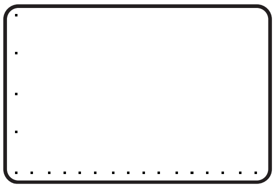


Figure 95

[1800, 2000, 20] by [5, 20, 5]

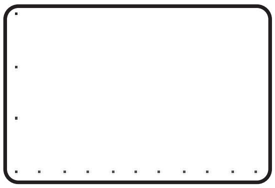


Figure 96

97. Graph b

98. Graph d

99. Graph a

100. Graph c

101. Plot the points $(1, 3)$, $(-2, 2)$, $(-4, 1)$, $(-2, -4)$ and $(0, 2)$ in $[-5, 5, 1]$ by $[-5, 5, 1]$. See Figure 101.

102. Plot the points $(6, 8)$, $(-4, -10)$, $(-2, -6)$, and $(2, -5)$ in $[-7, 7, 1]$ by $[-11, 11, 1]$. See Figure 102.

[–5, 5, 1] by [–5, 5, 1]

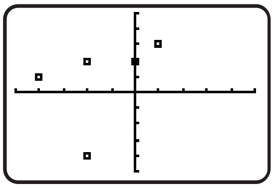


Figure 101

[–7, 7, 1] by [–11, 11, 1]

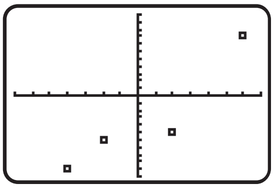


Figure 102

[–100, 100, 10] by [–100, 100, 10]

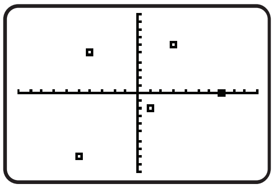


Figure 103

[–3, 3, 0.5] by [–2, 2, 0.5]

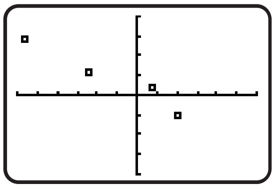


Figure 104

103. Plot the points $(10, -20)$, $(-40, 50)$, $(30, 60)$, $(-50, -80)$, and $(70, 0)$ in $[-100, 100, 10]$ by $[-100, 100, 10]$. See Figure 103.

104. Plot the points $(-1.2, 0.6)$, $(1.0, -0.5)$, $(0.4, 0.2)$, and $(-2.8,\ 1.4)$ in $[-3, 3, 0.5]$ by $[-2, 2, 0.5]$, See Figure 104.

105. (a) The maximum number of Netflix subscriptions is 19.4 million, and the minimum number is 6.1 million.

(b) x-min: 2006; x-max: 2010; y-min: 6; y-max: 19.5. [2005, 2001, 1] by [5, 20, 5]. Answers may vary.

(c) See Figure 105c

(d) See Figure 105d

106. (a) The maximum vehicle sales that are electric/hybrid is 8%, and the minimum number is 0%.

(b) x-min: 2011; x-max: 2025; y-min: 0; y-max: 8. [2005, 2030, 5] by [−2, 10, 2]. Answers may vary.

(c) See Figure 106c

(d) See Figure 106d

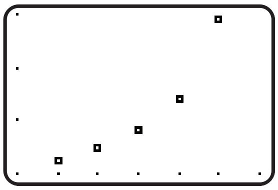


Figure 105c

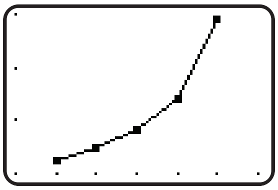


Figure 105d

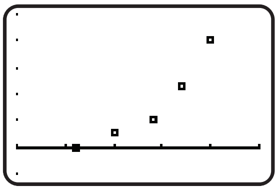


Figure 106c

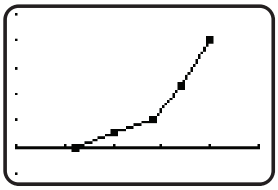


Figure 106d

107. (a) The maximum Myspace U.S. advertising revenue is \$590 million, and the minimum is \$180 million.

(b) x-min: 2006; x-max: 2011; y-min: 180; y-max: 590. [2005, 2012, 1] by [150, 600, 50]. Answers may vary.

(c) See Figure 107c

(d) See Figure 107d

108. (a) The maximum U.S. college enrollments who study Chinese is 60 thousand, and the minimum number is 26 thousand.

(b) x-min: 1995; x-max: 2009; y-min: 26; y-max: 60. [1990, 2015, 5] by [20, 70, 10]. Answers may vary.

(c) See Figure 108c

(d) See Figure 108d

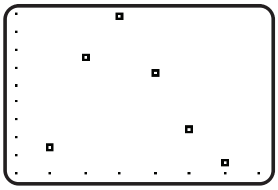


Figure 107c

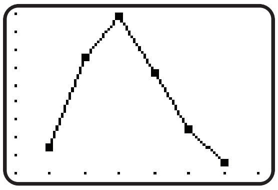


Figure 107d

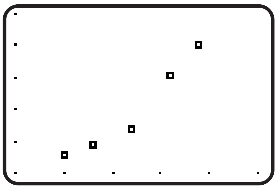


Figure 108c

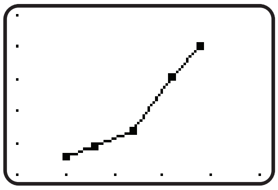


Figure 108d

Checking Basic Concepts for Sections 1.1 and 1.2

1. (a) $\sqrt{4.2(23.1 + 0.5^3)} \approx 9.88$

   (b) $\dfrac{23 + 44}{85.1 - 32.9} \approx 1.28$

2. (a) $5 - (-4)^2 \cdot 3 = 5 - 16 \cdot 3 = 5 - 48 = -43$

   (b) $5 \div 5\sqrt{2 + 2} = 5 \div 5\sqrt{4} = 5 \div (5)(2) = 1 \cdot 2 = 2$

3. (a) $348{,}500{,}000 = 3.485 \times 10^8$

   (b) $-1237.4 = -1.2374 \times 10^3$

   (c) $0.00198 = 1.98 \times 10^{-3}$

4. $d = \sqrt{(3 - (-3))^2 + (-5 - 1)^2} = \sqrt{6^2 + (-6)^2} = \sqrt{36 + 36} = \sqrt{36 \times 2} = 6\sqrt{2} \approx 8.49$

5. $M = \left(\dfrac{-2 + 4}{2}, \dfrac{3 + 2}{2}\right) = \left(1, \dfrac{5}{2}\right)$

6. $(x - h)^2 + (y - k)^2 = r^2 \Rightarrow (x - (-4))^2 + (y - 5)^2 = 8^2 \Rightarrow (x + 4)^2 + (y - 5)^2 = 64$

7. Mean $= \dfrac{13{,}215 + 12{,}881 + 13{,}002 + 3953}{4} = 10{,}762.75$; Median $= \dfrac{12{,}881 + 13{,}002}{2} = 12{,}941.5$. The mean average depth of the four oceans is 10,762.75 feet. Half of the oceans have average depths of more than 12,941.5 feet and half have average depths of less than 12.941.5 feet. The largest difference in average depths between any two oceans is 9262 feet.

8. $(-5, -4)$: Quad. III; $(-1, 2)$: Quad. II; $(2, -2)$: Quad. IV; $(3, 6)$: Quad. I

   See Figures 8a and 8b.

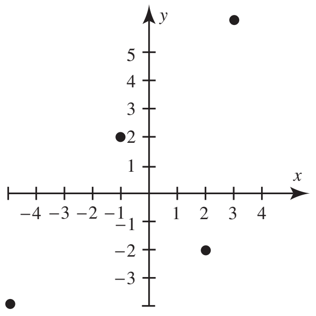


Figure 8a

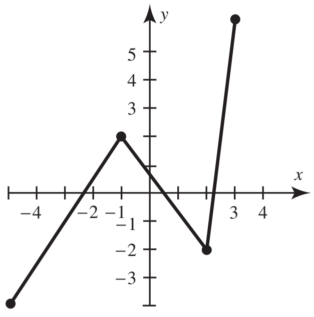


Figure 8b

1.3: Functions and Their Representations

1. If $f(-2) = 3$, then the point (–2, 3) is on the graph of *f*.
2. If $f(3) = -9.7$, then the point (3, –9.7) is on the graph of *f*.
3. If (7, 8) is on the graph of *f*, then $f(7) = 8$.
4. If (–3, 2) is on the graph of *f*, then $f(-3) = 2$.
5. See Figure 5.

6. See Figure 6.

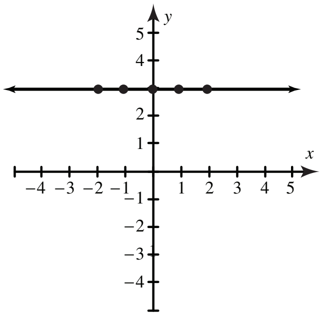


Figure 5

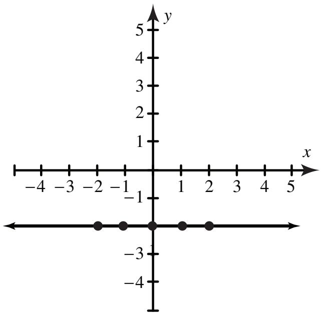


Figure 6

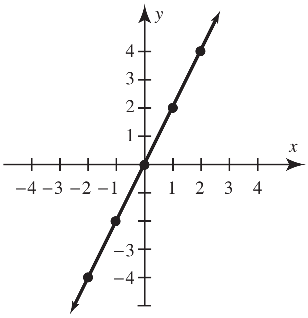


Figure 7

7. See Figure 7.

8. See Figure 8.

9. See Figure 9.

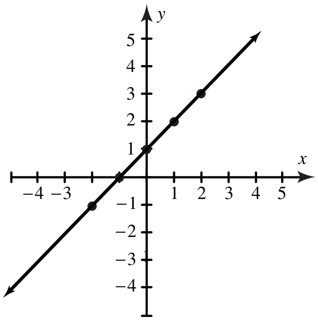
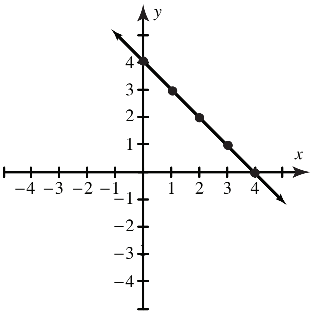


Figure 8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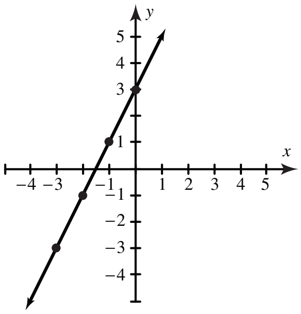


Figure 10

10. See Figure 10.

11. See Figure 11.

12. See Figure 12.

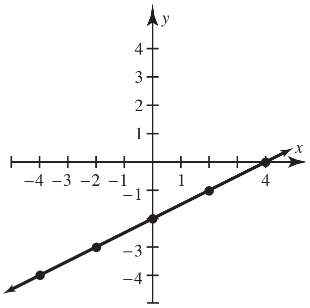


Figure 11

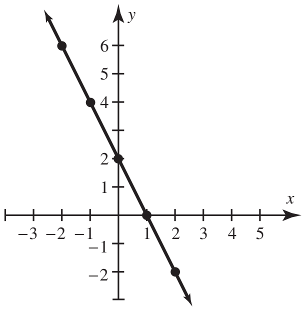


Figure 12

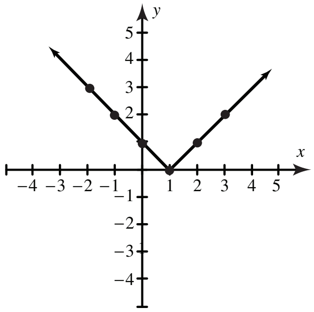


Figure 13

13. See Figure 13.

14. See Figure 14.

15. See Figure 15.

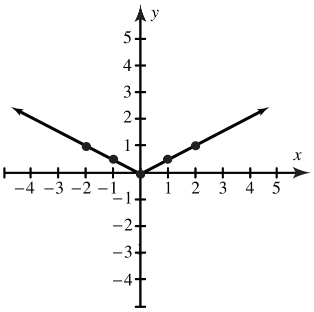


Figure 14

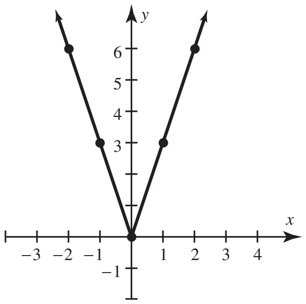


Figure 15

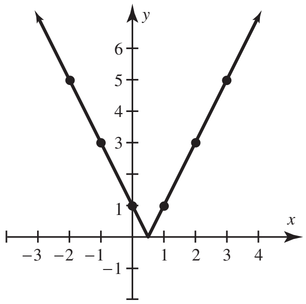


Figure 16

16. See Figure 16.

17. See Figure 17.

18. See Figure 18.

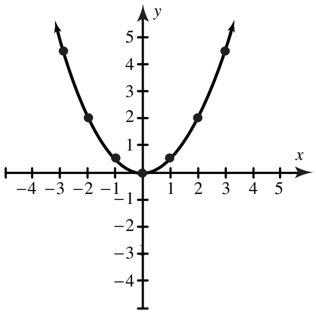


Figure 17

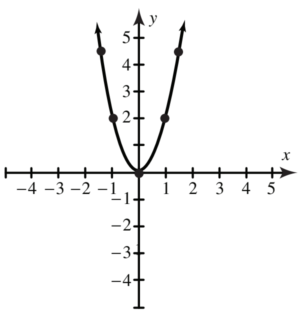


Figure 18

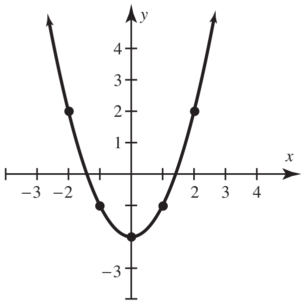


Figure 19

19. See Figure 19.

20. See Figure 20.

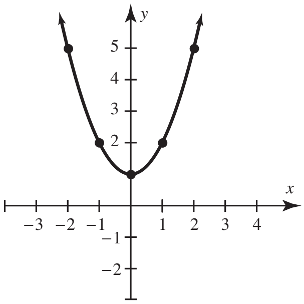


Figure 20

21. (a) $g = \{(-1, 0), (2, -2), (5, 7)\}$

    (b) $D = \{-1, 2, 5\}, R = \{-2, 0, 7\}$

22. (a) $g = \{(-2, 5), (3, 9), (4, -4)\}$

    (b) $D = \{-2, 3, 4\}, R = \{-4, 5, 9\}$

23. (a) $g = \{(1, 8), (2, 8), (3, 8)\}$

    (b) $D = \{1, 2, 3\}, R = \{8\}$

24. (a) $g = \{(-5, 0), (0, -5), (5, 0)\}$

(b) $D = \{-5, 0, 5\}, R = \{-5, 0\}$

25. (a) $g = \{(-1, 2), (0, 4), (1, -3), (2, 2)\}$

(b) $D = \{-1, 0, 1, 2\};\ R = \{-3, 2, 4\}$

26. (a) $g = \{(-4, 5), (0, -5), (4, 5), (8, 0)\}$

(b) $D = \{-4, 0, 4, 8\};\ R = \{-5, 0, 5\}$

27. (a) $f(x) = x^3 \Rightarrow f(-2) = (-2)^3 = -8$ and $f(5) = 5^3 = 125$.

(b) All real numbers.

28. (a) $f(x) = 2x - 1 \Rightarrow f(8) = 2(8) - 1 = 15$ and $f(-1) = 2(-1) - 1 = -3$.

(b) All real numbers.

29. (a) $f(x) = \sqrt{x} \Rightarrow f(-1) = \sqrt{-1}$ which is not a real number, and $f(a + 1) = \sqrt{a + 1}$.

(b) All non-negative real numbers.

30. (a) $f(x) = \sqrt{1 - x} \Rightarrow f(-2) = \sqrt{1 - (-2)} = \sqrt{3}$ and $f(a + 2) = \sqrt{1 - (a + 2)} = \sqrt{-1 - a}$.

(b) All real numbers less than or equal to 1 $(x \leq 1)$.

31. (a) $f(x) = 6 - 3x \Rightarrow f(-1) = 6 - 3(-1) = 6 + 3 = 9$ and

$f(a + 1) = 6 - 3(a + 1) = 6 - 3a - 3 = 3 - 3a$

(b) All real numbers.

32. (a) $f(x) = -7 \Rightarrow f(6) = -7$ and $f(a - 1) = -7$. (f is a constant function.)

(b) All real numbers.

33. (a) $f(x) = \dfrac{3x - 5}{x + 5} \Rightarrow f(-1) = \dfrac{3(-1) - 5}{-1 + 5} = -\dfrac{8}{4} = -2$ and $f(a) = \dfrac{3a - 5}{a + 5}$.

(b) All real numbers not equal to –5 $(x \neq -5)$.

34. (a) $f(x) = x^2 - x + 1 \Rightarrow f(1) = 1^2 - 1 + 1 = 1$ and $f(-2) = (-2)^2 - (-2) + 1 = 7$.

(b) All real numbers.

35. (a) $f(x) = \dfrac{1}{x^2} \Rightarrow f(4) = \dfrac{1}{4^2} = \dfrac{1}{16}$ and $f(-7) = \dfrac{1}{(-7)^2} = \dfrac{1}{49}$.

(b) All real numbers not equal to 0 $(x \neq 0)$.

36. (a) $f(x) = \dfrac{1}{1 - 9} \Rightarrow f(4) = \dfrac{1}{4 - 9} = -\dfrac{1}{5}$ and $f(a + 9) = \dfrac{1}{(a + 9) - 9} = \dfrac{1}{a}$

(b) All real numbers not equal to 9 $(x \neq 9)$.

37. (a) Domain and Range = All real numbers.

(b) $g(x) = 2x - 1 \Rightarrow g(-1) = 2(-1) - 1 = -2 - 1 = -3$ and $g(2) = 2(2) = 2(2) - 1 = 4 - 1 = 3$

(c) $g(-1) = 3$ and $g(2) = 3$

38. (a) Domain and Range = All real numbers.

(b) $g(x) = -\dfrac{1}{2}x - 1 \Rightarrow g(-1) = -\dfrac{1}{2}(-1) - 1 = -\dfrac{1}{2}$ and $g(2) = -\dfrac{1}{2}(2) - 1 = -1 - 1 = -2$

(c) $g(-1) = -\dfrac{1}{2}$ and $g(2) = -2$

39. (a) $D =$ All real numbers; $R = \{y \mid y \leq 2\}$

    (b) $g(x) = 2 - x^2 \Rightarrow g(-1) = 2 - (-1)^2 = 2 - 1 = 1$ and $g(2) = 2 - (2)^2 = 2 - 4 = -2$

    (c) $g(-1) = 1$ and $g(2) = -2$

40. (a) $D =$ All real numbers; $R = \{y \mid y \leq 3\}$

    (b) $g(x) = 3 - 2|x| \Rightarrow g(-1) = 3 - 2|-1| = 3 - 2 = 1$ and $g(2) = 3 - 2|2| = 3 - 4 = -1$

    (c) $g(-1) = 1$ and $g(2) = -1$

41. (a) $D = \{x \mid -2 \leq x \leq 2\}$; $R = \{-3 \leq y \leq 1\}$

    (b) $g(x) = x^2 - 3 \Rightarrow g(-1) = (-1)^2 - 3 = 1 - 3 = -2$ and $g(2) = (2)^2 - 3 = 4 - 3 = 1$

    (c) $g(-1) = -2$ and $g(2) = 1$

42. (a) $D = \{x \mid -2 \leq x \leq 2\}$; $R = \{-3 \leq y \leq 3\}$

    (b) $g(x) = x^3 - 4x \Rightarrow g(-1) = (-1)^3 - 4(-1) = -1 + 4 = 3$ and $g(2) = (2)^3 - 4(2) = 8 - 8 = 0$

    (c) $g(-1) = 3$ and $g(2) = 0$

43. $D = \{x \mid -3 \leq x \leq 3\}$

    $R = \{y \mid 0 \leq y \leq 3\}$

    $f(0) = 3.$

44. $D = \{x \mid -2 \leq x \leq 2\}$

    $R = \{y \mid -3 \leq y \leq 3\}$

    $f(0) = 0$

45. $D = \{$all real numbers$\}$

    $R = \{y \mid y \leq 2\}$

    $f(0) = 2$

46. $D = \{$all real numbers$\}$

    $R = \{$all real numbers$\}$

    $f(0) = 0$

47. $D = \{x \mid x \geq -1\}$

    $R = \{y \mid y \leq 2\}$

    $f(0) = 0$

48. $D = \{x \mid x \leq 2\}$

    $R = \{y \mid y \leq 3\}$

    $f(0) = 3$

49. (a) $f(2) = 7$

    (b) $f = \{(1, 7), (2, 7), (3, 8)\}$

    (c) $D = \{1, 2, 3\}$; $R = \{7, 8\}$

50. (a) $f(2) = 5$

(b) $f = \{(0, 1), (2, 5), (4, 3)\}$

(c) $D = \{0, 2, 4\};\ R = \{1, 3, 5\}$

51. Graph $f(x) = 0.25x^2$ in $[-4.7, 4.7, 1]$ by $[-3.1, 3.1, 1]$ by letting $Y_1 = 0.25X\text{^}2$. See Figure 51.

(a) From the graph, it appears that $f(2) = 1$.

(b) $f(2) = 0.25(2)^2 = 0.25(4) = 1$

(c) See Figure 51c.

[−4.7, 4.7, 1] by [−3.1, 3.1, 1]

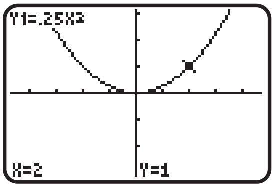
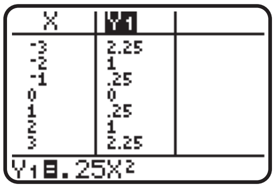


Figure 51



X	Y_1
-3	2.25
-2	1
-1	.25
0	0
1	.25
2	1
3	2.25

$Y_1=.25X^2$

Figure 51c

[−4.7, 4.7, 1] by [−3.1, 3.1, 1]

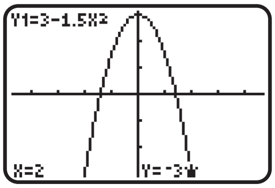
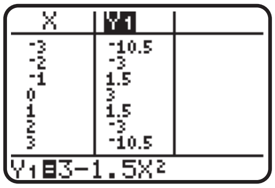


Figure 52



X	Y_1
-3	-10.5
-2	-3
-1	1.5
0	3
1	1.5
2	-3
3	-10.5

$Y_1=3-1.5X^2$

Figure 52c

52. Graph $f(x) = 3 - 1.5x^2$ in $[-4.7, 4.7, 1]$ by $[-3.1, 3.1, 1]$ by letting $Y_1 = 3 - 1.5X\text{^}2$. See Figure 52.

(a) From the graph, it appears that $f(2) = -3$.

(b) $f(2) = 3 - 1.5(2)^2 = 3 - 1.5(4) = 3 - 6 = -3$

(c) See Figure 52c.

53. Graph $f(x) = \sqrt{x + 2}$ in $[-4.7, 4.7, 1]$ by $[-3.1, 3.1, 1]$ by letting $Y_1 = \surd(X + 2)$. See Figure 53.

(a) From the graph, it appears that $f(2) = 2$.

(b) $f(2) = \sqrt{2 + 2} = \sqrt{4} = 2$

(c) See Figure 53c.

[−4.7, 4.7, 1] by [−3.1, 3.1, 1]

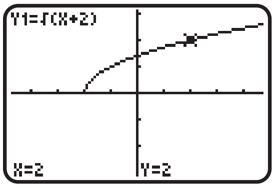
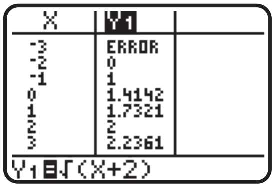


Figure 53



X	Y_1
-3	ERROR
-2	0
-1	1
0	1.4142
1	1.7321
2	2
3	2.2361

$Y_1=\surd(X+2)$

Figure 53c

[−4.7, 4.7, 1] by [−3.1, 3.1, 1]

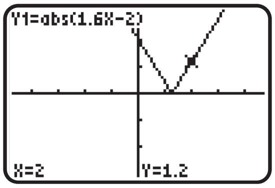
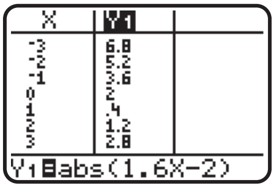


Figure 54



X	Y_1
-3	6.8
-2	5.2
-1	3.6
0	2
1	.4
2	1.2
3	2.8

Y_1=abs(1.6X-2)

Figure 54c

54. Graph $f(x) = |1.6x - 2|$ in $[-4.7, 4.7, 1]$ by $[-3.1, 3.1, 1]$ by letting $Y_1 = \text{abs}(1.6X - 2)$. See Figure 54.

(a) From the graph, it appears that $f(2) = 1.2$.

(b) $f(2) = |1.6(2) - 2| = |3.2 - 2| = |1.2| = 1.2$

(c) See Figure 54c.

55. Verbal: Square the input x.

Graphical: Graph $Y_1 = X\text{^}2$. See Figure 55.

Numerical:

x	-2	-1	0	1	2
y	4	1	0	1	4

$f(2) = 4$

56. Verbal: Multiply the input x by 2 and subtract 5 from the result.

Graphical: Graph $Y_1 = 2X - 5$. See Figure 56.

Numerical:

x	-2	-1	0	1	2
y	-9	-7	-5	-3	-1

$f(2) = -1$

[−10, 10, 1] by [−10, 10, 1]

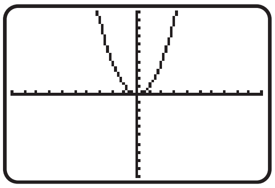


Figure 55

[−10, 10, 1] by [−10, 10, 1]

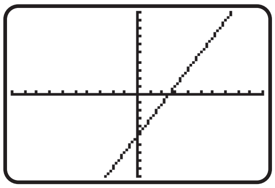


Figure 56

[−6, 6, 1] by [−4, 4, 1]

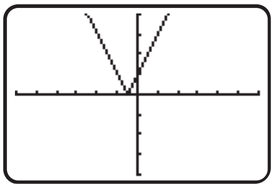


Figure 57

[−10, 10, 1] by [−10, 10, 1]

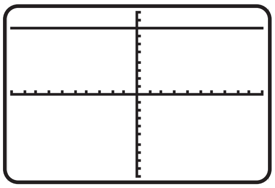


Figure 58

57. Verbal: Multiply the input x by 2, add 1, and then take the absolute value.

Graphical: Graph $Y_1 = \text{abs}(2X + 1)$. See Figure 57.

Numerical:

x	-2	-1	0	1	2
y	3	1	1	3	5

$f(2) = 5$

58. Verbal: Regardless of the input x, outputalways has a value of 8.

Graphical: $Y_1 = 8$. See Figure 58.

Numerical:

x	-2	-1	0	1	2
y	8	8	8	8	8

$f(2) = 8$

59. Verbal: Subtract the input x from 5.

Graphical: $Y_1 = 5 - X$. See Figure 59.

Numerical:

x	-2	-1	0	1	2
y	7	6	5	4	3

$f(2) = 3$

60. Verbal: Compute the absolute value of the input x.

Graphical: Graph $Y_1 = \text{abs}(X)$. See Figure 60.

Numerical:

x	-2	-1	0	1	2
y	2	1	0	1	2

$f(2) = 2$

[–10, 10, 1] by [–10, 10, 1]

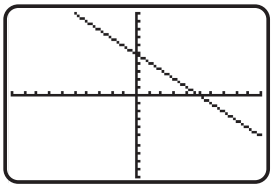


Figure 59

[–6, 6, 1] by [–4, 4, 1]

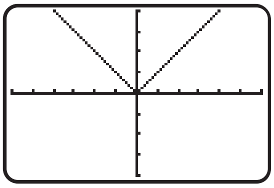


Figure 60

[–6, 6, 1] by [–4, 4, 1]

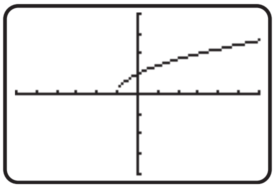


Figure 61

[–10, 10, 1] by [–10, 10, 1]

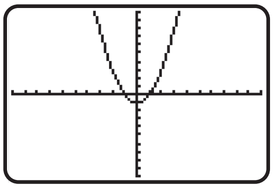


Figure 62

61. Verbal: Add 1 to the input x and then take the square root of the result.

Graphical: Graph $Y_1 = \sqrt{(X + 1)}$ See Figure 61.

Numerical:

x	−2	−1	0	1	2
y	—	0	1	$\sqrt{2}$	$\sqrt{3}$

$f(2) = \sqrt{3}$

62. Verbal: Square the input x and then subtract 1 from the result.

Graphical: $Y_1 = X \wedge 2 - 1$. See Figure 62.

Numerical:

x	−2	−1	0	1	2
y	3	0	−1	0	3

$f(2) = 3$

63. It costs about $0.50 per mile.

Symbolic: $f(x) = 0.50x$.

Graphical: See Figure 63a.

Numerical: See Figure 63b.

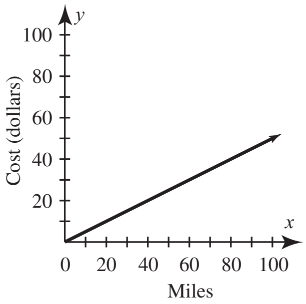


Figure 63a

Miles	1	2	3	4	5	6
Cost	0.50	1.00	1.50	2.00	2.50	3.00

Figure 63b

64.

Bills (millions)	0	1	2	3	4	5	6
Counterfeit Bills	0	9	18	27	36	45	54

65. This is a graph of a function because every vertical line intersects the graph at most once. Both the domain and the range are all real numbers.

66. This is a graph of a function because every vertical line intersects the graph at most once. Both the domain and the range are all real numbers.

67. This is not a graph of a function because some vertical lines can intersect the graph twice. Because a vertical line can intersect the graph twice, two functions are necessary to create this graph.
68. This is a graph of a function because every vertical line intersects the graph at most once.
    The domain is $\{x|-5 \le x \le 5\}$. The range is $\{y|-1 \le y \le 1\}$.
69. This is a graph of a function because every vertical line intersects the graph at most once.
    The domain is $\{x|-4 \le x \le 4\}$. The range is $\{y|0 \le y \le 4\}$.
70. This is not a graph of a function because it is possible for a vertical line to intersect the graph four times. Four functions would be necessary to create this graph.
71. a.) Yes
    b.) Each real number has exactly one real cube root.
72. a.) Yes.
    b.) Each unique person can have only one age.
73. a.)No.
    b.) More than one student can have score x.
74. a.)No.
    b.) Some people may have more than one child.
75. Yes, because the IDs are unique.
76. No, because two students could have the same height.
77. No. The ordered pairs (1, 2) and (1, 3) belong to the set *S*. The domain element 1 has more than one range element associated with it.
78. Yes. Each element in its domain is associated with exactly one range element.
79. Yes. Each element in its domain is associated with exactly one range element.
80. No. The ordered pairs (a, 2) and (a, 3) belong to the set *S*. The domain element a has more than one range element associated with it.
81. No. The ordered pairs (1, 10.5) and (1, –0.5) belong to the set *S*. The domain element 1 has more than one range element associated with it.
82. Yes. Each element in its domain is associated with exactly one range element.
83. No, for example, the ordered pairs (1, –1) and (1, 1) belong to the relation. The domain element 1 has more than one range element associated with it.
84. No, for example, the ordered pairs (3, 2) and (3, –2) belong to the relation. The domain element 3 has more than one range element associated with it.
85. Yes. Each element in the domain of *f* is associated with exactly one range element.
86. Yes. Each element in the domain of *f* is associated with exactly one range element.
87. No, for example, the ordered pairs$(0, \sqrt{70})$ and$(0, -\sqrt{70})$ belong to the relation. The domain element 0 has more than one range element associated with it.

88. No, for example, the ordered pairs $(1, 1)$ and $(1, -1)$ belong to the relation. The domain element 1 has more than one range element associated with it.

89. Yes. Each element in the domain of f is associated with exactly one range element.

90. Yes. Each element in the domain of f is associated with exactly one range element.

91. $g(x) = 12x \Rightarrow g(10) = 12(10) = 120$; there are 120 inches in 10 feet.

92. $g(x) = 4x \Rightarrow g(10) = 4(10) = 40$; there are 40 quarts in 10 gallons.

93. $g(x) = 0.25x \Rightarrow g(10) = 0.25(10) = 2.50$; there are 2.5 dollars in 10 quarters.

94. $g(x) = 4x \Rightarrow g(10) = 4(10) = 40$; there are 40 quarters in 10 dollars.

95. $g(x) = 60 \cdot 60 \cdot 24 \cdot x \Rightarrow g(x) = 86{,}400x \Rightarrow g(10) = 86{,}400(10) = 864{,}000$; there are 864,000 seconds in 10 days.

96. $g(x) = 5280x \Rightarrow g(10) = 5280(10) = 52{,}800$; there are 52,800 feet in 10 miles.

97. (a) $\{(R, 37), (N, 30), (S, 17)\}$

(b) $D = \{N, R, S\}$; $R = \{17, 30, 37\}$

98. (a) $P = \{(2006, 10.9), (2007, 11.1), (2008, 14.6), (2009, 14.7), (2010, 14.5)\}$

(b) $D = \{2006, 2007, 2008, 2009, 2010\}$; $R = \{10.9, 11.1, 14.5, 14.6, 14.7\}$

99. $f(x) = 40x \Rightarrow f(5) = 40(5) = 200$; About 200 million tons of electronic waste will be piled up after 5 years.

100. $W(x) = 8x \Rightarrow W(3) = 8(3) = 24$; After 3 years of ordering larger portions, the average American would be 24 pounds heavier.

101. $N(x) = 2200x$; $N(3) = 2200(3) = 6600$; in 3 years the average person uses 6600 napkins.

102. $W(x) = 40x$; $W(30) = 40(30) = 1200$; 30 loads of clothes use 1200 gallons of water.

103. Verbal: Multiply the input x by –5.8 to obtain the change in temperature.

Symbolic: $f(x) = -5.8x$.

Graphical: $Y_1 = -5.8X$. See Figure 103a.

Numerical: Table $Y_1 = -5.8X$. See Figure 103b.

[0, 3, 1] by [–20, 20, 5]

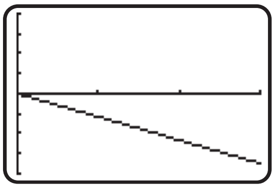


Figure 103a

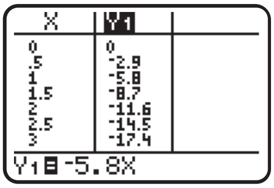


Figure 103b

104. a.) Sinced 6 ft 3 in. = 75 in, and $f(x) = 0.72x + 2$, then $f(75) = 0.72(75) + 2 = 56$ in.

b.) $f(x + 1) - f(x) = 0.72(x + 1) + 2 - [0.72x + 2] = 0.72x + 0.72 + 2 - 0.72x + 2 = 0.72$; for each 1-inch increase in a person's height, the recommended crutch length increases by 0.72 in.

1.4: Types of Functions and Their Rates of Change

1. $f(x) = 5 - 2x \Rightarrow f(x) = -2x + 5;\ m = -2, b = 5$

2. $f(x) = 3 - 4x \Rightarrow f(x) = -4x + 3;\ m = -4, b = 3$

3. $f(x) = -8x \Rightarrow f(x) = -8x + 0;\ m = -8, b = 0$

4. $f(x) = -6 \Rightarrow f(x) = 0x - 6;\ a = 0, b = -6$

5. $m = \dfrac{5 - 6}{2 - 4} = \dfrac{-1}{-2} = \dfrac{1}{2} = 0.5$

6. $m = \dfrac{-7 - 5}{-3 - (-8)} = \dfrac{-12}{5} = -2.4$

7. $m = \dfrac{-2 - 4}{5 - (-1)} = \dfrac{-6}{6} = -1$

8. $m = \dfrac{7 - (-4)}{-15 - 10} = \dfrac{11}{-25} = -0.44$

9. $m = \dfrac{-8 - (-8)}{7 - 12} = \dfrac{0}{-5} = 0$

10. $m = \dfrac{2 - (-5)}{8 - 8} = \dfrac{7}{0}$, undefined slope

11. $m = \dfrac{0.4 - (-0.1)}{-0.3 - 0.2} = \dfrac{0.5}{-0.5} = -1$

12. $m = \dfrac{1.1 - 0.6}{-0.2 - (-0.3)} = \dfrac{0.5}{0.1} = 5$

13. $m = \dfrac{7.6 - 9.2}{-0.3 - (-0.5)} = \dfrac{-1.6}{0.2} = -8$

14. $m = \dfrac{5 - 12}{1.6 - 1.6} = \dfrac{-7}{0} =$ undefined

15. $m = \dfrac{8 - 6}{-5 - (-5)} = \dfrac{2}{0} =$ undefined

16. $m = \dfrac{7 - 7}{19 - 17} = \dfrac{0}{2} = 0$

17. $m = \dfrac{\frac{7}{10} - (-\frac{3}{5})}{-\frac{5}{6} - \frac{1}{3}} = \dfrac{\frac{13}{10}}{-\frac{7}{6}} = \dfrac{13}{10} \cdot \left(-\dfrac{6}{7}\right) = -\dfrac{39}{35} \approx -1.143$

18. $m = \dfrac{\frac{3}{16} - (-\frac{7}{8})}{\frac{1}{10} - (-\frac{13}{15})} = \dfrac{\frac{17}{16}}{\frac{29}{30}} = \dfrac{17}{16} \cdot \dfrac{30}{29} = \dfrac{255}{232} \approx 1.0991$

19. Slope $= 2$; the graph rises 2 units for every unit increase in x.

20. Slope $= -1$; the graph falls 1 unit for every unit increase in x.

21. Slope $= -\dfrac{3}{4}$; the graph falls $\dfrac{3}{4}$ unit for every unit increase in x, or equivalently, the graph falls 3 units for every 4-unit increase in x.

22. Slope $= \dfrac{2}{3}$; the graph rises $\dfrac{2}{3}$ unit for every unit increase in x, or equivalently, the graph rises 2 units for every 3-unit increase in x.

23. Slope $= -1$; the graph falls 1 unit for every unit increase in x.

24. Slope $= 0$; the graph neither falls nor rises since the y-value is always 23.

25. (a) Buying no carpet should and does cost \$0.

(b)Slope $= \frac{100}{5} = 20$

(c) The carpet costs \$20 per square yard.

26. (a) Zero tons of rock would cost \$0.

(b) Slope $= \frac{25}{1} = 25$

(c) The rock costs \$25 per ton.

27. (a)$D(x) = 150 - 20x \Rightarrow D(5) = 150 - 20(5) = 50.$ After 5 hours the train is 50 miles from the station.

(b) Slope equals –20. The train is traveling toward the station at 20 mph.

28. (a) $C(5) = 29(5) = 145$; 5 gallons of paint costs \$145.

(b) Slope $= 29$; Paint costs \$29 per gallon.

29. (a)$D(2) = 75(2) = 150$ miles

(b)Slope $= 75$; the car is traveling away from the rest stop at 75 miles per hour.

30. (a) To find the median ages in 1980 and 2000, we must evaluate $A(1980)$ and $A(2000)$.

$A(1980) = 0.243(1980) - 450.8 = 30.34$ and $A(2000) = 0.243(2000) - 450.8 = 35.2$

In 1980, the median age was 30.34 years and in 2000, it increased to 35.2 years.

(b) Since $A(t) = 0.243t - 450.8$, the slope of its graph is $m = 0.243$. The value of 0.243 means that the median age in the United States is increasing by approximately 0.243 each year from 1970 to 2010.

31. $f(x) = -2x + 5$ is a linear function, but not a constant function, with a slope of $m = -2$. See Figure 31.

32. $f(x) = 3x - 2$ is a linear function, but not a constant function. See Figure 32.

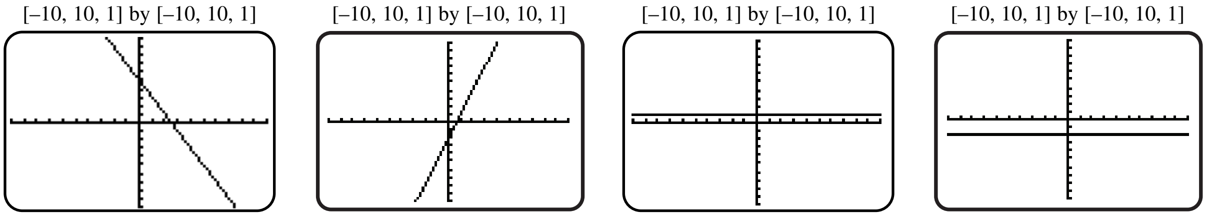


Figure 31 Figure 32 Figure 33 Figure 34

33. $f(x) = 1$ is a constant (and linear) function. See Figure 33.

34. $f(x) = -2$ is a constant (and linear) function. See Figure 34.

35. From its graph, we see that $f(x) = |x + 1|$ represents a nonlinear function. See Figure 35.

36. From its graph, we see that $f(x) = |2x - 1|$ represents a nonlinear function. See Figure 36.

[−10, 10, 1] by [−10, 10, 1]

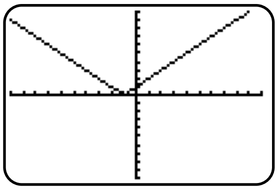


Figure 35

[−10, 10, 1] by [−10, 10, 1]

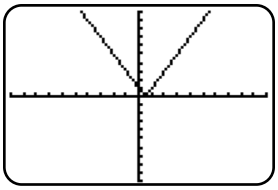


Figure 36

[−10, 10, 1] by [−10, 10, 1]

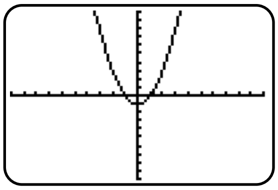


Figure 37

37. From its graph, we see that $f(x) = x^2 - 1$ represents a nonlinear function. See Figure 37.

38. From its graph, we see that $f(x) = \sqrt{x - 1}$ is a nonlinear function. See Figure 38.

[−10, 10, 1] by [−10, 10, 1]

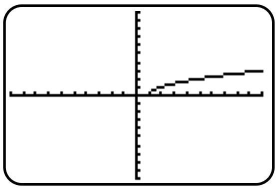


Figure 38

39. Between each pair of points, the y-values increase 4 units for each unit increase in x. Therefore, the data is linear. The slope of the line passing through the data points is 4.

40. The y-values decrease 1.5 units for every 2-unit increase in x. Therefore, the data is linear. The slope of the line passing through the data points is $-\frac{1.5}{2} = -0.75$.

41. The y-values do not increase by a constant amount for each 2-unit increase in x. The data is nonlinear.

42. The y-values do not increase by a constant amount for each 5-unit increase in x. The data is nonlinear.

43. (a) Slope $= \frac{\text{rise}}{\text{run}} = \frac{2}{1} = 2$; y-intercept: –1; x-intercept: 0.5

(b) $f(x) = ax + b \Rightarrow f(x) = 2x - 1$

(c) 0.5

44. (a) Slope $= \frac{\text{rise}}{\text{run}} = \frac{-2}{1} = -2$; y-intercept: 1; x-intercept: 0.5

(b) $f(x) = ax + b \Rightarrow f(x) = -2x + 1$

(c) 0.5

45. (a) Slope $= \frac{\text{rise}}{\text{run}} = \frac{-1}{3} = -\frac{1}{3}$; y-intercept: 2; x-intercept: 6

(b) $f(x) = ax + b \Rightarrow f(x) = -\frac{1}{3}x + 2$

(c) 6

46. (a) Slope $= \frac{\text{rise}}{\text{run}} = \frac{3}{4}$; y-intercept: –3; x-intercept: 4

(b) $f(x) = ax + b \Rightarrow f(x) = \frac{3}{4}x - 3$

(c) 4

47. $f(x) = ax + b \Rightarrow f(x) = -\frac{3}{4}x + \frac{1}{3}$

48. $f(x) = ax + b \Rightarrow f(x) = -122x + 805$

49. $f(x) = ax + b \Rightarrow f(x) = 15x + 0$, or $f(x) = 15x$

50. $f(x) = ax + b \Rightarrow f(x) = 1.68x + 1.23$

51. $[5, \infty)$

52. $(-\infty, 100)$

53. $[4, 19)$

54. $(-4, -1)$

55. $[-1, \infty)$

56. $(-\infty, -3]$

57. $(-\infty, 1) \cup [3, \infty)$

58. $(-\infty, -2] \cup [0, \infty)$

59. $(-3, 5]$

60. $[2, \infty)$

61. $(-\infty, -2)$

62. $[-4, 4]$

63. $(-\infty, -2) \cup [1, \infty)$

64. $(-\infty, -1] \cup [1, \infty)$

65. f is decreasing on $(-\infty, \infty)$. In set-builder notation, the interval is $\{x \mid -\infty < x < \infty\}$.

66. f is decreasing on $(0, \infty)$. In set-builder notation, the interval is $\{x \mid x \geq 0\}$.

67. f is increasing on $(2, \infty)$ and decreasing on $(-\infty, 2)$. In set-builder notation, these intervals are $\{x \mid x \geq 2\}$ and $\{x \mid x \leq 2\}$ respectively.

68. f is increasing on $(-\infty, -1)$ and decreasing on $(-1, \infty)$. In set-builder notation, these intervals are $\{x \mid x \leq -1\}$ and $\{x \mid x \geq -1\}$ respectively.

69. f is increasing on $(-\infty, -2), (1, \infty)$ and decreasing on $(-2, 1)$. In set-builder notation, these intervals are $\{x \mid x \leq 2 \text{ or } x \geq 1\}$ and $\{x \mid -2 \leq x \leq 1\}$ respectively.

70. f is increasing on $(-\infty, -2), (0, 1)$ and decreasing on $(-2, 0), (1, \infty)$. In set-builder notation, these intervals are $\{x \mid x \leq -2 \text{ or } 0 \leq x \leq 1\}$ and $\{x \mid -2 \leq x \leq 0 \text{ or } x \geq 1\}$ respectively.

71. f is increasing on $(-8, 0), (8, \infty)$ and decreasing on $(-\infty, -8), (0, 8)$. In set-builder notation, these intervals are $\{x \mid -8 \leq x \leq 0 \text{ or } x \geq 8\}$ and $\{x \mid x \leq -8 \text{ or } 0 \leq x \leq 8\}$ respectively.

72. f is increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$. In set builder notation, these intervals are $\{x \mid x \leq 0\}$ and $\{x \mid x \geq 0\}$ respectively.

73. The graph of this equation is linear with a slope of $2 \Rightarrow$ it is increasing: $(-\infty, \infty)$ and decreasing: never. In set builder notation, the interval is $\{x \mid -\infty < x < \infty\}$.

74. The graph of this equation is linear with a slope of $-1 \Rightarrow$ it is increasing: never and decreasing: $(-\infty, \infty)$. In set builder notation, the interval is $\{x \mid -\infty < x < \infty\}$.

75. The graph of this equation is a parabola with a vertex$(0, -2)$ $\Rightarrow$ it is increasing: $(0, \infty)$ and in set builder notation, these intervals are$\{x \mid x \geq 0\}$ and$\{x \mid x \leq 0\}$ respectively.
76. The graph of this equation is a parabola with a vertex (0, 0), because of the negative coefficient it opens downward $\Rightarrow$ it is increasing: $(-\infty, 0)$ and decreasing: $(0, \infty)$. In set builder notation, these intervals are$\{x \mid x \leq 0\}$ and$\{x \mid x \geq 0\}$ respectively.
77. The graph of this equation is a parabola with a vertex (1, 1), because of the negative coefficient beforex^2 it opens downward $\Rightarrow$ it is increasing: $(-\infty, 1)$ and decreasing: $(1, \infty)$. In set builder notation, these intervals are$\{x \mid x \leq 1\}$ and$\{x \mid x \geq 1\}$ respectively.
78. The graph of this equation is a parabola with a vertex$(2, -4)$ $\Rightarrow$ it is increasing: $(2, \infty)$ and decreasing: $(-\infty, 2)$. In set builder notation, these intervals are$\{x \mid x \geq 2\}$ and$\{x \mid x \leq 2\}$ respectively.
79. The square root equation graph has a starting point$(1, 0)$, all x-values less than 1 are undefined $\Rightarrow$ it is increasing: $(1, \infty)$ and decreasing: never. In set builder notation, the interval is$\{x \mid x \geq 1\}$.
80. The square root equation graph has a starting point$(-1, 0)$, because of the negative coefficient before $\sqrt{x + 1}$ the graph opens down to the right $\Rightarrow$ it is increasing: never and decreasing: $(-1, \infty)$. In set builder notation, the interval is$\{x \mid x \geq -1\}$.
81. The absolute value graph has a vertex$(-3, 0)$ $\Rightarrow$ it is increasing: $(-3, \infty)$ and decreasing: $(-\infty, -3)$. In set builder notation, these intervals are$\{x \mid x \geq -3\}$ and$\{x \mid x \leq -3\}$ respectively.
82. The absolute value graph has a vertex$(1, 0)$ $\Rightarrow$ it is increasing: $(1, \infty)$ and decreasing: $(-\infty, 1)$. In set builder notation, these intervals are$\{x \mid x \geq 1\}$ and$\{x \mid x \leq 1\}$ respectively.
83. The basic x^3 function is always increasing $\Rightarrow$ it is increaing: $(-\infty, \infty)$ and decreasing: never. In set builder notation, the interval is$\{x \mid -\infty < x < \infty\}$.
84. The basic $\sqrt[m]{x}$ function is always increasing $\Rightarrow$ it is increaing: $(-\infty, \infty)$ and decreasing: never. In set builder notation, the interval is$\{x \mid -\infty < x < \infty\}$.
85. The graph of this cubic equation has turning points$\left(-2, \dfrac{16}{3}\right)$ and $\left(2, \dfrac{-16}{3}\right)$ $\Rightarrow$ it is increasing: $(-\infty, -2)$ or $(2, \infty)$; and decreasing: $[-2, 2]$. In set builder notation, these intervals are$\{x \mid x \leq -2 \text{ or } x \geq 2\}$ and $\{x \mid -2 \leq x \leq 2\}$ respectively.
86. The graph of this cubic equation has turning points $(-1, 2)$ and $(1, -2)$ $\Rightarrow$ it is increasing: $(-\infty, -1)$ or $(1, \infty)$; and decreasing: $[-1, 1]$. In set builder notation, these intervals are$\{x \mid x \leq -1 \text{ or } x \geq 1\}$ and $\{x \mid -1 \leq x \leq 1\}$ respectively.
87. The graph of this x^4 graph has a negative lead coefficient therefore is reflected through the x-axis has turning points $\left(-1, \dfrac{5}{12}\right)$, $(0, 0)$ and $\left(2, \dfrac{8}{3}\right)$ $\Rightarrow$ it is increasing: $(-\infty, -1)$ or $(0, 2)$; and decreasing: In set builder notation, these intervals are $\{x \mid x \leq -1 \text{ or } 0 \leq x \leq 2\}$ and $\{x \mid -1 \leq x \leq 0 \text{ or } x \geq 2\}$ respectively.

88. The graph of this x^4 graph has turning points $(-2, -4)$, $(0, 0)$ and $(2, -4)$ $\Rightarrow$ it is increasing: $(-2, 0)$ or $(2, \infty)$; and decreasing: $(-\infty, -2]$ or $[0, 2]$. In set builder notation, these intervals are $\{x \mid -2 \le x \le 0 \text{ or } x \ge 2\}$ and $\{x \mid x \le -2 \text{ or } 0 \le x \le 2\}$ respectively.

89. According to the graph the water levels are increasing on the time intervals $(0, 2.4)$, $(8.7, 14.7)$ and $(21, 27)$. In set builder notation, these intervals are $\{x \mid 0 \le x \le 2.4 \text{ or } 8.7 \le x \le 14.7 \text{ or } 21 \le x \le 27\}$.

90. According to the graph the water levels are decreasing on the time intervals $(2.4, 8.7)$ and $(14.7, 21)$. In set builder notation, these intervals are $\{x \mid 2.4 \le x \le 8.7 \text{ or } 14.7 \le x \le 21\}$.

91. The average rate of change from –3 to –1 is $\dfrac{f(-1) - f(-3)}{-1 - (-3)} = \dfrac{3.7 - 1.3}{2} = \dfrac{2.4}{2} = 1.2$.

The average rate of change from 1 to 3 is $\dfrac{f(3) - f(1)}{3 - 1} = \dfrac{1.3 - 3.7}{2} = -\dfrac{2.4}{2} = -1.2$.

92. The average rate of change from –3 to –1 is $\dfrac{f(-1) - f(-3)}{-1 - (-3)} = \dfrac{-3.7 - (-1.3)}{2} = -\dfrac{2.4}{2} = -1.2$.

The average rate of change from 1 to 3 is $\dfrac{f(3) - f(1)}{3 - 1} = \dfrac{-1.3 - (-3.7)}{2} = \dfrac{2.4}{2} = 1.2$.

93. (a) $f(x) = x^2 \Rightarrow f(1) = 1^2 = 1$ and $f(2) = 2^2 = 4 \Rightarrow (1, 1), (2, 4)$; using the slope formula for rate of change we get $\dfrac{4 - 1}{2 - 1} = \dfrac{3}{1} = 3$.

(b) See Figure 93.

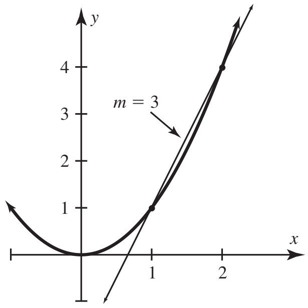


Figure 93

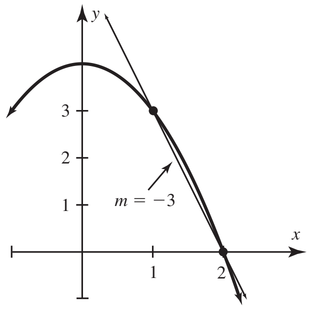


Figure 94

94. (a) $f(x) = 4 - x^2 \Rightarrow f(1) = 4 - (1)^2 = 4 - 1 = 3$ and $f(2) = 4 - (2)^2 = 4 - 4 = 0 \Rightarrow$ $(1, 3), (2, 0)$; using the slope formula for rate of change we get $\dfrac{0 - 3}{2 - 1} = \dfrac{-3}{1} = -3$.

(b) See Figure 94.

95. If $f(x) = 7x - 2$ then $\dfrac{f(4) - f(1)}{4 - 1} = 7$. The slope of the graph is 7.

96. If $f(x) = -8x + 5$ then $\dfrac{f(0) - f(-2)}{0 - (-2)} = -8$. The slope of the graph is -8.

97. If $f(x) = \sqrt{2x - 1}$ then $\dfrac{f(3) - f(1)}{3 - 1} \approx 0.62$. The slope of the line passing through the points is approximately 0.62.

98. If $f(x) = 0.5x^2 - 5$ then $\dfrac{f(4) - f(-1)}{4 - (-1)} = \dfrac{3 - (-4.5)}{5} = \dfrac{7.5}{5} = 1.5$. The slope of the line passing through the points $(-1, f(-1))$ and $(4, f(4))$ is 1.5.

99. (a)The average rate of change from 1900 to 1940 is calculated as $\dfrac{182 - 3}{1940 - 1900} = \dfrac{179}{40} = 4.475$; from 1940 to 1980:$\dfrac{632 - 182}{1980 - 1940} = \dfrac{450}{40} = 11.25$; from 1980 to 2010:$\dfrac{315 - 632}{2010 - 1980} = \dfrac{-317}{30} \approx -10.6$.

(b) The average rates of change in cigarette consumption in the time periods 1900 to 1940, 1940 to 1980, and 1980 to 2010 were 4.475, 11.25, and –10.06, respectively.

100.(a) The a verage rate of change from 1 to 1.5 is$\dfrac{A(1.5) - A(1)}{1.5 - 1} = \dfrac{49 - 64}{0.5} = -30$. From 1 to 1.5 minutes water is leaving the tank at a rate of 30 gallons per minute, on average. From 2 to 2.5, the average rate of change is$\dfrac{A(2.5) - A(2)}{2.5 - 2} = \dfrac{25 - 36}{0.5} = -22$. From 2 to 2.5 minutes water is leaving the tank at a rate of 22 gallons per minute, on average.

(b) The average rates of change are different because water drains faster when the tank is fuller.

101.See Figure 101.

102.See Figure 102.

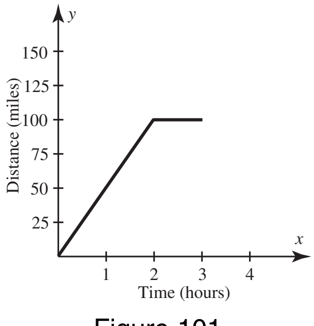


Figure 101

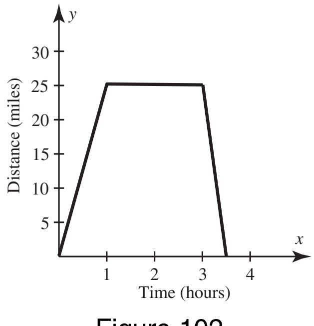


Figure 102

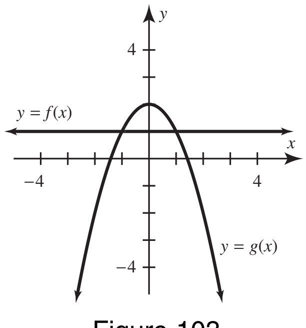


Figure 103

103.See Figure 103. *Answers may vary.*

104. See Figure 104. *Answers may vary.*

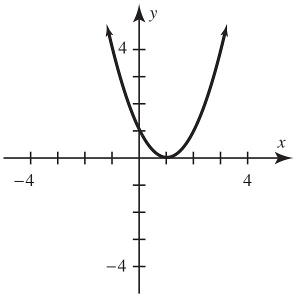


Figure 104

Extended and Discovery Exercises for Section 1.4

1. (a) $C(r) = 2\pi r \Rightarrow C(r+1) = 2\pi(r+1) = 2\pi r + 2\pi$; for every 1 inch increase in the radius, the circumference increases by 2π inches. So the circumference increases at a constant rate of 2π inches per second.

   (b) No, because the area function, $A(r) = \pi r^2$, depends on the radius squared. The area function is not linear and thus does not increase at a constant rate.

Checking Basic Concepts for Sections 1.3 and 1.4

1. Symbolic: $f(x) = 5280x$

   Numerical: Use a table f starting at $x = 1$, incrementing by 1. See Figure 1a.

   Graphical: Graph $Y_1 = 5280X$ as shown in Figure 1b.

x	1	2	3	4	5
$f(x)$	5280	10,560	15,840	21,120	26,400

Figure 1a

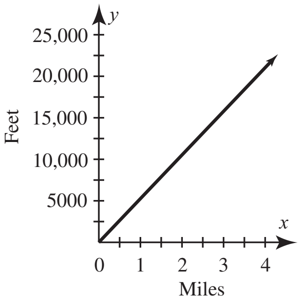


Figure 1b

2. (a) $f(2) = \dfrac{2(2)}{2-4} = \dfrac{4}{-2} = -2$; $f(a+4) = \dfrac{2(a+4)}{(a+4)-4} = \dfrac{2a+8}{a}$

   (b) The function is undefined when the denominator $x - 4 = 0$. This happens when $x = 4$; therefore, $D = \{x \mid x \neq 4\}$.

3. The slope is calculated as follows: $m = \dfrac{-5-4}{4-(-2)} = -\dfrac{9}{6} = -\dfrac{3}{2}$

   If the graph of the linear function $f(x) = ax + b$, passes through the points (–2, 4) and (4, –5), its slope must be equal to $-\dfrac{3}{2}$; therefore, $a = -\dfrac{3}{2}$.

4. (a) $f(x) = -1.4x + 5.1$ represents a linear function.

   (b) $f(x) = 25$ represents a constant (and linear) function.

   (c) $f(x) = 2x^2 - 5$ represents a nonlinear function.

5. (a) $(-\infty, 5]$

   (b)$[1, 6)$

6. The graph of f is a parabola opening up with vertex $(0, -2) \Rightarrow$ it is increasing on $[0, \infty)$ and decreasing on $(-\infty, 0]$

7. $\dfrac{f(-1) - f(-3)}{-1 - (-3)} = \dfrac{4 - 18}{2} = -7$

8. $\dfrac{f(x + h) - f(x)}{h} = \dfrac{4(x + h)^2 - 4x^2}{h} = \dfrac{4(x^2 + 2xh + h^2) - 4x^2}{h} =$

   $\dfrac{4x^2 + 8xh + 4h^2 - 4x^2}{h} = \dfrac{8xh + 4h^2}{h} = 8x + 4h$

Chapter 1 Review Exercises

1. –2 is an integer, rational number, and real number. $\dfrac{1}{2}$ is both a rational and a real number. 0 is an integer, rational number, and real number. 1.23 is both a rational and a real number. $\sqrt{7}$ is a real number. $\sqrt{16} = 4$ is a natural number, integer, rational number, and real number.

2. 55 is a natural number, integer, rational number, and real number. 1.5 is both a rational and a real number. $\dfrac{104}{17}$ is both a rational and a real number. $2^3 = 8$ is a natural number, integer, rational number, and real number. $\sqrt{3}$ is a real number. –1000 is an integer, rational number, and real number.

3. $1{,}891{,}000 = 1.891 \times 10^6$

4. $0.0001001 = 1.001 \times 10^{-4}$

5. $1.52 \times 10^4 = 15{,}200$

6. $-7.2 \times 10^{-3} = -0.0072$

7. (a) $\sqrt[m]{1.2} + \pi^3 \approx 32.07$

   (b) $\dfrac{3.2 + 5.7}{7.9 - 4.5} \approx 2.62$

   (c) $\sqrt{5^2 + 2.1} \approx 5.21$

   (d) $1.2(6.3)^2 + \dfrac{3.2}{\pi - 1} \approx 49.12$

8. (a) $(4 \times 10^3)(5 \times 10^{-5}) = (4)(5) \times 10^{3-5} = 20 \times 10^{-2} = 2 \times 10^{-1};\ 0.2$

   (b) The scientific notation is $\dfrac{3 \times 10^{-5}}{6 \times 10^{-2}} = \dfrac{3}{6} \times \dfrac{10^{-5}}{10^{-2}} = 0.5 \times 10^{-5-(-2)} = 0.5 \times 10^{-3} = 5 \times 10^{-4}$.

   This is equivalent to 0.0005 in standard form.

9 $.4 - 3^2 \cdot 5 = 4 - 9 \cdot 5 = 4 - 45 = -41$

10. $3 \cdot 3^2 \div \dfrac{3-5}{6+2} = 3 \cdot 9 \div \dfrac{-2}{8} = 27 \div \left(-\dfrac{1}{4}\right) = 27(-4) = -108$

11.

−23	−5	8	19	24

(a) Maximum $= 24$; Minimum $= -23$

(b) Mean $= \dfrac{-23 + (-5) + 8 + 19 + 24}{5} = 4.6$; Median $= 8$

12.

−3.8	−1.2	0.8	1.7	1.7	8.9

(a) Maximum $= 8.9$; Minimum $= -3.8$

(b) Mean $= \dfrac{-3.8 + (-1.2) + 0.8 + 1.7 + 1.7 + 8.9}{6} = 1.35$; Median $= \dfrac{0.8 + 1.7}{2} = 1.25$

13. (a) $S = \{(-15, -3), (-10, -1), (0, 1), (5, 3), (20, 5)\}$

(b) $D = \{-15, -10, 0, 5, 20\}$ and $R = \{-3, -1, 1, 3, 5\}$

14. (a) $S = \{(-0.6, 10), (-0.2, 20), (0.1, 25), (0.5, 30), (1.2, 80)\}$

(b) $D = \{-0.6, -0.2, 0.1, 0.5, 1.2\}$ and $R = \{10, 20, 25, 30, 80\}$

15. The relation {(10, 13), (–12, 40), (–30, –23), (25, –22), (10, 20)} is plotted in Figure 15. It is not a function since both (10, 13) and (10, 20) are contained in the set. Notice that these points are lined up vertically.

[–50, 50, 10] by [–50, 50, 10]

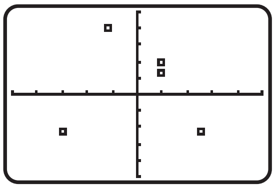


Figure 15

[–4, 4, 1] by [–4, 4, 1]

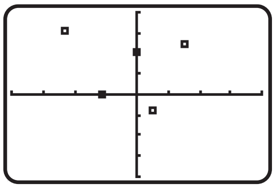


Figure 16

16. The relation {(1.5, 2.5), (0, 2.1), (–2.3, 3.1), (0.5, –0.8), (–1.1, 0)} is plotted in Figure 16. It is a function.

17. $d = \sqrt{(2 - (-4))^2 + (-3 - 5)^2} = \sqrt{6^2 + (-8)^2} = \sqrt{36 + 64} = \sqrt{100} = 10$

18. $d = \sqrt{(0.2 - 1.2)^2 + (6 - (-4))^2} = \sqrt{(-1)^2 + 10^2} = \sqrt{101}$

19. $M = \left(\dfrac{24 + (-20)}{2}, \dfrac{-16 + 13}{2}\right) = \left(\dfrac{4}{2}, \dfrac{-3}{2}\right) = \left(2, \dfrac{-3}{2}\right)$

20. $M = \left(\dfrac{\frac{1}{2} + \frac{1}{2}}{2}, \dfrac{\frac{5}{4} + \left(\frac{-5}{2}\right)}{2}\right) = \left(\dfrac{1}{2}, \dfrac{-5}{8}\right)$

21. Use the distance formula to find the side lengths:

$d_1 = \sqrt{(1 - (-3))^2 + (2 - 5)^2} = \sqrt{4^2 + (-3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$

$d_2 = \sqrt{((-3) - 0)^2 + (5 - 9)^2} = \sqrt{(-3)^2 + (-4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

$d_3 = \sqrt{(1 - 0)^2 + (2 - 9)^2} = \sqrt{1^2 + (-7)^2} = \sqrt{1 + 49} = \sqrt{50} = 7.07$

Two sides are 5 units long and the other side is 7.07 units long, therefore, the triangle is isosceles.

22. The equation is$(x + 5)^2 + (y - 3)^2 = 81$.

23. First calculate the midpoint of the diameter:$x = \frac{6 + (-2)}{2} = \frac{4}{2} = 2$ and $y = \frac{6 + 4}{2} = \frac{10}{2} = 5 \Rightarrow$ the center is (2, 5). Then calculate the length of the radius using points (2, 5) and (6, 6):

$d = \sqrt{(6 - 2)^2 + (6 - 5)^2} = \sqrt{4^2 + 1^2} = \sqrt{17}$. The equation is $(x - 2)^2 + (y - 5)^2 = 17$.

24. (a) $D = \{x \mid -2 \le x \le 2\}$ and $R = \{y \mid -2 \le y \le 0\}$; $f(-2) = 0$

(b) $D =$ all real numbers and $R = \{y \mid y \le 2\}$; $f(-2) = 2$

25. See Figure 25.

26. See Figure 26.

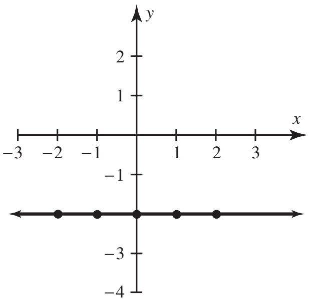


Figure 25

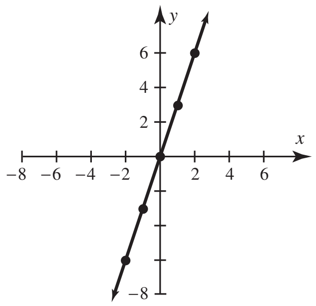


Figure 26

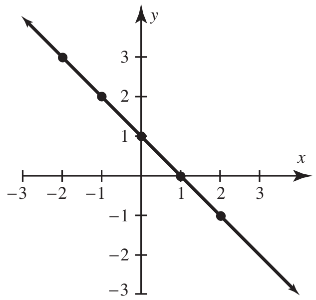


Figure 27

27. See Figure 27.

28. See Figure 28.

29. See Figure 29.

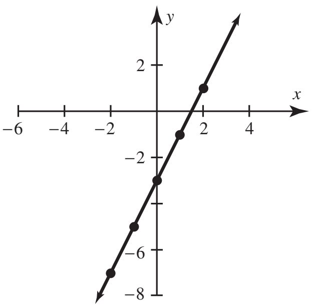


Figure 28

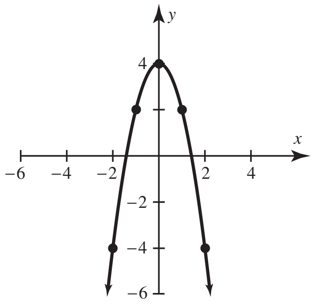


Figure 29

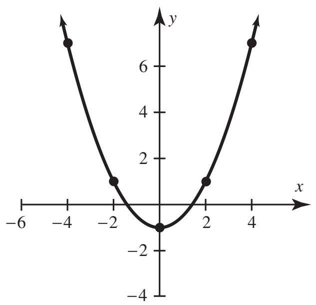


Figure 30

30. See Figure 30.

31. See Figure 31.

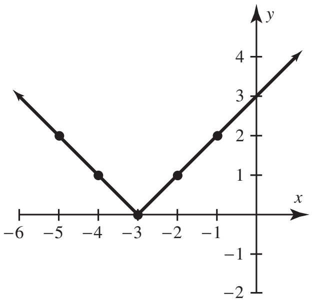


Figure 31

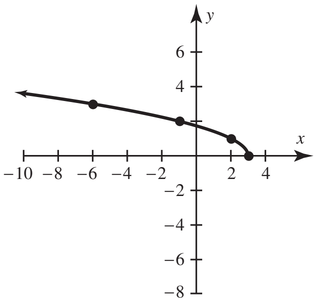


Figure 32

32. See Figure 32.

33. Symbolic: $f(x) = 16x$.

Numerical: Table f starting at $x = 0$, incrementing by 25. See Figure 33a.

Graphical: Graph $Y_1 = 16X$ in [0, 100, 10] by [0, 1800, 300]. See Figure 33b.

x	0	25	50	75	100
$f(x)$	0	400	800	1200	1600

Figure 33a

[0, 100, 10] by [0, 1800, 300]

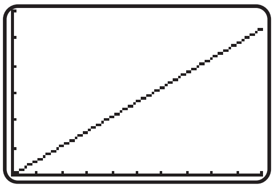


Figure 33b

34. Symbolic: $f(x) = x^2$.

Numerical: Table f starting at $x = 0$, incrementing by 25. See Figure 34a.

Graphical: Graph $Y_1 = X^2$ in [0, 100, 10] by [0, 12000, 1000]. See Figure 34b.

x	0	25	50	75	100
$f(x)$	0	625	2500	5625	10,000

Figure 34a

[0, 100, 10] by [0, 12000, 1000]

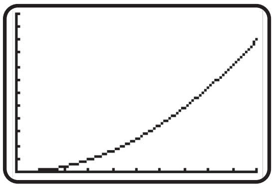


Figure 34b

35. (a) $f(x) = 5 \Rightarrow f(-3) = 5$ and $f(1.5) = 5$

(b) All real numbers

36. (a) $f(x) = 4 - 5x \Rightarrow f(-5) = 4 - 5(-5) = 29$ and $f(6) = 4 - 5(6) = -26$

(b) All real numbers

37. (a) $f(x) = x^2 - 3 \Rightarrow f(-10) = (-10)^2 - 3 = 97$ and $f(a+2) = (a+2)^2 - 3 =$

$a^2 + 4a + 4 - 3 = a^2 + 4a + 1$

(b) All real numbers

38. (a) $f(x) = x^3 - 3x \Rightarrow f(-10) = (-10)^3 - 3(-10) = -1000 + 30 = -970$ and $f(a+1) = (a+1)^3 - 3(a+1) = a^3 + 3a^2 + 3a + 1 - 3a - 3 = a^3 + 3a^2 - 2$

    (b) All real numbers

39. (a) $f(-3) = \dfrac{1}{-3-4} = -\dfrac{1}{7}$; $f(a+1) = \dfrac{1}{a+1-4} = \dfrac{1}{a-3}$

    (b) $D = \{x \mid x \neq 4\}$

40. (a) $f(x) = \sqrt{x+3} \Rightarrow f(1) = \sqrt{1+3} = \sqrt{4} = 2$ and $f(a-3) = \sqrt{(a-3)+3} = \sqrt{a}$

    (b) $D = \{x \mid x \geq -3\}$

41. No, for example, an input $x = 6$ produces outputs of $y = \pm 1$.

42. [5, 10)

43. (a) Using the points (0, 6) and (2, 2), $m = \dfrac{2-6}{2-0} = \dfrac{-4}{2} = -2$; y-intercept: 6; x-intercept: 3.

    (b) $f(x) = -2x + 6$

    (c) The zeros of f are the same as the x-intercepts. That is $x = 3$.

44. (a) Using the points (0, −40) and (10, 10), $m = \dfrac{10-(-40)}{10-0} = \dfrac{50}{10} = 5$; y-intercept: −40; x-intercept: 8.

    (b) $f(x) = 5x - 40$

    (c) The zeros of f are the same as the x-intercepts. That is $x = 8$.

45. Since any vertical line intersects the graph of f at most once, it is a function.

46. Some vertical lines will intersect the graph of f twice, so it is not a function.

47. Yes, it is a function. Each input produces a single output.

48. No, it is not a function. The input $x = -1$ produces two outputs, 3 and 7.

49. $a = 0$, so the slope $= 0$.

50. $a = \dfrac{1}{3}$, so the slope $= \dfrac{1}{3}$.

51. $m = \dfrac{4-7}{3-(-1)} = -\dfrac{3}{4}$

52. $m = \dfrac{10-(-4)}{2-1} = \dfrac{14}{1} = 14$

53. $m = \dfrac{4-4}{-2-8} = \dfrac{0}{-10} = 0$

54. $m = \dfrac{-\frac{5}{6} - \frac{2}{3}}{-\frac{1}{3} - \left(-\frac{1}{3}\right)} = \dfrac{\frac{-3}{2}}{0}$ is undefined

55. $f(x) = 8 - 3x$ represents a linear function.

56. $f(x) = 2x^2 - 3x - 8$ represents a nonlinear function.

57. $f(x) = |x + 2|$ represents a nonlinear function.

58. $f(x) = 6$ represents a constant (and linear) function.

59. See Figure 59.

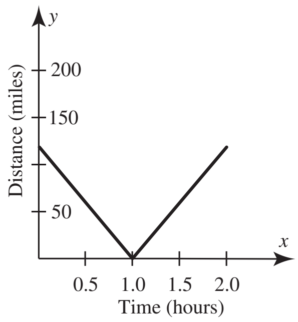


Figure 59

60. $f(x) = |x - 3|$ is an absolute value function opening up with vertex at (3, 0) $\Rightarrow$ f is increasing on $[3, \infty)$ and decreasing on $(-\infty, 3]$. In set-builder notation, these intervals are $\{x|x \geq 3\}$ and $\{x|x \leq 3\}$ respectively.

61. Yes, $m = \dfrac{50 - 26}{-2 - 4} = \dfrac{24}{-6} = -4$. The best model is linear, but not constant, since the y-values decrease 8 units for every 2-unit increase in x.

62. For $f(x) = x^2 - x + 1$, the average rate of change from 1 to 3 is $\dfrac{f(3) - f(1)}{3 - 1} = \dfrac{7 - 1}{2} = 3$.

63. $f(x + h) = 5(x + h) + 1 = 5x + 5h + 1$

$$\frac{f(x+h) - f(x)}{h} = \frac{5x + 5h + 1 - (5x + 1)}{h} = \frac{5h}{h} = 5$$

64. $f(x + h) = 3(x + h)^2 - 2 = 3(x^2 + 2xh + h^2) - 2 = 3x^2 + 6xh + 3h^2 - 2$

$$\frac{f(x+h) - f(x)}{h} = \frac{3x^2 + 6xh + 3h^2 - 2 - (3x^2 - 2)}{h} = \frac{6xh + 3h^2}{h} = 6x + 3h$$

65. $\dfrac{2.28 \times 10^8}{3 \times 10^5} = 760 \text{ seconds} = 12\dfrac{2}{3} \text{ minutes}$

66. The paint on the circular piece of plastic can be thought of as a thin cylinder.

The volume of a cylinder is given by $V = \pi r^2 h$. Substitute 0.25 in^3 for V and 10 in. for r:

$0.25 = \pi(10)^2 h \Rightarrow 0.25 = 100\pi h \Rightarrow h = \dfrac{0.25}{100\pi} \approx 7.96 \times 10^{-4}$

The thickness of the plastic is about 0.000796 inches.

67. (a) Sketching a diagram of the pool and sidewalk (Not Shown) gives the following dimensions:

$l = 62$ ft. and $w = 37$ ft. Thus, $P = 2(62) + 2(37) = 198$ ft.

(b) The area of the sidewalk would consist of the area of four 6×6 squares, two 50×6 rectangles, and two 25×6 rectangles. $A = 4(6 \cdot 6) + 2(50 \cdot 6) + 2(25 \cdot 6) = 4(36) + 2(300) + 2(150) = 1044 \text{ ft}^2$

68. (a) $D(2) = 280 - 70(2) = 280 - 140 = 140$ miles

(b) $D(1) = 280 - 70(1) = 280 - 70 = 210$. Now using the slope formula for $(1, 210)$ and $(2, 140)$ we get

$\dfrac{140 - 210}{2 - 1} = \dfrac{-70}{1} = -70$; the driver is moving toward the rest stop at 70 miles per hour.

69. (a) Plot the points (0, 100), (1, 10), (2, 6), (3, 3), and (4, 2) and make a line graph. See Figure 69. The survival rates decrease rapidly at first. This means that a large number of eggs never develop into mature adults.

(b) Since any vertical line could intersect the graph at most once, this graph could represent a function.

(c) From 0 to 1, $\frac{10 - 100}{1 - 0} = -90$; from 1 to 2, $\frac{6 - 10}{2 - 1} = -4$; from 2 to 3, $\frac{3 - 6}{3 - 2} = -3$; from 3 to 4, $\frac{2 - 3}{4 - 3} = -1$; during the first year, the population of sparrows decreased, on average, by 90 birds. The other average rates of change can be interpreted similarly.

[−1, 5, 1] by [0, 110, 10]

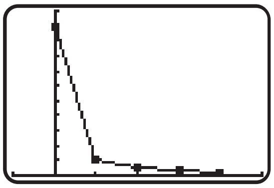


Figure 69

[1, 5, 1] by [40, 70, 5]

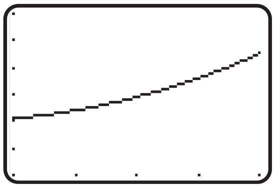


Figure 71a

[1, 5, 1] by [50, 64, 5]

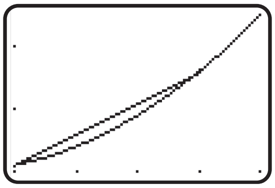


Figure 71c

70. (a) Taking zero credits would cost \$0.
    (b) $\frac{100 - 0}{1 - 0} = 100$
    (c) The cost per credit is \$100.
71. (a) See Figure 71a. f is nonlinear.
    (b) $f(x) = 0.5x^2 + 50 \Rightarrow \frac{f(4) - f(1)}{4 - 1} = \frac{58 - 50.5}{3} = 2.5$
    (c) The average rate of change in outside temperature from 1 P.M. to 4 P.M. was 2.5° F per hour. The slope of the line segment from (1, 50.5) to (4, 58) is 2.5. See Figure 71c. The temperature increased, on average, by 2.5° F per hour
72. Using the Pythagorean theorem to solve for the distance, we have one leg $20 + 30\left(\frac{3}{4}\right) = 42.5$ and the other leg $50\left(\frac{3}{4}\right) = 37.5 \Rightarrow$ Approximating $42.5^2 + 37.5^2 = c^2 \Rightarrow 3212.5 = c^2 \Rightarrow c \approx 57.$

Extended and Discovery Exercises for Chapter 1

1. For (–3, 1.8) and (–1.5, 0.45): $d = \sqrt{(-1.5 + 3)^2 + (0.45 - 1.8)^2} \approx 2.018$

   For (–1.5, 0.45) and (0, 0): $d = \sqrt{(1.5)^2 + (-0.45)^2} \approx 1.566$

   For (0, 0) and (1.5, 0.45): $d = \sqrt{(1.5)^2 + (0.45)^2} \approx 1.566$

   For (1.5, 0.45) and (3, 1.8): $d = \sqrt{(3 - 1.5)^2 + (1.8 - 0.45)^2} \approx 2.018$

   Curve length $\approx 2(2.018) + 2(1.566) \approx 7.17$ km.
2. For (–1, 1) and (0, 0): $d = \sqrt{(0 + 1)^2 + (0 - 1)^2} = \sqrt{2}$

   For (0, 0) and (1, 1): $d = \sqrt{(1 - 0)^2 + (1 - 0)^2} = \sqrt{2}$

   For (1, 1) and (2, 4): $d = \sqrt{(2 - 1)^2 + (4 - 1)^2} = \sqrt{10}$

   Curve length $\approx 2\sqrt{2} + \sqrt{10} \approx 5.991$. See Figure 2.

3. For (–1, –1) and (0, 0): $d = \sqrt{(0 + 1)^2 + (0 + 1)^2} = \sqrt{2}$

   For (0, 0) and (1, 1): $d = \sqrt{(1 - 0)^2 + (1 - 0)^2} = \sqrt{2}$

   For (1, 1) and $(2, \sqrt[m]{2})$:$d = \sqrt{(2 - 1)^2 + \sqrt[m]{2} - 1)^2} \approx 1.033$

   Curve length $\approx 2\sqrt{2} + 1.033 \approx 3.862$. See Figure 3.

[–4.5, 4.5, 1] by [–1, 5, 1]

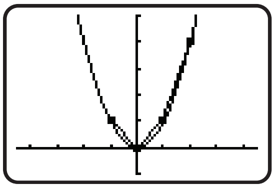


Figure 2

[–3, 3, 1] by [–2, 2, 1]

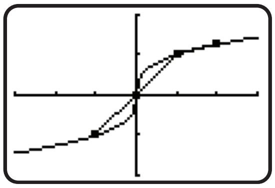


Figure 3

[–4.5, 4.5, 1] by [0, 6, 1]

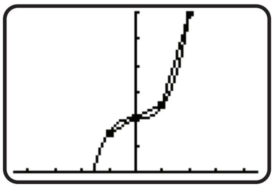


Figure 4

[–3, 3, 1] by [–1, 3, 1]

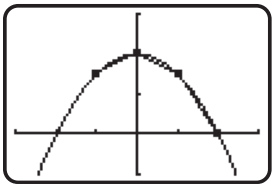


Figure 5

4. For (–1, 1.5) and (0, 2): $d = \sqrt{(0 + 1)^2 + (2 - 1.5)^2} \approx 1.118$

   For (0, 2) and (1, 2.5): $d = \sqrt{(1 - 0)^2 + (2.5 - 2)^2} \approx 1.118$

   For (1, 2.5) and (2, 6): $d = \sqrt{(2 - 1)^2 + (6 - 2.5)^2} \approx 3.640$

   Curve length $\approx 2(1.118) + 3.640 \approx 5.876$. See Figure 4.

5. For (–1, 1.5) and (0, 2): $d = \sqrt{(0 + 1)^2 + (2 - 1.5)^2} \approx 1.118$

   For (0, 2) and (1, 1.5): $d = \sqrt{(1 - 0)^2 + (1.5 - 2)^2} \approx 1.118$

   For (1, 1.5) and (2, 0): $d = \sqrt{(2 - 1)^2 + (0 - 1.5)^2} \approx 1.803$

   Curve length $\approx 2(1.118) + 1.803 \approx 4.039$. See Figure 5.

6. The graph of $y = 9 - x^2$ has the same shape as the graph of $y = x^2$ except that it has been reflected across the x-axis and shifted up 9 units. Note that if the point (0, 0) is reflected across the x-axis and shifted up 9 units it becomes the point (0, 9). Similarly, the point (3, 9), when reflected across the x-axis and shifted up 9 units, becomes the point (3, 0). Therefore the distances are identical. The distance from (0, 9) to (3, 0) along the curve $y = 9 - x^2$ is also approximately 9.747.

7. The graph of $y = \sqrt{x}$ is nearly linear from (1, 1) to (4, 2). It would be reasonable to estimate the distance along the curve by finding the linear distance from (1, 1) to (4, 2).
   $d = \sqrt{(4 - 1)^2 + (2 - 1)^2} = \sqrt{(3)^2 + (1)^2} = \sqrt{10} \approx 3.162$

8. (a) See Figure 8a.

   (b) Choosing (0, 3697) and (19, 1127), we get the average rate of change $= \dfrac{1127 - 3697}{19 - 0} \approx -135.$
   Since 3697 is the initial value, the function $f(x) = 3697 - 135x$ models the data approximately.

   (c) See Figure 8c.

   (d) For 1987, $x = 7 \Rightarrow f(7) \approx 2752$; for 2003, $x = 23 \Rightarrow f(23) \approx 592$.

[0, 20, 5] by [1000, 3800, 200]

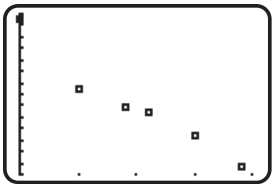


Figure 8a

[0, 20, 5] by [1000, 3800, 200]

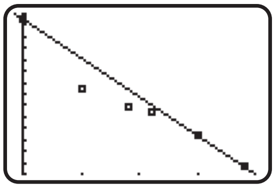


Figure 8c

9. (a) Determine the number of square miles of the Earth's surface that are covered by the oceans. Then divide the total volume of water from the ice caps by the surface area of the oceans to get the rise in sea level.

   (b) The surface area of a sphere is $4\pi r^2$, so the surface area of Earth is $4\pi(3960)^2 \approx 197{,}061{,}000$ sq. mi.; Part of surface covered by the oceans is $(0.71)(197{,}061{,}000) \approx 139{,}913{,}000$ sq. mi.;

   $$\text{rise in sea leve} = \frac{\text{volume of water}}{\text{surface area of the oceans}} = \frac{680{,}000 \text{ mi.}^3}{139{,}913{,}000 \text{ mi.}^2} \approx 0.00486 \text{ mi.} \approx 25.7 \text{ ftl}$$

   (c) Since the average elevations of Boston, New Orleans, and San Diego are all less than 25 feet, these cities would be under water without some type of dike system.

   (d) $\text{Rise in sea level} = \dfrac{6{,}300{,}000 \text{ mi.}^3}{139{,}913{,}000 \text{ mi.}^2} \approx 0.04503 \text{ mi.} \approx 238 \text{ ft.}$

10. Begin by assuming that $\sqrt{2}$ is rational. Then $\sqrt{2} = \dfrac{a}{b}$ for some integers a and b, where a and b have no common factors. Squaring both sides of this equation gives $2 = \dfrac{a^2}{b^2}$ and so $a^2 = 2b^2$. That is, a^2 is even, which implies that a itself must be even. If a is even, $a = 2c$ for some integer c. Substituting $2c$ for a in $a^2 = 2b^2$ results in $4c^2 = 2b^2$ or $2c^2 = b^2$; Thus, b^2 is even which implies that b itself must be even. Now, both a and b are even which is a contradiction since a and b have no common factors. We conclude that $\sqrt{2}$ is irrational.