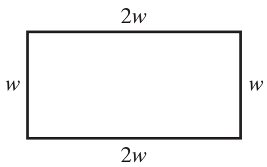
Chapter 1

Problems 1.1

1. Let w be the width and $2w$ be the length of the plot.



Then area = 800.

$(2w)w = 800$

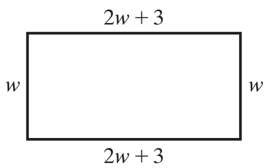
$2w^2 = 800$

$w^2 = 400$

$w = 20$ ft

Thus the length is 40 ft, so the amount of fencing needed is $2(40) + 2(20) = 120$ ft.

2. Let w be the width and $2w + 3$ be the length.



Then perimeter = 300.

$$\begin{aligned} 2w + 2(2w + 3) &= 300 \\ 6w + 6 &= 300 \\ 6w &= 294 \\ w &= 49 \text{ ft} \end{aligned}$$

Thus the length is $2(49) + 3 = 101$ ft.

The dimensions are 49 ft by 101 ft.

3. Let n = number of ounces in each part. Then we have

$4n + 5n = 145$

$9n = 145$

$n = 16\frac{1}{9}$

Thus there should be $4\left(16\frac{1}{9}\right) = 64\frac{4}{9}$ ounces of A and $5\left(16\frac{1}{9}\right) = 80\frac{5}{9}$ ounces of B.

4. Let n = number of cubic feet in each part.

Then we have

$1n + 3n + 5n = 765$

$9n = 765$

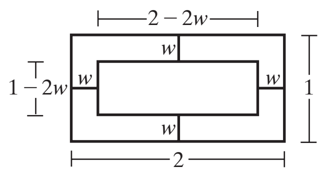
$n = 85$

Thus he needs $1n = 1(85) = 85$ ft^3 of portland cement, $3n = 3(85) = 255$ ft^3 of sand, and

$5n = 5(85) = 425$ft^3 of crushed stone.

5. From the data it follows that whipping cream accounts for 8/15 of a batch of ice cream. Thus for 3000 millilitres (3 litres) of ice cream, $(8/15)3000 = 1600$ millilitres of whipping cream will be needed.

6. Let w = width (in miles) of strip to be cut. Then the remaining forest has dimensions $2 - 2w$ by $1 - 2w$.



Considering the area of the remaining forest, we have

$(2 - 2w)(1 - 2w) = \frac{3}{4}$

$2 - 6w + 4w^2 = \frac{3}{4}$

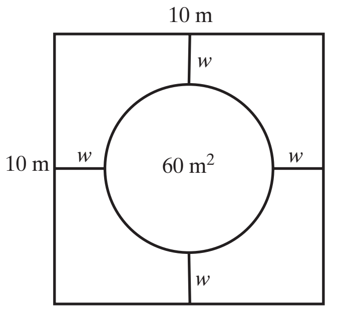
$8 - 24w + 16w^2 = 3$

$16w^2 - 24w + 5 = 0$

$(4w - 1)(4w - 5) = 0$

Hence $w = \frac{1}{4}, \frac{5}{4}$. But $w = \frac{5}{4}$ is impossible since one dimension of original forest is 1 mi. Thus the width of the strip should be $\frac{1}{4}$ mi.

7. Let w = "width" (in meters) of the pavement. Then $5 - w$ is the radius of the circular flower bed.



Thus

$$\pi r^2 = A$$
$$\pi(5-w)^2 = 60$$
$$w^2 - 10w + 25 = \frac{60}{\pi}$$
$$w^2 - 10w + \left(25 - \frac{60}{\pi}\right) = 0$$

$a = 1, b = -10, c = 25 - \dfrac{60}{\pi}$

$$w = \frac{-b \pm \sqrt{100 - 4(1)\left(25 - \frac{60}{\pi}\right)}}{2} \approx 9.37, 0.63$$

Since $0 < w < 5$, $w \approx 0.63$ m.

8. Since diameter of circular end is 140 mm, the radius is 70 mm. Area of circular end is $\pi(\text{radius})^2 = \pi(70)^2$. Area of square end is x^2.

Equating areas, we have $x^2 = \pi(70)^2$.

Thus $x = \pm\sqrt{\pi(70)^2} = \pm 70\sqrt{\pi}$. Since x must be positive, $x = 70\sqrt{x} \approx 124$ mm.

9. Let q = number of tons for \$560,000 profit.

Profit = Total Revenue – Total Cost

$560{,}000 = 134q - (82q + 120{,}000)$

$560{,}000 = 52q - 120{,}000$

$680{,}000 = 52q$

$\dfrac{680{,}000}{52} = q$

$q \approx 13{,}076.9 \approx 13{,}077$ tons.

10. Let the number of sales units required be n. Since profit is total revenue − total cost, we require

$$550n - (250n + 5{,}000{,}000) \geq 1{,}500{,}000$$

which is equivalent to $300n \geq 6{,}500{,}000$ amd hence $n \geq 21{,}666.666\ldots$. Since n must be an integer, we require $n \geq 21{,}667$.

11. Let x = amount at 6% and

$20{,}000 - x$ = amount at $7\frac{1}{2}\%$.

$x(0.06) + (20{,}000 - x)(0.075) = 1440$

$-0.015x + 1500 = 1440$

$-0.015x = -60$

$x = 4000$, so $20{,}000 - x = 16{,}000$. Thus the investment should be \$4000 at 6% and \$16,000 at $7\frac{1}{2}\%$.

12. Let x = amount at 4% and

$120{,}000 - x$ = amount at 5%.

$$0.04x + 0.05(120{,}000 - x) = 0.045(120{,}000)$$
$$-0.01x + 6000 = 5400$$
$$-0.01x = -600$$
$$x = 60{,}000$$

The investment consisted of \$60,000 at 5% and \$60,000 at 4%.

13. Let p = selling price. Then profit = $0.2p$. selling price = cost + profit

$p = 3.40 + 0.2p$

$0.8p = 3.40$

$p = \dfrac{3.40}{0.8} = \$4.25$

14. Following the procedure in Example 6 we obtain the total value at the end of the second year to be $1{,}000{,}000(1+r)^2$.

So at the end of the third year, the accumulated amount will be $1{,}000{,}000(1+r)^2$ plus the interest on this, which is $1{,}000{,}000(1+r)^2 r$.

Thus the total value at the end of the third year will be $1{,}000{,}000(1+r)^2 + 1{,}000{,}000(1+r)^2 r$

$= 1{,}000{,}000(1+r)^3$.

This must equal \$1,125,800.

$$1{,}000{,}000(1+r)^3 = 1{,}125{,}800$$
$$(1+r)^3 = \frac{1{,}125{,}800}{1{,}000{,}000} = 1.1258$$
$$1 + r \approx 1.04029$$
$$r \approx 0.04029$$

Thus $r \approx 0.04029 \approx 4\%$.

15. Let the required interest rate be r. They require $3{,}000{,}000(1+r)^3 \geq 3{,}750{,}000$ which is equivalent to $(1+r)^3 \geq \frac{375}{300}$ and hence $r \geq \sqrt[3]{\frac{375}{300}} - 1 = 0.077217345$. This means the rate must be greater than or equal to 7.7217345%.

16. Total revenue = variable cost + fixed cost
$100\sqrt{q} = 2q + 1200$

$50\sqrt{q} = q + 600$

$2500q = q^2 + 1200q + 360{,}000$

$0 = q^2 - 1300q + 360{,}000$

$0 = (q-400)(q-900)$

$q = 400$ or $q = 900$

17. Let n = number of bookings.

$0.90n = 81$

$n = 90$ seats booked

18. Let n = number of people polled.

$0.20p = 700$

$p = \frac{700}{0.20} = 3500$

19. Let s = monthly salary of deputy sheriff.

$0.30s = 200$

$s = \frac{200}{0.30}$

Yearly salary $= 12s = 12\left(\frac{200}{0.30}\right) = \$8000$

20. The lost wages, of each nurse, during the strike was $(21.50)(8)(27) = 4644$ dollars. Before the strike, a nurse earned $(21.50)(8)(260) = 44{,}720$ dollars in a year. In order to make up the lost salary in one year, the nurses will need to make $44{,}720 + 4644 = 49{,}364$ dollars a year. Because $\frac{4644}{44{,}720} = 0.103846154 = 10.3846154\%$, the nurses will need to get a 10.3846154% increase to make up for lost wages in a year. Notice that this percentage increase is actually $\frac{27}{260}$ and independent of a nurse's previous wage. It is also not necessary to calculate the new yearly wage to get the required percentage increase. Be sure to understand this point.

21. Let q = number of cartridges sold to break even.

total revenue = total cost

$$\begin{aligned} 21.95q &= 14.92q + 8500 \\ 7.03q &= 8500 \\ q &\approx 1209.10 \end{aligned}$$

1209 cartridges must be sold to approximately break even.

22. Let n = number of shares.

total investment $= 5000 + 20n$

$$\begin{aligned} 0.04(5000) + 0.50n &= 0.03(5000 + 20n) \\ 200 + 0.50n &= 150 + 0.60n \\ -0.10n &= -50 \\ n &= 500 \end{aligned}$$

500 shares should be bought.

23. Let v = total annual vision-care expenses (in dollars) covered by program. Then

$35 + 0.80(v - 35) = 100$

$0.80v + 7 = 100$

$0.80v = 93$

$v = \$116.25$

24. **a.** $0.031c$

b. $c - 0.031c = 600{,}000{,}000$

$0.969c = 600{,}000{,}000$

$c \approx 619{,}195{,}046$

Approximately 619,195,046 bars will have to be made.

25. From the formula for revenue it is evident that we require

$$((60-q)/3)q = 300$$

It follows that we must solve $60q - q^2 = 900$ which is equivalent to $(q-30)^2 = 0$ and has the unique solution $q = 30$.

26. If I = interest, P = principal, r = rate, and t = time, then $I = Prt$. To triple an investment of P at the end of t years, the interest earned during that time must equal $2P$. Thus $2P = P(0.045)t$

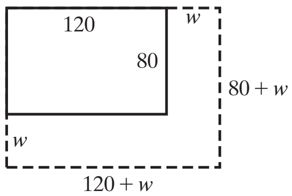
$2 = 0.045t$

$t = \frac{2}{0.045} \approx 44.4$ years

27. Let q = required number of units. Equate the incomes of each proposal.

$$5000 + 0.50q = 50{,}000$$
$$0.50q = 45{,}000$$
$$q = 90{,}000 \text{ units}$$

28. Let w = width of strip. The original area is 80(120) and the new area is $(120 + w)(80 + w)$.



Thus

$$(120 + w)(80 + w) = 2(80)(120)$$
$$9600 + 200w + w^2 = 19{,}200$$
$$w^2 + 20w - 9600 = 0$$
$$(w + 240)(w - 40) = 0$$
$$w = -240 \text{ or } w = 40$$

We choose $w = 40$ ft.

29. Let n = number of \$20 increases. Then at the rental charge of $400 + 20n$ dollars per unit, the number of units that can be rented is $50 - 2n$.

The total of all monthly rents is

$(400 + 20n)(50 - 2n)$, which must equal 20,240.

$$20{,}240 = (400 + 20n)(50 - 2n)$$
$$20{,}240 = 20{,}000 + 200n - 40n^2$$
$$40n^2 - 200n + 240 = 0$$
$$n^2 - 5n + 6 = 0$$
$$(n - 2)(n - 3) = 0$$
$$n = 2, 3$$

Thus the rent should be either

$\$400 + 2(\$20) = \$440$ or
$\$400 + 3(\$20) = \$460$.

30. Let the original blue-chip and glamour investments be b and g respectively. We have $b + g = 9{,}500{,}000$ and $(9/8)b + (11/12)g = 9{,}700{,}000$. (We need to find $(9/8)b$.) Since $g = 9{,}500{,}000 - b$, we have

$$(9/8)b + (11/12)(9{,}500{,}000 - b) = 9{,}700{,}000$$

It follows that $((27 - 22)/24)b = 9{,}700{,}000 - 11/12(9{,}500{,}000)$ from which we get $(5/24)b = (11{,}900{,}000)/12$ and $b = 4{,}760{,}000$. It follows that the *current* value of the blue chip investment is $(9/8)(4{,}760{,}000) = 5{,}355{,}000$

31. $10{,}000 = 800p - 7p^2$

$$7p^2 - 800p + 10{,}000 = 0$$
$$p = \frac{800 \pm \sqrt{640{,}000 - 280{,}000}}{14}$$
$$= \frac{800 \pm \sqrt{360{,}000}}{14} = \frac{800 \pm 600}{14}$$

For $p > 50$ we choose $p = \dfrac{800 + 600}{14} = \$100$.

32. Let p be the percentage change in market value.

$$(1 + 0.15)\left(\frac{P}{E}\right) = \frac{(1 + p)P}{(1 - 0.10)E}$$
$$1.15 = \frac{1 + p}{0.90}$$
$$1.035 = 1 + p$$
$$p = 0.035 = 3.5\%.$$

The market value increased by 3.5

33. To have supply = demand,

$$2p - 10 = 200 - 3p$$
$$5p = 210$$
$$p = 42$$

34. $2p^2 - 3p = 20 - p^2$

$$3p^2 - 3p - 20 = 0$$
$$a = 3, b = -3, c = -20$$
$$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(3)(-20)}}{2(3)}$$
$$= \frac{3 \pm \sqrt{249}}{6}$$

$p \approx 3.130$ or $p \approx -2.130$

The equilibrium price is $p \approx 3.13$.

35. Let l be the length of the side parallel to the enclosing wall of the building and w the length of the other two sides. We have $l + 2w = 250$ and $lw = 7762.5$. It follows that $l(250 - l)/2 = 7762.5$ so that we require

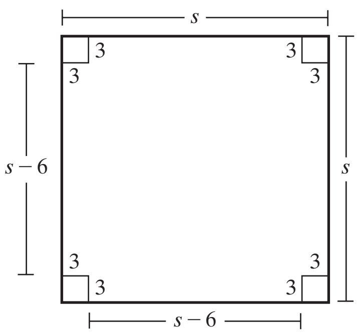
$$250l - l^2 = 15{,}525$$

which leads to $l =$

$$\begin{aligned}\frac{250 \pm \sqrt{(250)^2 - 62{,}100}}{2} &= \frac{250 \pm \sqrt{400}}{2} \\ &= 125 \pm 10 \\ &= 135 \text{ or } 115\end{aligned}$$

We reject 135 because the building is only 130 m long. (Consult your diagram. It follows that the dimensions of the rectangular area are 115 m by 67.5 m.

36. Let s = length in inches of side of original square.



Considering the volume of the box, we have
(length)(width)(height) = volume
$(s - 4)(s - 4)(2) = 50$

$(s - 4)^2 = 25$

$s - 4 = \pm\sqrt{25} = \pm 5$

$s = 4 \pm 5$

Hence $s = -1, 9$. We reject $s = -1$ and choose $s = 9$. The dimensions are 9 in. by 9 in.

37. Original volume $= (10)(5)(2) = 100 \text{ cm}^3$

Volume increase $= 0.50(100) = 50 \text{ cm}^3$

Volume of new bar $= 100 + 50 = 150 \text{ cm}^3$

Let x = number of centimeters that the length and width are each increased. Then

$2(x + 10)(x + 5) = 150$

$x^2 + 15x + 50 = 75$

$x^2 + 15x - 25 = 0$

$a = 1, b = 15, c = -25$

$$x = \frac{-15 \pm \sqrt{15^2 - 4(1)(-25)}}{2} \approx 1.51, -16.51$$

We reject -16.51 as impossible. The new length is approximately 11.51 cm, and the new width is approximately 6.51 cm.

38. Volume of old style candy
$= \pi(7.1)^2(2.1) - \pi(2)^2(2.1)$

$= 97.461\pi \text{ mm}^3$

Let r = inner radius (in millimeters) of new style candy. Considering the volume of the new style candy, we have

$$\begin{aligned}\pi(7.1)^2(2.1) - \pi r^2(2.1) &= 0.78(97.461\pi) \\ 29.84142\pi &= 2.1\pi r^2 \\ 14.2102 &= r^2 \\ r &\approx \pm 3.7696\end{aligned}$$

Since r is a radius, we choose $r = 3.77$ mm.

39. Let x = amount of loan. Then the amount actually received is $x - 0.16x$. Hence,

$x - 0.16x = 195{,}000$

$0.84x = 195{,}000$

$x \approx 232{,}142.86$

To the nearest thousand, the loan amount is \$232,000. In the general case, the amount received from a loan of L with a compensating balance of p% is $L - \frac{p}{100}L$.

$L - \frac{p}{100}L = E$

$\frac{100 - p}{100}L = E$

$L = \frac{100E}{100 - p}$

40. Let the number of machines sold be n.
If $n \geq 500$ then the commission paid is $n(50 + (n - 500)(0.05))$. For this to be at least \$33,000 we require $n(50 + (n - 500)(0.05)) \geq 33{,}000$, equivalently $0.05n^2 + 25n - 33{,}000 \geq 0$, equivalently $n^2 + 500n - 660{,}000 \geq 0$. The roots of the corresponding equation are easily seen to be -250 ± 1700. We reject the negative root and see, from the nature of the quadratic, that we require $n \geq 1450$.

41. Let n = number of acres sold. Then $n + 20$ acres were originally purchased at a cost of $\frac{7200}{n+20}$ each. The price of each acre sold was $30 + \left[\frac{7200}{n+20}\right]$. Since the revenue from selling n acres is \$7200 (the original cost of the parcel), we have

$$n\left[30 + \frac{7200}{n+20}\right] = 7200$$

$$n\left[\frac{30n + 600 + 7200}{n+20}\right] = 7200$$

$$n(30n + 600 + 7200) = 7200(n + 20)$$

$$30n^2 + 7800n + 7200n + 144{,}000$$

$$30n^2 + 600n - 144{,}000 = 0$$

$$n^2 + 20n - 4800 = 0$$

$$(n + 80)(n - 60) = 0$$

$n = 60$ acres (since $n > 0$), so 60 acres were sold.

42. Let q = number of units of product sold last year and $q + 2000$ = the number sold this year. Then the revenue last year was $3q$ and this year it is $3.5(q + 2000)$. By the definition of margin of profit, it follows that

$$\frac{7140}{3.5(q+2000)} = \frac{4500}{3q} + 0.02$$

$$\frac{2040}{q+2000} = \frac{1500}{q} + 0.02$$

$$2040q = 1500(q + 2000) + 0.02q(q + 2000)$$

$$2040q = 1500q + 3{,}000{,}000 + 0.02q^2 + 40q$$

$$0 = 0.02q^2 - 500q + 3{,}000{,}000$$

$$q = \frac{500 \pm \sqrt{250{,}000 - 240{,}000}}{0.04}$$

$$= \frac{500 \pm \sqrt{10{,}000}}{0.04}$$

$$= \frac{500 \pm 100}{0.04}$$

$$= 10{,}000 \text{ or } 15{,}000$$

So that the margin of profit this year is not greater than 0.15, we choose $q = 15{,}000$. Thus 15,000 units were sold last year and 17,000 this year.

43. Let q = number of units of B and

$q + 25$ = number of units of A produced.

Each unit of B costs $\frac{1000}{q}$, and each unit of A costs $\frac{1500}{q+25}$. Therefore,

$$\frac{1500}{q+25} = \frac{1000}{q} + 2$$

$$1500q = 1000(q + 25) + 2(q)(q + 25)$$

$$0 = 2q^2 - 450q + 25{,}000$$

$$0 = q^2 - 225q + 12{,}500$$

$$0 = (q - 100)(q - 125)$$

$$q = 100 \text{ or } q = 125$$

If $q = 100$, then $q + 25 = 125$; if $q = 125$, $q + 25 = 150$. Thus the company produces either 125 units of A and 100 units of B, or 150 units of A and 125 units of B.

Apply It 1.2

1. $200 + 0.8S \geq 4500$

$0.8S \geq 4300$

$S \geq 5375$

He must sell at least 5375 products per month.

2. Since $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$, and $x_4 \geq 0$, we have the inequalities,

$150 - x_4 \geq 0$

$3x_4 - 210 \geq 0$

$x_4 + 60 \geq 0$

$x_4 \geq 0$

Problems 1.2

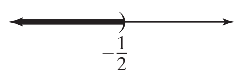
1. $x > 7$

2. $4x < -2$

$x < \frac{-2}{4}$

$x < -\frac{1}{2}$

$\left(-\infty, -\frac{1}{2}\right)$

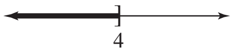


3. $5x - 11 \le 9$

$5x \le 20$

$x \le 4$

$(-\infty, 4]$

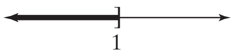


4. $5x \le 0$

$x \le \frac{0}{5}$

$x \le 0$

$(-\infty, 0]$

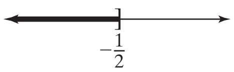


5. $-4x \ge 2$

$x \le \frac{2}{-4}$

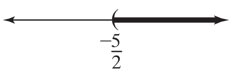
$x \le -\frac{1}{2}$

$\left(-\infty, -\frac{1}{2}\right]$



6. $2z > -5$

$z > \frac{-5}{2}$

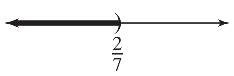


7. $5 - 7s > 3$

$-7s > -2$

$s < \frac{2}{7}$

$\left(-\infty, \frac{2}{7}\right)$

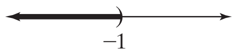


8. $4s - 1 < -5$

$4s < -4$

$s < -1$

$(-\infty, -1)$



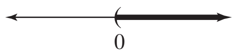
9. $3 < 2y + 3$

$0 < 2y$

$0 < y$

$y > 0$

$(0, \infty)$



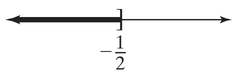
10. $4 \le 3 - 2y$

$1 \le -2y$

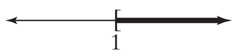
$-\frac{1}{2} \ge y$

$y \le -\frac{1}{2}$

$\left(-\infty, -\frac{1}{2}\right]$



11. $-t \le -1 \qquad t \ge 1$



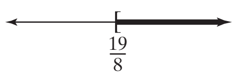
12. $-3 \ge 8(2 - x)$

$-3 \ge 16 - 8x$

$8x \ge 19$

$x \ge \frac{19}{8}$

$\left[\frac{19}{8}, \infty\right)$



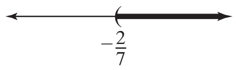
13. $3(2-3x) > 4(1-4x)$

$6-9x > 4-16x$

$7x > -2$

$x > -\frac{2}{7}$

$\left(-\frac{2}{7}, \infty\right)$



14. $8(x+1)+1 < 3(2x)+1$

$8x+9 < 6x+1$

$2x < -8$

$x < -4$

$(-\infty, -4)$

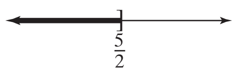
15. $2(4x-2) > 4(2x+1)$

$8x-4 > 8x+4$

$-4 > 4$, which is false for all x.

Thus the solution set is $\emptyset$.

16. $2x \le 5 \qquad x \le \frac{5}{2}$

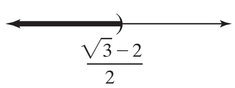


17. $x+2 < \sqrt{3}-x$

$2x < \sqrt{3}-2$

$x < \frac{\sqrt{3}-2}{2}$

$\left(-\infty, \frac{\sqrt{3}-2}{2}\right)$



18. $\sqrt{2}(x+2) > \sqrt{8}(3-x)$

$\sqrt{2}(x+2) > 2\sqrt{2}(3-x)$

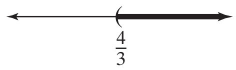
$x+2 > 2(3-x)$

$x+2 > 6-2x$

$3x > 4$

$x > \frac{4}{3}$

$\left(\frac{4}{3}, \infty\right)$

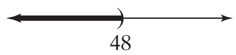


19. $\frac{5}{6}x < 40$

$5x < 240$

$x < 48$

$(-\infty, 48)$

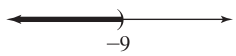


20. $-\frac{2}{3}x > 6$

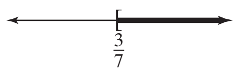
$-x > 9$

$x < -9$

$(-\infty, -9)$



21. $3y+1 \le 10y-2 \qquad -7y \le -3 \qquad y \ge \frac{3}{7}$



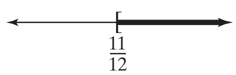
22. $\frac{3y-2}{3} \ge \frac{1}{4}$

$12y-8 \ge 3$

$12y \ge 11$

$y \ge \frac{11}{12}$

$\left[\frac{11}{12}, \infty\right)$



23. $-3x+1 \le -3(x-2)+1$

$-3x+1 \le -3x+7$

$1 \le 7$, which is true for all x. The solution is $-\infty < x < \infty$.

$(-\infty, \infty)$

24. $0x \le 0$

$0 \le 0$, which is true for all x. The solution is $-\infty < x < \infty$.

$(-\infty, \infty)$

25. $\dfrac{1-t}{2} < \dfrac{3t-7}{3}$

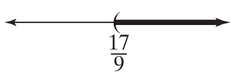
$3(1-t) < 2(3t-7)$

$3-3t < 6t-14$

$-9t < -17$

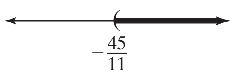
$t > \dfrac{17}{9}$

$\left(\dfrac{17}{9}, \infty\right)$



26. $18(t+2) > 3(t-3)+4(t)$

$11t > -45 \qquad t > \dfrac{-45}{11}$



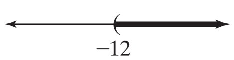
27. $2x+13 \ge \dfrac{1}{3}x-7$

$6x+39 \ge x-21$

$5x \ge -60$

$x \ge -12$

$[-12, \infty)$



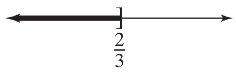
28. $3x - \dfrac{1}{3} \le \dfrac{5}{2}x$

$18x-2 \le 15x$

$3x \le 2$

$x \le \dfrac{2}{3}$

$\left(-\infty, \dfrac{2}{3}\right]$



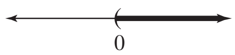
29. $\dfrac{2}{3}r < \dfrac{5}{6}r$

$4r < 5r$

$0 < r$

$r > 0$

$(0, \infty)$



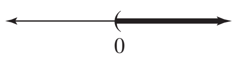
30. $\dfrac{7}{4}t > -\dfrac{8}{3}t$

$21t > -32t$

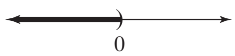
$53t > 0$

$t > 0$

$(0, \infty)$



31. $60y+6y < 15y+10y \qquad 41y < 0 \qquad y < 0$



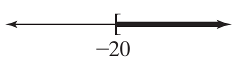
32. $9-0.1x \le \dfrac{2-0.01x}{0.2}$

$1.8-0.02x \le 2-0.01x$

$-0.01x \le 0.2$

$x \ge -20$

$[-20, \infty)$



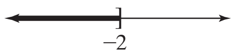
33. $0.1(0.03x + 4) \geq 0.02x + 0.434$

$0.003x + 0.4 \geq 0.02x + 0.434$

$-0.017x \geq 0.034$

$x \leq -2$

$(-\infty, -2]$



34. $\frac{3y-1}{-3} < \frac{5(y+1)}{-3}$

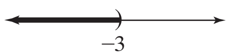
$3y - 1 > 5y + 5$

$-6 > 2y$

$-3 > y$

$y < -3$

$(-\infty, -3)$



35. $12(50) < S < 12(150)$

$600 < S < 1800$

36. $5 \leq t \leq 6$

37. The measures of the acute angles of a right triangle sum to 90°. If x is the measure of one acute angle, the other angle has measure $90 - x$.

$x < 3(90 - x) + 10$

$x < 270 - 3x + 10$

$4x < 280$

$x < 70$

The measure of the angle is less than 70°.

38. Let d be the number of disks. The stereo plus d disks will cost $219 + 18.95d$.

$219 + 18.95d \leq 360$

$18.95d \leq 141$

$d \leq \frac{141}{18.95} \approx 7.44$

The student can buy at most 7 disks.

Problems 1.3

1. Let q = number of units sold.

Profit > 0

Total revenue − Total cost > 0

$20q - (15q + 600{,}000) > 0$

$5q - 600{,}000 > 0$

$5q > 600{,}000$

$q > 120{,}000$

Thus at least 120,001 units must be sold.

2. Let n be the number of units and let $P(n)$ be the profit from n units. Since profit = total revenue − total cost, $P(n) = (8.20)n - (7000 + (6.50)n) = (1.70)n - 7000$ and for $P(n) \geq 0$ we require $(1.70)n - 7000 \geq 0$, equivalently $n \geq \frac{7000}{1.70} \approx 4117.647$. Thus we require at least 4118 units for a profit.

3. Let x = number of miles driven per year.

If the auto is leased, the annual cost is

$12(420) + 0.06x$.

If the auto is purchased, the annual cost is

$4700 + 0.08x$. We want Rental cost ≤ Purchase cost.

$12(420) + 0.06x \leq 4700 + 0.08x$

$5040 + 0.06x \leq 4700 + 0.08x$

$340 \leq 0.02x$

$17{,}000 \leq x$

The number of miles driven per year must be at least 17,000.

4. Let N = required number of shirts. Then

Total revenue = $3.5N$ and

Total cost = $1.3N + 0.4N + 6500$.

$$\begin{aligned} \text{Profit} &> 0 \\ 3.5N - (1.3N + 0.4N + 6500) &> 0 \\ 1.8N - 6500 &> 0 \\ 1.8N &> 6500 \\ N &> 3611.1 \end{aligned}$$

At least 3612 shirts must be sold.

5. Let q = number of magazines printed. Then the cost of publication is $1.30q$. The number of magazines sold is $0.80q$. The revenue from dealers is $(1.50)(0.80q)$. If fewer than 100,000 magazines are sold, the only revenue is from the sales to dealers, while if more than 100,000 are sold, there are advertising revenues of $0.20(1.50)(0.80q - 100{,}000)$. Thus,

$$\text{Revenue} = \begin{cases} 1.5(0.8)q & \text{if } 0.8q \le 100{,}000 \\ 1.5(0.8)q + 0.2(1.5)(0.8q - 100{,}000) & \text{if } 0.8q > 100{,}000 \end{cases}$$

$$= \begin{cases} 1.2q & q \le 125{,}000 \\ 1.44q - 30{,}000 & q > 125{,}000 \end{cases}$$

Profit = Revenue − Cost

$$= \begin{cases} 1.2q - 1.3q & q \le 125{,}000 \\ 1.44q - 30{,}000 - 1.3q & q > 125{,}000 \end{cases}$$

$$= \begin{cases} -0.1q & q \le 125{,}000 \\ 0.14q - 30{,}000 & q > 125{,}000 \end{cases}$$

Clearly, the profit is negative if fewer than 125,001 magazines are printed.

$$0.14q - 30{,}000 \ge 0$$
$$0.14q \ge 30{,}000$$
$$q \ge 214{,}286$$

Thus, at least 214,286 magazines must be printed in order to avoid a loss.

6. Let q = number of clocks produced during regular work week, so $11{,}000 - q$ = number produced in overtime. Then

$$2q + 3(11{,}000 - q) \le 25{,}000$$
$$-q + 33{,}000 \le 25{,}000$$
$$8000 \le q$$

At least 8000 clocks must be produced during the regular workweek.

7. Let x be the amount of money invested at 6.25%. The annual yield, in terms of x, is $Y(x) = (0.0625)x + (0.05)(70{,}000 - x) = (0.0125)x + 3{,}500$. Since the company wants

$$Y(x) = (0.0125)x + 3{,}500 \ge (0.055)(70{,}000) = 3{,}850$$

we must have $(0.0125)x \ge 350$, equivalently, $x \ge \dfrac{350}{0.0125} = 28{,}000$

8. Let L be current liabilities. Then

$$\text{Current ratio} = \frac{\text{current assets}}{\text{current liabilities}}$$
$$3.8 = \frac{570{,}000}{L}$$
$$3.8L = 570{,}000$$
$$L = \$150{,}000$$

Let x = amount of money they can borrow, where $x \ge 0$.

$$\frac{570{,}000 + x}{150{,}000 + x} \ge 2.6$$
$$570{,}000 + x \ge 390{,}000 + 2.6x$$
$$180{,}000 \ge 1.6x$$
$$112{,}500 \ge x$$

Thus current liabilities are \$150,000 and the maximum amount they can borrow is \$112,500.

9. Let q be the number of units sold this month at \$4.00 each. Then $2500 - q$ will be sold at \$4.50 each. Then

Total revenue $\geq 10{,}750$

$4q + 4.5(2500 - q) \geq 10{,}750$

$-0.5q + 11{,}250 \geq 10{,}750$

$500 \geq 0.5q$

$1000 \geq q$

The maximum number of units that can be sold this month is 1000.

10. Revenue = (no. of units)(price per unit)

$$q\left(\frac{200}{q} + 3\right) > 9000$$
$$200 + 3q > 9000$$
$$3q > 8800$$
$$q > 2933.\bar{3}$$

At least 2934 units must be sold.

11. For $t < 40$, we want

income on hourly basis

> income on per-job basis

$9t > 320 + 3(40 - t)$

$9t > 440 - 3t$

$12t > 440$

$t > 36.7$ hr

12. Let x be the employee's yearly sales. We need to know the values of x for which $(0.04)x \geq 50{,}000 + (0.02)x$, equivalently $(0.02)x \geq 50{,}000$ which gives $x \geq \frac{50{,}000}{0.02} = 2{,}500{,}000.$

13. Generalizing Example 4 we see that for fixed, positive, a and b with $a < b$ and positive c, we have

$$\frac{a}{b} < \frac{a+c}{b+c} < 1$$

Moreover, if $c < d$ then we can write $d = c + e$ for some positive e. Now if we replace a and b by $a + c$ and $b + c$ respectively, we can see that

$$\frac{a+c}{b+c} < \frac{a+c+e}{b+c+e} = \frac{a+d}{b+d} < 1$$

On the other hand, we have $1 - \frac{a+d}{b+d} = \frac{b-a}{b+d}$ and with a and b fixed we can make the difference $1 - \frac{a+d}{b+d}$ as small as we like by making d sufficiently large. The arguments show that by taking c to be very large, the fractions $\frac{a+c}{b+c}$ *approach* the number 1.

14. Let x = accounts receivable. Then

Acid test ratio $= \frac{450{,}000 + x}{398{,}000}$

$1.3 \leq \frac{450{,}000 + x}{398{,}000}$

$517{,}400 \leq 450{,}000 + x$

$x \geq 67{,}400$

The company must have at least \$67,400 in accounts receivable.

Apply It 1.4

3. $|w - 22| \leq 0.3$

Problems 1.4

1. $|-13| = 13$

2. $|2^{-1}| = \left|\frac{1}{2}\right| = \frac{1}{2}$

3. $|-2| = 2$

4. $\left|\frac{-3-5}{2}\right| = \left|\frac{-8}{2}\right| = |-4| = 4$

5. $\left|2\left(-\frac{7}{2}\right)\right| = |-7| = 7$

6. $|3-5| - |5-3| = |-2| - |2| = 2 - 2 = 0$

7. $|x| < 4, -4 < x < 4$

8. $\emptyset$ since $|x| \geq 0$

9. Because $3 - \sqrt{10} < 0$,
$\left|3 - \sqrt{10}\right| = -\left(3 - \sqrt{10}\right) = \sqrt{10} - 3.$

10. Because $\sqrt{5} - 2 > 0$, $\left|\sqrt{5} - 2\right| = \sqrt{5} - 2.$

11. a. $|x-7|<3$

b. $|x-2|<3$

c. $|x-7|\le 5$

d. $|x-7|=4$

e. $|x+4|<2$

f. $|x|<3$

g. $|x|>6$

h. $|x-105|<3$

i. $|x-850|<100$

12. $|f(x)-L|<\varepsilon$

13. $|p_1-p_2|\ge 5$ dollars

14. $|x-\mu|<3\sigma$

$-3\sigma<x-\mu<3\sigma$

$\mu-3\sigma<x<\mu+3\sigma$

15. $|x|=7$

$x=\pm 7$

16. $|-x|=2$

$-x=2$ or -2

$x=\pm 2$

17. $\left|\frac{x}{5}\right|=7$

$\frac{x}{5}=\pm 7$

$x=\pm 35$

18. $\frac{3}{x}=7$ or $\frac{3}{x}=-7$; $x=\frac{3}{7}$ or $x=-\frac{3}{7}$

19. $|x-5|=9$

$x-5=\pm 9$

$x=5\pm 9$

$x=14$ or $x=-4$

20. $|4+3x|=6$

$4+3x=\pm 6$

$3x=-4\pm 6$

$3x=-10$ or 2

$x=-\frac{10}{3}$ or $x=\frac{2}{3}$

21. $|5x-2|=0$

$5x-2=0$

$x=\frac{2}{5}$

22. $|7x+3|=x$

Here we must have $x\ge 0$.

$7x+3=x$ or $-(7x+3)=x$

$6x=-3$ or $-7x-3=x$

$x=-\frac{1}{2}<0$ $\quad x=-\frac{3}{8}<0$

There is no solution.

23. $3-5x=2$ or $3-5x=-2$; $x=\frac{1}{5}$ or $x=1$

24. $|5-3x|=7$

$5-3x=\pm 7$

$-3x=-5\pm 7$

$-3x=2$ or -12

$x=-\frac{2}{3}$ or $x=4$

25. $|x|<M$

$-M<x<M$

$(-M, M)$

Note that $M>0$ is required.

26. $|-x|<3$

$|x|<3$

$-3<x<3$

$(-3, 3)$

27. $\left|\frac{x}{4}\right| > 2$

$\frac{x}{4} < -2$ or $\frac{x}{4} > 2$

$x < -8$ or $x > 8$, so the solution is $(-\infty, -8) \cup (8, \infty)$.

28. $x > \frac{2}{3}$ or $x < -\frac{2}{3}$

29. $|x + 7| < 3$

$-3 < x + 7 < 3$

$-10 < x < -4$

$(-10, -4)$

30. $|2x - 17| < -4$

Because $-4 < 0$, the solution set is $\emptyset$.

31. $\left|x - \frac{1}{2}\right| > \frac{1}{2}$

$x - \frac{1}{2} < -\frac{1}{2}$ or $x - \frac{1}{2} > \frac{1}{2}$

$x < 0$ or $x > 1$

$(-\infty, 0) \cup (1, \infty)$

32. $|1 - 3x| > 2$

$1 - 3x > 2$ or $1 - 3x < -2$

$-3x > 1$ or $-3x < -3$

$x < -\frac{1}{3}$ or $x > 1$

The solution is $\left(-\infty, -\frac{1}{3}\right) \cup (1, \infty)$.

33. $-2 \le 3 - 2x \le 2$; $-5 \le -2x \le -1$; $\frac{5}{2} \ge x \ge \frac{1}{2}$; $\frac{1}{2} \le x \le \frac{5}{2}$

34. $|3x - 2| \ge 0$ is true for all x because $|a| \ge 0$ for all a. Thus $-\infty < x < \infty$, or $(-\infty, \infty)$.

35. $\left|\frac{3x - 8}{2}\right| \ge 4$

$\frac{3x - 8}{2} \le -4$ or $\frac{3x - 8}{2} \ge 4$

$3x - 8 \le -8$ or $3x - 8 \ge 8$

$3x \le 0$ or $3x \ge 16$

$x \le 0$ or $x \ge \frac{16}{3}$

The solution is $(-\infty, 0] \cup \left[\frac{16}{3}, \infty\right)$.

36. $\left|\frac{x - 7}{3}\right| \le 5$

$-5 \le \frac{x - 7}{3} \le 5$

$-15 \le x - 7 \le 15$

$-8 \le x \le 22$

$[-8, 22]$

37. $|d - 35.2\text{m}| \le 20$ cm or $|d - 35.2| \le 0.20$

38. $3 \le |T_1 - T_2| \le 5$

39. $|x - \mu| > h\sigma$

Either $x - \mu < -h\sigma$, or $x - \mu > h\sigma$. Thus either $x < \mu - h\sigma$ or $x > \mu + h\sigma$, so the solution is $(-\infty, \mu - h\sigma) \cup (\mu + h\sigma, \infty)$.

40. $|x - 0.01| \le 0.005$

Problems 1.5

1. The bounds of summation are 12 and 17; the index of summation is t.

2. The bounds of summation are 3 and 450; the index of summation is m.

3. $$\sum_{i=1}^{5} 3i = 3(1) + 3(2) + 3(3) + 3(4) + 3(5)$$
$$= 3 + 6 + 9 + 12 + 15$$
$$= 45$$

4. $7\sum_{q=0}^{3} q = 7(0 + 1 + 2 + 3) = 42$

5. $\sum_{k=3}^{9}(10k+16) = [10(3)+16]+[10(4)+16]+[10(5)+16]+[10(6)+16]+[10(7)+16]+[10(8)+16]+[10(9)+16]$

$= 46+56+66+76+86+96+106$

$= 532$

6. $\sum_{n=7}^{11}(2n-3) = [2(7)-3]+[2(8)-3]+[2(9)-3]+[2(10)-3]+[2(11)-3]$

$= 11+13+15+17+19$

$= 75$

7. $36+37+38+39+\cdots+60 = \sum_{i=36}^{60} i$

8. $1+8+27+64+125 = \sum_{k=1}^{5} k^3$

9. $\sum_{k=2}^{6} 3^k$

10. $11+15+19+23+\cdots+71 = \sum_{i=1}^{16}(7+4i)$

11. $2+4+8+16+32+64+128+256 = \sum_{i=1}^{8} 2^i$

12. $10+100+1000+\cdots+100{,}000{,}000 = \sum_{j=1}^{8} 10^j$

13. $\sum_{k=1}^{875} 10 = 10\sum_{k=1}^{875} 1 = 10(875) = 8750$

14. $10\sum_{k=1}^{875} 1 = 10(875) = 8750$

15. $\sum_{k=1}^{n}\left(5\cdot\frac{1}{n}\right) = \left(5\cdot\frac{1}{n}\right)\sum_{k=1}^{n} 1 = \left(5\cdot\frac{1}{n}\right)(n) = 5$

16. $\sum_{k=1}^{200}(k-100) = \sum_{k=1}^{200} k - 100\sum_{k=1}^{200} 1 = \frac{200(201)}{2} - 100(200) = 20{,}100 - 20{,}000 = 100$

17. $\sum_{k=51}^{100} 10k = 10\sum_{i=1}^{50}(i+50)$

$= 10\sum_{i=1}^{50} i + (10)(50)\sum_{i=1}^{50} 1$

$= 10 \cdot \frac{50(51)}{2} + 500(50) = 12{,}750 + 25{,}000$

$= 37{,}750$

18. $\sum_{k=1}^{n} \frac{n}{n+1}k^3 = \frac{n}{n+1}\sum_{k=1}^{n} k^3$

$= \frac{n}{n+1} \cdot \frac{n^2(n+1)^2}{4}$

$= \frac{n^3(n+1)}{4}$

19. $3\sum_{i=1}^{20} i^2 + 2\sum_{i=1}^{20} i = 3\frac{20(21)(41)}{6} + 2\frac{20(21)}{2}$

$= 9030$

20. $\sum_{k=1}^{100} \frac{3k^2 - 200k}{101} = \frac{3}{101}\sum_{k=1}^{100} k^2 - \frac{200}{101}\sum_{k=1}^{100} k$

$= \frac{3}{101} \cdot \frac{100(101)(201)}{6} - \frac{200}{101} \cdot \frac{100 \cdot 101}{2}$

$= 10{,}050 - 10{,}000 = 50$

21. $\sum_{k=51}^{100} k^2 = \sum_{i=1}^{50}(i+50)^2 =$

$\sum_{i=1}^{50}(i^2 + 100i + 2500)$

$= \sum_{k=1}^{50} i^2 + 100\sum_{i=1}^{50} i + 2500\sum_{i=1}^{50} 1$

$= \frac{50(51)(101)}{6} + 100\frac{50(51)}{2} + 2500(50)$

$= 42{,}925 + 127{,}500 + 125{,}000 = 295{,}425$

22. $\sum_{k=1}^{50}(k+50)^2 = \sum_{k=1}^{50}(k^2 + 100k + 2500)$

$= \sum_{k=1}^{50} k^2 + 100\sum_{k=1}^{50} k + 2500\sum_{k=1}^{50} 1$

$= \frac{50(51)(101)}{6} + 100\frac{50(51)}{2} + 2500(50)$

$= 42{,}925 + 127{,}500 + 125{,}000 = 295{,}425$

23. $\sum_{k=1}^{9}\left\{\left[3 - \left(\frac{k}{10}\right)^2\right]\left(\frac{1}{10}\right)\right\}$

$= \frac{1}{10}\sum_{k=1}^{9}\left(3 - \frac{k^2}{100}\right)$

$= \frac{3}{10}\sum_{k=1}^{9} 1 - \frac{1}{1000}\sum_{k=1}^{9} k^2$

$= \frac{3}{10}(9) - \frac{1}{1000} \cdot \frac{9(10)(19)}{6}$

$= \frac{3}{10}(9) - \frac{1}{100} \cdot \frac{3(19)}{2}$

$= \frac{483}{200}$

24. $\frac{3}{50}\sum_{j=1}^{100} 1 - \frac{1}{500{,}000}\sum_{j=1}^{100} j^2$

$= \frac{3}{50}(100) - \frac{1}{500{,}000}\frac{(100)(101)(201)}{6}$

$= 6 - \frac{(67)(101)}{10^4} = \frac{6 \cdot 10^4 - 67 \cdot 101}{10^4}$

$= \frac{53233}{10^4} = 5.3233$

25. $\sum_{k=1}^{n}\left\{\left[5 - \left(\frac{3}{n} \cdot k\right)^2\right]\frac{3}{n}\right\}$

$= \frac{3}{n}\sum_{k=1}^{n}\left(5 - \frac{9}{n^2}k^2\right)$

$= \frac{3}{n}(5)\sum_{k=1}^{n} 1 - \frac{3}{n}\left(\frac{9}{n^2}\right)\sum_{k=1}^{n} k^2$

$= \frac{15}{n}(n) - \frac{27}{n^3} \cdot \frac{n(n+1)(2n+1)}{6}$

$= 15 - \frac{9(n+1)(2n+1)}{2n^2}$

26. $\sum_{k=1}^{n} \frac{k^2}{(n+1)(2n+1)} = \frac{1}{(n+1)(2n+1)}\sum_{k=1}^{n} k^2$

$= \frac{1}{(n+1)(2n+1)} \cdot \frac{n(n+1)(2n+1)}{6} = \frac{n}{6}$

Apply It 1.6

4. Each term of the sequence is obtained by adding 18 to the previous term. The first term, c_1, is 183.

$c_1 = 183$

$c_2 = 183 + 18 = 201$

$c_3 = 201 + 18 = 219$

$c_4 = 219 + 18 = 237$

$c_5 = 237 + 18 = 255$

$c_6 = 255 + 18 = 273$

The terms of the sequence are 183, 201, 219, 237, 255, 273.

5. Each term of the sequence is obtained by multiplying the previous term by 1.06. Since the first term is \$9.57, the sequence can be written as $(9.57(1.06)^{k-1})_{k=1}^{4}$.

6. This is an arithmetic sequence of length 7 with first term $a = 1237 - 12 = 1225$ and common difference $d = -12$. We write e_k for the sequence of enrollments.

$e_1 = \qquad\qquad\qquad\qquad = 1225$

$e_2 = e_1 - 12 = 1225 - 12 = 1213$

$e_3 = e_2 - 12 = 1213 - 12 = 1201$

$e_4 = e_3 - 12 = 1201 - 12 = 1189$

$e_5 = e_4 - 12 = 1189 - 12 = 1177$

$e_6 = e_5 - 12 = 1177 - 12 = 1165$

$e_7 = e_6 - 12 = 1165 - 12 = 1153$

The sequence of enrollments is 1225, 1213, 1201, 1189, 1177, 1165, 1153.

7. This is a geometric sequence of length 4 with first term $a = 0.92(23{,}500) = 21{,}620$ and common ratio $r = 0.92$. We write p_k for the sequence of populations, rounding to the nearest person.

$p_1 = \qquad\qquad\qquad\qquad\qquad\qquad = 21{,}620$

$p_2 = (0.92)p_1 = (0.92)21{,}620 = 19{,}890.4 \approx 19{,}890$

$p_3 = (0.92)p_2 = (0.92)19{,}890 = 18{,}298.8 \approx 18{,}299$

$p_4 = (0.92)p_3 = (0.92)18{,}299 = 16{,}835.08 \approx 16{,}835$

The sequence of populations is 21,620, 19,890, 18,299, 16,835.

8. This is the sum s_7 of the first 7 terms of an arithmetic sequence b_k with first term $a = 27\text{M\$}$ and common difference $d = 1.5\text{M\$}$. Using the formula from Example 9, the sum is

$$\begin{aligned} s_7 &= \frac{n}{2}((n-1)d + 2a) \\ &= \frac{7}{2}((7-1)1.5\text{M\$} + 2(27\text{M\$})) \\ &= 3.5(9\text{M\$} + 54\text{M\$}) \\ &= 3.5(63\text{M\$}) \\ &= 220.5\text{M\$} \end{aligned}$$

The total revenue for 2009–2015, inclusive, is 220.5M\$.

9. This is the sum s_2, of the first 21 terms of a geometric sequence c_k with first term $a = 1000$ and common ratio $r = 1.07$. Using the formula from Example 10, the sum is

$$\begin{aligned} s_{21} &= \frac{a(1-r^n)}{1-r} \\ &= \frac{1000(1-1.07^{21})}{1-1.07} \\ &= -\frac{1000}{0.07}(1-1.07^{21}) \\ &\approx 44,865.18 \end{aligned}$$

On Bart's 21st birthday, there is \$44,865.18 in the account.

Problems 1.6

1. $a = \sqrt{2}, -\frac{3}{7}, 2.3, 57$

$a_3 = 2.3$

2. $b = 1, 13, -0.9, \frac{5}{2}, 100, 39$

$b_6 = 39$

3. $(a_k)_{k=1}^{7} = (3^k)$

$a_4 = 3^4 = 81$

4. $(c_k)_{k=1}^{9} = (3^k + k)$

$c_4 = 3^4 + 4 = 81 + 4 = 85$

5. $c_{15} = (3 + (15-5)2) = 23$

6. $(b_k) = (5 \cdot 2^{k-1})$

$b_6 = 5 \cdot 2^{6-1} = 5 \cdot 2^5 = 5 \cdot 32 = 160$

7. $(a_k) = (k^4 - 2k^2 + 1)$
$a_2 = 2^4 - 2(2^2) + 1 = 16 - 8 + 1 = 9$

8. $(a_k) = (k^3 + k^2 - 2k + 7)$
$a_3 = 3^3 + 3^2 - 2(3) + 7 = 27 + 9 - 6 + 7 = 37$

9. $-1, 2, 5, 8$
3 is added to each term.
$(-1 + (k-1) \cdot 3)_{k=1}^{4}$

10. $(10 - 3k)$

11. $2, -4, 8, -16$
Each term is multiplied by -2.
$((-1)^{k+1} \cdot 2^k)_{k=1}^{4}$

12. $5, \frac{5}{3}, \frac{5}{9}, \frac{5}{27}, \cdots$
Each term is multiplied by $\frac{1}{3}$.
$$\left(5\left(\frac{1}{3}\right)^{k-1}\right)_{k=1}^{\infty}$$

13. $((i+3)^3)$ and $(j^3 - 9j^2 + 9j - 27)$
$i = 1$ gives $4^3 = 64$.
$j = 1$ gives $1 - 9 + 9 - 27 = -26$
The sequences are not equal.

14. $(k^2 - 4)$ and $((k+2)(k-2))$
The sequences are equal since
$k^2 - 4 = (k+2)(k-2)$

15. not equal (second is 1/5 times first)

16. $(j^3 - 9j^2 + 27j - 27)_{j=1}^{\infty}$ and $((k-3)^3)_{k=1}^{\infty}$
For all k, $(k-3)^3 = k^3 - 9k^2 + 27k - 27$, so the sequences are equal.

17. $a_1 = 1, a_2 = 2, a_{k+2} = a_{k+1} \cdot a_k$
$a_3 = a_2 \cdot a_1 = 2 \cdot 1 = 2$
$a_4 = a_3 \cdot a_2 = 2 \cdot 2 = 4$
$a_5 = a_4 \cdot a_3 = 4 \cdot 2 = 8$
$a_6 = a_5 \cdot a_4 = 8 \cdot 4 = 32$
$a_7 = a_6 \cdot a_5 = 32 \cdot 8 = 256$

18. $a_1 = 1, a_{k+1} = a_{a_k}$
$a_2 = a_{a_1} = a_1 = 1$
$a_3 = a_{a_2} = a_1 = 1$
$a_4 = a_{a_3} = a_1 = 1$
$a_5 = a_{a_4} = a_1 = 1$
$a_6 = a_{a_5} = a_1 = 1$
$a_7 = a_{a_6} = a_1 = 1$
and so on…
Thus, $a_{17} = 1$.

19. $b_1 = 1, b_{k+1} = \dfrac{b_k}{k}$
$b_2 = \frac{b_1}{1} = \frac{1}{1} = 1$
$b_3 = \frac{b_2}{2} = \frac{1}{2}$
$b_4 = \frac{b_3}{3} = \frac{\frac{1}{2}}{3} = \frac{1}{6}$
$b_5 = \frac{b_4}{4} = \frac{\frac{1}{6}}{4} = \frac{1}{24}$
$b_6 = \frac{b_5}{5} = \frac{\frac{1}{24}}{5} = \frac{1}{120}$

20. $9 + 8 + 7 + 6 + 5 + 4 + 3 + c_1 = 42$

21. $a = 22.5, d = 0.9, b_{k+1} = d + b_k$.
$b_1 = 22.5$
$b_2 = 0.9 + b_1 = 0.9 + 22.5 = 23.4$
$b_3 = 0.9 + b_2 = 0.9 + 23.4 = 24.3$
$b_4 = 0.9 + b_3 = 0.9 + 24.3 = 25.2$
$b_5 = 0.9 + b_4 = 0.9 + 25.2 = 26.1$

22. $a = 0, d = 1, b_{k+1} = d + b_k$
$b_1 = 0$
$b_2 = 1 + b_1 = 1 + 0 = 1$
$b_3 = 1 + b_2 = 1 + 1 = 2$
$b_4 = 1 + b_3 = 1 + 2 = 3$
$b_5 = 1 + b_4 = 1 + 3 = 4$

23. $a = 96, d = -1.5, b_{k+1} = d + b_k$

$b_1 = 96$

$b_2 = -1.5 + b_1 = -1.5 + 9.6 = 94.5$

$b_3 = -1.5 + b_2 = -1.5 + 94.5 = 93$

$b_4 = -1.5 + b_3 = -1.5 + 93 = 91.5$

$b_5 = -1.5 + b_4 = -1.5 + 91.5 = 90$

24. $a = A, d = D, b_{k+1} = d + b_k$

$b_1 = A$

$b_2 = D + b_1 = D + A = +D$

$b_3 = D + b_2 = D + D + A = A + 2D$

$b_4 = D + b_3 = D + A + 2D = A + 3D$

$b_5 = D + b_4 = D + A + 3D = A + 4D$

25. $1/2, -1/2^2, 1/2^3, -1/2^4, 1/2^5$

26. $a = 50, r = (1.06)^{-1}, c_{k+1} = c_k \cdot r$

$c_1 = 50$

$$c_2 = c_1(1.06)^{-1} = \frac{50}{1.06} \approx 47.17$$

$$c_3 = c_2(1.06)^{-1} = \frac{50}{(1.06)^2} \approx 44.50$$

$$c_4 = c_3(1.06)^{-1} = \frac{50}{(1.06)^3} \approx 41.98$$

$$c_5 = c_4(1.06)^{-1} = \frac{50}{(1.06)^4} \approx 39.60$$

27. $a = 100, r = 1.05, c_{k+1} = c_k \cdot r$

$c_1 = 100$

$c_2 = c_1(1.05) = 100(1.05) = 105$

$c_3 = c_2(1.05) = 105(1.05) = 110.25$

$c_4 = c_3(1.05) = 110.25(1.05) = 115.7625$

$c_5 = c_4(1.05) = 115.7625(1.05) = 121.550625$

28. $a = 3, r = \frac{1}{3}, c_{k+1} = c_k \cdot r$

$c_1 = 3$

$$c_2 = \frac{c_1}{3} = \frac{3}{3} = 1$$

$$c_3 = \frac{c_2}{3} = \frac{1}{3} = \frac{1}{3}$$

$$c_4 = \frac{c_3}{3} = \frac{\frac{1}{3}}{3} = \frac{1}{9}$$

$$c_5 = \frac{c_4}{3} = \frac{\frac{1}{9}}{3} = \frac{1}{27}$$

29. 27th term, $a = 3, d = 2$

Arithmetic sequence

$b_k = (k-1)d + a$

$b_{27} = (27-1)(2) + 3 = 55$

30. -1

31. 11th term, $a = 1, r = 2$

Geometric sequence

$c_k = a \cdot r^{k-1}$

$c_{11} = 1 \cdot 2^{11-1} = 2^{10} = 1024$

32. 7th term, $a = 2, r = 10$

Geometric sequence

$c_k = a \cdot r^{k-1}$

$c_7 = 2 \cdot 10^{7-1} = 2 \cdot 10^6 = 2{,}000{,}000$

33.
$$\begin{aligned}\sum_{k=1}^{7}((k-1)3+5) &= \sum_{k=1}^{7}(3k+2)\\ &= 3\sum_{k=1}^{7}k + 2\sum_{k=1}^{7}1\\ &= 3\frac{7(7+1)}{2} + 2(7)\\ &= 98\end{aligned}$$

34.
$$\begin{aligned}\sum_{k=1}^{9}(k \cdot 2+9) &= 2\sum_{k=1}^{9}k + 9\sum_{k=1}^{9}1\\ &= 2\frac{9(9+1)}{2} + 9(9)\\ &= 171\end{aligned}$$

35. 6

36.
$$\begin{aligned}\sum_{k=1}^{34}((k-1)10+5) &= \sum_{k=1}^{34}(10k-5)\\ &= 10\sum_{k=1}^{34}k - 5\sum_{k=1}^{34}1\\ &= 10\frac{34(34+1)}{2} - 5(34)\\ &= 5780\end{aligned}$$

37. $$\sum_{k=1}^{10} 100\left(\frac{1}{2}\right)^{k-1} = \frac{a(1-r^n)}{1-r} = \frac{100\left[1-\left(\frac{1}{2}\right)^{10}\right]}{1-\frac{1}{2}} \approx 199.80$$

38. $$\sum_{k=1}^{10} 50(1.07)^{k-1} = \frac{a(1-r^n)}{1-r} = \frac{50(1-1.07^{10})}{1-1.07} \approx 690.82$$

39. $$\sum_{k=1}^{10} 50(1.07)^{1-k} = \sum_{k=1}^{10} 50(1.07)^{-(k-1)} = \sum_{k=1}^{10} 50\left(\frac{1}{1.07}\right)^{k-1} = \frac{a(1-r^n)}{1-r} = \frac{50\left[1-\left(\frac{1}{1.07}\right)^{10}\right]}{1-\frac{1}{1.07}} \approx 375.76$$

40. 186

41. $$\sum_{k=1}^{\infty} 3\left(\frac{1}{2}\right)^{k-1} = \frac{a}{1-r} = \frac{3}{1-\frac{1}{2}} = 6$$

42. $$\sum_{i=0}^{\infty}\left(\frac{1}{3}\right)^{i}$$

Let $j = i + 1$. Then $i = j - 1$. Thus

$$\sum_{i=0}^{\infty}\left(\frac{1}{3}\right)^{i} = \sum_{j=1}^{\infty}\left(\frac{1}{3}\right)^{j-1} = \frac{a}{1-r} = \frac{1}{1-\frac{1}{3}} = \frac{3}{2}$$

43. $$\sum_{k=1}^{\infty} \frac{1}{2}(17)^{k-1}$$

Since $|r| = |17| > 1$, it is not possible to find the sum.

44. $$\sum_{k=1}^{\infty} \frac{2}{3}(1.5)^{k-1}$$

Since $|r| = |1.5| > 1$, it is not possible to find the sum.

45. $$\frac{\frac{20}{1.01}}{1-1/1.01} = 2000$$

46. $$\sum_{j=1}^{\infty} 75(1.09)^{1-j} = \sum_{j=1}^{\infty} 75\left(\frac{1}{1.09}\right)^{j-1} = \frac{a}{1-r} = \frac{75}{1-\frac{1}{1.09}} \approx 908.33$$

47. Let the inventory level at the end of day k be I_k.

Then $I_k = 90 + (-3)k$.

$I_{19} = 90 + (-3)(19) = 33$

48. The inventory sequence is $95 + (-6)k$. The first seven terms are 89, 83, 77, 71, 65, 59, and 53. The tenth term is $95 + (-6)(10) = 35$.

49. The sequence for the account balance is $125.00 + (-5.00)k$, where k is the number of months. After 9 months, the account contains

$125.00 + (-5.00)(9) = \$80.00$.

50. $25(1.05)^7 \approx 35.178$

51. The population at the end of k years is $P_k = 50{,}000(1.08)^k$. The population at the end of 2020 is

$P_{11} = 50{,}000(1.08)^{11} = 116{,}582$.

52. The population k years from now is $P_k = 24{,}000(0.95)^k$.

53. We seek the sum of the sequence $(12{,}000 + k(1000))_{k=0}^{7}$. The sequence is arithmetic with first term 12,000 and last term $12{,}000 + 7(1000) = 19{,}000$. The sum is $\frac{8(12{,}000 + 19{,}000)}{2} = \$124{,}000$.

54. $\sum_{k=1}^{60} 300(1.01)^{-k}$

$= \sum_{k=1}^{60} 300\left(\frac{1}{1.01}\right)^{k} \cdot \left(\frac{1}{1.01}\right)^{-1} \cdot \frac{1}{1.01}$

$= \sum_{k=1}^{60} \frac{300}{1.01}\left(\frac{1}{10.1}\right)^{k-1}$

$= \frac{a(1-r^n)}{1-r}$

$= \frac{\frac{300}{1.01}\left[1-\left(\frac{1}{1.01}\right)^{60}\right]}{1-\frac{1}{1.01}}$

$\approx 13{,}486.51$

The selling price of the car is \$13,486.51.

55. $\sum_{k=1}^{60} 100(1.005)^{60-k} = 100\sum_{k=1}^{60}(1.005)^{k-1}$

$= 100\frac{1-1.005^{60}}{-0.005} \approx 6977.00$

56. $21 - 7 = 14$ years
$= 14 \cdot 12$ months
$= 168$ months

Let Lisa's deposit amount be R. The accumulated value of the deposits is

$R + R(1.004) + R(1.004)^2 + \cdots + R(1.004)^{167}$, which is the sum of 168 terms of a geometric series with first term $a = R$ and common ratio $r = 1.004$. The sum is $\frac{a(1-r^n)}{1-r} = \frac{R(1-1.004^{168})}{1-1.004}$.

Lisa wants this amount to equal \$1000. Solving for R, Lisa finds $R = \frac{1000(-0.004)}{1-1.004^{168}} \approx \$4.19$.

57. We need the present value of an infinite sequence of payments of \$500 each. The payment k years from now has a present value of $500(1.05)^{-k}$. Accordingly, we need the sum of the infinite sequence with first term $500(1.05)^{-1}$ with common ratio 1.05^{-1}. The sum of this series is $\frac{a}{1-r} = \frac{500(1.05)^{-1}}{1-1.05^{-1}} = \$10{,}000$.

58. In the calculation in Problem 57 we ultimately just divide the value of a payment by the interest rate. In the present case, we have $\frac{500}{0.10} = \$5000$.

59. No, it is not. The differences between successive terms are far from constant. In fact, the sequence of differences is 0, 1, 1, 2, 3, 5, 8, 13, . . . , the Fibonnaci sequence starting with 0, 1.

60. No, the ratio $\frac{a_{k+1}}{a_k} = \frac{ka_k}{(k-1)a_{k-1}}$ is not constant. (The first 5 terms are 1, 1, 2, 6, 24.)

61. $a = d = r = p = b = 2$

$b_{k+1} = d + b_k = 2 + b_k : 2, 4, 6, 8, 10$

$c_{k+1} = c_k \cdot r = c_k \cdot 2 : 2, 4, 8, 16, 32$

$d_{k+1} = (d_k)^p = (d_k)^2 : 2, 4, 16, 256, 65, 536$

$e_{k+1} = b^{e_k} = 2^{e_k} : 2, 4, 16, 65, 536, 2^{65,536}$

Chapter 1 Review Problems

1. $x \ge -2$

2. $2x - (7 + x) \le x$

$2x - 7 - x \le x$

$-7 \le 0$, which is true for all x, so $-\infty < x < \infty$, or $(-\infty, \infty)$.

3. $-(5x + 2) < -(2x + 4)$

$-5x - 2 < -2x - 4$

$-3x < -2$

$x > \frac{2}{3}$

$\left(\frac{2}{3}, \infty\right)$

4. $-2(x + 6) > x + 4$

$-2x - 12 > x + 4$

$-3x > 16$

$x < -\frac{16}{3}$

$\left(-\infty, -\frac{16}{3}\right)$

5. $3p(1-p) > 3(2+p) - 3p^2$

$3p - 3p^2 > 6 + 3p - 3p^2$

$0 > 6$, which is false for all x. The solution set is $\emptyset$.

6. $5 - (3/2)q < 2$; $-(3/2)q < -3$; $q > 2$

7. $\dfrac{x+5}{3} - \dfrac{1}{2} \le 2$

$2(x+5) - 3(1) \le 6(2)$

$2x + 10 - 3 \le 12$

$2x \le 5$

$x \le \dfrac{5}{2}$

$\left(-\infty, \dfrac{5}{2}\right]$

8. $\dfrac{x}{3} - \dfrac{x}{4} > \dfrac{x}{5}$

$20x - 15x > 12x$

$5x > 12x$

$0 > 7x$

$0 > x$

$(-\infty, 0)$

9. $\dfrac{1}{4}s - 3 \le \dfrac{1}{8}(3 + 2s)$

$2s - 24 \le 3 + 2s$

$0 \le 27$, which is true for all s. Thus

$-\infty < s < \infty$, or $(-\infty, \infty)$.

10. $\dfrac{1}{3}(t+2) \ge \dfrac{1}{4}$

$4(t+2) \ge 3t + 48$

$4t + 8 \ge 3t + 48$

$t \ge 40$

$[40, \infty)$

11. $2 - 3x = 7$ or $2 - 3x = -7$;
$-3x = 5$ or $-3x = -9$; $x = -5/3$ or $x = 3$

12. $\left|\dfrac{5x-6}{13}\right| = 0$

$\dfrac{5x-6}{13} = 0$

$5x - 6 = 0$

$x = \dfrac{6}{5}$

13. $|2z - 3| < 5$

$-5 < 2z - 3 < 5$

$-2 < 2z < 8$

$-1 < z < 4$

$(-1, 4)$

14. $4 < \left|\dfrac{2}{3}x + 5\right|$

$\dfrac{2}{3}x + 5 < -4$ or $\dfrac{2}{3}x + 5 > 4$

$\dfrac{2}{3}x > -9$ or $\dfrac{2}{3}x > -1$

$x < -\dfrac{27}{2}$ or $x > -\dfrac{3}{2}$

The solution is $\left(-\infty, -\dfrac{27}{2}\right) \cup \left(-\dfrac{3}{2}, \infty\right)$.

15. $|3 - 2x| \ge 4$

$3 - 2x \ge 4$ or $3 - 2x \le -4$

$-2x \ge 1$ or $-2x \le -7$

$x \le -\dfrac{1}{2}$ or $x \ge \dfrac{7}{2}$

The solution is $\left(-\infty, -\dfrac{1}{2}\right] \cup \left[\dfrac{7}{2}, \infty\right)$.

16. $\displaystyle\sum_{k=1}^{7}(k^2 + 10k + 25)$

$$= \sum_{k=1}^{7} k^2 + 10\sum_{k=1}^{7} k + 25\sum_{k=1}^{7} 1$$

$$= \frac{(7)(8)(15)}{6} + 10\frac{(7)(8)}{2} + 25(7)$$

$$= (7)(4)(3) + (5)(7)(8) + (7)(25)$$

$$= 7(12 + 40 + 25) = 7(77) = 539$$

17. $\displaystyle\sum_{i=4}^{11} i^3 = \sum_{i=1}^{11} i^3 - \sum_{i=1}^{3} i^3$

$$= \frac{11^2(11+1)^2}{4} - \frac{3^2(3+1)^2}{4}$$

$$= 4320$$

The second sum is a reindexing of the original sum. Considering Problem 16, let $i = k + 3$, then $k = 1$ gives $i = 4$ and $k = 8$ gives $i = 11$.

18. Let p = selling price, c = cost. Then

$p - 0.40p = c$

$0.6p = c$

$p = \frac{c}{0.6} = \frac{5c}{3} = c + \left(\frac{2}{3}\right)c$

Thus the profit is $\frac{2}{3}$, or $66\frac{2}{3}\%$, of the cost.

19. Let x be the number of issues with a decline, and $x + 48$ be the number of issues with an increase.

Then

$x + (x + 48) = 1132$

$2x = 1084$

$x = 542$

20. Let x = purchase amount excluding tax.

$$\begin{aligned} x + 0.065x &= 3039.29 \\ 1.065x &= 3039.29 \\ x &= 2853.79 \end{aligned}$$

Thus tax is $3039.29 - 2853.79 = \$185.50$.

21. Let n be the number of units produced at plant A. The company requires

$$(25{,}000 + (6)n) + (30{,}000 + (7.5)(10{,}000 - n)) \leq 115{,}000$$

equivalently $-1.5n \leq 115{,}000 - 25{,}000 - 30{,}000 - 75{,}000 = -15{,}000$ So $n \geq 10{,}000$.

22. Total volume of old tanks

$= \pi(10)^2(25) + \pi(20)^2(25)$

$= 2500\pi + 10{,}000\pi$

$= 12{,}500\pi \text{ ft}^3$

Let r be the radius (in feet) of the new tank.

Then

$$\begin{aligned} \frac{4}{3}\pi r^3 &= 12{,}500\pi \\ r^3 &= 9375 \\ r &= \sqrt[3]{9375} \approx 21.0858 \end{aligned}$$

The radius is approximately 21.0858 feet.

23. Let c = operating costs

$\frac{c}{236{,}460} < 0.90$

$c < \$212{,}814$

24. $a = 32, d = 3, b_{k+1} = d + b_k$

$b_1 = 32$

$b_2 = 3 + 32 = 35$

$b_3 = 3 + 35 = 38$

$b_4 = 3 + 38 = 41$

$b_5 = 3 + 41 = 44$

25. $a = 100, r = 1.02, c_{k+1} = c_k \cdot r$

$c_1 = 100$

$c_2 = 100(1.02) = 102$

$c_3 = 102(1.02) = 104.04$

$c_4 = 104.04(1.02) = 106.1208$

$c_5 = 106.1208(1.02) = 108.243216$

26. $12 + 17 + 22 + 27 + 32 = 110$

27. $a = 100, r = 1.02, c_{k+1} = c_k \cdot r, s_n = \dfrac{a(1 - r^n)}{1 - r}$

$$\sum_{k=1}^{5} 100(1.02)^{k-1} = s_5 = \frac{100(1 - 1.02^5)}{1 - 1.02} \approx 520.40$$