# Chapter 2 How to Calculate Present Values

### OVERVIEW

This chapter introduces the concept of present value and shows why a firm should maximize the market value of the stockholders’ stake in it. It describes the mechanics of calculating present values of lump sum amounts, perpetuities, annuities, growing perpetuities, growing annuities and unequal cash flows. Other related topics like simple interest, frequent compounding, continuous compounding, and nominal and effective interest rates are discussed. The net present value rule and the rate of return rule are explained in great detail.

### LEARNING OBJECTIVES

* To learn how to calculate present value of lump sum cash flows.
* To understand and use the formulas associated with the present value of perpetuities;

growth perpetuities; annuities; and growing annuities.

* To understand more frequent compounding including continuous compounding.
* To understand the important difference between nominal and effective interest rates.
* To understand value-additive property and the concept of arbitrage.
* To understand the net present value rule and the rate of return rule.

### CHAPTER OUTLINE

**Future values and present values**

The concepts of future value, present value, net present value (NPV) and the opportunity cost of capital (hurdle rate) are introduced. The authors show, using several numerical examples, that simple projects with rates of return exceeding the opportunity cost of capital have positive net present values. The “Net present value rule” and the “Rate of return rule” are stated here.

This chapter also extends the concept of discounting to assets, which produce a series of cash flows. The possibility of arbitrage restricts the relative values of discount factors DF1, DF2,…. DFt –.The main point is that money machines cannot exist in well-functioning financial markets. Using numerical examples it shows how to calculate PV and NPV of a series of cash flows over a number of periods (years).

**Looking for shortcuts – perpetuities and annuities**

This section is devoted to developing formulae for perpetuities and annuities. It explains the difference between an ordinary annuity and an annuity due. It also explains how the future value of an annuity is calculated. The present value of an annuity can be thought of as the difference between two perpetuities

beginning at different times. Using this simple idea, the formula for the present value of an annuity is derived. The future value of an annuity formula is also derived. These have numerous applications in pension funds, mortgages and valuation of financial assets.

**More shortcuts – growing perpetuities and annuities**

Some applications need the present value of a perpetual cash flow growing at a constant rate, as well as annuities that grow at a constant rate. The formula for the present value of a growing perpetuity is derived. The present value of a growing annuity can be thought of as the difference between two growing perpetuities starting at different times. Using this simple idea, the formula for the present value of a growing annuity is also derived. These formulas have many applications in the valuation of assets.

**How Interest Is Paid and Quoted**

This section explains the differences between compound interest and simple interest, as well as the differences between effective annual rates and annual percentage rates. It deals with how each interest rate is used in the market place and the math necessary to move between the two kinds of interest rates.

### TEACHING TIPS FOR POWERPOINT SLIDES

**Slide 1 - Title slide**

**Slide 2 – Topics covered**

**Slide 3**

Explain the terms “future value” and “present value”. The concept must be emphasized at this point. Consequently, it may be necessary to spend some time explaining real world examples of how present value and future value relate. A good example to use is retirement planning.

**Slide 4**

FV = PV× [(1 + r)^ t ]

Define the terms: FV = Future value  
 PV = Present value  
 r = interest rate  
 t = number of years (Periods)

Explain the time value of money and its importance to financial decision making.

**Slide 5**

Walk through each step in the math process and show how the value increases. If you plan to have your students use a financial calculator, you can skip the details of the basic math. Be aware, students often stumble when doing simple math calculations.

**Slide 6**

The longer the funds are invested, the greater the advantage with compound interest. Discuss the four examples and be sure to use the phrase “power of compounding.”

**Slide 7**

This slide contains the Present Value formula.

**Slide 8**

The Discount Factor (DF) is the present value of $1 expected to be received in the future. Here it is appropriate to introduce the use of the financial calculator to solve these problems.

**Slide 9**

Here we reverse the future value process from earlier. Show students how they can easily move between future value and present value with the basic formulae.

**Slide 10**

For visual learners, this graph illustrates the reverse of the future value compounding chart shown earlier. It is downwardly sloping, which can confuse students, so it may be necessary to spend time explaining the concept.

**Slide 11**

Explain how the present value concept discussed earlier is useful in valuing assets.

Cost of the building = $700,000

Sale price in Year –1 = $800,000

Opportunity cost of capital = 7%

**Slide 12**

Discount future cash flows at the opportunity cost of capital

PV = 800,000/(1.07) = 747,664

NPV = PV – required investment

NPV = 747,664 – 700,000 = 47,664

Explain the difference between PV and NPV. Explain sign conventions for cash flows.

**Slide 13**

Explain each variable in the equation. It is easy to tell the students that all present values come at a cost. That cost is the initial investment. This may help them easily transition from present value to net present value.

C0 = initial investment for the project. Normally it is a cash outflow and has a negative sign (-)

C1 = cash inflow from the project. Normally it has a positive sign (+)

r = opportunity cost of capital

Positive NPVs increase the value of a firm. Negative NPVs lower the value of a firm.

**Slide 14**

The concept of risk is introduced here. Briefly explain the idea of risk (lottery vs. bank deposit). Generally, investors do not like risk. In order to induce the investors invest in risky projects, a higher rate of return is needed. Higher rate of return causes lower PVs. Explain the relationship between discount rates and present values. Higher the discount rate, lower the present value.

**Slide 15 & 16**

Explain the relationship between discount rates and net present values. Higher the discount rate, the lower the net present value.

NPV at 12% : NPV = 714,286 - 700,000 = 14,286

NPV at 7% : NPV = 747,664 - 700,000 = 47,664

NPV at 5% : NPV = 761 - 700,000 = 61,904

**Slide 17**

Net Present Value Rule

Accept if NPV > 0: A very powerful financial decision-making rule. It looks simple but can get complicated quickly. This project is acceptable as the NPV > 0. Make sure that students understand this rule clearly.

**Slide 18**

This slide explains the rate of return rule.

**Slide 19**

Multiple cash flows occurring at different time periods can be evaluated using the DCF formula. It is a simple extension of the NPV formula, but can intimidate students because of the extra equations. Show how it is a minor extension of the prior Basic Point Value formula.

**Slide 20**

The graphic presentation of the net present value of multiple cash flows or sequential cash flows is given here. Here we extend the concept of PV to a series of cash flows by applying value-additive property of present values. These cash flows can be positive (cash inflows) or negative (cash outflows). We merely add the initial cost to make it NPV.

**Slide 21**

Depending on the type of cash flow you can use the formulas to simplify the calculations. There are formulas that can be used for finding the present values for cash flows with a pattern; for example perpetuities and annuities. Define perpetuity (same cash flow each year forever) and give an example of perpetuity.

**Slide 22**

Introduce the perpetuity concept as one in which you earn money forever. In doing so, you can easily demonstrate the return an investor earns.

**Slide 23**

Now, manipulate the formula to get the value of the infinite cash flow given a discount rate. Provide the formula for calculating the present value perpetuity. This formula is obtained using an algebraic technique; sum of an infinite geometric series.

**Slide 24**

Present value of $1 billion received forever at 10% is: PV = $1/0.1 = $10 billion. Using the formula will simplify the calculations.

**Slide 25**

The same example is used as in the previous slide, except the modification of time is added. Show the students how the value is reduced if you get the money later. This reinforces the time value of money concepts introduced earlier.

**Slide 26**

This slide provides the formula for the present value of an annuity. An annuity can be thought of as the difference between two perpetuities starting at different times. A slight derivation is presented, can be ignored if it is beyond the scope of the course, with no harm in understanding the broader concept.

**Slide 27**

This is the PVAF formula. Take some time to explain the variables. If a financial calculator is to be used in class, there is no need to cover the use in detail.

**Slide 28**

This slide is a more comprehensive example of an annuity and its relationship to perpetuities.

**Slide 29**

An asset that pays a fixed sum each period for a specified number of periods is called an annuity. For example: the present value of annual payments of $5,000 per year for five years is presented. In addition to the formula, using a financial calculator: PMT = 5,000; I = 7; N = 5; FV = 0 and compute PV = 20,501.

**Slide 30**

The state lottery pays the jackpot prize in 30 annual installments. If Internet access is available, it might be helpful to pull up an actual national lottery and determine how the lump sum payout was determined. The instructor should first calculate the internal rate of return and provided as a given number to the students.

**Slide 31**

This is the formula that reverses the PVAF math for future values.

**Slide 32**

This is an example of the Future Value of an annuity. It is highly recommended this example also be provided with a financial calculator.

**Slide 33**

This is another example of an annuity. In this case we are determining the payment necessary on a loan.

**Slide 34**

Has was done for the present value of an annuity earlier, this slide presents the future value of an annuity.

**Slide 35**

Continuing the theme of the prior slide, I hear we have an example of the future value of an annuity.

**Slide 36**

This formula is the present value of a perpetuity that is growing at a constant rate, where g is the annual growth rate of the cash flow; and r > g. It is useful and should be explained as a formula the students will use often.

**Slide 37**The formula can be used to evaluate a perpetuity starting at any point in time, where g is the annual growth rate of the cash flow; and r > g.

**Slide 38**

The present value of $1 billion (first payment starting one year from today) perpetuity that is growing at a constant rate of 4% and requires a rate of return of 10% is: PV0 = C1/(r-g) = 1/(0.1 - 0.04) = 16.667 billion. This is $6.667 billion more that the perpetuity without growth.

**Slide 39**

Go over each definition: EAR and APR. Students need to know how each is used in the market and why APR is quoted rather than EAR.

**Slide 40**

The basic formula for APR and EAR are presented. In reality, students are more likely to use the spreadsheet or financial calculator.

**Slide 41 & 42**

Go over the math of the problem. If students are comfortable with the use of a financial calculator, use the following method as a substitute for the formula. Using the financial calculator: PMT = 0; I = 1%; N = 12; PV = 1; FV = -1.1268. By dropping the (-1) the answer is arrived at of 12.68%.

**KEY TERMS AND CONCEPTS**

Present value, discount factor, discount rate, hurdle rate, opportunity cost of capital, net present value, net present value rule, rate of return rule, discounted cash flow, perpetuity, growing perpetuity, annuity, growing annuity, compound interest, simple interest, continuous compounding, annual percentage rate, effective annual rate.

### CHALLENGE AREAS

The link between financial markets and NPV

### WEB LINKS

[finance.yahoo.com](http://finance.yahoo.com)

[www.maxigrade.com/CorpFin1/corpfin1samplequestions2.php](http://www.maxigrade.com/CorpFin1/corpfin1samplequestions2.php)

[www.empowerosity.com/FinancePractice/](http://www.empowerosity.com/FinancePractice/)

### ADDITIONAL REFERENCES

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Copeland,T.E, J.F. Weston, and K. Shastri, “*Financial Theory and Corporate Policy*,” Fourth Edition,

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