

Elements of Quality Assurance and Acceptance Sampling

► 10.1 INTRODUCTION

The assurance of product quality in manufacturing has long been a problem of industrial and production engineers. Quality assurance, however, is of concern to all engineers. Compliance with minimum standards of construction and fabrication, and of quality in materials and workmanship, is necessary to ensure the design performance capability of an engineering system. For these purposes, acceptance criteria and acceptance sampling programs are necessary. For example, in the construction of highway pavements, acceptance criteria are necessary to ensure compliance with construction specifications; similarly, in stream pollution control, inspection plans are necessary to enforce water quality standards.

Probability concepts and statistical techniques are pertinent and useful to a variety of quality assurance problems. In this chapter we present and develop those statistical concepts that form the bases of some commonly used acceptance sampling programs. Such programs are of two types—*sampling by attributes* and *sampling by variables*.

► 10.2 ACCEPTANCE SAMPLING BY ATTRIBUTES

When a *lot* of material of size N is submitted for inspection, a sample of n items may be selected at random from the lot and subjected to inspection and testing. In acceptance sampling by attributes, each of these n items is classified as *good* (acceptable) or *bad* (defective) after the test. It is a common criterion that if more than r defective items are found from the sample of n , the lot will be rejected. Conversely, the lot will be accepted if there are r or less defectives. If among the lot of size N , the actual fraction of defectives is p , then the total number of defective items in the sample of size n is described by the hypergeometric distribution (see Section 3.2.9), and the *probability of accepting the lot* is accordingly given by

$$g(p) = \sum_{x=0}^r \frac{\binom{Np}{x} \binom{Nq}{n-x}}{\binom{N}{n}} \quad (10.1)$$

where $q = 1 - p$. If n is small relative to N , it can be shown (Hald, 1952) that $g(p)$ in Eq. 10.1 can be approximated by

$$g(p) \simeq \sum_{x=0}^r \binom{n}{x} p^x q^{n-x} \quad (10.2)$$

which involves the binomial distribution. Equation 10.2 can also be written as

$$g(p) = 1 - \sum_{x=r+1}^n \binom{n}{x} p^x q^{n-x} \quad (10.2a)$$

10.2.1 The Operating Characteristic (OC) Curve

The function $g(p)$ of Eq. 10.2 is referred to as the OC curve (*operating characteristic curve*). Examples of the OC curve are shown in Fig. 10.1 for various sampling plans (with different combinations of n and r). It can be observed from each of the OC curves in Fig. 10.1 that as the fraction of defective items increases, the probability that the lot will be accepted decreases. For example, according to the sampling plan of Fig. 10.1b, with $n = 15$ and $r = 1$, there is about 10% probability that a lot with 24% defective items will be accepted, whereas the probability of acceptance increases to 88% for a lot with 4% defective. From the appropriate OC curve, therefore, we can read off the probabilities of accepting and rejecting lots containing various percentages of defective items.

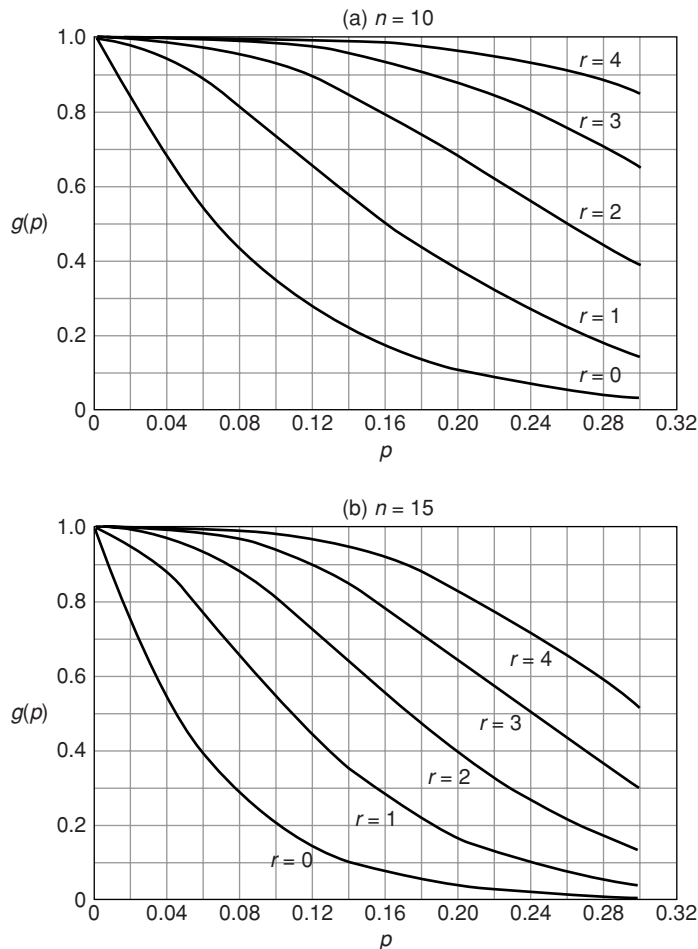


Figure 10.1a and b Operating characteristic (OC) curves. (a) $n = 10$. (b) $n = 15$.

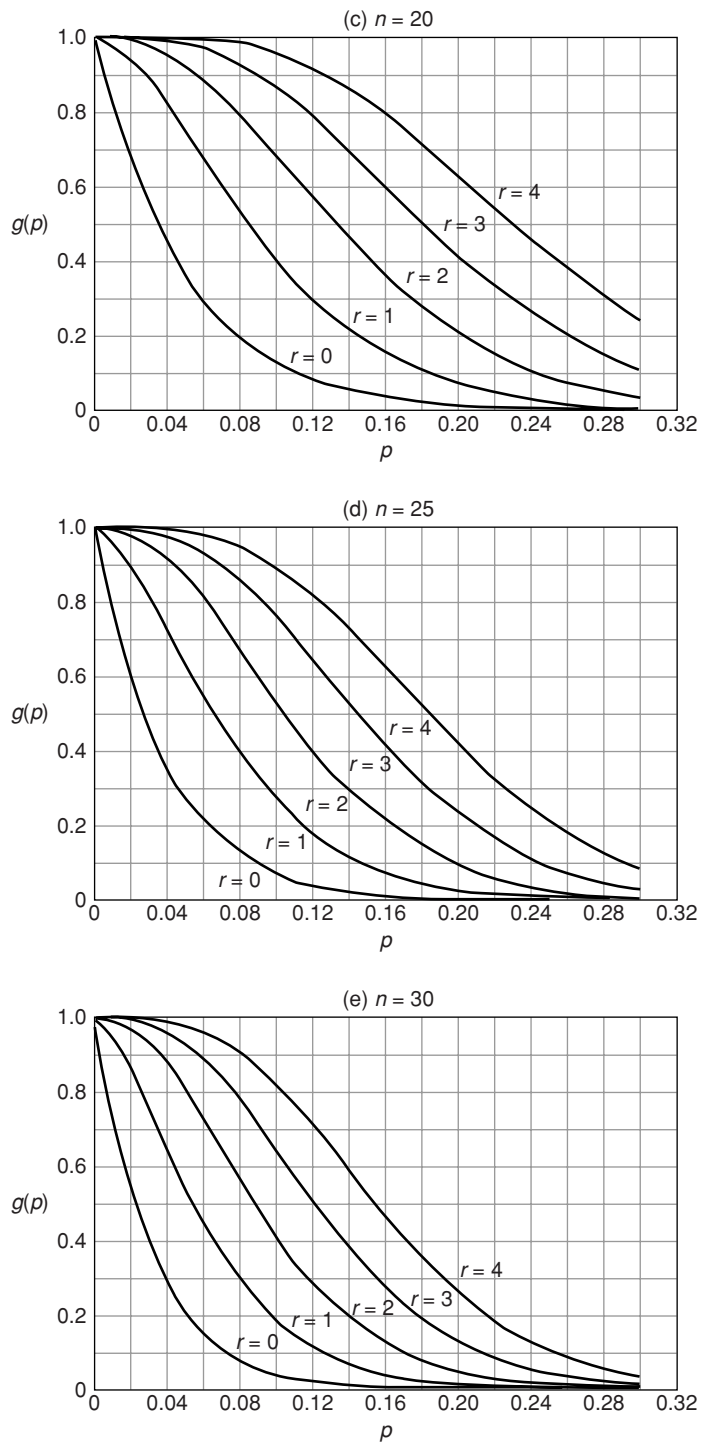


Figure 10.1c, d, and e Operating characteristic (OC) curves.
 (c) $n = 20$. (d) $n = 25$. (e) $n = 30$.

Generally, in determining the optimal inspection plan, it should be kept in mind that the plan has to be accepted by both the *supplier* and *receiver* of the lot.

- For the supplier, it is desirable that the plan have a low probability of rejecting a lot in which the actual fraction of defective items p is less than p_1 , the maximum fraction of defective units permitted in good-quality lots.
- For the receiver, it is desirable that there be a low probability of accepting a lot if p exceeds p_2 , the minimum fraction of defective units sufficient to define poor-quality lots.

The supplier's risk of rejecting good-quality lots and the receiver's risk of accepting inferior-quality lots may be referred to as the *producer's risk* and the *consumer's risk*, respectively. The optimal inspection plan should have values of n and r such that the corresponding OC curve will mutually satisfy these risk levels.

► EXAMPLE 10.1

In the construction of an earth embankment, the fill material is compacted to a specified CBR (California Bearing Ratio). Suppose that unsatisfactory performance will result if more than 15% of the fill falls below the specified CBR limit; however, the best-quality compaction that can be expected at the contract price would contain 1% of the embankment falling below the specified CBR limit. Assume a 5% risk for both the producer and consumer; what values of n and r should be used in the quality control program?

Using the approximation of Eq. 10.2, the conditions to be satisfied are

$$g(0.01) = \sum_{x=0}^r \binom{n}{x} (0.01)^x (0.99)^{n-x} = 0.95$$

and

$$g(0.15) = \sum_{x=0}^r \binom{n}{x} (0.15)^x (0.85)^{n-x} = 0.05$$

The required values of n and r may be determined by trial and error from the solutions to the two simultaneous equations given above. Alternatively, we can examine the available OC curves (such as Fig. 10.1) and select the appropriate one. It may be observed from Fig. 10.1e that $n = 30$ and $r = 1$ will suffice. ◀

► EXAMPLE 10.2

The density of asphalt concrete in a roadway is to be inspected. Twenty specimens (15-in. square each) were cored at random locations over a 5-mile stretch of the roadway. Laboratory tests show that only one of the specimens has a density below the specified limit. Suppose that the maximum permissible fraction of defective asphalt concrete is 15% and it is desired to limit the risk of accepting inferior quality material to 4%. Should the asphalt concrete be rejected?

The acceptance function $g(p)$ in Eq. 10.2 can be applied. In this case the criterion for accepting the asphalt concrete is

$$g(0.15) \leq 0.04$$

For $n = 20$ and $r = 1$, Eq. 10.2 gives

$$\begin{aligned} g(0.15) &\simeq \binom{20}{0} (0.15)^0 (0.85)^{20} + \binom{20}{1} (0.15)^1 (0.85)^{19} \\ &= 0.0388 + 0.1374 \\ &= 0.1762 \end{aligned}$$

Since $g(0.15)$ exceeds 0.04, the asphalt concrete should be rejected. In this case, for the roadway to be acceptable, all the 20 cored specimens must have at least the minimum specified density;